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Analyzing the Distributions of the Stochastic Firm Growth Approach

by

Toke Reichstein and Morten Berg Jensen

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Department of Business Studies – IKE Group Aalborg University tr@business.auc.dk & mbj@business.auc.dk

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Abstract

Recently there has been a renewed interest in the study of firm size distributions and firm growth rate distributions. The stochastic firm growth approach builds on the assumption that firm growth rates are independent identically distributed and size is determined by a first order autoregressive process leaving the size distribution log-normal. This paper analyzes these distributional patterns in an empirical context questioning the foundation of the stochastic growth approach. In a cross section analysis of four industries using Danish data it is shown that the foundation for and the outcome of the stochastic firm growth process as it has so far been conceived are empirically far-fetched. In particular significant deviations from normality are found with respect to third and fourth moments.

Key words: Firm Growth Rate and Size Distributions, Evolution of Industries

JEL Codes: C14, L11

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1 Introduction

When analyzing the dynamics of market structures it is necessary to realize that expressed by firm size distributions, these are inherently dependent on the firm growth processes and their characteristics. The stochastic firm growth approach, sometimes also specified as Gibrat's Law or the Law of Proportionate Effect, realizes this. It states that firm size is determined by a first order autoregressive growth process in which firm growth rates are independent identically distributed random variables. Consequently, firm size in logarithmic terms becomes equal to the logarithm of the initial size plus the sum of the growth rates since birth which in the limit tends to normality. Hence the size distribution of a given industry gravitates unavoidably toward the log-normal distribution. Early studies on firm size distributions supported this empirically (see e.g. Hart and Prais (1956) and Simon and Bonini (1958)).

Later the stochastic approach to firm growth has been rejected on numerous occasions (see e.g. Evans (1987a, 1987b), Hall (1987), Dunne & Hughes (1994), Hart and Oulton (1996) and Geroski (2000)). Even though skepticism concerning the stochastic approach to firm growth does exist it has had a strong foundation as it originally was introduced by presenting empirical regularities.

The stochastic approach is nevertheless almost useless with reference to its intended applications. Economics, defined as a science concerned with prediction of future states and the possibility of controlling the specific process in question, has a limited use of a proposition like this. The theorem is less suited for guiding managers and politicians in their quests as it solely and unconditionally highlights the classical Gaussian distribution as the explanatory element (Brock 1999).

Most efforts trying to challenge the stochastic approach do so by applying parametric analysis. Preferably regression analysis testing the independency of firm growth rates against a range of variables. But another option would be to apply nonparametric and parametric analysis questioning the distributional characteristics referred to by the stochastic approach. Such methods have been applied in papers by Bottazzi et al. (2001, 2002) and Bottazzi and Secchi (2002). By applying both empirical and simulation based analysis these papers show that the foundation of the stochastic approach only classifies as an approximation of the 'true' pattern. Three assertions, which rest on Italian data, are among others addressed throughout these papers; i) rather than being log-normal, the size distribution tends to be double-humped with an upper tail indicating a more or less oligopolistic market structure; ii) a growth rate distribution not corresponding to the Gaussian, but a symmetric distribution that resembles the Gaussian although with fatter tails and more tent shaped, like a Laplace; iii) while the size distributions tend to vary considerably, the growth rate distributions tend to be quite similar across industries.

The present paper has three aims. First, to analyze whether or not the size distribution can be categorized as being log-normal or if the deviation found from Italian data also applies to Danish data. Second, to test if the firm growth rate distribution may be categorized as being symmetric and mesokurtic and hence in line with the Gaussian shape. Third, to test if both firm size and growth rate distributions are similar across industries.

The outline is as follows. The data and the source of the data is addressed in Section 2. Considerations on the selected industries are discussed and moments summarized. Section 3 applies the formal statistical tests for distributional shapes and similarity across selected industries. Section 4 theorizes according to the results and gives some alternative interpretations to the observed patterns. The data are plotted in histograms and the corresponding Gaussian density function added as a reference line. Maximum likelihood estimation is used to estimate the parameters of a generalized distribution function – namely the Subbotin distribution – with reference to the firm growth rates. Theoretical deductions are put forward to explain the observed empirical patterns and their departures from the corresponding normal density functions as well as their fitted density shapes. Section 5 finalizes the paper by summarizing the results and considerations.

2 Data Source, Industries and Moments

The data used are Danish firm data covering firms from four different industries. They are found in the NewBiz database from Dansk Markeds Information A/S. From this database we have drawn information concerning firm net-revenue, firm balance and employment levels.¹ 1995 is used as the reference year for measuring firm size. The years 1995 and 1996 are used when calculating firm growth rates. Only firms that have reported data for both years are used. Outliers defined as the 5% of the observations that differs mostly from the mean in terms of growth have been deleted.

Bottazzi et al. (2002) propose to analyze the distributional structures according to the industry and the industry characteristics. By using four industries that represent each of the four sectors of the Pavitt-taxonomy² they claim to be analyzing industries that represent different production technologies and learning modes. The industries in question are the Textile, the Primary Metal, the Machine Tool and the Pharmaceutical industry representing Supplier Dominated, Scale In-

¹For clarification it should be mentioned that net-revenue refers to total revenue less discounts given that balance refers to total assets or liabilities and that employment refers to total number of employees working in the firm at a given point in time without taking into account the distinction between part-time and full-time employees.

²See Pavitt (1984) for the origin of the four sector types.

tensive, Specialized Suppliers and Science Based industries respectively.

Similar industries are represented in this paper for comparison. We will nevertheless refrain from giving reference to the Pavitt taxonomy. The industries represented here are the Textile, the Iron Metal, the Machine and the Pharmaceutical industry.³ While it is a reasonable assumption that the technological characteristics of the specific industry are important for understanding its distributional structure, it is nevertheless less likely that we should be looking toward the Pavitt Taxonomy for help in distinguishing between the industries. It is the dynamic characteristics within each industry that form the distributional structure and hence provide explanatory power in understanding such structures. Not a static framework that by using a technological bifurcation simply categorizes the industries into four different sectors. Instead industries should be viewed as going through different stages. The industry life cycle literature holds important perspectives in terms of the dynamics of a given industry.

Table 1 and 2 summarize the basic structure of the data used in terms of the logarithm of firm size and firm growth rates respectively. Unfortunately not all the observations are found for all the variables of interest and as mentioned some are deleted due to being categorized as outliers in terms of growth. Consequently the number of firms found for each variable of interest is reported. Both in absolute figures as well as a percentage of total number of firms found in the given industry.⁴

Table 1 and 2 reveal that firm balances are more often reported in the dataset than the two other measures. Only about 21-27% of the firms report net-revenue while the balance and employment figures are found for 84-89% and 65-69% of the firms respectively.

³Due to considerations of the number of observations a higher level of aggregation has been chosen in terms of the Machine Tool industry compared to Bottazzi et al. (2002). Instead the firms engaged in manufacturing machines in general have been drawn out. Also the Primary Metal industry has been converted to the Iron Metal industry.

⁴The firms are not legally obliged to report all the data. Some of them do so anyway and some do not. Given that it is a specific type of firm that chooses not to report in general it may lead to a bias in the analysis.

		Iron Metal	Machine	Pharmaceutical	Textile
	Number of Firms	1666	1324	409	611
Net-Revenue	Identified Firms	427	350	108	132
	Representation	25.63%	26.44%	26.41%	21.60%
	Mean	8.5264	9.4472	8.5901	9.2410
	Median	8.3209	9.2942	8.0382	9.2666
	Std. Dev.	1.8149	2.1431	2.2569	2.3381
	Skewness	0.3445	-0.1158	0.1607	-0.2359
	Kurtosis	0.3135	-0.6185	-0.7065	-0.9035
Balance	Identified Firms	1439	1148	347	542
	Representation	86.37%	86.70%	84.84%	88.71%
	Mean	8.2413	8.6891	8.2288	8.6441
	Median	8.1890	8.5939	8.0150	8.6171
	Std. Dev.	1.4194	1.6740	1.7264	1.6025
	Skewness	0.1932	0.1682	0.3074	0.0277
	Kurtosis	0.5826	0.2588	0.3724	0.0124
Employment	Identified Firms	1138	914	267	418
	Representation	68.41%	69.03%	65.28%	68.30%
	Mean	2.4857	2.7886	2.3073	2.7072
	Median	2.3979	2.7081	2.1972	2.6391
	Std. Dev.	1.2054	1.3654	1.4340	1.3079
	Skewness	0.3577	0.3179	0.5246	0.0997
	Kurtosis	0.3045	0.4214	0.0030	-0.2776

Table 1: Data structure of the four industries in terms of firm log(size)

5

Source: NewBiz Database, Version 98,4 - Plus X.

etal Ma 666
427 33%
369
184
533
492
420
439
17%
654
297
259
839
050
138
11%
302
000
603
911
912

Table 2: Data structure of the four industries in terms of firm growth

Source: NewBiz Database, Version 98,4 - Plus X.

Besides the statistics on the number of observations, the moments of the data sets are summarized. Considering all three measures, the third moment suggest that the log(size) distributions are symmetric while the fourth moment sends mixed signals. Some of the distributions exhibit statistics that witness a leptokurtic shape while others indicate that they are platykurtic. But these deviations are only limited and a statistical test should be applied if one wants to conclude anything on the peakedness of the distributions.⁵ All in all the distributions cannot be ruled out as being symmetric or even normally distributed according to these values.

One thing that may be noted is that the Machine industry tends to be inhabited by larger firms on the average compared to the other three industries. The firms in the Textile industry are larger with respect to the median in terms of balance. This points toward a higher level of right skewness or a lower level of left skewness in the Machine industry compared to the Textile industry. Even though the skewness measures are close to zero it is also what these indicate.

Considering Table 2 the growth rates are positive on the average in all the industries for all variables except for the Textile industry viz the net-revenue and the employment measures. The Textile industry seems to be coping less well than the rest. The Pharmaceutical industry has the highest growth rates on average.

It is obvious that the number of employees tends to be more stable than the other two measures. The absolute growth rates tend to be lower in this variable than in the other two variables. This points toward rigidities in the number of employees. Labor hoarding is one explanation why some consistency seems to exist in the staff level of firms. It is important to keep workers within the firm and benefit from the routines they build up over time.

While the skewness measure seems to be rather close to zero in all sectors and growth measures, it is nevertheless noteworthy that the kurtosis seems fairly high. A considerable number of the observations are located around the mean and

Skewness
$$(x) = \frac{1}{N\sigma(x)^3} \sum_{i=1}^{N} (x_i - \overline{x})^3$$
 (1)

$$\operatorname{Kurtosis}(x) = \frac{1}{N\sigma(x)^4} \sum_{i=1}^{N} (x_i - \overline{x})^4 - 3$$
(2)

x is the vector of the data. N the number of observations and \overline{x} the mean of the data array. σ refers to the standard deviation of the data string. A skewness measure at zero indicates a perfect symmetric distribution. A platykurtic distribution will get a negative kurtosis measure. A positive kurtosis measure indicates a leptokurtic distribution. A zero kurtosis measure tells us that the distribution is perfectly mesokurtic.

⁵In calculating skewness (third moment) and kurtosis (fourth moment) of the distributions the following equations have been used.

median. An indication that the firm distribution of growth rates may be more reminiscent of a Laplace distribution rather than a Gaussian distribution as proposed by Bottazzi and Secchi (2002).

Hymer and Pashigian (1962) and Mansfield (1962) found at the micro level that the level of variance in growth rates is negatively correlated with firm size. From Tables 1 and 2 it is interesting to see that this relationship is less obvious when considering across-industry data with reference to the average firm size. The Machine industry, which on average has the highest log(size) values does not seem to be the industry which exhibits the highest deviation in the growth rates. This might point toward the problematic issue of comparing firms across industries.

In the used measures it is the Pharmaceutical industry that exhibits the highest standard deviation in growth rates. This supports the industry life cycle hypothesis on market stability. The Pharmaceutical industry may be categorized as being in an early phase compared to the remaining three industries. Hence we would also expect this industry to be more turbulent.

3 Applying Statistical Tests on the Data

As stated in the introduction the aim is to directly test the propositions of the stochastic approach to firm growth by applying various statistical tools. Tables 3 and 4 summarize the Kolmogorov-Smirnov normality test statistics of the logarithm of the firm size and the firm growth rates respectively. The tables hold the D-statistics as well as their associated P-values.⁶

From Table 3 and 4 it is fairly obvious that neither the log(size) nor the growth rate may be categorized as being Gaussian distributed in general. Especially the growth rate distributions are far from Gaussian. The P-values are generally very low only surpassing a 5% level marginally in the case of the Textile industry using net-revenue growth. All other growth rate distributions must be categorized as having very little in common with a normal distribution.

The log(size) distributions have a higher probability of being drawn from a normal distribution than the growth rates. Especially the Textile and Pharmaceutical industries have high P-values throughout all three measures. It is not possible to rule out that the observations could have been drawn from a normal distribution. Only three data sets differ considerably from the normal distribution. In terms of net-revenue it is the Iron Metal industry that does not exhibit the normal distributional shape. The remaining two are found using employment data. Again it is the Iron Metal industry and also the Machine industry.

⁶The null hypothesis tested is that the input data values are a random sample drawn from a normal distribution.

	Net-Revenue	Balance	Employment
Iron Metal	1.4948	1.1284	1.4937
	(0.0229)	(0.1566)	(0.0231)
Machine	1.2058	0.9122	1.3984
	(0.1092)	(0.3761)	(0.0400)
Pharmaceutical	1.1567	1.0223	1.1488
	(0.1377)	(0.2468)	(0.1427)
Textile	1.2223	0.4628	0.7229
	(0.1008)	(0.9829)	(0.6728)

Table 3: Kolmogorov-Smirnov normality tests of log(size) distributions (P-values in parenthesis)

Source: NewBiz Database, Version 98,4 – Plus X. Note: Bold indicates distributions which cannot be categorized as normal according to a two-sided test using a 5% level of significance.

Table 4:	Kolmogorov-Smirnov	normality	tests of	growth	rate	distributions	(P-
values in	parenthesis)						

	Net-Revenue	Balance	Employment
Iron Metal	2.0020	3.5501	6.4618
	(0.0007)	(0.0000)	(0.0000)
Machine	2.6123	5.1085	5.0838
	(0.0000)	(0.0000)	(0.0000)
Pharmaceutical	1.9134	3.5377	4.0213
	(0.0013)	(0.0000)	(0.0000)
Textile	1.3530	4.2682	3.2677
	(0.0514)	(0.0000)	(0.0000)

Source: NewBiz Database, Version 98,4 – Plus X. Note: Bold indicates distributions which cannot be categorized as normal according to a two-sided test using a 5% level of significance.

All in all the results in terms of normality in the distributions are fairly weak. Only the log(size) distributions tend to something like normality giving some support for Hart and Prais (1956) and Simon and Bonini (1958). The next step is then to ask why the Kolmogorov-Smirnov test rules out that the data sets are normally distributed. Tables 5 and 6 do that by applying a parametric test on whether the distributions are significantly skewed or whether they differ significantly from zero in terms of kurtosis considering log(size) and firm growth rate respectively. For that purpose we use the Jarque-Bera test.

Table 5 holds the results of the log(size) datasets. The results are rather mixed.

		Net-Revenue	Balance	Employment
Skewness	Iron Metal	2.9061	2.9922	4.9263
	Machine	-0.8844	2.3268	3.9234
	Pharmaceutical	0.6819	2.3379	3.4993
	Textile	-1.1063	0.2633	0.8322
Kurtosis	Iron Metal	1.3225	4.5110	2.0968
	Machine	-2.3616	1.7902	2.6007
	Pharmaceutical	-1.4987	1.4160	0.0099
	Textile	-2.1189	0.0591	-1.1585

Table 5: Jarque-Bera test of skewness and kurtosis with reference to log(size) distributions

Source: NewBiz Database, Version 98,4 - Plus X.

Note: Bold indicates those distributions which are significantly skewed or peaked/fat tailed at a two-sided χ^2 test using a 5% level of significance.

Table 6: Jarque-Bera test of skewness and kurtosis with reference to growth rate distributions

		Net-Revenue	Balance	Employment
Skewness	Iron Metal	6.3200	13.6880	8.1412
	Machine	9.3474	30.5920	7.5101
	Pharmaceutical	6.3910	19.2790	15.4440
	Textile	-2.5003	27.3130	0.8278
Kurtosis	Iron Metal	10.3000	9.3303	10.2680
	Machine	15.0530	54.0970	9.6971
	Pharmaceutical	4.8878	32.3660	23.1020
	Textile	2.2679	62.2540	6.3751

Source: NewBiz Database, Version 98,4 – Plus X.

Note: Bold indicates those distributions which are significantly skewed or peaked/fat tailed at a two-sided χ^2 test using a 5% level of significance.

The balance and employment measures are asymmetrically distributed for three of the four industries counting the Iron Metal, the Machine and the Pharmaceutical industry. In terms of net-revenue three industries may be considered symmetrically distributed. These count the Machine, the Pharmaceutical and the Textile industry. Consequently the Textile industry tends to be symmetrically distributed and the Iron Metal industry asymmetrically distributed no matter which of the three log(size) measures applied. In terms of peakedness the Pharmaceutical industry tends to exhibit a mesokurtic shape no matter what measure is applied. In terms of net-revenues both the Machine and the Textile industry have a significant

platykurtic shape. The Iron Metal industry exhibits a significant leptokurtic shape when using balances and employment as measures of firm size.

Moving on to Table 6 and the shape of the growth rate distributions the Jarque-Bera statistics indicate that all the distributions are significantly leptokurtic and that all distributions but one are significantly asymmetric. Only the Textile industry, when using employment statistics, has a Jarque-Bera statistic that supports the proposition of a symmetric growth rate distribution. All distributions are more peaked than the normal distribution tending toward the Laplace shape with fat tails as proposed by Bottazzi and Secchi (2002). The majority of the growth rate distributions are significantly right skewed. Only the Textile industry using net-revenue data exhibits a significantly left skewed distributional shape.

Tables 7 and 8 test to what degree industries tend to exhibit a common firm size distribution and firm growth rate distribution. Again a Kolmogorov-Smirnov test has been applied to test similarity between the distributions across industries.

The log(size) distributions show a similarity between the Machine, the Pharmaceutical and the Textile industry no matter which measure we use. The level of the P-values is well above a 0.05 threshold. It may be concluded that the industrial structures of these industries are fairly similar. The Iron Metal industry on the other hand is significantly different from the Machine industry with respect to any of the log(size) measures considered. It does have some similarity to the Pharmaceutical industry when considering net-revenue as a size measure and to the Textile industry when considering either balance or employment as size measures.

Turning to the growth rate distributions the patterns are weaker. Even though some of the distributions seem similar, none of the industries share similarity across all three measures. None of the employment growth distributions have an acceptable level of similarity with the others.

Apart from the employment growth distributions some patterns do emerge. The Machine industry has similarities with the other industries in at least one of the growth measures. The remaining three industries do not seem to have similar distributions no matter which of the three growth measures used.

Some industries do tend to show some of the same patterns in terms of log(size) distributions. But these are by no means normally distributed. But what is interesting is that it is not the industries that have very similar log(size) distributions that have similar growth rate distributions. On the contrary. The stochastic firm growth approach would argue that this should be the case. But nothing points toward this.

logomi	cov-Smirnov test for simil	larities between log(<i>sıze</i>) disiribulio	ons (P-values in Faren	unesis)
		Iron Metal	Machine	Pharmaceutical	Textile
Iron Metal		0.0000	•	-	•
		(1.0000)	•		•
Machine		1.9046	0.0000		
		(0.0014)	(1.0000)		·
Pharmaceutical		1.0332	0.8527	0.0000	·
		(0.2361)	(0.4612)	(1.0000)	·
Textile		1.9748	0.8688	0.7331	0.0000
		(0.0008)	(0.4372)	(0.6557)	(1.0000)
Iron Metal		0.0000			•
		(1.0000)	•		•
Machine		1.3649	0.0000		
		(0.0482)	(1.0000)		
Pharmaceutical		1.3775	0.7387	0.0000	·
		(0.0450)	(0.6462)	(1.0000)	·
Textile		1.1679	0.5995	0.8061	0.0000
		(0.1307)	(0.8650)	(0.5343)	(1.0000)
Iron Metal		0.0000			
		(1.0000)			•
Machine		1.6624	0.0000		
		(0.0080)	(1.0000)		
Pharmaceutical		1.5709	1.2952	0.0000	
		(0.0144)	(0.0698)	(1.0000)	
Textile		1.2691	0.8682	1.0044	0.0000
		(0.0798)	(0.4381)	(0.2653)	(1.0000)

Source: NewBiz Database, Version 98,4 – Plus X. Note: The mean has been subtracted from all series. Note: Figures in bold correspond to those test statistics from which we conclude that the distributions are not similar using a 5% level of significance.

Textile							0.0000	(1.0000)							0.0000	(1.0000)							0.0000	
Pharmaceutical					0.0000	(1.0000)	2.3550	(0.0000)					0.0000	(1.0000)	1.6489	(0.0087)					0.0000	(1.0000)	4.3062	
Machine			0.0000	(1.0000)	1.2627	(0.0824)	1.7521	(0.0043)			0.0000	(1.0000)	1.1815	(0.1227)	0.9500	(0.3275)			0.0000	(1.0000)	5.1941	(0.000)	4.5864	
Iron Metal	0.0000	(1.0000)	0.4678	(0.8484)	1.4348	(0.0326)	1.6258	(0.0101)	0.0000	(1.0000)	1.3981	(0.0401)	1.8798	(0.0017)	1.7520	(0.0043)	0.0000	(1.0000)	7.3115	(0.000)	5.5246	(0.000)	4.8042	
	Iron Metal		Machine		Pharmaceutical		Textile		Iron Metal		Machine		Pharmaceutical		Textile		Iron Metal		Machine		Pharmaceutical		Textile	
	Net-Revenue								Balance								Employment							

Source: NewBiz Database, Version 98,4 – Plus X. Note: The mean has been subtracted from all series. Note: Figures in bold correspond to those test statistics from which we conclude that the distributions are not similar using a 5% level of significance.

4 Alternative Interpretations

Little seems to indicate that a common stochastic firm growth process may be defined for all industries. The present section will try to theorize further on the distributional shapes by looking at their actual shapes rather than their supposed shapes. Plotting the histogram for each variable presents additional information that may add to our understanding of how industries are structured and evolve. The following paragraphs interpret the observed distributions in an alternative way. With reference to the growth rate distribution an alternative distribution will be fitted to the data using maximum likelihood estimation of the parameters of the Subbotin distribution function.

4.1 The Size Distribution

Starting with log(size) the most interesting distributions are the net-revenue versions. These are depicted in Figure 1(a) through 1(d).⁷ The supposed normal distribution functions with the characteristics of the data sets have been added to the histograms.

It is obvious from the shape of the histograms that the data depict a pattern of more than one hump. In the case of the Iron Metal, Machine and Textile industries a pattern resembling that of Bottazzi et. al is found - namely a bimodal shape. It is even possible to distinguish three humps. The oligopolistic structure seems to be strongest in the case of the Iron Metal industry with a small but significant group in the right tail of the $\log(size)$ distribution. The other two industries tend to have more fuzzy patterns but still exhibit at least two humps.

The Pharmaceutical industry depicts a different pattern. The log(size) distribution of this industry is a bit difficult to describe by referring to a number of humps. Obviously there are considerable differences in the distributions but one should keep in mind the results from Table 7 suggesting that some similarities do exist.

So far, the four industries have been referred to in singular (e.g. the Iron Metal Industry). But in fact each of the four industries are composed of several subpopulations. These subpopulations represent subindustries that most likely represent several stages of the industry life cycle. The Machine industry may thus be disaggregated into a number of different industries which are spread over several different stages. Van Dijk (2000) showed for instance that the Dutch manufacturing of machinery for packing and wrapping may be categorized as going through an expanding stage, while manufacturing of wood and furniture machinery is in a

⁷See Figure A-1 in Appendix A for the log(size) distribution in terms of balances and employment.



Figure 1: log(size) distributions of the four industries (net-revenue)

contracting stage. The composite of the industry in terms of subpopulations then becomes important as we see differences in the demographic setups depending on which stage of the industry life cycle the specific subpopulation is in. Contemplating on an industry which is composed of many subpopulations that may be categorized as being in the early stages of the industry life cycle it is likely that a different pattern is observed than if the industry to a greater extent is composed of subpopulations at the mature stages. Additionally it has been argued that the number of subpopulations is relatively more stable in industries composed of matured subpopulations.

Whether entries to an industry stem from an existing firm from a different subpopulation like in the case of the U.S. Television industry (Klepper and Simons 2000) or simply are brand new firms popping up like in the case of the Automobile industry (Klepper 2002) makes no apparent difference. In the early stages firms tend to be of more or less the same size. The argument here is then that the peak and the high density of the upper right tail of the Pharmaceutical log(size) distribution consists of small and large firms respectively of subpopulations that

are in the shakeout phase or already have moved into the mature stage expressed by an oligopoly like stage. But the disturbance in the middle of the distribution, which may be one or more humps, illustrates subpopulations in which firms still compete on product innovations and that have not yet moved or are just about to move toward the process that leaves the industry in an oligopoly state that seems to be *a natural byproduct of technological change* (Klepper 2002, p. 58).

Andersen (2003) recently suggested that economics could apply the methods of biology in determining the relation between distinguishing between subpopulations. The argument is that subpopulations are related through evolution and may be traced through their genealogical structures. Molecular data suggest that humans and chimpanzees are closely related with gorillas, orangutans and further out gibbons. A mutation happens as a consequence of a stochastic shock on the gene-pool which creates a new specie. Similarly Andersen argues that industries may be related by a common genealogical structure measured by their input and output characteristics. Given this setup it is also reasonable to argue that subpopulations stem from a historical genealogical separation from existing or extinct subpopulations. The speed at which new species/subpopulations are introduced is determined by the level of instability in the gene-pool. In industrial economics the gene-pool may be expressed by technology. Hence it is expected that industries with a rough shaped size distribution are those in which new subpopulations emerge continuously due to rapid technological change (i.e. industries with high technological opportunities). Industries depicting the bimodal shape are those which mainly consist of subpopulations that have entered or that are about to enter the mature stage of the industry life cycle. If we were to reconsider the industrial aggregation according to the industry's genealogical structure as proposed by Andersen we would probably also be able to find more striking size distribution patterns.

Applying this logic to the four industries investigated it is apparent that the Pharmaceutical industry is composed of subpopulations that for a considerable part may be categorized as being in the early stages while the other three industries are less roughly shaped and hence have a lower relative level of subpopulations in the early stages. These three must be categorized as having reached a stable number of subpopulations. In these industries the emergence of new subpopulations is less likely to take place as the technological opportunities have been exhausted.

This is not to say that a similar pattern as the one found in the Pharmaceutical industry cannot appear in the other three industries in the future. Discovering a new technology that is applicable in a majority of subpopulations and that creates new and higher technological opportunities may hence create brand new subpopulations and put the industry into a new cycle. Existing large firms may not have a competitive advantage which then causes market shares between entrants and incumbent firms to be reshuffled again creating instability.

4.2 The Firm Growth Distribution

Turning to the growth rate distributions an even smaller chance of normality was found. The moments of the data and the Jarque-Bera tests suggested the distributions to be both asymmetric and considerably peaked. At least the peakedness and in some instances also the skewness is supported by the histograms. In terms of net-revenue the distribution with the highest probability of being normally distributed is the Textile industry. Figures 2(a) through 2(d) depict these distributions.⁸ The kurtosis measures in Table 2 indicated that the Textile net-revenue distribution depicts the least leptokurtic shape.

Figure 2(d) differs from 2(a), 2(b) and 2(c) in that it does not possess the same fat tails. This may explain the numerically small kurtosis measure in Table 2. The Textile industry simply holds less extreme growth rates which then results in a positive outcome in the normality test in Table 4. But considering the histogram it is evident that the distribution is peaked and should not be defined as Gaussian.



Figure 2: Firm growth rate distributions of the four industries (net-revenue)

⁸Corresponding distributions in terms of balances and employment are depicted in Figure A-2 Appendix A.

From Figures 2(a) through 2(d) we can neither deny nor acknowledge the idea of growth distributions having fat tails. Even though it seems as if more observations are located in the tails than suggested by the plotted corresponding normal density functions, no conclusion can be made from this simple visual evaluation. It does seem from the leptokurtic shapes that the distributions have a higher probability of being Laplace distributed as suggested by Bottazzi and Secchi (2002). Also the high kurtosis values shown in Table 2 indicate that the growth rate distributions have fat tails compared to a Gaussian distributions.

Explaining the peakedness of the distributions is difficult. One suggestion would be that the heterogeneity of firms has no considerable effect on firm growth rates. Firms tend to grow at the industry rate. This was also pointed out by Bottazzi et al. (2001). They conclude that there is no evidence of different life cycles and persistent forms of heterogeneity across firms in terms of innovative output having any effect on comparative growth performances.

Following Bottazzi and Secchi (2002) it may prove helpful to try and fit the growth rate distribution to the Subbotin probability density function.⁹ Using the asymmetric functional form requires three parameters to be estimated. A scalar parameter (α) and two parameters determining the shape of the tails (β_l for the left tail and β_r for the right tail). The original Subbotin function only had two parameters to be estimated as it was symmetric. It has already been shown that the growth rate distributions are asymmetric. Consequently, it is necessary to estimate the three parameters. Table 9 holds estimates of three parameters for the 12 growth rate distributions considered. Additionally the table holds the location parameter μ of the estimated density function and the log-likelihood estimate.

From the estimates it is obvious that the distributions are far from normal.

$$f(x) = \frac{e^{-\frac{1}{b} * |\frac{x-\mu}{a}|^b}}{2 * a * b^{\frac{1}{b}} * \Gamma(1+\frac{1}{b})}$$
(3)

This is the form used in Bottazzi and Secchi (2002). They found no reason to analyze the asymmetric characteristics of the distributions. But in his programs for estimation of the Subbotin distribution parameters, Bottazzi proposes the following function form for the asymmetric density function:

$$P(x) = \begin{cases} \frac{e^{-((x-\mu)/\alpha)^{\beta_{l+1}/\beta_{l}}}{A} & \text{for } x < \mu, \\ \frac{e^{-((x-\mu)/\alpha)^{\beta_{r+1}/\beta_{r}}}{A} & \text{for } x > \mu \end{cases}$$
(4)

where $A = \alpha * \left[\beta_l^{\frac{1}{\beta_l}} * \Gamma(1 + \frac{1}{\beta_l}) + \beta_r^{\frac{1}{\beta_r}} * \Gamma(1 + \frac{1}{\beta_r}) \right]$ and with $\alpha, \beta_l, \beta_r > 0$. If the β values are estimated to 2 the distribution becomes Gaussian while a β at 1 indicates a Laplace distribution.

⁹The Subbotin probability density function originates from Subbotin (1923). It has the following functional form:

		β_l	eta_r	α	μ	LL
Net-Revenue	Iron Metal	1.3840	0.8855	0.1875	0.0387	-0.0350
	Machine	0.9630	0.6104	0.1852	0.0476	0.0979
	Pharmaceutical	2.0830	0.6178	0.2474	0.1095	0.1850
	Textile	0.8546	1.3010	0.1204	-0.0172	-0.4446
Balance	Iron Metal	2.1240	0.9571	0.1879	0.0618	-0.1383
	Machine	2.0770	0.6547	0.1905	0.0975	-0.1033
	Pharmaceutical	2.3570	0.6302	0.1932	0.0978	-0.0538
	Textile	1.7430	0.5836	0.1615	0.0618	-0.2107
Employment	Iron Metal	0.2776	0.3810	0.0630	-0.0000	-1.0380
	Machine	0.2797	0.2321	0.0573	0.0000	-1.0900
	Pharmaceutical	0.1672	0.1935	0.0247	-0.0000	-1.9010
	Textile	0.2595	0.2597	0.0795	0.0000	-1.0050

Table 9: Maximum log-likelihood parameter estimates of the Subbotin distribution function

Source: NewBiz Database, Version 98,4 - Plus X.

None of the distributions seem to have both β 's estimated to 2. Neither does any of them hold β estimates which correspond to the Laplace shape (both of them being estimated to 1). The left and right tail parameters suggest asymmetric shapes as they are different for at least 8 of the 12 estimations. The remaining 4 are the employment data. In this case the low β and α estimates suggest the distribution to be a lot more peaked than the Laplace. This may be a result of the fact that we are dealing with firms of all sizes. Small one- and two men firms are perhaps more likely to stay one- and two men firms than firms of other sizes. We consequently get a great deal of observations with a zero growth rate. Especially the Pharmaceutical industry shows values which indicate an extremely peaked distribution.

In the case of the balance data it is interesting to see that there is a tendency for the left side of the distribution to be normal with β_l values close to 2 while the right hand parameter (β_r) is estimated to be less than 1 making it more steep than the Laplace distribution. The Textile industry has the lowest parameter estimates overall considering balance.

The pattern changes a bit when considering net-revenue growth rates. The Textile industry still has the lowest α and β_l estimates. But the right tail parameter has increased. In fact it has overtaken the corresponding left tail parameter making the distribution left skewed rather than right. This also corresponds to our findings in Table 2 in which the Textile industry had the only negative skewness measure using this specific measure of growth.

As the estimated distributional forms are rather difficult to imagine simply by looking at the numbers in Table 9, visual versions of the net-revenue growth rate distributions and balance growth rate distributions are depicted in Figure 3.¹⁰ Figure 3(a) and 3(b) verify our previous findings. The Textile industry seems to exhibit the most peaked distribution. But it should also be noticed that the right tail seems to be less fat than the other three. The Textile growth rate distribution also has a tendency to lean toward the left rather than the right considering netrevenue. Furthermore it is evident that the Pharmaceutical industry exhibits a distributional shape that is very Gaussian on the left side and Laplace like on the right. The result is that the distribution becomes less peaked and exhibits a fatter right tail than the others. As the left tail is lower than the others it is evident that Pharmaceutical firms are more likely than firms in any of the other three industries to experience extreme positive growth rates. This also accounts for the higher mean growth rate. It is tempting to conclude that this is linked to the industry life cycle literature on early stage instability. But we do not see the same pattern for the Pharmaceutical industry when using balance calculating growth rates. It is worth mentioning that balance is a problematic measure to use when considering growth rates. Assets or liabilities are often used to ensure a healthy growth path of a firm. Consequently the balance may exhibit a negative growth rate while the firm is actually expanding.

Considering Figure 3(a) it is tempting to theorize that as an industry evolves its growth rate distribution transforms from being close to mesokurtic and leaning a bit to the right to becoming very leptokurtic and slightly leaning to the left. In the early stages of the industry life cycle its firms may be experiencing above average growth rates. Also there is a tendency for greater dispersion in the growth rates in relatively young industries which results in a higher level of instability in terms of shifting market shares. This would suggest that the subpopulations of the Pharmaceutical industry mainly are in the early stages while the Textile subpopulations are in mature stages. The Iron Metal and Machine industries are somewhere in between. We leave this topic for future research.

5 Summary and Conclusions

This paper has empirically investigated the distributions of the stochastic firm growth approach. By applying simple statistical tests on the firm log(size) and firm growth rate distributions a number of highly interesting and revealing patterns have been presented. Shortly said it is evident that the very foundation of the

¹⁰Due to the numerically small parameter estimates in the employment data the figures have not been drawn as it produces incomparable shapes.



Figure 3: Firm growth rate distributions (estimated maximum likelihood parameters)

stochastic firm growth approach is violated. Firm growth rates cannot be considered random draws from an independent identical distribution. They are obviously very leptokurtic in their shapes and exhibit fat tails more likely to tend toward the tent-shaped Laplace distribution than the Gaussian. Also it has been shown that the log-normal size distribution is not as stylized as we think. It is evident that the log(size) distribution deviates considerably from the Gaussian pattern.

The above conclusions correspond to the findings by Bottazzi et al. (2001, 2002) and Bottazzi and Secchi (2002). But we do diverge from their conclusions on two issues. First, the Danish data clearly do not exhibit a symmetric firm growth rate distribution. Instead we are able to show that it is significantly skewed. Moreover the right tail may exhibit a considerable fatness. Secondly, it is the firm growth rate distributions that tend to vary considerably across industries as shown by testing similarities using the Kolmogorov-Smirnov two sample test. But it should be highlighted that these low levels of similarity between growth rate distributions may be a result of the peakedness as well as the asymmetry.

We suggest further analysis of the firm growth rate distribution and its asymmetry in terms of the estimated Subbotin parameters across a higher number of industries and across several countries. It may be that the stage of evolution of a given industry corresponds to a certain asymmetric shape as shortly discussed with reference to the net-revenue growth measure and the Textile and Pharmaceutical industries.

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A Appendix

Table 10: Sub-populations of the 4 industries **Iron Metal Industry**

28.11.00	Manufacture of metal structures and parts of structures
28.12.00	Manufacture of builders' carpentry and joinery of metal
28.21.00	Manufacture of tanks, reservoirs and containers of metal
28.22.00	Manufacture of central heating radiators and boilers
28.30.00	Manufacture of steam generators except central heating hot water boilers
28.40.00	Forging, pressing, stamping and roll-forming of metal; powder metallurgy
28.51.00	Treatment and coating of metals
28.52.00	General mechanical engineering
28.61.00	Manufacture of cutlery
28.62.00	Manufacture of tools
28.63.10	Manufacture of locks
28.63.20	Manufacture of hinges
28.71.00	Manufacture of steel drums and similar containers
28.72.00	Manufacture of tins and closures of metal
28.73.00	Manufacture of wire products
28.74.00	Manufacture of fasteners, screw machine products, chain and springs
28.75.10	Manufacture of metal sign plates
28.75.20	Manufacture of sanitary and household articles of metal
28.75.90	Manufacture of other fabricated metal products except metal sign plates, etal sani-
	tary and household articles
	•

	Machine Industry
29.11.10	Manufacture of marine engines
29.11.20	Repair of marine engines
29.11.90	Manufacture of other engines and turbines
29.12.10	Manufacture of air pumps and air compressors
29.12.20	Manufacture of pumps for liquids
29.12.30	Manufacture of hydraulic and pneumatic machinery
29.13.00	Manufacture of taps and valves
29.14.00	Manufacture of bearings, gears, gearing and driving elements
29.21.00	Manufacture of furnaces and furnace burners
29.22.10	Manufacture of conveyors and lifts
29.22.20	Manufacture of cranes, tackles and hoists
29.22.30	Manufacture of works trucks
29.22.90	Manufacture of other lifting and handling equipment
29.23.10	Manufacture of refrigerating and freezing industrial equipment
29.23.20	Manufacture of ventilating plants and air-conditioning for industrial use
29.24.10	Manufacture of weighing machinery
29.24.20	Manufacture of packing and wrapping machinery
29.24.30	Manufacture of high-pressure purifying machinery, fire extinguishers and sand &
	blasting machines, etc.
29.24.90	Manufacture of automatic goods vending machines, heat exchangers and cen-
	trifuges, etc.
29.31.00	Manufacture of agricultural tractors
29.32.10	Manufacture of harvesting machinery, etc.
29.32.20	Manufacture of agricultural machinery for soil preparation
29.32.30	Manufacture of agricultural and forestry machinery n.e.c.
29.32.40	Repair of agricultural and forestry machinery
29.41.00	Manufacture of portable hand held power tools
29.42.00	Manufacture of other metalworking machine tools
29.43.00	Manufacture of other machine tools n.e.c.
29.51.00	Manufacture of machinery for metallurgy
29.52.10	Manufacture of machinery for production of mortar, cement, concrete and articles of concrete
29.52.90	Manufacture of machinery for construction, etc.
29.53.10	Manufacture of machinery for the dairy industry
29.53.20	Manufacture of machinery for grain milling industry
29.53.30	Manufacture of machinery for production of sugar confectionery and bakery prod-
20 53 40	ucio Manufactura of machinary for processing of most noultry fish and shall fish
29.53.40	Manufacture of other machinery for food haverage and tobacco processing
29.53.90	Manufacture of machinery for textile apparel and leather production
29.54.00	Manufacture of machinery for paper and paperboard production
29.55.00	Manufacture of moulds
29.56.10	Manufacture of machinery for drying
29.56.20	Manufacture of industrial machinery n e c
29.50.90	Manufacture of measures and ammunition
29.00.00	Manufacture of domestic refrigerators and freezers 372 Danish Industrial Classifi-
29.11.10	eations
29 71 20	Manufacture of electric domestic cookers, cooking appliances and overs
29.71.20	Manufacture of domestic dishwashers, washing and drying machines
29 71 90	Manufacture of domestic vacuum cleaners water heaters electric radiators coffee
27.11.70	makers, etc.
29.72.00	Manufacture of non-electric domestic appliances

	Pharmaceutical Industry
33.10.10	Manufacture of syringes, needles, cathetres etc. used in medicine
33.10.20	Manufacture of hearing aids and parts thereof
33.10.30	Manufacture of electro-diagnostic apparatus
33.10.40	Manufacture of medical and dental furniture and fittings
33.10.90	Manufacture of X-ray apparatus, dental apparatus, respiration apparatus, or-
	thopaedic appliances, artificial limbs, etc.
33.20.10	Manufacture of navigation equipment
33.20.20	Manufacture of apparatus for measuring or checking the flow, level pressure or
	other variables of liquids or gases
33.20.30	Manufacture of apparatus for measuring and checking electrical quantities
33.20.40	Manufacture of apparatus for carrying out physical and chemical analyses
33.20.90	Manufacture of other measuring and checking equipment
33.30.00	Manufacture of industrial process control equipment
33.40.10	Manufacture of spectacle lenses and optical instruments, etc.
33.40.20	Manufacture of reproducing apparatus
33.40.90	Manufacture of photographic and cinematographic equipment n.e.c.
33.50.00	Manufacture of watches and clocks

Textile Industry

	Textile industry
17.10.00	Preparation and spinning of textile fibres
17.20.00	Textile weaving
17.30.00	Finishing of textiles
17.40.10	Manufacture of sails, flags, tents, etc.
17.40.20	Manufacture of made-up furnishing articles
17.40.90	Manufacture of other made-up textile articles
17.51.00	Manufacture of carpets and rugs
17.52.10	Manufacture of cordage, rope and twine
17.52.20	Manufacture of netting (seine makers)
17.53.00	Manufacture of nonwovens and articles made from nonwovens, except apparel
17.54.00	Manufacture of other textiles n.e.c.
17.60.00	Manufacture of knitted and crocheted fabrics
17.71.00	Manufacture of knitted and crocheted hosiery
17.72.00	Manufacture of knitted and crocheted pullovers, cardigans and similar articles
18.10.00	Manufacture of leather clothes
18.21.00	Manufacture of workwear
18.22.00	Manufacture of outerwear, dresses, trousers, etc.
18.23.10	Manufacture of shirts
18.23.90	Manufacture of other underwear
18.24.10	Manufacture of babies garments
18.24.90	Manufacture af other wearing apparel n.e.c.
18.30.00	Dressing and dyeing of fur; manufacture of articles of fur



Figure A-1: log(Size) distributions of the four industries where size is measured by balance for 1(a), 1(b), 1(c) and 1(d) and by employees for 1(e), 1(f), 1(g) and 1(h).



Figure A-2: Firm growth rate distributions of the four industries where size is measured by balance for 2(a), 2(b), 2(c) and 2(d) and by employees for 2(e), 2(f), 2(g) and 2(h).