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# Indicators of Electoral Victory* 

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#### Abstract

We study a two-party contest where candidates strategically allocate their campaign resources between two salient issues. We analyze to what extent the following indicators of a party' success predict the electoral victory: (1) the pre-campaign advantage, (2) the advantage on every salient issue, and (3) the advantage on campaign resources. We show that the electoral victory is guaranteed only when a party has a "sufficiently large" advantage on every salient issue. Otherwise no combination of these indicators ensures the electoral victory.


Key-words: Election campaign, salient issues, majority voting.
JEL classification numbers: D72, C70

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## 1 Introduction

Political parties' success indicators measure the chances that a party has of achieving a majority on election. Having more chances of winning the elections is usually attributed to (1) the pre-campaign advantage, (2) the advantage on every salient issue, and (3) the advantage on campaign resources.

In this paper we analyze from a theoretical viewpoint whether any of these indicators predicts the electoral victory of a political party. We consider a two-party, two-dimensional spatial model of political competition. For the duration of the political campaign, parties' platforms have already been decided. We follow the emphasis theory to describe parties' competition in the political campaign. The emphasis theory, as described by Page (1976), argues that "political information is imperfect and there are limits on the number of messages that candidates can transmit or that the average voter can or will receive. Candidates must allocate their emphasis (in time, energy, and money) among policy stands and other sorts of campaign appeals". As proposed by Page and formalized by Simon (2002) and Amorós and Puy (2007), parties' strategies for the duration of the political campaign aim at emphasizing the feature (in terms of issues) of the political party that attracts more votes. ${ }^{1}$ In our model, parties' campaign strategies consist of emphasizing some political aspects more than other.

According to our model, the electorate can be partitioned into three groups: partisan voters (those who will vote for one of the parties irrespectively of the parties' campaign strategies), issue voters (those whose vote depend on the campaign strategies, and whom the parties aim at influencing via campaign expenditures), and abstention voters (those who are indifferent between both political parties).

We show that the only indicator that can be used to predict the electoral victory is the advantage on the salient issues. A "sufficiently large" advantage on every salient issue guarantees the electoral victory as far as it guarantees that a party has a majority of partisan voters. When no party has a majority of partisan voters, the issue voters come into the scene and every kind of mischief regarding the electoral results can occur. In particular, we find that a party may be majority-defeated in the elections even if it is the precampaign winner, has more campaign resources than its opponent, and has

[^1]an advantage on every salient issue (as long as the sum of these advantages is not "too large").

The later result is related to the literature on the Ostrogorski's paradox (Daudt and Rae 1976), originating from Ostrogorski (1902). This literature postulates that the candidate with a majority on every single issue can be majority-defeated in a representative democracy. ${ }^{2}$ We depart from the assumptions of this paradox in two main respects: voters' preferences and parties' behavior. While the Ostrogorski's paradox assumes that voters vote for the party with which they agree on more rather than fewer issues, we assume that voters' preferences have intensities over issues and they vote for the party that match their own ideal policy more accurately. Concerning the political parties, while the Ostrogorski's paradox considers static parties, we consider that parties behave strategically allocating emphasis across issues.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the success indicators. Section 4 provides the conclusions.

## 2 The model

There is a society with a finite set of voters, $N=\{1, \ldots, n\}$, which will select by popular election a representative to serve in the legislature. There are two political parties, $A$ and $B$, that compete for winning a majority of the votes by spending campaign resources. There are two political issues, 1 and $2 .^{3}$

Each party $j \in\{A, B\}$ has a political position $x_{j}=\left(x_{j 1}, x_{j 2}\right) \in[0,1]^{2}$, where $x_{j r} \in[0,1]$ is the political position of party $j$ on issue $r \in\{1,2\}$. We assume that parties' political positions are different on both issues: $x_{A 1} \neq$ $x_{B 1}$ and $x_{A 2} \neq x_{B 2}{ }^{4}$ Each party $j$ is endowed with some fixed campaign funds $\bar{c}_{j} \geq 0$. Campaign funds are devoted to the advertising campaign. A campaign strategy of party $j$ is a vector $c_{j} \in C_{j}=\left\{\left(c_{j 1}, c_{j 2}\right) \in \mathbb{R}_{+}^{2}\right.$ : $\left.c_{j 1}+c_{j 2} \leq \bar{c}_{j}\right\}$, which indicates how the party allocates its funds between

[^2]the two different issues. Let $c=\left(c_{A}, c_{B}\right) \in C_{A} \times C_{B}=C$ denote a profile of campaign strategies. For each $c \in C$ and each $r \in\{1,2\}$, let $c_{r}=c_{A r}+c_{B r}$ be the total funds spent on issue $r$.

Each voter $i \in N$ has an ideal political position $\pi_{i}=\left(\pi_{i 1}, \pi_{i 2}\right) \in[0,1]^{2}$. Preferences of voter $i$ over political parties are represented by the following utility function:

$$
\begin{equation*}
u_{i}(j, c)=-\alpha_{1}\left(c_{1}\right)\left[x_{j 1}-\pi_{i 1}\right]^{2}-\alpha_{2}\left(c_{2}\right)\left[x_{j 2}-\pi_{i 2}\right]^{2} \tag{1}
\end{equation*}
$$

For each issue $r, \alpha_{r}($.$) is a strictly increasing function of the campaign$ expenditures on that issue, with $\alpha_{r}(0)>0$. We refer to $\alpha_{r}($.$) as the influence$ function on issue $r$. It indicates the weight that voters assign to that issue. The influence functions on each of the issues may be different.

Voter $i$ casts his ballot for party $j \neq k$ when $u_{i}(j, c)>u_{i}(k, c)$. We suppose that those voters who are indifferent between the two parties abstain from voting. Given any profile of campaign strategies $c \in C$, let $V_{j}(c)$ be the number of votes that party $j$ obtains in the elections, i.e., $V_{j}(c)=\#\left\{i \in N: u_{i}(j, c)>u_{i}(k, c)\right\}$.

The utility function of voter $i$ can be rewritten as:

$$
\begin{equation*}
u_{i}(j, c)=-T(c)\left[x_{j 1}-\pi_{i 1}\right]^{2}-\left[x_{j 2}-\pi_{i 2}\right]^{2} \tag{2}
\end{equation*}
$$

where $T(c)=\frac{\alpha_{1}\left(c_{1}\right)}{\alpha_{2}\left(c_{2}\right)}$ is the relative intensity of the preferences of voters over issue 1 when the profile of campaign strategies is $c$. The greater $T(c)$, the more relevant is issue 1 compared to issue 2 in voters' preferences.

Voter $i$ is indifferent between the two parties (and then he abstains from voting) when his ideal political position satisfies the following condition:

$$
\begin{equation*}
\pi_{i 2}=\frac{T(c)\left[x_{A 1}^{2}-x_{B 1}^{2}\right]+\left[x_{A 2}^{2}-x_{B 2}^{2}\right]}{2\left[x_{A 2}-x_{B 2}\right]}-\frac{T(c)\left[x_{A 1}-x_{B 1}\right]}{\left[x_{A 2}-x_{B 2}\right]} \pi_{i 1} . \tag{3}
\end{equation*}
$$

The line defined by Equation (3) divides the policy space $[0,1]^{2}$ into two regions based on the voters' most preferred party. Since each party $j$ can spend at most $\bar{c}_{j}$, we have $\frac{\alpha_{1}(0)}{\alpha_{2}\left(\bar{c}_{A}+\bar{c}_{B}\right)} \leqslant T(c) \leqslant \frac{\alpha_{1}\left(\bar{c}_{A}+\bar{c}_{B}\right)}{\alpha_{2}(0)}$ for all $c \in C$. We denote $T_{\text {min }}=\frac{\alpha_{1}(0)}{\alpha_{2}\left(\bar{c}_{A}+\bar{c}_{B}\right)}$ and $T_{\max }=\frac{\alpha_{1}\left(\bar{c}_{A}+\bar{c}_{B}\right)}{\alpha_{2}(0)}$ the minimum and maximum values of $T(c)$.

Every voter $i$ such that $u_{i}(j, c)>u_{i}(k, c)($ with $j \neq k)$ for all $c \in C$ always votes for party $j$, no matter what the profile of campaign strategies is. We call these voters partisan voters of party $j$.

There can be voters such that $u_{i}(A, c)>u_{i}(B, c)$ for some $c \in C$ and $u_{i}\left(B, c^{\prime}\right)>u_{i}\left(A, c^{\prime}\right)$ for some $c^{\prime} \in C$. These voters cast their ballots for one or the other political party, depending on the campaign strategies. We call these voters issue voters. ${ }^{5}$

Given any profile of campaign strategies $c \in C$, party $j$ wins the elections if $V_{j}(c)>V_{k}(c)$ (and therefore, party $k$ is majority-defeated). For simplicity, we assume that when parties are involved in a tie party $A$ governs.


Figure 1. An example of winning partition

Interval $\left[T_{\min }, T_{\max }\right]$ can be partitioned into different subintervals according to the party that wins the elections. We call it the winning partition of [ $T_{\min }, T_{\max }$ ]. Figure 1 provides an example to illustrate this concept. There

[^3]are three voters with ideal political positions $\pi_{1}, \pi_{2}$ and $\pi_{3}$. The political positions of the parties are represented by points $A$ and $B$. Abusing notation we use $T_{\min }$ (resp. $T_{\max }$ ) to denote the line defined by Expression (3) when $T(c)=T_{\min }\left(\right.$ resp. $\left.T(c)=T_{\max }\right)$. The winning partition of $\left[T_{\min }, T_{\max }\right]$ consists of two subintervals: $\left[T_{\min }, T_{1}\right)$ and $\left[T_{1}, T_{\max }\right]$. To see this note that for all $T(c) \in\left[T_{\min }, T_{1}\right)$ the winning party is $B$ (since voter 1 casts his ballot for party $A$ and voters 2 and 3 cast their ballots for party $B$ ). Similarly, for all $T(c) \in\left[T_{1}, T_{\max }\right]$ the winning party is $A$ (if $T(c)=T_{1}$ there is a tie).

Political parties aim at winning the elections. Preferences of each party $j \neq k$ are represented by the following utility function:

$$
\begin{align*}
w_{j}(c) & =\left\{\begin{array}{r}
1 ; \text { if } V_{j}(c)>V_{k}(c) \\
0 ; \text { if } V_{j}(c)<V_{k}(c)
\end{array}\right.  \tag{4}\\
\text { and } w_{A}(c) & =1, w_{B}(c)=0 \text { if } V_{A}(c)=V_{B}(c)
\end{align*}
$$

We are interested in analyzing under which circumstances a party can ensure its victory in the elections. This is the idea behind the concept of dominant strategies. A campaign strategy of party $j, c_{j}^{*} \in C_{j}$, is a (weakly) dominant strategy for that party if $w_{j}\left(c_{j}^{*}, c_{k}\right) \geqslant w_{j}\left(c_{j}^{\prime}, c_{k}\right)$ for all $c_{j}^{\prime} \in C_{j}$ and all $c_{k} \in C_{k}(j \neq k)$. Note that, in particular, $c_{j}^{*} \in C_{j}$ is a dominant strategy for party $j$ that ensures its victory if and only if $\left[\frac{\alpha_{1}\left(c_{j 1}^{*}\right)}{\alpha_{2}\left(c_{j 2}^{*}+\bar{c}_{k}\right)}, \frac{\alpha_{1}\left(c_{1}^{*}+\bar{c}_{k}\right)}{\alpha_{2}\left(c_{j 2}^{*}\right)}\right]$ is included in a subinterval of the winning partition where party $j$ wins. ${ }^{6}$

If a party achieves a majority of the votes for every $T(c) \in\left[T_{\min }, T_{\text {max }}\right]$ the problem is trivial. From now on, we assume that the distribution of voters' ideal political positions is such that the winning partition has at least two subintervals. ${ }^{7}$

[^4]
## 3 Results

Next, we analyze to what extent three success indicators ensure that a political party cannot be majority-defeated in the elections. These indicators are: the advantage on campaign resources, the pre-campaign advantage, and the advantage on every salient issue.

### 3.1 The advantage on campaign resources

Having more campaign funds than the opponent does not guarantee the electoral victory, even if the difference is "extremely large". To see this note that when a party has a majority of partisan voters, such party wins the elections for all $c \in C$, even if it has no campaign resources.

### 3.2 The pre-campaign advantage

The pre-campaign advantage measures the percentage of votes that a party would obtain if there were no political campaign. We write $c=0$ to denote the situation where there is no campaign expenditures. Let $T_{0}=\frac{\alpha_{1}(0)}{\alpha_{2}(0)}$ be the relative intensity of voters' preferences over issue 1 in this situation (note that $T_{\min }<T_{0}<T_{\max }$ ). The party that achieves a majority in this case is the pre-campaign winner. The pre-campaign advantage does not guarantee the electoral victory.

Proposition 1 There is no pre-campaign advantage that guarantees that a political party wins the elections.

Proof: Suppose that all voters' bliss points coincide, and that there is $T_{1} \in$ [ $\left.T_{\min }, T_{\max }\right]$ such that all voters vote for party $j$ when $T(c) \in\left[T_{\min }, T_{1}\right)$, and vote for party $k$ when $T(c) \in\left(T_{1}, T_{\max }\right]$. Suppose that $T_{1}<T_{0}$. Then party $k$ would obtain $100 \%$ of the votes if there where no political campaign (i.e., party $k$ has the maximum pre-campaign advantage). However, there always exist some influence functions and amounts of campaign funds such that $\frac{\alpha_{1}\left(\bar{c}_{k}\right)}{\alpha_{2}\left(\bar{c}_{j}\right)}<T_{1}$. In this case $c_{j}=\left(0, \bar{c}_{j}\right)$ is a dominant strategy for party $j$ that ensures its victory.

### 3.3 The advantage on every single issue

We say that party $j$ has a majority on issue $\mathbf{r}$ when it wins the hypothetical election where individuals only care about that issue. Let $n_{j r}$ be the number of voters that, on issue $r$, strictly prefer party $j$ to party $k$. Formally,
$n_{j r}=\#\left\{i \in N:\left[x_{j r}-\pi_{i r}\right]^{2}>\left[x_{k r}-\pi_{i r}\right]^{2}\right\}$. Party $j$ has a majority on issue $r$ when $n_{j r}>n_{k r}(j \neq k) .{ }^{8}$

Having a majority on both issues does not guarantee the electoral victory. The electoral victory is guaranteed only if a party holds a "sufficiently large" majority on both issues.

Proposition 2 Suppose that party $j$ has a majority on both issues. If $\frac{n_{j 1}+n_{j 2}}{n}>$ $\frac{3}{2}$, then party $j$ wins the elections for all $c \in C$ (whatever the campaign funds $\bar{c}_{A}, \bar{c}_{B} \geq 0$ and the influence functions $\alpha_{1}$ and $\alpha_{2}$ are). Otherwise, the electoral victory of party $j$ is not guaranteed. ${ }^{9}$

Proof. (See Figure 2) Let $n_{A r, B s} \geq 0$ be the number of voters that prefer party $A$ on issue $r$, but prefer party $B$ on issue $s$, i.e., $n_{A r, B s}=\#\{i \in N$ : $\left[x_{A r}-\pi_{i r}\right]^{2}<\left[x_{B r}-\pi_{i r}\right]^{2}$ and $\left.\left[x_{A s}-\pi_{i s}\right]^{2}>\left[x_{B s}-\pi_{i s}\right]^{2}\right\}$. Let $n_{j}^{*}=\#\{i \in$ $N:\left[x_{j r}-\pi_{i r}\right]^{2} \leq\left[x_{k r}-\pi_{i r}\right]^{2}$ for all $r \in\{1,2\}$, with strict inequality for some $r \in\{1,2\}\}$. Note that, since individuals take into account both issues simultaneously, the $n_{j}^{*}$ voters will always vote for party $j$ (i.e., the $n_{j}^{*}$ voters are partisan voters of party $j$ ). Suppose that, for some party $j, n_{j 1}+n_{j 2}>\frac{3}{2} n$. Since $n_{j 1} \leq n_{j}^{*}+n_{j 1, k 2}, n_{j 2} \leq n_{j}^{*}+n_{j 2, k 1}$, and $n_{j}^{*}+n_{j 1, k 2}+n_{j 2, k 1} \leq n$, we have $\frac{n}{2}<n_{j}^{*}$ and therefore party $j$ will win the elections for every $c \in C .{ }^{10}$

In Figure 2 we provide an example showing that, if $\frac{n_{j 1}+n_{j 2}}{n} \leq \frac{3}{2}$, then the electoral victory of party $j$ is not guaranteed even if it has a majority on both issues. The political positions of the parties are given by points $A$ and B. There are four groups of voters: $50 \%$ of the voters have ideal political position $\pi_{1}, 25 \%$ of the voters have ideal political position $\pi_{2}$, and $25 \%$ of the voters have ideal political position $\pi_{3}$. Note that $\frac{n_{B 1}}{n}=0.75$ and $\frac{n_{B 2}}{n}=0.75$. Suppose that the campaign funds, $\bar{c}_{A}, \bar{c}_{B} \geq 0$, and the influence functions, $\alpha_{1}$ and $\alpha_{2}$, are such that $T_{\min }$ and $T_{\max }$ are as depicted in Figure 2. In this case, for every $c \in C$, party $A$ obtains $50 \%$ of the votes.

Proposition 2 shows that a "sufficiently large" advantage on each of the issues guarantees the electoral victory. If the sum of votes that a party

[^5]obtains in the hypothetical one-issue elections is greater than $\frac{3}{2} n$, there is no way of defeating this party in the elections since it has a majority of partisan voters. Note that a particular instance for that condition is given when the advantage of a party on every single-issue is above the qualified majority of three-quarters.


Figure 2. Illustration of Proposition 2

When the advantage of a party on single issues does not satisfy the condition stated in Proposition 2, the paradox presented by Ostrogorski reappears. The Ostrogorski's paradox postulates that the party that has a majority on every single issue may be majority-defeated in the elections by its rival.

If we put together the three success indicators that we are analyzing, we find a stronger version of the Ostrogorski's paradox: a party may be majority-defeated in the elections even if it has a majority on every single issue, is the pre-campaign winner, and has more campaign funds than its rival.

Example 1 (Stronger version of Ostrogorski's paradox). The parties have the following political positions and campaign funds:

$$
\begin{array}{lll}
\text { Party } A: & x_{A}=(0.3,0.3) & \bar{c}_{A}=16 \\
\text { Party } B: & x_{B}=(0.7,0.7) & \bar{c}_{B}=9
\end{array}
$$

There are three voters with ideal political positions $\pi_{1}=(0.1,0.1), \pi_{2}=$ $(0.2,0.77)$, and $\pi_{3}=(0.8,0.44)$. The influence functions are $\alpha_{1}\left(c_{1}\right)=1+$ $\sqrt{c_{1}}, \alpha_{2}\left(c_{2}\right)=1+2 \sqrt{c_{2}}$. The relative intensity of voters' preferences over issue 1 may vary between $T_{\text {min }}=\frac{\alpha_{1}(0)}{\alpha_{2}\left(\bar{c}_{A}+\bar{c}_{B}\right)}=\frac{1}{11}$ and $T_{\max }=\frac{\alpha_{1}\left(\bar{c}_{A}+\bar{c}_{B}\right)}{\alpha_{2}(0)}=6$, depending on the campaign strategies $c \in C$. Given the preferences of voters over political parties, we have that:

Voter 1 is the only partisan voter of party $A$.
Voter 2 votes for party $B$ when $T(c) \in\left[T_{\min }, 0.9\right)$, and votes for party $A$ when $T(c) \in\left(0.9, T_{\max }\right]$.

Voter 3 votes for party $A$ when $T(c) \in\left[T_{\min }, 0.2\right)$, and votes for party $B$ when $T(c) \in\left(0.2, T_{\max }\right]$.

The winning partition of $\left[T_{\min }, T_{\max }\right]$ represented in Figure 3 has three subintervals. Note that $T_{0}=\frac{\alpha_{1}(0)}{\alpha_{2}(0)}=1$. Therefore party $A$ is the pre-campaign winner. Moreover, party $A$ has majority on both issues: on issue 1 it is supported by voters 1 and 2, while on issue 2 it is supported by voters 1 and 3. ${ }^{11}$ Nevertheless, $c_{B}^{*}=\left(c_{B 1}^{*}, c_{B 2}^{*}\right)=(2,7)$ is a dominant strategy for party $B$ that ensures its victory, since $\left[\frac{\alpha_{1}\left(c_{B 1}^{*}\right)}{\alpha_{2}\left(c_{B 2}^{*}+\bar{c}_{A}\right)}, \frac{\alpha_{1}\left(\bar{c}_{A}+c_{B 1}^{*}\right)}{\alpha_{2}\left(c_{B 2}^{*}\right)}\right]=\left[\frac{1+\sqrt{2}}{1+2 \sqrt{23}}, \frac{1+\sqrt{18}}{1+2 \sqrt{7}}\right] \subset$ $(0.2,0.9) .{ }^{12}$

In the previous example, two issue voters are crucial to win the elections. As we have shown, there is a subinterval of the winning partition where party $B$ can win the elections. This subinterval can be achieved by slightly influencing preferences on both political issues. The key idea is that by means of slightly influencing preferences in this way, party $B$ captures voter 2 without losing voter 3 . If instead party $B$ spends all its campaign funds on issue 1 or on issue 2 , then party $A$ wins. It is important to note that a necessary condition for the above result is that on one of the issues (issue 2),

[^6]the influence function is more vulnerable than on the other issue (issue 1). ${ }^{13}$ Thus, although party $A$ may try to compensate the strategy of party $B$, it is unable to reach the pre-campaign result.


Figure 3. Illustration of Example 1

## 4 Conclusions

In this paper we introduce a model that, in line with the emphasis theory, considers that candidates in the political campaign can emphasize some issues more than others. We analyze to what extent the electoral victory can

[^7]be predicted. We show that neither the pre-campaign advantage nor the amount of campaign resources can guarantee the electoral victory. Electoral victory is guaranteed only when a party holds a sufficiently large advantage on every salient issue, as far as its advantage guarantees that the party has a majority of partisan voters. The strategic emphasis of issues, however, does not prevent the misleading conclusion of the Ostrogorski's paradox: we find that a party that has an advantage on every salient issue may be majoritydefeated in representative democracy (even if it is the pre-campaign winner and has more campaign resources than its opponent), as long as the sum of the one-issue advantages is not too large.

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[^1]:    ${ }^{1}$ Among others, Laver and Hunt (1992), Budge (1993), Riker (1993), Petrocik (1996), Simon (2002), and Sigelman and Buell (2004) show that there is empirical evidence of candidates emphasizing some issues more than others.

[^2]:    ${ }^{2}$ There are several posterior analyses that compare the results obtained in the elections under representative democracy and those obtained by majority voting in issue-by-issue elections (e.g., Bezembinder and Van Acker, 1985, Kelly, 1989, and Laffond and Lainé, 2006).
    ${ }^{3}$ The results of this paper can be generalized to the case of $m \geq 2$ issues. For simplicity of exposition we focus on the case of two issues.
    ${ }^{4}$ If the political positions of both parties on issue $r$ are identical, then issue $r$ becomes "innocuous" according to the emphasis theory.

[^3]:    ${ }^{5}$ Some authors assume that the set of voters influenced by campaign expenditures is a fixed fraction of uninformed voters (see, e.g., Baron, 1994, and Grossman and Helpman, 1996).

[^4]:    ${ }^{6}$ The concepts of dominant strategies and Nash equilibria are closely linked: $\left(c_{j}^{*}, c_{k}^{*}\right)$ is a Nash equilibrium of the campaign game where party $j$ wins if and only if $c_{j}^{*}$ is a dominant strategy that ensures $j$ 's victory.
    ${ }^{7}$ In particular, if the winning partition has two subintervals, then there is $T_{1} \in$ $\left[T_{\min }, T_{\max }\right]$ such that party $j$ wins for all $T(c) \in\left[T_{\min }, T_{1}\right]$ and party $k$ wins for all $T(c) \in\left(T_{1}, T_{\max }\right]$. In this case, if $\frac{\alpha_{1}\left(\bar{c}_{k}\right)}{\alpha_{2}\left(\bar{c}_{j}\right)} \leq T_{1}, c_{j}=\left(0, \bar{c}_{j}\right)$ is a dominant strategy for party $j$ that ensures its victory, and if $\frac{\alpha_{1}\left(\bar{c}_{k}\right)}{\alpha_{2}\left(\bar{c}_{j}\right)}>T_{1}, c_{k}=\left(\bar{c}_{k}, 0\right)$ is a dominant strategy for party $k$ that ensures its victory.

[^5]:    ${ }^{8}$ We suppose that, if $\left[x_{A r}-\pi_{i r}\right]^{2}=\left[x_{B r}-\pi_{i r}\right]^{2}$, then voter $i$ abstains from voting in the hypothetical one-issue elections.
    ${ }^{9}$ The fact that $n_{k 1}+n_{k 2}<\frac{n}{2}$ does not necessarily imply that $n_{j 1}+n_{j 2}>\frac{3}{2} n$, since some voters could abstain in the hypothetical one-issue elections.
    ${ }^{10}$ This result can be easily generalized to the case of $m \geq 2$ issues: if $\sum_{r=1}^{m} n_{j r}>\frac{2 m-1}{2} n$, then party $j$ wins the elections, no matter how much campaign funds the parties have.

[^6]:    ${ }^{11}$ Since $\frac{2}{3}+\frac{2}{3}<\frac{3}{2} n$, the sufficient condition to win the elections does not hold.
    ${ }^{12}$ While three issues and five voters are the minimal requirements to prove the Ostrogorski's Paradox, when we account for voter's intensities on political issues and for parties strategically emphasizing issues, two issues and three voters are the minimal requirement to prove this paradox.

[^7]:    ${ }^{13}$ We say that issue $r$ is more vulnerable to campaign expenditures than issue $s \neq r$ when the growth rate of $\alpha_{r}$ when $c_{r}$ changes from 0 to $\bar{c}_{B}$ is greater than the growth rate of $\alpha_{s}$ when $c_{s}$ changes from 0 to $\bar{c}_{A}$ (where $\bar{c}_{B}<\bar{c}_{A}$ ), i.e., when $\frac{\alpha_{r}\left(\bar{c}_{B}\right)}{\alpha_{r}(0)}>\frac{\alpha_{s}\left(\bar{c}_{A}\right)}{\alpha_{s}(0)}$. In Example 1, $\frac{\alpha_{2}\left(\bar{c}_{B}\right)}{\alpha_{2}(0)}=7>5=\frac{\alpha_{1}\left(\bar{c}_{A}\right)}{\alpha_{1}(0)}$.

