Optimal R&D Investment Strategies Under the Threat of a Superior External Entry

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Abstract

This paper presents a model for strategic two-stage R&D investment of a firm facing the threat of a superior entry, subject to technological uncertainty about the outcome of the preliminary R&D project. The entry threat stimulates domestic innovation in the case of entry prevention, but discourages R&D if the entry is inevitable. Greater technical uncertainty stimulates starting of exploratory R&D and can result in implementation of more expensive research projects. The welfare analysis shows that the innovating monopoly can be preferred to competition if the innovation requires commitment of more resources.

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1 Introduction

In this paper we approach the issue of interaction between innovation and globalization, which in the first place results in a more competitive environment for various domestic firms. Those firms, which previously enjoyed relative safety inside their home markets, now face a possibility of external entry by often more efficient competitors. In the existing literature it was noted by Reinganum (1983) and Dasgupta and Stiglitz (1980) that the incumbent firm can avoid competition by innovating and, thus, discourage the potential entrants from entry with the innovative product. The object of our interest is the effect of a superior (technological process) entry threat on R&D investment decisions of the domestic firm. The "superiority" of an entrant is represented by the fact that the entrant already possesses a newer and more efficient production technology.

The case of a superior entry threat has special importance for domestic firm's decision-making in the context of globalization. As different countries gradually open their economies, many firms consider an option of establishing their production in another country. In order to strengthen its entry capabilities it is natural for the firm to enter the market holding an advantageous position, which can be provided by a superior technology.

Therefore the incumbent must decide whether or not to invest in research to develop such a new technology himself to counter the potential threat. Two important features of R&D investments are that an R&D project takes time to complete and that the outcome of R&D investments is uncertain. This makes that "the analysis of R&D investments is surely one of the most difficult problems of investment under uncertainty" (Schwartz and Moon (2000)). Still, the aim of this paper is to provide analytical results regarding incentives for R&D investments of firms dealing with superior competition. To do so we design a framework as simple as possible while it still contains the specific aspects of R&D: uncertainty and time to complete. After starting out with studying the monopoly benchmark case, a duopoly framework is considered. The paper is organized as follows. The model is presented in Section 2. Section 3 treats the monopoly case, while the effects of competition in the form of a duopoly are analyzed in Section 4. Section 5 contains the welfare analysis in the proposed setting. Conclusions and topics for further research are presented in Section 6.

2 The model

The model is based on the following set of assumptions. In the home country there is an Incumbent firm, which produces at a unit cost K. Another firm is located in a foreign country and considers to start a new plant in the home country of the Incumbent. This is a potential Entrant. The Entrant has a superior technology in the sense that it allows him to produce with a smaller unit cost, which for simplicity is put to zero. The cost of entry is fixed to f, which is exogenously set in the market.

In addition, we consider a two-step R&D process.

At time 0 the Incumbent has an opportunity to make an initial irreversible R&D investment βI, where it is assumed that $0 < \beta < 1$. The outcome of this investment is stochastic. After having carried out the initial R&D investment, at time 1 the Incumbent needs to invest $(1-\beta)I - h$ with probability $\frac{1}{2}$ in order to achieve the breakthrough, and with the same probability it needs to invest $(1-\beta)I + h$ to achieve the same breakthrough. This can be interpreted in a sense that a bad outcome means that the firm needs to apply more time, effort, or materials for R&D to be completed. The extra costs that have to be incurred in this case, compared to that of the good outcome, are $2h$. The total "planned" cost of R&D is, thus, equal to I, the first-stage share of this cost is β, and the parameter determining the second-stage investment's volatility is h. All these parameters are known beforehand, and it is assumed that $0 \leq h \leq (1-\beta)I$.[1]

In previous literature uncertainty about the R&D success was modelled using Poisson arrival process (for example Dasgupta and Stiglitz (1980) and Weeds (2002)). The drawbacks of this approach come from the facts that the current success probability is independent of investments in the past and that it is not possible to analyze the situation of increased uncertainty while keeping the mean constant. Our implementation approach for technical uncertainty allows to capture the uncertainty resolving nature of research and development and use the analytical advantages of a mean preserving spread.

When the technological breakthrough is achieved, the Incumbent is able to produce more efficiently from this moment onwards. In particular, it is assumed that the new technology developed by the Incumbent is equivalent to that of the Entrant so the R&D process results in a new technology where the unit production cost is reduced from K to zero.

$P(Q)$ is the normalized inverse demand function expressing the market price as a function of total supply Q:

$$P(Q) = 1 - Q.$$

To make our model closer to real life we assume that the Incumbent has a time lead over the Entrant. The Incumbent anticipates the entry and has one time period advantage in developing the response to the threat. Thus we assume that the Incumbent, while being a monopolist and producing with unit cost K, makes an R&D investment decision about starting up the process at time $t = 0$. The Entrant makes its entry decision at time $t = 1$. The perfect information condition implies that at that moment the Entrant has perfect knowledge concerning the Incumbent's decision whether to complete the R&D project or not. Based on the outcome of the first-stage R&D investment at time $t = 1$, the Incumbent decides about completing the R&D project, while it still produces with unit cost K. If the Incumbent develops a new technology (implying that the unit production cost drops from K to zero), it will start producing with it from time $t = 2$. If the Entrant decides to enter, it incurs the entry cost f and it will also start its production with zero unit cost from time

[1] The advantage of this formulation is that mean preserving spreads can be considered.

$t = 2$. Therefore, the final market structure will be realized from time $t = 2$ onwards.

This model is related to that of Kulatilaka and Perotti (1998) but differs in three aspects: (i) in Kulatilaka and Perotti the firm can carry out one investment expenditure in order to reduce unit production costs in the next period, while in our framework the firm needs to go through a two stage investment procedure; (ii) in Kulatilaka and Perotti there is demand uncertainty while we have R&D cost uncertainty, the impact of which can be derived unambiguously; and (iii) we put explicit difference between the Incumbent and the Entrant by allowing the Incumbent to have one time period lead over the Entrant, while Kulatilaka and Perotti use the Stackelberg setting to distinguish the leader and the follower. A similar approach, oriented at analyzing Cournot and Stackelberg competition, was employed in Smit and Trigeorgis (1998). Instead of the Stackelberg leader approach we use a straightforward time lead factor to give the incumbent advantage over the entrant. In our model the Stackelberg competition is less suitable, because there is no commitment of the Incumbent to its investment decision. Therefore, the time-lead introduction is a more realistic way to distinguish the players. In real life there are many opportunities for the incumbent firm to anticipate the entry and be able to prepare its reaction. For example, the study of Thomas (1999) provided empirical evidence of the incumbents preemptive actions under the threat of entry.

3 The Monopolist's Initial Decision Frontier and the Abandonment Decision Line

The monopoly case can be considered as the case of the Incumbent operating under no threat of entry. This firm operates in the market alone and makes plans about production and R&D investment subject to only technological uncertainty. In the game theory framework the factor of uncertainty can be considered as actions of player Nature (Tirole (1988)). In our case we have the monopolist playing against Nature, which plays "success" and "failure" of the first-stage R&D with equal probabilities. The structure of this game is given in Figure 1.

Suppose that at a given moment in time, when the firm decides about its production level, the unit production cost equals K. Then the optimal output is determined by solving:

$$\max_{Q_m} \ \pi_m(K) = \left[(1-Q)\,Q - KQ\right].$$

The first order condition gives

$$Q_m = \frac{1-K}{2}.$$

We see that in the monopoly case the firm will not produce if unit production cost $K \geq 1$. Thus, from now on we only need to consider the cases where

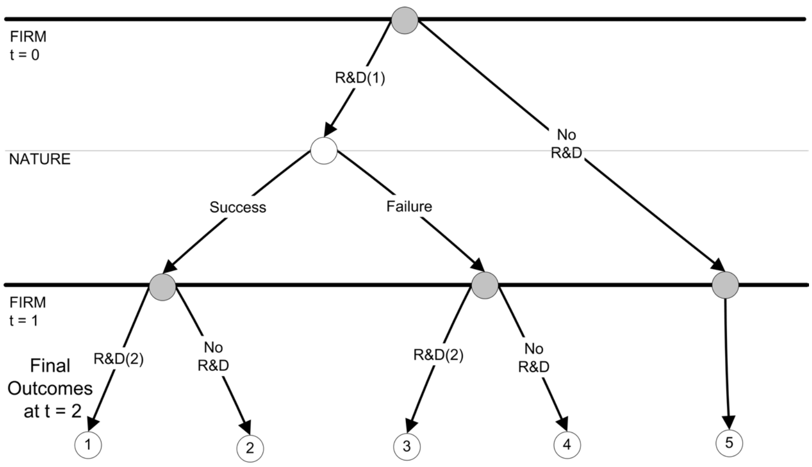

Figure 1: Structure of the Monopolist's Game

$0 \leq K < 1$. The monopolist's optimal profit per unit of time equals:

$$\pi_m(K) = \left(\frac{1-K}{2}\right)^2.$$

In case the firm already completed the two stage R&D investment process, the unit production cost is reduced to zero, so that profit is given by

$$\pi_m(0) = \frac{1}{4}.$$

From now on we will refer to $\pi(0)'s$ and to $\pi(K)'s$ as already maximized values of the firm's profit function. The complete enumeration of the firm's payoffs in the game is given in Table 1, where the outcome $v(i)$ refers to the corresponding outcome i occurring in the bottom row of Figure 1.

Outcome	**Firm's Payoff**
v(1)	$-\beta I - \frac{(1-\beta)I-h}{1+r} + \frac{1}{r(1+r)}\pi_m(0)$
v(2)	$-\beta I + \frac{1}{r(1+r)}\pi_m(K)$
v(3)	$-\beta I - \frac{(1-\beta)I+h}{1+r} + \frac{1}{r(1+r)}\pi_m(0)$
v(4)	$-\beta I + \frac{1}{r(1+r)}\pi_m(K)$
v(5)	$\frac{1}{r(1+r)}\pi_m(K)$

Table 1. Payoffs of the Monopolist Corresponding to Outcomes of the Game

To determine the optimal R&D investment strategy of a firm in the given setup, we observe its decision making backwards starting at time $t = 1$. If the first step R&D was *unsuccessful*, it costs $(1-\beta)I + h$ in the second stage to finish the R&D project successfully. This leads to the following investment gain (the difference between the net present value (NPV) of this investment and the NPV of not making the investment):

$$\triangle_{m,2}^{U} = -[(1-\beta)I + h] + \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t}\pi_m(0) - \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t}\pi_m(K)),$$

$$\Delta_{m,2}^{U} = -[(1-\beta)I + h] + \frac{1}{r}\Delta\pi_m, \tag{1}$$

where r is the discount rate, the superscript "u" refers to the unsuccessful first stage research, and the subscript "m,2" denotes the second-stage investment decision of the monopolist.

We define the new technology profit gain (or just the profit gain) at a given point in time as

$$\Delta\pi_m = \pi_m(0) - \pi_m(K). \tag{2}$$

When the first step R&D investment was *successful,* it only costs $(1-\beta)I - \triangle h$ to develop a new technology. Then the investment gain is

$$\triangle_{m,2}^{S} = -[(1-\beta)I - h] + \frac{1}{r}\Delta\pi_m. \tag{3}$$

The monopolist's decision to start the project is based on two different optimal investment criteria. One is the unconditional investment criterion, which is applied in case it is always optimal to complete the project:

$$\triangle_{m,NPV} = \frac{\frac{1}{2}\triangle_{m,2}^{S} + \frac{1}{2}\triangle_{m,2}^{U}}{1+r} - \beta I. \tag{4}$$

This criterion is the straightforward net present value of the R&D project.

The other criterion is a conditional (or a success-dependent) investment criterion:

$$\triangle_{m,NPV}^{S} = \frac{\frac{1}{2}\triangle_{m,2}^{S}}{1+r} - \beta I, \tag{5}$$

which is relevant when the R&D project will be finished only if the first stage was successful.

Proposition 1 *The monopolist's optimal R&D investment strategy is defined as:*
i) Start the first stage of the R&D project if the Initial Decision criterion

$$\Delta_{m,1} = \max\{\Delta_{m,NPV}, \Delta_{m,NPV}^{S}\} > 0 \tag{6}$$

is satisfied.
ii) Once the R&D is launched, carry out the second-stage R&D if the first-stage R&D is successful, or if the initial stage fails and $\Delta_{m,2}^{U} > 0$.

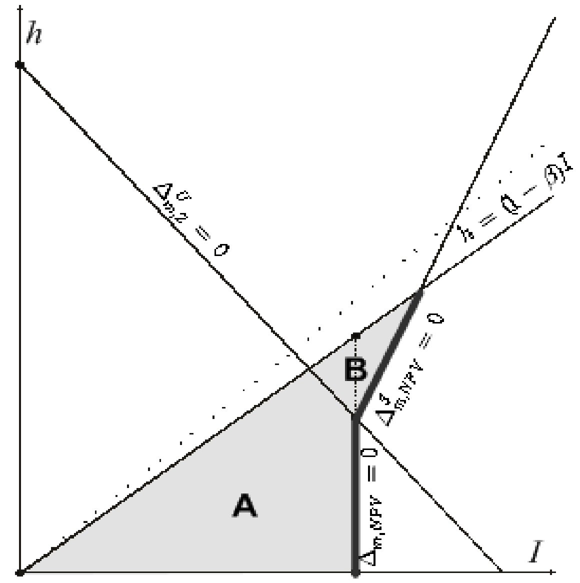

Figure 2: The Monopolist's R&D Decision Space in the $(I, \Delta h)$ Plane

Proof. see Appendix ■

As we can see, the initial decision criterion $\Delta_{m,1}$ allows for both flexibility to continue the R&D project only in the case of success and the possibility of having still a profitable R&D project in the case of an unsuccessful first stage.

In the following analyses we will concentrate our attention on two criteria: i) the initial investment criterion $\Delta_{m,1}$, and ii) the unsuccessful R&D abandonment criterion $\Delta^U_{m,2}$. In the decision space map we plot the zero-value lines of these two criteria and analyze different decision areas. Figure 2 presents the monopolist's decision area in the (I, h) plane and Figure 3 in the (K, h) plane correspondingly.

In both figures we distinguish two areas: **A** and **B**. Area **A** contains all the R&D projects, which will be started and finished regardless of the result of the first stage (the unconditional project implementation), i.e. it contains the pairs (I, h) or (K, h) such that

$$\Delta_{m,1} > 0, \quad and \quad \Delta^U_{m,2} > 0.$$

In area **B** we have the R&D projects, which are launched, but are abandoned if the preliminary stage fails (the conditional project implementation). This set contains (I, h) or (K, h) such that:

$$\Delta_{m,1} > 0, \quad and \quad \Delta^U_{m,2} < 0.$$

The critical border of the initial decision criterion $\Delta_{m,1} = 0$ is drawn as a

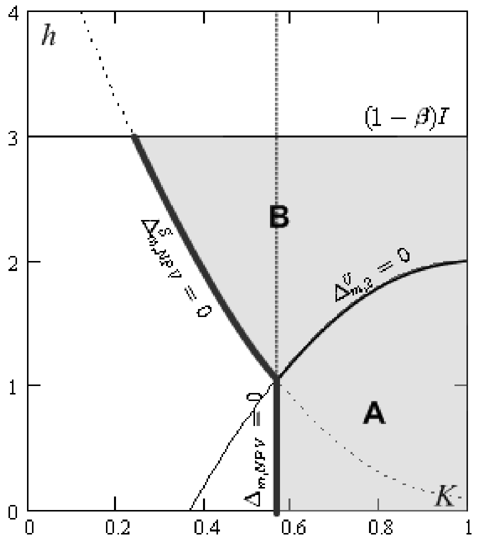

Figure 3: The Monopolist's R&D Decision Space in the $(K, \Delta h)$ Plane. $\beta =$ 25%, $r = 5\%$

thick gray line and can be called the Initial Decision Frontier. Line $\Delta^U_{m,2} = 0$ represents the Abandonment Decision line.

3.1 The Effect of Uncertainty on the R&D Investment Decision

Proposition 2 *Higher uncertainty about the outcome of R&D, while keeping the mean fixed:*
i) does not affect the firm's decision to start R&D with innovation levels lower then

$$K^*_m = \underset{K}{\arg}\Delta^S_{m,NPV}\Big|_{h = (1-\beta)I} = 0;$$

ii) positively affects the firm's decision to start R&D when it faces a project with negative NPV,
iii) negatively affects the firm's decision to continue the project with positive NPV.

Proof. see Appendix ■

The results of Proposition 2 is illustrated in Figure 3. Result i) refers to the case when $K \in [0, K^*_m]$. In figure 3 it is shown that irrespective of uncertainty level, the firm does start R&D. The reason is that the current unit production cost K is already low enough. Result ii) refers to the scenario where K is larger than K_m^* but so small that $\Delta^S_{m,NPV} < 0$. In Figure 3 we see that it is optimal for the firm to undertake the first-stage of the R&D project only when uncertainty is sufficiently large. The reason here is that the project is completed only when the first stage is successful; and in such a case the profitability increases with h. Result iii) refers to the case when K is large (corresponding to the right side relative to the line $\Delta^S_{m,NPV} = 0$ in Figure 3). We conclude that the firm always starts investing in R&D, and will always complete the project if uncertainty is small. In case uncertainty is large, it becomes too expensive to complete the project if the first-stage outcome is unsuccessful.

In addition to active and passive R&D investment strategies, we also distinguish between conditional and unconditional ones. The strategy is considered to be *unconditional* if it endorses a particular course of action regardless the outcome of the preliminary R&D. Correspondingly, the strategy is *conditional* if it endorses a particular decision depending on the result in the first-stage research. In this paper conditional variables and strategies are indicated with superscripts U or S.

For example in Figure 3, in the area beyond the region $(A \cup B)$, the firm will choose not to invest in R&D independent of the possible outcome of the preliminary research. This strategy is classified as unconditionally passive. The strategy, corresponding to region A is unconditionally active, because the firm will implement an R&D project in full no matter how its first stage came out. And in region B we find conditionally active strategies, because there the firm will finish R&D only if it is successful in the first step.

The effect of technical uncertainty in this model is different from the influence of market uncertainty, which is a more traditional type of uncertainty studied in the literature. The overview of empirical studies on investment under uncertainty done by Carruth *et al.* (2000) concludes that increased market uncertainty raises the value of an option to delay the investment, and thus leads to lower investment levels.

Technical uncertainty can not be resolved without engaging in research, and thus the delay option has no value. Due to the asymmetric nature of the R&D option in this model, increased uncertainty gives a greater value to the case of successful implementation of the first-stage R&D, while downward risk is limited (which also was observed by Lint and Pennings (1998))

4 The Incumbent's Entry Reaction Strategies in the Duopoly

4.1 Optimal Outputs in Duopoly. Non-drastic vs. Drastic Innovation.

Consider now the case when there can be two firms in the market. The normalized inverse market demand function now is $P(Q) = 1 - Q$, where $Q = q_{inc(umbent)} + q_{ent(rant)}$ We assume that the Entrant decided to enter and currently observe a duopolistic market structure.

In general, the Incumbent produces at a fixed unit production cost $K_{inc} \in \{0, K\}$. As the Entrant already has a new technology, its unit production cost is $K_{ent} = 0$. There are two different market structures that can emerge: One is symmetric competition with R&D completed by the Incumbent, which obtains the same production cost $K_{inc} = 0$ as the Entrant, and the other is the case of asymmetric competition in which the Incumbent has $K_{inc} = K$ and the Entrant has zero production cost.

At a given moment in time the Incumbent solves for the optimal production level:

$$\max_{\{q_{inc}\}} [1 - q_{inc} - q_{ent}]q_{inc} - K_{inc}q_{inc}.$$

In the duopoly the Incumbent's reaction function is:

$$q_{inc} = \frac{1 - K_{inc} - q_{ent}}{2}.$$

The Entrant solves for:

$$\max_{\{q_{ent}\}} [1 - q_{inc} - q_{ent}]q_{ent},$$

and has the reaction function

$$q_{ent} = \frac{1 - q_{inc}}{2}.$$

The corresponding optimal output of the Incumbent will be:

$$q_{inc}(K_{inc}) = \frac{1 - 2K_{inc}}{3},$$

and the Entrant will produce:

$$q_{ent}(K_{inc}) = \frac{1 + K_{inc}}{3}.$$

Suppose that it is profitable for the Incumbent to complete the R&D and obtain the new technology, so that $K_{inc} = 0$. Then, both the Incumbent and the Entrant will be producing with the advanced technology and at zero unit cost. Their optimal output at a given moment in time will be:

$$q_{inc}(0) = q_{ent}(0) = \frac{1}{3},$$

which leads to the profit level

$$\pi_{inc}(0) = \pi_{ent}(0) = \frac{1}{9}.$$

But if the Incumbent does not invest in R&D, the Entrant will have an advantageous position, because it is the only firm in the market with zero production cost. The Incumbent will produce with $K_{inc} = K$. In such a case we obtain the following pair of optimal outputs:

$$q_{inc}(K) = \frac{1-2K}{3}$$
$$q_{ent}(K) = \frac{1+K}{3},$$

and their corresponding profits:

$$\pi_{inc}(K) = \left(\frac{1-2K}{3}\right)^2$$
$$\pi_{ent}(K) = \left(\frac{1+K}{3}\right)^2$$

Looking at these results, it is necessary to make one special observation. If the unit production cost K lies $\frac{1}{2}$ and 1, then $q_{inc}(K)$ is negative, and the asymmetric R&D costs game automatically degrades to a standard monopoly situation with the Entrant pushing the Incumbent away. Here we can distinguish between the cases of **non-drastic** and **drastic** innovation (Tirole, (1988)). The **non-drastic innovation** corresponds to the case of a relatively low $K \in [0, \frac{1}{2})$. If one firm innovates and the other does not, the innovation is not strong enough to drive the not innovating firm out of the market. On the other hand, if we observe **drastic innovation** (bringing a relatively large $K \in [\frac{1}{2}, 1)$ to zero), the innovating agent gains so much that he actually pushes the "looser" from the market and gains a monopoly position.

When the Incumbent finds itself at the initial decision point, it observes values of (I, K, h, r, β), thinks of the uncertainty, and combines this information with the knowledge about its future profits, it can build own strategy. The logic of competitive decision-making with regard to uncertainty in R&D is the same as for the monopolistic firm. Nonetheless the Incumbent must consider different profit streams, which are dependent on the Entrant's decision whether to enter or not to enter the market.

The Incumbent vs. Superior Entrant Game's structure is given in Figure 4. Tables 2 and 3 in Appendix contain the corresponding payoffs of the Incumbent and the Entrant in this game, which correspond to the bottom row outcomes in Figure 4.

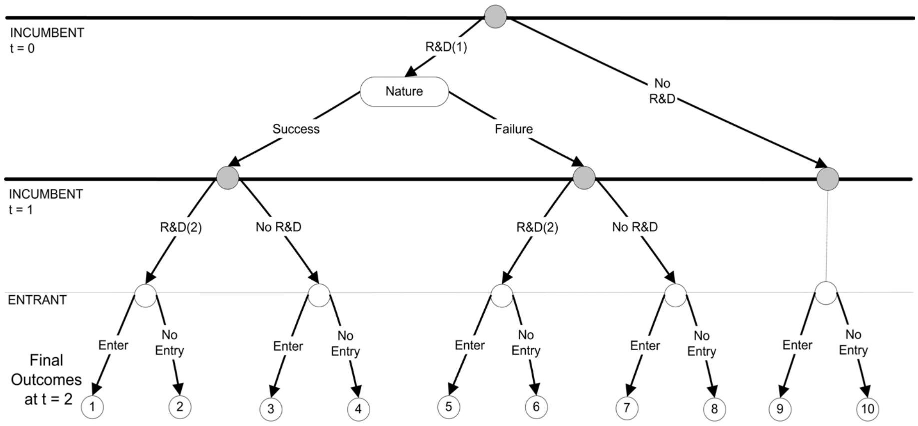

Figure 4: The Game of R&D Investment under the Threat of a Superior External Entry

Outcome	**Firms' Payoff**
$v(1)$	$\begin{cases} -\beta I - \frac{(1-\beta)I-h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi(0) \\ \frac{1}{r}\pi(0) - f \end{cases}$
$v(2)$	$\begin{cases} -\beta I - \frac{(1-\beta)I-h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_m(0) \\ 0 \end{cases}$
$v(3)$	$\begin{cases} -\beta I + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_{inc}(K) \\ \frac{1}{r}\pi_{ent}(K) - f \end{cases}$
$v(4)$	$\begin{cases} -\beta I + \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$
$v(5)$	$\begin{cases} -\beta I - \frac{(1-\beta)I+h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi(0) \\ \frac{1}{r}\pi(0) - f \end{cases}$
$v(6)$	$\begin{cases} -\beta I - \frac{(1-\beta)I+h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_m(0) \\ 0 \end{cases}$
$v(7)$	$\begin{cases} -\beta I + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_{inc}(K) \\ \frac{1}{r}\pi_{ent}(K) - f \end{cases}$
$v(8)$	$\begin{cases} -\beta I + \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$
$v(9)$	$\begin{cases} \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_{inc}(K) \\ \frac{1}{r}\pi_{ent}(K) - f \end{cases}$
$v(10)$	$\begin{cases} \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$

Table 2. Payoffs of the Duopoly Game with a Superior Entry Threat with Non-Drastic Innovation

Outcome	Firms' Payoff
$v(1)$	$\begin{cases} -\beta I - \frac{(1-\beta)I-h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi(0) \\ \frac{1}{r}\pi(0) - f \end{cases}$
$v(2)$	$\begin{cases} -\beta I - \frac{(1-\beta)I-h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_m(0) \\ 0 \end{cases}$
$v(3)$	$\begin{cases} -\beta I + \frac{2+r}{1+r}\pi_m(K) \\ \frac{1}{r}\pi_m(0) - f \end{cases}$
$v(4)$	$\begin{cases} -\beta I + \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$
$v(5)$	$\begin{cases} -\beta I - \frac{(1-\beta)I+h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi(0) \\ \frac{1}{r}\pi(0) - f \end{cases}$
$v(6)$	$\begin{cases} -\beta I - \frac{(1-\beta)I+h}{1+r} + \frac{2+r}{1+r}\pi_m(K) + \frac{1}{r(1+r)}\pi_m(0) \\ 0 \end{cases}$
$v(7)$	$\begin{cases} -\beta I + \frac{2+r}{1+r}\pi_m(K) \\ \frac{1}{r}\pi_m(0) - f \end{cases}$
$v(8)$	$\begin{cases} -\beta I + \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$
$v(9)$	$\begin{cases} \frac{2+r}{1+r}\pi_m(K) \\ \frac{1}{r}\pi_m(0) - f \end{cases}$
$v(10)$	$\begin{cases} \frac{1+r}{r}\pi_m(K) \\ 0 \end{cases}$

Table 3. Payoffs of the Duopoly Game with a Superior Entry Threat with Drastic Innovation

4.2 Entry Regimes and the Incumbent's Reactions

The strategic decision-making process of the Entrant is relatively straightforward. It will enter the market if profits are higher than the entry cost. In the situation where the Incumbent decides to invest in R&D, the Entrant will enter only if $\frac{1}{r}\pi_{ent}(0) - f > 0$. If the Incumbent does not invest in R&D and the innovation is non-drastic, the Entrant will enter if $\frac{1}{r}\pi_{ent}(K) - f > 0$. Correspondingly, in the case of drastic innovation and observing that the Incumbent does not invest in R&D, the Entrant enters only if $\frac{1}{r}\pi_m(0) - f > 0$.

Note that:

$$\begin{aligned} \pi_{ent}(K) &= \left(\frac{1+K}{3}\right)^2 > \pi(0) = \frac{1}{9}, \text{ for the case of non-drastic innovation} \\ \pi_m(0) &= \frac{1}{4} > \pi(0) = \frac{1}{9}, \text{ when the innovation is drastic.} \end{aligned}$$

This allows us to define the intervals of entry cost f that determine three types of entry threat, which are depicted in Figure 5:

i) **Inevitable Entry**. This type of entry threat occurs when $f \in [0, f^{EP}]$, where $f^{EP} = \frac{1}{r}\pi(0)$. The inevitability of entry does not depend on the innovation type and the Entrant will decide to enter the market regardless the actions of the Incumbent.

ii) The **Preventable Entry** situation exists when $f \in (f^{EP}, F^{EP}]$, where $F^{EP} = \frac{1}{r}\pi_{ent}(K)$ for non-drastic innovation and $F^{EP} = \frac{1}{r}\pi_m(0)$ under drastic innovation. The Entrant will enter the market only if the Incumbent does not invest in R&D.

iii) The **Non-Credible Entry Threat** is considered when the entry cost is prohibitively high: $f \in (F^{EP}, \infty)$.

It is evident that if f lies in the third interval, the situation becomes the same as in the monopoly's case discussed above.

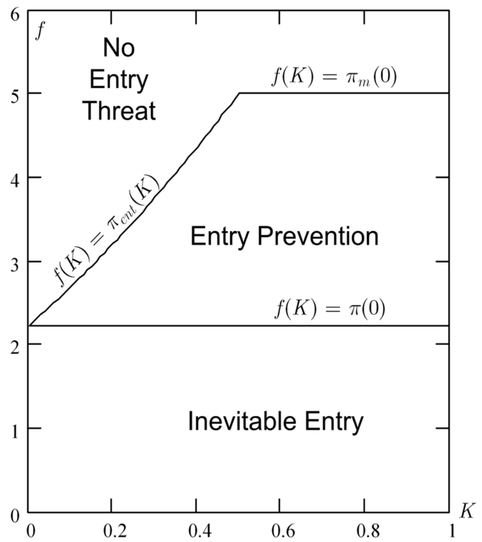

Figure 5: Entry Cost Regions

Facing the threat of entry, the Incumbent must develop an entry reaction strategy. Such a strategy describes whether or not to undertake R&D in order to counter the entry aggressively. The decision is made after considering the following investment gains:

$$
\begin{aligned}
\Delta^{U}_{inc,2} &= -[(1-\beta)I + h] + \frac{1}{r}\Delta\pi_{inc}, \\
\triangle^{S}_{inc,2} &= -[(1-\beta)I - h] + \frac{1}{r}\Delta\pi_{inc}, \\
\triangle_{inc,NPV} &= \frac{\frac{1}{2}\triangle^{S}_{inc,2} + \frac{1}{2}\triangle^{U}_{inc,2}}{1+r} - \beta I, \\
\triangle^{S}_{inc,NPV} &= \frac{\frac{1}{2}\triangle^{S}_{inc,2}}{1+r} - \beta I.
\end{aligned}
$$

Therefore we formulate another proposition, which defines the optimal strategy of the incumbent under the threat of inevitable entry.

Proposition 3 *The Incumbent's optimal R&D investment strategy is:*
i) Start the first stage of the R&D project if the Initial Decision criterion

$$\Delta_{inc,1} = \max(\Delta_{inc,NPV}, \Delta^{S}_{inc,NPV}) > 0. \tag{7}$$

ii) Once the R&D is launched, carry out the second-stage R&D if the first-stage R&D is successful, or if the initial stage fails and $\Delta^{U}_{inc,2} > 0$*.*

Proof. The Incumbent makes its decisions considering the current mode of entry. As the entry cost f is exogenously defined, at $t = 0$ the Incumbent knows exactly which entry strategy will be played by the Entrant. In this case the only opponent of the Incumbent becomes Nature and the optimal investment strategy is qualitatively equivalent to that of the Monopolist (see Proposition 1). ∎

If *entry is inevitable*, the Incumbent knows that the Entrant will enter at time $t = 1$. In this situation the Incumbent will calculate its criteria based on

$$\Delta\pi_{inc}(f \le f^{EP}, K \in [0, \frac{1}{2})) = \pi_{inc}(0) - \pi_{inc}(K) = \frac{1}{9} - \frac{1}{9}(1-2K)^2 > 0$$

under conditions of non-drastic innovation and

$$\Delta\pi_{inc}(f \le f^{EP} K \in [\frac{1}{2}, 1)) = \pi_{inc}(0) = \frac{1}{9} > 0$$

if innovation is drastic.

In the case of *preventable entry* the Incumbent is capable of locking the entrant out the market by obtaining the new technology. The new technology profit gain of the Incumbent depends on the innovation type. If innovation is non-drastic, the Incumbent builds its criteria based on

$$\Delta\pi_{inc}(f \in (f^{EP}, F^{EP}), K \in [0, \frac{1}{2})) = \pi_m(0) - \pi_{inc}(K) = \frac{1}{4} - \frac{1}{9}(1-2K)^2 > 0.$$

In the case of drastic innovation, the Incumbent risks being pushed out the market if it does not invest, therefore it must consider

$$\Delta\pi_{inc}(f \in (f^{EP}, F^{EP}), K \in [\frac{1}{2}, 1)) = \pi_m(0) = \frac{1}{4} > 0.$$

And if the entry threat is *non-credible*, then the positions of both the Incumbent and the Monopolist are exactly the same with

$$\Delta\pi_{inc}(f > F^{EP}) = \Delta\pi_m = \frac{2K - K^2}{4} > 0.$$

Using Propositions 2 and 3 combined we can formulate two other propositions about the effect of uncertainty on the Incumbent's decisions:

Proposition 4 *Under conditions of inevitable entry threat, the increased uncertainty about the outcome of R&D, while keeping the mean fixed*
i) does not affect the incumbents's decision to start R&D with innovation levels not greater than

$$K^*_{inc} \equiv \arg_K \left(\Delta^S_{inc,NPV} \Big|_{h = (1-\beta)I} = 0 \right);$$

ii) negatively affects the incumbent's decision to continue the project with positive NPV for $K > K^*_{inc}$*;*
iii) positively affects the incumbent's decision to start R&D when it faces a project with negative NPV for $K > K^*_{inc}$*.*

Proof. see Appendix. ■

Proposition 5 *Under conditions of entry prevention, the increased uncertainty about the outcome of R&D, while keeping the mean fixed*
i) does not affect the incumbents's decision to start R&D with innovation levels not greater than

$$K^*_{inc} \equiv \arg_K \left(\Delta^S_{inc,NPV} \Big|_{h = (1-\beta)I} = 0 \right),$$

given that $K^*_{inc} \geq K^{EP}(f)$*, where* $K^{EP}(f)$ *is the inverse of the above limit*$F^{EP} = \frac{1}{r}\pi_{ent}(K)$ *for entry prevention region under non-drastic innovation*
ii) negatively affects the incumbent's decision to continue the project with positive NPV for $K > K^*_{inc}$*;*
iii) positively affects the incumbent's decision to start R&D when it faces a project with negative NPV for $K > K^*_{inc}$*.*

Proof. see Appendix. ■

Proposition 2 shows that the effect of uncertainty on the decision of an individual firm can be positive or negative depending on the NPV of the underlying project. Propositions 3 and 4 show, the optimal R&D investment strategy of the Incumbent and the Monopolist is affected by uncertainty in the same way. When the NPV of the R&D project is negative, an increase in uncertainty can lead to a switch from the passive strategy to the active one. Correspondingly, if the NPV is positive, more uncertainty increases the chances that the project becomes abandoned in case of first-stage R&D failure, i.e. induces a switch from the active to passive entry reaction strategy.

The selection of the possible entry reaction strategies depends on the kind of strategic effect, determined by parameters f and K. If we consider K separately in the context of a superior entry threat, the degree of innovation can also be considered as a *technological distance* between the incumbent and the entrant in our game. Under conditions of inevitable entry, the Incumbent has the following set of strategies to choose from: active accommodation (AA), passive accommodation, and shut down. The shut down strategy is an equivalent of the strategy of passive accommodation when innovation is drastic.

From Proposition 4 it follows that higher uncertainty about the outcome of the first-stage R&D stimulates the firm to choose a more aggressive entry reaction strategy. For example under low uncertainty the firm's optimal strategy is passive accommodation, but if uncertainty is high enough, the firm will choose active accommodation, as illustrated in Figure 6.

If the opportunity for entry prevention exist, the scope of possible strategies becomes wider: passive accommodation, deterrence, blockade, and shut down. The uncertainty-driven change in the optimal entry prevention strategy of the incumbent depends on the following list of variables: $K^{EP}(f)$, K^*_{inc}, K^*_m, $\Delta_{m,NPV}$, and $\Delta_{inc,NPV}$.

4.3 Entry Accommodation under Inevitable Entry Conditions

The entry accommodation strategies appear under conditions of inevitable entry, when the entry cost is relatively low: $f \in (0, \frac{1}{9r})$. In this case the incumbent can carry out R&D and react actively, or be passive and stay with the current technology. If innovation is drastic, the incumbent must consider the shut-down option instead of passive accommodation, because the firm with an inferior technology must leave the market. The decision-making areas of the incumbent in such a situation are given in Figure 6.

Considering Proposition 3, we see that the Incumbent makes its strategic decision about the R&D driven by two main incentives. One is the willingness to obtain a better technology and more revenue (realized through bringing production cost K to zero). And the other is to strengthen its competitive position in the face of the entry threat by equalizing its own technology with that of the entrant.

Now let us compare the Incumbent's incentives to innovate under the inevitable threat of entry and when no such threat exists.

Proposition 6 *Under conditions of inevitable entry threat every R&D project being profitable for the incumbent is also profitable for the monopolist facing no*

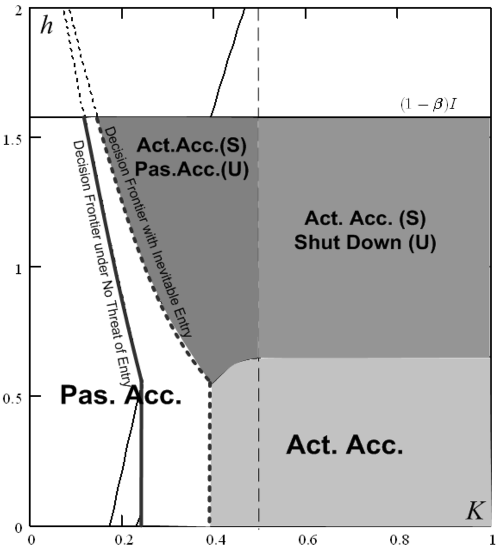

Figure 6: Active vs. Passive Entry Accomodation. $\beta = 25\%$, $r = 5\%$, $f = 1$, $I = 2$

such threat. The opposite does not hold.

Proof. Under conditions of inevitable entry and non-drastic innovation, the new technology profit gain of the incumbent is:

$$\Delta\pi_{inc}(K \in [0, \frac{1}{2})) = \pi(0) - \pi_{inc}(K) = \frac{1}{9} - \left(\frac{1-2K}{3}\right)^2 = \frac{2K - 2K^2}{4.5}.$$

If innovation is drastic the new technology profit gain is:

$$\Delta\pi_{inc}(K \in [\frac{1}{2}, 1)) = \pi(0) = \frac{1}{9}$$

The monopolist considers

$$\Delta\pi_m = \pi_m(0) - \pi_m(K) = \frac{1}{4} - \left(\frac{1-K}{2}\right)^2 = \frac{2K - K^2}{4}.$$

Profit gains of the monopolist are higher than those of the incumbent:

$$\Delta\pi_m > \Delta\pi_{inc}(K \in [0, 1)).$$

The investment gains of both types of agents are positively related to their profit gains, therefore

$$\begin{aligned} \Delta_{m,1} &> \Delta_{inc,1}, \\ \Delta^U_{m,2} &> \Delta^U_{inc,2}. \end{aligned}$$

■

Proposition 6 allows us to conclude that in the case of inevitable entry, the strategic effect is, in fact, negative. Inevitable entry narrows the scope of R&D projects being profitable for the incumbent and in this way becomes an impediment for the innovation incentives of the domestic firm.

4.4 Entry Blockade and Entry Deterrence under Entry Prevention Conditions

If an opportunity for entry prevention exist, the incumbent has a different set of strategies to choose from. The entry prevention case exists for $f \in [f^{EP}, F^{EP})$. The strategies are: Entry Blockade (BL), Entry Deterrence (DET), and Shut Down (SD). Entry Blockade occurs when the incumbent performs R&D and effectively blocks entry under such threat, but would have also carried out R&D if there was no entrant at all. Deterrence implies that the threat of entry creates an incentive for the incumbent to block it in a situation when, if unthreatened, the monopolist would not have carried out the R&D. If innovation is non-drastic, the Passive Accommodation strategy (PA) can be an option. In order to analyze the Entry Blockade and the Entry Deterrence strategies, let us consider behavior of the incumbent firm under the threat of entry in comparison to the behavior of the monopolist firm (illustrated in Figure 7).

The border between blockade and deterrence primarily depends on the monopolist's decision. Entry Blockade takes place if both the monopolist and the incumbent succeed in obtaining the new technology. If it is not optimal for the monopolist to perform R&D, Entry Deterrence takes place. These two reasons are:

1. The monopolist does not start R&D, i.e. $\Delta_{m,1} < 0$.

2. The monopolist starts R&D and abandons it when the first-stage fails, i.e. $\Delta^U_{m,2} < 0$.

But in order to analyze the general effect of competition on different entry reaction strategies under conditions of entry prevention, behavior of both the incumbent and the hypothetical monopolist must be considered. For that one more proposition must be proven.

Proposition 7 *If the opportunity for entry prevention exist, then it will be profitable for the incumbent to carry out any R&D project, which is profitable for the monopolist facing no threat of entry. The opposite does not hold.*

Proof. Under conditions of entry prevention, the new technology profit gains of the Incumbent are:

$$\Delta\pi_{inc}(K \in [0, \frac{1}{2})) = \pi_m(0) - \pi_{inc}(K) = \frac{1}{4} - \left(\frac{1-2K}{3}\right)^2 = \frac{5+16K-16K^2}{36},$$

if innovation is non-drastic, and

$$\Delta\pi_{inc}(K \in [\frac{1}{2}, 1)) = \pi_m(0) = \frac{1}{4},$$

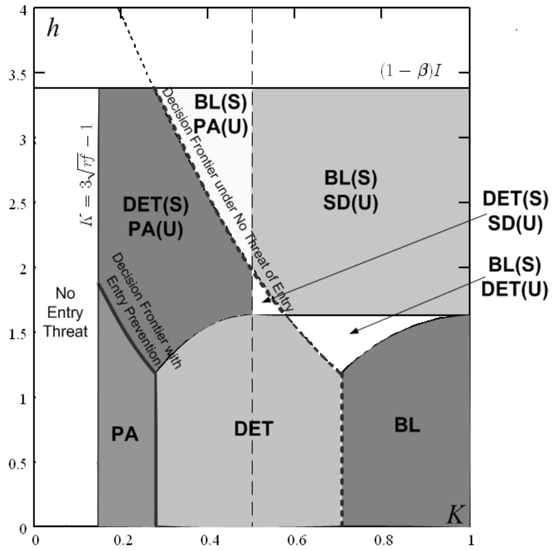

Figure 7: Entry Prevention Strategies Field. $\beta = 25\%$, $r = 5\%$, $f = 3$, $I = 4.5$

if innovation is a drastic one.

Correspondingly, the monopolist considers

$$\Delta\pi_m = \pi_m(0) - \pi_m(K) = \frac{1}{4} - \left(\frac{1-K}{2}\right)^2 = \frac{2K - K^2}{4}.$$

It can be shown that profit gains of the Incumbent are higher than those of the monopolist for corresponding types of innovation:

$$\Delta\pi_{inc}(K \in [0,1)) > \Delta\pi_m.$$

The investment gains of both types of agents are positively related to their profit gains, so that

$$\begin{aligned} \Delta_{inc,1} &> \Delta_{m,1}, \\ \Delta^U_{inc,2} &> \Delta^U_{m,2}. \end{aligned}$$

■

This proposition means that entry threat stimulates the Incumbent to undertake R&D projects, which it would not have carried out without such a threat. A preventable entry threat results in a positive strategic effect on the Incumbent's decisions. For a given arrangement of the underlying parameters it is possible to plot different optimal entry reaction strategies of the Incumbent depending on the degree of innovation and the level of uncertainty (see

Figure 7). The relative "size" of the Entry Deterrence area can serve as a measure for the strategic effect of R&D investment. Kulatilaka and Perotti (1998) demonstrated that the strategic influence of R&D investors increases the value of the investor's expansion option, thus inducing a more aggressive investment behavior. In Weeds (2002) it is stated that competition decreases the value of option to delay. In our model the effect of such strategic influence is not straightforward.

Two parameters in the model, K and f, determine the strength of the strategic effect of R&D investment. The value of technological distance, thus, determines the power of the strategic effect, and the value of the entry cost determines its entry prevention capability. If entry is inevitable, the strategic effect has a negative influence on incumbent's decisions, because investing in R&D does not result in a dramatic change of its competitive position. If the entry cost is high enough to allow for entry prevention, the strategic effect provides additional benefits for the Incumbent from investing in the new technology.

If the Incumbent decides to invest in R&D when entry is preventable, the monopoly is preserved. But the monopoly resulting from entry prevention is different from that in the case of no entry threat. In the situation when $\Delta_{inc,1} > \Delta_{m,1}$, the pure monopolist without potential competition stays with the same old technology, while the Incumbent with a threat entry threat conducts R&D in order to preserve its monopoly position. Therefore the potential competition has positive effect on innovating activities in the market.

5 The Welfare Analysis

In this section we consider welfare implications. In our the model welfare is affected via several different channels: competition, the cost and type of innovation, and the uncertainty about the outcome of innovation. These factors themselves are interconnected, making their welfare implications far from being straightforward.

Denoting producer and consumer surplus as $PS(\cdot)$ and $CS(\cdot)$, respectively, welfare can be defined as::

$$\begin{aligned} W(\cdot) &= PS(\cdot) + CS(\cdot), \\ W(\cdot) &= \pi_{inc}(\cdot) + \pi_{ent}(\cdot) - C(\cdot, f, I) + CS(\cdot), \end{aligned}$$

where

$$CS(\cdot) = \frac{(q_{inc}(\cdot) + q_{ent}(\cdot))^2}{2},$$

and $C(f, I)$ are the costs related to the R&D investment and entry in each particular case and subtracted from the producer surplus.

Knowing that $\pi_{inc}(\cdot) = (q_{inc}(\cdot))^2$ and $\pi_{ent}(\cdot) = (q_{ent}(\cdot))^2$ we obtain that

$$W(\cdot) = \frac{3}{2}[\pi_{inc}(\cdot) + \pi_{ent}(\cdot)] + q_{inc}(\cdot) q_{ent}(\cdot) - C(\cdot, f, I).$$

Let us consider several different scenarios in order to understand the effect of different factors on welfare in the game.

First, assume that the entry is inevitable. Under such an assumption, the Entrant will always enter and the game will yield the following list of outcomes depending on other conditions: $v(1)$, $v(3)$, $v(5)$, $v(7)$, $v(9)$ (see Tables 2 and 3 and Figure 2). The corresponding values of welfare function are:

$$\begin{aligned}
W(1) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{4}{9r(1+r)} - \frac{(1+r\beta)I-h}{1+r} - f; \\
W(3) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{11K^2-8K+8}{18r(1+r)} - \beta I - f, \text{ for non-drastic innovation;} \\
W(3) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{3}{8r(1+r)} - \beta I - f, \text{ for drastic innovation;} \\
W(5) &= W(1) - \frac{2h}{1+r}; \\
W(7) &= W(3); \\
W(9) &= W(3) + \beta I.
\end{aligned}$$

It can be easily shown that $W(1) > W(5)$, and $W(9) > W(3) = W(7)$. Outcomes $v(1)$ and $v(5)$ represents the case of two innovating firms competing in the market. Outcomes $v(9)$, $v(7)$, and $v(3)$ correspond to the asymmetric market arrangements.

In the case of entry prevention, the entrant will only enter if the Incumbent does not finish the R&D project. Therefore, only the outcomes $v(2)$, $v(3)$, $v(6)$, $v(7)$, and $v(9)$ are feasible. The corresponding functions for $v(2)$ and $v(6)$ are:

$$\begin{aligned}
W(2) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{3}{8r(1+r)} - \frac{(1+r\beta)I-h}{1+r}; \\
W(6) &= W(2) - \frac{2h}{1+r}.
\end{aligned}$$

If the entry cost is so high that no entry is feasible, then the following outcomes are to be considered: $v(2)$, $v(4)$, $v(6)$, $v(8)$, and $v(10)$, where

$$\begin{aligned}
W(4) &= \frac{1+r}{r}\frac{3(1-K)^2}{8} - \beta I; \\
W(8) &= W(4); \\
W(10) &= \frac{1+r}{r}\frac{3(1-K)^2}{8}.
\end{aligned}$$

The next proposition shows that more competition is not always good for welfare.

Proposition 8 *Under conditions of drastic innovation ($\frac{1}{2} \leq K < 1$) there exists a value $\bar{I} > 0$ such that for any $I > \bar{I}$ the monopoly provides a higher level of welfare than a duopoly.*

Proof. Under the assumption of $\frac{1}{2} \leq K < 1$, the duopoly can only occur in case entry is inevitable. If the opportunity for entry prevention exist, it is obvious that the game with drastic innovation will lead to monopolistic arrangements (either Incumbent's of Entrant's monopoly) in every feasible outcome. With inevitable entry $W(3)_{IE}$ is corresponds to the smallest possible value of welfare in monopoly, while $W(1)_{IE}$ gives the highest possible value of welfare in duopoly. Therefore we must show that there exists a value $\bar{I} > 0$ such that for any $I > \bar{I}$ the following holds:

$$W(3)_{IE} > W(1)_{IE}, \tag{8}$$

i.e. the lowest possible monopoly outcome is greater than the highest possible outcome of the duopoly.

Expression (8) gives:

$$\begin{aligned} \frac{3}{8r(1+r)} - \beta I &> \frac{4}{9r(1+r)} - \frac{(1+r\beta)I - h}{1+r}, \\ I &> \frac{5 + 72rh}{1-\beta} > 0. \end{aligned}$$

■

Corollary 9 *Under conditions of inevitable entry, $I > \bar{I}$, and drastic innovation ($\frac{1}{2} \leq K < 1$), $W(9)$ gives the highest value of welfare.*

Proposition 8 implies that for higher-cost R&D projects it is socially preferable to have only one firm in the market with a new technology. Moreover, according to the corollary, having the Entrant take over the market can be better than to have the Incumbent develop the new technology domestically.

In a similar manner the proposition above can be generalized.

Proposition 10 *For any type of innovation there exists a value $\bar{I} > 0$ such that for any $I > \bar{I}$ less competitive market structures such as monopoly and/or asymmetric duopoly, provide higher levels of welfare than symmetric competition.*

For large-scale research projects, the active reaction strategies of the Incumbent, such as entry deterrence or entry blockade are socially desirable and thus, the regulator does not have an incentive to promote competition. On the other hand, if the new technology is relatively easy accessible (requires few resources), entry deterrence and blockade policies are not the best strategies from the welfare point of view.

5.1 The Effects of Uncertainty on Welfare

The influence exerted by uncertainty on welfare directly depends on the decision making of the Incumbent. To describe it, we formulate the following proposition.

Proposition 11 *Increased uncertainty about the outcome of R&D, while keeping the mean fixed:*
i) does not affect expected welfare, if $\Delta_{inc,NPV} > 0$*;*
ii) positively affects expected welfare, if $\Delta_{inc,NPV} < 0$*, but* $\Delta^S_{inc,NPV} > 0$*;*
iii) does not affect expected welfare, if $\Delta_{inc,NPV} < 0$*, and* $\Delta^S_{inc,NPV} < 0$*.*

Proof. Consider the R&D project with $\Delta_{inc,NPV} > 0$. Therefore, the Incumbent will start the research and finish it regardless the outcome in the first stage. The expected welfare is:

$$\begin{aligned} E(W_{IE}) &= \frac{1}{2}W(1) + \frac{1}{2}W(5), \text{ if entry is inevitable;} \\ E(W_{EP}) &= \frac{1}{2}W(2) + \frac{1}{2}W(6), \text{ if entry is preventable, or not feasible.} \end{aligned}$$

Correspondingly:

$$\begin{aligned} E(W_{IE}) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{4}{9r(1+r)} - \frac{(1+r\beta)I}{1+r} - f; \\ E(W_{EP}) &= \frac{2+r}{1+r}\frac{3(1-K)^2}{8} + \frac{3}{8r(1+r)} - \frac{(1+r\beta)I}{1+r}. \end{aligned}$$

Both these expressions do not depend on h, thus are not influenced by uncertainty.

Now, if the project has negative NPV, but positive value in the case of success in the first stage ($\Delta^S_{inc,NPV} > 0$), then:

$$\begin{aligned} E(W_{IE}) &= \frac{1}{2}W(1), \text{ if entry is inevitable;} \\ E(W_{EP}) &= \frac{1}{2}W(2), \text{ if entry is preventable, or not feasible,} \end{aligned}$$

and the following holds:

$$\begin{aligned} \frac{\partial E(W_{IE})}{\partial h} &> 0, \\ \frac{\partial E(W_{EP})}{\partial h} &> 0. \end{aligned}$$

Finally, if $\Delta_{inc,NPV} < 0$, and $\Delta^S_{inc,NPV} < 0$, no R&D investment is made, therefore no uncertainty factor enters the problem. ■

Proposition 11 shows that uncertainty affects welfare in a manner similar to how it affects the R&D investment decisions of the Incumbent. For R&D projects with negative NPV existence of the option to continue the project, allows the firm to treat technical uncertainty asymmetrically. In this way larger variance in R&D costs is translated into a higher possible cost gain in the second stage of research. The social planner considers domestic R&D in a similar way, which should result in coherent actions of the Incumbent and the regulator. This advocates for a more liberal regulatory approach to R&D investment and competition among innovative firms.

5.2 Welfare and the Entry Cost

In this section we analyze the relationship between the market entry cost and the social benefit of the new technology's entry vs. domestic innovation decisions and monopoly vs. duopoly. We formulate two propositions concerning this issue. First, we analyze the situation when the Incumbent is capable of developing the new technology.

Proposition 12 *When it is optimal for the Incumbent to unconditionally invest in R&D ($\Delta_{inc,NPV} > 0$), there exist a value $f^* < f^{EP}$ such that for any $f^* < f < f^{EP}$ a superior entry is optimal but not socially desirable.*

Proof. If it is optimal for the Incumbent to develop new technology, the entry will take place only if it is inevitable and it will always result in a symmetric duopoly regardless of the degree of innovation.

Depending on value of the current entry cost, the social planner considers the expected welfare function $E(W)$, which is defined by following expressions:

$$\begin{aligned}
\text{if } f &< f^{EP},\ E(W_{IE}) = \frac{1}{2}W(1) + \frac{1}{2}W(5),\ \frac{\partial E(W_{IE})}{\partial f} < 0, \\
\text{if } f^{EP} &\leq f < \infty,\ E(W_{EP+NE}) = \frac{1}{2}W(2) + \frac{1}{2}W(6),\ \frac{\partial E(W_{EP+NE})}{\partial f} = 0.
\end{aligned}$$

It can be shown that $E(W_{IE}) > E(W_{EP+NE})$ for $f < f^*$, and $E(W_{IE}) < E(W_{EP+NE})$ for $f > f^*$, where $f^* = \frac{5}{72r}$. We know that $f^{EP} = \frac{1}{9r}$, thus $f^* < f^{EP}$. Therefore there indeed exist values of $f^* < f < f^{EP}$ in the inevitable entry region, for which duopoly provides lower levels of welfare than those of monopoly with prevented entry. ■

Secondly, if the Incumbent decides not to invest in R&D, the other proposition describes the desirability of bringing the new technology with the Entrant.

Proposition 13 *If it is optimal for the Incumbent not to innovate unconditionally ($\Delta^S_{inc,NPV} < 0$), then:*
i) under conditions of non-drastic innovation ($0 \leq K < \frac{1}{2}$), there exist a pair of values (K^*, f^*) such that for any $f^* < f < F^{EP}$ and $K < K^* < \frac{1}{2}$ a superior entry is optimal but not socially desirable;
ii) if innovation is drastic ($\frac{1}{2} \leq K < 1$), the entry is both optimal and socially desirable for any $f < F^{EP}$.

Proof. If it is not optimal for the Incumbent to develop a new technology, the entry will take place either when it is inevitable or preventable.

The social planner considers the following values of the expected welfare function:

$$\begin{aligned}
\text{if } f &< F^{EP},\ E(W_{IE+EP}) = W(9),\ \frac{\partial E(W_{IE+EP})}{\partial f} < 0, \\
\text{if } F^{EP} &\leq f < \infty,\ E(W_{NE}) = W(10),\ \frac{\partial E(W_{NE})}{\partial f} = 0.
\end{aligned}$$

It can be shown that $E(W_{IE+EP}) > E(W_{NE})$ for $f < f^*$, and $E(W_{IE+EP}) < E(W_{NE})$ for $f > f^*$, where $f^* = \frac{3K^2+2K-1}{24r}$ for $0 \leq K < \frac{1}{2}$ and $f^* = \frac{-3K^2+6K}{8r}$ for $\frac{1}{2} \leq K < 1$.

Assume that $0 \leq K < \frac{1}{2}$ and, therefore, $F^{EP} = \frac{(1+K)^2}{9r}$. Comparing f^* and F^{EP}, we can show that for $K < K^* = \frac{1}{3}$ it holds that $f^* < F^{EP}$. Correspondingly, for $K \geq K^*$ we have $f^* \geq F^{EP}$. Thus, for $K < K^*$ and $f^* < f < F^{EP}$ it holds that $E(W_{IE+EP}) < E(W_{NE})$, which proves statement i).

Now assume that $\frac{1}{2} \leq K < 1$. The corresponding non-feasible entry border is $F^{EP} = \frac{1}{4r}$. For any level of drastic innovation K, we have $f^* \geq F^{EP}$, therefore, when the entry cost is in the interval $[0, F^{EP}]$, it is true that $E(W_{IE+EP}) > E(W_{NE})$, which proves statement ii). ■

Both propositions imply that an increase in entry cost negatively affects the welfare when such an entry takes place. It results in a situation when it may not be socially desirable anymore to have the Entrant enter the market even though such an entry is optimal for the firms. Such case occur for any level of innovation if the Incumbent invests in R&D and only for a relatively smaller non-drastic innovation, if the Incumbent decides to stay with the old technology. The social planner, therefore, will have and incentive to decrease (if it is possible) the entry cost to the level, where the symmetric innovating duopoly becomes socially preferable, or increase it to the level, which makes the entry not feasible, resulting in a more socially desirable monopoly.

Then the obvious question arises. Which regulatory action is better for the regulator: to increase or to decrease the entry cost. It is obvious that the lower entry cost is better than the higher one, because decreasing f in the region below f^* will increase the welfare continuously above the level corresponding to prevented or non-feasible entry. The real limitation is the capability of the social planner to manipulate the entry cost.

6 Conclusions

Decisions of an incumbent firm having the possibility of carrying out an R&D project while facing the threat of superior external entry are determined by a combination of several factors: i) the degree of innovation, which determines the level of production cost reduction; ii) the uncertainty about R&D costs; iii) strategic decisions of the entrant; and iv) the size of the entry cost representing the barrier to entry.

We conclude that greater technical uncertainty positively affects the decision to start an R&D project. If there exist an opportunity to resolve uncertainty through exploratory research with an option to continue, higher initial uncertainty increases the positive effect of a success in the first-stage R&D, while the download risk in case of failure is limited. This finding illustrates the main difference between market payoff uncertainty, which induces a firm to wait for more information before undertaking the investment and technical uncertainty, which cannot be resolved just by simply waiting.

Our model demonstrates that the strategic effect of innovating is determined by both the degree of innovation and its entry prevention capability. Under inevitable entry the incentive for the incumbent to innovate is lower, because the strategic effect of innovation is too weak to provide the incumbent with an advantageous competitive position. Under preventable entry the incentive for the incumbent to innovate is higher compared to a monopoly situation, because of the strong strategic effect of innovating. Therefore, the threat of entry induces innovation only if the innovation can put the incumbent into an advantageous position.

The welfare considerations show that the incentives structure for innovation of the Incumbent is similar to that of the social planner only in the case of expensive innovations. In this case entry deterrence and entry blockade can be not only the optimal entry reaction strategy for the incumbent, but also the socially desirable outcome for the social planner. If R&D is cheap and innovation is easily accessible, the regulator is interested in a more competitive market structure. We also find that greater uncertainty about the outcome of exploratory research can provide greater expected welfare in the case of successful R&D. Therefore, the regulator has an incentive to stimulate riskier R&D projects.

The analysis of the effect of entry cost on welfare has shown that it is possible to have a scenario when a superior entry is optimal for the profit-maximizing firms, but is not socially desirable for the social planner. In such cases the regulator has an incentive to bring the entry cost lower to the level inducing the entry.

To summarize, we can conclude that the model presented in this paper has been proven capable of catching the complex relationships between factors of technological uncertainty and strategic interaction under superior entry threat, while preserving its simplicity and capability to produce analytically tractable implications. It is shown that besides capacity investment (Dixit and Stiglitz (1977)) and limit pricing, also R&D investment can be used as an entry deterrence strategy.

7 APPENDIX

Lemma 14 *If* $\Delta_{m,NPV} > 0$ *or* $\Delta_{m,NPV}^S > 0$*, then* $\Delta_{m,2}^S > 0$*.*

Proof.

$$\begin{aligned} \Delta_{m,NPV} &= \frac{\Delta_{m,2}^S}{2(1+r)} + \frac{\Delta_{m,2}^U}{2(1+r)} - \beta I, \\ \Delta_{m,NPV}^S &= \frac{\Delta_{m,2}^S}{2(1+r)} - \beta I. \end{aligned}$$

It holds that $\Delta_{m,2}^S > \Delta_{m,2}^U$, the lemma directly follows. ■

Proof. of Proposition 1

The dicision-making process discussed in Section 3 can be formalized in the following way.

The firm's production decision depends on the fact whether or not the new technology is developed. Therefore, we define the following decision variables:

1) i_1, which equals 1 if the firm decides to invest in the first-stage R&D and equals 0 otherwise;

2) i_2, which equals 1 if the firm decides to invest in the second-stage R&D and equals 0 otherwise.

To preserve of the sequencing properties of the decisions we assume that $i_1 = 1$ is a necessary, but not sufficient condition for $i_2 = 1$ to hold.

First, assume that the investment decision was taken ($i_1 = 1$) and the first stage R&D is completed.

If the first-stage R&D is successful, then $E(h|S) = -h$. Facing the second-stage R&D decision, the firm solves the following problem:

$$\max_{i_2} E(\pi_2|S) = \pi_m(K) + i_2 \frac{1}{r}\pi_m(0) + (1-i_2)\frac{1}{r}\pi_m(K) - i_2[(1-\beta)I - h]$$

The function $E(\pi_2)$ is linear in i_2. Therefore:

$$\begin{aligned} i_2 &= 1 = \arg\max_{i_2} E(\pi_2|S) \\ &\quad \text{only if} \\ \frac{\partial \pi_2}{\partial i_2} &= \frac{1}{r}\pi_m(0) - \frac{1}{r}\pi_m(K) - [(1-\beta)I - h] \geq 0, \end{aligned}$$

and

$$i_2 = 1 = \arg\max_{i_2} E(\pi_2|U), \text{ if } \Delta_{m,2}^S = \frac{1}{r}\Delta\pi_m - [(1-\beta)I - h] \geq 0.$$

Similarly, if the first-stage R&D is a failure, then $E(h|U) = h$ and

$$i_2 = 1 = \arg\max_{i_2} E(\pi_2|U), \text{ if } \Delta_{m,2}^U = \frac{1}{r}\Delta\pi_m - [(1-\beta)I + h] \geq 0.$$

It is also evident that if $\Delta^U_{m,2} > 0$, then $\Delta^S_{m,2} > 0$. Therefore the criterion value $\Delta^U_{m,2}$ is a decisive factor for second-stage R&D investment.

At the initial point in time the firm makes a decision about the first stage R&D and solves:

$$\begin{aligned}\max_{i_1} E(\pi_1) &= \frac{2+r}{1+r}\pi_m(K) + i_1 i_2 \frac{1}{r(1+r)}\pi_m(0) + (1 - i_1 i_2)\frac{1}{r(1+r)}\pi_m(K) - \\ & \quad i_1 \beta I - i_1 i_2 \frac{1}{1+r}[(1-\beta)I + E(h)], \\ & \quad \text{given} \\ i_2 &= \arg\max_{i_2} E(\pi_2).\end{aligned}$$

Similarly to the previous problem it holds that:

$$\begin{aligned} i_1 &= 1 = \arg\max_{i_1} E(\pi_1) \\ & \quad \text{only if} \\ \frac{\partial E(\pi_1)}{\partial i_1} &= i_2 \frac{1}{r}\Delta\pi_m - \beta I - i_2 \frac{1}{1+r}[(1-\beta)I + E(h)] \geq 0. \end{aligned}$$

If $\Delta^U_{m,2} > 0$, then $i_2 = 1 = \arg\max_{i_2} E(\pi_2)$ regardless of the outcome of the first-stage R&D. In this case $E(h) = 0$ and the following holds

$$i_1 = 1 = \arg\max_{i_1} E(\pi_1), \text{ if } \Delta_{m,NPV} = \frac{1}{r(1+r)}\Delta\pi_m - \beta I - \frac{1}{1+r}[(1-\beta)I] \geq 0. \tag{9}$$

If $\Delta^U_{m,2} < 0$ and $\Delta^S_{m,2} < 0$,then we always obtain $i_2 = 0 = \arg\max_{i_2} E(\pi_2)$ and the following holds

$$\text{if } i_2 = 0 = \arg\max_{i_2} E(\pi_2), \text{ then } i_1 = 0 = \arg\max_{i_1} E(\pi_1), \text{ because } \frac{\partial E(\pi_1)}{\partial i_1} = -\beta I < 0.$$

In the situation when $\Delta^U_{m,2} < 0$, but $\Delta^S_{m,2} > 0$ we obtain $i_2 = 1 = \arg\max_{i_2} E(\pi_2|S)$ and $i_2 = 0 = \arg\max_{i_2} E(\pi_2|U)$. Therefore it is required that the first-stage research is successful $(E(h|S) = -h)$ in order to continue the project, which yields

$$i_1 = 1 = \arg\max_{i_1} E(\pi_1|S), \text{ if } \Delta^S_{m,NPV} = \frac{1}{r(1+r)}\Delta\pi_m - \beta I - \frac{1}{2(1+r)}[(1-\beta)I - h] \geq 0. \tag{10}$$

Conditions (10) and (9) give us two specifications for the firm's pay-off maximization. It is evident that neither $\Delta_{m,NPV} \geq 0$, nor $\Delta^S_{m,NPV} \geq 0$ is the only pay-off maximization condition.

The relation between $\Delta_{m,NPV}$ and $\Delta^S_{m,NPV}$ is be described by the following expression:

$$\Delta_{m,NPV} = \Delta^S_{m,NPV} + \frac{\Delta^U_{m,2}}{2(1+r)} \tag{11}$$

If $\Delta^U_{m,2} > 0$, then $\Delta_{m,NPV} > \Delta^S_{m,NPV}$ and it is possible to have $\Delta_{m,NPV} > 0$, while $\Delta^S_{m,NPV} < 0$. On the other hand, if $\Delta^U_{m,2} < 0$, then $\Delta^S_{m,NPV} > \Delta_{m,NPV}$, and is is possible to have $\Delta^S_{m,NPV} > 0$, while $\Delta_{m,NPV} < 0$.

Specify:

$$\Delta_{m,1} = \max\{\Delta_{m,NPV}, \Delta^S_{m,NPV}\}.$$

Then the condition

$$i_1 = 1 = \arg\max_{i_1} E(\pi_1), \text{ if } \Delta_{m,1} = \max\{\Delta_{m,NPV}, \Delta^S_{m,NPV}\} \geq 0$$

allows to consider all the possible ways of obtaining a profitable R&D project, which proves statement i) of the above proposition.

Finally, using Lemma 14 we can show that if $\Delta_{m,1} \geq 0$, then it always holds that $\Delta^S_{m,2} > 0$, which finalizes the proof for statement ii) of the proposition. ∎

Proof. of Proposition 2

Define h^* such that $\Delta^S_{m,NPV} = 0$

$$\begin{aligned} h^* &= -\frac{1}{r}\Delta\pi_m + 2\beta(1+r)I + (1-\beta)I \\ &= -\frac{2K - K^2}{4r} + 2\beta(1+r)I + (1-\beta)I. \end{aligned}$$

Solving equation $h^* = (1-\beta)I$ for K gives us

$$K^*_m = 1 \pm \sqrt{1 - 8r\beta(1+r)I}.$$

The value of interest is $K^*_m = 1 - \sqrt{1 - 8r\beta(1+r)I}$, which lies inside the interval $[0, 1)$. This root takes up real values if the following condition holds:

$$I < \frac{1}{8r\beta(1+r)}. \tag{12}$$

The change in h^* corresponding to the change in K is:

$$\frac{\partial h^*}{\partial K} = \frac{K-1}{2r} < 0 \text{ for } K \in [0, 1).$$

For any $K < K^*_m$ we obtain $h^* > (1-\beta)I$. This means that for any feasible value of the mean preserving spread $h < (1-\beta)I$ the conditional investment gain $\Delta^S_{m,NPV}$ preserves its sign, i.e. an increase in uncertainty does not affect the firm's strategic choice.

In general it holds that:

$$\begin{aligned} \frac{\partial \Delta_{m,NPV}}{\partial h} &= 0, \\ \frac{\partial \Delta^S_{m,NPV}}{\partial h} &> 0. \end{aligned}$$

For values of $K > K_m^*$ we observe the following facts:

i) If $\Delta_{m,NPV} < 0$, then the research can start only if $\Delta_{m,1} = \Delta^S_{m,NPV} > 0$, which is positively affected by the increase in uncertainty and becomes positive as uncertainty exceeds h^*.

ii) If $\Delta_{m,NPV} > 0$, the project is launched regardless the level of uncertainty.

Once the project is started, the next relevant criterion is the project abandonment decision criterion $\Delta^U_{m,2}$, for which it holds that:

$$\frac{\partial \Delta^U_{m,2}}{\partial h} < 0.$$

which indicates the negative relationship between the technological uncertainty and the decision to continue research. ■

Proof. of Proposition 4.

Define h^* such that $\Delta^S_{inc,NPV} = 0$

$$h^* = -\frac{1}{r}\Delta\pi_{inc} + 2\beta(1+r)I + (1-\beta)I,$$

where

$$\Delta\pi_{inc}(f \leq f^{EP}, K \in [0, \frac{1}{2})) = \frac{K - K^2}{4.5}$$

and

$$\Delta\pi_{inc}(f \leq f^{EP}, K \in [\frac{1}{2}, 0)) = \frac{1}{9}.$$

Solving equation $h^* = (1-\beta)I$ for K gives us

$$K_m^* = \frac{1}{2} \pm \frac{\sqrt{1 - 18r\beta(1+r)I}}{2} \text{ for } f \leq f^{EP}, K \in [0, \frac{1}{2}),$$

and no solution for $f \leq f^{EP}, K \in [\frac{1}{2}, 1)$.

The value of interest is $K_m^* = \frac{1}{2} - \frac{\sqrt{1-18r\beta(1+r)I}}{2}$, which lies inside the interval $[0, \frac{1}{2})$ This root takes up real values if the following condition holds:

$$I < \frac{1}{18r\beta(1+r)}. \tag{13}$$

The change in h^* corresponding to the change in K is:

$$\frac{\partial h^*}{\partial K} = \frac{2K-1}{4.5r} < 0 \text{ for } K \in [0, \frac{1}{2}).$$

The conditions above yield the same path of proof as in the proof of Proposition (2) ■

Proof. of Proposition 5.

Define h^* such that $\Delta^S_{inc,NPV} = 0$

$$h^* = -\frac{1}{r}\Delta\pi_{inc} + 2\beta(1+r)I + (1-\beta)I,$$

where

$$\Delta\pi_{inc}(f \in (f^{EP}, F^{EP}), K \in [0, \frac{1}{2})) = \frac{5 + 16K - 16K^2}{36}$$

and

$$\Delta\pi_{inc}(f \in (f^{EP}, F^{EP}), K \in [\frac{1}{2}, 0)) = \frac{1}{4}.$$

Solving equation $h^* = (1-\beta)I$ for K gives us

$$K^*_m = \frac{1}{2} \pm \frac{\sqrt{2.25 - 18r\beta(1+r)I}}{2} \text{ for } f \in (f^{EP}, F^{EP}), K \in [0, \frac{1}{2}),$$

and no solution for $f \leq f^{EP}, K \in [\frac{1}{2}, 1)$.

The value of interest is $K^*_m = \frac{1}{2} - \frac{\sqrt{2.25 - 18r\beta(1+r)I}}{2}$, which lies inside the interval $[-0.25, \frac{1}{2})$ This root takes up real values if the following condition holds:

$$I < \frac{2.25}{18r\beta(1+r)}. \tag{14}$$

But we also must take into account the value $K^{EP}(f)$, which is the lowest level of innovation required for the entry prevention case to exist. Therefore we must consider only $K^*_m \in [K^{EP}(f), \frac{1}{2})$

The change in h^* corresponding to the change in K is:

$$\frac{\partial h^*}{\partial K} = \frac{8K - 4}{9r} < 0 \text{ for } K \in [K^{EP}(f), \frac{1}{2}).$$

The conditions above yield the same path of proof as in the proof of Proposition (2) ■

8 References

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