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Abstract

Advertising: "The Good, the Bad and the Ugly"

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We model the choice of firms competing in prices in a differentiated products market to bundle advertising messages with their goods in return for payment from advertisers. From the firms' perspective, the potential to earn revenue from advertisers, makes advertising a "good". However, because consumers in the product market dislike such advertising, the bundling dampens demand and in this sense is a "bad". There is also a third role played by advertising, however. Since a firm that bundles advertisements with its good sells a less attractive good, it has to price more aggressively than one that does not do such bundling. Thus, bundling advertisements with the good can lead to more aggressive product pricing and thereby intensify product market competition. In this sense, advertising can make things "ugly".

Advertising: The Good, the Bad, and the Ugly
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1. Introduction

Advertising is often, if not quite always, a bundled product. Until recently, such bundling has been done by media firms, notably, newspapers, radio, and television that combine entertainment products with advertising messages. Today, however, advertising is being bundled into an increasingly diverse range of consumer products, such as video games, movie screenings, cell phone services, and wrapped cars. Recent examples include **Microsoft** who, in September 2005, started sending mobile ads, showing the company's logo when customers viewed certain Web pages from their handsets. In that same month, **Visa USA** also launched a campaign in which consumers sending a five-letter text code to receive weather reports get them with an ad-promoting Visa. Even more recently, **Verizon Wireless** and **Sprint Nextel** have announced plans to test how consumers react to short video ads on their phones.[1]

What makes those traditional product-driven firms changes their strategies and begin to bundle ads into their products acting as firms in the media industry? In this paper, we provide a simple duopoly model that helps to isolate the factors behind this decision. The model investigates both the product market and the advertising market faced by the firms. When a firm decides to bundle another firm's ads into its product, it enters into a new market—the advertising market. This could of course bring extra revenue to the firm. In this sense, advertising is a good.   However, bundled ads are generally disliked by consumers. The ads interrupt the consumption of the product and thereby diminish consumer enjoyment. That is, the quality of the product is reduced when it is used as an advertising medium and this decreases the firm's product demand. In this sense, advertising is

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[1] See Matt Richtel "Their Ads Here-marketers want to appear on the small screen." *The New York Times*, January 16, 2006, p. C2

a bad.   Note, however, that as Kind (2005) et al, point out, the rival of the firm that bundles

advertising with its product may benefits from this activity because the *relative* quality of the rival's

product is now enhanced. The decision to bundle advertising must therefore include a consideration

of how this negative effect in the product market will be enhanced by additional revenues in the

advertising market. We show how this aspect of advertising can make competition more intense or,

as the firms may view it, uglier.

Previous authors, such as Liebowitz (1982), Chae and Flores (1998), Sumner (2001), and

Dukes and Gal-Or (2003) have also examined the bundling of advertising with news or electronic

(radio and television) media.   Somewhat closer to our work is Godes, Ofek and Sarvary (2003).

They investigate how firms' incentives to use their products as advertising media are affected by

competition in both the product market and advertising market.   Their approach though differs

from ours in a number of key ways.   First, their basic model has firms competing in quantities in an

homogeneous product market, and bundling advertising into the product does not adversely affect

consumer valuation of the product. In equilibrium, all firms will always choose to bundle some

positive amount of advertising into their products. Moreover this outcome continues to hold even

when they allow for consumer disutility from advertising. This outcome arises because the decision

to advertise is not a strategic decision that affects how products are differentiated in the product

market. Rather a firm's decision to bundle advertising is equivalent to incurring an additional "cost"

in the product market. In equilibrium all firms choose to advertise and the amount of bundled

advertising falls as the "cost" incurred increases.

In our model, on the other hand, the product market is a differentiated duopoly where firms

compete in prices, and a firm's decision to advertise affects not only consumer valuation of its product

3

but it also directly affects consumer valuation of the rival products. Specifically, if one firm decides to bundle advertising into its product then consumer valuation of its product falls but consumer valuation of a rival product increases. This strategic interaction on how advertising affects consumer valuation of all products in the market is important, and suggests the possibility of an asymmetric outcome where some firms bundle advertising and others do not. Thus, we include in the first stage of the game the firm decision of whether or not to advertise. This allows us to consider asymmetric outcomes in both the product and advertising markets.

Kind, Nilssen and Soregard (2004) also use a duopoly model to investigate how product market competition affects a firm's incentive to use its product as a medium for advertising. They also have the two firms competing in prices in the product market and compete in prices in the advertising market. However their approach to modeling differentiation and the degree of competition in the product market is notably different from ours. In Kind, Nilssen, and Soregard (2004) the degree of competition is measured by whether the rival's product is a close or poor substitute. The less differentiated the products the tougher is price competition for the same group of consumers. When the goods are perfect substitutes Bertrand price competition drives price to marginal cost. In their model, it is the toughness of competition in the product market that underlies the firms' incentives to use their products as advertising media.

In contrast, our approach to modeling price competition in the differentiated product market permits market niches. In our model, when a firm's good is highly differentiated it faces a relatively small demand for its product. The firm has a market niche, and as such its demand is not much affected by the rival firm's price. This makes differentiation a double-edged sword. It softens price competition but limits the size of the market. When a firm markets a less differentiated product, one

4

that is similar to the rival's product, firms compete intensely in price for customers but there are more customers.   A result is that price competition in our model is not necessarily tougher with undifferentiated products because the greater price sensitivity that increased product substitutability brings is offset by the fact that such greater substitutability is associated with a larger overall market for the firms' products.   Since audience size or product market sales is a key factor for advertisers this feature of our model permits us to capture an important relationship that influences whether and how much advertising firms will bundle with their products.

In sum, the previous literature on bundled advertising focuses mainly on media firms that are by assumption active in both the product and the advertising market.   This literature largely seeks to answer to the "how much" questions, i.e. the amount of revenues derived from the product and advertising markets, respectively. In contrast, our paper looks at the traditional product-making firms and instead tries to answer the "whether or not" question, i.e., under what conditions does a consumer product firm, e.g., Verizon Wireless, start to bundle advertising into its product. As we shall see, our analysis uncovers critical prisoner's dilemma problems faced by these firms. This in turn has some important policy implications.

The rest of the paper proceeds as follows. In the next section, we present the formal model. Section 3 discusses the model's implications for both conventional, single-product firms and for policy makers. Some concluding remarks follow in Section 4.

2. The model

The model is a two-firm and two-market dynamic game. Firms compete in prices to sell their differentiated products in the product market. Here, each has the same constant marginal cost of production, which, for simplicity, is set equal to zero. Firms also consider whether to use their

products as advertising media and earn additional revenue from the advertising market.

The product market demand function is:

$$q_i = V - p_i + \theta\, p_j - A_i + \alpha\, A_j, \qquad A_i = \{0,1\}, i = 1, 2;\ j \neq i \tag{1}$$

where $0 < \theta < 1$, $0 < \alpha < 1$, and V and is normalized to take the value of one. Firm i's

product demand depends on its own price p_i as well as upon the price of the rival p_j. The

parameter θ measures the positive effect on the firm's demand of a price increase by the rival firm.

It also measures indirectly how substitutable the two goods are. When θ is equal to zero, the two

goods are completely independent. As θ increases the two goods have a larger degree of

substitution. Observe as well that the larger is θ, the larger is the firm's overall market.

Increased substitutability or less differentiation is associated with operating in a bigger market

whereas low substitutability or a small value of θ is associated with selling a highly differentiated

product in a relatively small market niche.

Each firm's demand is also affected by its advertising strategy, denoted by A_i and its rival's

advertising strategy A_j. A_i or A_j takes the value of one if the firm chooses to advertise and 0

otherwise. The parameter α measures the impact of the other firm's advertising choice on the

firm's demand function. Because consumers generally dislike bundled advertising, the decision to

include such bundled ads reduces the firm's product quality and therefore has a negative impact on

its demand. On the other hand, if its rival does the "bad" advertising, it enhances the firm's relative

product quality and thereby raises demand.

Because the ads are bundled into the firm's product, the quantity of ads that the firm sells in

the advertising market is directly proportional to the quantity of the goods that the firm sells in the

product market. For convenience, we normalize this factor of proportionality to one[2]. The (inverse) demand function for advertising faced by the firms is:

$$r = kV - A_1 q_1 - A_2 q_2 \tag{2}$$

where $k \geq 1$. The price r denotes the price of advertising that firms receive for a bundle of advertising per unit product. The parameter k measures the size of the advertising market relative to the product market. We assume that $k \geq 1$ to ensure that the advertising market is not too small to be attractive to the firms. If both firms advertise, the two firms compete in the advertising market in a standard Cournot fashion. If only one firm advertises, it takes the whole market. As before, firms have the same constant marginal cost of advertising, which we again set equal to zero.

The game has two stages. In the first stage, each firm simultaneously decides whether or not to advertise, i.e., to bundle the "bad" in with its product. This is a discrete decision. In the second stage, the firms compete in prices, p_1 and p_2, in the differentiated product market having observed the advertising choices in stage one. We solve the game in the usual fashion by using backward induction.

Stage 2:

In stage two, the advertising strategies, A_1 and A_2 have already been determined. Given demand for the differentiated product i, $q_i(p_i, p_j; A_i, A_j)$, $i = 1, 2$, $j \neq i$, firm i chooses price p_i to maximize its profit $\pi_i(p_i, p_j; A_i, A_j)$:

$$\underset{p_1}{Max}\, \pi_1 = p_1 q_1 + A_1 r q_1 \tag{3}$$

$$\underset{p_2}{Max}\, \pi_2 = p_2 q_2 + A_2 r q_2 \tag{4}$$

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[2] Alternatively, we can use an extra parameter to capture the quantity of ads bundled into the product and let the firm decides this quantity. This however makes the model much more complicated while adding little additional insight.

The first order conditions with respect to p_i for these maximization problems imply the following best response functions:

$$2(1 - \theta A_1 A_2 + A_1^2)p_1^* = [\theta - (1 + \theta^2)A_1 A_2 + 2\theta A_1^2]p_2 + 1 - (1 + k)A_1 + \alpha A_2$$
$$+ (1 - \theta)A_1 A_2 + (3\alpha + \theta)A_1^2 A_2 - (1 + \theta\alpha)A_1 A_2^2 + 2A_1^2 - 2A_1^3 \tag{5}$$

$$2(1 - \theta A_1 A_2 + A_2^2)p_2^* = [\theta - (1 + \theta^2)A_1 A_2 + 2\theta A_2^2]p_1 + 1 - (1 + k)A_2 + \alpha A_1$$
$$+ (1 - \theta)A_1 A_2 + (3\alpha + \theta)A_2^2 A_1 - (1 + \theta\alpha)A_2 A_1^2 + 2A_2^2 - 2A_2^3 \tag{6}$$

Solving the functions (5) and (6) together, yields Nash equilibrium prices (p_1^*, p_2^*) in stage 2.

These are:

$$p_1^* = \frac{-2 - \theta + (-2\alpha + \theta + k\theta)A_2 + 2\theta(-1 + \alpha\theta)A_1^4 A_2 - 2(1 + \theta)A_2^2 - 2(\alpha - \theta)A_2^3}{-4 + \theta^2 - 2\theta(-1 + \theta^2)A_1^3 A_2 + 2(-2 + \theta^2)A_2^2 - 2\theta A_1 A_2(-3 + \theta^2 + (-1 + \theta^2)A_2^2) + A_1^2(2(-2 + \theta^2) + (-3 + 2\theta^2 + \theta^4)A_2^2)}$$

$$\frac{-A_1^3(-4 + 2\alpha\theta - 2\theta(1 + \theta)A_2 + (-3 + \theta^2 + \alpha\theta(1 + \theta^2))A_2^2)}{}$$

$$\frac{+A_1^2(-2(2 + \theta) + (-\theta + \alpha(-5 + 2\theta^2))A_2 - (3 + 3\theta + \theta^2 + \theta^3)A_2^2 + (\theta + \theta^3 + \alpha(-3 + \theta^2))A_2^3)}{}$$

$$\frac{+A_1(2 + 2k - \alpha\theta + (-1 + 3\theta + 2\theta^2)A_2 + (3 + k + \alpha\theta - 2\theta^2 - k\theta^2)A_2^2 + 2\theta(1 + \theta)A_2^3 + 2(\alpha - \theta)\theta A_2^4)}{} \tag{7}$$

$$p_2^* = \frac{-2 - \theta + (2 + 2k - \alpha\theta)A_2 - 2\theta(-\alpha + \theta)A_1^4 A_2 - 2(2 + \theta)A_2^2 - 2(-2 + \alpha\theta)A_2^3}{-4 + \theta^2 - 2\theta(-1 + \theta^2)A_1^3 A_2 + 2(-2 + \theta^2)A_2^2 - 2\theta A_1 A_2(-3 + \theta^2 + (-1 + \theta^2)A_2^2) + A_1^2(2(-2 + \theta^2) + (-3 + 2\theta^2 + \theta^4)A_2^2)}$$

$$\frac{+A_1^3(-2\alpha + 2\theta + \theta(1 + \theta)A_2 + (\theta + \theta^3 + \alpha(-3 + \theta^2))A_2^2)}{}$$

$$\frac{-A_1^2(2(1 + \theta) - (3 + k + \alpha\theta - (2 + k)\theta^2)A_2 + (3 + 3\theta + \theta^2 + \theta^3)A_2^2 + (-3 + \theta^2 + \alpha\theta(1 + \theta^2))A_2^3)}{}$$

$$\frac{+A_1(-2\alpha + \theta + k\theta + (-1 + 3\theta + 2\theta^2)A_2 + (-\theta + \alpha(-5 + 2\theta^2))A_2^2 + 2\theta(1 + \theta)A_2^3 + 2\theta(-1 + \alpha\theta)A_2^4)}{} \tag{8}$$

Corresponding to these prices are profits:

$$\pi_1^* = \frac{(1 + A_1^2 - \theta A_1 A_2)(2 + \theta + (2\alpha - (1 + k)\theta)A_2 - (\alpha(1 + \theta^2) + \theta(-2 + k - k\theta^2))A_1^2 A_2 + 2(1 + \theta)A_2^2 + 2(\alpha - \theta)A_2^3}{(-4 + \theta^2 - 2\theta(-1 + \theta^2)A_1^3 A_2 + 2(-2 + \theta^2)A_2^2 - 2\theta A_1 A_2(-3 + \theta^2 + (-1 + \theta^2)A_2^2) + A_1^2(2(-2 + \theta^2) + (-3 + 2\theta^2 + \theta^4)A_2^2))^2}$$

$$\frac{-A_1(2 - \alpha\theta + k(-2 + \theta^2) + (1 + \theta)^2 A_2 + (-1 + k)(-1 + \theta^2)A_2^2))^2}{} \tag{9}$$

8

$$\pi_2^* = \frac{(1+A_1^2-\theta A_1 A_2)(2+\theta+2(\alpha-\theta)A_1^3+(-2+\alpha\theta-k(-2+\theta^2))A_2-(1+\theta)A_1^2(-2+(-1+k)(-1+\theta)A_2)}{(-4+\theta^2-2\theta(-1+\theta^2)A_1^3 A_2+2(-2+\theta^2)A_2^2-2\theta A_1 A_2(-3+\theta^2+(-1+\theta^2)A_2^2)+A_1^2(2(-2+\theta^2)+(-3+2\theta^2+\theta^4)A_2^2))^2}$$

$$\frac{-A_1(-2\alpha+\theta+k\theta+(1+\theta)^2 A_2+(\alpha(1+\theta^2)+\theta(-2+k-k\theta^2))A_2^2))^2}{} \tag{10}$$

Stage 1:

In the first stage, the two firms simultaneously decide whether to bundle advertising into their product. This is a discrete yes or no decision. Either a firm advertises and sets $A_i = 1$, or it does not and $A_i = 0$. Observe that although the decision to bundle is discrete, the amount of advertising that firm i chooses when $A_i = 1$ is continuous, because one advertising unit is bundled with each unit sold in the product market. In making the choice of whether to use its product as an advertising medium, each firm anticipates the implications of these choices in terms of the payoffs in the stage two market game in which each firm earns revenue from potentially two markets –the product and the advertising market. We may analyze the stage one game with the following strategic form matrix:

<div align="center">

Firm 2

</div>

		$A_2 = 0$		$A_2 = 1$	
Firm 1	$A_1 = 0$	$\pi_{1(00)}^*$	$\pi_{2(00)}^*$	$\pi_{1(01)}^*$	$\pi_{2(01)}^*$
	$A_1 = 1$	$\pi_{1(10)}^*$	$\pi_{2(10)}^*$	$\pi_{1(11)}^*$	$\pi_{2(11)}^*$

The payoff $\pi_{i(A_i A_j)}^*$ associated with each pair of strategies that the two firms choose is shown in the appropriate cell of the matrix. Using equations (7),(8), (9) and (10)it is easy to show that the equilibrium product prices and profits in each possible strategy combination are as follows:

(A) $A_1^* = 0$ $A_2^* = 0$

$$p_{1(00)}^* = p_{2(00)}^* = \frac{1}{2-\theta}$$

$$\pi^*_{1(00)} = \pi^*_{2(00)} = \left[\frac{1}{2-\theta}\right]^2 \qquad (11)$$

(B) $\quad A^*_1 = 1 \quad A^*_2 = 0$

$$p^*_{1(10)} = \frac{3(1+\alpha)\theta - 2k}{8 - 3\theta^2}, \qquad\qquad \pi^*_{1(10)} = \frac{2(2k + \theta + \alpha\theta - k\theta^2)^2}{(8 - 3\theta^2)^2} \qquad (12)$$

$$p^*_{2(10)} = \frac{4 + 4\alpha - k\theta}{8 - 3\theta^2} \qquad\qquad \pi^*_{2(10)} = \frac{(4 + 4\alpha - k\theta)^2}{(8 - 3\theta^2)^2} \qquad (13)$$

(C) $\quad A^*_1 = 0 \quad A^*_2 = 1$

$$p^*_{1(01)} = p^*_{2(10)} = \frac{4 + 4\alpha - k\theta}{8 - 3\theta^2}, \qquad\qquad \pi^*_{1(01)} = \pi^*_{2(10)} = \frac{(4 + 4\alpha - k\theta)^2}{(8 - 3\theta^2)^2}$$

$$p^*_{2(01)} = p^*_{1(10)} = \frac{3(1+\alpha)\theta - 2k}{8 - 3\theta^2}, \qquad \pi^*_{2(01)} = \pi^*_{1(10)} = \frac{2(2k + \theta + \alpha\theta - k\theta^2)^2}{(8 - 3\theta^2)^2}$$

(D) $\quad A^*_1 = 1 \quad A^*_2 = 1$

$$p^*_{1(11)} = p^*_{2(11)} = \frac{\alpha(4-\theta) - k}{5 - 5\theta + \theta^2}$$

$$\pi^*_{1(11)} = \pi^*_{2(11)} = \frac{(2-\theta)(k + \alpha - k\theta)^2}{(5 - 5\theta + \theta^2)^2} \qquad (14)$$

Using the iterated elimination of strictly dominated strategies and comparing the profits under different

pair of strategies, we find that there are four possible Nash equilibria based on following ranking of the

profits $\pi^*_{i(A_i A_j)}$:

If $\pi^*_{1(00)} > \pi^*_{1(10)}$ and $\pi^*_{2(10)} > \pi^*_{2(11)}$, then neither firm bundles advertising, or $A_1 = A_2 = 0$.

If $\pi^*_{1(00)} > \pi^*_{1(10)}$ and $\pi^*_{2(10)} < \pi^*_{2(11)}$, then two equilibria exist. In one neither firm bundles

advertising, or $A_1 = A_2 = 0$, whereas in the second both firms do, or $A_1 = A_2 = 1$.

If $\pi^*_{1(00)} < \pi^*_{1(10)}$ and $\pi^*_{2(10)} > \pi^*_{2(11)}$, then there are two asymmetric equilibria. One firm

bundles advertising whereas the other one does not, or $A_1 = 1$, $A_2 = 0$ in one and $A_1 = 0$,
$A_2 = 1$ in the other.

If $\pi^*_{1(00)} < \pi^*_{1(10)}$ and $\pi^*_{2(10)} < \pi^*_{2(11)}$, then both firms bundle advertising or $A_1 = A_2 = 1$.

The main purpose of the paper is to answer the question of what drives traditional

product-driven firms to act as advertising media. The expression of the equations (11)-(14) and the analysis of potential Nash equilibrium show that this choice depends on the parameters, k, α and θ. Unfortunately, this is a complex parameter space to work with and one not easily susceptible to closed form solutions.

To analyze the problem and to see how changing the parameter values leads to different Nash equilibria, we examine the relationship between α and θ while holding k constant at 1. We then increase k by increments $\Delta k = 0.1$ and repeat the process. In each case, we set $\pi^*_{1(00)} = \pi^*_{1(10)}$ and $\pi^*_{2(10)} = \pi^*_{2(11)}$. In this way, we can, for the given k value, plot α and θ and derive the two critical lines $\pi^*_{1(00)} = \pi^*_{1(10)}$ and $\pi^*_{2(10)} = \pi^*_{2(11)}$. The areas divided by these two lines represent the regions in which the different Nash equilibria will obtain. We analyze how these regions change as k increases. These results are illustrated by the various graphs in Figure 1.

3. Discussion of the results

We organize our discussion of the results illustrated in Figure 1 in the following way: First, we present our main proposition and then briefly discuss the results of our findings regarding the relative size of the advertising market as reflected in the parameter k. We then describe briefly the conditions under which the firms confront a prisoner's dilemma in stage one and the extent to which the failure to maximize producer surplus is equivalent to a failure to maximize total surplus—i.e. consumers are also adversely affected in the prisoner's dilemma. This welfare analysis permits us to draw some tentative policy conclusions. We also comment briefly on the implications of the model for product prices.

3.1 The size of the advertising market relative to the size of the product market

As noted, the parameter k is a rough way to captures the size of the advertising market relative to the size of the product market. As the various graphs in Figure 1 show, this parameter is an important determinant of whether firms have an incentive to enter the media market and bundle advertising into

11

their products.   As k increases, it becomes more and more likely that both firms will enter the market and engage in such bundling.   For relatively low values of k ($k \leq 1.3$), the only equilibrium is one in which neither firm enters and bundles advertising into its product.   As k gets larger, it is possible that at least one firm enters the media market and includes bundled advertising. This initially occurs in the upper left corner of the θ, α parameter space where θ is small and α is large.   In this region the positive effect on the rival's demand of bundling is relatively large whereas the effect of the rival's price is relatively small. So if one firm enters the advertising market and the rival firm does not then the quality of rival firm's product increases but the two products are not very substitutable. As k increases and the potential revenues from advertising rise, it becomes much more likely that the equilibrium will include at least one firm bundles advertising with its product.

For still larger values of k, the parameter space includes a region in which both firms choose to enter the advertising market. This initially occurs in the region in which both θ and α are small. In this region the one firms decision to advertise does not spillover much to the other and the products are not that closely substitutable.   As k increases this region expand.   When k is 2.6, it covers 80 percent of the θ, α parameter space.   When $k \geq 4$, it covers 100 percent, and the outcome is that in equilibrium both firms enter the advertising market and bundle advertising into their products.   We summarize our results with the following proposition the proof of which is given in the Appendix.

Proposition 1 (a) When the advertising market is relatively small, $1 \leq k < 1.36903$, no firm will enter the advertising market and bundle advertising with its product.   When the advertising market is relatively large, $k \geq 4$, both firms enter the advertising market and will bundle advertising.   For values of k such that $1.36903 < k < 4$, various equilibria are possible in which either no firm, one firm or both firms advertise.

3.2 Prisoner's dilemmas, producer surplus, and social welfare

As the graphs in Figure 1 show there are values of the parameters, k, α and θ, for which the Nash equilibrium will be one in which neither firm includes bundled advertising with its products or both do. Some of these equilibria, from the point of view of firm profitability, are not first best outcomes. Instead, they reflect prisoner's dilemma type outcomes and as such both firms could be made better off if they could cooperate and achieve another outcome. To determine whether the game is a prisoner's dilemma, we compare $\pi_{1(00)}^*$ and $\pi_{1(11)}^*$ in each game. Again, in the θ-α plane, we draw the critical curve along which $\pi_{1(00)}^* = \pi_{1(10)}^*$. This curve separates the equilibrium region into prisoner's dilemma and non-prisoner's dilemma categories.

First, we investigate the non-advertising case that arises for relatively small k ($k \leq 1.3$). Figure 2(a) illustrates this outcome for the case in which k is exactly 1.3. As shown, the upper left part of the parameter space shaded in green is the region of the prisoner's dilemma. Although neither firm has entered the advertising market, and neither bundles advertising in its product, both would be better of if each did. The underlying intuition is straightforward. When the two goods are less substitutable and the consumer base is not that large but the positive spillover from the rival entering into advertising is relatively large, then if one firm enters into advertising the other firm finds more profitable not to enter. The outcome is that neither firm advertises but if both did they would both be more profitable.

The no-advertising equilibrium may also not be best for society overall. Figure 2(b) again shows the case of a relatively small advertising market, $k = 1.3$. As before, everything to the northwest of the lower curve, here shown as the combination of a green-shaded and orange-shaded

region, indicates the area in which producer surplus is lower than it would be if both firms advertised. The orange-shaded subset of this region is the portion of this space in which not only producer surplus but the total surplus is reduced because neither firm advertises. This region reveals a potential for market failure in which there is no advertising even though both producers and consumers could potentially be better off if both firms bundled advertising with their products. Recall consumers in the product market can benefit from the lower prices, or perhaps free or even subsidized consumption of the product when both firms enter into the advertising market.

As k increases and equilibria occur in which both firms include bundled advertising, the prisoner's dilemma cases and potential social welfare losses occur when both firms enter the advertising market. Figures 3a and 3b illustrate this possibility for the case of $k = 1.9$. Here, the green-shaded region of Figure 3a indicates that portion of the parabola-shaped region of the parameter space in which both firms advertise but where producer surplus would be larger if neither did. The large, orange-shaded portion in Figure 3b then shows the portion of the shaded region in 3a for which this failure to maximize total producer surplus also means that the total surplus is not maximized. As the figure suggests, the region in which total welfare would be raised by a ban on bundled advertising is relatively large, especially in comparison with the region in which such a ban would raise producer surplus. This is a common feature of all the cases in which bundled advertising is excessive. It suggests that public policy to ban bundled advertising will very frequently be in the public interest.

Figures 4a and 4b illustrates the potential for excessive advertising for those cases in which the only possible Nash equilibria are those in which both firms enter the advertising market and choose to bundle advertising into their primary products. As in Figures 3a and 3b, there are many

parameter configurations in which firms would do better if there were some statutory ban that prohibited them from including advertising with their products. Here again, just as in the earlier figures, there is a large likelihood that such a ban would raise welfare overall.

3.2 Pricing

In discussing the various cases, we have not explicitly addressed the nature of the equilibrium product prices. In any equilibrium in which neither firm advertises, it is clear that products are goods and prices are positive. However, in an equilibrium in which at least one firm advertises, there is the possibility that the depressing effect this has on consumer product valuation could lead to the product being a "bad" and a negative product price.   Negative pricing should not in this case be ruled out. Effectively negative pricing amounts to paying the consumer to "listen" to the bundled ad, and this phenomenon is frequently observed.   For example, real estate firms often pay consumers by giving them goods in return for listening to an hour-long presentation on a time-sharing opportunity. Similarly, so-called "wrapped" cars give consumers the free use of an automobile but one that is covered in advertising.   The Disney Company offers yet another example. In early 2005, it gave consumers free access to a number of on-line games that were based on Disneyland attractions (e.g., the Haunted Mansion) in an effort to lure consumers to the theme park in celebration of Disneyland's 50th anniversary.[3]   Accordingly, we have not rule out equilibria that include negative product prices. Moreover as k gets large and entering the advertising market and selling a lot of bundled advertising becomes increasingly profitable, the portion of the θ, α parameter space that includes such negative pricing also increases.   If we restrict attention to only those equilibria with non-negative product prices then the product becomes accordingly a free good in a great many cases.

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[3] See Michael McCarthy "Disney plans to mix ads, video games to target kids, teens" *USA Today*, January 17, 2005.

4. Conclusion

Increasingly the age of the information highway is one in which information is bundled with other goods. In recent years we have witnessed more and more traditional product firms making a decision to bundle advertising with their products and thereby implicitly serve as a communications or information medium. In this paper, we use a simple duopoly model to study this phenomenon. We assume the two firms compete in prices in the product market and compete in quantity in the advertising market. By bundling or selling advertising "space" in their products, firms gain additional revenues. However, such bundling reduces consumers' valuation of the firm's product leading consumers to switch to the firm's competitor while the bundling firm's product demand decreases accordingly.

We show that the equilibrium depends on three key parameters measuring, respectively, the relative size of the advertising market, the degree of substitution between the rival products or the size of the consumer base, and the impact that bundled advertising has on product market demand. When the advertising market is relatively small, not surprisingly firms do not have an incentive to enter the advertising market. This is true even when both firms would be more profitable, and total surplus would rise, if each firm entered the advertising media market and did engage in such bundling. As the size of the advertising market becomes relatively larger, the likelihood that at least one firm will enter the advertising market and use its product as an advertising medium increases. A threshold value of the relative size of the advertising market is reached beyond which the only possible equilibria are those in which both firms enter the advertising media market and bundle advertising with their products. Here as well there is the possibility of a market failure in which there is too much advertising and firms would both be better if neither entered the advertising market. Moreover,

the parameter range over which there is a consumer welfare loss because of too much advertising is relatively large.

An interesting feature of many equilibria in which both firms advertise is that product prices are negative, or constrained to be zero. The product firms have an incentive to pay people or to give their products away to use with a view toward increasing "circulation" and raising advertising revenue. This seems to be consistent with a number of real world cases involving such give-a-ways, such as those employed by real estate firms.

5. References

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6. Appendix

Proof of proposition 1:

For neither firm to include bundled advertising requires that profits when neither advertises exceed those when at least one advertises, i.e., that $\pi^*_{1(00)} \leq \pi^*_{1(10)}$. We first solve for the conditions under which these two values are equal. This implies:

$$\left[\frac{1}{2-\theta}\right]^2 = \frac{2(2k+\theta+\alpha\theta-k\theta^2)^2}{(8-3\theta^2)^2} \tag{A1}$$

Setting $\alpha = 1$, yields:

$$k = \frac{-32\theta+32\theta^2+8\theta^3-16\theta^4+4\theta^5}{2(16-16\theta-12\theta^2+16\theta^3-4\theta^5+\theta^6)}$$
$$+\frac{\sqrt{2}\sqrt{(1024-1024\theta-1536\theta^2+1792\theta^3+720\theta^4-1168\theta^5-44\theta^6+336\theta^7-48\theta^8-36\theta^9+9\theta^{10})}}{2(16-16\theta-12\theta^2+16\theta^3-4\theta^5+\theta^6)} \quad \text{or}$$

$$\tag{A2}$$

$$k = \frac{-32\theta+32\theta^2+8\theta^3-16\theta^4+4\theta^5}{2(16-16\theta-12\theta^2+16\theta^3-4\theta^5+\theta^6)}$$
$$-\frac{\sqrt{2}\sqrt{(1024-1024\theta-1536\theta^2+1792\theta^3+720\theta^4-1168\theta^5-44\theta^6+336\theta^7-48\theta^8-36\theta^9+9\theta^{10})}}{2(16-16\theta-12\theta^2+16\theta^3-4\theta^5+\theta^6)}$$

$$\tag{A3}$$

A3 is not a valid solution, as k is negative when $0<\theta<1$. Minimize k in A2 by setting $\frac{\partial k}{\partial \theta}=0$ yields:

$$\frac{-348\sqrt{2}\theta^4-24\sqrt{2}\theta^5+96\sqrt{2}\theta^6-9\sqrt{2}\theta^8+4\theta^3(28\sqrt{2}-\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2})}{2(-2+\theta)(-2+\theta^2)^2\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2}}$$
$$+\frac{16(-16\sqrt{2}+\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2})-8\theta(16\sqrt{2}+\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2})}{2(-2+\theta)(-2+\theta^2)^2\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2}}$$
$$+\frac{8\theta^2(64\sqrt{2}+\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2})}{2(-2+\theta)(-2+\theta^2)^2\sqrt{(32-16\theta-28\theta^2+14\theta^3+6\theta^4-3\theta^5)^2}}=0$$

$$\tag{A4}$$

The numerical solution is 1.36903, when $0<\theta<1$. This is the minimum value of k for which $\pi^*_{1(00)} = \pi^*_{1(10)}$ in the permissible (0, 1) space of α and θ. Therefore, when $k < 1.36903$, no advertising is the only possible equilibrium in the market.

19

An equilibrium in which both firms advertise requires that the profit either gets in this state exceeds the profit one would earn if that firm unilaterally stopped bundling advertising with its product. That is, we must have $\pi^*_{1(00)} \leq \pi^*_{1(10)}$ Now return to equation (A1) and set $\alpha = 0$, to obtain:

$$k = \frac{-16\theta + 16\theta^2 + 4\theta^3 - 8\theta^4 + 2\theta^5}{2(16 - 16\theta - 12\theta^2 + 16\theta^3 - 4\theta^5 + \theta^6)}$$
$$-\frac{\sqrt{2}\sqrt{(1024 - 1024\theta - 1536\theta^2 + 1792\theta^3 + 7200\theta^4 - 1168\theta^5 - 44\theta^6 + 336\theta^7 - 48\theta^8 - 36\theta^9 + 9\theta^{10})}}{2(16 - 16\theta - 12\theta^2 + 16\theta^3 - 4\theta^5 + \theta^6)} \quad \text{or}$$

$$(A5)$$

$$k = \frac{-16\theta + 16\theta^2 + 4\theta^3 - 8\theta^4 + 2\theta^5}{2(16 - 16\theta - 12\theta^2 + 16\theta^3 - 4\theta^5 + \theta^6)}$$
$$+\frac{\sqrt{2}\sqrt{(1024 - 1024\theta - 1536\theta^2 + 1792\theta^3 + 7200\theta^4 - 1168\theta^5 - 44\theta^6 + 336\theta^7 - 48\theta^8 - 36\theta^9 + 9\theta^{10})}}{2(16 - 16\theta - 12\theta^2 + 16\theta^3 - 4\theta^5 + \theta^6)}$$

$$(A6)$$

The solution in (A5) is not valid as k is negative when $0 < \theta < 1$. Focusing on the solution in (A6) and setting $\theta = 1$, we obtain $k = \frac{5}{\sqrt{2}} - 1 \approx 2.53553$. This is the maximum k value for which the critical line $\pi^*_{1(00)} = \pi^*_{1(10)}$ is still in the (0, 1) space of α and θ. Any k larger than that will make the asymmetric equilibrium not possible in the market. Along this critical line we have $\pi^*_{2(10)} = \pi^*_{2(11)}$, or

$$\frac{(4 + 4\alpha - k\theta)^2}{(8 - 3\theta^2)^2} = \frac{(2 - \theta)(k + \alpha - k\theta)^2}{(5 - 5\theta + \theta^2)^2} \tag{A7}$$

Set $\alpha = 0$ yields

$$k = \frac{-100\theta + 200\theta^2 - 90\theta^3 - 60\theta^4 + 66\theta^5 - 20\theta^6 + 2\theta^7}{72 - 80\theta - 80\theta^2 + 144\theta^3 - 48\theta^4 - 23\theta^5 + 26\theta^6 - 11\theta^7 + 2\theta^8}$$
$$-\frac{\sqrt{2}\sqrt{(2300\theta^2 - 14400\theta^3 + 24480\theta^4 - 16080\theta^5 - 4222\theta^6 + 13015\theta^7 - 6940\theta^8 - 4\theta^9 + 1500\theta^{10} - 672\theta^{11} + 126\theta^{12} - 9\theta^{13})}}{72 - 80\theta - 80\theta^2 + 144\theta^3 - 48\theta^4 - 23\theta^5 + 26\theta^6 - 11\theta^7 + 2\theta^8} \quad \text{or}$$

$$(A8)$$

$$k = \frac{-100\theta + 200\theta^2 - 90\theta^3 - 60\theta^4 + 66\theta^5 - 20\theta^6 + 2\theta^7}{72 - 80\theta - 80\theta^2 + 144\theta^3 - 48\theta^4 - 23\theta^5 + 26\theta^6 - 11\theta^7 + 2\theta^8}$$
$$+\frac{\sqrt{2}\sqrt{(2300\theta^2 - 14400\theta^3 + 24480\theta^4 - 16080\theta^5 - 4222\theta^6 + 13015\theta^7 - 6940\theta^8 - 4\theta^9 + 1500\theta^{10} - 672\theta^{11} + 126\theta^{12} - 9\theta^{13})}}{72 - 80\theta - 80\theta^2 + 144\theta^3 - 48\theta^4 - 23\theta^5 + 26\theta^6 - 11\theta^7 + 2\theta^8}$$

$$(A9)$$

Only A8 is a valid solution consistent with $0<\theta<1$. Now, minimize k by setting $\dfrac{\partial k}{\partial \theta}=0$. The

numerical solution is then $k=1.741753$, when $0<\theta<1$. This is the minimum value of k for which

the critical line $\pi_{2(10)}^{*} = \pi_{2(11)}^{*}$ lies in the permissible (0, 1) parameter space of α and θ, i.e., for

which both firms advertising, as opposed to just one, " becomes a possible equilibrium. In a

similar fashion, it is easy to show that the point at which this line lies to the southeast of the entire

permissible parameter space is when $k = 4$.

Figure 1

k≤1.3: Neither Advertises

k=1.4: Either One Advertises Or Neither Advertises

k=1.6: Either One Advertises Or No One Advertises

k=1.9: Either Both Advertise, One Advertise, or No One Advertises

k=2.6: Either Both Advertise Or Just One Advertises

k ≥ 4.0:   Both Firms Advertise

22
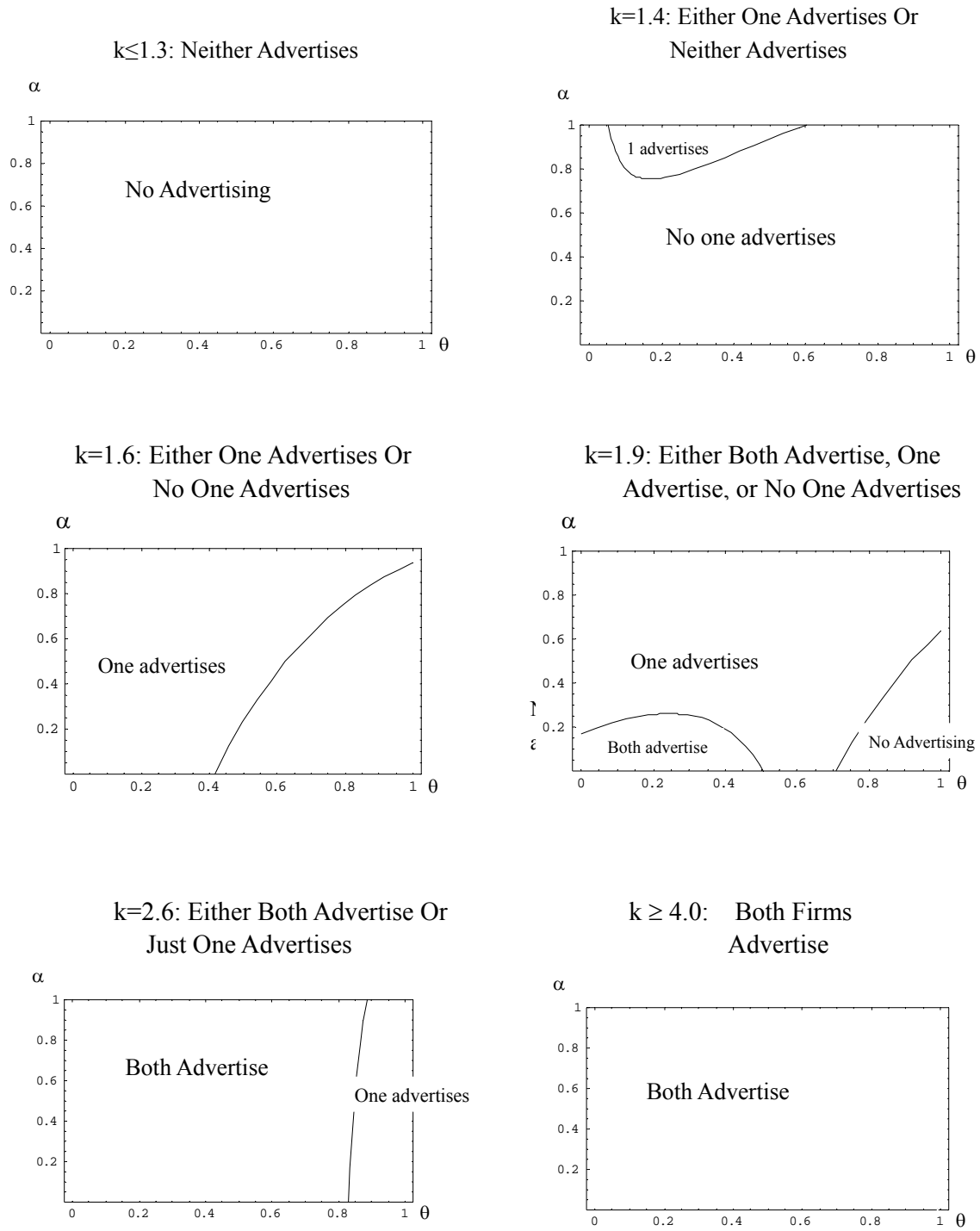
Figure 2(a)

Small size advertising market, $k = 1.3$

Shaded area reflects region of Prisoner's
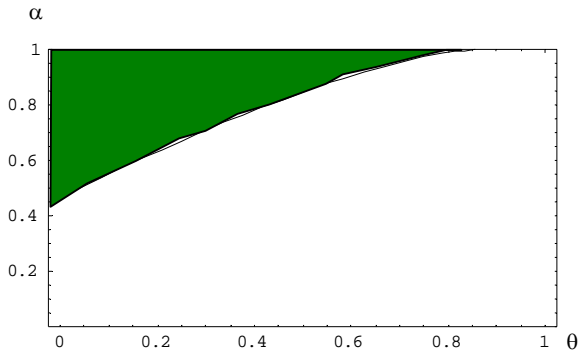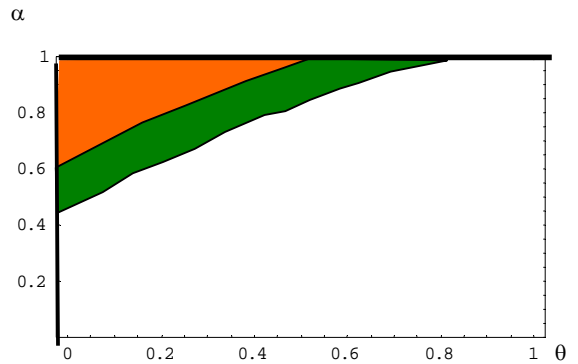Dilemma—Neither Advertises but Both Should

Figure 2(b)

Potential for Market Failure Due to Insufficient
Advertising When $k = 1.3$

Figure 3(a)

Medium size advertising market, $k = 1.9$

Shaded area reflects region of Prisoner's
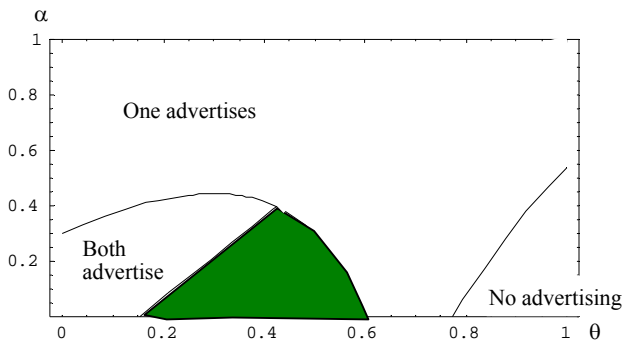Dilemma—Both Advertise but Neither Should

Figure 3(b)

Potential for Market Failure Due to Excessive
Advertising When $k = 1.9$
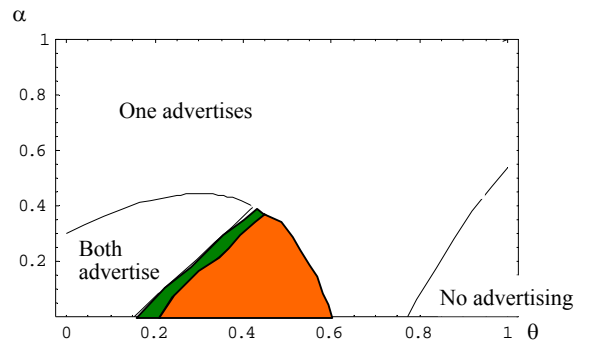
Total Surplus rises in Orange-Shaded Area

Figure 4a

Large advertising market, $k = 4.0$

Shaded area reflects region of Prisoner's
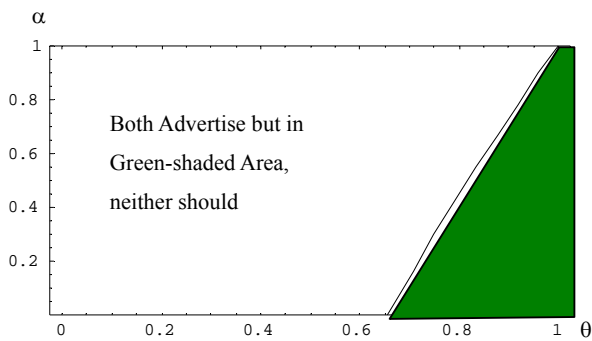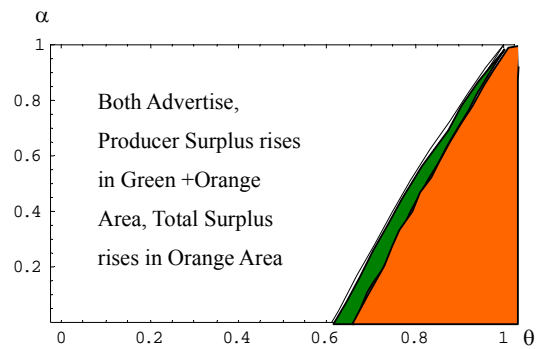Dilemma—Both Advertise but Neither Should

Figure 4a

Potential for Market Failure Due to Excessive
Advertising when $k = 4.0$

23

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