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Durable Goods Monopoly, Learning-by-doing and "Sleeping Patents"

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Abstract

We analyze a durable–good monopolist's decision to adopt a new and more efficient technology that is readily available at no cost. After an initial period of learning by doing, the new technology can either lower the cost of production, or make the good more attractive to consumers. We show that for certain parameter values, the monopolist finds it optimal to continue using the inferior production technology. An implication for welfare purposes is that a durable–good monopolist may hold onto a "sleeping patent" when its use is socially desirable. However, we also show that sometimes the monopolist innovates too much relative to the socially optimal level.

1 Introduction

Economists have long been skeptical about arguments in popular press according to which a monopoly acquires a patent to an innovation, only to subsequently put it away into a safe and never use it. If the innovation improves the quality of the good that the monopolist sells or decreases his production costs, why would the monopolist not adopt it if he can do that at no additional cost? In this paper, we show that in the presence of learning–by–doing, a monopolist may indeed prefer not to use an existing, readily available innovation, even though he may have previously invested in its development. We build a model in which a durable–good monopolist decides whether to adopt a new technology that is readily available at no cost and that will either reduce the marginal cost of production or improve upon the quality of the good.[1] Although the adoption of the new technology is costless, an initial period of production is necessary before the full benefits of this innovation are realized in the next period. This captures the notion of learning

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[1] Thus, we are explicitly considering innovations that require no investments. One possible interpretation of this situation is that the monopolist has access to two different technological processes, as in Karp and Perloff (1996). Alternatively, the monopolist may have acquired a patent for the superior technology through an earlier race to innovate so as to prevent potential competitors from obtaining the patent (as in Gilbert (1981) and Gilbert and Newbery (1982)). We discuss the monopolist's R&D incentives later in the paper.

by doing, which we consider to constitute an essential part of any production process involving a new technology.

A well known result in the literature is that any durable good monopolist who cannot commit to future prices faces the familiar 'Coase problem': The monopolist engages in an intertemporal price discrimination, charging a lower price to those consumers who purchase the good later. However, as Coase (1972) and subsequent researchers show,[2] the monopolist may be hurt by her own ability to price discriminate intertemporally — rational consumers correctly anticipate a future reduction in price and therefore postpone their purchases, which lowers the monopolist's overall profit.

In our paper, adoption of a new technology in the presence of learning by doing intensifies the monopolist's time inconsistency problem. Switching to a *cost reducing technology* decreases the next period's expected price even further, which induces even more consumers to postpone their purchases. Learning by doing plays a crucial role here, because it decreases the second period marginal cost more than the first period marginal cost, thus causing a relative decrease in the future price of the good. Similarly, switching to a *quality improving technology* makes the units purchased today economically obsolete (or inferior) in the next period. This decreases the value that the consumers place on the units produced in the first period, which is reflected in the lower price they are willing to pay. These effects make the adoption of the new technology less desirable, which leads to the main result of our paper: For some parameter values, the durable good monopolist strategically shelves an existing innovation that is available to her at no cost, even though it would be socially efficient to adopt it.

Interestingly enough, we also show that, if there is a small cost associated with adoption of the innovation, sometimes the monopolist can innovate too much — more than what is socially desirable. This result has a relatively simple intuition: In the presence of learning by doing, the benefits of an innovation materialize mainly in the second period. But if the good is sold at the marginal cost, as is socially optimal, then sometimes the second period residual demand is not high enough to warrant production in that period. This means that it may not be socially efficient to adopt the innovation if the adoption is not completely costless. In contrast, under monopoly the second–period residual demand is higher than in the first best scenario because the monopoly charges a first period price that is above marginal cost. Therefore, sometimes the innovation may be more valuable to a monopolist than to a social planner, which can lead the monopolist to inefficiently adopt the innovation.

The paper proceeds as follows. The model is presented in the next section (Section 2), with the cost reduction and product innovation technologies analyzed separately in Sections 3 and 4 respectively. Section 5 contains a discussion of two strands of related literature. Section 6 concludes the paper. All proofs are in the Appendix.

2 The Model

Consider a durable good monopolist that operates in two periods. The monopolist chooses her production level and prices so as to maximize the present value of profits. There is no leasing of the product and it is assumed that the monopolist cannot commit to the price path in advance.

At the beginning of the first period, the monopolist can choose between two known production technologies: the existing, old technology or a new one. Production with the old technology incurs a marginal cost of $c_1 < 1$ in both periods. Alternatively, the monopolist can adopt the new technology which, after a first period use, will *either* lower the cost of production *or* make the good more attractive in the second period.

[2]See, for example, Bulow (1982), Gul, Sonnenschein, and Wilson (1986), and Stokey (1981).

Although the intuition for our results is similar under both, we choose to analyze them separately, for two reasons. First, we do not think it is immediately obvious that the results in one setting extrapolate into the other one. Second, each of these cases seems to be connected to different literature. In the case of the cost reducing innovation the closest paper appears to be Karp and Perloff (1996), while the product innovation case appears to be more closely related to Waldman (1996a) and Lee and Lee (1998).[3]

Throughout the most of the paper, we concentrate on the monopolist's incentives to adopt the new technology when it is already developed and freely available. Although it may have been costly to the monopolist to develop this new technology, there is *no extra cost of adopting it once it is developed*. We discuss the monopolist's incentives to invest in developing the new technology in Subsection 4.2.4.

We also assume that the production process with the new technology must be learnt, albeit costlessly, even though this technology is readily available: specifically, we assume that the new production technology must be used in the first period to produce some minimum amount of production in order for the full potential to be realized later. In particular, we assume that if at least $q_{min} > 0$ units are produced in the first period, then the new technology results in a second period cost saving (or quality improvement; whichever applies), otherwise it does not. This is a simple version of *learning by doing*.[4] Throughout most of the analysis, we maintain informational symmetry regarding the technology. This means that if the monopolist decides to adopt the new technology, consumers can observe this decision and use it to form their expectations about future price. We discuss the effects of informational asymmetry in Subsection 3.2.4.

Each consumer lives for two periods and wishes to buy at most one unit of the good immediately, or after a delay. If a consumer purchases the good in the first period, he enjoys it for two periods or sells it at the secondhand market in period two.[5] It is also assumed that a good produced in period 2 has only one period of economic life, although it may have two periods of physical life.

Consumers differ in their valuation for the good. Let θ_1 be a representative consumer's one-period valuation of the good produced in the first period and θ_2 the one-period valuation of the good produced in the second period. A consumer that does not make a purchase in any period gets zero utility. As mentioned earlier, we investigate two possible cases, one where the innovation decreases costs of production and one where it makes the product more valuable to consumers. In the case of the cost–reducing innovation, the innovation has no effect on the valuation of the good by consumers, so that $\theta_1 = \theta_2 = \theta$. In the case of the quality improving innovation, we assume that the new technology makes the good more attractive to consumers by raising their valuation for the good: by a factor of $s_1 \geq 1$ in the first period and by a factor of $s_2 > s_1$ in the second period. Hence, if the new technology is adopted, then $\theta_1 = s_1\theta$ and $\theta_2 = s_2\theta$. The condition $s_2 > s_1$ reflects the learning-by-doing feature of our model. In both cases we assume that θ is distributed uniformly on $[0, 1]$. Consumers know their valuations, but the monopolist only knows the distribution of θ.

Although we allow for quality improvements, we are not interested in a related question of durability choice.[6] In our model, the good is perfectly durable, which is reflected in θ being

[3]See also Levinthal and Purohit (1989), who analyze alternative ways of selling a durable good when new and better units are being introduced onto the market. We provide a more detailed discussion of the relationship between our model and the existing literature in Section 5.

[4]Assumption 1 below guarantees that, in equilibrium, this minimum production requirement is non–binding. This means that our assumption of costless adoption of the new technology is preserved.

[5]Alternatively, we could have assumed that there exists no secondhand market for the good. Both of these assumptions are used in the literature. For example, Sobel (1990), Lee and Lee (1998), and Chi (1999) do not allow for second hand markets, while Bulow (1982), Waldman (1996a), and Karp and Perloff (1996) do. We have checked that our qualitative results do not depend on this assumption.

[6]As shown by other researchers (Bulow, 1986; Waldman, 1996a), the monopolist can use planned obsolescence to

constant over time. Both the firm and the consumers are risk neutral and discount the future using a common discount factor $\delta \in (0, 1]$.

In order to reduce the number of cases that need to be analyzed under both types of innovation, we adopt the following restriction on the model's parameters:

Assumption 1. $q_{\min} \leq \min\left\{\frac{2(1-c_1+\delta c_2)}{4+\delta}, \frac{2[s_2-c_1(s_2-\delta)]}{4s_2(1+\delta)-3\delta}\right\}$.

This assumption guarantees that in equilibrium the monopolist's unconstrained first period output is always high enough for learning by doing to be effective.

Each of the two settings that we investigate begins with an analysis of a social planner's problem of whether to adopt the new technology or not, followed by an analysis of the monopolist's incentives to adopt the innovation. The cost innovation is examined in the next section, while the analysis of the product–quality innovation is presented in Section 4.

3 Cost–reducing innovation

Consider a readily available technological innovation that reduces the marginal cost of production to $c_2 < c_1$ per unit in the second period, but only if adopted (at no cost) in the first period. To model learning–by–doing in a simple way, we assume that the innovation does not decrease the first period marginal cost.[7] The innovation has no effect on the valuation of the good by consumers, *i.e.,* $\theta_1 = \theta_2 = \theta$. The question is whether this innovation will be adopted by a monopolistic firm in a socially optimal way.

3.1 Socially efficient adoption of a cost-reducing innovation

We start by analyzing the benchmark case, the socially efficient adoption of a cost reducing technology. Suppose the technology is adopted. Then it is socially optimal for a consumer with valuation θ to get the good in the first rather than in the second period if and only if $\theta(1+\delta) - c_1 \geq \delta(\theta - c_2)$, *i.e.,* if and only if $\theta \geq c_1 - \delta c_2$. Define θ^* by $\theta^* = c_1 - \delta c_2$. Then all consumers with valuations $\theta \geq \theta^*$ will get the good in the first period. Note that $\theta^* < 1$, so that the measure of consumers that will be served in the first period, $1 - \theta^* = 1 - c_1 + \delta c_2$, is positive.

In the second period, the good is produced only if $\theta^* > c_2$; *i.e.,* $c_2 < \frac{c_1}{1+\delta}$. If this condition is satisfied, then all consumers with valuations between c_2 and θ^* will be served in the second period. Otherwise, no production takes place in that period. We thus have the following result.

mitigate the Coase problem by building a lesser degree of durability into each unit. See also Chi (1999), who shows that one way for the monopolist to alleviate the Coase problem is to make the good more attractive by choosing a higher (than optimum) quality in the first period, and lower the price–quality ratio in order to make high–demand consumers purchase immediately. Finally, Basu (1988) offers a model in which the monopolist chooses to make the product less durable in order to price discriminate between consumers.

[7] We could make the analysis slightly more general by assuming that the first period marginal cost decreases too if the innovation is adopted, although less so than the second period marginal cost. This would not affect our qualitative results.

Also, at the cost of substantially complicating the exposition, we could let the extent of cost reduction (or, in the case of quality innovation, the degree of quality–improvement) depend upon the amount produced in the first period. That is, the marginal cost in period 2 would be $c_2(q_1)$, where $c_2(0) = c_1$ and $c_2'(q_1) < 0$. We believe that, after putting some additional structure on $c_2(q_1)$, the qualitative nature of our results would not be altered by this generalization.

Lemma 1. *(i) If $c_2 \geq \frac{c_1}{1+\delta}$, then it is not socially efficient to adopt the innovation. The good should be produced only in the first period; no production should take place in the second period.*[8]

(ii) If $c_2 < \frac{c_1}{1+\delta}$, then it is socially efficient to always adopt the innovation and the good should be produced in both periods.

Lemma 1 simply states that it is efficient to adopt a cost-saving technology only if it leads to a substantial reduction in the marginal cost of production. The intuition is most easily seen when there is no discounting. Suppose the costs of production are the same in both periods. Then any consumer who gets the good will get it in the first, rather than in the second period, because then the consumer enjoys the good longer. Also, because each consumer who gets the good enjoys it for two periods, everyone who has a valuation of at least $\frac{c_1}{2}$ should get the good. Thus, in the second period all consumers with valuations high enough to warrant production already got the good in the first period. The innovation thus has no value unless it decreases the second period cost substantially.

3.2 Adoption of the cost–reducing innovation under monopoly

We solve the monopolist's optimal production problem by backwards induction. First, we take the share of consumers who purchased the good in the first period as given and derive the optimal second period output as a function of this share. Then we find the optimal first period quantity produced under the equilibrium condition that the consumers' expectation of the second period price is correct. This also determines the equilibrium share of consumers who purchase the good in the first period.

3.2.1 The second period quantity and profit

Let q_1 be the quantity that was sold in period 1 and q_2 the quantity produced in period 2. Then the total supply in period 2 is $q_1 + q_2$, so that the market price is $1 - q_1 - q_2$. Given a q_1, the monopolist chooses the second period production q_2 so as to maximize

$$\max_{q_2} q_2(1 - q_1 - q_2 - c_2),$$

which yields $q_2^* = \frac{1-q_1-c_2}{2}$. The equilibrium price in the second period is then given by $p_2^* = \frac{1-q_1+c_2}{2}$, and the second period profit level is

$$\pi_2(q_1, c_2) = \left(\frac{1 - q_1 - c_2}{2}\right)^2.$$

3.2.2 The first period price and total profit

A consumer i buys the good in period 1 if and only if $\theta_i - p_1 + \delta E(p_2) \geq 0$, where $E(p_2)$ is the price that the consumer expects to prevail in period 2. Let $\hat{\theta}$ be the valuation of the consumer who is indifferent between buying and not buying in the first period. Then the first period sales

[8] Strictly speaking, a social planner would be indifferent between adopting the technology or not in this case, because the first period cost is assumed to be the same under both technologies. However, it can easily be seen that whenever there is a small cost, $\varepsilon > 0$, associated with adoption of the new technology, then the social planner strictly prefers the old technology.

are $q_1 = 1 - \hat{\theta} = 1 - p_1 + \delta E(p_2)$, so that $p_1 = 1 - q_1 + \delta E(p_2)$. Since equilibrium beliefs must be correct, it must be that $E(p_2) = p_2^* = \frac{1-q_1+c_2}{2}$, which implies $p_1 = (1-q_1)\left(1+\frac{\delta}{2}\right) + \frac{\delta c_2}{2}$.

The monopolist's problem at $t = 1$ is then to choose her first period output q_1 so as to maximize the total profit over the two periods:

$$\max_{q_1} \pi(c_1, c_2) = \max_{q_1} q_1 \left[(1-q_1)\left(1+\frac{\delta}{2}\right) + \frac{\delta c_2}{2} - c_1\right] + \delta\left(\frac{1-q_1-c_2}{2}\right)^2, 1 \tag{1}$$

which yields $q_1^* = \frac{2(1-c_1+\delta c_2)}{4+\delta}$.

3.2.3 Adoption of the cost–reducing innovation

The analysis above allows us to evaluate the monopolist's incentives to adopt a cost–reducing innovation.

Lemma 2. *(i) Suppose $c_1 > \frac{1}{5}$. Then there exists a $c_2^* < c_1$ such that if $c_2 \in (c_2^*, c_1)$, the monopolist does not adopt the innovation, and if $c_2 < c_2^*$, the innovation is adopted.*

(ii) Suppose $c_1 < \frac{1}{5}$. Then the monopolist adopts the new technology for all values of $c_2 < c_1$.

To see what drives the results in the above lemma, consider an innovation that decreases the second period cost slightly below c_1. As in the case of the Coase' time inconsistency problem, the monopolist in our model competes with herself across the two periods because the good is durable. If the monopolist adopts the innovation, this intertemporal competition is intensified. The reason is that, in the presence of learning–by–doing, the innovation decreases the second period price of the good relatively more than the first period price. This induces more consumers to postpone their purchase in order to take advantage of the lower future price. This, in turn, forces the monopolist to decrease her first period price, which decreases her first period profit. Thus, a cost reducing innovation has two effects on the firm's overall profit: $-\frac{\partial \pi(c_1,c_2)}{\partial c_2} = \delta\left(\frac{1-q_1-c_2}{2}\right) - \delta\frac{q_1}{2}$.

(i) The first term on the right hand side, proportional to $1-q_1-c_2$, represents a profit–increasing effect due to a lower cost of production in the second period.

(ii) The second term, proportional to q_1, represents a profit–decreasing effect due to a decrease in the first period price.

The magnitude of each of these effects depends on the level of the second period marginal cost c_2. The magnitude of the first effect depends on the level of c_2 directly, as the increase in profit due to this effect is caused directly by cost savings in the second period. On the other hand, the magnitude of the profit–decreasing effect depends on the level of c_2 only indirectly — through the first period production q_1, given by $q_1^* = \frac{2(1-c_1+\delta c_2)}{4+\delta}$.

The first effect is therefore relatively stronger (in comparison with the second effect), when c_2 is very small. Thus, when c_1 is small, then also c_2 must be small and the direct, profit–increasing, effect prevails, making the innovation profitable. On the other hand, when c_2 is relatively large, then the indirect, profit–decreasing, effect dominates. Therefore, for a large initial cost c_1 and a small innovation, the firm's overall profit may decrease if the innovation is adopted, even if the innovation is costless.

Next, we show that these forces can give rise to cases in which the monopolist fails to adopt the better technology when it is socially desirable to adopt it. Moreover, it turns out that there are also cases where the monopolist innovates too much: As we have seen in Lemma 1, if $c_2 > \frac{c_1}{1+\delta}$ it is not socially efficient to adopt the innovation, because there should be no production in the second period. In contrast, the monopolist sometimes adopts the innovation even when $c_2 > \frac{c_1}{1+\delta}$, as part (ii) in Proposition 1 below shows.

Proposition 1. *(i) Suppose $c_1 > \frac{1}{5}$. Then there exists a $c_1^* < 1$ such that if $c_1 > c_1^*$, the monopolist does not adopt the innovation even though it is socially efficient to adopt it.*

(ii) Suppose $c_1 < \frac{1}{5}$ and $c_2 \in (\frac{1}{5(1+\delta)}, c_1)$. Then the monopolist adopts the innovation even though it is not socially efficient to adopt it.[9]

The first part of the above proposition shows that sometimes the monopolist suppresses a socially desirable innovation that is freely available. This provides a possible rationale for the existence of "sleeping patents". The intuition is similar to the one behind Lemma 2: If the monopoly adopts the innovation, it means that prices will tend to be high in the first period and low in the second period, when the monopolist benefits from learning–by–doing. This induces consumers to postpone their consumption until the second period, which in turn decreases the monopolist's first period profit. The decrease in the first period profit may be so large that it deters the monopolist from adopting the innovation where it would be socially desirable to adopt it. As in Lemma 2, this is likely to happen when the initial marginal cost, c_1, is relatively large.

The second result, saying that sometimes the monopolist adopts an innovation that is not socially desirable seems more surprising, but the intuition is simple. The residual demand in the second period is higher under monopoly than it is in the first best situation, because the monopoly charges a first period price that is higher than the efficient price (which is equal to marginal cost). Therefore, sometimes there is production under monopoly in the second period when there should be no production from the efficiency point of view. But the new technology is valuable only if there is production in the second period. This means that the monopolist can sometimes consider the innovation to be more valuable than a social planner would.

3.2.4 The effects of asymmetric information

A crucial assumption in the reasoning behind the result of Proposition 1 is that the consumers know about the innovation and its price reducing effects, an assumption that we share with Karp and Perloff (1996). In the case of a cost–reducing innovation, this informational requirement may appear rather strong. It may be relatively easy for the monopolist to conceal from consumers the fact that she is adopting the innovation. In fact, in our model the monopolist has an incentive to do exactly that, in order to prevent the consumers from postponing their purchases till the second period. Of course, if the consumers know about the existence of the innovation, they form rational beliefs about whether the monopolist adopts it or not, which in equilibrium have to be correct. It is relatively easy to see that in this asymmetric information case, Lemma 2 would require only a small change in formulation: In part (i), for parameter values such that $c_2 \in (c_2^*, c_1)$, the monopolist now adopts the innovation with some positive probability $\alpha < 1$.

To see this, suppose that the consumers believe that for $c > \frac{1}{5}$ and $c_2 \in (c_2^*, c_1)$, the monopolist never adopts the innovation. Then, following the reasoning preceding Lemma 2, the consumers expect a second period price equal to $E(p_2) = \frac{1-q_1+c_1}{2}$. This implies that the monopolist

[9]The same caveat applies here as in Lemma 1. Again, whenever there is a small cost, ε, associated with the adoption of the new technology, the social planner *strictly* prefers the old technology while the monopoly still prefers the new one if ε is small enough.

will choose the first period price $p_1 = (1-q_1)\left(1+\frac{\delta}{2}\right) + \frac{\delta c_1}{2}$, which yields the expected profit of $q_1\left[(1-q_1)\left(1+\frac{\delta}{2}\right)+\frac{\delta c_1}{2}-c_1\right]+\delta\left(\frac{1-q_1-c_2}{2}\right)^2$. It follows immediately that, for every feasible level of q_1, profits are greater with c_2 (*i.e.,* with the new technology) than c_1. Hence, in this case the monopolist has an incentive to always adopt the innovation, making the consumers beliefs incorrect. On the other hand, if the consumers believe that the monopolist is always going to adopt the innovation, then the original analysis leading to Lemma 2 applies. This means that the monopolist prefers not to adopt the innovation when $c_2 \in (c_2^*, c_1)$, which again makes the consumers beliefs incorrect.

Thus, the only possibility is that the monopolist adopts the innovation with some probability $\alpha \in (0,1)$. Then the consumers' equilibrium expectation regarding the second period price is $E(p_2,\alpha) = \frac{1-q_1+\alpha c_2+(1-\alpha)c_1}{2}$, which leads to the profit for the monopolist of

$$\pi(\alpha, c_2, q_1) = q_1\left[E(p_2,\alpha) - c_1\right] + \delta\left(\frac{1-q_1-c_2}{2}\right)^2$$

if she adopts the innovation, and

$$\pi(\alpha, c_1, q_1) = q_1\left[E(p_2,\alpha) - c_1\right] + \delta\left(\frac{1-q_1-c_1}{2}\right)^2$$

if she does not adopt. Let $q_1^*(\alpha, c_2)$ and $q_1^*(\alpha, c_1)$ be the monopolist's profit maximizing first period outputs in the above two cases respectively, i.e., $q_1^*(\alpha, c_2) = \arg\max_{q_1} \pi(\alpha, c_2, q_1)$ and $q_1^*(\alpha, c_1) = \arg\max_{q_1} \pi(\alpha, c_1, q_1)$. Then the monopolist adopts the innovation with probability α^*, given by $\pi(\alpha^*, c_1, q_1^*(\alpha, c_1)) = \pi(\alpha^*, c_2, q_1^*(\alpha, c_2))$.[10] That is, in equilibrium, the monopolist is willing to randomize in her decision whether to adopt the innovation or not, because this decision does not affect her total profit.

Using the above analysis, we immediately obtain the following proposition, which extends the results of Proposition 1 to the asymmetric information setting.

Proposition 2. *Suppose that consumers cannot observe the monopolist's decision to adopt the innovation.*

(i) Let $c_1 > \frac{1}{5}$. Then there exists a $c_1^* < 1$ such that if $c_1 > c_1^*$, the monopolist does not adopt the innovation with some probability $\alpha^* < 1$, even though it is socially efficient to adopt it with probability 1.

(ii) Let $c_1 < \frac{1}{5}$ and $c_2 \in (\frac{1}{5(1+\delta)}, c_1)$. Then the monopolist adopts the innovation even though it is not socially efficient to adopt it.

Proposition 2 demonstrates that the insights of Proposition 1 continue to hold even if we relax the assumption that the firm's choice of technology is observable.

4 Quality–improving technology

In this section, we revert to our original assumption that the monopolist's choice of technology can be observed by the consumers. Apart from the fact, demonstrated above, that this assumption is not crucial for our qualitative results, we believe it is quite realistic in the case of a product innovation.

[10]The existence of such an α is guaranteed by the continuity of the profit function $\pi(\alpha)$.

When the innovation involves an improvement in the quality of the good or an introduction of an entirely new product, it would probably be impossible for the firm to conceal this innovation: Once the product is brought to the market in the first period, consumers can immediately observe whether it is an old product or a new one.[11] This section focuses on such an innovation.

As mentioned earlier, we formalize a quality–improving innovation by assuming that the new technology makes the good more attractive to consumers by raising their valuation for the good to $\theta_1 = s_1\theta$ in period 1 and to $\theta_2 = s_2\theta$ in period 2, where $s_2 > s_1 \geq 1$. θ is again uniform on $[0,1]$, as before. For simplicity, we let the marginal cost of production in both periods be c, where $c \in [0,1)$.

We will again focus on a situation where the new technology is readily available and can be adopted at no cost. As in the previous section, we start by investigating the socially efficient adoption of the innovation.

4.1 Socially efficient adoption of a quality-improving innovation

Suppose that the innovation is adopted and that q_1 units are produced in period 1 and q_2 in period 2. Then the social welfare $W(s_1, s_2)$ is given by

$$W(s_1,s_2) = \int_{1-q_1}^{1} (s_1\theta - c)d\theta + \delta \int_{1-q_2}^{1} (s_2\theta - c)d\theta + \delta \int_{1-q_1-q_2}^{1-q_2} s_1\theta d\theta.2 \qquad (2)$$

The first term in (2) represents the consumer surplus from producing q_1 units in period 1 and selling them at marginal cost. The second term is the consumer surplus from producing q_2 of higher quality units in period 2 and selling them at marginal cost to high valuation consumers. The last term is the consumer surplus derived in period 2 from the transfer of old, lower quality units to low valuation consumers at zero cost.

Lemma 3. *(i) Suppose* $s_1 = 1$ *and let* $\hat{s} \equiv 1 + \frac{\delta c}{1+\delta}$. *Then it is efficient to adopt the innovation if and only if* $s_2 > \hat{s}$.

(ii) Suppose $s_1 > 1$. *Then the innovation should always be adopted.*

Again, as in the case of a cost–reducing innovation, it is not efficient to adopt a quality–enhancing innovation that only improves quality in the second period, unless it is sufficiently large. The intuition is similar to that behind Lemma 2: If the innovation is relatively small, then in the second period there are no consumers left who would have high enough valuation for the new units to offset the cost of producing them. No production therefore takes place in the second period, which means that the innovation has no value.

4.2 Adoption of the product innovation under monopoly

We now proceed with the analysis of the monopolist's decision to adopt a new, quality enhancing technology. Suppose that the innovation was adopted by the monopolist. As before, we start with production and profit in period two.

4.2.1 The second period price and profit

Assume there are q_1 low quality and q_2 high quality units offered for sale in the second period and let p_2^L and p_2^H be their respective prices in this period. A consumer with valuation θ buys the high

[11]Perhaps with the exception of experience goods.

quality unit in this period if and only if $s_2\theta - p_2^H \geq s_1\theta - p_2^L$, whether or not he bought the good in period 1. If the reverse of the above inequality is true, then the consumer buys the low quality unit in the secondhand market, as long as $s_1\theta - p_2^L \geq 0$.

Let $\overline{\theta}$ be the valuation of a consumer who is indifferent in period 2 between buying the old unit and buying a new one. Then $\overline{\theta}$ is defined by $s_2\overline{\theta} - p_2^H = s_1\overline{\theta} - p_2^L$; that is, $\overline{\theta} = \frac{p_2^H - p_2^L}{s_2 - s_1}$. Since the second period output is given by $q_2 = 1 - \overline{\theta}$, we have $p_2^H = p_2^L + (1-q_2)(s_2 - s_1)$. Similarly, define $\underline{\theta}$ by $s_1\underline{\theta} - p_2^L = 0$, so that $\underline{\theta}$ is the valuation of a consumer who is indifferent between buying a used unit in period 2 and not buying at all. Then $\underline{\theta}$ is also the measure of consumers with valuations below $\underline{\theta}$, who will not purchase the good in either period. Therefore, it must be that the total output over the two periods, $q_1 + q_2$, is determined by $1 - q_1 - q_2 = \underline{\theta}$. The respective second period prices of low and high quality goods are then $p_2^L = s_1(1 - q_1 - q_2)$ and $p_2^H = s_1(1 - q_1 - q_2) + (1 - q_2)(s_2 - s_1)$.

The monopolist chooses the second period output so as to maximize this period's profit, according to

$$\max_{q_2} \pi_2 = q_2 \left[s_1 (1 - q_1 - q_2) + (1 - q_2)(s_2 - s_1) - c \right].$$

This yields $q_2^{**}(q_1) = \frac{s_2 - s_1 q_1 - c}{2s_2}$, and a second period profit of

$$\pi_2^{**}(q_1) = \frac{1}{s_2} \left(\frac{s_2 - s_1 q_1 - c}{2} \right)^2.$$

4.2.2 The first period price and total profit

Let p_1 denote the first period price. A consumer with valuation θ buys in this period if and only if $s_1\theta - p_1 + \delta E(p_2^L) \geq 0$, where $E(p_2^L)$ is the price of old units that the consumer expects to prevail in the second-hand market in period 2. Let $\tilde{\theta}$ be the valuation of a consumer who is indifferent between buying and not buying in period 1, which means that $\tilde{\theta}$ is given by $s_1\tilde{\theta} - p_1 + \delta E(p_2^L) = 0$. This yields the first period quantity $q_1 = 1 - \tilde{\theta}$, so that $p_1 = s_1(1 - q_1) + \delta E(p_2^L)$. Finally, imposing the equilibrium condition $E(p_2^L) = p_2^L$, the first period price is

$$p_1 = s_1(1 - q_1) + \delta s_1[1 - q_1 - q_2^{**}(q_1)] = s_1 \left[(1 - q_1)(1 + \delta) - \delta \left(\frac{s_2 - q_1 - c}{2s_2} \right) \right].$$

The monopolist's first period problem is then to choose q_1 so as to maximize the total profit over the two periods:

$$\pi(s_1, s_2) = \max_{q_1} q_1 s_1 \left[(1 + \delta)(1 - q_1) - \delta \left(\frac{s_2 - s_1 q_1 - c}{2s_2} \right) - \frac{c}{s_1} \right] + \frac{\delta}{s_2} \left(\frac{s_2 - s_1 q_1 - c}{2} \right)^2 .3 \quad (3)$$

The optimal first period output is therefore given by $q_1^{**} = \frac{2[s_1 s_2 - c(s_2 - \delta s_1)]}{s_1[4s_2(1+\delta) - 3\delta s_1)]}$.

4.2.3 Adoption of the quality improving innovation

Using the expressions obtained above, we get the following result.

Lemma 4. *Suppose $s_1 = 1$. There exists a $\bar{c} \in (0, \frac{1}{3})$ such that:*

(i) If $c \leq \bar{c}$, then the monopolist always adopts a quality–improving innovation.

(ii) If $c > \bar{c}$, then there exist $s_2^{**} > 1$ and $\delta^{**} \in (0, 1)$ such that if $s_2 \in (1, s_2^{**})$ and $\delta \in (\delta^{**}, 1)$ the monopolist does not adopt the innovation.

To see the intuition underlying Lemma 4, consider the effect on the firm's profit of an innovation that improves only the second period quality. This innovation has two opposing effects: First, profit is lowered due to the delay of purchase by some consumers hoping to acquire an old unit at the secondhand market in period two. On the other hand, a quality–enhancement in the second period allows the monopolist to charge higher prices for new units of the good.

The magnitude of both of these effects increases as the marginal cost of production, c, becomes smaller. A smaller c means that both, the first period price and output, as well as the second period price and output are relatively large. A decrease in the first period price, caused by the innovation, therefore decreases the first period profit more when c is small, as it affects larger output. Similarly, the increase in the second period price affects larger output if c is small and causes a greater increase in the second period profit. However, since the second period effect is more direct than the first period effect, it is also affected more by the magnitude of c. Hence, the second period, profit-increasing, effect tends to outweigh the first period, profit-decreasing effect when c is small — the monopolist adopts the innovation. The reverse is true when c is relatively large and consumers are patient enough (δ is large): the loss of revenue due to the delay in consumption more than offsets the benefit from selling at a higher price in the second period. In this case, the monopolist is better off not adopting the new technology.

On the other hand, if consumers are impatient (δ is small), then the high–demand types delay consumption until period 2, when they buy new units at a higher price. In such a case, the monopolist always finds it worthwhile to innovate. Hence, as in the case of the cost–reducing technology, we can show that not only does the monopolist fail to innovate when it is socially optimal to do so, but sometimes she innovates too much.

Proposition 3. *(i) There exist* $c^+ > 0$, $\delta^+ < 1$, $s_1^+ > 1$ *and* $s_2^+ > 1$ *such that if* $c \geq c^+$, $\delta \in (\delta^+, 1)$, $s_1 \in (1, s_1^+)$ *and* $s_2 \in (1, s_2^+)$, *the monopolist does not adopt the innovation even though it is socially efficient to adopt it.*

(ii) Suppose $s_1 = 1$. *There exist* $c^- \in (0, 1)$ *and* $s_2^-(c) > 1$ *such that if* $c \in (0, c^-)$ *and* $s_2 \in (1, s_2^-(c))$ *then the monopolist adopts the innovation even though it is not socially efficient to adopt it.*

Proposition 3 is a counterpart of Proposition 1 in the setting of a quality-improving innovation. It thus serves to demonstrate that the paper's main results are robust to the type of innovation considered.

The intuition here is analogous to that behind Proposition 1. Again, part (i) is possible because in the presence of learning by doing the innovation amplifies the monopolist's time inconsistency problem. The time inconsistency problem tends to decrease the monopolist's overall profit and therefore works against the adoption of the innovation. At this point, it is useful to contrast our finding with the result obtained by Waldman (1996a). In Waldman's model, the time inconsistency problem is manifested in *excessive* innovation by the monopolist (compared to her optimal *ex ante* level of innovation). This difference in conclusions is mainly driven by different timings in the two models. In Waldman's model, the monopolist decides how much to innovate only after selling the first period's production. She therefore does not internalize the effects of her decision on the first period profit. In our model, the monopolist makes her decision at the beginning of the first period, and therefore cares about the effect of this decision on her first period profit.

The second part of Proposition 3 can hold because the monopolist charges a higher first period price than is socially optimal. She therefore faces a residual demand in the second period that is larger than would be the residual demand in the first best situation. Because, the benefits of the innovation are reaped in the second period, this larger second period demand makes the innovation relatively more valuable to the monopolist than to a social planner.

4.3 Investing in the development of the new technology

The analysis in the preceding sections starts at the point where the new technology is already developed and available to the monopolist for adoption at no additional cost. We have seen that sometimes, the monopolist chooses to inefficiently shelve such a technology and continue to use the old one. This naturally raises the following question: Why would the monopolist ever invest in the development of the innovation if she is going to shelve it once it is developed? It is easy to extend our model to answer this question and incorporate costly development of innovation while preserving the result that the innovation will be sometimes shelved once it is developed.

Suppose that at time $t = 0$, the monopolist can invest a fixed amount $I > 0$ in R&D which will generate a technology improvement by the beginning of time $t = 1$. To be specific, consider the case of a cost–reducing innovation. Assume also that I is not too high, so that for innovations that decrease costs to $c_2 \leq 1/5$ it is optimal for the monopolist to invest I. Finally, suppose that the R&D process is uncertain, and that the resulting reduced cost, c_2, is distributed according to a cumulative distribution function $G(\cdot)$, with the pdf $g(\cdot)$ strictly positive everywhere on its support $[0, c_1]$.

In this setting, the monopolist will find it profitable to invest I and undertake the R&D if the probability that $c_2 \leq 1/5$ is large enough, *i.e.,* if $G(\frac{1}{5})$ is close enough to 1. However, even in this case it will happen with positive probability that the realization of the cost reduction is too small, *i.e.,* $c_2 \in (c_2^*, c_1)$, where c_2^* is as in Lemma 2. In such a case, Lemma 2 implies that the monopolist optimally shelves the resulting innovation, even though she has previously invested to obtain it.

5 Related literature

Our central finding, that a durable good monopolist may inefficiently ignore an existing, freely available innovation, is closely related to Karp and Perloff (1996). They show that a monopolist may buy the rights to a superior production technology but suppress it and continue to use an inferior technology. In addition to this main result, our paper shares with Karp and Perloff (as well as with some other papers in this strand of literature) some common modeling features. In particular, we too assume that the monopolist cannot commit to future prices to mitigate the Coase problem. Similarly, she cannot use any other commitment device such as leasing, planned obsolescence, capacity constraint, and so on.

However, the economics behind the present model differs from that behind the model developed by Karp and Perloff. First, in Karp and Perloff's (1996) analysis it is important that the monopoly has an increasing marginal cost of production. This allows them to build on a previous result by Kahn (1986), according to which an increasing marginal cost curve has a commitment value to the monopolist because it forces her to decrease production in every period. Karp and Perloff use this effect to demonstrate that the monopolist will sometimes keep using an old technology with a steeper marginal cost curve, rather than adopting a new one which has a lower marginal cost, but for which the marginal cost curve is less steep. In contrast, the key feature of our model is the presence of learning-by-doing, which, over time, causes a *decrease* in the firm's marginal cost.

Second, all the interesting results in Karp and Perloff are derived for parameter values where the marginal cost under the old technology is lower than the marginal cost under the new technology for some production levels. In our model, the cost of production under the new technology is never higher than the cost of production under the old one. Thus, in the case of the cost decreasing innovation in our model, the monopolist sometimes sticks with an old technology that always has at least as high a marginal cost as the new one. Finally, Karp and Perloff only consider cost decreasing technological innovations, while our results hold also for the case of improvements in product quality.

Our analysis of the monopolist's decision to adopt a quality-improving technology is related to the extensive literature on technological innovation and quality–improving R&D. We relate our results to two models that we consider to be most closely related to our analysis.

The first of these papers is due to Waldman (1996a). Waldman investigates a durable good monopolist's incentive to invest in R&D that will improve the quality of the good. He shows that the monopolist faces a time inconsistency problem in her R&D investment decisions: she invests too much compared to her optimal level of investment, to which she would commit if she could.[12] This is analogous to the classical time inconsistency problem identified by Coase, whereby a durable good monopolist produces too much in the second period. As Waldman shows, this high investment level improves upon welfare, which is again analogous to the effect of the high second–period production in the classical Coase problem.

The second paper, due to Lee and Lee (1998), also builds a model in which a durable good monopolist invests into a quality-improving technological innovation, which makes old units economically obsolete. Lee and Lee concentrate on a setting were there is no second hand market for old units. This makes the valuation of new units dependent on consumers' purchase history — those who bought the good in the first period can use it also in the second period and therefore do not value the new units as much as those consumers who did not purchase in the first period. The focus of their model is on the possibility of price discrimination by the monopolist, based on the consumers' purchase history: A consumer who purchased in the first period is charged a lower price for the higher quality good in the second period, as long as he shows a proof of first period purchase, as in a product-upgrade policy.

Apart from the fact that neither of the above two papers considers learning-by-doing, the main difference between our analysis of the quality-improving innovation and the models developed by Waldman (1996b) and Lee and Lee (1998) is in the questions these papers are addressing. Unlike the other two papers, our main focus is on the possibility of "sleeping patents" — the incentive of the monopolist to suppress an existing superior technology.

6 Conclusion

The issue of a durable–good monopolist's decisions to undertake product or technological innovation has been widely discussed in the literature. In this paper, we approach this issue in a slightly different setting and ask a slightly different question than most other papers. First, we introduce learning by doing, which we believe adds more realism into the analysis. Second, in our main analysis, we do *not* consider innovations that require investments (although we show that the innovation process can be easily incorporated in our model). Rather, we concentrate on existing innovations that are available at no (or very small) cost and ask how a monopolist's incentives to adopt such innovations compare to the social optimum.

There are two effects that influence the monopolist's incentives to adopt an innovation in the presence of learning by doing, and these work in opposite directions. First, the adoption of the

[12] Waldman (1993) and Choi (1994) reach a somewhat similar conclusion, but in a different context. In a model characterized by positive network externalities, they show that from the standpoint of both the monopolist's own profitability and social welfare, the monopolist's incentive to introduce new goods that are incompatible with old units is too high.

innovation makes the monopolist's time inconsistency problem more severe, which tends to discourage innovation. Our first result is that this effect can lead to situations where the monopolist suppresses a better product or a dominant technology in a socially inefficient way.

Second, owing to the presence of learning by doing, the gains from the innovation are realized mainly in the second period. Because the monopolist's pricing leads to a higher second period residual demand than is socially optimal, the monopolist may view the innovation as more desirable than a social planner would. This sometimes makes the monopolist adopt an innovation that optimally should not be adopted, which is our second main result. These two findings are robust to the type of innovation considered: they hold both in the case of a cost-reducing innovation, as well as in the case of a quality-improving innovation.

There are many ways in which our analysis could be extended. Probably the most interesting would be to investigate the effect of learning by doing on investment incentives in oligopolies. We may return to this question in future research.

A Appendix

Proof of Lemma 1. (i) We have already proved the first two statements in this part. The last statement follows immediately from the fact that the innovation is valuable only if there is production in the second period.
(ii) Again, the first part of this statement has already been proved. To obtain the second claim, suppose the second period cost c_2 can be chosen so as to maximize social welfare, W:

$$\max_{c_2} W(c_2) = \int_{\theta^*}^{1}((1+\delta)\theta - c_1)d\theta + \int_{c_2}^{\theta^*} \delta(\theta - c_2)d\theta.$$

The first integral is the welfare generated in the first period and the second integral is the welfare generated in the second period. Straightforward differentiation yields

$$\frac{\partial W(c_2)}{\partial c_2} = -\delta\left(c_1 - (1+\delta)c_2\right) < 0$$

if $c_2 < \frac{c_1}{1+\delta}$. Moreover, if $c_2 = \frac{c_1}{1+\delta}$, we have $\theta^* = c_2$ so that

$$W(c_2) = W(c_1) = \int_{\theta^*}^{1}((1+\delta)\theta - c_1)d\theta.$$

These two results together imply $W(c_2) > W(c_1)$ for all $c_2 < \frac{c_1}{1+\delta}$, which concludes the proof. Q.E.D.

Proof of Lemma 2. (i) Fix c_1 and look at $\pi(c_1, c_2)$ as a function of c_2. Using the Envelope Theorem we get $\frac{\partial \pi(c_1,c_2)}{\partial c_2} > 0$ if and only if $q_1^* > \frac{1-c_2}{2}$, which holds if and only if $c_2 > \frac{4c_1+\delta}{4+5\delta} \equiv \hat{c}_2$. Thus, $\pi(c_1, c_2)$ is a convex function of c_2, minimized at $\hat{c}_2$. Hence, whenever $\hat{c}_2 < c_1$, there exists a c_2^*, $c_1 > c_2^* \geq 0$, such that $\pi(c_1, c_2) < \pi(c_1, c_1)$ if and only if $c_2 \in (c_2^*, c_1)$, while $\pi(c_1, c_2) > \pi(c_1, c_1)$ if $c_2 < c_2^*$. The proof of this part is finished by noting that $\hat{c}_2 < c_1$ if and only if $c_1 > \frac{1}{5}$.
(ii) If $c_1 < \frac{1}{5}$, then $\frac{4c_1+\delta}{4+5\delta} > \frac{1}{5}$. In that case, $\frac{\partial \pi(c_1,c_2)}{\partial c_2} < 0$ for all $c_2 < c_1 \leq \frac{1}{5}$. Hence, the monopolist adopts the cost–reducing technology. Q.E.D.

Proof of Proposition 1. (i) If $c_1 > \frac{1}{5}$, then from Lemma 2 we know that the monopolist does not adopt the innovation if $c_2 \in (c_2^*, c_1)$. Also, if $c_2 < \frac{c_1}{1+\delta}$ then, by part (ii) in Lemma 1, it is efficient to adopt the innovation. It therefore suffices to show that there exists a $c_2' < \frac{c_1}{1+\delta}$ such that $c_2' > c_2^*$. Notice that c_2^* in Lemma 2 was defined by $\pi(c_1, c_2^*) = \pi(c_1, c_1)$. Because the monopolist's profit function is quadratic in c_2 and minimized at $\hat{c}_2 = \frac{4c_1+\delta}{4+5\delta}$ (as derived in the proof of Lemma 1), it is symmetrical in c_2 around $\hat{c}_2$. Therefore, we have $c_2^* = c_1 - 2(c_1 - \hat{c}_2) = 2\hat{c}_2 - c_1$. Since $c_1 < \frac{c_1}{1+\delta}$, it is sufficient to prove that $c_2^* = 2\hat{c}_2 - c_1 < \frac{c_1}{1+\delta}$, or $\hat{c}_2 < \frac{2+\delta}{2(1+\delta)}c_1$. This holds if and only if $c_1 > \frac{2(1+\delta)}{6+5\delta} \equiv c_1^*$. The proof is finished by noting that $c_1^* < 1$ for all $\delta \in (0, 1]$.
(ii) This follows immediately from combining parts (i) in Lemma 1 and Lemma 2. These two parts hold simultaneously when $c_1 < \frac{1}{5}$ and $c_2 > \frac{1}{5(1+\delta)} > \frac{c_1}{1+\delta}$. Q.E.D.

Proof of Lemma 3. (i) Let $s_1 = 1$. Differentiate $W(s_2)$ with respect to s_2 and use the Envelope Theorem to get

$$\frac{\partial W(s_2)}{\partial s_2} = \delta \int_{1-q_2^e}^{1} \theta d\theta.$$

Thus, $\frac{\partial W(s_2)}{\partial s_2} > 0$ if and only if $q_2^e > 0$, where q_2^e stands for the efficient second period output. The first order conditions for q_1 and q_2 yield $q_2^e|_{s_1=1} = \frac{(s_2-1)(1+\delta)-\delta c}{s_2(1+\delta)-\delta}$, which is greater than zero if and only if $s_2 > 1 + \frac{\delta c}{1+\delta} \equiv \hat{s}$. Since the welfare with the innovation is equal to the welfare without the innovation when $s_2 = \hat{s}$ (because $q_2^e = 0$), the above analysis means that the innovation improves welfare if and only if $s_2 > \hat{s}$. Finally, it can easily be checked that the efficient level of output in the first period under the new technology satisfies our constraint that at least the amount $q_{\min}$ be produced in this period: $q_1^e|_{s_1=1} = \frac{s_2(1+\delta c)}{s_2(1+\delta)-\delta} - c > q_{\min}$.

(ii) Differentiate $W(s_1, s_2)$ with respect to s_1 to get

$$\frac{\partial W(s_1, s_2)}{\partial s_1} = \int_{1-q_1^e}^{1} \theta d\theta + \delta \int_{1-q_1^e-q_2^e}^{1-q_2^e} \theta d\theta > 0,$$

where the inequality follows because $q_1^e = \frac{s_2(s_1-c)+\delta c s_1}{s_1[s_2(1+\delta)-\delta s_1]} > 0$. Since $\frac{\partial W(s_1,s_2)}{\partial s_2} \geq 0$, the innovation strictly increases welfare whenever it improves the first period quality. Finally, the efficient choice of output in the first period under the new technology satisfies the constraint $q_1^e > q_{\min}$, because $q_1^e > q_1^e|_{s_1=1}$. The last inequality can be checked by verifying that $\frac{\partial q_1^e}{\partial s_1} > 0$. Q.E.D.

Proof of Lemma 4. (i) Let $s_1 = 1$. Using the Envelope Theorem, differentiate the expression for profit in (3) with respect to s_2 to get $\frac{\partial \pi(s_2)}{\partial s_2} < 0$ if and only if

$$s_2^2 - (q_1 + c)(3q_1 + c) < 0.4 \tag{4}$$

It can easily be checked that q_1^{**} decreases in s_2, so that the left hand side of (4) increases with s_2. Hence, if (4) does not hold for $s_2 = 1$, it does not hold for any $s_2 > 1$. It is therefore enough to concentrate on the validity of (4) when $s_2 = 1$.

Let $s_2 = 1$ and solve for the x_1 and x_2 such that (4) holds with equality when $q_1 \in \{x_1, x_2\}$. It can be verified that $\min\{x_1, x_2\} < 0$. Thus, (4) holds for a positive quantity if and only if $\frac{2(1-c(1-\delta))}{4+\delta} = q_1^{**}|_{s_1=1,s_2=1} > \hat{x} \equiv \max\{x_1, x_2\} = \frac{\sqrt{3+c^2}-2c}{3}$. Rearranging, we get that (4) holds if and only if

$$6 + 2c + 8c\delta - (4+\delta)\sqrt{3+c^2} > 0.5 \tag{5}$$

Differentiate the left hand side of (5) with respect to δ to get $\frac{\partial LHS\ (5)}{\partial \delta} > 0$ if and only if $c > \frac{1}{\sqrt{21}}$. We thus get two cases:

Case 1: $c \leq \frac{1}{\sqrt{21}}$. In this case the LHS of (5) decreases in δ, which means that if (5) does not hold for $\delta = 0$, it never holds. Plugging $\delta = 0$ into (5) yields LHS (5) $= -(1-c)^2 < 0$, so that (5) never holds in this case.

Case 2: $c > \frac{1}{\sqrt{21}}$. Here, the LHS of (5) increases in δ, which means that if (5) does not hold for $\delta = 1$, it never holds. Using $\delta = 1$, we can see that (5) holds if and only if $75c^2 + 120c - 39 > 0$. Because this inequality holds when $c = 1/3$ and does not hold when $c = 0$, there exists a $\bar{c} \in (0, \frac{1}{3})$ such that (4) holds if and only if $c > \bar{c}$. It can be checked that $\bar{c} > \frac{1}{\sqrt{21}}$. Hence, whenever $c \leq \bar{c}$, we have $\frac{\partial \pi(s_2)}{\partial s_2} > 0$ for all $s_2 \geq 1$, which means that the monopoly always adopts a quality–improving innovation. This proves part (i).

(ii) For part (ii), note that if (4) holds for some s_2 when $\delta = 1$, then by continuity there exists a $\delta^{**} < 1$ such that (4) also holds for all $\delta \in (\delta^{**}, 1)$. Similarly, continuity implies that if (4) holds for $s_2 = 1$, then there exists an $s_2^{**} > 1$ such that (4) holds for all $s_2 \in (1, s_2^{**})$. But we have already proved above that $\left.\frac{\partial \pi(s_2)}{\partial s_2}\right|_{s_2=1,\delta=1} < 0$ whenever $c > \bar{c}$. Hence, there exist $\delta^{**} \in (0,1)$ and $s_2^{**} > 1$ such that $\frac{\partial \pi(s_2)}{\partial s_2} < 0$ whenever $\delta \in (\delta^{**}, 1)$ and $s_2 \in (1, s_2^{**})$. This implies that for $\delta \in (\delta^{**}, 1)$ and $s_2 \in (1, s_2^{**})$ the profit with innovation is lower than the profit without innovation, which concludes the proof of this part. Q.E.D.

Proof of Proposition 3. (i) Suppose $s_1 = 1$, and let $c \in [\bar{c}, 1]$, $\delta \in (\delta^{**}, 1)$, and $s_2 \in (1, s_2^{**})$, where $\bar{c}$, s_2^{**}, and δ^{**} are as in Lemma 4. Then the monopolist does not adopt the innovation. By continuity, the argument in the proof of Lemma 4, part (ii), extends also to s_1 slightly greater than 1. Hence, there exist $c^+ > 0$, $\delta^+ < 1$, $s_1^+ > 1$ and $s_2^+ > 1$ such that if $c \geq c^+$, $\delta \in (\delta^+, 1)$, $s_1 \in (1, s_1^+)$ and $s_2 \in (1, s_2^+)$, then the monopolist does not adopt the innovation. On the other hand, Lemma 3 implies that it is socially efficient to adopt the innovation, because $s_1 > 1$.
(ii) Let $s_1 = 1$ and $c^- = \bar{c}$, where $\bar{c}$ is as in Lemma 4. Then if $c < c^-$, the monopolist always adopts the innovation (by part (i) in Lemma 4), but it is inefficient to do so if $s_2 < 1 + \frac{c\delta}{1+\delta} \equiv \hat{s}_2(c)$ (by part (i) in Lemma 3). Q.E.D.

References

[1] Basu, Kaushik. (1988), "Why Monopolists Prefer to Make their Goods Less Duarble," *Economica* 55, pp. 541-6.

[2] Bulow, J. (1982), "Durable–Goods Monopolists," *Journal of Political Economy,* 90, 314–332.

[3] ——— (1986), "An Economic Theory of Planned Obsolescence," *Quarterly Journal of Economics,* 101(4), 729–750.

[4] Chi, W. C. (1999), "Quality Choice and the Coase Problem," *Economics Letters,* 64, 107–115.

[5] Choi, J. P. (1994), "Network Externality, Compatibility Choice and Planned Obsolescence," *Journal of Industrial Economics,* 42, 167–182.

[6] Coase, R. H. (1972), "Durability and Monopoly," *Journal of Law and Economics,* 15, 143–149.

[7] Gilbert, R. J. (1981), "Patents, Sleeping Patents, and Entry Deterrence," in Steven C. Salop (ed), *Strategy, Predation and Antitrust Analysis,* Washington, D.C: Federal Trade Commission, 205–269

[8] ———, and Newbery, D. M. G. (1982), "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 72 (3), 514–526

[9] Gul, F., Sonnenschein, H, and Wilson, R. (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory,* 39, 155–190.

[10] Kahn, C. M. (1986), "The Durable Good Monopolists and Consistency with Increasing Costs", *Econometrica* 54, 275–294.

[11] Karp, L. S. and Perloff J. M. (1996), "The Optimal Suppression of a Low–Cost Technology by a durable–good Monopoly," *Rand Journal of Economics,* 27(2), 346–364.

[12] Lee, I. H. and Lee J. (1998), "A Theory of Economic Obsolescence", *Journal of Industrial Economics*, 46(3), 383–401.

[13] Levinthal, D. A., and Purohit, D. (1989), "Durable Goods and Product Obsolescence," *Marketing Science,* 8, 35–56.

[14] Sobel, J. (1991), "Durable Goods Monopoly with Entry of New Consumers", *Econometrica*, 59 (5), 1455–85.

[15] Stokey, N.L. (1981), "Rational Expectations and Durable Good Pricing," *Bell Journal of Economics* 12, 112-28.

[16] Waldman, M. (1996a), "Planned Obsolescence and the R&D Decision," *Rand Journal of Economics,* 27 (3), 583–595.

[17] —— (1996b), "Durable Goods Pricing when Quality Matters," *Journal of Business,* 69 (4), 489–510.

[18] —— (1993), "A New Perspective on Planned Obsolescence," *Quarterly Journal of Economics,* 108, 273–283.

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