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Abstract

We collect simple and pragmatic exact formulae for the convexity adjustment of irregular interest rate cash flows as Libor-in-arrears or payments of a swap rate (CMS rate) at an irregular date. The results are compared with the results of an approximative approach available in the popular literature.

For options on Libor-in-arrears or CMS rates like caps or binaries we derive an additional new convexity adjustment for the volatility to be used in a standard Black & Scholes model. We study the quality of the adjustments comparing the results of the approximative Black & Scholes formula with the results of an exact valuation formula.

Further we investigate options to exchange interest rates which are possibly set at different dates or admit different tenors.

We collect general quanto adjustments formulae for variable interest rates to be paid in foreign currency and derive valuation formulae for standard options on interest rates paid in foreign currency.

Key words: interest rate options, convexity, quanto adjustment, change of numeraire

JEL Classification: G13

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1 Introduction

Increasing the return of a variable interest rate investment or cheapening the costs of borrowing can often be achieved by taking advantage of certain shapes of the forward yield curve. A well known and frequently used technique is a delayed setting of the variable index (Libor-in-arrears) or the use of long dates indices like swap rates in place of Libor (CMS rate products) or the use of foreign floating interest rate indices to be paid in domestic currency. It is well known that payments of variable interest rates like Libor or swap rates at dates different from their natural payment dates or in currencies different from their home currency imply certain convexity effects which result in adjusted forward rates.

Pricing those structures in the framework of a fully calibrated term structure model would take the convexity effects automatically into account. However, for many applications this seems to be a modelling overkill. Many convexity adjustment formulae are available in the literature ([2], [4],[1]), some of them based on more or less theoretically sound arguments. Here we collects simple and pragmatic exact formulae for the convexity adjustment for arbitrary irregular interest rate cash flows and compare the results with the outcomes of an approximative approach available in the popular literature.

Then we extend our analysis to options like caps, floors or binaries on irregular interest rates. Given the market model of log-normality for standard interest rate options the consistent model for options on irregular rates is certainly different from a log-normal one. We derive a new additional volatility adjustment that allows an approximation of the true distribution by a log-normal distribution with the same second moment. The results of the approximative pricing are then compared with the exact but more involved valuation. The advantage of the volatility adjustment is that standard pricing libraries can be applied with good accuracy in this context as well. It seems that this new volatility adjustment is so far ignored in practice.

Another application consists of options to exchange interest rates, e.g. Libor-in-arrears versus Libor. For example, those options are implicitly contained in structures involving the maximum or minimum of Libor-in-arrears and Libor.

The final section of this notes collects general formulae for quanto adjustments on floating interest rates paid in a different currency and related options.

2 Notation

Denote by $B(t, T)$ the price of a *zero bond* with maturity T at time $t \leq T$. The zero bond pays one unit at time T, $B(T, T) = 1$. At time $t = 0$ the zero bond price $B(0, T)$ is just the discount factor for time T.

The *Libor* $L(S, T)$ for the interval $[S, T]$ is the money market rate for this interval as fixed in the market at time S, this means

$$\begin{aligned} B(S, T) &= \frac{1}{1 + L(S, T)\Delta} \\ L(S, T) &= \frac{\frac{1}{B(S,T)} - 1}{\Delta}, \end{aligned}$$

with Δ as the length of the period $[S, T]$ in the corresponding day count convention.

The *forward Libor* $L^0(S, T)$ for the period $[S, T]$ as seen from today, $t = 0$, is given by

$$L^0(S, T) = \frac{\frac{B(0,S)}{B(0,T)} - 1}{\Delta}. \tag{1}$$

The *swap rate* or *CMS rate* X for a swap with reference dates $T_0 < T_1 < \cdots < T_n$ as fixed at time T_0 is defined as

$$X = \frac{1 - B(T_0, T_n)}{\sum_{i=1}^{n} \Delta_i B(T_0, T_i)}, \tag{2}$$

with Δ_i as length of the period $[T_{i-1}, T_i]$ in the corresponding day count convention. The tenor of X is the time $T_n - T_0$.

The *forward swap rate* X^0 as seen from today is then

$$X^0 = \frac{B(0, T_0) - B(0, T_n)}{\sum_{i=1}^{n} \Delta_i B(0, T_i)}. \tag{3}$$

In the theory of derivative pricing the notion of a numeraire pair $(N, \mathbf{Q}_N)$ plays a central role. In the context of interest rate derivatives the basic securities are the zero bonds of all maturities. A *numeraire pair* (N, Q_N) then consists of a non-negative process N and an associated probability distribution $\mathbf{Q}_N$ such that all basic securities are martingales under $\mathbf{Q}_N$ if expressed in the numeraire N as base unit, i.e.,

$$\frac{B(t, T)}{N_t}, \; t \leq T, \; \text{is a } \mathbf{Q}_N \text{ martingale for all } T > 0. \tag{4}$$

The *price* $V_0(Y)$ today for a contingent claim[1] Y to be paid at time p is then

$$V_0(Y) = N_0 \mathbf{E}_{\mathbf{Q}_N} \left(\frac{Y}{N_p} \right). \tag{5}$$

For two numeraire pairs $(N, \mathbf{Q}_N)$ and $(M, \mathbf{Q}_M)$ the transformation (Radon-Nikodym density) between the distributions $\mathbf{Q}_N$ and $\mathbf{Q}_M$ on the information structure up to time p is

$$\mathrm{d}\mathbf{Q}_M = \frac{N_0 M_p}{M_0 N_p} \mathrm{d}\mathbf{Q}_N. \tag{6}$$

The time T *forward measure* is the distribution $\mathbf{Q}_T$ referring to the numeraire being the zero bond with maturity T, $N_t = B(t,T)$, and we write

$$\mathbf{Q}_T = \mathbf{Q}_{B(.,T)}. \tag{7}$$

An immediate consequence of (5) applied with $B(.,p)$ as numeraire is

$$V_0(Y) = B(0,p) \mathbf{E}_{\mathbf{Q}_p} Y, \tag{8}$$

i.e., the price of the claim Y today is its discounted expectation under the time p forward measure.

3 Convexity adjusted forward rates

3.1 Libor-in-arrears

In standard interest rate derivatives on Libor the claim on the Libor $L(S,T)$ for period $[S,T]$ is paid at the end of the interval, i.e., at time T. Since the Libor is set at time S this is named "set in advance, pay in arrears". Under $\mathbf{Q}_T$ the process $\frac{B(.,S)}{B(.,T)}$ is a martingale and we get

$$\mathbf{E}_{\mathbf{Q}_T} L(S,T) = \mathbf{E}_{\mathbf{Q}_T} \left(\frac{\frac{B(S,S)}{B(S,T)} - 1}{\Delta} \right) = \frac{\frac{B(0,S)}{B(0,T)} - 1}{\Delta} = L^0(S,T). \tag{9}$$

This implies from (8) the well-known expression for the price of a Libor,

$$V_0(L(S,T)) = B(0,T) L^0(S,T), \tag{10}$$

as discounted forward Libor.

[1] We assume that Y can be hedged with the basic securities.

In a Libor-in-arrears payment the Libor $L(S,T)$ for period $[S,T]$ is now paid at the time S of its fixing. According to (8) its price today is

$$B(0,S)\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right). \tag{11}$$

Our goal is to express $\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right)$ in terms of the forward rate $L^0(S,T)$ plus some "convexity" adjustment, the convexity charge.

Proposition 1 *The following general valuation formula holds*

$$\boxed{\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right) = L^0(S,T)\left(1+\frac{\Delta}{L^0(S,T)}\frac{\mathrm{Var}_{\mathbf{Q}_T}L(S,T)}{(1+\Delta L^0(S,T))}\right),} \tag{12}$$

with $\mathrm{Var}_{\mathbf{Q}_T}L(S,T)$ *as the variance of* $L(S,T)$ *under the distribution* $\mathbf{Q}_T$.

Proof: Using (6) we obtain

$$\begin{aligned}
\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right) &= \mathbf{E}_{\mathbf{Q}_T}\left(L(S,T)\frac{B(S,S)B(0,T)}{B(S,T)B(0,S)}\right)\\
&= \mathbf{E}_{\mathbf{Q}_T}\left(L(S,T)(1+\Delta L(S,T))\frac{B(0,T)}{B(0,S)}\right)\\
&= \frac{\mathbf{E}_{\mathbf{Q}_T}\left((L(S,T)+\Delta L(S,T)^2\right)}{1+\Delta L^0(S,T)}\\
&= \frac{(L^0(S,T)+\Delta \mathrm{Var}_{\mathbf{Q}_T}L(S,T)+\Delta L^0(S,T)^2)}{1+\Delta L^0(S,T)}\\
&= L^0(S,T)\left(1+\frac{\Delta}{L^0(S,T)}\frac{\mathrm{Var}_{\mathbf{Q}_T}L(S,T)}{1+\Delta L^0(S,T)}\right).
\end{aligned}$$

◇

Under the so-called market model which is the model underlying the market valuation for caps, the Libor $L(S,T)$ is lognormal under $\mathbf{Q}_T$ with volatility σ,

$$L(S,T) = L^0(S,T)\exp(\sigma W_S - \frac{1}{2}\sigma^2 S), \tag{13}$$

with some Wiener process (W_t). In this case the convexity adjustment can be expressed in terms of the volatility and (12) reduces to

$$\boxed{\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right) = L^0(S,T)\left(1+\frac{\Delta L^0(S,T)(\exp(\sigma^2 S)-1)}{1+\Delta L^0(S,T)}\right).} \tag{14}$$

Example. To illustrate the magnitude of the convexity charge we show below as an example the convexity adjusted forward rates $\mathbf{E}_{\mathbf{Q}_s}\left(L(S,T)\right)$ for various maturities S and volatilities σ. We assume $L^0(S,T) = 5\%$ and $\Delta = 0.5$.

$S\|\sigma$	10%	15%	20%
1	5,001%	5,003%	5,005%
2	5,002%	5,006%	5,010%
3	5,004%	5,009%	5,016%
4	5,005%	5,011%	5,021%
5	5,006%	5,015%	5,027%
6	5,008%	5,018%	5,033%
7	5,009%	5,021%	5,039%
8	5,010%	5,024%	5,046%
9	5,011%	5,027%	5,053%
10	5,013%	5,031%	5,060%

In the general case, for arbitrary payment times $p \geq S$ a somewhat more involved formula can be derived, see [5]. However, using an idea similar to the assumption of a linear swap rate model (cf. [3]), we can easily derive a formula even for $p \geq S$ following the same line of arguments as above.

Let us assume a linear model of the from

$$\frac{B(S,p)}{B(S,T)} = \alpha + \beta_p L(S,T), \; \forall p \geq S. \tag{15}$$

The constants α and β_p are straightforward to determine in order to make the model consistent. Since $\frac{B(.,p)}{B(.,T)}$ is a $\mathbf{Q}_T$ martingale and using (9)

$$\begin{aligned} \frac{B(0,p)}{B(0,T)} &= \mathbf{E}_{\mathbf{Q}_T} \frac{B(S,p)}{B(S,T)} \\ &= \mathbf{E}_{\mathbf{Q}_T} (\alpha + \beta_p L(S,T)) \\ &= (\alpha + \beta_p L^0(S,T)), \end{aligned}$$

which implies $\beta_p = \left(\frac{B(0,p)}{B(0,T)} - \alpha \right) / L^0(S,T)$. Also $\alpha = 1$ as a consequence of $1 = \frac{B(S,T)}{B(S,T)} = \alpha + \beta_T L(S,T)$, so finally,

$$\beta_p = \left(\frac{B(0,p)}{B(0,T)} - 1 \right) / L^0(S,T). \tag{16}$$

Now we can formulate the result for $p \geq S$.

Proposition 2 *Under the assumption of a linear Libor model (15) we have the following general formula for payments of the Libor $L(S,T)$ at arbitrary times $p \geq S$*

$$\boxed{\mathbf{E}_{\mathbf{Q}_p} \left(L(S,T) \right) = L^0(S,T) \left(1 + \frac{1 - \frac{B(0,T)}{B(0,p)}}{(L^0(S,T))^2} \mathrm{Var}_{\mathbf{Q}_T} L(S,T) \right),} \tag{17}$$

with $\mathrm{Var}_{\mathbf{Q}_T} L(S,T)$ *as the variance of* $L(S,T)$ *under the distribution* $\mathbf{Q}_T$*.*

Remarks.
1. For $p = S$ formula (17) obviously reduces to the general formula (12).
2. In case of the market model (13) for caps formula (17) gets more explicit:

$$\boxed{\mathbf{E}_{\mathbf{Q}_p}\left(L(S,T)\right) = L^0(S,T)\left(1 + \left(1 - \frac{B(0,T)}{B(0,p)}\right)(\exp(\sigma^2 S) - 1)\right)} \tag{18}$$

3.2 CMS

The market standard valuation formula for swaptions is closely related to a particular numeraire pair called the *swap numeraire* or *PV01 numeraire* pair $(\mathrm{PV01}, \mathbf{Q}_{\mathrm{Swap}})$ with numeraire

$$N_t = \mathrm{PV01}_t = \sum_{i=1}^{n} \Delta_i B(t,T_i),\; t \leq T_1. \tag{19}$$

Under $\mathbf{Q}_{\mathrm{Swap}}$ the expectation of the swap rate X is just the forward swap rate X^0 which is again a consequence of the martingale property of $B(.,T_0), B(.,T_n)$ if expressed in the numeraire PV01

$$\begin{aligned}
\mathbf{Q}_{\mathrm{Swap}} X &= \mathbf{Q}_{\mathrm{Swap}}\left(\frac{B(T_0,T_0) - B(T_0,T_n)}{\sum_{i=1}^{n} \Delta_i B(T_0,T_i)}\right) \\
&= \mathbf{Q}_{\mathrm{Swap}}\left(\frac{B(T_0,T_0) - B(T_0,T_n)}{\mathrm{PV01}_{T_0}}\right) \\
&= \frac{B(0,T_0) - B(0,T_n)}{\mathrm{PV01}_0} = X^0.
\end{aligned} \tag{20}$$

In a CMS based security, e.g., a CMS swap or cap, the rate X is paid only once and at a time $p \geq T_0$. According to (8) we are interested in an explicit valuation of

$$\mathbf{E}_{\mathbf{Q}_p}(X)$$

in terms of the forward swap rate X^0 and some "convexity" adjustment.

Applying (6) we get

$$\mathbf{E}_{\mathbf{Q}_p}(X) = \mathbf{E}_{\mathbf{Q}_{\mathrm{Swap}}}\left(X \frac{B(T_0,p)}{\mathrm{PV01}_{T_0}} \frac{\mathrm{PV01}_0}{B(0,p)}\right) = \frac{\mathrm{PV01}_0}{B(0,p)} \mathbf{E}_{\mathbf{Q}_{\mathrm{Swap}}}\left(X \frac{B(T_0,p)}{\mathrm{PV01}_{T_0}}\right). \tag{21}$$

In order to calculate the right hand side explicitly one has to express or approximate $\frac{B(T_0,p)}{\mathrm{PV01}_{T_0}}$ in terms of simpler objects like Libor or the swap rate

X itself. Several approximations are studied in [5]. We rely here on a very elegant approximation based the assumption of a *linear swap rate model*, see [3] or [4],

$$\frac{B(T_0,p)}{\mathrm{PV01}_{T_0}} = \alpha + \beta_p \cdot X,\ p \geq T_0. \tag{22}$$

The constant α and the factor β_p have to be determined consistently. Since $\frac{B(.,p)}{\mathrm{PV01}_.}$ is a $\mathbf{Q}_{\mathrm{Swap}}$-martingale using (20) we obtain

$$\begin{aligned}
\frac{B(0,p)}{\mathrm{PV01}_0} &= \mathbf{E}_{\mathbf{Q}_{\mathrm{Swap}}}\left(\frac{B(T_0,p)}{\mathrm{PV01}_{T_0}}\right) \\
&= \mathbf{E}_{\mathbf{Q}_{\mathrm{Swap}}}(\alpha + \beta_p X) \\
&= (\alpha + \beta_p X^0),
\end{aligned}$$

and thus

$$\beta_p = \frac{\frac{B(0,p)}{\mathrm{PV01}_0} - \alpha}{X^0}. \tag{23}$$

To determine α observe that

$$\begin{aligned}
1 &= \frac{\sum_{i=1}^n \Delta_i B(T_0,T_i)}{\mathrm{PV01}_{T_0}} \\
&= \sum_{i=1}^n \Delta_i \alpha + \sum_{i=1}^n \Delta_i \beta_{T_i} X \\
&= \sum_{i=1}^n \Delta_i \alpha + (1 - \sum_{i=1}^n \Delta_i \alpha)\frac{X}{X^0},
\end{aligned}$$

which yields

$$\alpha = \frac{1}{\sum_{i=1}^n \Delta_i}. \tag{24}$$

Proposition 3 *Under the assumption (22) of the linear swap rate model we have the following general valuation formula*

$$\boxed{\mathbf{E}_{\mathbf{Q}_p} X = X^0 \left(1 + \frac{1 - \frac{(B(0,T_0)-B(0,T_n))}{X^0 B(0,p)\sum_{i=1}^n \Delta_i}}{(X^0)^2} \mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X)\right)}, \tag{25}$$

with $\mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X)$ *as the variance of* X *under the swap measure* $\mathbf{Q}_{\mathrm{Swap}}$.

Proof: Using (21), (22) and (20) we obtain

$$\begin{aligned}
\mathbf{E}_{\mathbf{Q}_p} X &= \frac{\mathrm{PV01}_0}{B(0,p)} \mathbf{E}_{\mathbf{Q}_{\mathrm{Swap}}} \left(X(\alpha + \beta_p X) \right) \\
&= \frac{1}{\alpha + \beta_p X^0} (\alpha X^0 + \beta_p (X^0)^2 + \beta_p \mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X)) \\
&= X_0 \left(1 + \frac{\beta_p \mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X))}{X^0(\alpha + \beta_p X^0)} \right).
\end{aligned}$$

Substituting

$$\alpha + \beta_p X_0 = \frac{B(0,p)}{\mathrm{PV01}_0} = \frac{B(0,p)X^0}{(B(0,T_0) - B(0,p))}$$

and using (24) yields the assertion. ◇

Remark. In the special case of a one period swap the CMS adjustment formula (25) reduces to the respective formula (17) for Libor payments.

Under the market model for swaptions it is known that the swap rate X is lognormal under $\mathbf{Q}_{\mathrm{Swap}}$,

$$X = X^0 \exp(\sigma W_{T_0} - \frac{1}{2}\sigma^2 T_0), \tag{26}$$

with some Wiener process (W_t). In this case the variance of X under $\mathbf{Q}_{\mathrm{Swap}}$ is just

$$\mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X) = (X^0)^2(\exp(\sigma^2 T_0) - 1) \tag{27}$$

and (25) reduces to

$$\boxed{\mathbf{E}_{\mathbf{Q}_p} X = X^0 \left(1 + \left(1 - \frac{B(0,T_0) - B(0,T_n)}{X^0 B(0,p) \sum_{i=1}^{n} \Delta_i} \right) (\exp(\sigma^2 T_0) - 1) \right).} \tag{28}$$

Example. Again we illustrate the size of the CMS adjustment by an example. We assume that the swap curve is flat at 5% for all maturities. Payment of the CMS rate is at $p = T_0 + 1$ as it happens in most cases in practice.

σ	10,00%			15%			20%		
$T_0 \mid T_n - T_0$	5	10	20	5	10	20	5	10	20
1	5,005%	5,010%	5,017%	5,010%	5,022%	5,039%	5,019%	5,039%	5,071%
2	5,009%	5,019%	5,035%	5,021%	5,044%	5,080%	5,038%	5,079%	5,144%
3	5,014%	5,029%	5,053%	5,032%	5,066%	5,121%	5,058%	5,121%	5,220%
4	5,019%	5,039%	5,071%	5,043%	5,089%	5,163%	5,079%	5,164%	5,300%
5	5,023%	5,048%	5,089%	5,054%	5,113%	5,206%	5,101%	5,209%	5,382%
6	5,028%	5,058%	5,107%	5,066%	5,137%	5,250%	5,123%	5,256%	5,469%
7	5,033%	5,068%	5,125%	5,077%	5,161%	5,295%	5,147%	5,305%	5,558%
8	5,038%	5,079%	5,144%	5,089%	5,186%	5,341%	5,170%	5,356%	5,652%
9	5,043%	5,089%	5,163%	5,101%	5,212%	5,388%	5,196%	5,409%	5,749%
10	5,048%	5,099%	5,182%	5,114%	5,238%	5,436%	5,222%	5,464%	5,850%

Proposition 3 yields an interesting corollary.

Corollary 1 *For payment times p ranging from T_0 to T_n the convexity charge*

$$C(p) = X^0 \left(\frac{1 - \frac{(B(0,T_0)-B(0,T_n))}{X^0 B(0,p) \sum_{i=1}^n \Delta_i}}{(X^0)^2} \mathrm{Var}_{\mathbf{Q}_{\mathrm{Swap}}}(X) \right)$$

is monotonously decreasing in p changing its sign from positive at $p = T_0$ to negative at $p = T_n$. The charge changes sign and vanishes exactly at the point $p \in (T_0, T_n)$ where $B(0,p) \sum_{i=1}^n \Delta_i = \sum_{i=1}^n \Delta_i B(0, T_i)$. Moreover,

$$\sum_{i=1}^{n} C(T_i) = 0.$$

Intuitively, this is not surprising, since applying the valuation formula (25) multiplied with $B(0,p)$ for all $p = T_1, \ldots, T_n$ and summing up we end up with the valuation of a full interest rate swap and all convexity adjustments should cancel out.

Observe that the qualitative statement of the Corollary remains true also without the assumption of a linear swap rate model, see [5].

3.3 Unified approach under the linear rate model

Assuming a linear rate model (15) or (22) we derived a closed valuation formula for a Libor or CMS rate paid at an arbitrary date. Both derivations can be unified under one umbrella.

Write Y_S for a floating rate which is set at time S. Examples of particular interest are $Y_S = L(S,T)$, the Libor for the interval $[S,T]$, or, $Y_S = X$ with $S = T_0$ and X the swap rate with reference dates $T_0 < T_1 < \cdots < T_n$. Let $N, \mathbf{Q}_N$ denote the natural ("market") numeraire pair associated with Y_S and all we need is that

$$\mathbf{E}_{\mathbf{Q}_N} Y_S = Y_0, \tag{29}$$

where Y_0 is known and a function of the yield curve $B(0,.)$ today.

We are interested in todays price of the rate Y_S to be paid at some time $p \geq S$,

$$B(0,p)\mathbf{E}_{\mathbf{Q}_p} Y_S.$$

Assume a linear rate model of the from

$$\frac{B(S,p)}{N_S} = \alpha + \beta_p Y_S, \tag{30}$$

with some deterministic α, β_p which have to be determined accordingly to make the model consistent, see (16) resp. (24), (23) for α and β_p in case of a linear Libor resp. swap rate model. From the martingale property of $\frac{B(.,p)}{N.}$ and (29) we get immediately

$$\frac{B(0,p)}{N_0} = \alpha + \beta_p Y_0.$$

Proposition 4 *Under the assumption of a linear model (30) we have the following general valuation formula*

$$\boxed{\mathbf{E}_{\mathbf{Q}_p} Y_s = Y_0 \left(1 + \frac{\beta_p}{Y_0(\alpha + \beta_p Y_0)} \mathrm{Var}_{\mathbf{Q}_N}(Y_S)\right),} \tag{31}$$

with $\mathrm{Var}_{\mathbf{Q}_N}(Y_S)$ *as the variance of* Y_S *under the measure* $\mathbf{Q}_N$*.*
If in addition, the distribution of Y_S *under* $\mathbf{Q}_N$ *is lognormal with volatility* σ_Y*,*

$$Y_S = Y_0 \exp(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S),$$

then

$$\boxed{\mathbf{E}_{\mathbf{Q}_p} Y_S = Y_0 \left(1 + \frac{\beta_p Y_0}{(\alpha + \beta_p Y_0)} (\exp(\sigma_Y^2 S) - 1)\right).} \tag{32}$$

Proof: On the information up to time S the density of the time p forward measure $\mathbf{Q}_p$ w.r.t. $\mathbf{Q}_N$ is according to (6)

$$\mathrm{d}\mathbf{Q}_p = \frac{N_0}{B(0,p)} \frac{B(S,p)}{N_S} \mathrm{d}\mathbf{Q}_N = \frac{\alpha + \beta_p Y_S}{\alpha + \beta_p Y_0} \mathrm{d}\mathbf{Q}_N. \tag{33}$$

Therefore,

$$\begin{aligned} \mathbf{E}_{\mathbf{Q}_p} Y_S &= \mathbf{E}_{\mathbf{Q}_N} \left(Y_S \frac{\alpha + \beta_p Y_S}{\alpha + \beta_p Y_0} \right) \\ &= Y_0 \left(1 + \frac{\beta_p}{Y_0(\alpha + \beta_p Y_0)} \mathrm{Var}_{\mathbf{Q}_N}(Y_S) \right). \end{aligned}$$

◇

In case of a linear Libor model or a linear swap rate model formula (31) reduces to (17) or (25), respectively.

3.4 Comparison study

In this section we compare the results of the above convexity adjustment formulae with the results of a popular formula which can be found, for example, in [2], Section 16.11.

Suppose we are interested in a derivative involving a yield Y at time p of a bond with yield to price function $P(Y)$. To the forward price P_0 as of today for a forward contract maturing at time p there corresponds a forward yield Y_0,

$$P_0 = P(Y_0).$$

Then [2] gives the following approximative formula

$$\boxed{\mathbf{E}_{\mathbf{Q}_p}(Y) \approx Y_0 - \frac{1}{2} Y_0^2 \sigma^2 p \frac{P''(Y_0)}{P'(Y_0)},} \tag{34}$$

with σ as the volatility of the yield Y.

Applying this to a Libor-in-arrears, $Y = L(S,T), p = S, P(y) = \frac{1}{1+\Delta y}$ yields the formula

$$\boxed{\mathbf{E}_{\mathbf{Q}_s}(L(S,T)) \approx L^0(S,T)\left(1 + \frac{\Delta L^0(S,T)\sigma^2 S}{1+\Delta L^0(S,T)}\right),} \tag{35}$$

which is in line with the exact formula (14) as long as one approximates $(\exp(\sigma^2 S) - 1)$ to the first order

$$\exp(\sigma^2 S) - 1) \approx \sigma^2 S.$$

It is obvious that (35) underestimates the true convexity as quantified by (14) which becomes apparent in particular for long dated Libor-in-arrears structures and relatively high volatilities.

For an interest rate level of $L^0(S,T) = 5\%$[2] the following table shows a comparison of the convexity charges as resulting from equation (14) or (35), respectively,

[2] On an act/360 basis.

	$\sigma = 10\%$		$\sigma = 20\%$	
S	(14)	(35)	(14)	(35)
1	0,001%	0,001%	0,005%	0,005%
2	0,002%	0,002%	0,010%	0,010%
3	0,004%	0,004%	0,016%	0,015%
4	0,005%	0,005%	0,021%	0,020%
5	0,006%	0,006%	0,027%	0,025%
6	0,008%	0,007%	0,033%	0,030%
7	0,009%	0,009%	0,040%	0,035%
8	0,010%	0,010%	0,046%	0,039%
9	0,012%	0,011%	0,053%	0,044%
10	0,013%	0,012%	0,061%	0,049%
11	0,014%	0,014%	0,068%	0,054%
12	0,016%	0,015%	0,076%	0,059%
13	0,017%	0,016%	0,084%	0,064%
14	0,019%	0,017%	0,093%	0,069%
15	0,020%	0,018%	0,101%	0,074%
16	0,021%	0,020%	0,111%	0,079%
17	0,023%	0,021%	0,120%	0,084%
18	0,024%	0,022%	0,130%	0,089%
19	0,026%	0,023%	0,140%	0,094%
20	0,027%	0,025%	0,151%	0,099%

Now we apply formula (34) to the situation of CMS rate. In this case $Y = X$ and Y can be interpreted as the yield of a coupon bond with coupon dates $T_1, \dots, T_n$ and coupon $C = X^0$. At time $p = T_0$ the forward bond price P_0 is at par and the corresponding forward yield is $Y_0 = X^0$. For a bond with annual coupons, $T_i = T_0 + i$, and face value of 1 we have

$$
\begin{aligned}
P(Y) &= \sum_{i=1}^{n} \frac{C}{(1+Y)^i} + \frac{1}{(1+Y)^n} \\
P'(Y) &= \sum_{i=1}^{n} \frac{-i\,C}{(1+Y)^{i+1}} - \frac{n}{(1+Y)^{n+1}} \\
P''(Y) &= \sum_{i=1}^{n} \frac{i(i+1)C}{(1+Y)^{i+2}} + \frac{n(n+1)}{(1+Y)^{n+2}}.
\end{aligned}
$$

Observe that the formula (34) can be applied correctly only in case of a payment of the CMS rate at fixing, i.e., $p = T_0$. This is not what is of interest in most practical applications, where usually $p = T_1$. However, ignoring this inconsistency, equation (34) is often applied for $p = T_1$ which yields a higher convexity charge contrary to what should be the case according to the result of Corollary 1. As we shall see below in the numerical examples, equation (34) underestimates the convexity, so overall, luckily the inconsistency corrects some other error.

Comparing the convexity charges of equation (34) applied to the case of a CMS rate on one hand and of equations (25) and (27) on the other hand we get the following results for a flat interest rate environment of $X^0 = 5\%$ and a tenor of 5 years for the underling CMS rate X:

	$\sigma = 10\%$		$\sigma = 20\%$	
$p = T_0$	(34)	(25) & (27)	(34)	(25) & (27)
1	0,138%	0,137%	0,553%	0,558%
2	0,276%	0,271%	1,106%	1,116%
3	0,415%	0,408%	1,659%	1,709%
4	0,553%	0,549%	2,211%	2,332%
5	0,691%	0,687%	2,764%	2,967%
6	0,829%	0,829%	3,317%	3,635%
7	0,968%	0,972%	3,870%	4,330%
8	1,106%	1,116%	4,423%	5,053%
9	1,244%	1,258%	4,976%	5,789%
10	1,382%	1,405%	5,529%	6,570%
11	1,520%	1,553%	6,081%	7,383%
12	1,659%	1,703%	6,634%	8,230%
13	1,797%	1,854%	7,187%	9,111%
14	1,935%	2,007%	7,740%	10,028%
15	2,073%	2,162%	8,293%	10,982%
16	2,211%	2,318%	8,846%	11,975%
17	2,350%	2,475%	9,399%	13,009%
18	2,488%	2,634%	9,951%	14,085%
19	2,626%	2,795%	10,504%	15,205%
20	2,764%	2,958%	11,057%	16,371%

The same analysis repeated for a tenor of 10 years for the CMS rate yields

	$\sigma = 10\%$		$\sigma = 20\%$	
$p = T_0$	(34)	(25) & (27)	(34)	(25) & (27)
1	0,243%	0,230%	0,971%	0,934%
2	0,486%	0,460%	1,943%	1,897%
3	0,728%	0,694%	2,914%	2,904%
4	0,971%	0,929%	3,885%	3,952%
5	1,214%	1,167%	4,856%	5,039%
6	1,457%	1,407%	5,828%	6,173%
7	1,700%	1,650%	6,799%	7,354%
8	1,943%	1,895%	7,770%	8,583%
9	2,185%	2,142%	8,741%	9,855%
10	2,428%	2,392%	9,713%	11,185%
11	2,671%	2,644%	10,684%	12,569%
12	2,914%	2,899%	11,655%	14,011%
13	3,157%	3,157%	12,626%	15,510%
14	3,399%	3,417%	13,598%	17,072%
15	3,642%	3,680%	14,569%	18,696%
16	3,885%	3,946%	15,540%	20,387%
17	4,128%	4,214%	16,511%	22,148%
18	4,371%	4,485%	17,483%	23,980%
19	4,613%	4,759%	18,454%	25,886%
20	4,856%	5,035%	19,425%	27,871%

Again, (34) consistently underestimates the convexity charge, which is particularly significant for very long dated payments and high volatilities.

Overall, for both, the convexity charge in the Libor-in-arrears case and in the CMS case, the size of the charge increases with the time to the payment and with the volatility of the underlying rate.

4 Options on Libor-in-arrears and CMS rates

In this section we investigate European options on interest rates like Libor $L(S,T)$ for period $[S,T]$ or CMS rates X with reference dates $T_0 < T_1 < \cdots < T_n$. The payment date of the option is an arbitrary time point p with $p \geq S$ or $p \geq T_0$, respectively. Of particular interest are caps and floors or binaries. For standard caps and floors on Libor we have $p = T$ and the standard market model postulates a lognormal distribution of $L(S,T)$ under the forward measure $\mathbf{Q}_T$. For standard options on a swap rate X, i.e. swaptions, the market used a lognormal distribution for X under the swap measure $\mathbf{Q}_{\text{Swap}}$. However in the general case, i.e., for options on Libor or CMS with arbitrary payment date p a lognormal model would be inconsistent with the market model for standard options.

In Sections 4.1 and 4.2 we follow again the general setup of Section 3.3. Y_S is a floating interest rate which is set at time S and $(N, \mathbf{Q}_N)$ denotes the "market" numeraire pair associated with Y_S. We assume that the distribution of Y_S under $\mathbf{Q}_N$ is lognormal with volatility σ_Y,

$$Y_S = Y_0 \exp(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S). \tag{36}$$

For a payment date $p \geq S$ we further assume a linear rate model of the form (30)

$$\frac{B(S,p)}{N_S} = \alpha + \beta_p Y_S. \tag{37}$$

Recall that for the case of $Y_S = L(S,T), N_S = B(S,T)$ and $p = S$, i.e., the case of Libor-in-arrears, the assumption of a linear model is trivially satisfied and no restriction.

4.1 Volatility adjustments

In this section we derive a simple lognormal approximation for the distribution of a rate to be paid at an arbitrary date p which is based on a suitably adjusted volatility.

The main motivation for this approximation is the desire to be able to use standard valuation formulae also for options on interest rates which are irregularly paid. As we shall see below in Section 4.2 there exists also exact valuation formulae but these are somewhat more involved.

Proposition 5 *Suppose (36) and (37). Then for arbitrary $p \geq S$ under $\mathbf{Q}_p$ the rate Y_S is approximately lognormal*

$$Y_S \approx E_{\mathbf{Q}_p}(Y_s)\ \exp(\sigma_Y^* W_S - \frac{1}{2}(\sigma_Y^*)^2 S) \tag{38}$$

with volatility

$$\boxed{(\sigma_Y^*)^2 = \sigma_Y^2 + \ln\left[\frac{(\alpha + \beta_p Y_0)(\alpha + \beta_p Y_0 \exp(2\sigma_Y^2 S))}{(\alpha + \beta_p Y_0 \exp(\sigma_Y^2 S))^2}\right]/S,} \tag{39}$$

and $E_{\mathbf{Q}_p}(Y_S)$ given by (32).

Proof: We calculate the second moment of Y_S under $\mathbf{Q}_p$. Using (33) and (36) we obtain

$$\begin{aligned}
\mathbf{E}_{\mathbf{Q}_p} Y_S^2 &= \mathbf{E}_{\mathbf{Q}_N} \left(Y_S^2 \frac{\alpha + \beta_p Y_S}{\alpha + \beta_p Y_0} \right) \\
&= \frac{1}{\alpha + \beta_p Y_0} \mathbf{E}_{\mathbf{Q}_N} \left(\alpha Y_S^2 + \beta_p Y_S^3 \right) \\
&= \frac{1}{\alpha + \beta_p Y_0} Y_0^2 \left(\alpha \exp(\sigma_Y^2 S) + \beta_p Y_0 \exp(3\sigma_Y^2 S) \right).
\end{aligned}$$

In view of (32) we get for the variance of Y_S under $\mathbf{Q}_p$

$$\mathrm{Var}_{\mathbf{Q}_p} Y_S = (\mathbf{E}_{\mathbf{Q}_p} Y_S)^2 \left(\frac{\exp(\sigma_Y^2 S)(\alpha + \beta_p Y_0)(\alpha + \beta_p Y_0 \exp(2\sigma_Y^2 S))}{(\alpha + \beta_p Y_0 \exp(\sigma_Y^2 S))^2} - 1 \right).$$

Assuming a hypothetical lognormal distribution for Y_S under $\mathbf{Q}_p$ with volatility σ_Y^* as on the right hand side of (38) we would have

$$\mathrm{Var}_{\mathbf{Q}_p} Y_S = (\mathbf{E}_{\mathbf{Q}_p} Y_S)^2 \left(\exp((\sigma_Y^*)^2 S) - 1 \right).$$

Now matching moments yields the assertion. ◇

4.2 Exact valuation under the linear rate model

Assuming a linear rate model it is relatively straightforward to derive exact valuation formulae for standard European options like calls, puts or binaries on Libor or CMS rates to be paid at an arbitrary date. However, the resulting formulae are somewhat more involved.

We use again the setup of a general linear model , i.e. (37). Also we assume a lognormal distribution (36) of Y_S under $\mathbf{Q}_N$.

We are interested in the valuation of standard options on the rate Y_S but the option payout is at some arbitrary time $p \geq S$. The value of a call option with strike K is then

$$\begin{aligned}
&B(0,p) \mathbf{E}_{\mathbf{Q}_p} \max(Y_s - K, 0) \\
&= N_0 \mathbf{E}_{\mathbf{Q}_N} \left(\max(Y_s - K, 0) \frac{B(S,p)}{N_S} \right) \\
&= N_0 \mathbf{E}_{\mathbf{Q}_N} \left(\max(Y_s - K, 0)(\alpha + \beta_p Y_S) \right) \\
&= N_0 \mathbf{E}_{\mathbf{Q}_N} \left[\max \left(Y_0 \exp(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S) - K, 0 \right) \right. \\
&\qquad \left. \left(\alpha + \beta_p Y_0 \exp(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S) \right) \right].
\end{aligned}$$

This expectation is straightforward to calculate explicitly.

Proposition 6 *Under the assumptions (37) and (36) the value of a call option on the rate Y_S with payment at time $p \geq S$ is given by*

$$
\begin{aligned}
& B(0,p)\mathbf{E}_{\mathbf{Q}_p} \max(Y_s - K, 0) \qquad (40)\\
= \quad & B(0,p)\frac{Y_0 \mathrm{N}(d_1)(\alpha - \beta_p K) - \alpha K \mathrm{N}(d_2) + \beta_p Y_0^2 \mathbf{e}^{\sigma_Y^2 S} \mathrm{N}(d_1 + \sigma_Y \sqrt{S})}{\alpha + \beta_p Y_0},
\end{aligned}
$$

with

$$
\begin{aligned}
d_1 &= \frac{\ln(\frac{Y_0}{K}) + \frac{1}{2}\sigma_Y^2 S}{\sigma_Y \sqrt{S}}\\
d_2 &= \frac{\ln(\frac{Y_0}{K}) - \frac{1}{2}\sigma_Y^2 S}{\sigma_Y \sqrt{S}}
\end{aligned}
$$

For the special case of an in-arrears option on a Libor $Y_S = L(S,T)$, $p = S$, the assumption of a linear model is obviously satisfied and the corresponding valuation has been noted in [1], Section 10.2.1

4.3 Accuracy study and examples

In this section we study the accuracy of the lognormal volatility approximations derived in Section 4.1 thereby also giving some numerical examples on the size of the volatility adjustment. We compare the prices of caps and binaries on Libor-in-arrears calculated from the approximating lognormal model with the results of the exact but more involved evaluation (40). It turns out that the approximation proposed delivers results of high accuracy.

The following table shows some numerical results for the price (in basis points) of a caplet[3]

$$\Delta \, \mathbf{E}_{\mathbf{Q}_S} \max(L(S,T) - K, 0)$$

for a forward rate $L^0(S,T) = 5\%$ and various scenarios for the time S to maturity and the volatility σ. The approximate price is based on a lognormal distribution with adjusted volatility according to (39) and adjusted forward rates $\mathbf{E}_{\mathbf{Q}_S} L(S,T)$ from (14).

[3]This is the price of the caplet up to a multiplication with the discount factor $B(0,S)$.

S	10			S	10		
σ	20,00%			σ	40,00%		
σ^*	20,14%			σ^*	43,27%		
$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,061%			$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,490%		
K	approx.	exact	% error	K	approx.	exact	% error
1%	406,334	406,327	0,000000%	1%	462,996	460,630	0,000051%
2%	312,399	312,327	0,000002%	2%	402,164	396,910	0,000132%
3%	233,987	233,839	0,000006%	3%	356,308	349,029	0,000209%
4%	173,590	173,409	0,000010%	4%	320,070	311,471	0,000276%
5%	128,767	128,595	0,000013%	5%	290,500	281,079	0,000335%
6%	95,965	95,825	0,000015%	6%	265,802	255,904	0,000387%
7%	72,021	71,920	0,000014%	7%	244,804	234,668	0,000432%
8%	54,484	54,422	0,000011%	8%	226,697	216,492	0,000471%
9%	41,560	41,531	0,000007%	9%	210,902	200,747	0,000506%
10%	31,965	31,962	0,000001%	10%	196,992	186,968	0,000536%

S	20			S	20		
σ	20,00%			σ	40,00%		
σ^*	20,42%			σ^*	50,23%		
$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,15%			$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	7,92%		
K	approx.	exact	% error	K	approx.	exact	% error
1%	417,952	417,780	0,000004%	1%	733,461	722,494	0,000152%
2%	336,494	335,867	0,000019%	2%	697,997	679,488	0,000272%
3%	273,187	272,205	0,000036%	3%	670,784	647,240	0,000364%
4%	224,353	223,187	0,000052%	4%	648,358	621,272	0,000436%
5%	186,347	185,128	0,000066%	5%	629,158	599,497	0,000495%
6%	156,394	155,205	0,000077%	6%	612,312	580,746	0,000544%
7%	132,483	131,374	0,000084%	7%	597,275	564,288	0,000585%
8%	113,165	112,161	0,000090%	8%	583,681	549,633	0,000619%
9%	97,386	96,497	0,000092%	9%	571,267	536,433	0,000649%
10%	84,369	83,596	0,000093%	10%	559,839	524,435	0,000675%

Errors are shown here as percentage errors of time value.

The differences between the true distribution and the approximating log-normal distribution become more transparent when comparing binary options, i.e. options with payout $\mathbf{1}_{\{L(S,T)>K\}}$ at time S. Here are some numerical comparisons:

S	10			S	10		
σ	20,00%			σ	40,00%		
σ^*	20,14%			σ^*	43,27%		
$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,061%			$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,490%		
K	approx.	exact	% error	K	approx.	exact	% error
1%	9870	9873	0,03%	1%	7123	7447	4,34%
2%	8727	8735	0,10%	2%	5214	5459	4,49%
3%	6924	6929	0,08%	3%	4042	4205	3,89%
4%	5203	5204	0,02%	4%	3253	3357	3,10%
5%	3823	3821	-0,06%	5%	2690	2753	2,28%
6%	2790	2787	-0,14%	6%	2269	2303	1,49%
7%	2039	2035	-0,20%	7%	1944	1958	0,73%
8%	1498	1494	-0,24%	8%	1687	1687	0,02%
9%	1108	1105	-0,27%	9%	1479	1470	-0,65%
10%	826	824	-0,28%	10%	1308	1292	-1,27%

S	20			S	20		
σ	20,00%			σ	40,00%		
σ^*	20,42%			σ^*	50,23%		
$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	5,152%			$\mathbf{E}_{\mathbf{Q}_S} L(S,T)$	7,916%		
K	approx.	exact	% error	K	approx.	exact	% error
1%	9097	9137	0,44%	1%	4198	5135	18,25%
2%	7190	7233	0,60%	2%	3047	3653	16,57%
3%	5540	5567	0,48%	3%	2447	2864	14,57%
4%	4288	4299	0,26%	4%	2063	2362	12,68%
5%	3359	3359	0,01%	5%	1791	2011	10,94%
6%	2665	2659	-0,23%	6%	1587	1751	9,36%
7%	2141	2132	-0,45%	7%	1426	1549	7,90%
8%	1740	1729	-0,65%	8%	1297	1388	6,57%
9%	1429	1417	-0,82%	9%	1189	1256	5,33%
10%	1184	1173	-0,97%	10%	1099	1147	4,18%

4.4 Options to exchange interest rates

Consider two interest rates Y_1 and Y_2 which are set (fixed) at times S_1 and S_2, respectively. For example, Y_1 and Y_2 could be Libor rates $L(S_1, T_1)$ and $L(S_2, T_2)$ referring to different fixing dates S_1, S_2, e.g. Libor and Libor-in-arrears. One could also think of two CMS rates to be set at the same date but with different tenors.

We are interested in an option to exchange both interest rates

$$\max(Y_2 - Y_1, 0) \tag{41}$$

with payout to be paid at time $p \geq \max(S_1, S_2)$. To keep the notation simple let us suppose that $S_2 \geq S_1$.

We assume that both interest rates are lognormal under $\mathbf{Q}_p$

$$\begin{aligned} Y_i &= Y_i^0 \exp(\sigma_i W^i_{S_i} - \sigma_i^2 S_i/2) & (42) \\ Y_i^0 &= \mathbf{E}_{\mathbf{Q}_p}(Y_i). & (43) \end{aligned}$$

with $\mathbf{E}_{\mathbf{Q}_p}(Y_i)$ given by (17), (25) or (14), (28). According to our analysis in Section 4.1 the assumption of log-normality is at least approximately satisfied under the market model if the market volatility is adjusted according to (39).

Proposition 7 *Let the driving Brownian motions W^1 and W^2 be correlated with dynamic correlation ρ, i.e.,*

$$\mathbf{E}_{\mathbf{Q}_p}(W^1_t W^2_t) = \rho t, \quad t \geq 0.$$

The fair price of the exchange option is then given by

$$B(0,p)\left[Y_2^0 \mathrm{N}(b_1) - Y_1^0 \mathrm{N}(b_2)\right] \tag{44}$$

with

$$
\begin{aligned}
b_1 &= \frac{\ln(\frac{Y_2^0}{Y_1^0}) + \frac{1}{2}(\sigma_1^2 S_1 + \sigma_2^2 S_2 - 2\sigma_1\sigma_2\rho S_1)}{\sqrt{\sigma_1^2 S_1 + \sigma_2^2 S_2 - 2\sigma_1\sigma_2\rho S_1}} \\
b_2 &= \frac{\ln(\frac{Y_2^0}{Y_1^0}) - \frac{1}{2}(\sigma_1^2 S_1 + \sigma_2^2 S_2 - 2\sigma_1\sigma_2\rho S_1)}{\sqrt{\sigma_1^2 S_1 + \sigma_2^2 S_2 - 2\sigma_1\sigma_2\rho S_1}}
\end{aligned}
$$

Proof. The price of the exchange option is given by

$$B(0,p)\mathbf{E}_{\mathbf{Q}_p} \max(Y_2 - Y_1, 0).$$

The calculation of this expectation is rather standard. We represent $W^1_{S_1}, W^2_{S_2}$ via independent standard Gaussian random variables ξ_1, ξ_2

$$
\begin{aligned}
W^1_{S_1} &= \sqrt{S_1}\xi_1 \\
W^2_{S_2} &= \sqrt{S_2}(\lambda\xi_1 + \sqrt{1-\lambda^2}\xi_2) \\
\lambda &= \rho\sqrt{\frac{S_1}{S_2}}.
\end{aligned}
$$

First taking the expectation w.r.t. ξ_2 leads us to a Black & Scholes type expression

$$\mathbf{E}_{\mathbf{Q}_p} \max(Y_2 - Y_1, 0) = \mathbf{E}_{\mathbf{Q}_p}(X_2 \mathrm{N}(d_1) - X_1 \mathrm{N}(d_2)) \tag{45}$$

with

$$
\begin{aligned}
X_2 &= Y_2^0 \exp(\sigma_2\sqrt{S_2}\lambda\xi_1 - \frac{1}{2}\sigma_2^2 S_2 \lambda^2) \\
X_1 &= Y_1^0 \exp(\sigma_1\sqrt{S_1}\xi_1 - \frac{1}{2}\sigma_1^2 S_1) \\
d_{1,2} &= \frac{\ln\left(\frac{X_2}{X_1}\right) \pm \frac{1}{2}\sigma_2^2 S_2(1-\lambda^2)}{\sqrt{\sigma_2^2 S_2(1-\lambda^2)}}.
\end{aligned}
$$

The expectation on the right hand side of (45) is now further explored integrating w.r.t. ξ_1 and applying the following well-known formula for integrals w.r.t. the standard normal density $\varphi = \mathrm{N}'$

$$\int_{-\infty}^{\infty} \mathrm{N}(ax+b)\exp(cx)\varphi(x)\mathrm{d}x = \mathrm{N}\left(\frac{ac+b}{\sqrt{1+a^2}}\right)\exp(c^2/2).$$

◊

Remark. As expected the formula (44) is related to the well-known Margrabe formula (see e.g. [2]) on the difference of two assets. Since in our case the two underlying quantities Y_2 and Y_1 are not necessarily set at the same point in time one has to adopt the inputs to Margrabes formula appropriately to take these effects into account and derive our formula (44) from Margrabes formula.

5 Variable interest rates in foreign currency

Consider a domestic floating interest rate Y_S^d which is set (fixed) in the market at time S. Examples of interest are $Y_S^d = L(S,T)$, the Libor for the interval $[S,T]$, and, $Y_S^d = X$ with $S = T_0$ and X the swap rate with reference dates $T_0 < T_1 < \cdots < T_n$.

We are interested in the price of the rate Y_S^d to be paid in foreign currency units at some time $p \geq S$.

5.1 General relationships

Let N^c denote a numeraire process with associated martingale measure $\mathbf{Q}^c$ for the domestic ($c = d$) and foreign ($c = f$) economy. The foreign exchange rate X_t at time t is the value of one unit foreign in domestic currency at time t. Any foreign asset S_t^f is considered to be a traded asset in the domestic economy if multiplied with the exchange rate: $S_t^f \cdot X_t$.

The value today of a domestic payoff Z_T^d to be paid in foreign units and at time T is by the general theory

$$N_0^f \mathbf{E}_{\mathbf{Q}^f} \frac{Z_T^d}{N_T^f}.$$

On the other hand, the same payoff translated back into domestic currency with the exchange rate at time T should trade at the same price, therefore

$$N_0^f \mathbf{E}_{\mathbf{Q}^f} \frac{Z_T^d}{N_T^f} = \frac{N_0^d}{X_0} \mathbf{E}_{\mathbf{Q}^d} \frac{Z_T^d X_T}{N_T^d}.$$

Consequently, the density of the two martingale measures $\mathbf{Q}^f$ and $\mathbf{Q}^d$ on the information structure up to time T is given by

$$\begin{aligned} \frac{\mathrm{d}\mathbf{Q}^d}{\mathrm{d}\mathbf{Q}^f} &= \frac{N_T^d}{X_T N_T^f} \frac{X_0 N_0^f}{N_0^d} \qquad (46) \\ \frac{\mathrm{d}\mathbf{Q}^f}{\mathrm{d}\mathbf{Q}^d} &= \frac{X_T N_T^f}{N_T^d} \frac{N_0^d}{X_0 N_0^f}. \end{aligned}$$

This implies the following

Corollary 2 *The process* $(\frac{X_t N_t^f}{N_t^d})_{t\geq 0}$ *is a* $\mathbf{Q}^d$ *martingale.*

5.2 Quanto adjustments

Now we come back to the valuation of a variable interest rate Y_S^d paid at time p in foreign units. Using the time p maturity foreign zero bond $B^f(.,p)$ as numeraire together with the foreign time p forward measure and relation (46) for $T = S$ as well as Corollary 2 the price is

$$\begin{aligned} B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f}\frac{Y_S^d}{B^f(p,p)} &= \frac{N_0^d}{X_0}\mathbf{E}_{\mathbf{Q}^d}\left(Y_S^d\frac{X_S B^f(S,p)}{N_S^d}\right) \\ &= \frac{N_0^d}{X_0}\mathbf{E}_{\mathbf{Q}^d}\left(Y_S^d\frac{X_S B^f(S,p)}{B^d(S,p)}\frac{B^d(S,p)}{N_S^d}\right). \end{aligned} \tag{47}$$

By definition the ratio $\frac{B^d(S,p)}{N_S^d}$ is a $\mathbf{Q}^d$ martingale in the time variable $S \leq p$. The expression $\frac{X_S B^f(S,p)}{B^d(S,p)}$ is the time S forward foreign exchange rate for delivery at time $p \geq S$.

Our goal is to make (47) more explicit in terms of market observable quantities like forward rates, volatilities etc. To this end we need to impose some modelling assumptions. From now on we assume that our domestic numeraire N^d is the natural numeraire associated with the interest rate Y_S^d, i.e., if $Y_S^d = L(S,T)$ then $N_\cdot^d = B^d(.,T)$ and $\mathbf{Q}^d = \mathbf{Q}_T^d$ or if $Y_S^d = X$ then $N_\cdot^d = \sum \Delta_i B^d(.,T_i)$ and $\mathbf{Q}^d = \mathbf{Q}_{\text{Swap}}^d$. Under the market model, which we assume from now on, the distribution of Y_S^d under the measure $\mathbf{Q}^d$ is then lognormal

$$Y_S^d = Y_0^d \exp(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S) \tag{48}$$

with expectation

$$Y_0^d = \mathbf{E}_{\mathbf{Q}^d} Y_S^d.$$

The rate Y_0^d is the forward Libor $L^0(S,T)$ (cf. (9)) or the forward swap rate X^0 (cf. (3)), respectively, as of today.

For the last term on the right hand side of equation (47) we suppose (as we have already done in previous sections, equations (15), (22)) a linear model of the form

$$\frac{B^d(S,p)}{N_S^d} = \alpha + \beta_p Y_S^d \tag{49}$$

with α and β_p given by (16) or (24) and (23), respectively.

The most critical assumption we are going to require is on the distribution of the forward foreign exchange rate. We suppose a lognormal distribution under $\mathbf{Q}^d$

$$X_S \frac{B^f(S,p)}{B^d(S,p)} = X_0^\star \exp(\sigma_{\text{fx}} W_S^{\text{fx}} - \frac{1}{2}\sigma_{\text{fx}}^2 S) \tag{50}$$

with expectation

$$X_0^\star = \mathbf{E}_{\mathbf{Q}^d}\left(X_S \frac{B^f(S,p)}{B^d(S,p)}\right)$$

to be calculated below. Observe that the assumption of log-normality of the forward foreign exchange rate is in general not compatible with the assumption of a lognomal rate Y_S^d. In practice, we will identify the volatility σ_{fx} with the implied volatility of a foreign exchange rate option with maturity S. This would be a crucial simplification as long as the payment date p is not close to S, which is, however, the case in most applications.

To calculate the expectation $X_0^\star$ we make use of the fact that according to Corollary 2 ($\frac{X_S B^f(S,p)}{N_S^d}$) is a $\mathbf{Q}^d$ martingale in $S \le p$. Together with (49) and (48) this yields

$$\begin{aligned}
&\frac{X_0 B^f(0,p)}{N_0^d} \\
&= \mathbf{E}_{\mathbf{Q}^d}\left(\frac{X_S B^f(S,p)}{N_S^d}\right) = \mathbf{E}_{\mathbf{Q}^d}\left(X_s \frac{B^f(S,p)}{B^d(S,p)} \frac{B^d(S,p)}{N_S^d}\right) \\
&= \mathbf{E}_{\mathbf{Q}^d}\left[X_0^\star \exp(\sigma_{\text{fx}} W_S^{\text{fx}} - \frac{1}{2}\sigma_{\text{fx}}^2 S)(\alpha + \beta_p Y_0^d \exp(\sigma_Y W_s - \frac{1}{2}\sigma_Y^2 S))\right] \\
&= X_0^\star(\alpha + \beta_p Y_0^d \exp(\rho\sigma_{\text{fx}}\sigma_Y S)),
\end{aligned}$$

with ρ as correlation between the driving Brownian motions W^{fx} and W. As a consequence the expectation of the forward foreign exchange rate under $\mathbf{Q}^d$ is

$$X_0^\star = \frac{X_0 B^f(0,p)}{N_0^d(\alpha + \beta_p Y_0^d \exp(\rho\sigma_{\text{fx}}\sigma_Y S))}. \tag{51}$$

Now we are ready to follow up with an explicit calculation of (47). Substituting (48), (49) and (50) we obtain

$$\begin{aligned}
& B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f}\frac{Y_S^d}{B^f(p,p)} \\
&= \frac{N_0^d}{X_0}Y_0^d\frac{X_0B^f(0,p)}{N_0^d(\alpha+\beta_pY_0^d\exp(\rho\sigma_{\text{fx}}\sigma_YS))}\mathbf{E}_{\mathbf{Q}^d}\Big(\exp(\sigma_YW_S-\frac{1}{2}\sigma_Y^2S) \\
&\quad \exp(\sigma_{\text{fx}}W_S^{\text{fx}}-\frac{1}{2}\sigma_{\text{fx}}^2S)(\alpha+\beta_pY_0^d\exp(\sigma_YW_S-\frac{1}{2}\sigma_Y^2S))\Big) \\
&= B^f(0,p)Y_0^d\,\frac{\exp(\rho\sigma_{\text{fx}}\sigma_YS)(\alpha+\beta_pY_0^d\exp(\rho\sigma_{\text{fx}}\sigma_YS+\sigma_Y^2S))}{\alpha+\beta_pY_0^d\exp(\rho\sigma_{\text{fx}}\sigma_YS)}.
\end{aligned}$$

Proposition 8 *Under the conditions (48), (49) and (50) the quanto adjusted forward rate for a payment of the variable rate Y_S^d set at time S and paid at time $p \geq S$ in foreign currency units is given by*

$$\boxed{\mathbf{E}_{\mathbf{Q}_p^f}Y_S^d = Y_0^d\,\frac{\exp(\rho\sigma_{\text{fx}}\sigma_YS)(\alpha+\beta_pY_0^d\exp(\rho\sigma_{\text{fx}}\sigma_YS+\sigma_Y^2S))}{\alpha+\beta_pY_0^d\exp(\rho\sigma_{\text{fx}}\sigma_YS)}} \tag{52}$$

with α and β_p as in (16) or (24) and (23), respectively, and ρ as correlation between the driving Brownian motions.

Remark. In the special case of the foreign unit being the domestic unit, i.e. $\sigma_{\text{fx}} = 0$, formula (52) reduces to (18) or (28) as expected.

Example. To illustrate the impact and size of the quanto adjustment consider as an example a diff swap, i.e. $Y_S^d = L(S,T)$ to be paid at time $p = T$ in foreign currency units. In this particular case we have $\alpha = 1$ and $\beta_p = \beta_T = 0$ and the adjustment reduces to

$$\mathbf{E}_{\mathbf{Q}_T^f}L(S,T) = L^0(S,T)\exp(\rho\sigma_{\text{fx}}\sigma_YS).$$

For $S = 5, \sigma_{\text{fx}} = 15\%, \sigma_Y = 18\%$ and $\rho = 50\%$ this gives an adjustment factor on the forward rate of

$$\exp(\rho\sigma_{\text{fx}}\sigma_YS) = 1,0698.$$

5.3 Quantoed options on interest rates

In this section we extend the analysis of the previous section to standard options on a domestic interest rate Y_S^d with payment at an arbitrary time $p \geq S$ but in foreign currency units. This can be seen also as an extension of the result in Section 4.2.

We use the same notation and assumptions (48), (49) and (50) as in the previous section.

Consider a call option on the domestic rate Y_S^d with strike K paid at $p \geq S$ in foreign currency. By the general theory the price of this option is (compare also (47))

$$\begin{aligned}
& B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f} \frac{\max(Y_S^d - K, 0)}{B^f(p,p)} \\
&= \frac{N_0^d}{X_0}\mathbf{E}_{\mathbf{Q}^d}\left(\max(Y_S^d - K, 0)\frac{X_S B^f(S,p)}{B^d(S,p)}\frac{B^d(S,p)}{N_S^d}\right) \\
&= \frac{N_0^d}{X_0}\mathbf{E}_{\mathbf{Q}^d}\left(\max(Y_S^d - K, 0)\frac{X_S B^f(S,p)}{B^d(S,p)}(\alpha + \beta_p Y_S^d)\right) \\
&= \frac{N_0^d}{X_0}\mathbf{E}_{\mathbf{Q}^d}\left[\max\left(Y_0^d \exp\left(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S\right) - K, 0\right)\right. \\
& \qquad \left. X_0^\star \exp\left(\sigma_{\mathrm{fx}} W_S^{\mathrm{fx}} - \frac{1}{2}\sigma_{\mathrm{fx}}^2 S\right)\left(\alpha + \beta_p Y_0^d \exp\left(\sigma_Y W_S - \frac{1}{2}\sigma_Y^2 S\right)\right)\right].
\end{aligned}$$

The last expectation can be calculated again explicitly and we obtain the following result.

Proposition 9 *Under the conditions (48), (49) and (50) the call option on the domestic variable rate Y_S^d set at time S and paid at time $p \geq S$ in foreign currency units is given by*

$$\begin{aligned}
& B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f} \max(Y_S^d - K, 0) \qquad (53) \\
&= \frac{B^f(0,p)}{\alpha + \beta_p Y_0^d \mathbf{e}^{\sigma_{\mathrm{fx}}\sigma_Y \rho S}}\left[Y_0^d \mathbf{e}^{\sigma_{\mathrm{fx}}\sigma_Y \rho S}\mathrm{N}(d_1 + \sigma_{\mathrm{fx}}\rho\sqrt{S})(\alpha - \beta_p K)\right. \\
& \quad \left. + \beta_p (Y_0^d)^2 \mathbf{e}^{(\sigma_y^2 + 2\sigma_{\mathrm{fx}}\sigma_y\rho)S}\mathrm{N}(d_1 + (\sigma_{\mathrm{fx}}\rho + \sigma_Y)\sqrt{S}) - \alpha K \mathrm{N}(d_2 + \sigma_{\mathrm{fx}}\rho\sqrt{S})\right]
\end{aligned}$$

with

$$\begin{aligned}
d_1 &= \frac{\ln(\frac{Y_0^d}{K}) + \frac{1}{2}\sigma_Y^2 S}{\sigma_Y\sqrt{S}} \\
d_2 &= \frac{\ln(\frac{Y_0^d}{K}) - \frac{1}{2}\sigma_Y^2 S}{\sigma_Y\sqrt{S}}
\end{aligned}$$

and α and β_p as in (16) or (24) and (23), respectively, and ρ as correlation *between the driving Brownian motions. The corresponding put formula is*

$$B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f}\max(K - Y_S^d, 0) \tag{54}$$
$$= \frac{B^f(0,p)}{\alpha + \beta_p Y_0^d \mathbf{e}^{\sigma_{\text{fx}}\sigma_Y \rho S}}\Big[- Y_0^d \mathbf{e}^{\sigma_{\text{fx}}\sigma_Y \rho S} \mathrm{N}(-d_1 - \sigma_{\text{fx}}\rho\sqrt{S})(\alpha - \beta_p K)$$
$$-\beta_p (Y_0^d)^2 \mathbf{e}^{(\sigma_y^2 + 2\sigma_{\text{fx}}\sigma_y\rho)S}\mathrm{N}(-d_1 - (\sigma_{\text{fx}}\rho + \sigma_Y)\sqrt{S}) + \alpha K \mathrm{N}(-d_2 - \sigma_{\text{fx}}\rho\sqrt{S})\Big].$$

Finally, the formula for a digital is

$$B^f(0,p)\mathbf{E}_{\mathbf{Q}_p^f}\left(\mathbf{1}_{\{Y_S^d > K\}}\right) \tag{55}$$
$$= \frac{B^f(0,p)}{\alpha + \beta_p Y_0^d \mathbf{e}^{\sigma_{\text{fx}}\sigma_Y \rho S}}\Big[\beta_p Y_0^d \mathbf{e}^{\sigma_{\text{fx}}\sigma_Y \rho S} \mathrm{N}(d_1 + \sigma_{\text{fx}}\rho\sqrt{S}) + \alpha \mathrm{N}(d_2 + \sigma_{\text{fx}}\rho\sqrt{S})\Big].$$

Remarks 1. For the special case of $Y_S = L(S,T)$, $p = T$, the assumption (49) of a linear model is trivially satisfied and the corresponding valuation reduces to formula (11.27) in [1], Section 11.4.2.
2. In case that the option is not quantoed, i.e., we can assume $\sigma_{\text{fx}} = 0$ formula (54) reduces to the general formula (40) for options on interest rates with arbitrary payment date.

6 Empirical correlation estimates

The convexity and quanto adjustment formulae in the previous sections involve several correlations between interest rates and interest rates and foreign exchange rates. In this section, as a first indication and for reference, we collect the results of some empirical correlation estimates for the most important interest rate tenors and currencies.

The data underlying the analysis are daily data ranging from Aug 1, 2001 to Sep 1, 2003. We estimate correlations of daily logarithmic changes between 1M, 3M, 6M, 12M cash and 2Y,..., 10Y swap rates in each currency.[4]

[4] We use Euribor/Libor fixings for cash rates and closing swap rates from Reuters. A similar analysis on forward rates instead of spot rates would be desirable, however, to calculate sufficiently stable forward rates from spot rates requires sophisticated interpolation techniques and is beyond the scope of the present analysis.

Correlations between EUR interest rates.

	1M	3M	6M	12M	2Y	3Y	4Y
1M	100,00%	83,08%	53,54%	30,61%	6,49%	3,64%	2,78%
3M	83,08%	100,00%	80,04%	57,95%	16,28%	12,32%	9,91%
6M	53,54%	80,04%	100,00%	91,06%	31,61%	24,78%	23,75%
1Y	30,61%	57,95%	91,06%	100,00%	38,95%	31,72%	31,74%
2Y	6,49%	16,28%	31,61%	38,95%	100,00%	91,04%	93,43%
3Y	3,64%	12,32%	24,78%	31,72%	91,04%	100,00%	88,24%
4Y	2,78%	9,91%	23,75%	31,74%	93,43%	88,24%	100,00%
5Y	2,77%	8,91%	20,66%	27,91%	92,50%	88,15%	98,57%
6Y	2,86%	8,45%	19,51%	26,51%	91,23%	86,71%	97,75%
7Y	1,93%	8,20%	19,76%	27,10%	90,02%	85,46%	95,84%
8Y	2,66%	8,81%	19,34%	26,41%	88,89%	84,51%	94,80%
9Y	2,63%	7,77%	17,44%	24,15%	86,97%	82,86%	93,61%
10Y	2,76%	7,55%	16,49%	22,82%	86,17%	81,99%	92,77%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	2,77%	2,86%	1,93%	2,66%	2,63%	2,76%	
3M	8,91%	8,45%	8,20%	8,81%	7,77%	7,55%	
6M	20,66%	19,51%	19,76%	19,34%	17,44%	16,49%	
12M	27,91%	26,51%	27,10%	26,41%	24,15%	22,82%	
2Y	92,50%	91,23%	90,02%	88,89%	86,97%	86,17%	
3Y	88,15%	86,71%	85,46%	84,51%	82,86%	81,99%	
4Y	98,57%	97,75%	95,84%	94,80%	93,61%	92,77%	
5Y	100,00%	99,26%	97,54%	96,63%	95,68%	94,79%	
6Y	99,26%	100,00%	98,21%	97,56%	96,80%	95,92%	
7Y	97,54%	98,21%	100,00%	99,23%	98,73%	98,02%	
8Y	96,63%	97,56%	99,23%	100,00%	99,48%	98,74%	
9Y	95,68%	96,80%	98,73%	99,48%	100,00%	99,24%	
10Y	94,79%	95,92%	98,02%	98,74%	99,24%	100,00%	

Correlations between USD interest rates.

	1M	3M	6M	1Y	2Y	3Y	4Y
1M	100,00%	85,40%	64,78%	43,54%	7,35%	9,98%	7,36%
3M	85,40%	100,00%	88,41%	69,29%	13,20%	16,30%	13,88%
6M	64,78%	88,41%	100,00%	91,63%	17,67%	19,72%	17,74%
1Y	43,54%	69,29%	91,63%	100,00%	20,45%	22,97%	21,36%
2Y	7,35%	13,20%	17,67%	20,45%	100,00%	93,83%	91,04%
3Y	9,98%	16,30%	19,72%	22,97%	93,83%	100,00%	97,40%
4Y	7,36%	13,88%	17,74%	21,36%	91,04%	97,40%	100,00%
5Y	10,09%	16,86%	19,98%	23,31%	89,54%	96,13%	96,82%
6Y	16,26%	24,63%	33,28%	41,02%	68,00%	75,26%	75,81%
7Y	16,55%	24,67%	32,89%	39,46%	69,23%	76,69%	77,78%
8Y	16,54%	24,41%	32,90%	39,35%	65,86%	73,22%	74,38%
9Y	15,29%	22,45%	30,87%	37,30%	64,05%	71,17%	72,76%
10Y	15,35%	23,39%	31,23%	36,62%	64,65%	72,41%	73,79%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	10,09%	16,26%	16,55%	16,54%	15,29%	15,35%	
3M	16,86%	24,63%	24,67%	24,41%	22,45%	23,39%	
6M	19,98%	33,28%	32,89%	32,90%	30,87%	31,23%	
1Y	23,31%	41,02%	39,46%	39,35%	37,30%	36,62%	
2Y	89,54%	68,00%	69,23%	65,86%	64,05%	64,65%	
3Y	96,13%	75,26%	76,69%	73,22%	71,17%	72,41%	
4Y	96,82%	75,81%	77,78%	74,38%	72,76%	73,79%	
5Y	100,00%	79,39%	81,32%	78,39%	76,80%	77,98%	
6Y	79,39%	100,00%	95,79%	95,98%	94,96%	91,20%	
7Y	81,32%	95,79%	100,00%	96,72%	95,12%	94,43%	
8Y	78,39%	95,98%	96,72%	100,00%	98,04%	95,23%	
9Y	76,80%	94,96%	95,12%	98,04%	100,00%	95,35%	
10Y	77,98%	91,20%	94,43%	95,23%	95,35%	100,00%	

Correlations between GBP interest rates.

	1M	3M	6M	1Y	2Y	3Y	4Y
1M	100,00%	77,24%	51,27%	36,86%	12,61%	8,36%	12,39%
3M	77,24%	100,00%	83,50%	66,47%	26,12%	19,80%	25,42%
6M	51,27%	83,50%	100,00%	92,63%	44,76%	38,17%	42,27%
1Y	36,86%	66,47%	92,63%	100,00%	53,78%	48,27%	52,15%
2Y	12,61%	26,12%	44,76%	53,78%	100,00%	95,07%	89,12%
3Y	8,36%	19,80%	38,17%	48,27%	95,07%	100,00%	91,04%
4Y	12,39%	25,42%	42,27%	52,15%	89,12%	91,04%	100,00%
5Y	10,27%	22,30%	38,17%	48,14%	87,24%	89,79%	99,04%
6Y	9,54%	20,17%	34,43%	44,17%	84,99%	88,02%	97,65%
7Y	8,83%	18,55%	31,31%	41,14%	82,73%	86,31%	95,67%
8Y	9,00%	17,21%	28,64%	37,92%	80,42%	84,07%	94,40%
9Y	8,40%	16,09%	26,43%	35,38%	78,27%	81,92%	92,55%
10Y	7,84%	14,76%	24,14%	32,90%	76,59%	80,36%	90,97%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	10,27%	9,54%	8,83%	9,00%	8,40%	7,84%	
3M	22,30%	20,17%	18,55%	17,21%	16,09%	14,76%	
6M	38,17%	34,43%	31,31%	28,64%	26,43%	24,14%	
1Y	48,14%	44,17%	41,14%	37,92%	35,38%	32,90%	
2Y	87,24%	84,99%	82,73%	80,42%	78,27%	76,59%	
3Y	89,79%	88,02%	86,31%	84,07%	81,92%	80,36%	
4Y	99,04%	97,65%	95,67%	94,40%	92,55%	90,97%	
5Y	100,00%	99,22%	97,74%	96,81%	95,42%	94,12%	
6Y	99,22%	100,00%	98,84%	98,46%	97,43%	96,39%	
7Y	97,74%	98,84%	100,00%	98,82%	98,13%	97,39%	
8Y	96,81%	98,46%	98,82%	100,00%	99,53%	99,02%	
9Y	95,42%	97,43%	98,13%	99,53%	100,00%	99,64%	
10Y	94,12%	96,39%	97,39%	99,02%	99,64%	100,00%	

Correlations between JPY interest rates.

	1M	3M	6M	1Y	2Y	3Y	4Y
1M	100,00%	31,11%	29,58%	36,91%	2,11%	4,75%	2,69%
3M	31,11%	100,00%	61,47%	53,09%	6,42%	5,92%	8,27%
6M	29,58%	61,47%	100,00%	70,30%	5,54%	7,52%	6,06%
1Y	36,91%	53,09%	70,30%	100,00%	4,31%	7,37%	6,29%
2Y	2,11%	6,42%	5,54%	4,31%	100,00%	67,83%	68,25%
3Y	4,75%	5,92%	7,52%	7,37%	67,83%	100,00%	88,47%
4Y	2,69%	8,27%	6,06%	6,29%	68,25%	88,47%	100,00%
5Y	0,74%	6,25%	4,62%	5,70%	60,45%	81,24%	89,23%
6Y	1,14%	6,75%	4,35%	5,29%	56,51%	77,84%	86,81%
7Y	-2,90%	3,73%	-0,14%	2,21%	55,37%	67,64%	77,82%
8Y	-3,15%	1,57%	-2,36%	-0,82%	51,44%	64,10%	74,89%
9Y	-3,64%	0,29%	-2,38%	-0,83%	48,23%	59,65%	69,83%
10Y	-4,63%	0,58%	-0,46%	-0,53%	53,55%	65,08%	74,95%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	0,74%	1,14%	-2,90%	-3,15%	-3,64%	-4,63%	
3M	6,25%	6,75%	3,73%	1,57%	0,29%	0,58%	
6M	4,62%	4,35%	-0,14%	-2,36%	-2,38%	-0,46%	
1Y	5,70%	5,29%	2,21%	-0,82%	-0,83%	-0,53%	
2Y	60,45%	56,51%	55,37%	51,44%	48,23%	53,55%	
3Y	81,24%	77,84%	67,64%	64,10%	59,65%	65,08%	
4Y	89,23%	86,81%	77,82%	74,89%	69,83%	74,95%	
5Y	100,00%	94,57%	85,48%	79,03%	72,33%	79,88%	
6Y	94,57%	100,00%	88,63%	85,16%	78,31%	83,16%	
7Y	85,48%	88,63%	100,00%	91,67%	85,34%	91,13%	
8Y	79,03%	85,16%	91,67%	100,00%	95,82%	87,75%	
9Y	72,33%	78,31%	85,34%	95,82%	100,00%	86,87%	
10Y	79,88%	83,16%	91,13%	87,75%	86,87%	100,00%	

Correlations between CHF interest rates.

	1M	3M	6M	1Y	2Y	3Y	4Y
1M	100,00%	91,65%	81,47%	60,07%	18,77%	13,02%	13,39%
3M	91,65%	100,00%	90,92%	71,07%	28,86%	22,31%	21,63%
6M	81,47%	90,92%	100,00%	83,77%	37,26%	31,07%	28,96%
1Y	60,07%	71,07%	83,77%	100,00%	45,36%	42,55%	38,18%
2Y	18,77%	28,86%	37,26%	45,36%	100,00%	90,08%	81,28%
3Y	13,02%	22,31%	31,07%	42,55%	90,08%	100,00%	90,51%
4Y	13,39%	21,63%	28,96%	38,18%	81,28%	90,51%	100,00%
5Y	12,46%	19,99%	26,60%	35,78%	80,93%	91,15%	98,51%
6Y	10,67%	18,23%	24,17%	33,23%	78,91%	88,95%	95,15%
7Y	10,76%	18,04%	23,86%	32,19%	77,81%	88,01%	93,52%
8Y	10,79%	17,93%	23,91%	31,85%	78,77%	89,14%	93,60%
9Y	10,51%	17,54%	23,00%	30,61%	77,80%	88,43%	92,38%
10Y	11,27%	17,92%	23,24%	31,04%	77,36%	87,57%	90,72%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	12,46%	10,67%	10,76%	10,79%	10,51%	11,27%	
3M	19,99%	18,23%	18,04%	17,93%	17,54%	17,92%	
6M	26,60%	24,17%	23,86%	23,91%	23,00%	23,24%	
1Y	35,78%	33,23%	32,19%	31,85%	30,61%	31,04%	
2Y	80,93%	78,91%	77,81%	78,77%	77,80%	77,36%	
3Y	91,15%	88,95%	88,01%	89,14%	88,43%	87,57%	
4Y	98,51%	95,15%	93,52%	93,60%	92,38%	90,72%	
5Y	100,00%	97,27%	96,06%	96,42%	95,46%	93,98%	
6Y	97,27%	100,00%	98,65%	96,49%	95,75%	94,23%	
7Y	96,06%	98,65%	100,00%	97,47%	96,69%	95,18%	
8Y	96,42%	96,49%	97,47%	100,00%	99,06%	97,70%	
9Y	95,46%	95,75%	96,69%	99,06%	100,00%	98,53%	
10Y	93,98%	94,23%	95,18%	97,70%	98,53%	100,00%	

Correlations between NOK interest rates.

	1M	3M	6M	1Y	2Y	3Y	4Y
1M	100,00%	68,75%	54,16%	44,44%	12,45%	9,27%	7,11%
3M	68,75%	100,00%	86,58%	76,33%	36,18%	31,47%	27,60%
6M	54,16%	86,58%	100,00%	92,49%	47,93%	43,62%	40,25%
1Y	44,44%	76,33%	92,49%	100,00%	56,14%	52,05%	48,39%
2Y	12,45%	36,18%	47,93%	56,14%	100,00%	95,20%	91,87%
3Y	9,27%	31,47%	43,62%	52,05%	95,20%	100,00%	97,57%
4Y	7,11%	27,60%	40,25%	48,39%	91,87%	97,57%	100,00%
5Y	4,92%	24,64%	37,22%	46,43%	90,81%	92,42%	94,99%
6Y	3,04%	21,45%	33,76%	42,11%	85,49%	92,33%	96,05%
7Y	1,99%	18,40%	31,11%	39,94%	82,85%	90,17%	94,19%
8Y	1,34%	17,59%	30,99%	39,59%	80,61%	88,01%	92,57%
9Y	-0,27%	16,05%	29,37%	37,72%	78,27%	86,48%	91,35%
10Y	-1,28%	13,99%	26,80%	35,20%	76,22%	84,83%	89,62%
	5Y	6Y	7Y	8Y	9Y	10Y	
1M	4,92%	3,04%	1,99%	1,34%	-0,27%	-1,28%	
3M	24,64%	21,45%	18,40%	17,59%	16,05%	13,99%	
6M	37,22%	33,76%	31,11%	30,99%	29,37%	26,80%	
1Y	46,43%	42,11%	39,94%	39,59%	37,72%	35,20%	
2Y	90,81%	85,49%	82,85%	80,61%	78,27%	76,22%	
3Y	92,42%	92,33%	90,17%	88,01%	86,48%	84,83%	
4Y	94,99%	96,05%	94,19%	92,57%	91,35%	89,62%	
5Y	100,00%	95,46%	93,52%	91,96%	90,54%	88,96%	
6Y	95,46%	100,00%	97,94%	96,72%	95,75%	94,38%	
7Y	93,52%	97,94%	100,00%	98,53%	97,40%	96,19%	
8Y	91,96%	96,72%	98,53%	100,00%	98,87%	97,56%	
9Y	90,54%	95,75%	97,40%	98,87%	100,00%	99,05%	
10Y	88,96%	94,38%	96,19%	97,56%	99,05%	100,00%	

The quanto adjustments for variable domestic interest rates paid in foreign currency require certain correlations between interest rates and foreign exchange rates. In estimating these quantities from data on has to be careful using the data in the right way. Remember that our foreign exchange rate X_t is always understood as the price of one unit foreign expressed in domestic currency. Here are the empirical estimation results of correlations for the given data sets.

	Foreign currency exchange rate				
EUR interest rates	USD	GBP	JPY	CHF	NOK
1M	0,65%	-4,73%	3,49%	-5,27%	2,44%
3M	2,19%	-4,21%	-1,62%	-1,72%	4,82%
6M	5,24%	-3,52%	-3,79%	-4,44%	4,35%
1Y	5,07%	-3,63%	-5,40%	-4,83%	3,91%
2Y	38,37%	23,40%	15,09%	-12,97%	13,38%
3Y	35,07%	21,00%	13,91%	-10,59%	8,80%
4Y	41,07%	26,45%	18,70%	-12,68%	13,87%
5Y	41,73%	26,25%	18,90%	-12,33%	14,48%
6Y	41,47%	26,88%	18,21%	-11,36%	13,84%
7Y	39,96%	25,30%	17,07%	-9,40%	14,02%
8Y	39,35%	25,27%	16,20%	-9,68%	13,88%
9Y	39,35%	25,19%	16,38%	-9,04%	14,07%
10Y	39,21%	24,70%	15,96%	-8,92%	14,08%

	Foreign currency exchange rate				
USD interest rates	EUR	GBP	JPY	CHF	NOK
1M	-8,02%	-11,60%	-9,91%	-11,07%	-8,84%
3M	-5,98%	-12,21%	-11,41%	-8,47%	-6,67%
6M	-6,54%	-12,37%	-12,36%	-9,40%	-4,70%
1Y	-6,89%	-9,89%	-13,64%	-10,58%	-4,52%
2Y	-34,04%	-22,07%	-22,93%	-35,73%	-29,50%
3Y	-34,13%	-23,37%	-23,46%	-36,40%	-30,28%
4Y	-33,62%	-22,29%	-23,41%	-35,18%	-29,78%
5Y	-33,56%	-22,68%	-24,00%	-35,89%	-30,86%
6Y	-30,43%	-21,44%	-23,69%	-32,74%	-28,25%
7Y	-32,37%	-23,94%	-25,27%	-34,39%	-30,67%
8Y	-31,36%	-22,48%	-25,35%	-33,32%	-28,99%
9Y	-30,26%	-21,47%	-24,47%	-32,15%	-27,42%
10Y	-28,13%	-20,78%	-22,52%	-30,22%	-26,48%

GBP interest rates	Foreign currency exchange rate USD	EUR	JPY	CHF	NOK
1M	-2,58%	-1,08%	-1,76%	-1,83%	-0,22%
3M	-1,08%	-0,26%	-5,44%	-1,33%	-1,74%
6M	-1,52%	-4,13%	-5,91%	-4,98%	-3,31%
1Y	0,02%	-6,59%	-8,77%	-8,70%	-3,51%
2Y	16,21%	-23,95%	-6,16%	-27,87%	-13,20%
3Y	16,29%	-24,51%	-5,42%	-27,30%	-13,09%
4Y	15,78%	-22,92%	-7,19%	-24,01%	-11,52%
5Y	17,01%	-24,21%	-7,40%	-24,79%	-12,45%
6Y	17,67%	-24,81%	-7,73%	-24,97%	-12,73%
7Y	18,73%	-23,89%	-7,38%	-24,41%	-11,71%
8Y	19,50%	-25,00%	-8,23%	-25,31%	-12,93%
9Y	20,14%	-24,35%	-7,73%	-25,23%	-12,15%
10Y	20,69%	-23,86%	-6,93%	-24,56%	-12,11%

JPY interest rates	Foreign currency exchange rate USD	EUR	GBP	CHF	NOK
1M	-0,21%	-1,57%	-1,80%	-5,75%	-3,10%
3M	12,93%	1,99%	6,94%	-0,67%	0,14%
6M	0,89%	4,57%	1,13%	0,58%	0,65%
1Y	1,30%	4,11%	1,51%	2,38%	0,82%
2Y	4,06%	-3,50%	1,03%	-3,70%	1,90%
3Y	-1,18%	-9,14%	-2,16%	-10,72%	-5,56%
4Y	2,01%	-8,18%	-0,12%	-8,09%	-4,82%
5Y	4,17%	-8,82%	-0,23%	-9,05%	-6,70%
6Y	3,66%	-7,82%	0,24%	-7,97%	-4,57%
7Y	7,55%	-7,09%	0,27%	-5,31%	-4,17%
8Y	8,75%	-6,04%	1,01%	-5,27%	-2,12%
9Y	7,87%	-4,32%	0,51%	-3,66%	-0,49%
10Y	6,43%	-4,71%	-0,19%	-2,46%	-0,96%

CHF interest rates	Foreign currency exchange rate USD	EUR	GBP	JPY	NOK
1M	-5,28%	-9,03%	-7,38%	-6,80%	2,21%
3M	-3,30%	-7,53%	-8,53%	-5,22%	6,46%
6M	-3,47%	-6,23%	-8,76%	-7,66%	7,14%
1Y	0,25%	-2,70%	-9,10%	-5,09%	8,50%
2Y	24,17%	9,97%	17,40%	6,90%	18,98%
3Y	22,26%	8,31%	14,67%	6,24%	14,19%
4Y	21,24%	8,57%	16,44%	6,49%	14,56%
5Y	23,76%	9,95%	18,21%	7,40%	14,85%
6Y	24,16%	11,73%	18,27%	7,00%	16,19%
7Y	24,16%	11,14%	18,95%	6,96%	16,32%
8Y	24,64%	10,12%	19,34%	7,60%	15,75%
9Y	24,03%	8,69%	18,35%	6,47%	13,96%
10Y	23,77%	8,86%	17,61%	6,42%	13,69%

NOK interest rates	Foreign currency exchange rate USD	EUR	GBP	JPY	CHF
1M	-12,00%	-7,78%	-3,77%	-12,92%	-8,66%
3M	-13,83%	-11,09%	-12,47%	-12,33%	-8,76%
6M	-12,83%	-13,46%	-12,61%	-12,16%	-10,42%
1Y	-11,33%	-16,17%	-12,51%	-10,87%	-14,09%
2Y	-0,23%	-23,27%	-7,90%	-10,86%	-24,93%
3Y	2,28%	-23,39%	-6,93%	-9,94%	-24,76%
4Y	4,09%	-22,09%	-4,41%	-7,81%	-23,18%
5Y	3,40%	-20,90%	-3,71%	-10,23%	-23,62%
6Y	4,33%	-21,15%	-3,32%	-8,26%	-22,05%
7Y	6,77%	-19,81%	-2,15%	-7,70%	-21,35%
8Y	7,06%	-18,87%	-1,78%	-7,22%	-20,04%
9Y	7,49%	-17,39%	-0,81%	-6,54%	-19,21%
10Y	8,17%	-17,29%	-0,85%	-5,79%	-18,87%

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