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Product Differentiation, Cost-Reducing Mergers, and Consumer Welfare

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ABSTRACT

Cost synergies are an explicitly recognized justification for a two-firm merger and empirical techniques are now widely used to assess the impact of cost-reducing mergers on prices and welfare in the post-merger market. We show that if the merger occurs in a vertically product differentiated market then the merger will lead to a reduction in product offerings that limits the usefulness of pre-merger empirical estimates. Indeed, we further show that in such markets, two-firm merges will lead to higher prices regardless of the merger's cost-savings. We show that our results may obtain even when we allow for post-merger entry.

Keywords: mergers, cost synergies, vertical product differentiation

JEL Classifications: L10, L41

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1. Introduction

This paper is concerned with the effects of horizontal mergers on prices and consumer welfare. The last sixteen years has witnessed an impressive growth in the use of empirical techniques to evaluate such effects. The typical approach has been to build a model of the post-merger market by using estimates of the pre-merger demand elasticities and cost parameters. Simulations of the empirical model then yield best guesses of the likely post-merger outcomes.[1] Such techniques can be extremely useful for evaluating the impact of any cost efficiencies that the merger is alleged to generate. This is important, particularly in light of the increased emphasis on cost efficiencies in the 1997 horizontal merger guidelines jointly published by the U.S. Federal Trade Commission (FTC) and the Department of Justice (DOJ). The importance of evaluating the impact of cost efficiencies takes on even greater significance given the substantial weight placed on such savings by Timothy Muris, current Chairman of the FTC, who has publicly argued that mergers that both raise price and reduce costs are rare, even when entry is difficult.[2]

Yet while we think that empirical modeling and simulation will continue to play a vital and necessary role in merger analysis, we also believe that theory plays an equally important role. This is particularly true for cases in which a merger of two firms could result in a change in the mix of products offered in the post-merger market. In this case the relevance of the pre-merger estimates of own- and cross-price demand elasticities becomes a questionable guide to assessing the post-merger market outcome. Eliminating product lines may be an unlikely scenario in

[1] See, for example, Baker and Bresnahan (1985), Werden and Froeb (1994), Shapiro (1996). Hausman and Leonard (1997), and Nevo (2000).

markets whose products are best thought of as horizontally differentiated. In horizontally differentiated product markets consumers do not agree on what is the most preferred good and so each consumer has her own preferred product specification. Firms that merge in this setting have an incentive to maintain their pre-merger product lines as a means to reach as many consumers as possible. However, firm incentives are quite different when products are vertically rather horizontally differentiated. In vertically differentiated product markets, consumers do agree on the ranking of the different brands of products, but they have different willingness to pay for the goods. Thus the incentive to maintain the pre-merger product offering is different in the post merger market.

In this paper we use a vertically differentiated product market to explore the issue of post-merger product selection in the welfare analysis of a horizontal mergers. Our analysis demonstrates that a little bit of theory can sometimes go a long way in illuminating the likely impact of a merger on consumer welfare. Specifically, we show that in the absence of any new entry a merger in a vertically differentiated market can lead to a reduction in the number of products available in the market and a rise in consumer prices *no matter how large* are the associated cost efficiencies. Even when we allow for the possibility of entry into the post merger market our results continue to hold for a range of cost efficiencies.

The remainder of the paper is structured as follows. Our basic model is presented in section 2. In sections 3 and 4 we analyze the impact of a two-firm merger on prices and profits under two scenarios with respect to the pre-merger costs of the merging firms. Section 5 summarizes our main conclusions.

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[2] See Muris, Statement before Federal Trade Commission Hearings on Global and Innovation-Based Competition, (November, 1995). See also Muris (1999).

2. The Model

We adopt the vertically differentiated model first developed by Mussa and Rosen (1978) for a product market in which there are initially 3 products.[3] Consumers are assumed to value some characteristic of each product i, measured by the variable z_i, where $z_i \in [\underline{z}, \bar{z}]$ with $\underline{z} > 0$, i=1,2,3. Consumers know the characteristic content z_i of each of the products and they have identical rankings of the products. In particular, consumers all agree that more of the characteristic z is better. However, consumer willingness to pay for this characteristic differs across the population of consumers. The characteristic z_i is like an index of quality agreed to by all consumers. More specifically, z_i could be interpreted as the strength of the brand image of good i, as in Pepall and Richards (2002). Consumers agree that products with a stronger brand image, as measured by the index z, are more highly valued, but consumers differ in terms of their willingness to pay for a stronger brand.[4]

In the pre-merger market there are three incumbent firms, where firm i offers a product i with characteristic z_i at price p_i. We define the parameter θ to be a consumer's marginal willingness to pay for the characteristic and assume that θ is uniformly distributed at unit density over the interval $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} > 0$. We define the indirect utility consumer θ obtains from buying one unit of product i with characteristic content z_i at price p_i to be:

$$V_i(\theta) = \max\{0, V + \theta z_i - p_i\} \qquad i = 1,2,3 \qquad (1)$$

Each consumer buys exactly one unit of the product that offers the highest utility provided, of course, that this utility is non-negative. We assume that V is sufficiently high that the market is fully covered in any price equilibrium.

[3] See also Tirole (1988), Gabszewicz and Thisse (1979, 1986) and Anderson, de Palma and Thisse (1992) and Shaked and Sutton (1982).

We also assume the following:

(A.1) $\underline{z} = \bar{z} - 1; \underline{\theta} = \bar{\theta} - 1,$

(A.2) $z_1 = \bar{z}; z_2 = \bar{z} - \Delta z; z_3 = \underline{z}$

In other words, firm 1 markets the most highly valued product, firm 3 the least valued, and firm 2 offers an intermediate brand of product. The term Δz is the difference in brand strength between the top and middle brand. Note that (A.1) implies that the extent of product differentiation between brands 1 and 2, as measured by Δz, is such that $0 < \Delta z < 1$.[5] In addition, $(1 - \Delta z) = z_2 - \underline{z}$ is a measure of the difference in brand strength between the middle brand and low valued brand.

Under assumptions (A.1), (A.2) our model of consumer behavior implies that, for prices p_1, p_2 and p_3 such that each good has a positive market share, the market demands are given by:

$$D_1(p_1, p_2) = \bar{\theta} - \frac{p_1 - p_2}{\Delta z} \tag{2a}$$

$$D_2(p_1, p_2, p_3) = \frac{p_1(1 - \Delta z) - p_2 + p_3 \Delta z}{\Delta z(1 - \Delta z)} \tag{2b}$$

$$D_3(p_2, p_3) = \frac{p_2 - p_3}{1 - \Delta z} - \bar{\theta} + 1 \tag{2c}$$

When the incumbent firms entered the market they each incurred a sunk cost to create the brand identity of their products. This sunk entry cost includes the product development and design costs. When z_i is interpreted as a measure of brand strength, these sunk costs also include the promotional costs of marketing the brand. For now, we assume that once established, firms

[4] This brand interpretation is particularly appealing if we think of the consumers as retailers buying from manufacturers. Some retailers serve consumer markets where brand image plays a stronger role than for other retailers.

[5] It should be emphasized that (A.2) is not restrictive. Suppose that in establishing this market the three firms were initially involved in a two-stage quality-price game, with qualities being chosen sequentially in the order z_1, z_3, z_2 and prices being chosen simultaneously after qualities are observed. Then the firms would choose $z_1 = \bar{z}$, $z_2 = \bar{z}$ - 1/2, $z_3 = \bar{z}$ - 1. So allowing a range on Δz actually increases rather than decreases the generality of our analysis.

cannot easily change their choice of z_i. The brand strength interpretation of z_i is particularly appealing in this regard. Business consultants increasingly view brand names as strategic assets that are key to long-term performance. These analysts further argue that changing a well-established brand identity can be both expensive and potentially damaging to the firm, (e.g., Aaker, 1996).

For each firm the marginal cost of producing the good is assumed to be constant and independent of the characteristic z_i. This is not unreasonable if the product characteristic z_i is related to product design, image or brand strength. These are characteristics whose costs are typically sunk and not variable costs. We do assume, however, that there are cost asymmetries among the firms in the pre merger market. Specifically, there are potentially two kinds of producers, high cost producers with unit cost $c_H = c > 0$, and low cost producers with unit cost $c_L = 0$. A firm might be a low cost producer because it is a multi-product firm and as such benefits from economies of scope in the production of this good. Alternatively, a firm might be a high cost producer because it has a relatively small market share and therefore its best practice technology has a higher unit cost of production.

We are interested in proposed mergers that have cost synergies, i.e., those in which one or more high cost producers have lower costs as a result of the merger. Section 4 of the FTC/DOJ Merger Guidelines makes explicit reference to such mergers as the type that could potentially enhance competition in the market. However, it is well known that in the vertically differentiated framework that we employ, the firm with the highest quality product tends to enjoy a distinct advantage and a dominant market share. We therefore focus on cost-reducing mergers between the firms with the smallest market shares in the market because it is efficiency gains among these two firms that are most likely to permit them to challenge the top brand firm's dominant position.

We distinguish two important cases that seem most relevant to merger policy. In the first case, the firm that produces the intermediate brand has a cost disadvantage relative to the two other firms. This case is, in fact, motivated by a real world example. In the summer of 2000, the Heinz Corporation proposed to acquire the Milnot Corporation, a firm whose major product was the well-known Beech Nut brand of baby food. Heinz was also a major player in the U.S. jarred baby food industry. Beech Nut and Heinz each had about a 15 to 17 percent share of the market. However, the remaining market share was virtually all controlled by the industry's dominant firm, Gerber. Gerber was also recognized as the industry's premium line—the strongest brand identity and the baby food consumers routinely identified as high quality. By contrast, Heinz was widely known as the discount brand or low quality product. Beech Nut was an old, established line positioned somewhat between Gerber and Heinz. Partly as a result of its age, however, it was widely agreed that while Gerber and Heinz had similar unit costs, those of Beech Nut were about 20 to 25 percent higher. The proposed merger was in fact largely defended by the claimed cost efficiencies that would, it was alleged, permit production of the Beech Nut line at a unit cost similar to that incurred by Gerber and Heinz.

Our second case is a logical extension of the first. Here we consider a setting in which ***both*** the intermediate and lower brand firms have higher pre-merger unit costs than the industry's top branded firm does. We then assume that the merger-generated cost efficiencies allow the two product lines to be produced at the same low cost as that incurred by the dominant firm. The cost synergies in the second case are therefore stronger than those assumed in the first.

3. *Merger Case A: The intermediate brand firm is a high cost producer*

In this case the top and low-end branded firms have low unit production costs $c_1 = c_3 = 0$, while for the intermediate brand the unit cost is high, or $c_2 = c > 0$. We make two further

assumptions to ensure that the two lower ranked brands have positive market shares in the pre-merger market. They are:

(A.3) $c < \Delta z(1 - \Delta z)$

(A.4) $\bar{\theta} \leq \dfrac{(3 + \Delta z)(1 - \Delta z) + 2c}{3(1 - \Delta z)}$

3.1 The Pre-Merger Equilibrium

The firms independently choose the prices of their brands to maximize profits, which from (2a)-(2c) are defined respectively by $\pi_1 = p_1 D_1$, $\pi_2 = (p_2 - c)D_2$, and $\pi_3 = p_3 D_3$. It can be shown that there exists a unique non-cooperative price equilibrium in this type of model (see, for example, Gabszewicz *et al* 1981(Is this the correct reference?). The equilibrium prices are:

$$p_1^*(\Delta z, c, \bar{\theta}) = (2c + \Delta z(1 - \Delta z + 3\bar{\theta}))/6 \qquad (3a)$$

$$p_2^*(\Delta z, c, \bar{\theta}) = (2c + \Delta z(1 - \Delta z))/3 \qquad (3b)$$

$$p_3^*(\Delta z, c, \bar{\theta}) = (2c + (1 - \Delta z)(3 + \Delta z - 3\bar{\theta}))/6 \qquad (3c)$$

It is straightforward to show, as we would expect, that the equilibrium product prices are ordered in terms of consumer ranking of the three goods: $p_1^* > p_2^* > p_3^*$. Note also that the prices of all three brands are increasing in firm 2's cost disadvantage c. The market shares of the three firms in the pre-merger equilibrium are given by:

$$D_1^*(\Delta z, c, \bar{\theta}) = (2c + \Delta z(1 - \Delta z + 3\bar{\theta}))/6z \qquad (4a)$$

$$D_2^*(\Delta z, c, \bar{\theta}) = (\Delta z(1 - \Delta z) - c)/3\Delta z(1 - \Delta z) \qquad (4b)$$

$$D_3^*(\Delta z, c, \bar{\theta}) = (2c + (1 - \Delta z)(3 + \Delta z - 3\bar{\theta}))/6(1 - \Delta z) \qquad (4c)$$

The market shares of the top and low brand are increasing in the cost disadvantage c of the middle brand. As a result, when costs differ across firms the market shares of the firms do not

necessarily conform to the consumer ranking of brands. However, it is easy to show that $D_1^*(\Delta z, c, \bar{\theta}) > D_2^*(\Delta z, c, \bar{\theta})$ and $D_1^*(\Delta z, c, \bar{\theta}) > D_3^*(\Delta z, c, \bar{\theta})$ so that the top brand $z_1 = \bar{z}$ has the largest market share, i.e., the high quality firm is the dominant firm in this industry. Whether or not the market share of the intermediate brand is greater than the market share of the low brand depends on the magnitude of the intermediate brand firm's cost disadvantage c. Specifically, if this cost disadvantage lies in the range $\frac{3\bar{\theta} - 1 - \Delta z}{4} < c < \Delta z(1 - \Delta z)$ then

$D_2^*(\Delta z, c, \bar{\theta}) < D_3^*(\Delta z, c, \bar{\theta})$: firm 2's market share is less than that of the low valued brand.

The profits of the three firms in the pre-merger market are:

$$\pi_1^*(\Delta z, c, \bar{\theta}) = \left(2c + \Delta z\left(1 - \Delta z + 3\bar{\theta}\right)\right)^2 / 36\,\Delta z \qquad (5a)$$

$$\pi_2^*(\Delta z, c, \bar{\theta}) = \left(\Delta z(1 - \Delta z) - c\right)^2 / 9\Delta z(1 - \Delta z) \qquad (5b)$$

$$\pi_3^*(\Delta z, c, \bar{\theta}) = \left(2c + (1 - \Delta z)\left(3 + \Delta z - 3\bar{\theta}\right)\right)^2 / 36(1 - \Delta z) \qquad (5c)$$

Observe that the profits of the two low-cost firms, firms 1 and 3, are increasing in the cost disadvantage c of firm 2, whereas the profit of firm 2 is, of course, decreasing in c.

3.2 The Post-Merger Equilibrium

Now suppose that the cost-disadvantaged firm 2 is able to merge with the low-cost firm 3 and that in doing so firm 3 is able to extend its low-cost production technology to firm 2. The marginal cost of the intermediate brand 2 then becomes $c_2 = 0$. Because this type of merger creates cost efficiencies between the two firms with the smaller market shares, one might think that competition in the market will be enhanced and, therefore, that the merger will be welfare improving for consumers. This presumption turns out to be wrong largely because, as suggested in the introduction, the newly merged firm has a strong profit incentive to change its product strategy and withdraw a brand from the market.

To see why, first suppose that in the post-merger market the merged firm continues to market the two brands, $z_2 = \bar{z} - \Delta z$ and $z_3 = \underline{z}$, and to set the prices p_2 and p_3 to maximize the joint profit of offering the two brands. The merged firm chooses prices $\tilde{p}_2^m$, $\tilde{p}_3^m$ to maximize:

$$\tilde{\pi}^m(p_2^m, p_3^m, p_{,1}) = p_2^m D_2(p_1, p_2^m, p_3^m) + p_3^m D_3(p_2^m, p_3^m)$$

subject to the constraint that $D_3(p_2^m, p_3^m) \geq 0$, i.e., that the low-valued brand has a non-negative market share. This constraint is important because the merged firm has an incentive to raise the price of the low-valued brand in order to set a higher price for the middle brand. However, if the merged firm raises the price of the low-valued brand too high then the demand for the low-valued good will fall to zero. This happens when the price differential between its two brands is less than the smallest consumer willingness to pay for the difference in the strength of the two brands. In other words, there is positive demand for the low brand $D_3(p_2^m, p_3^m) \geq 0$ if and only if the price differential $(p_2^m - p_3^m) \geq \underline{\theta}(z_2 - \underline{z}) = (\bar{\theta} - 1)(1 - \Delta z)$. As a result, the maximum price that the merged firm can set for the low valued brand is $p_3^m = p_2^m - (\bar{\theta} - 1)(1 - \Delta z)$. The rival firm, firm 1, in the post merger market chooses a price $\tilde{p}_1^m$ to maximize its profit $\tilde{\pi}^m(p_1, p_2) = p_1 D_1(p_1, p_2)$. The equilibrium post-merger prices for this case, subject to the constraint that good 3 has non-negative market share, are then:

$$\tilde{p}_1^m(\Delta z) = \Delta z\left(\bar{\theta} + 1\right)/3 \qquad (6a)$$

$$\tilde{p}_2^m\left(\Delta z, \bar{\theta}\right) = \Delta z(2 - \bar{\theta})/3 \qquad (6b)$$

$$\tilde{p}_3^m\left(\Delta z, \bar{\theta}\right) = \left(3 - \Delta z - \bar{\theta}(3 - 2\Delta z)\right)/3 \qquad (6c)$$

It is clear that if the merged firm continues to market both brands of goods, 2 and 3, then the firm will set prices $\widetilde{p}_2^m$ and $\widetilde{p}_3^m$ such that the constraint on the market share of the low valued brand is binding or that $D_3(\widetilde{p}_2^m, \widetilde{p}_3^m) = 0$. The profits earned by each firm in this case are:

$$\widetilde{\pi}_1^m(\Delta z) = \Delta z\left(\overline{\theta} + 1\right)^2 / 9 \tag{7a}$$

$$\widetilde{\pi}_2^m\left(\Delta z, \overline{\theta}\right) = \Delta z(2 - \overline{\theta})^2 / 9 \tag{7b}$$

Because the merged firm has a profit incentive essentially to price its low-valued brand out of the market the supposition that the firm would choose to market both brands in the post merger outcome is questionable. Suppose instead that the merged firm competes with the rival firm, firm 1, by offering a single brand $z_m \in \{\bar{z} - \Delta z, \underline{z}\}$ that is either the intermediate brand or the low-valued brand. The demands facing the two firms are:

$$D_1^m(p_1, p_2) = \overline{\theta} - \frac{p_1 - p_2}{\bar{z} - z_m} \tag{8a}$$

$$D_2^m(p_1, p_2) = \frac{p_1 - p_2}{\bar{z} - z_m} - \overline{\theta} + 1 \tag{8b}$$

The two rival firms choose prices non-cooperatively to maximize profits $p_1 D_1^m(\ )$ and $p_2 D_2^m(\ )$. In this case, the price equilibrium is:

$$p_1^m\left(\bar{z}, z_m, \overline{\theta}\right) = \left(\bar{z} - z_m\right)\left(1 + \overline{\theta}\right)/3 \tag{9a}$$

$$p_2^m\left(\bar{z}, z_m, \overline{\theta}\right) = \left(\bar{z} - z_m\right)\left(2 - \overline{\theta}\right)/3 \tag{9b}$$

The post-merger demands to the two firms in this case are:

$$D_1^m\left(\overline{\theta}\right) = \left(1 + \overline{\theta}\right)/3 \tag{10a}$$

$$D_2^m\left(\overline{\theta}\right) = \left(2 - \overline{\theta}\right)/3 \tag{10b}$$

where (A.3) and (A.4) also guarantee that $D_2^m\left(\overline{\theta}\right) > 0$. The post-merger profits are:

$$\pi_1^m\left(\bar{z}, z_m, \bar{\theta}\right) = \left(\bar{z} - z_m\right)\left(1 + \bar{\theta}\right)^2 / 9 \qquad (11a)$$

$$\pi_2^m\left(\bar{z}, z_m, \bar{\theta}\right) = \left(\bar{z} - z_m\right)\left(2 - \bar{\theta}\right)^2 / 9 \qquad (11b)$$

The profits of both firms are decreasing in the choice of brand strength z_m of the merged firm's product. As a result, if the merged firm offers only one brand then it is more profitable to market the low-valued brand $\underline{z}$ because of the softer price competition to which this maximum differentiation gives rise. The post-merger equilibrium prices in this case are:

$$\tilde{p}_1^m\left(\bar{\theta}\right) = \left(1 + \bar{\theta}\right)/3 \qquad (12a)$$

$$\tilde{p}_2^m\left(\bar{\theta}\right) = \left(2 - \bar{\theta}\right)/3 \qquad (12b)$$

and post-merger profits to the two firms are:

$$\tilde{\pi}_1^m\left(\bar{\theta}\right) = \left(1 + \bar{\theta}\right)^2 / 9 \qquad (13a)$$

$$\tilde{\pi}_2^m\left(\bar{\theta}\right) = \left(2 - \bar{\theta}\right)^2 / 9 \qquad (13b)$$

Since $\Delta z < 1$, comparison of (7b) and (13b) allows us to conclude:

Proposition 1: Given (A.1) – (A.4) the merged firm will offer only the low-valued brand in the post-merger market.

3.3 Welfare Effects of Merger

The relevant questions to ask at this point are first, whether the merger is profitable for the merged and non-merged firms and secondly, how does the merger affect consumer welfare.[6]

Proposition 2: Given (*A*.1) – (*A*.4) *a merger of firms 2 and 3*

(i) increases profits of both the merged and the non-merged firms;

(ii) benefits the non-merged firm more than the merged firm.

The merger between the high-cost firm 2 and low-cost firm 3 creates a positive externality for the non-merged firm 1. The externality arises because of the decision of the merged firm to eliminate one of the brands from the market. The decision to drop the middle brand increases the

differentiation between brands, softens price competition between the firms and so increases the profit of the firm that is not party to the merger. This reduction in product variety and the softer price competition that it implies combine to reduce consumer welfare unambiguously.

Proposition 3: Given (*A*.1) – (*A*.4) *a merger of firms 2 and 3 that leads to lower unit costs of production leads to* ***higher*** *post-merger prices*.

Proposition 3 is a powerful result. Without any estimation of the structure of market demand and absent any investigation of the validity of claimed cost efficiencies, Proposition 3 says that in a vertically differentiated market that is dominated by a strong brand with a cost advantage over its closest rival, ***no*** merger that eliminates the cost disadvantage of that second-best rival will be beneficial to consumers. Moreover, this is not because the reduction in firms will enable any collusion in the post-merger market. Rather it is because the surviving firms in the post merger market act independently and choose their pricing and product strategy to maximize profits. Even though our theoretical framework is only an approximation to real world product competition, Proposition 3 nonetheless serves to establish a strong prior against the ability of alleged cost efficiencies to justify the merger.

Given the higher prices and profits accruing to both the merged firm and non-merged firm it would be surprising if firm 1 were to object to the merger of firms 2 and 3. However, it is worth noting that while firm 1 will enjoy a price and profit increase as a result of the merger of firms 2 and 3, the implications for its market share are more ambiguous. Specifically, firm 1's market share can rise or fall as a result of the merger depending on the size of the merger's cost savings. We address this issue in the proposition below.

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[6] Calculations were performed using *Mathematica*®. The notebook is available from the authors on request. Proofs are presented in the Appendix.

Proposition 4: Given (*A*.1) – (*A*.4) *a merger of firms 2 and 3 decreases the market share of the non-merged firm and increases the market share of the merged firms if and only if* $c > \Delta z\left(1 + \Delta z - \bar{\theta}\right)/2$.

Since $\bar{\theta} > 1$, a sufficient condition for the merger of firms 2 and 3 to reduce the market share of the non-merged firm 1 is that $c > \Delta z^2/2$, which is consistent with (A.3) if $\Delta z < 2/3$. The merger therefore reduces the market share of the firm 1 when the cost synergies to which the merger gives rise are sufficiently great and when the intermediate brand, which is eliminated from the market, is not too similar to the low brand good, i.e. the one that is offered in the post-merger market.

In sum, the merger results in consumers being offered less product selection at higher prices. Thus, *all* consumers are worse off as a result of this merger despite the cost synergies that it brings. By contrast, the merger is unambiguously beneficial for the merged and non-merged firms. Once again, a simple intuition—independent of econometric estimation—underlies these results. The cost synergies generated by the merger put a downward pressure on prices. But the reduction in product selection reduces competitive pressures between the remaining firms allowing them to increase prices. For consumers, the loss of variety and the softening of competition outweigh the effect of the cost synergy. Firms gain but consumers lose from the merger.

3.4 Entry

The analysis in the previous section ignores the possibility of post-merger entry by a new firm. Ignoring entry is reasonable if entry into the market is blockaded because, say, the incumbent firms control the technology, the distribution networks or the access to retailers. In the baby food case, it was widely agreed that for these and other reasons post-merger entry was unlikely. Indeed, there had been no significant entry in the industry's previous fifty-year plus history. However, a merger that reduces the range of product selection, as we have shown is

likely in the present case, does leave a gap in the market which suggests at least the potential for entry. In turn, this threat of entry may give the merged firm an incentive to position its brand strategically so as to deter the emergence of a new rival. At the same time, such strategic entry deterrence will affect both the profitability of the merger and the impact of the merger on consumer welfare.

First, consider entry into the post-merger market when the two brands marketed by the incumbent firms are $z_1 = \bar{z}$ and $z_m = \underline{z}$, that is, the post-merger equilibrium described above. Suppose that if the entrant invests a sunk cost f_e then it can enter the market with a new brand of good $z_e \in [\underline{z}, \bar{z}]$. Further suppose that the marginal cost of producing a new good is zero.[7] When the entrant enters the post-merger market with brand $z_e \in [\underline{z}, \bar{z}]$, we can use the demand functions in (3a), (3b), and (3c) to find the resulting price equilibrium. It is:

$$p_1^* = \frac{(\bar{z} - z_e)(3\bar{z}\bar{\theta} + z_e - \underline{z}(1+\bar{\theta}))}{6}$$

$$p_e^* = \frac{(\bar{z} - z_e)(z_e - \underline{z})}{3}$$

$$p_m^* = \frac{(z_e - \underline{z})(\bar{z}(4 - 3\bar{\theta}) - z_e + 3\underline{z}(\bar{\theta} - 1))}{6}$$

By substituting these prices into the profit function of the new entrant and differentiating with respect to z_e we find that the optimal brand to offer satisfies the condition:

$$\frac{\partial \pi_e}{\partial z_e} = \frac{\bar{z} - 2z_e + \underline{z}}{9} = 0$$

The incentive to minimize the intensity of price competition leads the entrant to position its brand at $z_e = (\bar{z} + \underline{z})/2$, midway between the two incumbent firms' brands, generating profit $\pi_e = 1/36 - f_e$. Clearly, if the sunk cost of establishing a new brand name in this market is greater than 1/36, it follows that entry is essentially blockaded and the merger analysis of Section 3.3 holds.

If, however, $f_e < 1/36$ then the merged firm can deter entry by strategically changing its post-merger brand identity.[8] In particular, by choosing a quality closer to the product of firm 1 the merged firm can reduce the entrant's potential market and thereby make entry less likely. In turn, such strategic positioning will affect both the merger's profitability and its impact on consumers.

3.5 Strategic Deterrence:

Suppose that the merged firm strategically selects a brand identity in the post merger market of $z_m \in [\underline{z}, \bar{z}]$.[9] Lemma 1 identifies the brand strength that will deter a potential entrant.

Lemma 1: An entrant firm in the post merger market will market a brand $z_e = (\bar{z} + z_m)/2$ *and earn profit* $\pi_e = (\bar{z} - z_m)/36 - f_e$. *To deter entry the merged firm must therefore market a brand* $z_m = \bar{z} - 36 f_e$.

Lemma 1 indicates that the lower is the entry cost f_e, the higher is the brand strength that the merged firm must market if entry is to be deterred. However, the stronger is the post-merger brand image of the merged firm, the tougher is the competition with the rival firm 1, which adversely affects profitability of the merged firm. Will the merged firm wish to deter entry given that this change in branding strategy reduces its profit?

Proposition 5: The merged firm chooses its post merger brand to deter entry only if

$$\frac{1}{36} > f_e > \frac{\left(7 - 6\bar{\theta}\right)^2}{1152\left(2 - \bar{\theta}\right)^2}.$$

While Proposition 5 identifies a lower limit on the sunk entry cost f_e above which the merged firm prefers to deter rather than accommodate entry this is a necessary but not sufficient condition for entry deterrence. Simply put, Proposition 5 begs the larger question of whether the

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[7] Assuming $c = 0$ for the entrant provides a sufficient but not necessary constraint on f_e for entry to be deterred.

[8] Donnenfeld and Weber (1992, 1995) also discuss vertical differentiation and entry in slightly different but related models of product differentiation.

[9] We ignore the costs of changing brand strength, as a result of which our analysis identifies necessary but not necessarily sufficient conditions for the merger to be profitable with strategic entry deterrence.

merger should take place at all given the threat of entry. If the merger goes ahead and entry is deterred the merged firm makes profit (ignoring any costs of changing its brand identity) equal to $\pi_m^d = 4f_e\left(2-\bar{\theta}\right)^2$. By contrast, if the merger does not occur then there is no entry and firms 2 and 3 continue together to make a profit equal to $\pi_2^*\left(\Delta z, c, \bar{\theta}\right) + \pi_3^*\left(\Delta z, c, \bar{\theta}\right)$ defined in 5(a)-(c).

Proposition 6: Suppose that there is a threat of post-merger entry. Then the merger is unprofitable if the sunk entry cost f_e is less than

$$f\left(\Delta z, c, \bar{\theta}\right) = \frac{\pi_2^*\left(\Delta z, c, \bar{\theta}\right) + \pi_3^*\left(\Delta z, c, \bar{\theta}\right)}{4\left(2-\bar{\theta}\right)^2} < \frac{1}{36}.$$

We provide a range of values for the critical sunk entry cost $f\left(\Delta z, c, \bar{\theta}\right)$ in the Appendix. We note here that $\left.\frac{\partial f\left(\Delta z, c, \bar{\theta}\right)}{\partial c}\right|_{c=0} < 0$ and $\left.\frac{\partial f\left(\Delta z, c, \bar{\theta}\right)}{\partial c}\right|_{c=z(1-z)} > 0$. This means that when the incumbent firms anticipate the possibility of post-merger entry, then a merger of firms 2 and 3 is less likely to arise under two scenarios. The first is that the merger generates very little cost efficiencies. The product repositioning necessary to prevent entry intensifies price competition with the dominant firm. In turn, this results in losses that outweigh any benefits achieved from the small amount of cost savings that the merger generates when *c* is small.

The second scenario that limits the likelihood of a merger is if it generates cost efficiencies that are too large. This is really a way of saying that *c* is quite high. From the standpoint of firm 3, there is little incentive to remove a high-cost product if there is a real risk of its being replaced by a low-cost competitor.

The finding that, in the presence of a potential entrant, very large cost savings act as a disincentive for firms 2 and 3 to merge is worth noting. Effectively, it says that in the vertically

differentiated framework of our model, a merger defense based on very large cost efficiencies should perhaps be greeted with some skepticism if post-merger entry is feasible. Such large cost efficiencies in the presence of post-merger entry could render the merger unprofitable at the start. To put it another way, if the two firms find it profitable to merge the resultant cost efficiencies are not likely to be very big.

Allowing for the threat of entry in the model does generate some pro-competitive features. As noted above, it givens the merged firm an incentive to choose a brand or quality greater than $\underline{z}$ as an entry-deterring measure. This then strengthens price competition and thereby mitigates the harmful effect on consumer welfare. There is in fact a range, albeit small, of f_e values such that consumers could gain from the merger. To see this, suppose that $1/36 > f_e > f\left(\Delta z, c, \bar{\theta}\right)$, in which case entry is not blockaded but the merged firm would find it profitable to market a new brand image to deter entry. If f_e is "close to" 1/36 the merged firm's brand image is "close to" $\underline{z}$ and the merger increases aggregate profit and increases prices. By contrast, if f_e is "close to" $f\left(\Delta z, c, \bar{\theta}\right)$ then the merged firm chooses a brand image closer to that of the rival firm, and this reduces the both the profit and the price of firm 1. We can show the following:[10]

Proposition 7: There exists an ε *such that if* $f_e \in [\, f\left(\Delta z, c, \bar{\theta}\right), f\left(\Delta z, c, \bar{\theta}\right) + \varepsilon\,]$ *then the merger decreases aggregate profit, decreases the price of brand 1 and increases aggregate consumer surplus.*

We summarize our results thus far as follows. In markets in which there is a dominant brand and two other brands, one middle-valued and the other low-valued, and in which the middle brand has a cost disadvantage, then a merger of the two lower valued rival brands that eliminates the cost disadvantage generally does not benefit consumers. If post-merger entry is

not possible, the merger will unambiguously harm consumers though it will leave firms better off. Even if entry is feasible, the newly merged firm may well be able to position its brand strategically in a way that deters entry and, again, reduces consumer welfare. Only if entry is very likely, so that deterring it requires that the newly merged firm and its remaining rival compete vigorously to prevent the emergence of a third rival, is it possible that the merger brings consumers some gains.

There is one final issue to address. In Section 3.1, we analyzed a pre-merger equilibrium, but we did not show there that entry into the pre-merger market was not profitable. Clearly, this would be the case if the sunk cost of entry in the pre-merger market were sufficiently high. The question then is how this sunk cost of entry relates to the sunk cost f_e of the potential entrant in the post-merger market. Suppose in the pre-merger market that only three products are sustainable and so a new entrant into the market would drive out the lowest ranked brand.[11] Then the entrant in the pre-merger market would choose to locate its brand midway between brands 1 and 2. In this case there is a level of sunk entry costs $\hat{f}\left(\Delta z, c, \bar{\theta}\right)$ such that this type of entry would be unprofitable. We can show that $\frac{1}{36} > \hat{f}\left(\Delta z, c, \bar{\theta}\right) > f\left(\Delta z, c, \bar{\theta}\right)$ provided that $\Delta z >$ 0.33. In other words, if the sunk cost $\hat{f}\left(\Delta z, c, \bar{\theta}\right)$ in the pre-merger market is sufficiently high to insure that that pre-merger structure is indeed an equilibrium, then strategic entry-deterrence post-merger is both feasible and profitable.

4. Merger Case B: Both the middle and low valued brands are high cost producers

We now investigate the case in which both the middle brand and the low-valued brand in the pre-merger market are produced by high cost firms. The top-valued brand, on the other hand,

[10] Given the complicated nature of the equations, these results are based upon an extensive numerical grid search. Details can be obtained from the authors on request.

continues to be produced by a low cost producer. We assume for this case that the unit production cost of firm 1 is $c_1 = 0$, whereas the unit costs of firm 2 and firm 3 are $c_2 = c_3 = c > 0$. This kind of cost structure makes sense for industries in which firms with small market shares are the high cost producers. The rationale for a merger in this case is that it allows the small firms to increase market share and perhaps thereby utilize a lower cost technology.

In order to ensure that the three brands have positive market share in the pre-merger market for this case we assume that:

(A.5) $$c < \Delta z - 3\left(\overline{\theta} - 1\right)$$

Note that since $\overline{\theta} > 1$ (A.5) implies that the cost disadvantage $c < \Delta z$, where Δz is a measure of the difference in brand strength between the top and middle-valued brands.

4.1 Pre-Merger Equilibrium

Again we identify the pre-merger equilibrium prices and profits. Initially, the firms choose independently the prices of their brands to maximize profits, which from (2a)-(2c) are now defined respectively by $\pi_1 = p_1 D_1$, $\pi_2 = (p_2 - c)D_2$, and $\pi_3 = (p_3 - c)D_3$. The pre-merger equilibrium prices are:

$$p_1^*\left(\Delta z, c, \overline{\theta}\right) = \left(2c + \Delta z\left(1 + c - \Delta z + 3\overline{\theta}\right)\right)/6 \qquad (14a)$$

$$p_2^*\left(\Delta z, c, \overline{\theta}\right) = \left(2c + \Delta z\left(1 + c - \Delta z\right)\right)/3 \qquad (14b)$$

$$p_3^*\left(\Delta z, c, \overline{\theta}\right) = \left(5c + \Delta z\left((c + 3\overline{\theta} - 2) - \Delta z\right)\right) - 3(\overline{\theta} - 1)/6 \qquad (14c)$$

As before, the top brand sells at a price $p_1^* > p_2^*$ and the middle brand sells at price greater than the price of the low brand, or $p_2^* > p_3^*$. The prices of all three brands are increasing in c.

The market shares of the three firms in the pre-merger equilibrium are given by:

$$D_1^*\left(\Delta z, c, \overline{\theta}\right) = \left(2c + \Delta z\left(1 + c - \Delta z + 3\overline{\theta}\right)\right)/6\,z \qquad (15a)$$

[11] Gabszewicz and Thisse (1989) discuss this type of entry.

$$D_2^*\left(\Delta z, c, \bar{\theta}\right) = \left(\Delta z - c\right)/3\Delta z \tag{15b}$$

$$D_3^*\left(\Delta z, c, \bar{\theta}\right) = \left(3 - c + \Delta z - 3\bar{\theta}\right)/6 \tag{15c}$$

Again (A.5) guarantees that brands 2 and 3 have positive market shares in the pre-merger market. In this case the market share of the top brand is increasing whereas the market shares of the lower ranked brands are decreasing in the cost disadvantage *c*. Also in this case the market shares are unambiguously ranked by brand strength.

The pre-merger profits of the three firms are:

$$\pi_1^*\left(\Delta z, c, \bar{\theta}\right) = \left(2c + \Delta z\left(1 + c - \Delta z + 3\bar{\theta}\right)\right)^2/36\,\Delta z \tag{16a}$$

$$\pi_2^*\left(\Delta z, c, \bar{\theta}\right) = \left(\Delta z - c\right)^2\left(1 - \Delta z\right)/9\Delta z \tag{16b}$$

$$\pi_3^*\left(\Delta z, c, \bar{\theta}\right) = \left(1 - \Delta z\right)\left(3 - c + \Delta z - 3\bar{\theta}\right)^2/36 \tag{16c}$$

The post-merger equilibrium for Case 2 is the same as that analyzed in Case 1, and accordingly is defined by equations (9)-(13). Thus *Proposition 1 (In the post-merger market the merged firm offers only the low valued brand)* applies to this case as well.

4.2 Welfare Effects of Merger

Because the merger now lowers the unit cost of two firms instead of just one, the cost efficiencies in Case 2 are much stronger than those in Case 1. Accordingly, one might think that in this case a merger of firms 2 and 3 would not raise market prices. However, this supposition is not correct. While it is true that the welfare impact of a merger in this case is less clear-cut, it also true that there is a non-trivial set of mergers that benefit the merged firms but hurt consumers. Moreover, this result again obtains even when we permit entry into the post-merger market.

To see the effects of a merger we begin by first defining three different levels of cost disadvantage, $\bar{c}_1 \geq \bar{c}_2 \geq \bar{c}_3$, that are each a function of Δz and $\bar{\theta}$. They are:

$$\bar{c}_1(\Delta z, \bar{\theta}) = \left(\Delta z^2 - \Delta z + 2 + \bar{\theta}(2 - 3\Delta z)\right)/(2 + \Delta z) \qquad (17a)$$

$$\bar{c}_2(\Delta z, \bar{\theta}) = \left(\Delta z^2 + 2\sqrt{\Delta z}(1 + \bar{\theta}) - \Delta z(1 + 3\bar{\theta})\right)/(2 + \Delta z) \qquad (17b)$$

$$\bar{c}_3(\Delta z, \bar{\theta}) = \left((\Delta z + 1)^2 + \bar{\theta}(1 - 3\Delta z)\right)/(5 + \Delta z) \qquad (17c)$$

With these definitions, we now present two propositions, Propositions 8 and 9.

Proposition 8: Given (*A*.1), (*A*.2) *and* (*A*.5) *a merger of firms 2 and 3 in Case 2*

(i) increases profits of the merged firms;

(ii) decreases the profits of the non-merged firm if $c > \bar{c}_2(\Delta z, \bar{\theta})$.

If the pre-merger cost disadvantage of the merged firms is sufficiently great, i.e., if the cost efficiencies generated by the merger are sufficiently large then the non-merged firm no longer enjoys a positive externality. While it still gains from the fact the merged firms reduce their product offerings, this benefit is more than offset by the increased competitiveness of the merged firm due to its now lower cost of production.

Proposition 9: Given (*A*.1), (*A*.2) *and* (*A*.5) *a merger of firms 2 and 3 in Case 2*

(i) leads to higher post-merger prices if $c < \bar{c}_3(\Delta z, \bar{\theta})$;

(ii) increases the price of firm 1 *but decreases the price of the low-valued brand if* $\bar{c}_3(\Delta z, \bar{\theta}) < c < \bar{c}_1(\Delta z, \bar{\theta})$;

(iii) decreases the price of firm 1 and the low-valued brand if $c > \bar{c}_1(\Delta z, \bar{\theta})$.

We illustrate Propositions 8 and 9 in Figure 1. In this figure, the shaded area indicates the levels of pre-merger cost disadvantage *c* that do not satisfy our assumption (A.5), that is, values for which $c > \Delta z - 3(\bar{\theta} - 1)$. The three different levels of the pre-merger cost disadvantage of firms 1 and 2 are also indicated. The heavy dark lines illustrate how the parameter space is partitioned for a

given value of the parameter $\overline{\theta}$, which measures the maximum willingness to pay at the margin for an increase in z. The light lines show how the partition changes as $\overline{\theta}$ increases.

The primary insight revealed by Figure 1 is that despite the strong cost synergies of Case B, there is still a large region in the parameter space over which consumer welfare declines. This is clearly the case for values of $c < \overline{c}_3(\Delta z, \overline{\theta})$ because here, all product prices in the post-merger market are higher. It is also the case for much of the region in which $\overline{c}_3(\Delta z, \overline{\theta}) < c < \overline{c}_1(\Delta z, \overline{\theta})$ in which the price of the top-valued brand rises while that of low-valued brand falls. The intuition behind this result is equally straightforward. The bulk of the pre-merger market is captured by the top-valued brand. Therefore, after the merger, consumers tend to lose more from the rise in the price of brand 1 than they gain from the fall in the price of low valued brand. Moreover, since the intermediate brand is eliminated in all these cases there is reduced product selection and this also tends to reduce consumer welfare.

(Figure 1 near here)

4.3 Entry

As in Case A, the negative impact of the merger might be reversed if there were the possibility of entry in the post-merger market. However, it is clear that Lemma 1 also applies to Case B. Hence, the essential argument that underlies the results of the previous case continues to hold. If the potential entrant incurs a sunk cost greater than 1/36, the threat of entry is moot. The merger simply leads to the elimination of the intermediate brand with the net welfare consequences depending on the value of cost disadvantage c as described above.

If $f_2(\Delta z, c, \overline{\theta})$ < 1/36, then variants of Propositions 6 and 7 apply to Case B, as well. In other words, for a sunk entry cost $f_2(\Delta z, c, \overline{\theta})$ below 1/36 (but not too low) the merged firm finds it

profitable to market a brand z_m strategically to deter entry. We provide a range of values for $f_2\left(\Delta z, c, \overline{\theta}\right)$ in the Appendix. Note again that while strategic entry deterrence will intensify the post-merger price competition it is still quite possible that consumer welfare declines despite such tactics.

Conclusions

Much of the recent work evaluating the impact of mergers in product-differentiated markets is empirically based and relies on estimates of pre-merger demand and cost parameters to infer the likely nature of the post-merger equilibrium. The development of such models has been a welcome addition to the economist's arsenal of analytical techniques. It may be especially useful in assessing the ability of merger-related cost efficiencies to offset any adverse impact on consumer welfare that the merge may have. At the same time, the insights of more purely theoretical models can be an equally powerful tool in determining the welfare implications of mergers with cost efficiencies. Indeed, the insights of a formal model may be all upon which policy makers can rely in settings in which the merging firms have an incentive to change their product offerings so that the applicability of post-merger simulations based on pre-merger demand estimates is questionable at best.

We have shown that changes in the product mix are particularly likely following a merger in a vertically differentiated product market. We have further shown that, in the absence of any entry threat, mergers in such markets will tend to reduce product variety and raise consumer prices virtually regardless of the level of merger related cost efficiencies. We have also shown that this same adverse impact on consumer welfare remains likely to characterize mergers in vertically differentiated markets even when entry is a possibility.

The intuition behind our findings is straightforward. In vertically differentiated markets, the merging firms have a strong incentive to discontinue one of their two pre-merger product

lines as a means to soften competition with the non-merging rival. It is this post-merger product selection decision and its implications for consumer prices that underlie our results.

From a policy perspective, our analysis makes clear that mergers that result in lower costs may nevertheless lead to higher prices. Indeed, the likelihood of this outcome is very high in vertically differentiated markets. It follows that in markets where vertical differentiation is a primary feature, a merger defense based on alleged cost efficiencies should be viewed with considerable skepticism.

References

Aaker, D. A., ***Building Strong Brands*****,** New York: The Free Press (1996).

Anderson, S., A. de Palma and J-F Thisse, "Social Surplus and Profitability under Different Spatial Pricing Policies", ***Southern Economic Journal*** 58 (Number 4, 1992), 934-49.

Baker, J. and T. Bresnahan¸ "The Gains from Merger or Collusion in Product Differentiated Industries," ***Journal of Industrial Economics***, 33 (June, 1985), 427-44,

Donnenfeld, S. and S. Weber, "Vertical Product Differentiation with Entry", with S. Weber, ***International Journal of Industrial Organization***, 10 (September 1992), 449-72

_____, and _____, "Limit Qualities and Entry Deterrence", ***Rand Journal of Economics***, 26 (Spring 1995), 113-130.

Gabszewicz, J-J and J-F Thisse, "Price Competition, Quality, and Income Disparities", ***Journal of Economic Theory***, 2 (March, 1979), 340-59.

_____, and _____, "On the Nature of Competition with Differentiated Products". ***Economic Journal***, 96 (March, 1986), 160-72.

Hausman, J. A. and G. Leonard, "Economic Analysis of Mergers in Differentiated Products Industries: Mergers Using Real World Data". ***George Mason Law Review***, 5 (April, 1997), 321-46.

Muris, T., "The Government and Merger Efficiencies: Still Hostile After All These Years" 7 ***George Mason Law Review,*** 729 (Spring, 1999).

Mussa, M. and S. Rosen, "Monopoly and Product Quality." ***Journal of Economic Theory***, 18 (March, 1978), 301-17.

Nevo, A. "Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry". ***Rand Journal of Economics***, 31 (Autumn, 2000), 395-421.

Pepall, L. and D. J. Richards (2002). "The Simple Economics of Brand-Stretching." ***Journal of Business*** (July 2002), 535-52.

Shaked, A. and J. Sutton, "Relaxing Price Competition through Product Differentiation", ***Review of Economic Studies*** 49 (January, 1982), 3-13.

Shapiro, C. Mergers with Differentiated Products. ***Antitrust Magazine***, (Spring 1996), 23-30. .

Tirole, J., ***The Theory of Industrial Organization***. Cambridge, MA: MIT Press, 1988.

Werden, G. J. and L. M. Froeb, "The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy." ***Journal of Law, Economics, and Organization*** 10 (June, 1994), 407-26.

Appendix

Proof of Proposition 2:

(i) Define the impact of the merger on the firms' profits as:

$\Delta\pi_1(\Delta z, c, \bar{\theta}) = \tilde{\pi}_1^m(\bar{\theta}) - \pi_1^*(\Delta z, c, \bar{\theta})$; $\Delta\pi_2(\Delta z, c, \bar{\theta}) = \tilde{\pi}_2^m(\bar{\theta}) - \pi_2^*(\Delta z, c, \bar{\theta}) - \pi_3^*(\Delta z, c, \bar{\theta})$. It is easy to show that $\left.\frac{\partial \Delta\pi_2}{\partial c}\right|_{c=0} = \frac{1}{9}(3\bar{\theta} - 1 - \Delta z) > 0$; $\left.\frac{\partial \Delta\pi_2}{\partial c}\right|_{c=\Delta z(1-\Delta z)} = \frac{1}{3}(\bar{\theta} - 1 - \Delta z) < 0$ and $\frac{\partial^2 \Delta\pi_2}{\partial c^2} < 0$ so $\Delta\pi_2(\Delta z, c, \bar{\theta})$ is minimized either when $c = 0$ or $c = \Delta z(1 - \Delta z)$ (Recall A.4).

$\frac{\partial \Delta\pi_1}{\partial c} = \frac{1}{9\Delta z}(-2c - \Delta z(3\bar{\theta} + 1 - \Delta z)) < 0$ so $\Delta\pi_1(\Delta z, c, \bar{\theta})$ is minimized at $c = \Delta z(1 - \Delta z)$.

Evaluation of $\Delta\pi_2(\Delta z, 0, \bar{\theta})$; $\Delta\pi_2(\Delta z, \Delta z(1-\Delta z), \bar{\theta})$ and $\Delta\pi_1(\Delta z, \Delta z(1-\Delta z), \bar{\theta})$ over the permissible ranges of Δz and $\bar{\theta}$ confirms that they are positive.

(ii) $\frac{\partial}{\partial c}(\Delta\pi_2 - \Delta\pi_1) > 0$ so $\Delta\pi_2(\Delta z, c, \bar{\theta})$ - $\Delta\pi_1(\Delta z, c, \bar{\theta})$ is maximized at $c = \Delta z(1 - \Delta z)$.

Evaluation of $\Delta\pi_2(\Delta z, \Delta z(1-\Delta z), \bar{\theta}) - \Delta\pi_1(\Delta z, \Delta z(1-\Delta z), \bar{\theta})$ over the permissible ranges of Δz and $\bar{\theta}$ confirms that it is negative.

Proof of Proposition 3:

Since pre-merger prices are increasing in c the merger is most likely to decrease prices when c is at its upper limit. Evaluation of $\tilde{p}_1(\bar{\theta}) - p_1^*(\Delta z, \Delta z(1-\Delta z), \bar{\theta})$ and $\tilde{p}_2(\bar{\theta}) - p_3^*(\Delta z, \Delta z(1-\Delta z), \bar{\theta})$ over the permissible range of parameters indicates that these price differentials are positive.

Proof of Proposition 4:

Follows immediately from (4a) and (10a).

Proof of Lemma 1:

Consider three brands with strength $z_1 > z_2 > z_3$ that are marketed in the post merger market. The resulting price equilibrium is

$$p_1^* = \frac{(\bar{z} - z_2)(3\bar{z}\bar{\theta} + z_2 - z_3(1+\bar{\theta}))}{6(\bar{z} - z_3)}; \; p_2^* = \frac{(\bar{z} - z_2)(z_2 - z_3)}{3(\bar{z} - z_3)};$$

$$p_3^* = \frac{(z_2 - z_3)(\bar{z}(4 - 3\bar{\theta}) - z_2 + 3z_3(\bar{\theta} - 1))}{6(\bar{z} - z_3)}$$

Substituting these prices into the demand functions and calculating profits, we find that:

$$\frac{\partial \pi_2}{\partial z_2} = \frac{\bar{z} - 2z_2 + z_3}{9(\bar{z} - z_3)}$$

The strength of brand 2 is $z_2 = (\bar{z} + z_3)/2$ midway between brands 1 and 3.

Suppose that the entrant chooses brand strength z_3. Then its profit, ignoring entry costs, is $\pi_e = \frac{1}{288}(\bar{z} - z_3)(7 - 6\bar{\theta})^2$. By contrast, if the entrant chooses brand $z_2 = (\bar{z} + z_3)/2$ its profit, ignoring entry costs, is $\pi_e = \frac{1}{36}(\bar{z} - z_3)$. As a result, for any brand strength z_m of the merged firm the entrant will always choose $z_e = (\bar{z} + z_m)/2$ and earn profit $\pi_e = (\bar{z} - z_m)/36 - f$. For the merged firm to deter entry it must market a brand strength $z_m = \bar{z} - 36f$.

Proof of Proposition 5:

If the merged firm chooses to deter entry its profits are, from (11b)

$$\pi_m^d = \frac{1}{9}(\bar{z} - z_m^d)(2 - \bar{\theta})^2 = 4f(2 - \bar{\theta})^2.$$

On the other hand if the merged firm accommodates entry it markets brand $z_m = \underline{z}$ while the entrant chooses quality $z_e = \bar{z} - 1/2$. Profit to the merged firm is, from (5c)

$$\pi_m^a = \frac{1}{288}(7 - 6\bar{\theta})^2$$

The merged firm chooses its quality to deter entry only if $\pi_m^d > \pi_m^a$.

Appendix Table 1a: $f\left(\Delta z, c, \bar{\theta}\right);\ \Delta z = 1/2$

		$\bar{\theta}$			
c	0	.00781	.00817	.00874	.00962
	0.05	.00615	.00609	.00617	.00645
	0.10	.00531	.00493	.00463	.00443
	0.15	.00531	.00470	.00412	.00357
	0.20	.00615	.00540	.00463	.00386
	0.25	.00781	.00701	.00617	.00530

Appendix Table 2: $f_2\left(\Delta z, c, \bar{\theta}\right);\ \Delta z = 1/2$

		$\bar{\theta}$			
c	0	.00781	.00817	.00874	.00962
	0.05	.00633	.00656	.00704	.00779
	0.10	.00500	.00517	.00553	.00616
	0.15	.00383	.00392	.00421	.00476
	0.20	.00281	.00286	.00309	.00357
	0.25	.00195	.00196	.00215	.00260

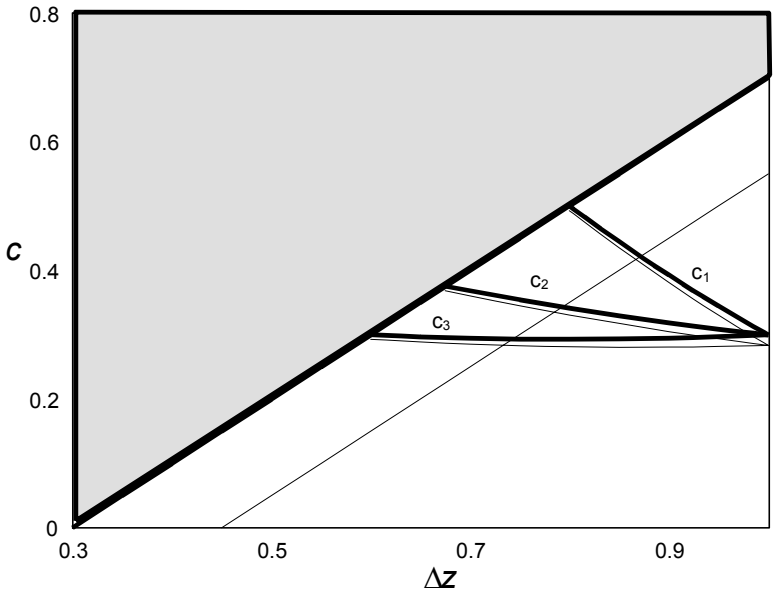

Figure 1: Propositions 8 and 9

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