

# Prices versus Exams as Strategic Instruments for Competing Universities

Elena Del Rey\* and Laura Romero†

October 2004

## Abstract

In this paper we investigate the optimal choice of prices and/or exams by universities in the presence of credit constraints. We first compare the optimal behavior of a public, welfare maximizing, monopoly and a private, profit maximizing, monopoly. Then we model competition between a public and a private institution and investigate the new role of exams/prices in this environment. We find that, under certain circumstances, the public university may have an interest to raise tuition fees from minimum levels if it cares for global welfare. This will be the case provided that (i) the private institution has higher quality and uses only prices to select applicants, or (ii) the private institution has lower quality and uses also exams to select students. When this is the case, there are efficiency grounds for raising public prices.

Keywords: Analysis of education.

JEL Classification: I21

---

\*Universitat de Girona. Campus de Montilivi 17071 Girona (Spain). Tel: ++ 34 972 41 87 49  
Fax: ++ 34 972 41 80 32 E-mail: elena.delrey@udg.es

†Universidad Carlos III de Madrid

The authors gratefully acknowledge financial support from the Spanish Science and Technology Ministry (Project SEC 2002-02606) and the hospitality of Université de Toulouse. We are grateful to Dolors Berga, Nicolas Bocard and seminar participants at the University of Adelaide (Australia) for their helpful comments. Errors remain ours.

# 1 Introduction

European public universities have traditionally relied on exams, rather than prices, to select their students, while operating costs have been covered largely by general taxation. The often poor attainment of these universities has recently called for higher fees and decentralization, whereas private institutions are being welcome to enter the higher education sector.

These tendencies raise a new interest in the role of prices in higher education and of competition among private and increasingly decentralized public universities. Are there any reasons other than the obvious increasing budgetary constraints for a public university to raise prices? Can this decision be conditional on the existence and behavior of a private university?

The aim of this paper is to study the strategic choice of exams and prices by educational institutions (e.g. universities) in competition.

In order to do that, let us start by recalling the main difference between prices and exams as allocation devices: while fees make students self-select according to willingness and ability to pay, exams are used by schools to select students according to revealed ability to learn. Willingness to pay is a good approximation for student ability to learn as long as the marginal return to education of higher quality is larger for higher ability students. This is the case if student ability and school quality are complements in the production of education, which seems a reasonable assumption in the case of higher education. In this case, and in the absence of credit constraints, students make adequate choices regarding their enrollment to existing schools. Markets and exams could then be equivalent allocation devices for the universities. However, it is generally acknowledged that the existence of credit constraints prevents a pure price mechanism from attaining the optimal allocation of students to schools of different quality. It is in this context that Fernandez (1998) and Fernandez and Gali (1999) show the superiority of exams in terms of welfare when the exam technology is sufficiently powerful.<sup>1</sup>

Still, the reaction to the existence of borrowing constraints may differ according to the objectives of the university and the market conditions. Concerning the first,

---

<sup>1</sup>There are other reasons why a price mechanism may not attain the optimal allocation. For example, higher education is a risky investment and the aversion to risk makes that, even in absence of borrowing constraints, individual decisions are not optimal. Therefore, even if several countries are introducing financing schemes that aim at eliminating credit constraints, this does not guarantee that prices alone will be optimal. We then consider credit constraints as a simple way to model general mismatches between decentralized and optimal decisions.

we consider two limiting cases: a public university that maximizes total student surplus and a private institution that maximizes profits. This specification of public/private firm payoffs is extreme, though usual in the literature on mixed oligopolies (see De Fraja and Delbono (1990), Cremer et al. (1991) and Grilo (1994)). It is also the specification used by Gary-Bobo and Trannoy (2004), who study the role of prices and exams in presence of asymmetries of information concerning student ability and with borrowing constraints. There is no competition in their work, but they consider two types of university: a philanthropic university and a for-profit or rent-seeking university.

In what can be considered a simplified version of their model, we study the behavior of these institutions when isolated and in presence of borrowing constraints. In order to explore the strategic value of exams and prices, we next let the two types of institutions share the market.

In this paper, public universities care for the welfare of the whole student body. Indeed, being totally funded by public money, it seems reasonable to assume their not caring for own students alone. It is, in this sense, as if we considered the public sector in competition with the private.

We show that, in presence of credit constraints, the public university sets the lowest possible price while the use of exams allows to maintain the quality of enrollments. In contrast, the private university may choose to combine prices and exams. These results are consistent with Gary-Bobo and Trannoy (2004) and are used here as a benchmark.

With competition, we show that the public university may have an interest in raising its price above the minimum when it cares for global welfare. Such higher fee has the effect of increasing enrollments at the private institution while limiting the access to the public institution, and may increase global welfare provided that

1. the private university has higher quality and it does not use exams to select students
2. the private university has lower quality and it does use exams to select applicants

There are, hence, efficiency grounds for raising public prices. This is also what Gary-Bobo and Trannoy (2004) concludes, albeit for different reasons. In their case, the mix of prices and exams proves useful when there is bilateral asymmetric information: both students and universities have some information about student ability that the other does not have.

The paper is organized as follows: section 2 presents the model. We describe the characteristics and behavior of students and universities and introduce the alternative allocation devices. Section 3 analyzes the monopoly benchmark referred to both a public and a private monopoly. Section 4 considers competition and section 5 concludes.

## 2 The Model

### 2.1 Individuals

The economy consists of a continuum of individuals  $i$  of measure one, characterized by a different ability,  $a^i$ , and by an initial income endowment,  $w^i$ , both uniformly and independently distributed over the interval  $[0, 1] \times [a^-, a^+]$  with  $a^+ - a^- = 1$ . Individuals derive utility from their total lifetime income, which consists of their initial income endowment,  $w^i$ , and their earnings if they become educated,  $a^i Q_j$ , where  $Q_j$  is the educational quality provided by university  $j$ . Thus, ability and educational quality are complements in the production of human capital.<sup>2</sup> Therefore, a student  $i$  with ability  $a^i$  and income  $w^i$  and enrolled at university  $j$  of quality  $Q_j$  and charging a price  $p_j$  enjoys the following utility:

$$U_j^i = w_i - p_j + a^i Q_j. \quad (1)$$

### 2.2 Universities

We consider universities that produce educational services of a *given quality*.<sup>3</sup> In this sense, the analysis can be considered to take a short run approach, with quality changes taking place only in the long run.

Public and private universities differ in their objectives: while the public university maximizes welfare (the difference between utility generated and costs incurred), the private institution maximizes profits.

---

<sup>2</sup>This assumption is crucial to characterize the efficient allocation of students in our model and standard in the literature.

<sup>3</sup>In ongoing research, we consider a three stage game in which quality is chosen first, then prices and exams (see Romero and Del Rey, 2004). In that work, the public university cares only for own students' surplus. It is easy to verify that, in this case, there is no competition as such. The high quality institution behaves as a monopoly and the other one takes leftovers at choice, provided that some market share is left available, without further interacting with the first.

University  $j$  (where  $j = pb, pv$  stand respectively for public and private) incurs per capita costs  $C_j(\bar{a}_j)$ , where  $\bar{a}_j$  is the average ability of the student body at university  $j$ . This cost is assumed to be decreasing and convex in the average ability of the student body  $\bar{a}_j$ ,  $C'_j < 0$ ,  $C''_j > 0$ . In order to guarantee the concavity of the payoffs, the cost function is assumed to be sufficiently convex.<sup>4</sup>

The timing is the following: given quality, universities choose a price and observe demand. Then, they decide whether to run an exam in order to limit the quantity/quality of admissions. Prices and/or limiting admission grades are characterized as allocation mechanisms as follows.

**Exams** Suppose first that some regulation in place compels universities from optimally setting their fees. In such a case, the university may use an entry exam to select the best students among those who are willing to attend the university. In order to do so, it establishes a minimum score,  $s^E$  such that those who obtain a score equal or higher,  $s^i \geq s^E$  are accepted by the university. We assume the exam technology to be able to perfectly reveal the student's ability, which means that the students obtaining a score  $s \geq s^E$  at university  $j = pb, pv$  are those with an ability  $a^i \geq a_j^E$ .

**Prices** Suppose now that universities are not allowed to use exams as selection devices but must fix the price of the services they provide. By choosing a price  $p_j$ , the university indirectly determines the type of students (characterized by their ability and income) who are willing and able to enroll, given quality  $Q_j$ .

When there is only one school, students compare their utility with and without education. Let  $\hat{a}_j$  be the ability of the student who is indifferent between attending school  $j$  and remaining uneducated:

$$w^i = w^i - p_j + \hat{a}_j Q_j \Rightarrow \hat{a}_j = p_j / Q_j.$$

All students with ability  $a > \hat{a}_j$  are willing to attend university  $j$ . Among those, only students with income  $w > p_j$  are able to do so in the presence of liquidity constraints.

---

<sup>4</sup>This simplification rules out potentially interesting results. Although a complete analysis of the game between two universities would require accounting for all possible solutions, we leave this matter for further research.

### 3 The monopoly benchmark

We want to compare the reaction of public and private universities to the existence of liquidity constraints in a situation in which there is only one institution, either public or private, active in the higher education market. The objective of the public institution is to maximize total surplus:

$$U_{pb} = \int_{p_{pb}}^{a^+} \int_{a_{pb}}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw, \quad (2)$$

where  $p_{pb}$  is the public price,  $a_{pb} = \max\{\hat{a}_{pb}, a_{pb}^E\}$  is the limiting admission grade in the public university and  $\bar{a}_{pb} = \frac{a^+ + a_{pb}}{2}$  is the mean ability of students attending the public university.

On the other hand, the private university maximizes profits:

$$U_{pv} = \int_{p_{pv}}^{a^+} \int_{a_{pv}}^{a^+} (p_{pv} - C(\bar{a}_{pv})) da dw, \quad (3)$$

where  $p_{pv}$  and  $a_{pv} = \max\{a_{pv}^E, \hat{a}_{pv}\}$  stand respectively for prices and the minimum ability at the private school and  $\bar{a}_{pv} = \frac{a^+ + a_{pv}}{2}$  is the mean ability of students attending the private university.

Since we solve the problem backwards, we first look at the condition that defines the optimal admission grade given prices. This may be larger than or equal to the ability of the last student willing to attend the university, but will always be smaller than  $a^+$  if the university takes students at all. Then we look at the choice of prices. We need at this stage to consider two cases: either exams are going to be used or not. In some cases we will find two optimal strategies, one with exams, the other without. Which is the one resulting at equilibrium depends on the benefits from limiting admissions, i.e. the cost technology. In other cases we will be able to eliminate one of them by contradiction.

We first consider a public monopoly.

#### 3.1 Public Monopoly

The optimal admission grade  $a_{pb}^E \geq \hat{a}_{pb}$  is given by

$$-a_{pb}^E Q_{pb} + C_{pb} - (a^+ - a_{pb}^E) C'_{pb}/2 \leq 0. \quad (4)$$

At stage 1, the objective function of the public university is (2) with  $a_{pb} = \max\{\widehat{a}_{pb}, a_{pb}^E\}$  and  $\bar{a}_{pb} = \frac{a^+ + a_{pb}}{2}$ . The optimal public price is given by:

$$-\left(\frac{a^+ - a_{pb}^2}{2}Q_{pb} - (a^+ - a_{pb})C_{pb}\right) + \frac{\partial a_{pb}}{\partial p_{pb}}(1 - p_{pb})(-a_{pb}Q_{pb} + C_{pb} - (a^+ - a_{pb})C'_{pb}/2) = 0. \quad (5)$$

If  $a_{pb}^E \geq \widehat{a}_{pb}$  is not binding, (4) is satisfied with equality and (5) is negative provided that the university payoff is positive,<sup>5</sup>

$$\frac{a^+ - a_{pb}^2}{2}Q_{pb} - (a^+ - a_{pb})C_{pb} > 0 \quad (6)$$

If, on the other hand,  $a_{pb}^E \geq \widehat{a}_{pb}$  is binding, (4) is negative and so is (5). Therefore, (5) is always negative and the optimal price goes to zero. The allocation of students is determined by (4).

### 3.2 Private Monopoly

The optimal limiting admission grade  $a_{pv}^E \geq \widehat{a}_{pv}$  is given by

$$-p_{pv} + C_{pv} - (a^+ - a_{pv}^E)C'_{pv}/2 \leq 0. \quad (7)$$

The optimal private price is, in turn, given by:

$$(1 - 2p_{pv} + C_{pv})(a^+ - a_{pv}) - \frac{\partial a_{pv}}{\partial p_{pv}}(1 - p_{pv})(p_{pv} - C_{pv} + (a^+ - a_{pv})C'_{pv}/2) = 0. \quad (8)$$

We may identify two types of optimal policy depending on the production technology represented by the cost function.

In the first type, the benefits of limiting admissions in terms of lower costs are not enough to compensate the loss of revenues. Then  $a_{pv}^E = a_{pv}$  and selection via exams does not take place. (7) is negative and (8) has an interior solution at some  $p_{pv} < (1 + C_{pv})/2$ .

In the second type of optimum  $a_{pv}^E > \widehat{a}_{pv}$ . (7) is satisfied with equality and exams take place at the last stage. The optimal price is  $p_{pv} = (1 + C_{pv})/2$ .<sup>6</sup> Whether one type or the other results in equilibrium depends entirely on the cost technology.

<sup>5</sup>Note that, otherwise, the operating costs of the public university would exceed its contribution to welfare, so that its mere existence would be unjustified.

<sup>6</sup>Note that we are unable to compare the optimal price corresponding to each case since  $C_{pv}$  is lower the higher the average ability of enrolled students.

Therefore, when we have a monopoly of higher education, if it is a public monopoly it will set the lowest possible price and select students via exams. If it is a private monopoly, it may set only prices or combine prices and exams depending on the production technology, which determines the benefits of limiting enrollments in terms of lower costs.

Let us now see how these choices may be affected by competition.

## 4 Competition between Public and Private Universities

When there are two universities in the market, the shape of demand and hence of the payoffs of the universities differ depending on students' relative preferences for both institutions (denoted by  $\succ$ ), which, in turn, depend upon chosen prices and exogenous qualities. In order to determine the optimal choices of the institutions at stake, we first need to characterize the set of possible market partitions and corresponding payoffs.

We then start with the characterization of students' preferences given qualities and prices.

We denote by  $\tilde{a}_j \in [a^-, a^+]$  the ability of the individual who is indifferent between both universities when  $Q_j \geq Q_{-j}$ . From (1):

$$\tilde{a}_i = \frac{p_j - p_{-j}}{Q_j - Q_{-j}} \text{ for } j = pb, pv.$$

On the other hand, there may also exist an individual with ability  $\hat{a}_j \in [a^-, a^+]$ ,  $j = pb, pv$ , who is indifferent between attending the school  $j$  or remaining uneducated. As we saw before,  $\hat{a}_j = p_j/Q_j$ .

Let Case A refer to  $Q_{pb} \geq Q_{pv}$  and Case B to  $Q_{pb} < Q_{pv}$ .

**Case A.1.** If  $Q_{pb} \geq Q_{pv}$ , and the public, high-quality institution sets a lower price,  $p_{pb} < p_{pv}$  all students who want to attend university prefer the public rather than the private university:  $\tilde{a}_{pb} < a^- < \hat{a}_{pb} < \hat{a}_{pv}$ , which corresponds to the following ordering of preferences:

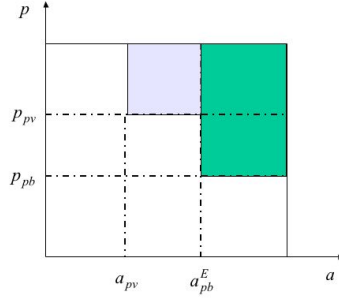
- $a^i \in (a^-, \hat{a}_{pb}) : 0 \succ pb \succ pv$ .
- $a^i \in (\hat{a}_{pb}, \hat{a}_{pv}) : pb \succ 0 \succ pv$ .
- $a^i \in (\hat{a}_{pv}, a^+) : pb \succ pv \succ 0$ .

Of those rejected from the public university ( $a < a_{pb}^E$ ), some are willing and can afford to attend the private university. These are individuals of ability  $a \geq \hat{a}_{pv}$  and



initial endowment  $w \geq p_{pv}$ . Of course, for this case to be relevant, we need that the limiting admission grade at the public university  $a_{pb}^E > \hat{a}_{pv}$ . Otherwise, students rejected from the public university would prefer to remain uneducated and we would be facing a public monopoly.

Figure 1 represents the allocation of students to universities in this case.



The payoffs of the universities are then:

$$U_{pb} = \int_{p_{pb}}^1 \int_{a_{pb}}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw + \int_{p_{pv}}^1 \int_{a_{pv}}^{a_{pb}} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw, \quad (9)$$

and

$$U_{pv} = \int_{p_{pv}}^1 \int_{a_{pv}}^{a_{pb}} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw, \quad (10)$$

where  $\bar{a}_{pb} = (a^+ + a_{pb})/2$ ,  $\bar{a}_{pv} = (a_{pb} + a_{pv})/2$ ,  $a_{pb} = \max\{\hat{a}_{pb}, a_{pb}^E\}$  and  $a_{pv} = \max\{\hat{a}_{pv}, a_{pv}^E\}$  and we assume  $a_{pb}^E > \hat{a}_{pv}$ .

Conversely, if the price of the public high-quality university is higher than the price of its private competitor, we must differentiate two cases:

- $\frac{p_{pb}}{p_{pv}} > \frac{Q_{pb}}{Q_{pv}}$ , which is equivalent to  $\hat{a}_{pb} > \hat{a}_{pv}$  and also to  $\tilde{a}_{pb} > \hat{a}_{pb}$  by definition of  $\tilde{a}_j, \hat{a}_j$  for  $j = pb, pv$ . Therefore, we obtain the following ordering of ability thresholds  $\hat{a}_{pv} < \hat{a}_{pb} < \tilde{a}_{pb}$ , which corresponds to the following ordering of preferences:

- $a^i \in (a^-, \hat{a}_{pv}) : 0 \succ pv \succ pb$
- $a^i \in (\hat{a}_{pv}, \hat{a}_{pb}) : pv \succ 0 \succ pb$ .
- $a^i \in (\hat{a}_{pb}, \tilde{a}_{pb}) : pv \succ pb \succ 0$ .
- $a^i \in (\tilde{a}_{pb}, a^+) : pb \succ pv \succ 0$ .

- $\frac{p_{pb}}{p_{pv}} < \frac{Q_{pb}}{Q_{pv}}$ , which is equivalent to  $\hat{a}_{pb} < \hat{a}_{pv}$  and also to  $\tilde{a}_{pb} < \hat{a}_{pb}$ . The ordering of ability thresholds is thus,  $\hat{a}_{pv} > \hat{a}_{pb} > \tilde{a}_{pb}$  and the preference ordering is the following:

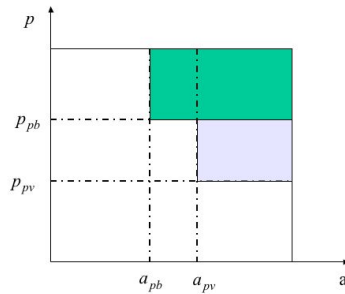
- $a^i \in (a^-, \tilde{a}_{pb}) : 0 \succ pv \succ pb$ .
- $a^i \in (\tilde{a}_{pb}, \hat{a}_{pb}) : 0 \succ pb \succ pv$ .
- $a^i \in (\hat{a}_{pb}, \hat{a}_{pv}) : pb \succ 0 \succ pv$ .
- $a^i \in (\hat{a}_{pv}, a^+) : pb \succ pv \succ 0$ .

Therefore, when both public prices and quality are higher, enrollments can follow one of these patterns:

$$p_{pb} > p_{pv} \begin{cases} \frac{p_{pb}}{p_{pv}} > \frac{Q_{pb}}{Q_{pv}} \Rightarrow \hat{a}_{pv} < \hat{a}_{pb} < \tilde{a}_{pb} \begin{cases} a_{pb}^E > a_{pv}^E, & [1] \\ a_{pb}^E < a_{pv}^E, & [2] \end{cases} \\ \frac{p_{pb}}{p_{pv}} < \frac{Q_{pb}}{Q_{pv}} \Rightarrow \tilde{a}_{pb} < \hat{a}_{pb} < \hat{a}_{pv} \begin{cases} a_{pb}^E > a_{pv}^E, & [3] \\ a_{pb}^E < a_{pv}^E, & [4] \end{cases} \end{cases}$$

where all  $a_j^E$  are assumed to be non binding or interior (otherwise they would not be responsible for the shape of the payoff). What is meant by this table is to make clear that, whatever the ordering of student preferences, it is always possible for the universities to change the pattern of enrollments by means of the limiting admission grade. Ultimately, we are only concerned by such final patterns of enrollments or market partitions. Therefore, we group subcases together as follows.

**Case A.2.** Since the shape of the payoff functions is the same in both cases (albeit for different reasons), we group [2] and [4] in Case A.2 which hence studies the possibility that the private institution enrolls students of higher average quality when public quality is larger. Graphically,



The payoffs of the universities are in this case

$$U_{pb} = \int_{p_{pb}}^1 \int_{a_{pb}^E}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw + \int_{p_{pv}}^{p_{pb}} \int_{a_{pv}^E}^{a^+} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw, \quad (11)$$

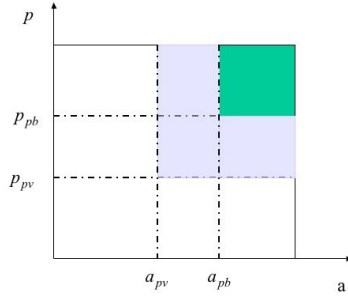
where  $\bar{a}_{pb} = (a^+ + a_{pb}^E) / 2$  and  $\bar{a}_{pv} = (a^+ + a_{pv}^E) / 2$ .

$$U_{pv} = \int_{p_{pv}}^{p_{pb}} \int_{a_{pv}^E}^{a^+} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw. \quad (12)$$

Although this case is in principle feasible, note that if the objective of the public university is to maximize global welfare, it will always be better off by setting prices at zero than by letting some good students attend the private low quality school. We can, for this reason, disregard this case.

**Case A.3.** Similarly, we group [1] and [3] together in Case A.3, which analyzes the possibility that the public university is more selective when its price is higher.

Figure 3 represents the market partition corresponding to this case:



The payoffs of the universities are

$$U_{pb} = \int_{p_{pb}}^1 \int_{a_{pb}}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) dw da + \int_{p_{pv}}^{p_{pb}} \int_{a_{pb}}^{a^+} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw + \int_{p_{pv}}^1 \int_{a_{pv}}^{a_{pb}} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw, \quad (13)$$

and

$$U_{pv} = \int_{p_{pv}}^{p_{pb}} \int_{a_{pb}}^{a^+} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw + \int_{p_{pv}}^1 \int_{a_{pv}}^{a_{pb}} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw, \quad (14)$$

where  $\bar{a}_{pb} = (a^+ + a_{pb}^E) / 2$  and

$$\bar{a}_{pv} = \frac{(a^+ - a_{pb}^2)(p_{pb} - p_{pv}) + (a_{pb}^2 - a_{pv}^2)(1 - p_{pv})}{2((a^+ - a_{pb})(p_{pb} - p_{pv}) + (a_{pb} - a_{pv})(1 - p_{pv}))}, \quad (15)$$

where  $a_{pb} = \max \{a_{pb}^E, \hat{a}_{pb}, \tilde{a}_{pb}\}$  and  $a_{pv} = \max \{a_{pv}^E, \hat{a}_{pv}\}$ .

A similar procedure identifies all possible enrollment patterns when  $Q_{pv} > Q_{pb}$  (case B). By substituting subscripts  $pv$  and  $pb$  by each other we obtain cases B.1, B.2 and B.3.

In the following sections, we successively assume to be in each case in order to study the corresponding optimal choice of prices and exams. Note that, as we vary prices (and/or limiting admission grades), the demand schedules change smoothly, without giving rise to discontinuities in the changes of regime. The payoffs are hence continuous.

We start our analysis with Case A.1, characterized by  $Q_{pb} > Q_{pv}$  and  $p_{pb} < p_{pv}$ .

#### 4.1 Case A.1

Since  $Q_{pb} > Q_{pv}$  and  $p_{pb} < p_{pv}$  the public university is preferred by all the potential students. Then, for the private university to have access to the market, it has to be the case that the public university uses selective exams. Moreover, the private school cannot be more selective, otherwise it would not even be in the market. For case A.1 to result in equilibrium we then need that  $a_{pb}^E > a_{pv}$ , where  $a_{pv} = \max \{a_{pv}^E, \hat{a}_{pv}\}$ .

Hence, some students willing to attend the public university are rejected admission there and turn to the private university, as they prefer to pay for lower quality rather than being left uneducated (see Figure 1).

**Exams** At this stage, prices, and hence demand, are taken as given by the university.

The objective of the public university is to maximize (9) subject to  $a_{pb}^E \geq 0$ . The objective of the private university is to maximize (10) subject to  $a_{pv}^E \geq \hat{a}_{pv}$ .

The first order condition that determines the limiting admission grade at the public university is satisfied with equality for the reasons just explained and writes

$$\begin{aligned} & (1 - p_{pb}) (-a_{pb}^E Q_{pb} + C_{pb} - (a^+ - a_{pb}^E) C'_{pb} / 2) \\ & + (1 - p_{pv}) (a_{pb}^E Q_{pv} - C_{pv} - (a_{pb}^E - a_{pv}) C'_{pv} / 2) = 0. \end{aligned} \quad (16)$$

The private university chooses a limiting admission grade  $a_{pv}^E \geq \hat{a}_{pv}$  such that

$$-p_{pv} + C_{pv} - (a_{pb}^E - a_{pv}^E)C'_{pv}/2 \leq 0. \quad (17)$$

Intuitively, as before, the constraint  $a_{pv}^E \geq \hat{a}_{pv}$  is binding if the reduction in costs when raising the minimum ability is small compared to the loss in revenues. If the reduction in costs due to the enrollment of more able students is not too large, it is not worth being selective (which after all has a cost since it reduces enrollments and hence revenues) and all students willing to enroll are accepted. If, on the contrary, the benefits of raising the average ability of the student body are larger than the costs in terms of reduced revenues, the private university will use selective exams.

**Prices** The public university chooses  $p_{pb}$  so as to maximize (9). The optimal price for the public university  $p_{pb}$  is given by the following condition:

$$\begin{aligned} & - \left( \frac{a^+ - a_{pb}^2}{2} Q_{pb} - (a^+ - a_{pb}) C_{pb} \right) + \\ & (1 - p_{pv}) \left( -a_{pv} Q_{pv} + C_{pv} - (a_{pb} - a_{pv}) C'_{pv}/2 \right) \frac{\partial a_{pv}}{\partial p_{pb}} = 0 \end{aligned} \quad (18)$$

since  $a_{pb} = a_{pb}^E$  and with  $a_{pv} = \max\{\hat{a}_{pv}, a_{pv}^E\}$ .

The contribution to welfare by the public university must be positive, since otherwise its existence would be unjustified (global welfare would be larger without it):

$$(1 - p_{pb})(a^+ - a_{pb}) \left( \frac{a^+ + a_{pb}}{2} Q_{pb} - C_{pb} \right) > 0.$$

The first term in (18) is then negative. Still, the optimal price may be positive. For this to be the case, the private institution needs to use exams, since  $\frac{\partial \hat{a}_{pv}}{\partial p_{pb}} = 0$ .

In contrast, by a simple comparative static analysis,  $\frac{\partial a_{pv}^E}{\partial p_{pb}} < 0$ . Moreover, if  $a_{pv} = a_{pv}^E$ , (17) is satisfied with equality, which implies that  $a_{pb}^E > \hat{a}_{pv}$  and hence  $p_{pv} < a_{pv}^E Q_{pv}$ . We also know that the public university has higher quality than the private and needs to be more selective for the two universities to share the market. Then,  $p_{pv} < a_{pv}^E Q_{pv} < a_{pb}^E Q_{pv}$ . As a result, the term  $-a_{pv} Q_{pv} + C_{pv} - (a_{pb} - a_{pv}) C'_{pv}/2$  in (18) is negative. Therefore, the necessary (not sufficient) conditions for a positive public price are satisfied when the private institution uses exams to select students.

We thus find that optimal public prices are zero if the lower quality private institution does not use exams and may be positive if the private institution uses

exams as a way to select applicants. The intuition underlying this result is the following: public prices do not affect the willingness to attend the private school (in any case, it is less preferred than the public) but may affect the choice of the limiting admission grade at the private institution. If the public university cares for global welfare, it will use its prices to ensure that a maximum of students get access to higher education. There may be a trade-off: higher prices reduce enrollments at public university of higher quality and may increase admissions at private university of lower quality. The latter must be a large effect to make it worthwhile for the public university to raise prices.

In turn, private prices are chosen to maximize (10) according to

$$\begin{aligned} & \frac{\partial a_{pb}^E}{\partial p_{pv}} (1 - p_{pv}) (p_{pv} - C_{pv} - (a_{pb} - a_{pv}) C'_{pv} / 2) + \quad (19) \\ & (1 - 2p_{pv} + C_{pv}) (a_{pb} - \hat{a}_{pv}) - \frac{\partial a_{pv}}{\partial p_{pv}} (1 - p_{pv}) (p_{pv} - C_{pv} + (a^+ - a_{pv}) C'_{pv} / 2) = 0 \end{aligned}$$

where  $a_{pb} = a_{pb}^E$  and  $a_{pv} = \max\{\hat{a}_{pv}, a_{pv}^E\}$ . This condition, similar in qualitative terms to (8) implies no fundamental difference in the behavior of the private institution as result of competition, as long as the choice of strategic variable is concerned.

## 4.2 Case A.3

The allocation of students to schools is represented in this case by Figure 3. The objective functions are given by (13) and (14). Average ability of students at the public institution continues to be  $\bar{a}_{pb} = \frac{a^+ + a_{pb}}{2}$ .

Unfortunately, there is not much additional insight we can provide for this case, which results when the benefits of raising public prices are so large that they become larger than private prices. It is of course still necessary that the private institution uses exams to select students for public prices to be positive.

The following proposition therefore summarizes our conclusions regarding the use of exams and prices when there is competition between a public and a private university, public quality is higher and there are credit constraints.<sup>7</sup>

**Proposition 1** *In the presence of borrowing constraints, if a public university shares the market with a private university of lower quality (i) the private institution may use both exams and prices (ii) public prices may be positive, but only if the lower quality private institution uses exams as selective devices.*

<sup>7</sup>Recall that quality is exogenously given in this paper.

We then conclude that the public university may raise the price with respect to the monopoly benchmark in order to influence the private's behavior and thus maximize welfare. For this to be the case, we need the lower quality private university to use exams to select students.

### 4.3 Case B.2

We now turn to study the case in which private quality is higher. The symmetric to case A.1, B.1, corresponds to  $Q_{pv} > Q_{pb}$  and  $p_{pb} > p_{pv}$ . In this case, all students prefer the private school and the only possibility for the public school to stay in the market with higher prices is that the private university uses selective exams. Still, in this case, it can be simply verified (as will be made clearer in the following sections) that the optimal public price is zero, which contradicts the initial assumptions. We therefore disregard case **B.1**.

Otherwise, with  $Q_{pv} > Q_{pb}$  and  $p_{pv} > p_{pb}$  we have the possibility that the public university accepts students who cannot afford to attend the private school but have high ability. More specifically, we first study the case in which the average ability of students in the public school is larger (symmetric to A.2). The payoffs of the schools are

$$U_{pv} = \int_{p_{pv}}^1 \int_{a_{pv}}^{a^+} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw,$$

$$U_{pb} = \int_{p_{pv}}^1 \int_{a_{pv}}^{a^+} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw + \int_{p_{pb}}^{p_{pv}} \int_{a_{pb}}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw,$$

where  $\bar{a}_{pb} = \frac{a^+ + a_{pb}}{2}$  and  $\bar{a}_{pv} = \frac{a^+ + a_{pv}}{2}$ .

Public exams are set according to (4) while the optimal exam at the private institution is such that (7) is satisfied.

In fact, the private university also behaves as a monopoly when choosing prices, that are chosen according to (8). If exams are used, the limiting admission grade  $a_{pv}^E$  depends only on  $p_{pv}$ . Still, if  $a_{pv} = \tilde{a}_{pv}$  there may be some interaction with the lower quality public institution.

Public prices now satisfy

$$- \left( \frac{a^+ - a_{pb}^2}{2} Q_{pb} - (a^+ - a_{pb}) C_{pb}' \right) \tag{20}$$

$$+ \frac{\partial a_{pb}}{\partial p_{pb}} (p_{pv} - p_{pb}) (-a_{pb} Q_{pb} + C_{pb} - (a^+ - a_{pb}) C_{pb}' / 2)$$

$$+ \frac{\partial a_{pv}}{\partial p_{pb}} (1 - p_{pv}) (-a_{pv} Q_{pv} + C_{pv} - (a^+ - a_{pv}) C_{pv}' / 2) = 0.$$

The first term of this expression is negative, as before, if the contribution to welfare by the public university is positive. The second term is smaller than or equal to zero, depending on whether exams are used or not by the public university. Therefore, public prices can only be positive provided that the last term is. We know that, if the private university uses exams,  $\partial a_{pv}^E / \partial p_{pb} = 0$  from (7) with  $\bar{a}_{pv} = \frac{a^+ + a_{pv}^E}{2}$ . Then, public prices are zero and the private institution acts in complete isolation from the public university, as a pure monopoly.

The presence of the public institution only affects the behavior of the private when the latter uses prices alone to select students and  $p_{pv}/p_{pb} > Q_{pv}/Q_{pb}$  since only in this case the last student enrolled at the private university is  $\tilde{a}_{pv}$ .<sup>8</sup> From (7), the last expression between brackets in (20) is smaller than zero. Hence, a public university of lower quality may decide to set positive prices in order to induce students to enroll at the high quality institution ( $\partial \tilde{a}_{pv} / \partial p_{pb} < 0$ ), although at the cost of leaving some good students uneducated. In contrast with the case where private quality was lower, for a positive public price to be optimal we now need the private school to use prices alone as a student allocation device.

#### 4.4 Case B.3

Finally, case B.3 is the symmetric to A.3 and the payoffs of the schools are

$$U_{pv} = \int_{p_{pv}}^1 \int_{a_{pv}}^{a^+} (p_{pv} - C_{pv}(\bar{a}_{pv})) da dw,$$

$$U_{pb} = \int_{p_{pb}}^{p_{pv}} \int_{a_{pv}}^{a^+} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw + \int_{p_{pb}}^1 \int_{a_{pb}}^{a_{pv}} (aQ_{pb} - C_{pb}(\bar{a}_{pb})) da dw$$

$$+ \int_{p_{pv}}^1 \int_{a_{pv}}^{a^+} (aQ_{pv} - C_{pv}(\bar{a}_{pv})) da dw,$$

where  $\bar{a}_{pv} = (a^+ + a_{pv})/2$  and  $\bar{a}_{pb} = \frac{(a^+ - a_{pv}^2)(p_{pv} - p_{pb}) + (a_{pv}^2 - a_{pb}^2)(1 - p_{pb})}{2((a^+ - a_{pv})(p_{pv} - p_{pb}) + (a_{pv} - a_{pb})(1 - p_{pb}))}$ .

Taking this into account, and given prices and limiting admission grades at the private institution, the optimal admission grade at the public institution is given by

---

<sup>8</sup>To the left of  $\hat{a}_{pv}$  no schooling is preferred to attending the private institution. Therefore, for  $\tilde{a}_{pb}$  to be the ability of the last admitted at the private institution, this threshold needs to be larger than  $\hat{a}_{pv}$ . See page 11 and shift the subscripts to account for the case where private quality is higher.



the condition:

$$-a_{pb}^E Q_{pb} + C_{pb} - (a_{pv} - a_{pb}) C'_{pb} \frac{\partial \bar{a}_{pb}}{\partial a_{pb}^E} \leq 0. \quad (21)$$

On the other hand, given  $p_{pv}$ , the private university chooses the limiting admission grade under competition according to (7), just as under monopoly.

Finally, the optimal public price is now given by

$$\begin{aligned} & - \left( \frac{a^+ - a_{pb}^2}{2} Q_{pb} - (a^+ - a_{pb}) C_{pb} \right) \\ & - C'_{pb} \frac{\partial \bar{a}_{pb}}{\partial p_{pb}} \left( (1 - p_{pb})(a_{pv} - a_{pb}) + (p_{pv} - p_{pb})(a^+ - a_{pv}) \right) \\ & + \frac{\partial a_{pb}}{\partial p_{pb}} (1 - p_{pb}) \left( -a_{pb} Q_{pb} + C_{pb} - (a_{pv} - a_{pb}) C'_{pb} \frac{\partial \bar{a}_{pb}}{\partial a_{pb}} \right) \\ & + \frac{\partial a_{pv}}{\partial p_{pb}} \left( a_{pv}(1 - p_{pv})(Q_{pb} - Q_{pv}) - (1 - p_{pb})(a^+ - a_{pb}) C'_{pv}/2 \right) = 0. \quad (22) \end{aligned}$$

When the private university uses exams to select students ( $a_{pv} = a_{pv}^E$ ) (22) is unambiguously negative for the same reasons already exposed in the previous case. Hence, the optimal public price tends to zero.

That the private institution does not use exams to select students is necessary, although not sufficient, for positive prices to be welfare improving. On the one hand, the ability of the last admitted at the private school must be, as before,  $a_{pv} = \tilde{a}_{pv}$ . This is true, as we have seen, if and only if the difference in prices is larger than the difference in qualities.

Moreover, the only positive term in this case is

$$a_{pv}(1 - p_{pv})(Q_{pb} - Q_{pv}) \frac{\partial \tilde{a}_{pv}}{\partial p_{pb}} > 0,$$

since  $Q_{pv} > Q_{pb}$ . Thus, on the one hand, the larger the difference in qualities, the larger are the benefits of raising the public tuition fee, as more students are induced to attend a university considerably better. But if the difference in qualities becomes larger than the difference in prices, the private institution will start using exams to determine admissions, thus remaining isolated from competition. The benefits of increasing the public price will be lost.

**Proposition 2** *In the presence of borrowing constraints, if a public university competes with a private university of higher quality*

1. *when the difference in quality is larger than the difference in price, the public university sets the price equal to zero and does not affect the behavior of the private, that behaves as a monopoly*

2. *otherwise, the public university may set positive prices in order to maximize global surplus provided that the high quality private university does not use exams*

Although private prices are determined at the margin simply as under the monopoly benchmark (condition (8)), if  $\tilde{a}_{pv} > \hat{a}_{pv} \Leftrightarrow Q_{pv}/Q_{pb} < p_{pv}/p_{pb}$  the last student admitted at the private university is of higher ability when there is a public competitor of lower quality. The reason is that less students are willing to pay  $p_{pv}$  when there is a substitute for private education, even if it is of lower quality. Enrollments at the private institution are in this case sensitive to the choice of price by the public institution. It is for this reason that optimal public prices may be positive. Raising the public price certainly leaves some good students out of the market due to credit constraints, but may induce the private institution to expand, increasing overall access to higher education.

If, on the other hand, the difference in qualities is larger than the difference in prices  $Q_{pv}/Q_{pb} > p_{pv}/p_{pb}$ , there is no strategic interaction or effect of competition. The existence of a public competitor of lower quality does not affect the behavior of the private monopoly.

Summing up, we can conclude that optimal public prices can rise from zero under certain circumstances when a public university that cares for global welfare competes with a private university that maximizes profits. A necessary condition for this to be the case is, if private quality is lower than public quality, that the private institution uses exams to select students and, if private quality is higher than public quality, that the private institution uses only prices.

## 5 Concluding Remarks

In this paper, we have investigated the strategic role of prices and exams for public and private universities in the presence of credit constraints. First, we have compared the optimal choices of a public and a private monopoly. While the public university sets minimum prices and selects students via exams, the private institution may choose to use both instruments, provided that the selection of the best students lowers operating costs more than revenues.

When there is competition, the choice of instrument by the public university does depend strongly on the instruments chosen by the private university. Positive (i.e. higher) public prices can be optimal. For this to be the case we need that the private

university has lower quality and uses exams to select students, or that it has higher quality and uses only prices to select students. In these cases, the public university can, by raising its price, increase enrollments at the private university. This will clearly increase global welfare when the private university has higher quality and the number of students who gain access to this higher quality is larger than the number of students who become unable to afford the public university. Global welfare can also increase when private quality is lower if the total number of university enrollments increases as a result of the rise in the public price.

Although the model proposed is simple and some of the assumptions made are debatable, this paper constitutes a first attempt in the literature to formalize the choice of strategic variable by educational institutions in competition. Our conclusions are original and shed a new light of the role of public prices in higher education.

## References

- [1] Cremer, H., Marchand, M. and Thisse, J.F. (1991). “Mixed Oligopoly with Differentiated Products”, *International Journal of Industrial Organization* 9, pp. 43-53.
- [2] De Fraja, G. and Delbono, F. (1990): Game theoretic models of mixed oligopoly. *Journal of Economic Surveys*, 4.
- [3] Fernandez, R. (1998). “Education and Borrowing Constraints: Tests versus Prices”, *NBER Working Paper* No 6588.
- [4] Fernandez, R. and Gali, J. (1999). “To Each According to...? Tournaments, Markets and the Matching Problem under Borrowing Constraints”, *Review of Economic Studies* 66 (4), pp.799-824.
- [5] Gary-Bobo, R. and Trannoy, A. (2004): Efficient tuition fees, examinations, and subsidies. Mimeo University of Paris 1 and EHESS, CREQAM.
- [6] Grilo, I. (1994). “Mixed Duopoly under Vertical Differentiation”, *Annales d’Economie et de Statistique* No 33.
- [7] Romero, L. and Del Rey, E. (2004). Competition between Public and Private Universities: Quality, Prices and Exams. Mimeo, Universidad Carlos III de Madrid.