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Mergers in a Cournot Model of Spatial Competition: Urban Sprawl and Product Specialization

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Abstract

This paper investigates the profitability and locational effects of mergers when firms play a Cournot game and compete in spatially differentiated markets. A two-firm merger is generally profitable in these types of markets because the merged partners can coordinate their location decisions. The merged firm locates its plants outside the market quartiles with distance from the market center being an increasing function of the number of non-merged firms remaining at the market center. Profitable two-firm mergers reduce competitive pressure, leading to higher prices and reduced consumer surplus. The increased profits of both the merged firm and the other firms are such that total surplus rises as a result of the merger. When we allow for the possibility of several two-firm mergers, three sets of agglomerations emerge. The merged firms agglomerate their activities at locations outside the market quartiles while the non-merged firms remain agglomerated at the center. This outcome is consistent with the idea that multi-facility production will lead to urban sprawl. In the context of product differentiation, mergers lead to greater product diversity as the merged firm(s) tailor products to serve better the more peripheral segments of the market.

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1. Introduction

The need to understand the market impact of mergers has become a central issue in antitrust policy and enforcement in the wake of an unprecedented wave of merger activity in the U.S. and abroad. Indeed, it is hard to find an area of public policy where the theory of industrial organization is more needed. A recent report by the Federal Trade Commission, "Anticipating the 21st Century", cites the prevention of anti-competitive mergers as one its four key activities. Policy-makers' interest has been focused in particular on the competitive effects of *horizontal* mergers, or on those mergers that make it profitable for merging firms to reduce output and cause market price to increase unilaterally, that is, in the absence of collusion.

Unfortunately, the currently available theory of horizontal mergers is not yet quite up to task. The standard Cournot model of quantity competition that appears to be becoming the *de facto* paradigm is beset by the paradoxical result that the majority of horizontal mergers are simply not profitable.[1] The logic behind this paradox goes as follows: A merger among two firms converts an *n*-firm pre-merger Cournot game into an *n*-1-firm post-merger Cournot game. In the standard setting with constant unit costs and no capacity constraints there is no credible commitment that the merged firm can make to exploit its potentially greater size in the post-merger game. The merged firm ends up looking just like any other firm in the industry. As a result, such a merger is almost certainly unprofitable and should, therefore, not occur. This is a clear drawback to the use of the standard Cournot model as a foundation for the formulation of merger policy.[2]

A two-firm merger, by combining the assets of the two firms, should in some sense lead to a "bigger" or "better" firm than either the two firms were pre-merger. But as we have just seen, this is precisely the limitation of the standard Cournot model with constant marginal costs. A two-firm merger in the standard model will just lead to one of the merged firms effectively being closed in the post-merger game. Now consider, however, a perfectly reasonable situation in which the Cournot firms compete across a set of spatially separated markets. A two-firm merger could in this context lead to a quite different outcome. Cournot firms in such a spatial setting choose not just the quantity of output to supply to the spatially differentiated markets. They must also choose where to locate their

1 The profitability paradox of horizontal mergers in the Cournot model is most often associated with Salant, Switzer and Reynolds (1983). The paradox was also shown to hold in a more general Cournot model in Szidarovzky and Yakowitz (1982).

2 One way to resolve the paradox is to assume that the firms have increasing rather than constant marginal costs of production. This is done in Perry and Porter (1985), Farrell and Shapiro (1990), and McAfee and Williams (1992).

production plants to serve these markets. If instead we interpret "space" not in a geographic sense but in the sense of product characteristic space, Cournot firms must choose where to anchor their basic product design within this space. Location choice then provides a means by which two firms can merge *and* can commit to being a "bigger and better" firm. This is because, in contrast to the standard model, a merger between two Cournot firms in the spatial context need not lead to the shutting down of one of the firms. Firms that have different locations will have different locational advantages in serving the set of spatially separated markets. A merged firm can then be potentially "bigger". More importantly, a merger between two firms also allows the location decisions of the two production plants to be coordinated, and so the merger may serve the market "better".

The goal of this paper is to investigate whether or not horizontal mergers can in fact lead to "bigger and better" firms in a Cournot model in which the firms compete across a set of spatially differentiated markets. We build on the work of Anderson and Neven (1991) who show that, when demand and costs are such that each Cournot firm wishes to serve the entire set of spatially separated markets, then all firms will choose to cluster or agglomerate in the center of the market area. We take this tendency to agglomeration as the starting point of our analysis. We then proceed by assuming that merger confers a leadership advantage on the merging firms in that they can choose first whether and where they wish to relocate their production plants. In doing so the merged firm is able to anticipate correctly the outcome of the quantity game that will be played with its non-merged rivals.

We show that in this context a two-firm merger will result in the merged firm relocating its plants away from the market center, whereas the non-merged firms will remain at the center. It is the advantage of being able to coordinate the two plants' locations that explains why, in sharp contrast to the standard Cournot model, a two-firm merger can be profitable. Because profitable two-firm mergers reduce competitive pressure across the set of markets, they lead to higher prices across the set of markets. However, and again in sharp contrast to non-spatial analysis, the increased profit of both the merged firm and the non-merged firms is such that total surplus rises as a result of the merger. The introduction of an explicitly spatial context means that merger can lead to a "bigger firm", which so far as total welfare is concerned, serves the market "better".

We next allow for the possibility that there will be more than one two-firm merger. This scenario accords with ample anecdotal evidence that more than one merger typically takes place in an industry. When more than one two-firm merger occurs we find that the merged firms agglomerate their activities outside the market center, while the non-merged firms remain clustered at the center. In other words, a series of two-firm mergers leads to a particular kind of urban sprawl, as the multi-plant merged firms relocate away from the clustered center of the market area.[3] However, these multi-plant firms also choose to cluster their activities, creating new hubs of production activity away from the center.

Alternatively, in a product specification context, we begin initially with all firms producing the same basic product design and each tailoring the add-ons in the same way to meet consumer demand for product variety. Mergers lead the merging partners each to choose a different basic product design, which more efficiently meet the demands of consumers. However, once again when there is more than one two-firm merger the multi-product merged firms choose the same new basic product designs. In this context, merging leads to a pattern of product clustering in differentiated product markets.

The remainder of the paper is organized as follows. In Section 2 we present the Cournot model of spatial competition with all firms located at the center of the market and derive the equilibrium conditions that will characterize a two-firm merger. In the following section we identify the distance from the center that the merged firm will locate its plants and the conditions that make such a merger profitable. In Section 4 we evaluate the welfare effects of a two-firm merger in the spatial model. Then in Section 5 we consider what happens in terms of location choice, profitability and welfare if there is more than one two-firm merger. Concluding remarks are presented in the final section.

2. The Model

There are n identical firms located on a Hotelling line whose length is normalized to unity. Each firm i produces a homogeneous product at constant marginal costs which are normalized (without loss of generality) to zero. We also ignore any set-up costs that the firms might incur. Demand at each point x on the line is identical, given by the inverse demand function $p(x) = \hat{v} - Q(x)$ where $p(x)$ is the product price and $Q(x)$ is aggregate output supplied to x. Firms incur shipping costs that are linear in distance and quantity, $\tau(|x_i - x|) = \tau|x_i - x|$, $i = 1,\ldots, n$, where x_i is the location of firm i. We eliminate the transport cost parameter by working with the transport-cost

[3] There is an alternative interpretation of our model that we leave to subsequent analysis. Assume that firms entering a market can choose to operate either a single plant or two plants. For any mix of firms, the resulting location equilibrium will be just that identified in this paper, with the single-product firms agglomerated at the market center and the two-product firms agglomerated outside the market quartiles, once again exhibiting urban sprawl.

adjusted reservation price $v = \hat{v}/t$.[4] In the product-differentiation analogy of this model, *v* can be interpreted as an inverse measure of the extent to which consumer tastes are strongly localized.

The *n* firms compete at each location *u* by playing a Cournot game in quantities. There is no arbitrage across local markets as a result of which the firms can and will price discriminate across markets. Effectively, competition between the firms results in a set of independent Cournot equilibria, one for each location on the line.

Anderson and Neven (1991) consider a two-stage game in this framework. In the first stage the *n* firms simultaneously choose their locations knowing that in the second stage the firms will compete in quantities in the manner we have described above. Their main result is to show that the subgame perfect location equilibrium has all *n* firms located at the market center. The intuition behind this agglomeration result is straightforward. Each firm is assumed to be able to supply the entire market no matter the location it chooses.[5] Moreover, a Cournot market game softens, to at least some extent, competition among firms in each local market. As a result, every firm has a strong incentive to adopt a central location and to supply the entire market from that location.

We take agglomeration at the center of the market as the starting point in our analysis. That is, we assume that the *n* firms are initially located at the center of the market, or $x_i^* = 1/2$, $i = 1,\ldots, n$. In the Appendix we show that if $v > n/2$, then no firm individually will have an incentive to deviate and change its location by moving from the center, given that the remaining n–1 firms are located at the center. We assume throughout the remaining analysis that the parameters of the model satisfy the condition $v > n/2$.

Given the starting point in which all *n* firms are agglomerated at the center of the line, the Cournot equilibrium firm output, aggregate output and price at each localized market *x* are:

(1a) $$q_i^C(x) = \frac{v - |x - 1/2|}{n+1}$$

(1b) $$Q_i^C(x) = n \cdot \frac{v - |x - 1/2|}{n+1}$$

(1c) $$p^C(x) = \frac{v + n.|x - 1/2|}{n+1}$$

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4 Rowthorn (1992) makes a similar normalization.

5 To guarantee that each firm is active in each local market irrespective of its location, Anderson and Neven assume that the maximum consumer reservation price *v* is greater than *n*, the number of firms. This ensures that even in the case where one firm is located at one end of the market spectrum, $x = 0$, and the remaining n - 1 firms are located at the other end, x =1, it is still true that the firm located at x=0 will be active in the local market $x = 1$.

From (1) we can calculate firm *i*'s profit from supplying the entire set of markets:

$$\pi_i^C = 2.\int_0^{1/2}\left(\frac{v-|x-1/2|}{n+1}\right)^2 dx = \frac{12v^2-6v+1}{12(n+1)^2}.$$

Consider now the possibility of a merger between two of the *n* firms. If the two merging firms do not change their locations then such a merger is subject to the Salant, Switzer, Reynolds (1983) paradox discussed above. Specifically, if there are $n \geq 3$ firms and two firms merge without changing their locations, then the merger will be unprofitable. The post-merger profit of the merged firm in serving every local market on the line is less than the sum of the two firms' pre-merger profit from serving these markets. A simple revealed preference argument suggests that such a merger cannot be expected to occur.[6]

Suppose instead that the two merging firms can change their production locations. In particular, suppose that the merging firms can coordinate their location decisions by becoming "location leaders" in a two-stage game. That is, in the first stage the merged firm choose to locate what are now its two divisions or branches a distance *d* from the market center, with, of course, one branch to the left and the other to the right of the market center. In the second stage the merged firm and the non-merged firms simultaneously choose their outputs to supply the local markets along the line. It might be thought that we should consider a three-stage game, in the second stage of which the remaining n - 2 non-merged firms choose their locations. As we show in the Appendix, however, we need not consider this stage explicitly since the non-merged firms will not want to change their location decisions in response to the merger.

It follows from the structure of the model that if the merged firm relocates its divisions a distance *d* from the market center it will want the left-hand division to supply all consumer markets in the interval [0,1/2) and the right-hand division all consumer markets in (1/2,1]. Denote the merged firm by a subscript *m* and the remaining firms by a subscript *i*. By symmetry we need only consider consumer markets in the left-hand half of the set of markets. Given the merged firm's decision to relocate to x = ½ - d we can calculate the new Cournot-Nash equilibrium quantities at each consumer location in the left side of the market line in the second-stage of the game. They are:

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6 The precise statement of the paradox is that with linear demand a merger of Cournot firms will be unprofitable unless it involves at least 80 percent of the firms operating in the relevant market. The paradox applies in our model because of the Cournot-at-every-point nature of the equilibrium.

(2a) $$q_m^C(x;d) = \begin{cases} \dfrac{v-(n-1)(1/2-d-x)+(n-2)(1/2-x)}{n} & \text{for } x < \dfrac{1}{2} - d \\ \dfrac{v-(n-1)(x-1/2+d)+(n-2)(1/2-x)}{n} & \text{for } x > \dfrac{1}{2} - d \end{cases} \quad x \in [0,1/2)$$

2(b) $$q_i^C(x;d) = \begin{cases} \dfrac{v-(1/2-x)+(1/2-d-x)}{n} & \text{for } x < \dfrac{1}{2} - d \\ \dfrac{v-(1/2-x)+(x-1/2+d)}{n} & \text{for } x > \dfrac{1}{2} - d \end{cases} \quad x \in [0,1/2), i = 1,..,(n-2)$$

The post-merger product price at each location x on the line is:

(3) $$p^M(x;d) = \begin{cases} \dfrac{v+(1/2-d-x)+(n-2)(1/2-x)}{n} & \text{for } x < \dfrac{1}{2} - d \\ \dfrac{v+(x-1/2+d)+(n-2)(1/2-x)}{n} & \text{for } x > \dfrac{1}{2} - d \end{cases}.$$

The equilibrium price falls as we move from the market periphery towards the merged firm's branch location. If $n = 3$ the consumer markets in the interval (½ - d, ½) are charged a uniform delivered price. However, for $n > 3$ the price in that interval continues to fall as we move from the branch location towards the market center.[7]

The profit earned by the left-hand branch of the merged firm and the profit of each non-merged firm for supplying this side of the market are respectively:

(4a) $$\pi_m^C(d) = \int_0^{1/2} \left(q_m^C(d,x)\right)^2 dx$$

(4b) $$\pi_i^C(d) = \int_0^{1/2} \left(q_i^C(d,x)\right)^2 dx \qquad (i = 1,\ldots,n-2).$$

3. Location Equilibrium for a Single 2-Firm Merger

Using equations (2a) and (4a) we can identify the subgame perfect Nash equilibrium location choice d of the left-hand branch of the merged firm, given by the solution to the first-order profit-maximizing condition:[8]

(5) $$\frac{d\pi_m^C(d)}{dd} = 2\frac{(n-1)}{n^2}\left[\int_0^{1/2-d} q_m^{Cl}(d,x)dx - \int_{1/2-d}^{1/2} q_m^{Cr}(d,x)dx\right]$$

7 See Greenhut and Greenhut (1975) for an analysis of this type of price discrimination.
8 Second-order conditions are satisfied for $v > (n-1)/4$.

where $q_m^{Cl}(d,x)$ is the equilibrium quantity supplied by the merged firm to each consumer market *x* to the left of its branch location and $q_m^{Cr}(d,x)$ is output supplied to each consumer market *x* to the right of its branch location. From equation (5) we have the following proposition:

PROPOSITION 1: *With linear demand and transport costs, a two-firm merger will result in the merged firm locating its branches such that for each branch aggregate sales to the left of the branch equals aggregate sales to the right of the branch.*

Solving equation (5) for the profit-maximizing location *d** (*v*,*n*) gives the following:

PROPOSITION 2: *With linear demand and transport costs a two-firm merger will result in the merged firm locating its two branches a distance d*(v,n) from the market center where:*

(6) $$d^*(v,n) = \frac{1}{4} + \sqrt{\left(\frac{4v-1}{4(n-2)}\right)^2 + \frac{1}{16}} - \frac{4v-1}{4(n-2)}.$$

It follows immediately from (6) that $d^*(v,n) > 1/4$ for $n \geq 3$. The merged firm will relocate its two branches nearer to the market periphery than to the market center. Furthermore, $d^*(v,n)$ is a decreasing function of the maximum reservation price *v*, and an increasing function of the initial number of firms *n*. Table 1 below gives $d^*(v,n)$ for a range of values of *v* and *n*.[9]

The intuition behind these results is straightforward. The left-hand branch of the merged firm faces much stronger competition for consumers to its right, that is, consumer markets located between itself and the non-merged firms at the market center, than it does for consumer markets to its left. Examination of equation (2a) indicates that aggregate sales of the branch fall much more slowly with distance for consumer markets located to its left than they do with distance from consumer markets located to its right. Therefore, because of the stronger competition near the

9 For reasons that will be made clear below, we report $d^*(v,n)$ only for $v > n/2$.

market center, the branch is willing to forego more sales in these local markets and to locate outside the market quartile.

An increase in n increases the degree of competition the merged firm faces from the non-merged firms, which strengthens its desire to distance itself from the market center. By contrast, an increase in the maximum consumer reservation price v has a greater proportionate effect on sales at local markets where sales are relatively low (locations near the market center) than where sales are relatively high. An increase in v therefore makes a location nearer to the quartile more desirable.

(Table 1 near here)

Although we have shown that a two-firm merged firm will find it profitable to relocate its branches away from the center we have not as yet established whether merging is in fact profitable. For mergers to occur it is reasonable to suppose that the profit of the merged firm in the post-merger game must be at least as great as the sum of the profits of the two firms in the pre-merger game. The question is whether the advantage of coordinating location and serving the market "better" or in a more cost-effective way is sufficiently strong to outweigh the disadvantage of the standard Cournot paradox.

The complex nature of equations (1), (4) and (6) makes analytical investigation of merger profitability intractable. However, it is clear from these equations that the profitability of a two-firm merger depends solely upon the two parameters of the model, the maximum reservation price v and the number of firms n. As a result, numerical simulation gives us considerable insight into the potential profitability of a two-firm merger.

Profits to both the non-merged firm and the merged firm are increasing in v. However, our numerical simulations indicate that the profit of each non-merged firm increases more quickly with v than the profit of the merged firm. In other words, we can identify an upper limit $\tilde{v}(n)$ on the maximum consumer reservation price *below* which a two-firm merger is profitable. Table 2 presents the values of $\tilde{v}(n)$ and of the associated equilibrium distance $d^*(v,n)$ for values of n up to 10, from which we can conclude the following:

PROPOSITION 3: *With linear demand and transport costs there exists a critical value $\tilde{v}(n)$ of the maximum consumer reservation price such that a two-firm merger will be profitable for the merged firms if and only if $v < \tilde{v}(n)$. This critical value lies within the feasible range of v for $n \leq 8$.*

(Table 2 near here)

A merger that confers upon the two firms the ability to coordinate their location decisions, correctly anticipating the outcome of the subsequent quantity competition with their non-merged rivals, gives the merged firm sufficient market power to overcome the merger paradox of the standard Cournot model. By contrast with the standard model, a merger results in a "bigger" firm, which has combined the assets of the two merging firms. However, there is a countervailing force: competition from the non-merged firms. It is not surprising, therefore, that two-firm mergers are profitable only if the industry is relatively concentrated, or not too competitive; in our model, for industries such that $n \leq 8$.

What impact does the merger have on the profits of the non-merged firms? There are two conflicting forces at work as a result of the merger. On the one hand, there are now only $(n - 1)$ firms in direct competition with each other at each local market x, and this reduction in the degree of competition over the entire market has a positive effect on the non-merged firms' profitability. On the other hand, for all consumer markets located at a distance greater than $d^*(v, n)/2$ from the market center, the non-merged firms are at a cost disadvantage relative to the merged firm's divisions, and this has a negative impact on profitability. It turns out that the former effect outweighs the latter. *A profitable two-firm merger in this model always increases the profits of the firms outside the merger*.

4. Welfare Effects of a Single 2- Firm Merger

4.1 Prices

In the standard Cournot model a two-firm merger always results in increased consumer prices in the absence of what Farrell and Shapiro (1990) refer to as "cost synergies". A two-firm merger in the spatial Cournot model exhibits such cost synergies since the branches of the merged firm move closer to consumer markets on the market periphery, reducing the costs of the merged firm in supplying those consumer markets. The important question is whether these cost synergies are strong enough to offset the reduced degree of competition that results from the merger. Once again, we need only consider prices in local markets in the interval [0,1/2].

Comparison of equations (1) and (3) indicates that the impact of a two-firm merger on consumer prices in this interval, as measured by the difference between the post-merger and the pre-merger price, is:

(7) $$p^M(x;d) - p^C(x) = \begin{cases} \dfrac{2v + 2x - (2dn + 2d + 1)}{2n(n+1)} & \text{for } x < \dfrac{1}{2} - d \\[2ex] \dfrac{2v + 2x(2n+3) - (3-2d) - 2n(1-d)}{2n(n+1)} & \text{for } x > \dfrac{1}{2} - d \end{cases}.$$

Equation (7) indicates that the consumer markets most likely to benefit from a two-firm merger are those on the market periphery. However, when we substitute the optimal distance $d^*(v,n)$ from Table 1 into equation (7) we can confirm that the only case in which a two-firm merger might actually lead to some consumers being charged lower prices is when $n = 3$, i.e. when there is a relatively high degree of concentration at the center. Also, and not surprisingly, when we substitute $x = 1/2$ in (7) it is easy to confirm that *a two-firm merger always increases prices for consumers located at the market center.*

4.2 Output

Aggregating the output supplied in each local market x over the relevant market areas (integrating equations in (1) and (2)) allows us to calculate the impact of a two-firm merger on the total output supplied by the merged and by the non-merged firms. Denote by q_i^C the aggregate output of each firm in the pre-merger game. For the post-merger game denote by $q_m^C(d)$ the aggregate output of each branch of the merged firm and by $q_i^C(d)$ the aggregate output of each non-merged firm. The differences in output supplied in the post-merger to pre-merger game are as follows:

(8) $$\begin{cases} (a) \quad 2q_m^C(d) - q_i^C(d) = d(1-2d) > 0 \\ (b) \quad q_m^C(d) - q_i^C(d) = -F(v,n,d)/8n < 0 \\ (c) \quad q_m^C(d) - q_i^C = -(n-1)F(v,n,d)/8n(n+1) < 0 \\ (d) \quad q_i^C(d) - q_i^C = F(v,n,d)/4n(n+1) > 0 \end{cases}$$

where $F(v,n,d) = 4v - 4d(1-2d)(n+1) - 1$ and $F(.) > 0$ in the range of the parameters v and n for which mergers are profitable.

In contrast to the standard Cournot model, the merged firm in the spatial model has a greater total output than its non-merged competitors (refer to (8a)). In this sense a two-firm merger results in a "bigger" firm because the merged firm in the spatial model can, by relocating its divisions, exploit its potentially greater size. Nevertheless, each branch of the merged firm produces less total output post-merger than does a non-merged firm (refer to (8b)). Moreover, each non-merged firm produces more total output post merger than it produced prior to the merger (refer to 8(d)). Finally, each branch of the merged firm produces less total output than either of the two firms did pre-merger (8(c)).

4.3 Surplus

In a spaceless model we have the strong result that, "in the absence of fixed costs, all mergers in quantity competition are socially undesirable." (Gaudet and Salant, 1992, p. 153). This is not the case once we introduce spatial competition between the merged and non-merged firms.

Given the impact of the merger on consumer prices noted above, it is clear that a two-firm merger reduces consumer surplus. Furthermore, the loss in consumer surplus increases as both the consumer reservation price v and the number of firms n increase in the parameter range for which a two-firm merger is profitable. Since we confine our attention to two-firm mergers that are profitable, it is not surprising that a two-firm merger leads to an overall increase in aggregate profit of the industry. Aggregate profit is also an increasing function of the maximum reservation price. Because a two-firm merger can be profitable in the spatial model while increasing the overall profitability of the industry we are able to show that a profitable two-firm merger is generally socially efficient.

Figure 1 illustrates the effects of a (profitable) two-firm merger on total surplus, which can be summarized in the following proposition:

PROPOSITION 4: *With linear demand and transport costs, a profitable two-firm merger will increase social surplus even if there are no fixed costs:*

(i) *for the case of* $n = 3$ *and* $v < 2.75$

(ii) *for* $n \geq 4$.

The introduction of strategic location choice in the merger decision has two important effects that give rise to Proposition 4. First, a two-firm merger is potentially profitable for the merged firm as well as for the non-merged firms. Secondly, while consumer surplus is reduced by the merger, the price-increasing effects of the merger are considerably moderated as compared to what would have happened to price if the merging firms had not been able to change their locations. It is easy to see that the change of location by the merged firms results in all consumers in the intervals $[0,(1-d^*(v,n))/2]$ and $[(1+d^*(v,n))/2,1]$ paying lower post-merger prices than they would have paid if the merged firms had instead remained at the market center. As a result, the profit-increasing effect of the merger is almost always sufficient to offset the consumer surplus reducing effect. The only exception, when $n = 3$, arises because, when the number of pre-merger firms is this small, the competition-reducing effects of the merger are particularly great.

5 Extension to More than One 2-Firm Merger

An important question raised by the possibility that a single two-firm merger is profitable is whether the same will be true of several two-firm mergers. Certainly there is ample anecdotal evidence of waves of two-firm mergers in particular industries. In addition, it is interesting to investigate the location pattern of production that will characterize a market in which more than one pair of firms merge. Are the groups of merging firms subject to similar agglomerative forces as those identified by Anderson and Neven (1991) for single-plant firms?

Let us suppose then that there is a set of $2k$ firms, each of which undertakes a two-firm merger to form k two-branch merged firms in an industry that will then contain $n - k$ firms. For this to be different from the single two-firm merger case, we are interested in situations where $n \geq 4$ and $k \geq 2$. As in Section 3, we assume that the merged firms are location leaders in that they simultaneously choose the locations for their branch plants, correctly anticipating the outcome of the subsequent game in which the merged and non-merged firms simultaneously choose their outputs.

5.1 Location Equilibrium

As in Section 3, we assume that each merged firm locates its branches symmetrically around the market center. We can, therefore, confine our attention to the left-hand half of the market. The location of the left-hand branch of merged firm m is denoted d_m ($m = 1,\ldots,k$). We label the merged firms such that $0 \leq d_1 \leq d_2 \leq \ldots \leq d_k \leq 1/2$ in a candidate equilibrium and denote $d = (d_1, d_2,\ldots, d_k)$. Standard Cournot analysis gives the following expressions for equilibrium output of each merged and non-merged firm at each consumer market $x \in [0,1/2]$ in the second-stage output game:

$$\text{(9a)} \qquad q_m^C(x;d) = \frac{\left(v - (n-k+1)\left|d_m - x\right| + \sum_{j=1}^{k}\left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right)}{n-k+1} \qquad m = 1,\ldots, k$$

(9b) $$q_i^C(x;d) = \frac{\left(v - (k+1)\left(\frac{1}{2} - x\right) + \sum_{j=1}^{k} \left|d_j - x\right|\right)}{n-k+1} \qquad i = 1,\ldots, n-2k.$$

The product price in local market x is:

(10) $$p^M(x;d) = \frac{\left(v + \sum_{j=1}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right)}{n-k+1}.$$

Profit of the left-hand branch of a merged firm h is then:

(11) $$(n-k+1)^2 \pi_h(,d,k,n) = \int_{x=0}^{1/2} \left(v - (n-k)\left|d_h - x\right| + \sum_{\substack{j=1 \\ j \neq h}}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right)^2 dx\,.$$

The first-order condition determining the profit-maximizing location $d_h{}^*(v,n,k)$ of this branch is:

(12) $$\frac{(n-k+1)^2}{2(n-k)} \frac{\partial \pi_h}{\partial d_h} = -\int_0^{d_1} \left(v - (n-k)(d_h - x) + \sum_{\substack{j=1 \\ j \neq h}}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right) dx - ..$$

$$-\int_{d_{h-1}}^{d_h} \left(v - (n-k)(d_h - x) + \sum_{\substack{j=1 \\ j \neq h}}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right) dx$$

$$+\int_{d_h}^{d_{h+1}} \left(v - (n-k)(x - d_h) + \sum_{\substack{j=1 \\ j \neq h}}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right) dx + ..$$

$$+\int_{d_k}^{1/2} \left(v - (n-k)(x - d_h) + \sum_{\substack{j=1 \\ j \neq h}}^{k} \left|d_j - x\right| + (n-2k)\left(\frac{1}{2} - x\right)\right) dx = 0.$$

We have an extension of Proposition 1:

Proposition 5: With linear demand and transport costs, if 2k firms in an n firm industry each form a two-firm merger, the merged firms will locate their branches such that for each of the merged firms aggregate sales to the left of each branch equals aggregate sales to the right of each branch.

We can use the first order condition in (12) to derive an agglomeration result that has much in common with that derived in Anderson and Neven (1991).[10]

Proposition 6: With linear demand and transport costs, a set of k two-firm mergers in an n-firm industry will result in the merged firms each locating their two branches the **same** *distance* $d^*(v,n,k)$ *from the market center, where:*

$$d^*(v,n,k)=\frac{1}{4}+\sqrt{\left(\frac{4v-1}{4(n-2k)}\right)^2+\frac{1}{16}}-\frac{4v-1}{4(n-2k)}. \tag{13}$$

As with a single two-firm merger, we can also show that the non-merged firms will not change their locations in response to the location choices of the merged firms.

Table 3 gives values for $d^*(v,n,k)$ for a range of values of v, n and k. It follows from (13) that $d^*(v,n,k) \geq 1/4$ and that $\lim_{k\to n/2} d^*(v,n,k)=1/4$. Also, $d^*(v,n,k)$ is an increasing function of the reservation price v and the number of merged firms k. In other words, we have three agglomerations of firms with the merged firms concentrating their activities outside the market quartiles and the non-merged firms concentrated at the market center. As the number of pairs of merged firms increases, they choose an agglomerated equilibrium closer to the market quartiles. This result is suggestive of a pattern of urban sprawl. Mergers that create multi-product firms also generate clusters of production activity towards the periphery of the initially concentrated center of the market area.

(Table 3 near here)

5.2 Profitability of 2-Firm Mergers

The question to which we now turn is whether or not such a wave of two-firm mergers is profitable. We adopt a similar approach as for a single two-firm merger. For the case of a merger wave we can identify an upper

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[10] The formal proof mirrors that in Anderson and Neven. Second-order conditions are satisfied for $v > n/2$.

limit $\tilde{v}(n,k)$ on the maximum consumer reservation price such that, if *k*-1 pairs of firms are considering merging, then if the maximum reservation price $v < \tilde{v}(n,k)$ an additional two-firm merger is profitable. The results are summarized in Table 4 in which the *non-shaded* cells are those for which the critical value $\tilde{v}(n,k)$ lies within the feasible range. We can conclude the following:

PROPOSITION 7: With linear demand and transport costs there exists a critical value $\tilde{v}(n,k)$ for the maximum consumer reservation price such that k two-firm mergers will be profitable for the merged firms if and only if $v < \tilde{v}(n,k)$. This critical value lies within the feasible range of parameter values for v and n when $n \leq 6$ and $k = 2$.

We have at least some support for the stylized fact on merger waves. Proposition 7 suggests that we are likely to see more than one two-firm merger in relatively concentrated industries.[11] However, and as for the case of a single two-firm merger, the profit advantages of merging are moderated as a result of the external benefits that the non-merged firms enjoy from mergers among their rivals. For in this case as well, an additional two-firm merger leads to an increase in output of the non-merged firms and increases their profits. We explore further the market impact of a second two-firm merger in the next section.

5.3 Price, Output and Surplus Effects of a Second 2-Firm Merger

The effects of a second two-firm merger on consumer prices and individual firm outputs are similar to those for a single two-firm merger. Prices increase in every consumer market, with the result that the second merger further reduces consumer surplus. When there are *two* two-firm mergers in the industry, the reduction in competitive pressures in the local markets leads to each merged firm's total output being greater than if there is a single two-firm merger. Moreover, each merged firm has a greater total output than the remaining non-merged firm(s). However, again similar to the single merger case, each branch of the merged firms is smaller in terms of total output than a non-merged firm. Finally, although location leadership confers some advantages on the firms

11 A similar effect is identified in Daughety (1990). In his model, each pair of merging firms joins a group of industry leaders where the leadership advantage is conferred in the *quantity* game. It turns out that this type of leadership confers such strong market power that *every* two-firm merger is profitable.

participating in the merger wave, these are not so great as to affect adversely the profitability of the firms outside the merger.

Figure 2 illustrates the impact of a second two-firm merger on total surplus and is summarized in the following proposition:

Proposition 8: With linear demand and transport costs, a second two-firm merger that is profitable will increase total surplus even if there are no fixed costs if

(i) $n = 4$ *and* $v \leq 2.786$*;*

(ii) $n = 5$ *or* 6.

(Figure 2 near here)

Once again we see that, when we confine our attention to profitable mergers, the profit-increasing effect of a second two-firm merger is, in general, sufficient to offset the reduction in consumer surplus to which such a merger gives rise. Moreover, the increase in total surplus is greater the greater the initial number of firms (n) in the industry. The intuition behind this is simply explained. A second two-firm merger reduces the number of firms competing at any given consumer location from n-1 to n-2. The *proportionate* impact of such a merger on competitiveness at any given consumer location is, therefore, inversely related to the initial number of firms n.

6 Conclusions

U.S. policy-makers current interest in the unilateral competitive effects of horizontal mergers has no doubt been motivated by the more than twofold increase in number of pre-merger filings over the past five years. It is clearly important to policy-makers to understand what market forces underlie this surge in merger activity, for only then can they evaluate the likely impact mergers will have upon competition. We have shown how horizontal mergers confer profitable advantages to firms who compete in quantities across a set of spatially separated markets. The spatial aspect of competition is critically important because it provides the means by which a horizontal merger can, by combining the assets of two firms, credibly create a bigger firm serving the market area. This firm is smaller than the combined output of the merged firms pre-merger but this should not be surprising since merger can be seen as a particularly effective, since legal, form of collusion between the merging firms.

More importantly, we have also identified the conditions under which such two-firm mergers are profitable and can therefore be expected to occur. Specifically, profitable two-firm mergers are more likely to be found in industries that are relatively concentrated.

Horizontal mergers in spatially differentiated markets generate cost synergies in that the resulting relocation of plants or products leads to the market area being served in a more cost-effective way. However, these cost synergies are not sufficient to offset the price-increasing effects of the reduction in competitive pressures in the local markets to which the merger gives rise. As a result, the merger reduces consumer surplus. Nevertheless, since we confine our attention to profitable mergers *and* both the merged and non-merged firms benefit from the merger, we find that a profitable two-firm merger will almost always increase total surplus and so can be efficiency enhancing.

When we extend the analysis to allow for the possibility of more than one two-firm merger we find something of the same story. With six or fewer firms in the industry pre-merger, we find that provided demand is not too strong, as measured by the maximum consumer reservation price, two two-firm mergers are profitable. The reduction in the upper limit on the number of firms for which merger is profitable arises because of the external benefits the non-merged firms derive from a merger. Once one pair of firms has merged the hurdle for a second two-firm merger to be profitable is raised. Nevertheless, we have a story that is at least to some extent consistent with the evidence that mergers tend to spawn further mergers. Moreover, an additional, profitable two-firm merger can also be expected to be efficiency enhancing.

With respect to location choice, we start from a scenario in which all firms are agglomerated at the center of the market area. A single two-firm merger leads the merging firms to disperse their branches, locating the branch plants nearer to the market peripheries than to the market center. The degree to which the merging firms disperse is an increasing function of the initial number of firms in the market, reflecting the fact that dispersal is desired in order to moderate competition with the non-merged firms remaining at the market center.

When the analysis is extended to consider more than a single two-firm merger, we find that the merged firms face the same pressure to agglomerate as characterize the non-merged firms. In other words, a set of two-firm mergers leads to the pairs of merged firms choosing to agglomerate their branch plants while the non-merged firms remain agglomerated at the market center. This implies that, while two-firm mergers improve locational efficiency to some extent, the improvement is far from perfect. A single two-firm merger generates locations that are "too far" from the market center in pure efficiency terms, and, as we have indicated, several two-firm mergers generate a further locationally inefficient agglomeration of firms. In other words, mergers are consistent with the phenomenon of urban sprawl, as satellite hubs of production activity emerge that are located away from the center of the market area.

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Table 1: Equilibrium Distance *d(*v*,*n*)**

		Number of Firms (*n*)					
		3	4	5	6	7	8
Reservation Price (*v*)	2	0.2678	0.2850				
	3	0.2613	0.2725	0.2835	0.2940		
	4	0.2583	0.2666	0.2748	0.2828	0.2906	0.2981
	5	0.2566	0.2631	0.2696	0.2760	0.2823	0.2885
	6	0.2554	0.2608	0.2662	0.2716	0.2769	0.2821
	7	0.2546	0.2592	0.2638	0.2684	0.2730	0.2774

Table 2: Critical Value of Reservation Price
For a Profitable Two-Firm Merger

Number of Firms (*n*)	Upper Limit on Reservation Price $(\tilde{v})$	Distance *d**(*v*,*n*)
3	4.4089	0.2575
4	3.2390	0.2708
5	3.2782	0.2805
6	3.5124	0.2875
7	3.8132	0.2926
8	4.1443	0.2965
9	4.4917	0.2995
10	4.8489	0.3020

Table 3: Equilibrium Distance *d(*v*,*n*,*k*)**

(a) *n* = 4

Reservation Price (*v*)	*k* = 1	*k* = 2
2	0.2850	0.25
2.5	0.2774	0.25
3	0.2725	0.25
3.5	0.2691	0.25
4	0.2666	0.25
4.5	0.2647	0.25

(b) *n* = 5

Reservation Price (*v*)	*k* = 1	*k* = 2
2.5	0.2906	0.2638
3	0.2835	0.2613
3.5	0.2785	0.2596
4	0.2748	0.2583
4.5	0.2719	0.2573

(c) *n* = 6

Reservation Price (*v*)	*k* = 1	*k* = 2	*k* = 3
3	0.2940	0.2725	0.25
3.5	0.2876	0.2691	0.25
4	0.2828	0.2666	0.25
4.5	0.2790	0.2647	0.25

(d) *n* = 7

Reservation Price (*v*)	*k* = 1	*k* = 2	*k* = 3
3.5	0.2964	0.2785	0.2596
4	0.2906	0.2748	0.2583
4.5	0.2860	0.2719	0.2573

Table 4: Critical Value of Reservation Price for a Profitable Two-Firm Merger

		Number of Pairs of Merged Firms (*k*)			
		1	2	3	4
Initial Number of Firms (*n*)	3	4.4089			
	4	3.2390	4.2831		
	5	3.2782	3.1006		
	6	3.5124	3.1470	2.9118	
	7	3.8132	3.3903	2.9677	
	8	4.1443	3.6988	3.2276	2.6944

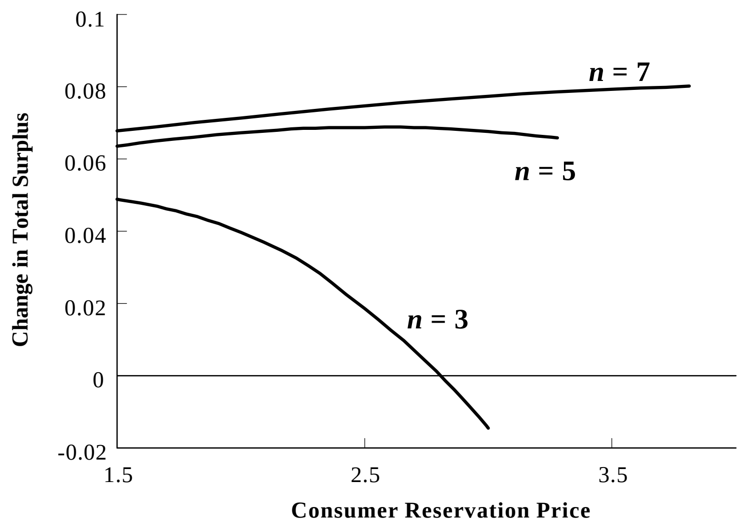

Figure 1: Impact of a Two-Firm Merger on Total Surplus

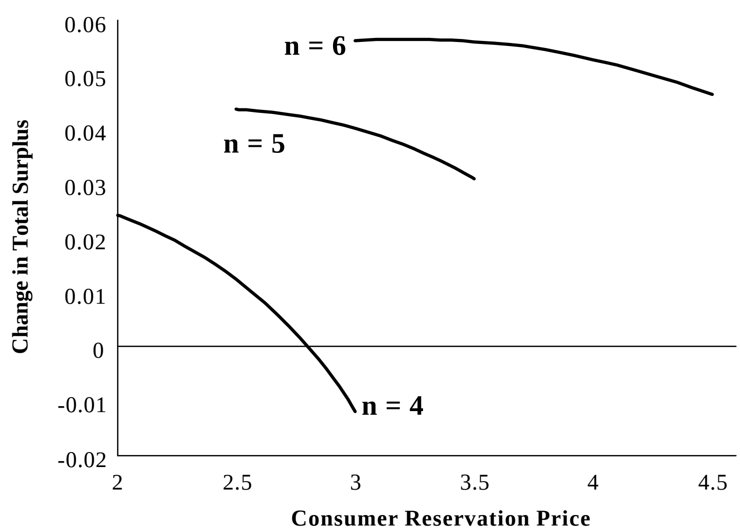

Figure 2: Impact of a Second Two-Firm Merger on Total Surplus

APPENDIX

Claim 1: If $v \geq n/2$ and if all n firms are located in the center of the market then no one single firm finds it profitable to deviate from its location at the center of the market, given that the remaining (n-1) firms are at the center.

Assume that firm 1 locates a distance d from the left-hand side of the market, with $0 < d \leq 1/2$. We confine explicit analysis to the case in which the deviating firm can supply the entire set of markets along the line. This implies that $d \geq \frac{(n+1)-2v}{2n}$, a not particularly restrictive constraint given that $v \geq n/2$. The proof can easily be generalized to the remaining values of d.

Profit to firm 1 is:

$$\pi_1(d) = \int_0^d \frac{\left(v - n(d-r) + (n-1)(1/2 - r)\right)^2}{(n+1)^2} dr + \int_d^{1/2} \frac{\left(v - n(r-d) + (n-1)(1/2 - r)\right)^2}{(n+1)^2} dr + \int_{1/2}^{1} \frac{\left(v - n(r-d) + (n-1)(r - 1/2)\right)^2}{(n+1)^2} dr$$

The first-order condition on d is:

$$\frac{\partial \pi_1(d)}{\partial d} = \frac{n(1-2d)(4v - n - 2d(n-1) - 1)}{2(n+1)^2}.$$

This is positive for $v \geq n/2$ and $d < 1/2$.

Claim 2: After the merged firm has relocated its two plants a distance d^* from the center, no non-merged firm has an incentive to locate away from the market center, anticipating the subsequent quantity competition game to be played between itself and all of its rivals, merged and non-merged.

We deal explicitly with the case in which there is one two-firm merger and the potential deviating firm locates somewhere between the center and merged firm’s location on the line, which for ease of exposition we denote here by d^*. Once again, the proof can be easily extended to choosing locations between 0 and d^*. If the deviating firm 1 locates distance u from the left-hand edge of the market its profit is:

$$\pi_1(u) = \int_0^{d*} \frac{(v-(n-1)(u-r)+(d*-r)+(n-3)(1/2-r))^2}{n^2}\,dr +$$
$$\int_{d*}^{d} \frac{(v-(n-1)(u-r)+(r-d*)+(n-3)(1/2-r))^2}{n^2}\,dr +$$
$$\int_{d}^{1/2} \frac{(v-(n-1)(r-u)+(r-d*)+(n-3)(1/2-r))^2}{n^2}\,dr +$$
$$\int_{1/2}^{1-d*} \frac{(v-(n-1)(r-u)+(1-d*-r)+(n-3)(r-1/2))^2}{n^2}\,dr +$$
$$\int_{1-d*}^{1} \frac{(v-(n-1)(r-u)+(r-1+d*)+(n-3)(r-1/2))^2}{n^2}\,dr$$

The first-order condition on *u* is:

$$\frac{\partial \pi_1(u)}{\partial u} = \frac{(1-2u)(n-1)(4v-n-2u(n-4)-4d*)}{2n^2}.$$

Since $v \geq n/2$, $d* < 1/4$ and $d \leq 1/2$, this is positive for any $u < 1/2$.

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