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## **THE KEYNESIAN MULTIPLIER AND THE PIGOU EFFECT UNDER SUBSTITUTION BETWEEN PRIVATE AND PUBLIC CONSUMPTION**

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### ***Abstract***

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In this paper we present a fixprice model in which private and public consumption show some degree of substitution. We offer formulae for the Keynesian multiplier which depend on this degree of substitution. We also show that there is a Pigou effect and that, sometimes, this effect is larger than the Keynesian multiplier.

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## 1. INTRODUCTION

Current economic events have fueled a lively discussion about the best way to fight unemployment. This paper is an attempt to discuss this issue in a simple static macroeconomic model in which:

1. Agents maximize utility subject to budget constraint and given prices.
2. Goods and money markets clear but, for exogenous reasons that are not discussed in this paper, the labor market does not clear.<sup>1</sup>

Our model is very stylized: There is only one input, labor, which produces a consumption good under constant returns to scale. The latter assumption simplifies the supply side of the economy by making supply infinitely elastic and together with our assumption of perfect competition implies that there are no profits. Since neither the responsiveness of supply to demand nor the distribution of income are at the stake in current discussions this is not a harmful assumption. Money is the only asset and it is demanded because it is an argument in the utility function. This choice of modelling is dictated by simplicity, see Blanchard and Fischer (1989). The government obtains all revenue on a linear income tax again an assumption brought about by simplicity. Finally, monetary wages are given.<sup>2</sup> Thus, important considerations for the understanding of the crisis such as productivity growth, capital/investment, indirect taxation, monopolistic competition, stock market and increasing/decreasing returns to scale are outside the scope of this paper.

Following an idea of Bailey (1971), see also Barro (1981), we consider the possible substitution between private and public consumption. For instance if publicly built schools are as good as privately built schools, an increase in the former should have some effect on the private demand of the latter. Or if the government runs a deficit, the citizens may feel that this deficit, sooner or later, will fall on them.<sup>3</sup> Thus in our model the utility of the representative consumer is a function of two variables. On the one hand, a linear combination of the privately and the publicly provided consumption good with weights 1 and  $a \in [0, 1]$ . On the other hand on money and the budget surplus of the government with weights 1 and  $b \in [0, 1]$ . If  $a = 0$ , the consumer feels that the publicly provided good, say bridges, is not related to the privately provided good, say, beef. If  $a = 1$ , the consumer takes the publicly provided consumption good, say public health care, as a perfect substitute for the privately provided consumption good, say private health care. If  $b = 0$ , the consumer feels that the debt (resp. surplus) incurred by the government will not be paid (resp. received) by her, because it will be paid (resp. received) in a distant future or by a future

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<sup>1</sup>For a possible explanation of why wages do not adjust supply and demand of labor see Bewley (1999).

<sup>2</sup>Our model falls into the fixprice literature pioneered by Barro and Grossman (1976), Benassy (1975), Drèze (1975), Malinvaud (1977) and Younès (1975). See Silvestre (1982) for a general view on this literature.

<sup>3</sup>This is the issue behind the "Ricardian equivalence".

generation. If  $b = 1$ , the consumer takes the government debt (resp. surplus) as an immediate decrease (resp. increase) in her wealth.  $b$  can be interpreted as the percentage of the current deficit paid by the current generation. Similar preferences have been considered by Heijdra, Ligthart and van der Ploeg (1998) in the framework of Monopolistic Competition and by Linnemann and Schabert (2003) in a New Keynesian model. Our main results are:

1. When the increase in public expenditure is financed entirely by taxes or  $b = 1$ , i.e. the deficit is discounted as an immediate reduction in wealth, the Keynesian multiplier is  $1 - a$ . When publicly and privately provided consumption goods are not related ( $a = 0$ ) this is the well-known result that the multiplier of the balanced budget is one.
2. When the increase in public expenditure is financed by a deficit the multiplier is decreasing in both  $a$  and  $b$ . When the goods offered by the government and the private sector are not related ( $a = b = 0$ ) we obtain a formula that is identical to the textbook Keynesian multiplier.
3. When written in terms of elasticities, instead of the usual formulation of a ratio of increments, the Keynesian multiplier is always smaller than one. This means that an increase in public expenditure of  $x\%$  increases total output in less than  $x\%$ .
4. The Pigou effect, namely the effect on real wealth of a decrease in prices, in terms of elasticities is, also, smaller than one. When the increase in public expenditure is financed entirely by taxes or  $a = b = 1$  the elasticity associated with the Pigou effect is larger than the elasticity of the Keynesian multiplier. If  $a = b = 0$  the Keynesian multiplier is larger than its Pigouvian counterpart for "reasonable" values of the parameters.

## 2. THE MODEL

There are two goods consumed with prices denoted by  $p$  and 1. The second good will be called money from now on. There is an input, labor, whose price is denoted by  $\omega$ . Good 1 is produced under constant returns to scale and units are chosen such that the marginal cost of this good is  $\omega$ . Assuming that the economy is perfectly competitive,

$$p = \omega. \tag{1}$$

The government raises funds by a tax on income ( $I$ ) with a constant tax rate  $t$ , and buys goods 1 and 2 in quantities  $G$  and  $S$ . While  $G$  is positive,  $S$  can be positive (surplus) or negative (deficit). The government budget constraint is

$$pG + S = tI. \tag{2}$$

There is a representative consumer with preferences representable by a Cobb-Douglas utility function

$$U = A(x + aG)^\alpha(M + bS)^{1-\alpha}$$

where  $x$  is the consumption of good 1,  $M$  is the consumption of money,  $A > 0$ ,  $\alpha \in (0, 1)$  and  $a, b \in [0, 1]$ . When  $a = b = 1$  the consumer considers that  $G$  and  $S$  are perfect substitutes of private consumption and wealth. When  $a = b = 0$  the consumer does not take into consideration  $G$  and  $S$ . This may be because she considers public expenditure as a total waste (like certain public works) and that the debt arising from the deficit will be paid in a distant future, perhaps by a different generation. Another interpretation of  $a = 0$  is that the government produces a different good (bridges) whose price, by taking units, can also be taken as  $\omega$ . This good affects  $A$  making the consumer better off but it does not affect her consumption choices. Compare with Heijdra, Ligthart and van der Ploeg (1998) equation (1) and Linnemann and Schabert (2003) equation (2).

The representative consumer has endowments of labor and money of  $\bar{L}$  and  $\bar{M}$  respectively. Her budget constraint is

$$px + M = I(1 - t). \quad (3)$$

Adding (2) and (3), we get that  $pG + S + px + M = I$ . Since profits are zero and wages are obtained producing  $G$  or  $x$ , income is defined as  $I = \omega G + \omega x + \bar{M}$ . Taking into account (1), the last two equalities imply that  $S + M = \bar{M}$ . Thus, when the public sector runs a surplus ( $S > 0$ ),  $M < \bar{M}$ . A public sector deficit ( $S < 0$ ) implies  $M > \bar{M}$ .

From utility maximization at given prices and income, demand functions are

$$x = \frac{\alpha(I(1 - t) + bS)}{p} - (1 - \alpha)aG, \quad (4)$$

$$M = (1 - \alpha)(I(1 - t) + apG) - \alpha bS \quad (5)$$

## 2. 1. Market Clearing Equilibrium

In a market-clearing equilibrium all markets clear. Denoting the variable, say,  $z$  in a market-clearing equilibrium as  $z^C$  ( $C$  for clearing) we have that

$$x^C + G = \bar{L}. \quad (6)$$

$$I^C = \omega^C \bar{L} + \bar{M}. \quad (7)$$

Notice that the real GDP of our economy is  $x^C + G$  which, given our choice of units, is just  $\bar{L}$ . From (4), (6) and (7) prices and wages in a market-clearing equilibrium are

$$p^C = \omega^C = \frac{\alpha(\bar{M} - S(1 - b))}{(1 - \alpha)(\bar{L} - G(1 - a))}. \quad (8)$$

Thus prices and wages increase with the money supply, the deficit (unless  $b = 1$ ) and the public expenditure (unless  $a = 1$ ) and decrease with the labor supply.

From (6) consumption of good 1 in a market-clearing equilibrium is

$$x^C = \bar{L} - G \quad (9)$$

Thus, consumption of good 1 increases with resources and decreases with public expenditure in a one-by-one basis independently of the parameter  $a$ .

## 2.2. Non Clearing Equilibrium

In a no clearing labor market equilibrium, non clearing for short, the supply of labor exceeds the demand of labor denoted by  $L^N$ . All other markets clear. Let us denote the variables in such an equilibrium by the superscript  $N$  ( $N$  for non clearing). Thus,

$$L^N = x^N + G. \quad (10)$$

$$p^N = \omega^N. \quad (11)$$

Now the income of the consumer is determined by the actual production, namely

$$I^N = \omega^N(x^N + G) + \bar{M}. \quad (12)$$

Taking into account (11) and (4) we have that

$$x^N = \frac{\alpha(I^N(1-t) + bS)}{\omega^N} - (1-\alpha)aG. \quad (13)$$

Plugging (12) and (2) in (13) and using (10) we have that

$$x^N = \frac{\alpha(\bar{M} - S(1-b))}{\omega^N(1-\alpha)} - aG \quad (14)$$

$$L^N = \frac{\alpha(\bar{M} - S(1-b))}{\omega^N(1-\alpha)} + G(1-a) \quad (15)$$

From (15) we see that  $\bar{L} > L^N$  iff

$$\omega^N > \frac{\alpha(\bar{M} - S(1-b))}{(1-\alpha)(\bar{L} - G(1-a))} = \omega^C \quad (16)$$

Thus it can be said that, in this model, unemployment arises because wages are too high.<sup>4</sup> This is just a way of saying that our model embodies a Pigou effect where a decrease in prices may increase GDP.

Let us make a simple calibration exercise. Despite the fact that, as noticed in the introduction, our economy has very stylized features, this exercise might throw some light on our model. In order to obtain possible values for the parameters, divide both sides of (15) by the GDP,  $L^N$ . We interpret  $\frac{L^N \omega^N}{\bar{M}}$  as

<sup>4</sup>The same happens in the venerable IS-LM model.

the velocity of money and we will denote this magnitude by  $v$ . Defining  $g \equiv \frac{G}{L^N}$  and  $s \equiv \frac{S}{L^N} = t - g$  as, respectively, the fractions of public expenditure and deficit in GDP, (15) yields

$$(1 - \alpha)(1 - g(1 - a)) = \alpha\left(\frac{1}{v} - s(1 - b)\right). \quad (17)$$

From (13) we interpret  $\alpha$  as the marginal propensity to consume. Letting  $\alpha = .8$ ,  $g \in [.3, .5]$  and  $s \in [-.03, -.05]$  in (17) we see that depending on the values of  $a$  and  $b$ ,  $v$  ranges between 4 and 13.333, not very different from what is shown in real data. Notice that (17) implies that if  $g$  and  $s$  are constant the velocity of money is constant too.

### 3. THE KEYNESIAN MULTIPLIER

Recall that the government can not choose  $S$ ,  $G$  and  $t$  simultaneously. Equation (15) involves the deficit and public expenditure, so it is the appropriate equation to work with when the strategies of the government are these two variables. Thus in this case, the tax rate is adjusted to satisfy (2). In this case we see that the Keynesian multiplier is

$$\frac{\partial L^N}{\partial G} = 1 - a \quad (18)$$

which in the best case scenario (i.e. when public consumption does not affect the private consumption or  $a = 0$ ) is one and in the extreme case in which public and private consumption are perfect substitutes it is zero. The former corresponds to the well-known result that the multiplier with a balanced budget is one. Writing the multiplier in terms of elasticities we have that

$$\frac{\partial L^N}{\partial G} \frac{G}{L^N} = (1 - a)g < 1.$$

Suppose now that the strategies of the government are  $t$  and  $G$ . In this case the deficit (or surplus) takes the toll of the adjustment. After lengthy calculations we obtain that

$$L^N = \frac{\alpha \frac{\bar{M}}{\omega^N} (1 - (1 - b)t) + G(a\alpha - a - b\alpha + 1)}{(1 - \alpha) + \alpha(1 - b)t} \quad (19)$$

Now the Keynesian multiplier is

$$\frac{\partial L^N}{\partial G} = \frac{-a(1 - \alpha) + 1 - b\alpha}{(1 - \alpha) + \alpha(1 - b)t}. \quad (20)$$

Notice that, as intuition suggests, the larger  $a$  the smaller the value of the multiplier. In the extreme case in which  $a = b = 0$  the multiplier is  $\frac{1}{1 - \alpha(1 - t)}$  which, interpreting again  $\alpha$  as the marginal propensity to consume, is just the

textbook Keynesian multiplier. But when  $b = 1$ , i.e. when the effect of debt is fully anticipated, the multiplier is, again,  $1 - a$ , never larger than one. Our expressions (18) and (20) generalize those obtained by Bailey (1971), Chapter 9, Table 1 for the limiting cases where  $a$  and  $b$  are either 0 or 1.

Again writing the multiplier in terms of elasticity

$$\frac{\partial L^N}{\partial G} \frac{G}{L^N} = \frac{(-a(1-\alpha) + 1 - b\alpha)p^N G}{\alpha\bar{M}(1 - (1-b)t) + p^N G(-a(1-\alpha) + 1 - b\alpha)} < 1. \quad (21)$$

#### 4. THE PIGOU EFFECT

The fixprice model is an interesting scenario to study the so called "Pigou effect" in which a decrease in prices/wages increases employment via increases in real wealth.<sup>5</sup> Let us consider first the case in which the deficit is constant. In this case, from (15) the multiplier associated with the Pigou effect is:

$$-\frac{\partial L^N}{\partial p^N} = \frac{\alpha(\bar{M} - (1-b)S)}{(1-\alpha)(p^N)^2}. \quad (22)$$

Writing the previous expression in terms of elasticities,

$$-\frac{\partial L^N}{\partial p^N} \frac{p^N}{L^N} = \frac{\alpha(\bar{M} - (1-b)S)}{\alpha(\bar{M} - (1-b)S) + (1-\alpha)p^N G(1-a)} = 1 - g(1-a). \quad (23)$$

Clearly, the elasticity associated with the Keynesian multiplier is larger than the elasticity associated with the Pigouvian multiplier iff

$$2g(1-a) > 1. \quad (24)$$

Even in the most favorable case for the Keynesian multiplier, i.e.  $a = 0$ , the inequality (24) requires that the ratio public expenditure/GDP be larger than a half. So if the deficit is constant, we should expect better results on employment from a pro-competitive policy which facilitates price decreases rather than from public expenditure.

Suppose now that the strategies of the government are  $t$  and  $G$ . In this case the Pigouvian multiplier is

$$-\frac{\partial L^N}{\partial p^N} = \frac{\alpha\bar{M}(1 - (1-b)t)}{((1-\alpha) + \alpha(1-b)t)(\omega^N)^2}. \quad (25)$$

Writing the Pigou multiplier in terms of elasticities, we have that

$$-\frac{\partial L^N}{\partial p^N} \frac{p^N}{L^N} = \frac{\alpha\bar{M}(1 - (1-b)t)}{\alpha\bar{M}(1 - (1-b)t) + p^N G(a\alpha - a - b\alpha + 1)} < 1. \quad (26)$$

<sup>5</sup>The current crisis has caused a moderate deflation in many countries. Thus, prices decreased in Great Britain .4% in March 2009 and .1% in Spain in the same period.



From (21) and (26) we obtain that the elasticity associated with the Keynesian multiplier is larger than the elasticity associated with the Pigouvian multiplier iff  $(-a(1 - \alpha) + 1 - b\alpha)p^N G > \alpha\bar{M}(1 - (1 - b)t)$ , or

$$(-a(1 - \alpha) + 1 - b\alpha)gv > \alpha(1 - (1 - b)t) \quad (27)$$

In this case the comparison is ambiguous. If  $a = b = 1$  the Pigouvian multiplier is larger than its Keynesian counterpart. If  $a = b = 0$  the Keynesian multiplier is larger than its Pigouvian counterpart if and only if  $gv > \alpha(1 - t)$ . This inequality holds for the kind of values of  $g, v$  and  $\alpha$  considered in the simple calibration exercise performed at the end of Section 4.

## 5. CONCLUSIONS

In this paper we have presented a simple fixprice model in which public and private consumption might be substitutes. We have shown that the value of the multiplier depends on this degree of substitution. In particular when deficits are fully incorporated into wealth we get a "Ricardian Equivalence" result, see Benassy (2007) for a similar result in a different framework. We also have shown that the Pigou effect exists in our fixprice model and that, sometimes, it is stronger than the Keynesian multiplier. This is, of course, not an argument against public expenditure as a means of increasing employment. But it raises the point that, sometimes, a policy of deflation may have even better results in terms of increasing employment. The latter policy was considered dangerous by Keynes ([1936], Chapter 19) because of its effects on price expectations. We plan to study this question more carefully in a subsequent paper.

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