Corporate Investment under Uncertainty and Competition: A Real Options Approach

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Proefschrift

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Contents

Chapter 1

Introduction

On Saturday morning, 18th March 2000, Iridium LLC, "The World's First Hand-held Global Satellite Telephone and Paging Service Provider" shut down its operations.[1] The USD 5bn project, launched in November 1998 by the consortium of, among others, Motorola, Lockheed Martin, Sprint and Raytheon, turned out to be one of the biggest financial disasters in the history of technically advanced communication projects. Iridium was established to provide a telephone connection from each point on the globe, including the peaks of Himalaya, Amazonian forests and African deserts. In order to meet this objective Iridium placed 66 satellites on the orbit located 781 kilometers above the Earth. Via a portable phone that transmitted a signal directly to/from one of the satellites, the user was able to obtain a connection with any operating telephone network. The Iridium analysts believed that this additional flexibility offered by their new system would be highly appreciated by the target users and would compensate them for relatively high costs (USD 3,000 for a telephone and up to USD 7 per call per minute). The offer was initially directed to businesspeople, explorers, and wealthy travelers. The market potential was estimated as high. However, by the end of 1999 the firm managed to get only 50 thousand out of 700 thousand planned subscribers. The loss reported in the first quarter of 1999 alone amounted to USD 500m with miserable revenue of USD 1.5m. The book value of the company's debt already exceeded USD 4.4bn. Eventually, the investment made by Iridium LLC appeared to be far from what is in the finance textbooks meant by 'a value-creating project'.[2]

[1] The citation and the relevant data are based on the information available at the time at the website www.iridium.com.

[2] Finally, the assets of Iridium LLC have been purchased by a newly established firm Iridium Satellite LLC that continues to provide satellite telecommunication services (see also www.iridium.com).

1.1 NPV vs. Irreversibility and Uncertainty

What lesson for capital budgeting managers emerges from the Iridium case? Most of the finance textbooks present the net present value (NPV) rule as a valid criterion for evaluating capital investment projects. According to this rule, one needs to estimate the present value of the expected stream of cash flow generated by a new factory or a product line. Subsequently, the present value of the expenditures necessary to launch the factory or the product line has to be deducted from the discounted cash inflow. A positive difference (a positive net present value) implies that the project should be undertaken. In other words, NPV analysis suggests that minimal present value of cash inflows necessary for undertaking the project, V^*, must be equal to

$$V^* = I, \tag{1.1}$$

where I is the investment cost.

Of course, there are a lot of technical issues arising while calculating NPV of an investment project. The problems associated with determining the probabilities of particular scenarios, finding an appropriate discount rate or even with quantifying inflation and exchange rate risk need to be resolved (cf. Schockley and Arnold, 2002). However, the basic principle remains very simple: the sign of NPV determines whether a given project should be undertaken or not.

As pointed out by Dixit and Pindyck (1996), the idea of NPV is based on one of the following crucial and often overlooked assumptions:

- investment is either fully reversible (i.e. the invested money can be recovered if the uncertain market conditions turn out to be unfavorable ex post), or
- a firm is facing a now-or-never decision.

In most real-life situations, however, none of the above conditions is met.

The Iridium project, which comprised of a network of 66 satellites with virtually no alternative use, was totally irreversible (NASA or any other organization was not interested in acquiring the satellites after the bankruptcy was announced). Moreover, after the decision to stop the project, all the satellites ought to be destroyed at an additional cost of USD 30-50m over the subsequent two years. Another aspect of this particular investment is related to its timing. The decision to start the project was obviously not a now-or-never choice; some flexibility to postpone the project existed, especially until more reliable estimates concerning highly uncertain demand were available. And just the demand uncertainty constitutes the third distinct feature of the project.

1.2 Real Options and Investment under Uncertainty

The need of developing valuation models that are capable of capturing such features of investment as irreversibility, uncertainty as well as timing flexibility has resulted in a vast amount of literature on real options and investment under uncertainty.[3] In his seminal paper Myers (1977) draws attention to the optimal exercise strategies of real options as being the significant source of corporate value. Brennan and Schwartz (1985) are one of the first to adopt the modern option pricing techniques (see Black and Scholes, 1973, and Merton, 1973) to evaluate natural resource investments. The price of the commodity is used as an underlying stochastic variable upon which the value of the investment project is contingent. McDonald and Siegel (1986) derive the optimal exercise rule for a perpetual investment option when both the value of the project and the investment costs follow correlated geometric Brownian motions. The authors show that for realistic values of model parameters it can be optimal to wait with investing until the present value of the project exceeds the present cost of investment by a factor of 2. This reflects substantial value of waiting in the presence of irreversibility and uncertainty. Majd and Pindyck (1987) contribute to the literature by considering the effect of a time to build on the optimal exercise rule. The optimal choice of the project's capacity is analyzed by Pindyck (1988) and Dangl (1999). Dixit (1989) analyzes the effects of uncertainty on the magnitude of hysteresis in the models with entry and exit. Dixit and Pindyck (1996) present a detailed overview of this early literature and constitute an excellent introduction to the techniques of dynamic programming and contingent claims analysis, which are widely applicable in the area of real options and investment under uncertainty. An introduction to real options, which is closer in the spirit to the financial options theory, is presented by Trigeorgis (1996).

The 1990s brought a vast number of applications of the existing real options framework. They include, among others, managing R&D projects (Pennings, 1998), natural resources investment (Trigeorgis, 1990), real estate (Williams, 1993), energy (Kulatilaka, 1993, and Pindyck, 1993), aerospace industry (Sick, 1999), banking (Panayi and Trigeorgis, 1998), technology adoption (Grenadier and Weiss, 1997), merger policy (Mason and Weeds, 2002) and biotechnology sector (Ottoo, 1998, and Woerner, 2001).[4] Shackleton and Wojakowski (2001) analyze a finite-maturity real

[3] A reader being unfamiliar with this approach is referred to the Appendix where a standard real option model is analyzed.

[4] For a variety of real options applications see the 1998 special issue of the *Quarterly Review of Economics and Finance*, 38, entitled: "Real Options: Developments and Applications" (ed. G.E. Pinches). The collections of papers compiled by Grenadier (2000), Brennan and Trigeorgis (2000),

option to switch among two streams of revenues when the switching is costless.

Some recent contributions relax the assumption concerning perfect information about the project's value and introduce learning effects. Thijssen et al. (2001) analyze the optimal investment timing when the information about the project's value is driven by a Poisson process. Bernardo and Chowdhry (2002) analyze optimal option exercise policy when the firm is learning about its capabilities by applying the filtering approach of Liptser and Shiryayev (1978). Finally, Decamps et al. (2001) investigate the optimal investment rule when the firm observes the value of the process (market index) which is imperfectly correlated with unobservable demand process. In Chapter 2 we introduce incomplete information and learning about the firm's investment cost.

The empirical literature on real options is quite limited but growing, as the project level data become more easily available. The classic contributions include Paddock et al. (1988) who analyze the valuation of offshore petroleum leases, Quigg (1993) investigating the behavior of real estate prices in Seattle, and Berger et al. (1996), who on the basis of the differences between the firms' market values and their discounted cash flow (DCF) valuations try to estimate the value of the option to abandon operations.

1.3 Imperfect Competition

The extensive process of deregulation taking place in the last decade, combined with a wave of mergers and acquisitions, has resulted in an oligopolistic structure of a large number of sectors. A shift towards such a structure takes place not only in traditional regulated markets (telecommunications, energy, transportation) but also in more competitive industries (fast-moving consumer goods, car manufacturing, pharmaceuticals). Imperfect competition in the firm's product market requires that strategic interactions with other firm(s) are taken into account. The gap between capital budgeting and strategic planning has already been recognized by Myers (1987) and has been confirmed by Zingales (2000).

Real option models taking into account imperfect competition among the firms are based on several contributions on timing games within the area of non-cooperative game theory. The first model describing the optimal timing of entry has been presented by Reinganum (1981). In this paper, the author derives the optimal strategies of the leader and the follower and shows that the leader realizes a positive relative surplus.

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and Schwartz and Trigeorgis (2001) are also of interest.

This result is due to the assumption that firms use open-loop strategies, i.e. their roles are predetermined. We use the same assumption in Chapter 5. The problem of endogenous selection mechanism has been addressed by Fudenberg and Tirole (1985). In their set-up, in which the firms play closed-loop strategies, the roles of the firms are not predetermined and, as a consequence, the firms' strategies are history-dependent (time-consistent). Fudenberg and Tirole (1985) show that there is rent equalization of the leader and the follower, which is the result of the preemption game played by the firms. This framework is applied in Chapters 3 and 4.

The first model that combines these game-theoretical insights with the optimal option exercise rule is Smets (1991). He analyzes the trade-off between the value of waiting with constructing a production facility in an emerging economy and the threat of being preempted by a competitor. Grenadier (1996) applies a version of this model to analyze an increase in construction activity during market downturn. Huisman and Kort (1999) present an endogenous selection mechanism based on which the roles of the leader and the follower are determined. Mason and Weeds (2003) extend this framework and allow for positive externalities among the competitors. The latter feature allows them for obtaining a negative relationship between uncertainty and the leader's investment threshold. Boyer et al. (2002) develop a general model of evolution of duopolists' capacities, which nests, as its special cases, the new market model and the model with firms already competing in the product market. Applications of strategic real option games in the internet and aircraft manufacturing sectors have been presented by Perotti and Rossetto (2000), and Shackleton et al. (2003), respectively. Discrete time strategic real option models include Smit and Ankum (1993), Smit and Trigeorgis (1998), and Kulatilaka and Perotti (1998).

Games of incomplete information constitute a fruitful avenue of contemporary strategic real options research. Grenadier (1999) considers informational cascades in a situation where multiple agents optimally exercise their options not only on the basis of their private noisy signals but also taking into account the actions of the others. Decamps and Mariotti (2000) and Thijssen et al. (2001) consider games in which firms learn about the profitability of the market by observing their competitors. Lambrecht (2000) analyzes optimal strategic investment in patents when the type of the competitor is unknown and shows that it may be optimal to let patents "sleep" for some time before the commercialization phase takes place. Finally, Lambrecht and Perraudin (2003), develop a model of a preemption game under incomplete information, in which the payoff of the follower drops to zero after the investment of the leader.

Another class of real options contributions are the models of industry equilibrium. These models include Williams (1993), Leahy (1993), and Grenadier (2002) with an all-equity financing assumption and Fries et al. (1997) with a debt and equity financing assumption.

1.4 Debt Financing

There are two types of agency problems that result in a suboptimal investment policy in the presence of debt financing. First, as shown by Jensen and Meckling (1976), debt financing results in the owner-manager shifting towards more risky projects and, as an effect, in the expropriation of the debtholders' wealth. Another effect of debt on the firm's investment policy has been discussed by Myers (1977). It is shown that since investment is associated with a wealth transfer from the equityholders to the debtholders, some of the good investment opportunities (those whose NPV does not fully compensate for the wealth transfer) will expire unexercised.

The impact of debt financing on investment has been analyzed in a dynamic real options framework by a number of authors. Mello and Parsons (1992) analyze the binary decision to abandon or resume a production process and estimate the agency costs of debt. For reasonable parameter values they obtain that a suboptimal operating policy lowers the value of the firm by more than 4% of the total debt value. The magnitude of the agency costs has also been estimated by Mauer and Ott (2000), who essentially develop a dynamic version of the model of Myers (1977). For some scenarios they obtain that the agency cost of debt associated with underinvestment amount to up to 3% of the debt value. Titman and Tsyplakov (2002) show in a dynamic model with a continuous investment flow that an equity value-maximizing firm has a lower investment rate than a firm maximizing the value of all its claims. A similar result is obtained by Moyen (2002). Lambrecht (2001), and Khadem and Perraudin (2003) build upon strategic models in the spirit of Brander and Lewis (1986), and Maksimovic (1995), and perform a dynamic analysis of exit strategies where duopolists are financed by equity and debt. Their theoretical results support the empirical evidence that "the fittest and the fattest" firms, i.e. those with the highest profitability and the highest interest coverage, are more likely to remain in the market (cf. Zingales, 1999). However, in equilibrium an exit of the less levered firm occurs with positive probability. Finally, Fries et al. (1997) investigate the optimal capital structure in the industry equilibrium taking into account market volatility and possibility of a free entry. A common feature of all the above mentioned contributions assumed a single type of non-renegotiable debt. As we show in Chapter 6, relaxing this assumption may lead to new interesting insights.

1.5 Outline of the Thesis

The thesis consists of the introduction, which is followed by five chapters. In Chapters 2, 3, 4, and 5, the investment decisions of all-equity financed firms are analyzed. Consequently, all the investment decisions are made optimally as to maximize the value of the firm. In Chapter 6, the impact of debt financing on investment is considered.

In Chapter 2 we develop a non-strategic model in which the impact of a policy change on investment behavior is analyzed. Withdrawal of the investment tax credit, or a change in the preferential tax treatment of foreign investor constitute some examples of the policy change that is of our interest. The policy change is modeled as an upward jump in the effective investment cost (cf. Hassett and Metcalf, 1999, for a tax credit interpretation) and is triggered by the value of the project reaching an upper barrier. The firm has incomplete information concerning the trigger value of the process for which the jump occurs and updates its beliefs according to Bayes' rule. The uncertainty concerning the moment of the change can be explicitly accounted for by changing the variance parameter of the underlying probability distribution. The optimal investment threshold maximizing the value of the firm is derived and non-monotonicity of this threshold in trigger value uncertainty is shown.

Chapter 3 contains an analysis of a firm's decision to replace an existing production facility with a new, more cost-efficient one. Kulatilaka and Perotti (1998) find that, in a two-period model, increased product market uncertainty could encourage the firm to invest strategically in the new technology. We extend their framework to a continuous-time model and show that, in contrast with the two-period model, more uncertainty always increases the expected time to invest. Furthermore, it is shown that under increased uncertainty the probability of the optimal production facility replacement within a given time period always decreases for time periods longer than the time to reach the optimal Jorgensonian threshold calculated for the deterministic case. For smaller time periods there are contrary effects so that the relationship between uncertainty and the probability of investing is in this case humped (cf. Sarkar, 2000, who first documents the non-monotonicity of the investment-uncertainty relationship in a real options framework).

Chapter 4 considers the impact of investment cost asymmetry on the value of the firm and optimal real option exercise strategies of firms under imperfect competition (cf. Grenadier, 1996, for a limiting case with identical firms). Both firms have an opportunity to invest in a project enhancing *ceteris paribus* the profit flow. We show that three types of equilibria exist (which extends, e.g., Huisman, 2001, Ch. 8, who obtains two types of equilibria in a new market model). Furthermore, we derive critical levels of cost asymmetry separating the equilibrium regions. The presence of strategic interactions leads to counterintuitive results. First, a marginal increase in the investment cost of the firm with the cost disadvantage can increase this firm's own value. Second, such a cost increase can result in a decrease in the value of the competitor. Subsequently, we discuss the welfare implications of the optimal exercise strategies and show that firms being identical can result in a socially less desirable outcome than if one of the competitors has a significant investment cost disadvantage. Finally, we prove that profit uncertainty always delays investment, even in the presence of a strategic option of becoming the first investor.

Chapter 5 addresses the issue of the value of flexibility in quality choice (cf. Pennings, 2002, for a model addressing similar issues but using a different model formulation). Firms decide about quality of their products when they enter the market upon incurring a sunk cost. Flexibility in quality choice induces *ceteris paribus* earlier investment, and the value of flexible quality increases with demand uncertainty. We find that the possibility of competitive entry more than doubles the relative value of flexibility. We also show that flexible quality serves as an entry deterrent control, while it can still be set at the optimal monopoly level. Furthermore, we extend the theory of strategic real options, from which it is known that the follower's investment timing is irrelevant for the decision of the leader if the roles of the firms are predetermined. The addition of a second control (quality) results in the leader's investment timing being influenced by the follower's expected entry. Finally, we show that the follower can be driven out of the market due to an "aggressive" quality choice of the leader in high states of demand.

Chapter 6 analyzes the firm's optimal investment and liquidation policy in the presence of debt financing and the equityholders' option to renegotiate the debt. We show that the presence of the renegotiation option ("soft debt") exacerbates the underinvestment problem described by Myers (1977). The detrimental impact of the renegotiation option on the investment policy results from the fact that in the presence of the renegotiation option the wealth transfer to the debtholders, which occurs upon investment, is greater. This is due to a significant reduction in the probability of strategic default occurring upon undertaking the investment project. Furthermore, we find that the liquidation policy in the presence of debt differs from the optimal liquidation policy under all-equity financing. Even after removing the effects of the tax shield by excluding taxes, it holds that the liquidation policy is affected by the second-best investment policy, thus liquidation occurs inefficiently early. Also, the impact of a growth opportunity on the optimal bankruptcy and renegotiation timing is analyzed and it is shown that high shareholders' bargaining power combined with the presence of growth options can make strategic default more likely.

1.6 Appendix: Standard Real Options Model

In this section we present the standard investment model as described by McDonald and Siegel (1986), and extensively analyzed by Dixit and Pindyck (1996). The basic problem is to find the optimal timing of an irreversible investment, I, given that the value of the investment project follows a geometric Brownian motion (GBM)

$$dV(t) = \alpha V(t)\,dt + \sigma V(t)\,dw(t), \tag{1.2}$$

where parameter α denotes the deterministic drift parameter, σ is the instantaneous standard deviation, and dw is the increment of a Wiener process.

Technically speaking, the uncertainty in the model is described by a complete filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in(0,\infty)}, \mathbb{P}\right)$, where Ω is the state space, $\mathcal{F}$ is the σ-algebra representing measurable events, and $\mathbb{P}$ is the actual probability measure. The filtration is the augmented filtration generated by the Brownian motion and satisfies the usual conditions.[5] The deterministic riskless interest rate is r and the drift rate α satisfies $\alpha < r$ so that finite valuations can be obtained. The firm is risk-neutral and maximizes the value of the investment option, $F(V)$, by choosing the threshold value of V at which the project is undertaken.

Since there are no intermediate payoffs to the holder of the investment option, the Bellman equation in the continuation region (i.e. before exercising the option) can be written as

$$rFdt = E\left[dF(V)\right]. \tag{1.3}$$

Equation (1.3) means that for a risk-neutral firm, the expected rate of change in the value of the investment opportunity over the time interval dt equals the riskless rate. Applying Itô's lemma to the RHS of (1.3), and dividing both sides of the equation by dt results in the following ordinary differential equation (ODE):

[5] A filtration $\{\mathcal{F}_t\}$ satisfies the usual conditions if it is right continuous and $\mathcal{F}_0$ contains all the $\mathbb{P}$-null sets in $\mathcal{F}$ (see Karatzas and Shreve, 1991, p. 10).

$$rF = \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + \alpha V \frac{\partial F}{\partial V}. \tag{1.4}$$

The general solution to (1.4) has the following form:

$$F(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2}, \tag{1.5}$$

where A_1 and A_1 are constants, and

$$\beta_{1,2} = -\frac{\alpha}{\sigma^2} + \frac{1}{2} \pm \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{1.6}$$

Moreover, it holds that $\beta_1 > 1$ and $\beta_2 < 0$. In order to find the value of the investment option, $F(V)$, and the optimal investment threshold, V^m, the following boundary conditions are applied to (1.5):

$$F(V^m) = V^m - I, \tag{1.7}$$

$$F'(V^m) = 1, \tag{1.8}$$

$$F(0) = 0. \tag{1.9}$$

Conditions (1.7) and (1.8) are called the value-matching and the smooth-pasting conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition (1.9) ensures that the investment option is worthless at the absorbing barrier $V = 0$. Consequently, it implies that $A_2 = 0$.

Substitution of (1.5) into (1.7)-(1.9) and some algebraic manipulation yield the value of the optimal investment threshold:

$$V^m = \frac{\beta_1}{\beta_1 - 1} I. \tag{1.10}$$

Since $\beta_1 > 1$, the optimal investment threshold is strictly larger than 1 (cf. NPV rule given by (1.1)). This reflects the value of waiting associated with the uncertainty of the project's value and the irreversibility of the investment decision. The value of the option to invest, $F(V)$, is given by

$$F(V) = (V^m - I)\left(\frac{V}{V^m}\right)^{\beta_1}, \tag{1.11}$$

where $V^m - I$ is just a NPV of the project at the moment of undertaking the investment. The second factor is a stochastic discount factor which reflects the present value of $1 received when the cash flow process hits the investment threshold V^m.

The value of the optimal investment threshold is positively related both to the volatility of the project's value as well as to its growth rate (the higher σ and α are, the higher V must be reached for the project to be undertaken). $F(V)$ increases with the volatility of the value of the project (β_1 is a decreasing function of σ and F is decreasing with β_1) which results from the convex payoff of the investment opportunity. Moreover, F is increasing with the growth rate, α, since the effective discount rate of future cash flow decreases linearly with α.

Finally, the expected time to hit the investment threshold V^m starting from level V, denoted by T^m, equals[6]

$$E\left[T^m\right] = \begin{cases} -\frac{1}{\alpha-\frac{1}{2}\sigma^2}\ln\left(\frac{V}{V^m}\right) & \text{for} \quad \sigma^2 < 2\alpha, \\ \infty & \text{for} \quad \sigma^2 \geq 2\alpha. \end{cases} \tag{1.12}$$

Since the expected time to reach the investment threshold is infinite for a sufficiently high volatility of process (1.2), another measures are often used to characterize investment timing. They include the probability of investing within a certain time horizon (cf. Sarkar, 2000, and Chapters 3 and 6 of this thesis), and the median time to invest (cf. Grenadier, 1996).

[6]For a derivation of the probability distribution of the first passage time see Harrison (1985) for a formal exposition and Dixit (1993) for a heuristic approach.

Chapter 2

Discrete Change in Investment Cost

2.1 Introduction

Corporate investment opportunities may be represented as a set of (real) options to acquire productive assets. In the literature it is widely assumed that the present values of cash flows generated by these assets are uncertain and that their evolution can be described by a stochastic process. Consequently, identification of the optimal exercise strategies for real options plays a crucial role in capital budgeting and in the maximization of a firm's value.

So far, the real options literature provides relatively little insight into the impact of structural changes of the economic environment on the investment decisions of a firm. The existing papers (see overview in Chapter 1) mainly consider continuous changes in the value of relevant variables. Most of the time, this results in the assumption that the entire uncertainty in the economy can be described by a geometric Brownian motion process.

It is often more realistic to model an economic variable as a process that makes infrequent but discrete jumps.[1] In such cases use is made of a Poisson (jump) process. An interesting application is provided by Hassett and Metcalf (1999), who analyze the impact of an expected reduction in the investment tax credit. In their setting a Poisson process describes the changes in the tax regime that affect the value of the investment opportunity. Within such a framework the implicit assumption is made that the firm has virtually no information about the mechanisms governing the shocks in the economy.

[1]For instance, recent tax debates across Europe are a significant source of uncertainty associated with discountinuous changes in the economic environment.

When a change in the economic environment reflects a new policy implemented by the authority, it may be more realistic to assume that the firm has some conjecture about the expected moment of the change. Referring to the example of the investment tax credit, the firm typically expects the reduction to be imposed when the economy is booming and an active pro-investment policy is no longer needed or desired. Conversely, applying the Poisson based methodology is equivalent to assuming that it is time itself and not the state of economic environment that governs the change.

Moreover, the firm can to some extent assess the precision of its conjecture concerning the moment of change, i.e. the variance of the estimate of the timing of the future event. A Poisson based approach does not allow for including this type of uncertainty in the analysis since it entails a single parameter characterizing the arrival rate of the jump. Consequently, such a modelling approach lacks degrees of freedom necessary for capturing both the expectation and the precision of this expectation.

In this chapter we propose a method to model the impact of a policy change on the investment strategy of the firm that takes into account the type of information possessed by the firm while making the investment decision. In our approach the subjective expectation concerning the moment of the change as well as the level of imprecision of such a conjecture serve as input parameters. We model the policy change as being triggered by a sufficiently high realization of a stochastic process related to the value of the investment opportunity. This, for instance, reflects the fact that - as we already argued - a tax credit reduction is more likely to occur when the economy is booming. Hence, the moment of the reduction depends on the state of the economy. This is in contrast with the models based on the Poisson process where the probability of the change is constant over time.[2]

There are other economic situations in which it is realistic to impose a certain relationship between the occurrence of the shock and the state of the economy. A foreign direct investment decision to purchase a privatized enterprise where the local government may increase the offering price after the performance of the enterprise improves, can also be perceived as an option with an embedded risk of an increase in the strike price. A non-exclusive investment opportunity for which a competitive bid can be expected can serve as another example.[3]

[2]Hassett and Metcalf (1999) try to correct this by letting the arrival rate depend on the output price. But still it is then possible that an investment subsidy is reduced for low output prices, while the subsidy was maintained under high output prices. This kind of inconsistency in the authority's behavior is no longer possible under our approach.

[3]See Smets (1991) and Cherian and Perotti (1999) for a discussion of the effects of strategic interactions and political risk.

We consider the possibility of an upward jump in the (net) investment cost. This jump is caused, for instance, by the reduction of an investment tax credit. It occurs at the moment that an underlying variable reaches a certain trigger. Here, the underlying variable is the value of the investment project. The firm is not aware of the exact value of the trigger but it knows the probability distribution underlying the trigger. Taking into account consistent authority behavior, the firm knows that a jump will not occur as long as the current value of the variable remains below the maximum that this variable has attained in the past. When the underlying variable reaches a new maximum and still the jump does not occur, the firm updates its conjecture about the value of the barrier.

Consequently, our objective is to determine the optimal timing of an irreversible investment when the investment cost is subject to change and the firm has incomplete information about the moment of the change. It is clear that the value of the investment opportunity drops to zero at the moment that the investment cost jumps to infinity. However, we mainly consider scenarios where the cost of investment is still finite after the upward jump occurred. In this respect this work generalizes Berrada (1999), Schwartz and Moon (2000), and Lambrecht and Perraudin (2003), in which the value of the project drops to zero at the unknown point of time.[4]

Our main results are the following. An equation is derived that implicitly determines the value of the project at which the firm is indifferent between investing and refraining from the investment. This value is the optimal investment threshold and it is shown that this threshold is decreasing with the hazard rate of the cost-increase trigger. For the most frequently used density functions it holds that, for a given value of the project, the hazard rate first increases and then decreases with trigger value uncertainty. This leads to the conclusion that the investment threshold decreases with the *trigger value* uncertainty when the uncertainty is low, while it increases with uncertainty for high uncertainty levels. Hence, for a policy maker interested in accelerating investment, an optimal (strictly positive) level of the trigger value uncertainty can be identified which is the level corresponding to the minimal investment threshold. Furthermore, it is shown that the uncertainty concerning the *magnitude* of the change delays investment. This implies that an effective policy stimulating early investment should minimize the investors' uncertainty about the size of the expected change.

In Section 2.2 the model with the investment cost jump resulting from a policy change is introduced. Section 2.3 provides the major results and Section 2.4 contains a

[4]However, the unknown trigger in Lambrecht and Perraudin (2003) is chosen endogenously by the firm's competitor.

numerical analysis including some comparisons with Poisson based models. Section 2.5 extends the model to allow for a stochastic size of the jump in the cost. In Section 2.6 we present the implications of our model for the authority that considers an investment tax credit policy change, and Section 2.7 concludes.

2.2 Framework of the Model

In this section we develop the model that allows for incorporating the impact of the expected policy change on the firm's investment strategy. The value of the investment project follows a geometric Brownian motion

$$dV(t) = \alpha V(t)\, dt + \sigma V(t)\, dw(t), \tag{2.1}$$

where parameter α denotes the deterministic drift parameter, σ is the instantaneous standard deviation, and dw is the increment of a Wiener process. The riskless rate is r and it holds that $\alpha < r$. The firm is assumed to be risk-neutral and it maximizes the value of the investment option, $F(V)$. If the value of the investment project reaches a critical level, a change in the value of a certain policy instrument is imposed and, as a result, an effective increase in the investment cost occurs.[5] This instrument can be interpreted, among others, as a reduction in the investment tax credit, an increase in the cost of capital via lending rates or an increase in the offering price for a privatized enterprise. Allowing for a broader interpretation, an arrival of a competitive firm offering a higher bid for a particular project belongs to the set of potential sources of the investment cost shock as well.

We denote by V^* such a realization of the process for which the new policy is imposed and the investment cost changes from I_l to I_h, where $I_h > I_l$. At this stage we assume that I_h is deterministic. Later we consider I_h to be stochastic and discuss implications of such an extension. The firm does not know the value of V^* but knows only its cumulative density function, $\Psi(V^*)$. $\Psi(\cdot)$ is continuous and twice differentiable everywhere in the interior of its domain. To provide a simple interpretation, we assume that $\Psi(\cdot)$ is completely defined by its first two moments and is time-independent. Consequently, if the investment cost has not increased by time τ, while $\widehat{V}$ is the highest realization of the process so far, the cost will not increase at any $u > \tau$ as long as

[5] If, instead, a downward change in investment cost is considered, the same solution methodology can be applied as in the remainder of the paper. Consequently, a unique realization of the underlying process has to be found for which the marginal cost of waiting beyond the optimal investment threshold equals the benefit of waiting associated with the expected decrease in the investment cost.

$V(t) \leq \widehat{V}$ for all $t \leq u$. Hence, the probability of the jump in investment cost is a function of V alone.

In order to restrict our analysis to the most interesting case, we impose the following assumptions on the values of the variables used in the model:

$$\begin{cases} \overline{V} > V(0) & (i) \\ \underline{V} < \frac{\beta_1}{\beta_1 - 1} I_l & (ii) \\ 1_{\{V^* < V_h\}} (V_h - I_h) \left(\frac{V^*}{V_h}\right)^{\beta_1} < V^* - I_l, & (iii) \end{cases} \tag{2.2}$$

where $\underline{V}$ and $\overline{V}$ are the lower and the higher bound of the domain of $\Psi(\cdot)$, respectively. $V(0)$ denotes the initial value of the project, β_1 is given by

$$\beta_1 = -\frac{\alpha}{\sigma^2} + \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{2.3}$$

and V_h ($\equiv \beta_1 I_h / (\beta_1 - 1)$) is the unconditional optimal investment threshold corresponding to the cost I_h.[6] Assumptions (i) and (ii) ensure that the problem is relevant, i.e. that the policy change has not occurred yet and that there is a positive probability that the change will take place before the optimal threshold corresponding to I_l is reached. Assumption (iii) states that ex post it is never optimal to wait with investing until the upward change in cost occurs.

2.2.1 Value of the Investment Opportunity

Since the value of the project that triggers the increase in the investment cost is not known beforehand, two scenarios are possible. In the first scenario the investment occurs *before* the change in the investment cost, and in the second scenario the investment takes place *after* the upward change. Consequently, the value of the investment opportunity reflecting the structure of the expected payoff, has the following form:

$$\begin{aligned} F_s(V, \widehat{V} | I &= I_l) = p_s(\widehat{V}) E\left[(V(T_s) - I_l) e^{-rT_s}\right] + \\ &+ \left(1 - p_s(\widehat{V})\right) E\left[(V(T_h) - I_h) e^{-rT_h}\right], \end{aligned} \tag{2.4}$$

where $p_s(\widehat{V})$ is the conditional (on the highest realization of V, $\widehat{V}$) probability that the investment cost will not increase *before* the investment is made optimally, and T_s and T_h denote the first passage time corresponding to the optimal investment threshold

[6] 1_B denotes an indicator function of B such that $1_B(x) = \begin{cases} 1 & x \in B \\ 0 & x \notin B \end{cases}$.

at the low and at the high cost, respectively. After rearranging and including these expectations, we obtain the following maximization problem that allows for finding the optimal investment threshold:

$$\begin{aligned} F_s(V,\widehat{V}|I &= I_l) = \max_{V_s}\left[(V_s - I_l)\left(\frac{V}{V_s}\right)^{\beta_1}\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}+\right. \\ &\left.+(V_h - I_h)\left(\frac{V}{V_h}\right)^{\beta_1}\left(1-\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}\right)\right]. \end{aligned} \tag{2.5}$$

V_s is the optimal investment threshold in case the investment takes place before the change in cost, and $\widehat{V}$ is the highest realization of the process so far. Hence, the ratio $(1-\Psi(V_s))/(1-\Psi(\widehat{V}))$ is the probability that the jump in the investment cost will not occur by the moment V is equal to V_s, given that the shock has not occurred for V smaller than $\widehat{V}$. Equation (2.5) is therefore interpreted as follows: the value of the investment opportunity is equal to the weighted average of the values of two investment opportunities. They correspond to the investment cost I_l and I_h, respectively, given that the investment is made optimally (at V_s if the cost is still equal to I_l and at V_h if the upward change has already occurred).[7]

The value of the investment opportunity depends on the highest realization of the process, $\widehat{V}$. A higher $\widehat{V}$ (thus a one closer to V_s) implies a lower probability of the cost-increase trigger falling into the interval $(\widehat{V},\, V_s)$ and, as a consequence, a higher probability of making the investment at the lower cost, I_l. In order to calculate the value of the investment opportunity, we first need to establish the value of V_s by solving the maximization problem.

2.2.2 Optimal Investment Threshold

The optimal investment threshold, V_s, is determined by maximizing the value of the investment opportunity or the RHS of the Equation (2.5).

[7]It is worth pointing out that for $I_h \to \infty$ the value of the investment opportunity boils down to:

$$F_s(V,\widehat{V}|I=I_l) = \max_{V_s}(V_s - I_l)\left(\frac{V}{V_s}\right)^{\beta}\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}, \tag{2.6}$$

which directly corresponds to the result of Lambrecht and Perraudin (2003). In the other limiting case, i.e. for $I_h \to I_l$, the value of investment opportunity converges to

$$F_s(V,\widehat{V}|I=I_l) = (V_l - I_l)\left(\frac{V}{V_l}\right)^{\beta} \tag{2.7}$$

which is the formula obtained by McDonald and Siegel (1986).

Proposition 2.1 *Under the sufficient condition that*

$$V_s \left. \frac{\partial h(V)}{\partial V} \right|_{V=V_s} + h(V_s) \geq 0, \tag{2.8}$$

the investment is made optimally at V_s which is the solution to the following equation:

$$h(V_s)V_s^2 + (\beta_1 - 1)V_s - (V_s h(V_s) + \beta_1)I_l - h(V_s)\frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta}} \frac{V_s^{\beta_1 + 1}}{I_h^{\beta_1 - 1}} = 0, \tag{2.9}$$

where $h(x) = \frac{\psi(x)}{1-\Psi(x)}$ denotes the hazard rate and $\psi(x) \equiv \frac{\partial \Psi(x)}{\partial x}$.[8]

Proof. See the Appendix. ∎

A sufficient condition for (2.8) to hold is that the hazard rate has to be non-decreasing in V.[9] Condition (2.8) is satisfied for most of the common density functions as, e.g., exponential, uniform and Pareto.[10]

2.3 Solution Characteristics

In this section we analyze how the optimal threshold is affected by changes in the parameters characterizing the dynamics of the project value. In particular, we determine the direction of the impact of the project value uncertainty and of the changes in the investment costs under both policy regimes. Subsequently, we examine how the uncertainty concerning the moment of imposing the change influences the firm's optimal investment rule.

2.3.1 Changing the Parameters of the Investment Opportunity

We are interested in how potential changes in the characteristics of the investment opportunity influence the optimal investment rule. For this purpose we formulate the following proposition.

---

[8]In our case, the hazard rate has the following interpretation. The probability of the upward change in the investment cost occurring during the nearest increment of the value of the project, dV, (given that the cost-increase has not occurred by now) is equal to the appropriate hazard rate multiplied by the size of the value increment, i.e. to $h(V;\cdot)dV$.

[9]More precisely, the elasticity of the hazard rate with respect to the value of the process evaluated at the optimal investment threshold has to be larger than or equal to -1.

[10]In fact, the hazard rate based on the Pareto function is decreasing at the order of $1/x$ and the property (2.8) is still met.

Proposition 2.2 *The effects on the investment threshold level of the changes in the different parameters are as follows:*

$$\frac{dV_s}{dI_l} > 0, \tag{2.10}$$

$$\frac{dV_s}{dI_h} < 0, \tag{2.11}$$

$$\frac{dV_s}{d\beta_1} < 0, \tag{2.12}$$

$\forall I_l, I_h$ *satisfying* $0 < I_l < I_h$, $\forall \beta_1 \in (1, r/\alpha)$ *if* $\alpha > 0$ *and* $\forall \beta_1 \in (1, \infty)$ *if* $\alpha \leq 0$.

Proof. See the Appendix. ■

Consequently, the optimal threshold (*ceteris paribus*) increases with the initial investment cost and decreases with the magnitude of the potential cost-increase as well as in the parameter β_1. The latter implies that the threshold increases with uncertainty of the value of the project and decreases with the wedge between interest rate and the project's growth rate.

2.3.2 Impact of Policy Change

The optimal investment rule depends not only on the characteristics of the project itself but also on the firm's conjecture about the probability distribution underlying the expected policy change. The parameters of this distribution can be influenced by actions of the authority. For instance, an information campaign about the expected changes in the investment tax credit leads to a reduction of the variance (often to zero) of the distribution underlying the value triggering the change. Therefore, it is important to know how changes in the uncertainty related to the project value triggering the jump in the investment cost influence the firm's optimal investment rule. Knowing that the firms are going to act optimally, the authority can implement a desired policy, which is, for instance, accelerating the investment expenditure, by changing the level of the firms' uncertainty about the tax strategy. We come back to this point in Section 2.6, where policy implications for the authority are considered.

Hazard Rate

The hazard rate of the arrival of the cost-increase trigger is one of the basic inputs for calculating the optimal investment threshold. Although it is exogenous to the firm, it may well be controlled by another party such as the authority. Here, we determine the impact of its change on the firm's investment rule. Later, we discuss some of the policy implications of the obtained result.

From (2.9) the following result can be obtained.

Proposition 2.3 *The optimal investment threshold is decreasing with the corresponding hazard rate, i.e. the following inequality holds:*

$$\left.\frac{dV_s}{dh\,(V)}\right|_{V=V_s} < 0. \tag{2.13}$$

Proof. See the Appendix. ■

This result implies that an increasing incremental probability of the jump leads to an earlier optimal exercise. The intuition is quite simple: an increasing probability of a partial deterioration of the investment opportunity after a small appreciation in the project value reduces the value of waiting.

Furthermore, (2.13) implies that for any parameter of the density function underlying the jump, θ, the following condition holds:

$$\forall\theta \in \{a, b\} \quad \text{sgn}\left.\frac{\partial h\,(V)}{\partial\theta}\right|_{V=V_s} = -\text{sgn}\,\frac{dV_s}{d\theta}. \tag{2.14}$$

Using (2.14) we can establish how the investment threshold is affected by changes in the parameters of the distribution function underlying the occurrence of the jump.

Trigger Value Uncertainty

Now the aim is to analyze how the optimal investment threshold is affected by uncertainty related to the value of the cost-increase trigger. To do so, due to (2.14), we only need to establish the sign of the relationship between the hazard rate and the uncertainty related to the value of the trigger. We measure the trigger-value uncertainty by applying a mean-preserving spread (see Rotschild and Stiglitz, 1970)

If the cost-increase trigger, V^*, is known with certainty, the investment is made optimally at an infinitesimal instant before V^* is reached. At this point, the hazard rate is zero (there is no risk that the cost increases before this trigger is reached). As the uncertainty marginally increases, the hazard rate is affected by: 1) the value of the density function underlying the trigger, denoted by $\psi(V^*)$, and 2) a change in the value of the survival function, $1 - \Psi(V^*)$. It can be shown that, for the most frequently used density functions, such as normal, uniform, exponential and Pareto, the value of the hazard rate, for any $V \in [V\,(0)\,, E\,[V^*])$, first increases and then decreases with the mean-preserving spread. An example for the normal density function is shown in Figure 2.1.[11]

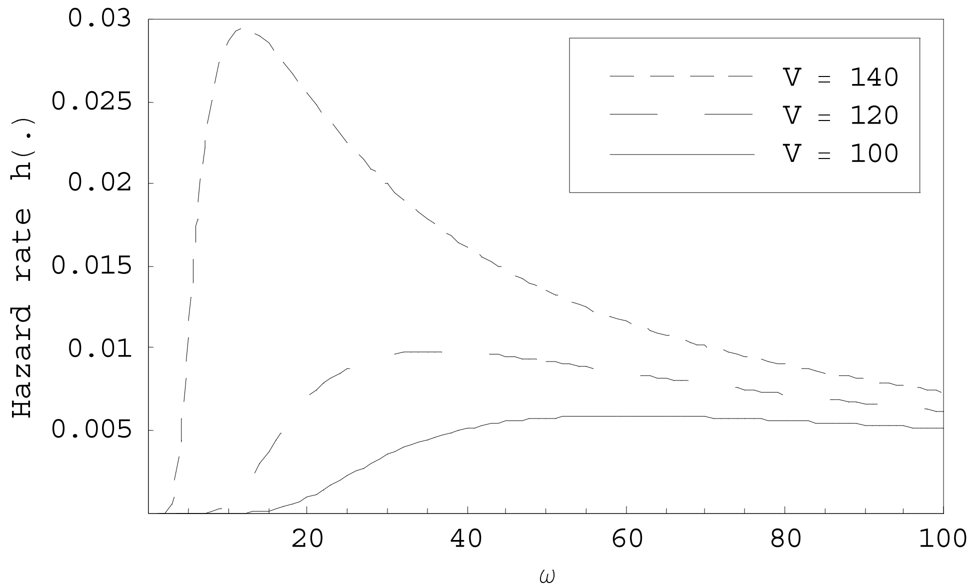

Figure 2.1: The relationship between the hazard rate and standard deviation of a normal density function $N(150, \omega^2)$. Hazard rates are plotted for $V = 100$, 120 and 140.

We conclude that, for each degree of the trigger value uncertainty, there exists such a value of $V < E[V^*]$, say $\widetilde{V}$, that for $V \in [V(0), \widetilde{V})$ the hazard rate increases, and for $V \in (\widetilde{V}, E[V^*])$ decreases, with this uncertainty. This form of the relationship between the hazard rate and the uncertainty implies (via Proposition 2.3) that V_s decreases with the uncertainty if it falls into the interval $[V(0), \widetilde{V})$ and increases otherwise. Consequently, in order to determine the sign of the effect of uncertainty on V_s, we need to establish the relative position of V_s with respect to $\widetilde{V}$.

We denote the standard deviation of the density function underlying the cost-increase trigger by ω. Since the expression for V_s is already known (see (2.9)), all we have to calculate is $\widetilde{V}$ as a function of ω, such that, for each pair (V, ω), the following condition holds:[12]

$$\left.\frac{\partial h(V)}{\partial \omega}\right|_{V=\widetilde{V}} = 0. \tag{2.15}$$

[11]Although the concepts of the mean-preserving spread and increased standard deviation are, in general, not equivalent, they may be treated as such for the types of density functions referred to in this chapter.

[12]Although $\widetilde{V}(\omega)$ cannot be written explicitly in a general form, its values corresponding to a given density function may be easily found numerically.

For the most frequently used density functions it can be shown that $\widetilde{V}$ decreases with uncertainty. Consequently, for a relatively low degree of uncertainty, it holds that $V_s < \widetilde{V}$ $(< E[V^*])$. Since for $V < \widetilde{V}$ the hazard rate increases in ω, V_s falls when the uncertainty rises. After the uncertainty reaches a critical level, say ω^e, at which $V_s = \widetilde{V}$, the hazard rate at V_s decreases with ω and the optimal threshold begins to increase. This implies that optimal investment threshold attains its minimum for $\omega = \omega^e$. Now, we are able to formulate the following proposition.

Proposition 2.4 *Consider the following unrestrictive conditions*

$$\begin{aligned} &\lim_{\omega \to \infty} \psi(V, \cdot) = 0, \; \forall V \quad \textit{and} \\ &\psi(V, \cdot) \textit{ is unimodal.} \end{aligned} \tag{2.16}$$

Then, there exists a non-monotonic relationship between the optimal investment threshold and the trigger value uncertainty. At a low degree of uncertainty, the marginal increase in uncertainty leads to an earlier optimal investment. The reverse is true for a high degree of uncertainty. There exists a unique ω^e*, such that* $V_s(\omega^e) = \widetilde{V}(\omega^e)$*, which separates the areas of low and high uncertainty levels.*

Proof. Proposition 2.4 directly follows from the analysis performed so far. ■

The interpretation of the proposition is relatively simple. At low levels of uncertainty concerning the policy change the firm responds to an increase of this uncertainty by investing earlier (i.e. at a lower V). This is because the chance of earlier implementation of the policy change increases. However, when this uncertainty becomes sufficiently high, the firm is more willing to ignore the information about the expected change since the quality of this information has deteriorated too much. The marginal impacts of a higher probability of an early change and of the increased "noisiness" of the firm's conjecture offset exactly at the level of uncertainty equal to ω^e.

Figures 2.2 and 2.3 show the relationship between the uncertainty, ω, and the optimal investment threshold. From Figure 2.2 it can be seen that the optimal investment threshold is first decreasing and then increasing with the uncertainty concerning the value of the trigger. The minimum is always reached when $V_s(\omega)$ intersects $\widetilde{V}(\omega)$. The hazard rate increases with ω in the area located to the south-west from $\widetilde{V}(\omega)$ and decreases in the north-eastern region. The opposite holds for V_s. Moreover, the optimal threshold is higher if the expected change in the investment cost is smaller (cf. Proposition 2.2).

From Figure 2.3 it can be noticed that the point, $\widetilde{V}$, at which the derivative of the hazard rate is equal to zero decreases when the trigger uncertainty increases.

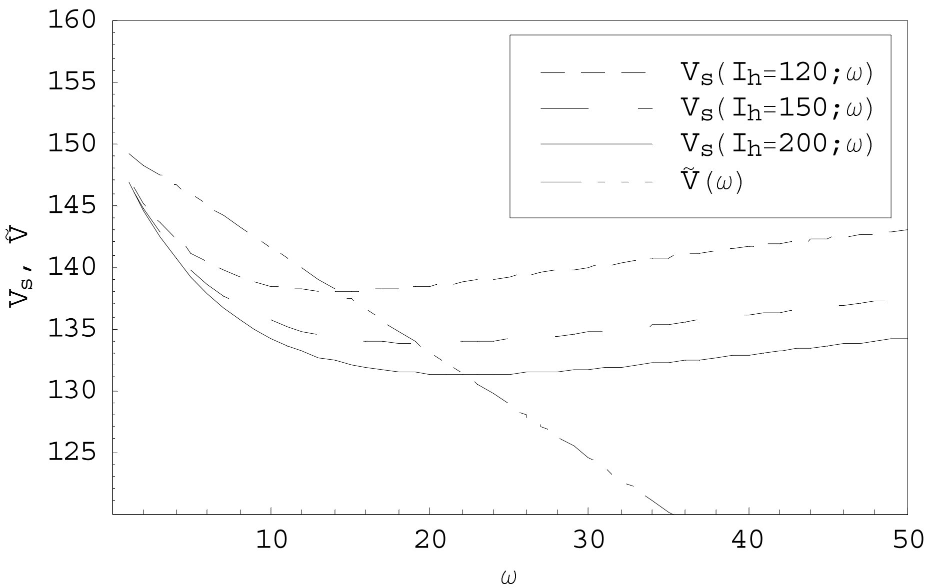

Figure 2.2: The relationship between the uncertainty, ω, and the optimal investment threshold, V_s, for different magnitudes of the high investment cost ($I_h = 120, 150$ and 200). The values are calculated for a normal density function with mean 150. The original investment cost, I_l equals 100. An intersection of V_s and $\widetilde{V}$ corresponds to the minimal investment threshold, $V_s(\omega^e)$. The parameters of the underlying process are: $\alpha = 0$, $r = 0.025$ and $\sigma = 0.1$.

As long as $V_s < \widetilde{V}$, the optimal threshold also decreases (cf. the location of V_s^L). When the standard deviation is equal to ω^e, V_s equals $\widetilde{V}$. After a further increase in the uncertainty, $\widetilde{V}$ continues to decrease and V_s starts to increase (cf. V_s^H). For a sufficiently high degree of uncertainty V_s tends to the unconditional threshold, denoted by V_l $(\equiv \beta_1 I_l / (\beta_1 - 1))$.[13]

2.4 Comparative Statics

In this section we provide a numerical illustration of the results of our model. In Table 2.1 the relationship between the uncertainty about the timing of the jump in the investment cost and the optimal investment threshold is shown for different levels of the after-shock investment cost. The results are grouped in three panels corresponding to the different combinations of the rate of growth and volatility of the project's value.

[13]The necessary and sufficient condition for $\lim_{\omega \to \infty} V_s = V_l$ is $\lim_{\omega \to \infty} h(V_s, \cdot) = 0$.

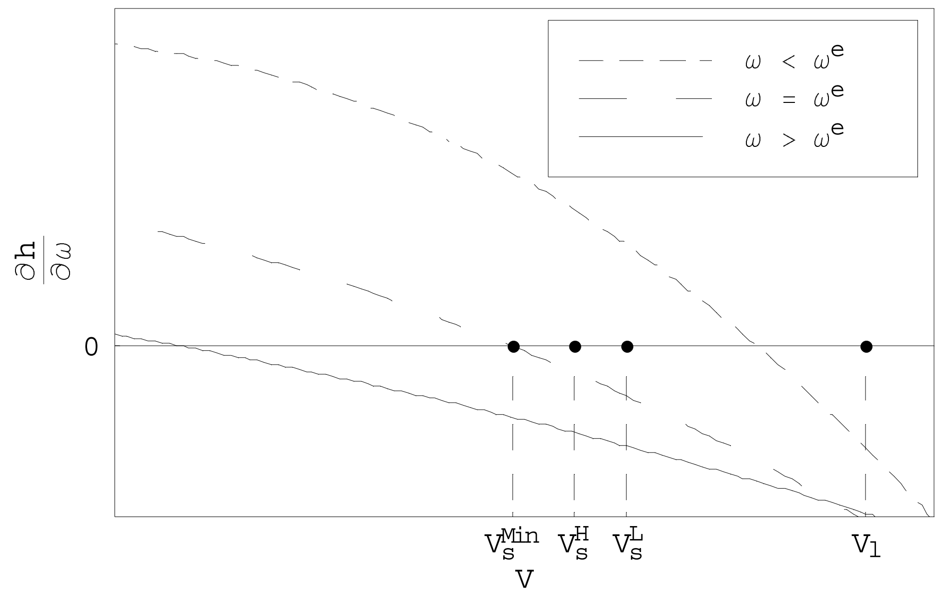

Figure 2.3: The relationship between V and the derivative of the hazard rate with respect to the trigger value uncertainty. The optimal investment thresholds, V_s^{Min}, V_s^H, and V_s^L, correspond to uncertainty levels equal to, higher that, and lower than, respectively, the level of uncertainty triggering the earliest investment. V_l is the optimal investment threshold in the absence of the expected policy change.

The results indicate a clear non-monotonic dependence of the optimal investment threshold on the uncertainty related to the occurrence of the shock. For example, consider the case where $\alpha = 0.02$ and $\sigma = 0.1$. When the firm's conjecture about the expected occurrence of the shock is relatively precise ($\omega = 5$), the possibility of doubling the effective investment cost results in the expected timing of undertaking the project being equal to 4.91 years.[14] When the uncertainty concerning the occurrence of the jump becomes moderately higher ($\omega = 25$), the firms is expected to invest within 2.78 years. Finally, when the firm's conjecture about the moment of the shock is highly imprecise ($\omega = 100$), the expected time to invest equals 9.67 years. If the project is about to deteriorate completely after the shock in the economy, the expected timing of investment shortens significantly, especially if the uncertainty concerning the occurrence of the shock is high. For $\omega = 5$ it is equal to 4.13 years, and for $\omega = 25$ it is

[14]The result is obtained by substituting appropriate values into (1.12), i.e.

$$-\frac{1}{0.02 - \frac{1}{2}0.01} \ln \frac{140}{150.71} \approx 4.91.$$

optimal to invest immediately. In the case corresponding to a very high imprecision of the conjecture ($\omega = 100$) the expected time to invest equals 3.80 years.

$E[V^*] = 160$	V_s					
I_h	ω	100	50	25	10	5
110		186.48	177.91	169.62	162.32	159.57
125		176.96	166.88	158.90	153.95	154.10
150		169.02	158.64	151.65	149.62	151.99
200		161.85	151.68	145.98	146.76	150.71
500		152.76	143.32	139.64	143.93	149.47
∞		148.22	*NOW*	*NOW*	142.74	148.94
$V_l = 200$	$\alpha = 0.02$		$\sigma = 0.1$		$r = 0.05$	
110		153.41	150.18	147.36	147.40	150.42
125		149.08	144.74	142.11	144.45	149.04
150		145.39	140.59	138.53	142.69	148.25
200		142.21	*NOW*	*NOW*	141.45	147.69
500		*NOW*	*NOW*	*NOW*	140.35	147.20
∞		*NOW*	*NOW*	*NOW*	140.11	147.10
$V_l = 158.77$	$\alpha = 0.01$		$\sigma = 0.1$		$r = 0.05$	
110		302.09	281.54	271.10	302.07	302.07
125		270.82	248.50	236.21	230.52	201.37
150		246.79	223.99	210.45	203.01	201.22
200		225.19	202.79	188.74	179.47	176.70
500		194.54	174.42	162.24	155.32	154.80
∞		160.54	145.73	140.46	144.60	149.97
$V_l = 371.85$	$\alpha = 0.02$		$\sigma = 0.3$		$r = 0.05$	

Table 2.1: The optimal investment thresholds calculated for three different combinations of the rate of growth and volatility of the project's value. *NOW* means that investment takes place immediately. The results are presented for the following parameter values: investment cost before the jump $I_l = 100$, investment cost after the jump ranging from 110 to infinity, standard deviation of the probability distribution underlying the policy change, ω, ranging from 5 to 100. The initial value of the process equals $V(0) = 140$.

The direction of the impact of change in the growth rate and/or volatility of the project's value is consistent with the conclusions in the existing real options literature: the change in both parameters results in an increase in the optimal investment threshold.

In Table 2.2 we show the values corresponding to the investment opportunity and probabilities that the investment is made before the increase in the investment cost (provided that the cost still equals I_l at $V(0)$). It can be seen that a higher magnitude of the change in the investment cost results in *i*) deteriorating the value of the investment opportunity, and *ii*) an increased probability of investing before the shock occurs (which is a direct consequence of the lower optimal threshold).

$E[V^*] = 160$	$F(V), P(V_s < V^* \mid V^* > V(0))$					
I_h	ω	100	50	25	10	5
110		61.54	66.65	70.58	71.24	66.00
		0.68	0.55	0.44	0.42	0.53
125		55.82	57.11	56.94	53.25	48.66
		0.75	0.68	0.66	0.74	0.88
150		50.93	50.01	48.28	46.28	45.27
		0.80	0.78	0.80	0.87	0.95
200		46.69	44.70	42.93	43.01	43.92
		0.85	0.86	0.90	0.93	0.97
500		42.16	40.51	40.00	40.86	42.98
		0.91	0.96	1.00	0.97	0.98
∞		40.62	40.00	40.00	40.30	42.66
		0.94	1.00	1.00	0.98	0.97
$V_l = 200$	$\alpha = 0.02$		$\sigma = 0.1$		$r = 0.05$	

Table 2.2: The values of the investment opportunity and probabilities of investing at I_l for the following parameter values: investment cost before the jump $I_l = 100$, investment cost after the jump ranging from 110 to infinity, standard deviation of the probability distribution underlying the policy change ranging from 5 to 100. The initial value of the process equals $V(0) = \widehat{V} = 140$.

An interesting observation can be made upon analyzing the relationship between the trigger-value uncertainty and the value of the investment opportunity. The non-monotonicity of this relationship results from the interaction of two opposite effects. First, increasing the variance, ω, implies lower quality of the firm's information about the moment of the policy change. This factor affects the value of the investment opportunity negatively. On the other hand, higher uncertainty makes the probability of survival on the interval $[V(0), V_s]$ become higher. This enhances the value of the investment opportunity.[15] It appears that in situations where the magnitude of the change in the investment cost is small, the value of the project is the highest for a moderate precision of the conjecture about the timing of the change. Conversely, if the investment opportunity is to deteriorate completely upon the occurrence of the shock, the value of the project is most likely to be equal to its static NPV, i.e. the value of the project minus investment cost, for a moderate precision of the conjecture.

To provide some intuition of how the results of our model correspond to the outcome of Poisson based models, in which the whole information about the shock is aggregated in a single arrival parameter, we present some comparative statics comparing both approaches in Table 2.3.[16]

$V_l = 200.00$		$I_h = 150$			
λ	$E[V^*]$	V_P	$\frac{1}{\lambda}$	$\omega(\lambda)$	$V_s(\omega(\lambda))$
0.01	627.44	191.64	100	1031.90	196.61
0.05	188.98	172.25	20	56.91	166.49
0.10	162.66	161.11	10	24.42	152.51
0.25	148.66	148.48	4	8.92	142.49
0.33	146.51	145.47	3	6.66	140.98
0.50	144.26	141.67	2	4.33	*NOW*
$\alpha = 0.02$		$\sigma = 0.1$		$r = 0.05$	

Table 2.3: The optimal investment thresholds based on the model with the policy change triggered by trigger V^*, $V_s(\omega(\lambda))$, compared with the outcomes of the Poisson based model, V_P, with the arrival rate λ ranging from 0.01 to 0.50 where the initial value of the process equals $V(0) = 140$ and the investment cost before the jump $I_l = 100$. $\omega(\lambda)$ is a geometric average of an upward and downward deviation from $E[V^*]$, that are associated with the expected first passage time $\frac{1}{\lambda}$.[17]

[15] The positive impact on the value of the investment opportunity results from the fact that conditional on $V^* > V(0)$ the cumulative density function of V^* is decreasing in ω for sufficiently large ω. This is equivalent, by definition, to the increase of the value of the conditional survival function.

[16] In order to calculate the optimal thresholds based on the Poisson arrivals, we apply a similar methodology as Dixit and Pindyck (1996), pp. 305-306.

[17] Consequently, $\omega(\lambda)$ is defined as $\omega(\lambda) \equiv E\sqrt{(E[V^*] - V^{sd_-})(V^{sd_+} - E[V^*])}$, where V^{sd_+} (V^{sd_-}) is the upward (downward) deviation from $E[V^*]$ such that the expected first-passage time of reaching V^{sd_+} ($E[V^*]$) when the process originates at $E[V^*]$ (V^{sd_-}) equals $\frac{1}{\lambda}$.

In Table 2.3 $E[V^*]$ is selected in such a way that its expected first passage time is equal to the expected time of a Poisson jump of a given arrival rate. Moreover, the level of uncertainty concerning the cost-increase trigger corresponds to the standard deviation of the trigger implied by the Poisson process. It appears that the slope of the relationship between the cost-increase trigger uncertainty and the optimal investment threshold is higher when our model is used than in the Poisson based approach. In other words, the resulting investment thresholds will be more responsive to the changes in ω. Consequently, for high levels of cost-increase trigger uncertainty, the optimal investment threshold under our approach will be higher than for Poisson based models (a cost increase trigger combined with very noisy information will not have a substantial effect on the firm's investment behavior). Conversely, if the prediction of the policy change is more reliable, the firm will invest more carefully (therefore earlier).

Finally, in Table 2.4 we show the outcomes of the Poisson based model in which the arrival rate is positively related to the value of the project.

$V_l = 200.00$			$\lambda_{Var}\|_{V=V_{PVar}} = \lambda$		$\lambda_{Var}\|_{V=V_0} = \lambda$	
λ	$E[V^*]$	V_P	d	V_{PVar}	d	V_{PVar}
0.01	627.44	191.64	5.195×10^{-5}	192.52	7.143×10^{-5}	190.20
0.05	188.98	172.75	2.875×10^{-4}	173.71	3.511×10^{-4}	170.69
0.10	162.66	161.11	6.160×10^{-4}	162.49	7.143×10^{-4}	160.33
0.25	148.66	148.48	1.675×10^{-3}	149.24	1.178×10^{-3}	148.53
0.33	146.51	145.47	2.284×10^{-3}	145.96	2.357×10^{-3}	145.64
0.50	144.26	141.67	3.520×10^{-3}	142.07	3.571×10^{-3}	141.95
			$\alpha = 0.02$		$\sigma = 0.1$	$r = 0.05$

Table 2.4: The optimal investment threshold, V_P, and V_{PVar}, calculated according to the Poisson based model with a constant and a variable arrival rate $\lambda = Vd$, respectively. The initial value of the process equals $V(0) = 140$, the investment cost before the jump $I_l = 100$, and the investment cost after the jump $I_h = 150$. Parameter d corresponding to the variable arrival rate is a solution to $\lambda = V_{PVar}d$ in column 4 and $\lambda = V_0 d$ in column 6, while the relevant λ is presented in column 1.

Table 2.4 illustrates the impact on the optimal investment threshold of introducing a variable arrival rate. The arrival rate increases with the value of the project. For the first set of solutions (columns 4-5) the variable $\lambda(V)$ equals λ in column 1 exactly at the level of V triggering the investment, i.e. $\lambda(V_{PVar}) = \lambda$. Analogously, the second set of solutions (columns 5-6) correspond to such a normalization upon which the variable rate $\lambda(V)$ equals to a constant λ in column 1 at $V(0)$. Despite the fact that the variable λ has been normalized in two extreme ways, the differences in outcomes are relatively small. Therefore, we conclude that introducing a variable arrival rate in the Poisson-based model does not significantly alter the firm's investment rule.

2.5 Extension: Stochastic Jump Size

In this section we relax the assumption that the magnitude of the change in the investment cost is known beforehand. The firm is assumed to know only the density function of the size of the jump. Consequently, the random variable I_h is distributed according to the cumulative density function $\Phi(I_h)$ with a support $\left[\underline{I_h}, \overline{I_h}\right]$ and $\underline{I_h} > 0$. Moreover, we impose a condition

$$\left(\int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi\left(I_h\right)\right)^{\frac{1}{1-\beta_1}} \geq I_l \tag{2.17}$$

that ensures that the firm prefers incurring the cost I_l to spending the stochastic amount I_h.[18]

Like in the deterministic case, the value of the investment opportunity, F_s, reflects the structure of the expected payoffs maximized with respect to the optimal investment threshold, V_s. For stochastic I_h, the value of the investment opportunity becomes (cf. (2.5)):

$$\begin{aligned} F_s(V, \widehat{V}|I &= I_l) = \max_{V_s}\left[(V_s - I_l)\left(\frac{V}{V_s}\right)^{\beta_1}\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}+\right. \\ &\left.+\int_{\underline{I_h}}^{\overline{I_h}}(V_h - I_h)\left(\frac{V}{V_h}\right)^{\beta_1}\left(1-\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}\right)d\Phi\left(I_h\right)\right]. \end{aligned} \tag{2.18}$$

Equation (2.18) can be interpreted analogously to (2.5), where the second component is the expected value of the option to invest after the upward change in the investment cost occurs. We prove that the following proposition holds.

Proposition 2.5 *In case of a stochastic size of the jump in the investment cost, the optimal investment rule can be determined by replacing the deterministic counterpart* I_h *by*

$$I_h^* = \left(\int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi\left(I_h\right)\right)^{\frac{1}{1-\beta_1}} \tag{2.19}$$

[18] The LHS of (2.17) has a natural interpretation presented in the remainder of the section.

in expression (2.9) for the optimal threshold.

Proof. See the Appendix. ■

Formula (2.19) can be interpreted as a certainty equivalent of the high investment cost. In other words, the investment policy of the firms is identical in the following two cases: i) investment cost I_h is stochastic and distributed according to $\Phi(I_h)$, and ii) I_h is deterministic and equal to I_h^*. This allows for a relatively simple analysis of the impact on the optimal investment timing of the uncertainty concerning the magnitude of the jump.

The impact of the uncertainty concerning the magnitude of the jump can be analyzed by applying Jensen's inequality. It holds that

$$\int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi(I_h) > \left(\int_{\underline{I_h}}^{\overline{I_h}} I_h d\Phi(I_h) \right)^{1-\beta_1}, \tag{2.20}$$

since the function $f(x) = x^a, a < 0$, is convex for all $x > 0$. From (2.20) it is easily obtained that

$$I_h^* < \int_{\underline{I_h}}^{\overline{I_h}} I_h d\Phi(I_h). \tag{2.21}$$

Since, by (2.11), $\frac{\partial V_s}{\partial I_h} < 0$, the threshold is higher in the case of a stochastic jump.

This result can be explained in the following way. The value of the investment opportunity is a convex function of the new investment cost, I_h (cf. (2.4)). Therefore, the gains from below average realizations of the jump are assigned a larger weight by the firm than the symmetric losses resulting from above-average realizations. Consequently, the firm is going to wait longer if the realizations are random than in the case when all of them are equal to the average.

Compared to the basic model where investment cost is constant, the threat of an upward change in the investment cost reduces the optimal investment threshold. Now, we can see that the uncertainty in the size of the jump mitigates this reduction of the threshold value. Again, it holds that increased uncertainty raises the option value of waiting.

Apart from the overall difference between the uncertain and deterministic outcome, we are interested in the marginal impact of uncertainty on the optimal investment strategy. In other words, we aim at establishing how the investment threshold behaves for different degrees of uncertainty concerning the size of the jump. Therefore, we compare the investment triggers corresponding to a relatively small and a high degree of uncertainty. For this purpose, we use the concept of mean preserving spread (Rotschild and Stiglitz, 1970). In this setting, the effect of increasing uncertainty is examined by replacing the original random variable I_h ('low uncertainty' case) by a new random variable $I_h + \xi$ ('high uncertainty' case), where $E[\xi] = 0$ and $\sigma_\xi \in (0, \infty)$. By applying Jensen's inequality it can be proved that the expected value of a convex function (in our case $f(I_h) = I_h^{1-\beta_1}$) increases as its argument undergoes a mean preserving spread (cf. Hartman, 1976). Consequently, an increase in the uncertainty leads to a higher expected value of $I_h^{1-\beta_1}$ which corresponds to a lower I_h^*. This observation results in the following corollary.

Corollary 2.1 *Increasing the uncertainty concerning the magnitude of the jump of the investment cost (in a mean-preserving spread sense) leads to a higher optimal investment threshold and is equivalent to decreasing the expected magnitude of the jump.*

The impact on the optimal investment rule of uncertainty related to the magnitude of the change in the cost is monotonic. Furthermore, (2.11) implies that a lower potential increase in the investment cost is associated with a higher optimal investment threshold. In Table 2.5 we present the numerical results illustrating the impact of the uncertainty related to the magnitude of the change on the optimal investment threshold.

$V^* = 160$		V_s					
$\underline{I_h}$	$\overline{I_h}$	ω	100	50	25	10	5
150	150		169.02	158.64	151.65	149.62	151.99
125	175		169.34	158.96	151.92	149.76	152.06
100	200		170.39	160.02	152.83	150.26	152.29
50	250		177.29	167.25	159.23	154.18	154.21
25	275		192.81	186.54	179.08	171.64	168.60
$V_l = 200$		$\alpha = 0.02$			$\sigma = 0.1$		$r = 0.05$

Table 2.5: The impact of the uncertainty concerning the magnitude of the change in the investment cost on the optimal investment threshold, where investment cost before the jump $I_l = 100$.

The numerical results in Table 2.5 illustrate that a higher degree of uncertainty associated with the magnitude of the potential cost-increase results in a later investment (the first row of Table 2.5 corresponds to the third row of Table 2.1). Therefore, in the investment credit example, increasing this type of uncertainty has the same effect on the investment as the reduction of the magnitude of the change.

2.6 Implications for the Investment Credit Tax Policy Change

In our setting, the way in which the policy change is implemented by the authority can be expressed as a triple $\left\{\frac{I_h}{I_l}, V^*, \omega\right\}$. Consequently, as a result of the policy change, the investment cost is subject to increase by a proportion $\frac{I_h}{I_l}$. The increase is triggered by the project's value reaching the level V^* and ω corresponds to the precision of the firm's conjecture concerning the moment of change. To simplify the example we assume that the ratio $\frac{I_h}{I_l}$ is predetermined by the current amount of the tax credit (and is *a priori* common knowledge). The variables V^* and ω are the authority's decision variables.

As we already know, in case of a single firm whose investment opportunity satisfies (2.2), a decrease in a deterministic V^* results in a lower optimal threshold. Consequently, a reduction in the trigger value is going to accelerate this firm's investment. However, in case of multiple heterogenous firms, due to the fact that a reduction of V^* makes condition (2.2, *iii*) tighter, lowering the trigger has two opposite effects. First, as in the single-firm case, it leads to an earlier investment for those firms for which Assumption (*iii*) (cf. (2.2)) is still satisfied. On the other hand, it results in the other firms waiting longer and investing at a high cost (i.e. those firms for which Assumption (*iii*) does not hold any longer). Hence, if the firms are sufficiently heterogeneous, reducing V^* does not yield the desired effect of accelerating aggregate investment.

Therefore, the authority may prefer to resort to another instrument, such as ω. From Proposition 2.4 it can be concluded that there exists a U-shaped relationship between ω and the optimal threshold, V_s. Since V_s reaches a minimum for a certain (strictly positive) degree of uncertainty, ω^e, the optimal strategy of the authority interested in accelerating the investment is to generate sufficiently (but not excessively) imprecise information about the conditions triggering the change. In purely analytical terms, this corresponds to setting the standard deviation of the density function associated with the conjecture about the policy change trigger, $\Psi(V^*)$, to ω^e.

Since finding the true value of ω^e can be difficult in practice, we briefly discuss the impact on the investment behavior of misspecifying the optimal ω. A small deviation from ω^e results in a small relative delay in investment. Consequently, it is still desirable for the authority to create informational noise. However, if the misspecification of ω^e is large, it can happen that the resulting optimal investment threshold, V_s, is higher than the threshold corresponding to the case where V^* is known to the firm. In this case the authority is better off by revealing the value of V^* to the investing firm. It is possible to find a critical level of ω, defined as $\overline{\omega}$, above which the optimal threshold is greater than the one corresponding to the known V^*. According to Proposition 2.1, $\overline{\omega}$ satisfies the following equation[19]

$$\begin{aligned} 0 &= h(V^*;\overline{\omega},\cdot)\,(V^*)^2 + (\beta_1 - 1)V^* - (V^* h(V^*;\overline{\omega},\cdot) + \beta_1)I_l - \\ &\quad - h(V^*;\overline{\omega},\cdot)\frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta}}\frac{(V^*)^{\beta_1 + 1}}{I_h^{\beta_1 - 1}}. \end{aligned} \tag{2.22}$$

If it is assumed that increasing the uncertainty by the authority is equivalent to applying a mean preserving spread, the change in the optimal investment threshold at $\overline{\omega}$ is discontinuous. Since the mean preserving spread implies that a policy change occurs at $V^* = E\left[V^*\right]$, imposing a level of uncertainty $\omega > \overline{\omega}$ results in the investment being made after the change in the cost occurs, i.e. at $V_h\,(\gg V_s)$. Therefore, increasing ω beyond $\overline{\omega}$ leads to a considerable delay of the investment.

We conclude that the level of uncertainty concerning the value of the policy change trigger can be characterized in the following way:

$$\omega \in \begin{cases} [0,\overline{\omega})\backslash\omega^e & : \text{ feasible (suboptimal) level of uncertainty,} \\ \omega^e & : \text{ optimal level of uncertainty,} \\ [\overline{\omega},\infty) & : \text{ excess uncertainty resulting in an investment delay.} \end{cases}$$

The threat of the policy change accelerates investment most significantly if the degree of uncertainty concerning the moment of the change is equal to ω^e. Therefore, from the point of view of the authority, this is the optimal level of the trigger value uncertainty. Revealing the value of V^* by the authority ($\omega = 0$) makes the firm invest an instant before V^* is reached. Excessive uncertainty (above $\overline{\omega}$) implies that information concerning the policy change is too unreliable to trigger investment before V^* is hit. As an effect, the optimal investment threshold exceeds the threshold corresponding to the known V^*. Consequently, there exists a set of feasible, though suboptimal, levels of uncertainty $\omega \in [0,\overline{\omega})\backslash\omega^e$ for which the optimal investment threshold is lower than V^*. For this set the threat of change remains high enough to trigger early investment.

The implications related to uncertainty in the *magnitude* of the policy change are straightforward, thus not requiring additional analysis. As shown in Section 2.5, an increase in the uncertainty concerning the *magnitude* of the change leads to a delay

[19] Equation (2.22) is also satisfied for $\omega = 0$, since the optimal threshold in the deterministic case is equal to V^*.

of the moment of investment. Consequently, ensuring that the magnitude of the policy change is known beforehand to potential investors lies in the interest of the authority interested in accelerating investment.

2.7 Conclusions

In this chapter we consider an investment opportunity of a firm, where the investment cost is irreversible and subject to an increase resulting from a policy change. The value of the cost-increase trigger is unknown to the firm but the firm knows the underlying density function instead. This corresponds to a situation where the firm has some information concerning the authority's future policy and this information is incomplete. Moreover, it is taken into account that a policy change mainly occurs under certain economic conditions.

We show that the threat of a policy change resulting in a higher investment cost leads to a reduction in the option value of waiting. Consequently, the firm invests earlier than in the case of a constant investment cost. The optimal investment threshold decreases with the magnitude of the change in investment cost and increases with market volatility (the latter result also holds for the Dixit and Pindyck, 1996, framework). One of our main results is that the impact of trigger value uncertainty on the optimal investment threshold is non-monotonic. If the uncertainty is sufficiently low, then the investment threshold is negatively related to the trigger value uncertainty. However, a rise in the uncertainty beyond a certain critical point reverses this relationship and leads to an increase of the optimal investment threshold.

Moreover, we extend the analysis by considering the case where the magnitude of the change is stochastic. This additional source of uncertainty results in a delay of investment. Increasing the uncertainty concerning the magnitude of the change leads to an outcome that is closer to the unconditional optimal threshold.

We apply our results to determine the optimal design of a change in the authority's policy, where the authority's aim is to accelerate investment undertaken by the firm. There exists a certain (strictly positive) level of the uncertainty concerning the policy change trigger that is associated with the earliest investment. Hence, a policy maker interested in accelerating investment should aim at achieving that particular level of uncertainty. In addition, in order to stimulate the firm to invest early, the authority should make sure that the magnitude of the policy change is known beforehand to potential investors.

2.8 Appendix

Proof of Proposition 2.1. The implicit solution for the optimal investment threshold is found by calculating the first order condition of (2.5). By differentiating (2.5) with respect to V_s, and equalizing to zero, we obtain:

$$\frac{V^{\beta_1}}{V_s^{\beta_1+1}}\left(V_s-\beta_1 V_s+\beta_1 I_l\right)\frac{1-\Psi(V_s)}{1-\Psi(\widehat{V})}-\left(V_s-I_l\right)\left(\frac{V}{V_s}\right)^{\beta_1}\left(\frac{\psi\left(V_s\right)}{1-\Psi(\widehat{V})}\right)$$
$$+\left(V_h-I_h\right)\left(\frac{V}{V_h}\right)^{\beta_1}\frac{\psi\left(V_s\right)}{1-\Psi(\widehat{V})}=0. \tag{2.23}$$

Further simplification yields:

$$\frac{1}{V_s^{\beta_1+1}}\left(V_s-\beta_1 V_s+\beta_1 I_l\right)\left(1-\Psi(V_s)\right)-\left(V_s-I_l\right)\left(\frac{1}{V_s}\right)^{\beta_1}\psi\left(V_s\right)$$
$$+\left(V_h-I_h\right)\left(\frac{1}{V_h}\right)^{\beta_1}\psi\left(V_s\right)=0,$$

thus

$$-\left(\beta_1-1\right)V_s+\beta_1 I_l-h\left(V_s\right)\left(V_s-I_l\right)V_s+\left(V_h-I_h\right)V_s^{\beta_1+1}\left(\frac{1}{V_h}\right)^{\beta_1}h\left(V_s\right)=0.$$

Since $V_h=\frac{\beta_1}{\beta_1-1}I_h$ (after the jump the McDonald-Siegel problem is left), this is equal to

$$-h\left(V_s\right)V_s^2-\left(\beta_1-1\right)V_s+\left(h\left(V_s\right)V_s+\beta_1\right)I_l+V_s^{\beta_1+1}h\left(V_s\right)\frac{I_h}{\beta_1-1}\left(\frac{\beta_1-1}{\beta_1 I_h}\right)^{\beta_1}=0,$$

which in a straightforward way leads to (2.9).

In order to prove that (2.9) is the expression for the maximal value of the project, we calculate the second order condition, which is equal to the following derivative:

$$\frac{\partial}{\partial V_s}\left(-h(V_s)V_s^2-(\beta_1-1)V_s+(V_s h(V_s)+\beta_1)I_l+h(V_s)\frac{(\beta_1-1)^{\beta_1-1}}{\beta_1^{\beta_1}}\frac{V_s^{\beta_1+1}}{I_h^{\beta_1-1}}\right).$$

After differentiating, we obtain an expression for the second order condition of (2.5):

$$\begin{aligned}\frac{\partial^2 F_s}{\partial V_s^2} &= -\left(h'\left(V_s\right)V_s+h\left(V_s\right)\right)\left(V_s-I_l-\frac{V_s}{\beta_1}\left(\frac{\beta_1-1}{\beta_1}\frac{V_s}{I_h}\right)^{\beta_1-1}\right)\\ &\quad -h\left(V_s\right)V_s\left(1-\left(\frac{\beta_1-1}{\beta_1}\frac{V_s}{I_h}\right)^{\beta_1-1}\right)-(\beta_1-1).\end{aligned} \tag{2.24}$$

The sign of the second component is negative since

$$1-\left(\frac{\beta_1-1}{\beta_1}\frac{V_s}{I_h}\right)^{\beta_1-1} > 1-\left(\frac{\beta_1-1}{\beta_1}\frac{V_h}{I_h}\right)^{\beta_1-1} = 0. \tag{2.25}$$

The sign of the first component can be determined by noting that the lower bound of V_s, denoted by $\underline{V_s}$, is a solution to the following equation (cf. (2.2, *iii*)):

$$\underline{V_s} - I_l = (V_h - I_h)\left(\frac{\underline{V_s}}{V_h}\right)^{\beta_1}. \tag{2.26}$$

For $V_s = \underline{V_s}$ the second factor in the first component of (2.24) is equal to zero and for $V_s > \underline{V_s}$ it is positive. Therefore the whole expression is negative if (2.8) holds. ■

Proof of Proposition 2.2. Let us define the LHS of (2.9) as a function:

$$\begin{aligned} & H(V_s, I_l, I_h, \beta_1) \qquad (2.27)\\ = \; & h(V_s)V_s^2 + (\beta_1-1)V_s - (V_s h(V_s)+\beta_1)I_l - h(V_s)\frac{(\beta_1-1)^{\beta_1-1}}{\beta_1^{\beta_1}}\frac{V_s^{\beta_1+1}}{I_h^{\beta_1-1}}. \end{aligned}$$

Differentiating (2.27) with respect to I_l, I_h and β_1, respectively, yields:

$$\begin{aligned} \frac{\partial H}{\partial I_l} &= -(V_s h(V_s)+\beta_1) < 0,\\ \frac{\partial H}{\partial I_h} &= (\beta_1-1)h(V_s)\frac{(\beta_1-1)^{\beta_1-1}}{\beta_1^{\beta_1}}\frac{V_s^{\beta_1+1}}{I_H^{\beta_1}} > 0, \qquad (2.28)\\ \frac{\partial H}{\partial \beta_1} &= V_s - I_l - h(V_s)\frac{(\beta_1-1)^{\beta_1-1}}{\beta_1^{\beta_1}}\frac{V_s^{\beta_1+1}}{I_h^{\beta_1-1}}\ln\left(\frac{\beta_1-1}{\beta_1}\frac{V_s}{I_h}\right) > 0, \end{aligned}$$

$\forall I_l, I_h$ satisfying $0 < I_l < I_h$, $\forall \beta_1 \in (1, r/\alpha)$ if $\alpha > 0$ and $\forall \beta_1 \in (1,\infty)$ if $\alpha \leq 0$. Furthermore, differentiating (2.27) with respect to V_s gives:

$$\begin{aligned} \frac{\partial H}{\partial V_s} &= (h'(V_s)V_s + h(V_s))\left(V_s - I_l - (V_h - I_h)\left(\frac{V_s}{V_h}\right)^{\beta_1}\right) + \qquad (2.29)\\ &\quad + h(V_s)V_s\left(1-\left(\frac{\beta_1-1}{\beta_1}\frac{V_s}{I_h}\right)^{\beta_1-1}\right) + (\beta_1-1). \end{aligned}$$

From the proof of Proposition 2.1 (cf. (2.26)) it is known that under condition (2.8) $\frac{\partial H}{\partial V_s}$ is positive. Finally, by observing that

$$\frac{dV_s}{dz} = -\frac{\frac{\partial H}{\partial z}}{\frac{\partial H}{\partial V_s}}, \tag{2.30}$$

where z is an arbitrary parameter of our interest, we know that

$$\operatorname{sgn}\frac{dV_s}{dz} = -\operatorname{sgn}\frac{\partial H}{\partial z}, \tag{2.31}$$

which completes the proof. ■

Proof of Proposition 2.3. By differentiating (2.27) with respect to the hazard rate, while taking into account that $V_h = \frac{\beta_1}{\beta_1 - 1} I_h$, we obtain:

$$\frac{\partial H}{\partial h} = V_s \left(V_s - I_l - (V_h - I_h) \left(\frac{V_s}{V_h} \right)^{\beta_1} \right) > 0. \tag{2.32}$$

The inequality holds since both factors are positive (cf. (2.26)). Since $\frac{\partial H}{\partial V_s}$ is also positive, we directly obtain the sign of (2.13). ■

Proof of Proposition 2.5. Equation (2.19) requires the optimal investment threshold with a deterministic size of the jump to be equal to the threshold with a jump with a stochastic size distributed according to $\Phi(I_h)$. Since the maximization problem with a stochastic size of the jump can be expressed as follows:

$$\begin{aligned} F_s(V, \widehat{V} | I &= I_l) = \max_{V_s} \left[(V_s - I_l) \left(\frac{V}{V_s} \right)^{\beta_1} \frac{1 - \Psi(V_s)}{1 - \Psi(\widehat{V})} + \right. \\ &\left. + \left(1 - \frac{1 - \Psi(V_s)}{1 - \Psi(\widehat{V})} \right) \frac{(\beta_1 - 1)^{\beta_1 - 1} V^{\beta_1}}{\beta_1^{\beta_1}} \int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi\left(I_h\right) \right], \end{aligned} \tag{2.33}$$

the expression for the optimal investment threshold is a slight modification of (2.9):

$$\begin{aligned} & h(V_s)V_s^2 + (\beta_1 - 1)V_s - (V_s h(V_s) + \beta_1) I_l - \\ & - V_s^{\beta_1 + 1} h(V_s) \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi\left(I_h\right) = 0. \end{aligned} \tag{2.34}$$

Comparing (2.34) with (2.9) allows for observing that the threshold values are equal if:

$$h(V_s) \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \frac{V_s^{\beta_1 + 1}}{I_h^{*\beta_1 - 1}} = \int_{\underline{I_h}}^{\overline{I_h}} h(V_s) \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \frac{V_s^{\beta_1 + 1}}{I_h^{\beta_1 - 1}} d\Phi\left(I_h\right). \tag{2.35}$$

A simple algebraic manipulation yields:

$$I_h^{*1-\beta_1} = \int_{\underline{I_h}}^{\overline{I_h}} I_h^{1-\beta_1} d\Phi\left(I_h\right). \tag{2.36}$$

which is equivalent to (2.19). ■

Chapter 3

Demand Uncertainty in a Cournot Model

3.1 Introduction

In this chapter we consider a continuous-time model in which a firm makes a decision to replace a production facility with a new, cost-efficient one. The firm operates in an uncertain economic environment under imperfect competition. The model follows Smets (1991) and Grenadier (1996) in assuming that *i*) there are two identical firms competing in the product market, and *ii*) the value of the firm depends on the value of a stochastic process but is otherwise time independent. The payoff functions are derived from the firm's reaction curves in the oligopolistic market. We determine the optimal replacement strategies, calculate the expected replacement timing and determine the probabilities of making optimal replacement within given time intervals.

Under either perfect competition or a monopolistic market structure, the modern theory of investment under uncertainty (cf. Section 1.6 of this thesis and Dixit and Pindyck, 1996, Ch. 5) predicts that the firm will wait longer with investing if uncertainty is higher. This is due to the fact that investment is irreversible and the firm has an option to postpone it until some uncertainty is resolved. However, if (*i*) more than one firm holds the investment opportunity, and (*ii*) the firm's investment decision directly influences payoffs of its competitor(s), opposite effects of increasing uncertainty with respect to the investment timing can arise. First, increasing uncertainty enhances the value of the option to wait. Second, the value of an early strategic investment (made in order to achieve the first mover advantage) can significantly increase as well. Huisman and Kort (1999) show that in a continuous-time duopoly model with *profit* *uncertainty* (cf. Smets, 1991, and Grenadier, 1996) the effect of a change in the value of the option to wait on the optimal investment threshold is always stronger than the impact of strategic interactions.[1] This implies a negative relationship between uncertainty of the firm's profit flow and investment. On the contrary, Kulatilaka and Perotti (1998) find that *product market uncertainty* may, in some cases, stimulate investment.[2] The latter authors consider a two-period setting in which (one of the) duopolistic firms can invest in a cost-reducing technology. The payoff from investment is convex in the size of the demand since an increase of demand has a more-than-proportional effect on the realized duopolistic profits (firms are responding to higher demand by increasing both output and price). Taking into account Jensen's inequality, Kulatilaka and Perotti (1998) conclude that higher volatility of the product market can accelerate investment.

The aim of this chapter is to determine the effects of product market uncertainty on investment in a continuous-time setting. To do so, we begin the analysis by describing the equilibrium strategies that occur in the resulting real option game in the model with product market uncertainty. We show that, contrary to the models based on profit uncertainty, the type of equilibrium depends on the investment cost: if this cost is sufficiently low (high), a preemptive (simultaneous) equilibrium occurs.[3] Furthermore, we prove that the minimal demand level triggering the investment increases with uncertainty for both firms. This result holds both for the case in which the investment is associated with replacing an existing asset and for the case in which the firms have to decide when to start up production. Moreover, we show that, in expectation, product market uncertainty delays investment. We thus can conclude that the result of Kulatilaka and Perotti (1998) does not carry over to a continuous-time setting. Finally, we analyze the probability of asset replacement within a given time interval. It turns out that the replacement probability decreases with uncertainty for time intervals longer than the time to reach the optimal Jorgensonian threshold calculated for the deterministic case.[4] For shorter intervals there are two opposite effects which leads

[1] In fact, the option effect and strategic the effect may work in the same direction. Boyer et al. (2002) show that an increase in uncertainty can result in a the equilibrium in which firms opt for a late simultaneous investment instead of an earlier sequential entry.

[2] Profit uncertainty is associated with the profit function following a geometric Brownian motion, whereas product market uncertainty relates to random shifts in the demand curve.

[3] This implies that the type of equilibrium can easily be affected by e.g. the authority. The rule imposed by Germany's telecom regulator enabling six companies which acquired the third generation mobile-phone licenses to share the costs of building a new infrastructure, may serve as an example of such an action. See *The Economist*, 9th June 2001

[4] At such a threshold the flow revenues are equal to the flow costs associated with investment.

to a humped relationship between uncertainty and the probability of replacement (cf. Sarkar, 2000).

The model is presented in Section 3.2, while the value functions and replacement thresholds are derived in Section 3.3. Section 3.4 contains a description of the equilibria and in Section 3.5 the effect of uncertainty on replacement thresholds is determined. In Section 3.6 the decision to start production in a new market is analyzed. Section 3.7 examines how uncertainty influences replacement timing and the probability of investment within a given time interval. Section 3.8 concludes.

3.2 Framework of the Model

Consider a risk-neutral firm that has an investment opportunity to replace its existing production facility with a technologically superior one. The firm operates in a duopoly, in which, in line with basic microeconomic theory, the following inverse linear demand function holds:

$$p(t) = A(t) - Q(t). \tag{3.1}$$

For each $t \in [0, \infty)$, $p(t)$ is the price of a non-durable good/service offered by the firm and can be interpreted as the instantaneous cash flow per unit sold, $A(t)$ is a measure of the size of the demand, and $Q(t)$ is the total amount of the good supplied to the market. Parameter $A(t)$ follows a geometric Brownian motion

$$dA(t) = \alpha A(t)\, dt + \sigma A(t)\, dw(t), \tag{3.2}$$

where α is the instantaneous drift parameter, σ is the instantaneous standard deviation, dt is the time increment, and $dw(t)$ is the standard Wiener increment.[5]

The other firm operating in the market is identical to the first, both are profit-maximizers and compete in quantities (*à la* Cournot).[6] The initial constant marginal cost of supplying a unit of the good is K and setting up the new production facility reduces this cost from K to k. In order to start using the new facility, Firm i, $i \in \{1, 2\}$, has to incur an irreversible cost I. Simple algebraic manipulation results in the

[5]Such a formulation implies that demand is driven by consumers' tastes (a varying maximal valuation) and by replication of consumers (a varying mass of consumers). Models with multiplicative profit uncertainty, such as Huisman (2001), Ch. 7-9, and Boyer et al. (2002), are equivalent, under zero marginal cost assumption, to demand driven by the changes in consumers' tastes.

[6]Quantity competition yields the same output as a two-stage game in which the capacities are chosen first and, subsequently, the firms are competing in prices (see Tirole, 1988, p. 216).

following instantaneous profits, π^{ij}, of Firm i (the other firm is denoted by j, $j \neq i$)[7]

$$\pi^{00} = \frac{1}{9}(A-K)^2, \tag{3.3}$$

$$\pi^{10} = \frac{1}{9}(A+K-2k)^2, \tag{3.4}$$

$$\pi^{01} = \frac{1}{9}(A-2K+k)^2, \text{ and} \tag{3.5}$$

$$\pi^{11} = \frac{1}{9}(A-k)^2. \tag{3.6}$$

Superscript 1 (0) in π^{ij} indicates which firm replaced (did not replace) its production facility, where i indicates the own firm and j the competitor. It is seen immediately that

$$\pi^{10} > \pi^{11} > \pi^{00} > \pi^{01}. \tag{3.7}$$

The profit of the only firm which replaced the production facility is higher than in the profit of a firm in a situation where two firms made the replacement. In turn, the latter profit exceeds the profit of symmetric firms operating the existing facility, which is still higher than the profit of the only firm which did not replace its production asset.

Admittedly, the chosen model formulation is one out of many possibilities. We choose this specification in order to be able to make comparisons with the results of the two-period model of Kulatilaka and Perotti (1998). This chapter can be seen therefore as a first fully dynamic investigation of the impact of product market uncertainty on investment timing. Extensions can include multiplicative rather than additive demand uncertainty, variable marginal costs, and Bertrand competition.

We summarize the problem by describing the strategy space of the firms. Define a simple strategy of Firm i ($i \in \{1,2\}$) as a tuple of real-value functions $(G_i(\cdot), p_i(\cdot)) : [0,\infty) \times \Omega \longrightarrow [0,1] \times [0,1]$, such that for all $\omega \in \Omega$ it holds that (cf. Thijssen et al., 2002, and Boyer et al., 2002):

(*i*) $G_i(\cdot;\omega)$ is non-decreasing and right-continuous with left limits,

(*ii*) $p_i(\cdot;\omega)$ is right differentiable and right-continuous with left limits,

(*iii*) if $p_i(t;\omega) = 0$ and $t = \inf\{u | p_i(u;\omega) > 0\}$, then the right derivative of $p_i(t;\omega)$ is positive.

Now the strategy space for Firm i is given by the set $S_i = \{(G_i(\cdot), p_i(\cdot)) | G_i(\cdot)$ satisfies (*i*), and $p_i(\cdot)$ satisfies (*ii*) and (*iii*)$\}$. The strategy space of the game is then

[7]We assume that $K \ll A(0)$, so that the probability weighted discount factor associated with the event $\{A(t) < 2K - k\}$ is negligible. Waiving this assumption would not significantly contribute to our results and would be done at the expense of explicit analytical formulae for the optimal investment thresholds (cf. Dixit and Pindyck, 1996, p. 191).

$\mathcal{S} = S_1 \times S_2$. To determine the firms' optimal policies we use the subgame perfect equilibrium concept, while the firms' strategies are assumed to satisfy intertemporal consistency and α-consistency conditions (for the definitions see Fudenberg and Tirole, 1985, p. 393, and Thijssen et al., 2002, p. 9, respectively).[8]

3.3 Value Functions and Replacement Thresholds

In this section we establish the value of the firms and their optimal replacement thresholds. There are three possibilities concerning the timing of Firm i's investment relatively to the decision of the competitor (Firm j). First, Firm i may invest before Firm j does and, therefore, become the leader. Alternatively, Firm j may invest sooner and Firm i becomes the follower. Finally, the firms may invest simultaneously.

The standard approach used to solve dynamic games is to analyze the problem backwards in time. Consequently, we begin with the optimal strategy of the follower. Then, the decision of the leader is analyzed. Finally, we discuss the case of joint investment.

3.3.1 Follower

Consider the case of the firm that replaces as second (follower). Since the other firm (leader) has already replaced its production facility, the follower's replacement decision is not affected by strategic interactions (the follower chooses its optimal threshold as if the roles of the firms are preassigned). From (3.5) and (3.6) it is obtained that after replacing the asset by the leader, the value of the follower at the moment of making the investment by the leader, t, equals

$$V^F(t) = E\left[\int_t^{T^F} \frac{1}{9}(A(s) - 2K + k)^2 e^{-r(s-t)} ds\right] \tag{3.8}$$
$$+E\left[e^{-r\left(T^F - t\right)}\left(\int_{T^F}^{\infty} \frac{1}{9}(A(s) - k)^2 e^{-r\left(s-T^F\right)} ds - I\right)\right],$$

where T^F is the random stopping time associated with replacing the production facility by the follower. The first row of (3.8) is the expected discounted cash flow received until replacement. At T^F the follower makes the replacement and from now on produces against a lower marginal cost k. The expected discounted cash flow after replacement is captured by the second row of (3.8).

[8] Our notation differs from Thijssen et al. (2002), where our function p_i is denoted by α_i.

Let us consider the optimal replacement strategy of the follower. In investment problems of this type (cf. Section 1.6) a threshold value of A exists at which the firm is indifferent between investing and refraining from investment. Consequently, the value of the firm is maximized when replacement of the production facility takes place as soon as A exceeds this threshold value. Using standard dynamic programming methodology we arrive at the following Bellman equation

$$rV^F = \frac{1}{2}\sigma^2 A^2 \frac{\partial^2 V^F}{\partial A^2} + \alpha A \frac{\partial V^F}{\partial A} + \pi^{01}. \tag{3.9}$$

Solving the differential equation (3.9) and excluding the existence of speculative bubbles gives

$$V(A) = \underbrace{CA^{\beta_1}}_{\text{Value of flexibility}} + \underbrace{\frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2(2K-k)A}{\delta} + \frac{(2K-k)^2}{r}\right)}_{\text{PV of expected cash flow}}, \tag{3.10}$$

where C is a constant, β_1 is the positive root of the equation (cf. Section 1.6):

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0, \tag{3.11}$$

and

$$\varrho \equiv r - 2\alpha - \sigma^2, \tag{3.12}$$

$$\delta \equiv r - \alpha. \tag{3.13}$$

From (3.10) it can be seen that there are two components contributing to the value of the firm. The first component corresponds to the value of the flexibility to replace the production facility. The remainder of the RHS of (3.10) reflects the present value of the expected cash flow given that the firm produces with the existing technology forever. Convexity of the value of the firm in A implies that a finite valuation is obtained only if the condition $r - 2\alpha - \sigma^2 > 0$ is satisfied.[9]

[9] In order to assess how restrictive the condition $r - 2\alpha - \sigma^2 > 0$ is, we calculate the maximum feasible growth rate of demand using the parameters of Dixit and Pindyck (1996), Ch. 6, and of a representative US Standard and Poor's 500 firm (as reported in Morellec, 2001). By applying Itô's lemma one can show that the volatility of the process proportional to the square of the original process equals two times the volatility of the original process. Hence, since the cash flow of the firm is proportional to A^2, its instantaneous volatility equals twice the volatility of A. This implies that, for the parameter set of Dixit and Pindyck ($r = 4\%$ and $\sigma = 20\%$), the standard deviation of the demand can be estimated at the 10% level (0.20/2), whereas for the representative S&P 500 firm ($r = 6\%$ and $\sigma = 25\%$), it equals 12.5%. Then, finite valuations are ensured for values of parameter α ranging from minus infinity to $\left(0.04 - (0.1)^2\right)/2 = 1.5\%$, and approx. 2.2%, respectively.

To derive the optimal replacement threshold we apply the value-matching and smooth-pasting conditions to (3.10) and the value of the firm after the replacement net of the investment cost.[10] This leads to

$$CA^{\beta_1} = \frac{1}{9}\left(\frac{4(K-k)A}{\delta} - \frac{4K(K-k)}{r}\right) - I, \tag{3.14}$$

$$\beta_1 CA^{\beta_1 - 1} = \frac{4}{9}\frac{K-k}{\delta}. \tag{3.15}$$

From (3.14) and (3.15) we obtain the optimal replacement threshold of the follower:

$$A^F = \frac{\beta_1}{\beta_1 - 1}\frac{I + \frac{4K}{9r}(K-k)}{\frac{4}{9}(K-k)}\delta. \tag{3.16}$$

The optimal time of replacement made by the follower is denoted by

$$T^F \equiv \inf\left(t | A(t) \geq A^F\right). \tag{3.17}$$

Note that the optimal threshold (3.16) is increasing with uncertainty (via β_1) and in the wedge δ.[11] The value of the follower (at the moment at which the leader invests) can now be calculated by substituting C, as derived from (3.14) and (3.15), into (3.10). Such a substitution yields

$$V^F(A) = \begin{cases} \frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2(2K-k)A}{\delta} + \frac{(2K-k)^2}{r}\right) \\ +\left(\frac{1}{9}\left(\frac{4(K-k)A^F}{\delta} - \frac{4K(K-k)}{r}\right) - I\right)\left(\frac{A}{A^F}\right)^{\beta_1} & \text{if } A \leq A^F, \\ \frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2kA}{\delta} + \frac{k^2}{r}\right) - I & \text{if } A > A^F. \end{cases} \tag{3.18}$$

3.3.2 Leader

Having established the optimal replacement rule of the follower, we are ready to determine the payoff of the firm that invests as the leader. The value function of the

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[10] The value matching condition equalizes the value of the firm before the replacement (including the replacement option), as in (3.10), and the value after the replacement net of the associated sunk cost. Upon observing that the value after the replacement corresponds to the expected cash flow from new assets in place and equals

$$\frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2kA}{\delta} + \frac{k^2}{r}\right),$$

condition (3.14) is obtained. Condition (3.15) is obtained by taking the derivatives of (3.14) with respect to A.

[11] Increasing the wedge δ also has an indirect effect because it positively affects β_1, but that effect is dominated by the direct effect on A^F.

leader, evaluated at the moment of investing, t, is

$$V^L(t) = E\left[\int_t^{T^F} \frac{1}{9}(A(s)+K-2k)^2 e^{-r(s-t)}ds - I + \int_{T^F}^{\infty} \frac{1}{9}(A(s)-k)^2 e^{-r(s-t)}ds\right]. \tag{3.19}$$

The first two components of (3.19) correspond to the present value of the leader's profits realized until the moment of the follower's investment, net of the leader's sunk cost. The second integral corresponds to the discounted perpetual stream of profits obtained after the investment of the follower.

Analogous to expression (3.18) of the follower problem, we can express the value of Firm i as the leader at the moment of its investment in the following way:

$$V^L(A) = \begin{cases} \frac{1}{9}\left(\frac{A^2}{\varrho} + \frac{2(K-2k)A}{\delta} + \frac{(K-2k)^2}{r}\right) - I \\ -\frac{1}{9}\left(\frac{2(K-k)A^F}{\delta} + \frac{(K-2k)^2-k^2}{r}\right)\left(\frac{A}{A^F}\right)^{\beta_1} & \text{if } A \leq A^F, \\ \frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2kA}{\delta} + \frac{k^2}{r}\right) - I & \text{if } A > A^F. \end{cases} \tag{3.20}$$

The first row of (3.20) corresponds to the net present value of the leader profits without the follower ever making the investment. The second row reflects the present value of future profits lost due to the follower's investment. This loss is caused by the fact that after the follower has invested, the follower can produce in a cheaper way, which makes it a stronger competitor for a leader. The last row represents the net present value of profits in a situation where it is optimal for the follower to invest immediately.

3.3.3 Simultaneous Investment

It is possible that the firms decide to invest simultaneously. The value function of both firms investing optimally at the joint investment threshold, calculated at $t \leq T^S$, is

$$\begin{aligned} V^S(t) &= E\left[\int_t^{T^S} \frac{1}{9}(A(s)-K)^2 e^{-r(s-t)}ds - Ie^{-r\left(T^S-t\right)}\right] \\ &\quad + E\left[\int_{T^S}^{\infty} \frac{1}{9}(A(s)-k)^2 e^{-r(s-t)}ds\right], \end{aligned} \tag{3.21}$$

where

$$T^S \equiv \inf\left(t | A(t) \geq A^S\right) \tag{3.22}$$

and

$$A^S = \frac{\beta_1}{\beta_1 - 1} \frac{I + \frac{1}{9}\frac{K^2-k^2}{r}}{\frac{2}{9}(K-k)}\delta. \tag{3.23}$$

Expression (3.21) can be interpreted analogous to (3.8) and (3.19). Consequently, the value of Firm i when the investment is made simultaneously equals

$$V^S(A) = \begin{cases} \begin{array}{c} \frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2KA}{\delta} + \frac{K^2}{r}\right) \\ + \left(\frac{1}{9}\left(\frac{2(K-k)A^S}{\delta} - \frac{K^2-k^2}{r}\right) - I\right)\left(\frac{A}{A^S}\right)^{\beta_1} \end{array} & \text{if } A \leq A^S, \\ \frac{1}{9}\left(\frac{A^2}{\varrho} - \frac{2kA}{\delta} + \frac{k^2}{r}\right) - I & \text{if } A > A^S. \end{cases} \tag{3.24}$$

The last row equals the value of the firm when the simultaneous investment is made immediately. In such a case, the value of the firm is denoted by $V^J(A)$.

3.4 Equilibria

Since both firms are ex ante identical, it is natural to consider symmetric replacement strategies and assume the firms' roles being endogenous, i.e. that it is not determined beforehand which firm will be the first to replace. There are two types of equilibria that can occur under this choice of strategies. We start by presenting the preemptive equilibrium, which is followed by a description of the simultaneous equilibrium.

3.4.1 Preemptive Equilibrium

The first type of equilibrium is a preemptive equilibrium where Firm i is the leader and Firm j is the follower. Let us define A^P to be the root of

$$\xi(A) \equiv V^L(A) - V^F(A). \tag{3.25}$$

on the interval $(0, A^F)$. In the Appendix we prove that the root exists, it is unique, $\xi(A) < 0$ for $A < A^P$, and $\xi(A) > 0$ for $A \in (A^P, A^F)$. Assume for the moment that $A(0) < A^P$. Since on the interval (A^P, A^F) the payoff of the leader is higher than the payoff of the follower, each firm will have an incentive to be the leader at the moment that $A(t) \in (A^P, A^F)$. In the search for an equilibrium we reason backwards in terms of the values of A (note that equation (3.2) does not imply that A increases monotonically over time). Consider a value of A such that $A \in (A^P, A^F)$. Then it holds that the leader's payoff is higher than the payoff of the follower. This implies that (without loss of generality) Firm i has an incentive to be the first investor there. Firm j anticipates this and would invest at $A - \varepsilon$. Repeating this reasoning we reach an equilibrium in which Firm i invests at A^P and Firm j waits with replacement until demand equals A^F. Note that if both firms invest at A^P with probability one, they end up with the low payoff $V^J(A^P)$. At $A = A^P$ simultaneous replacement is not profitable because demand is insufficient.

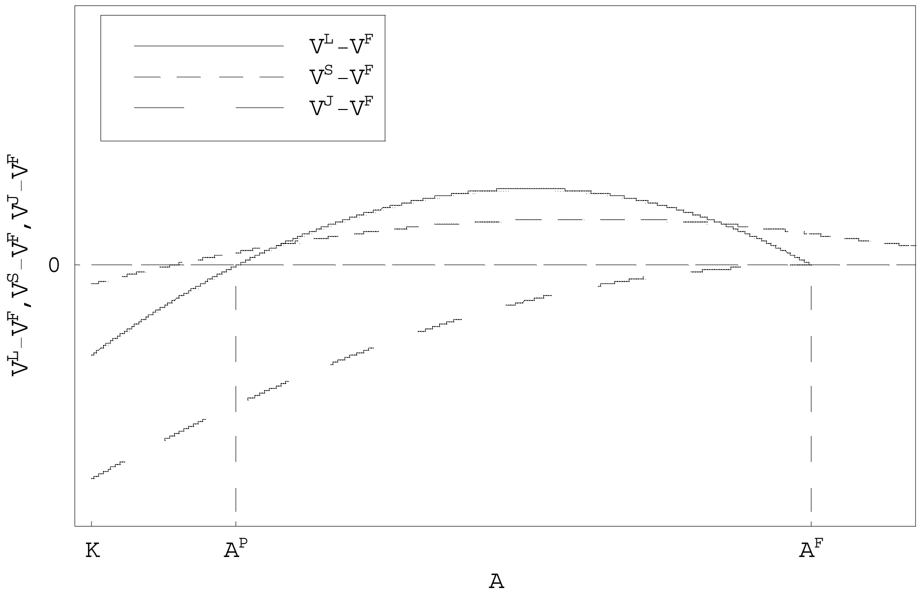

Figure 3.1: The values of the leader, V^L, optimal simultaneous replacement, V^S, and early simultaneous replacement, V^J, relative to the value function of the follower, V^F, for a set of parameter values resulting in a preemptive replacement at A^P (leader) and A^F (follower).

Figure 3.1 depicts the payoffs (relative to the follower payoff) associated with the preemptive equilibrium. Since the firms are identical it is not clear beforehand which of them will be the leader. In order to formalize the analysis of how the roles of the firms are determined, we adopt the approach of Thijssen et al. (2002). This approach extends the perfect equilibrium concept of Fudenberg and Tirole (1985) to stochastic games. As in Fudenberg and Tirole (1985), the firms use mixed strategies in which the expected payoff is equal to the payoff of the follower (recall that the firms are risk-neutral). It is argued there (see also Torvund, 1999) that in continuous-time preemption games a closed-loop strategy of Firm i consists of a collection of simple strategies $(G_i(\cdot), p_i(\cdot))$. $G_i(t)$ is the probability that Firm i has invested by time t given that Firm j has not invested. The function $p_i(t)$ is the measure of the intensity of atoms in the interval $[t, t+dt]$. It can be interpreted as the probability of playing the first row and the first column (for Firm 1 and Firm 2, respectively) in the following 2×2 game: {{*replace*, *replace*}, {*replace*, *don't replace*}, {*don't replace*, *replace*}, {*don't replace*, *don't replace*}}. Playing this game costs no time and the game is repeated until at least one firm invests.

In case $A \in \left(A^P, A^F\right)$, the value of p_i (p_j) is determined as follows. Since p_i (p_j) is the probability that Firm i (j) replaces its asset, Firm i sets p_i such that

$$V_i = \max_{p_i} \left[p_i\left(1-p_j\right)V^L + \left(1-p_i\right)p_jV^F + p_ip_jV^J + \left(1-p_i\right)\left(1-p_j\right)V_i\right]. \quad (3.26)$$

Since Firm i replaces its asset with probability p_i and Firm j with probability p_j, the probability that Firm i obtains the leader role is $p_i\left(1-p_j\right)$. Similarly, with probability $\left(1-p_i\right)p_j$ Firm i is the follower, p_ip_j is the joint investment probability, and with probability $\left(1-p_i\right)\left(1-p_j\right)$ nothing happens and the game is repeated. After writing down the first-order conditions for Firm i and Firm j, and imposing symmetric strategies, we obtain that

$$p = p_i = p_j = \frac{V^L - V^F}{V^L - V^J}. \quad (3.27)$$

It holds that the equilibrium strategy of Firm i equals

$$p\left(t\right) = \begin{cases} 0 & \text{if} \quad A\left(t\right) < A^P, \\ \frac{V^L(A(t)) - V^F(A(t))}{V^L(A(t)) - V^J(A(t))} & \text{if} \quad A\left(t\right) \in \left[A^P, A^F\right], \\ 1 & \text{if} \quad A\left(t\right) > A^F, \end{cases} \quad (3.28)$$

and

$$G\left(t\right) = \begin{cases} 0 & \text{if} \quad A\left(t\right) < A^P, \\ \frac{V^L(A(t)) - V^J(A(t))}{V^L(A(t)) - 2V^J(A(t)) + V^F(A(t))} & \text{if} \quad A\left(t\right) \in \left[A^P, A^F\right). \\ 1 & \text{if} \quad A\left(t\right) \geq A^F \end{cases} \quad (3.29)$$

After substituting $p = p_i = p_j$ in (3.26), the value of Firm i can be expressed as

$$V_i = \frac{p\left(1-p\right)V^L + p(1-p)V^F + p^2V^J}{2p - p^2} = V^F. \quad (3.30)$$

Consequently, for $A\left(t\right) \in \left[A^P, A^F\right]$, the probability that one of the firms invests at time t equals

$$\Pr\left(\text{one firm has invested}|t\right) = \frac{2 - 2p\left(t\right)}{2 - p\left(t\right)}, \quad (3.31)$$

while the firms invest simultaneously with probability

$$\Pr\left(\text{two firms have invested}|t\right) = \frac{p\left(t\right)}{2 - p\left(t\right)}. \quad (3.32)$$

If $A\left(0\right) < A^P$, the leader payoff curve lies below the follower curve which implies that it is optimal for both firms to refrain from investment. At $A = A^P$, the leader and the follower values are equal. This implies that (3.28) and (3.31) yield the probability of being the leader (or follower) equal to $\frac{1}{2}$. The probability of simultaneous investment at $A = A^P$ is therefore equal to zero. The leader invests at the moment that $A = A^P$, which is the smallest solution of $V^L(A) = V^F(A)$, and the follower waits until A^F is reached.

If the stochastic process starts at $A(0) \geq A^P$, at least one of the firms invests immediately. The probability of an immediate joint investment leading to the low payoff $V^J(A(0))$ is $\frac{p(0)}{2-p(0)}$ (cf. (3.32)). In this case, according to (3.28), $p(0) > 0$ since the payoff of the leader exceeds the payoff of the follower. This makes the probability of investing jointly, and ending up with a low payoff of $V^J(A(0))$, become positive.

In order to be able to translate the derived mixed strategies into applicable decision rules (since "real-world decision makers do not flip coins"), we refer to the approach of Harsanyi (1973). He has shown that a mixed-strategy equilibrium of a complete information game, such as the one analyzed in this paper, can be interpreted as the limit of a pure-strategy equilibrium of a slightly perturbed game of incomplete information (see also Tirole, 1988).[12] Consequently, instead of assuming that firms play mixed strategies in the described above 2×2 game, one can assume that the actual payoff resulting from becoming the leader equals $V^L(A(t)) + \varepsilon$, and ε is distributed according to a density function $\varphi(\varepsilon)$ with a bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$, $\underline{\varepsilon} < 0 < \overline{\varepsilon}$.[13] The firm observes its own realization of ε but not the one of its competitor. Now, it can be shown that a symmetric Bayesian equilibrium in pure strategies exists. There is a critical value of $\varepsilon = \varepsilon^*$ such that the optimal strategy for Firm i is to invest if and only if $\varepsilon_i > \varepsilon^*$. Consequently, the firms do not have to invoke randomizing devices in the implementation of optimal actions.

3.4.2 Simultaneous Equilibrium

The other type of outcome that can occur in the analyzed real option game is the simultaneous replacement equilibrium. In such a case, the firms replace their production facilities at the same point in time defined by $T^S \equiv \inf\left(t|A(t) \geq A^S\right)$. A graphical illustration of the simultaneous equilibrium is depicted in Figure 3.2. From this figure it can be concluded that no firm has an incentive to deviate from this equilibrium since the payoff of this strategy exceeds all other payoffs.[14]

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[12]For sufficient conditions on the payoff functions and information structure when such an interpretation is possible, see Milgrom and Weber (1986).

[13]Here, the uncertainty about the value of parameter ε is just a reduced form representation of uncertainty about the value of (the one of) the firm-specific primitive parameters of the model.

[14]Of course, the payoffs resulting from the preemptive equilibrium in Section 3.4.1 may be lower than those associated with the optimal joint replacement. However, the occurrence of the preemptive

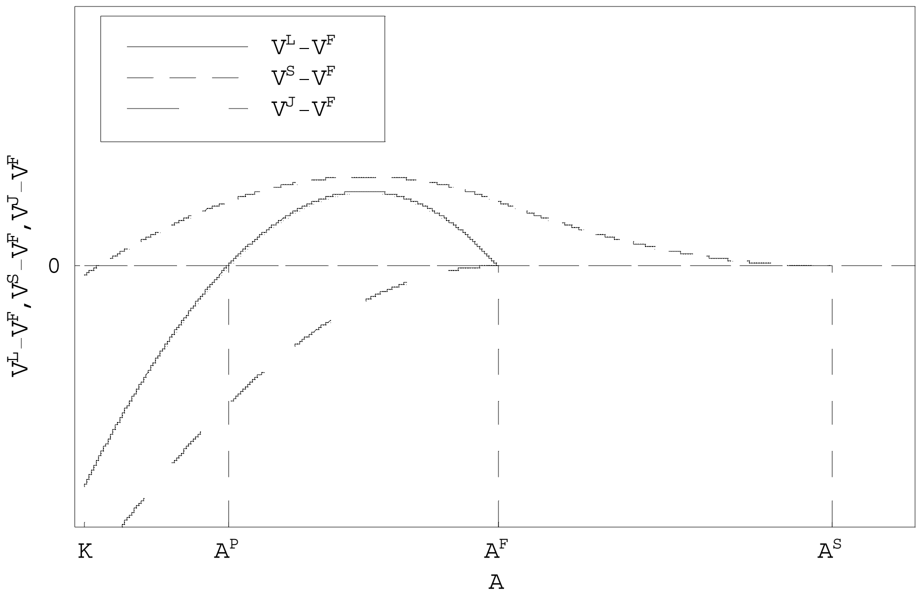

Figure 3.2: The values of the leader, V^L, optimal simultaneous replacement, V^S, and early simultaneous replacement, V^J, relative to the value function of the follower, V^F, for a set of parameter values resulting in the optimality of a simultaneous replacement at A^S.

The occurrence of a particular type of equilibrium is determined by the relative payoffs. The preemptive equilibrium occurs when

$$\exists A \in \left(A^P, A^F\right) \text{ such that } V^L(A) > V^S(A), \tag{3.33}$$

i.e. when for some A it is more profitable to become the leader than to replace production facilities simultaneously. Otherwise, simultaneous replacement is the Pareto-dominant equilibrium. In the latter case, the strategy of Firm i can be formalized as

$$p(t) = \begin{cases} 0 & \text{if} \quad A(t) < A^S, \\ 1 & \text{if} \quad A(t) \geq A^S, \end{cases} \tag{3.34}$$

and

$$G(t) = \begin{cases} 0 & \text{if} \quad A(t) < A^S, \\ 1 & \text{if} \quad A(t) \geq A^S. \end{cases} \tag{3.35}$$

equilibrium, as in Section 3.4.1, is due to the fact that values of A exist that the corresponding leader payoff exceeds the value from the joint replacement strategy. It is the lack of coordination among the firms (with possible transfer of excess value) that leads to *ex post* Pareto-inefficient outcomes. In the case of the simultaneous equilibrium the payoff of the leader never exceeds the payoff from optimal joint replacement and therefore the preemptive equilibrium, while still existent, is Pareto-dominated (see Fudenberg and Tirole, 1985)

The following proposition implies that firms replace their production facilities simultaneously if the investment cost is sufficiently high.

Proposition 3.1 *A unique I^* exists such that $\forall I > I^*$ simultaneous replacement is the Pareto-dominant equilibrium.*

Proof. See the Appendix. ■

This proposition constitutes an important result with respect to the comparison between the real option exercise game with *profit* uncertainty and the situation where the firms face *product market* uncertainty. In the first case the occurrence of either of the equilibria does not depend on the irreversible cost associated with the investment decision (see Huisman and Kort, 1999). This results from the fact that the optimal threshold under profit uncertainty is proportional to the investment cost I. This proportionality is a consequence of the multiplicative way in which uncertainty enters the profit function. Conversely, introducing market uncertainty in a Cournot model results in the optimal threshold being no longer proportional to I. This is the reason why the resulting equilibrium regions depend on the sunk cost.[15]

3.5 Uncertainty and Asset Replacement Thresholds

Since the firms' decisions to replace production assets are irreversible (sunk cost I cannot be recovered) and they have the flexibility in timing the replacement, they replace their production assets later than a simple NPV rule would indicate. In a non-strategic framework, there exists an option value of waiting for better (but never complete) information which is taken into account before committing the corporate resources. As uncertainty about the demand grows, the firm is going to wait with replacement for a higher level of demand, as the classical real option theory suggests. However, it also has to take into account the interactions in the product market, that may substantially reduce the value of the timing flexibility. Kulatilaka and Perotti (1998) obtained in a two-period model that these interactions may in fact result in a negative relationship between the required level of demand at which resources are committed and uncertainty. In this section, we examine how uncertainty influences the level of demand triggering investment in the continuous-time model.

[15] In general, the investment cost affects the boundaries of the equilibrium regions. Therefore, the lack of such a relationship in a profit uncertainty model is rather a coincidence than a rule.

First, we investigate the impact of volatility on the optimal asset replacement thresholds of the follower and of optimal simultaneous replacement. In these cases the optimal threshold, A^{opt}, can be expressed as

$$A^{opt} = \frac{\beta_1}{\beta_1 - 1} f(I, K, k, r, \alpha). \tag{3.36}$$

It is straightforward to derive that

$$\frac{\partial A^{opt}}{\partial (\sigma^2)} = -\frac{1}{(\beta_1 - 1)^2} f(I, K, k, r, \alpha) \frac{\partial \beta_1}{\partial (\sigma^2)} > 0, \tag{3.37}$$

so that the optimal replacement thresholds of the follower and of optimal simultaneous replacement increase in uncertainty. In the case of the follower's decision, the competitor has already replaced its asset. Hence, what is left to do for the remaining firm is to choose the optimal replacement timing. Since the opponent has already taken its decision, strategic interactions do not play a role here. So, as in standard real options theory, also here the threshold goes up with uncertainty, which reflects the value of waiting argument. In determining the optimal simultaneous replacement timing strategic interactions do not play a role either. Therefore, analogous to the follower's case, the value of waiting argument also prevails here.

The impact of volatility on the production facility replacement threshold of the leader requires an additional analysis. Let us set the marginal cost k to zero to simplify the notation.[16] The replacement threshold of the leader equals $\max\left(A(0), A^P\right)$, where A^P is the smallest root of $\xi(A) = 0$. To determine the effect of market uncertainty on A^P, we calculate the derivative of $\xi(A)$ with respect to σ. The change of (3.25) resulting from a marginal increase in σ^2 can be decomposed as follows

$$\frac{d\xi(A)}{d(\sigma^2)} = \left(\underbrace{\frac{\partial \xi(A)}{\partial \beta_1}}_{\text{"Waiting" effect}} + \underbrace{\frac{\partial \xi(A)}{\partial A^F} \frac{dA^F}{d\beta_1}}_{\text{Strategic effect}} \right) \frac{\partial \beta_1}{\partial (\sigma^2)}. \tag{3.38}$$

The derivative $\frac{\partial \xi(A)}{\partial \beta_1} \frac{\partial \beta_1}{\partial (\sigma^2)}$ directly measures the influence of uncertainty on $\xi(A)$, thus on the net benefit of being the leader. The product $\frac{\partial \xi(A)}{\partial A^F} \frac{dA^F}{d\beta_1} \frac{\partial \beta_1}{\partial (\sigma^2)}$ reflects the impact on the net benefit of being the leader of the fact that the follower replacement threshold increases with uncertainty.

[16] An additional motivation for this simplification is provided by the fact that for, e.g., the majority of intangible/information products the marginal cost of a unit of good or service is negligible (cf. Shapiro and Varian, 1998).

It can be shown that

$$\frac{\partial \xi(A)}{\partial \beta_1}\frac{\partial \beta_1}{\partial(\sigma^2)} < 0, \quad (3.39)$$

$$\frac{\partial \xi(A)}{\partial A^F}\frac{dA^F}{d\beta_1}\frac{\partial \beta_1}{\partial(\sigma^2)} > 0. \quad (3.40)$$

At first sight, the joint impact of both effects is ambiguous. (3.39) represents the simple value of waiting argument: if uncertainty is large, it is more valuable to wait for new information before replacing the existing production facility (cf. Dixit and Pindyck, 1996). As we have just seen, this also holds for the follower. The implication for the leader of the follower replacing later is that the leader has a cost advantage for a longer time. This makes an earlier replacement of the leader potentially more beneficial. This effect is captured by (3.40), which can thus be interpreted as an increment in the strategic value of becoming the leader vs. the follower resulting from the delay in the follower's implementation of the superior technology. Obviously, the latter effect is not present in monopolistic/perfectly competitive markets, where the impact of uncertainty is unambiguous.

It is possible to show that the direct effect captured by (3.39) dominates, irrespective of the values of the input parameters.

Proposition 3.2 *When uncertainty in the product market increases, the threshold value of the demand at which the leader replaces its production facility increases too.*

Proof. See the Appendix. ■

From Proposition 3.2 it can be concluded that the leader threshold responds to volatility in a qualitatively similar way as a non-strategic threshold, i.e. it increases with uncertainty. The reason for this result is the following. First, in our model we introduced the possibility of postponing the replacement of the production facility. Increased uncertainty raises the profitability of replacement (because the follower replaces later) but the value of the option to wait rises even more. Second, uncertainty could be beneficial for earlier replacement because of the convex shape of the net gain function, resulting in a power option-like type of payoff (cf. Kulatilaka and Perotti, 1998). Then, while performing a mean preserving spread, downside losses are more than compensated by upside gains. However, unlike the two-period framework of Kulatilaka and Perotti (1998), in our continuous-time model the net gain function is always linear in the stochastic variable A. If the leader invests, the profit flow π^{00} is replaced by the profit flow π^{10}, and it is clear from (3.3) and (3.4) that $\pi^{10} - \pi^{00}$ is linear. The same holds for the follower investment ($\pi^{11} - \pi^{01}$ linear) and simultaneous investment (linearity of $\pi^{11} - \pi^{00}$). To see whether the convexity argument could also work here, in Section 3.6 we consider the decision to start production. In this case the firms are not active initially and can start up production only upon investing. Consequently, the net gain flows for the leader, and the follower, being equal to π^{10} and π^{01}, respectively, are convex in A.

3.6 Decision to Start Production

Consider two firms having the possibility to start production in a new market where there is no incumbent. The new market assumption implies, in contrast with Sections 3.3-3.4, that the firms can only start realizing profits after incurring a sunk cost I. It still holds that demand follows the stochastic process (3.2). With little loss of generality the marginal cost of a unit of output after starting production is set to $k = 0$.

First, we calculate the value of the demand parameter for which it is optimal for the follower to start production. After, by now, familiar steps it is obtained that

$$A^{FN} = 3\sqrt{\frac{\beta_1}{\beta_1 - 2} I \varrho}. \tag{3.41}$$

Given that for positive ϱ it holds that $\beta_1 > 2$, it can be shown that

$$\frac{\partial A^{FN}}{\partial \left(\sigma^2\right)} > 0. \tag{3.42}$$

The optimal follower threshold (3.41) exists only for $\sigma^2 < r - 2\alpha$. For a relatively high degree of uncertainty, i.e. for $\sigma^2 \geq r - 2\alpha$ (which corresponds to $\beta_1 \in (1, 2]$), the follower will never start production since for such levels of uncertainty the value of the option to invest always exceeds the net present value of investment. In the limiting case, the optimal follower threshold (3.41) is equal to

$$\overline{A^{FN}} \equiv \lim_{\sigma^2 \uparrow r - 2\alpha} A^{FN} = 3\sqrt{(3r - 4\alpha) I} \tag{3.43}$$

(for a derivation see the Appendix). Equation (3.43) corresponds to the maximal value of A^{FN} provided that it is finite. In case information about the uncertainty level is imperfect, the investment problem is solved by first calculating the uncertainty implied by the threshold $\overline{A^{FN}}$. Subsequently, the decision maker can decide whether the true level of uncertainty is more likely to lie below or above the implied value. In the latter case, he should refrain from entering the market.

Now, let t to be the moment at which the leader starts producing in the new market. The value of the follower at t is equal to

$$V^{FN}(A) = \begin{cases} \left(\frac{\frac{1}{9}\left(A^{FN}\right)^2}{\varrho} - I\right)\left(\frac{A}{A^{FN}}\right)^{\beta_1} & \text{if } A \leq A^{FN}, \\ \frac{1}{9}\frac{A^2}{\varrho} - I & \text{if } A > A^{FN}, \end{cases} \tag{3.44}$$

The value of the leader at t can be expressed as

$$V^{LN}(A) = \begin{cases} \begin{array}{c} \frac{1}{4}\frac{A^2}{\varrho} - I \\ -\frac{5}{36}\frac{\left(A^{FN}\right)^2}{\varrho}\left(\frac{A}{A^{FN}}\right)^{\beta_1} \end{array} & \text{if } A \leq A^{FN}, \\ \frac{1}{9}\frac{A^2}{\varrho} - I & \text{if } A > A^{FN}. \end{cases} \tag{3.45}$$

From (3.44) and (3.45) it is obtained that indeed the leader and follower values are convex in A. The threshold of the leader, being the preemption point, is the smallest solution of the following equation

$$V^{LN}(A) - V^{FN}(A) = \frac{1}{4}\frac{A^2}{\varrho} - I - I\left(\frac{9}{4}\frac{\beta_1}{\beta_1 - 2} - 1\right)\left(\frac{A}{A^{FN}}\right)^{\beta_1} = 0. \tag{3.46}$$

The impact of uncertainty on the threshold of the leader is not straightforward. Similar as in the model with the firms initially competing in the product market, there are two effects: the effect of the waiting option and of the strategic option. Let us denote $V^{LN}(A) - V^{FN}(A)$ by $\xi^N(A)$. We have

$$\frac{d\xi^N(A)}{d(\sigma^2)} = \tag{3.47}$$
$$\underbrace{\frac{\partial\xi(A)}{\partial(\sigma^2)} + \frac{\partial\xi(A)}{\partial A^{FN}}\frac{dA^{FN}}{d(\sigma^2)}}_{\text{Discount rate effect}} + \underbrace{\left(\frac{\partial\xi(A)}{\partial\beta_1} + \frac{\partial\xi(A)}{\partial A^{FN}}\frac{dA^{FN}}{d\beta_1}\right)\frac{\partial\beta_1}{\partial(\sigma^2)}}_{\text{Direct uncertainty effect}}.$$

Uncertainty affects the magnitude of each of the mentioned effects via parameter β_1, as in Section 3.5, and via the effective discount rate, ϱ. The latter contribution results from the convexity of the profit function, i.e. its proportionality to the square of the underlying stochastic variable A (see (3.45)).

After substituting the functional forms of $V^{LN}(A)$ and $V^{FN}(A)$ into $\xi^N(A)$ and calculating the derivative explicitly, the following result is obtained.

Proposition 3.5 *The threshold value of the demand at which the leader starts production increases with uncertainty.*

Proof. See the Appendix. ■

Analogous to the follower case, there exists a critical level of uncertainty, $\sigma^2 = r - 2\alpha$, above which it is optimal for the leader never to invest. In the limit, where $\sigma^2 \to r - 2\alpha$, the leader threshold is the smaller root of the equation (for the proof see the Appendix)

$$\left(\left(\frac{A}{A^{FN}}\right)^2 - 1\right) I - \frac{A^2}{2\,(3r - 4\alpha)} \ln\left(\frac{A}{A^{FN}}\right) = 0. \tag{3.48}$$

The conclusion is that also in the case of a new market, uncertainty raises the threshold levels of market demand at which it is optimal for firms to invest. Moreover, the resulting convexity of the payoff functions not only raises the threshold of the firms but also results in a subset of parameters for which no replacement is optimal.

3.7 Uncertainty and Replacement Timing

Until now we analyzed the impact of uncertainty and strategic interactions on the optimal replacement *threshold* of the firm. Although threshold values and *timing* have a lot to do with each other, it cannot be concluded in general that the relation between the two is monotonic (cf. Sarkar, 2000). After having determined the dependency of threshold values on uncertainty in Section 3.5 and 3.6, in this section we investigate the relationship between uncertainty, expected timing of replacement and the probability with which the threshold is reached within a time interval of a given length.

First, let us observe that the expectation of the first passage time equals

$$E\left[T^*\right] = \frac{1}{\alpha - \frac{1}{2}\sigma^2} \ln \frac{A^*\left(\sigma^2\right)}{A}, \tag{3.49}$$

where $A^*\left(\sigma^2\right)$ denotes the optimal replacement threshold as a function of uncertainty. We note that expectation (3.49) tends to infinity for $\sigma^2 \to 2\alpha$ and does not exist for $\sigma^2 \geq 2\alpha$.[17] For $\sigma^2 < 2\alpha$ it holds that

$$\frac{\partial E\left[T^*\right]}{\partial\left(\sigma^2\right)} = \frac{1}{2\left(\alpha - \frac{1}{2}\sigma^2\right)^2} \ln \frac{A^*\left(\sigma^2\right)}{A} + \frac{1}{\alpha - \frac{1}{2}\sigma^2} \frac{\frac{dA^*}{d(\sigma^2)}}{A^*\left(\sigma^2\right)} > 0. \tag{3.50}$$

The expected timing of replacement increases with uncertainty due to two effects. First, for any given threshold, the associated expected first passage time is increasing with

[17] Increasing σ^2 beyond 2α implies that the probabilities of surviving without reaching the threshold before a given time do not fall sufficiently fast for longer hitting times. Since the expectation is the sum of the product of the first passage times and their probabilities, an insufficient decay in the survival probabilities (without reaching the threshold) results in the divergence of the expectation.

uncertainty (cf. the first component of the RHS of (3.50)). Second, for a fixed level of uncertainty, an increase in the optimal investment threshold leads to an increase in the expected time to reach (cf. second component of RHS of (3.50)). Based on (3.50) it can be concluded that *whenever the threshold goes up due to more uncertainty, it also holds that the expected time to replace the production facility increases.*

An alternative approach to measure the impact of uncertainty on the timing of replacement is to look at the probability with which the threshold is reached within a time interval of a given length, say τ. Contrary to the expected first passage time, this approach does not impose any restrictions on the values of σ. The probabilities of optimal asset replacement within a given interval are particularly useful when this interval coincides with a budgeting period.[18]

After substituting $y = \ln \frac{A^*}{A}$ in the formula (8.11) in Harrison (1985) and rearranging, we obtain

$$P(T < \tau) = \Phi\left(\frac{-\ln\frac{A^*}{A} + \left(\alpha - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right) + \left(\frac{A^*}{A}\right)^{\frac{2\alpha}{\sigma^2}-1}\Phi\left(\frac{-\ln\frac{A^*}{A} - \left(\alpha - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right), \tag{3.51}$$

where T denotes the time to reach the threshold and Φ is the standard normal cumulative density function. As already pointed out by Sarkar (2000), the derivative $\frac{\partial P(T<\tau)}{\partial\sigma}$ does not have an unambiguous sign and it can thus be shown that, in general, uncertainty can affect the probability of reaching the threshold within a given time in both directions.

First, we illustrate the relationship between the first passage time, volatility and related probabilities for the follower threshold since this threshold is unaffected by strategic considerations. Subsequently, we present results of simulations related to the threshold of the leader. In this part we use the model of Sections 3.3-3.4. The results for the decision to start production are qualitatively similar and are not reported.[19]

From Figure 3.3 it can be concluded that the form of the relationship between the uncertainty and the probability of reaching the threshold depends on the length of the time interval. For sufficiently long time intervals, the probability of reaching the threshold decreases with volatility. Intuitively, this can be explained by the fact that

[18] For a discussion of the capital budgeting process at the corporate level see Kaplan and Atkinson (1998), Ch. 14 and Bower (1986), Ch. 1-3.

[19] In this case a restriction on σ has to be imposed in order to ensure the positive sign of ϱ (cf. (3.12)).

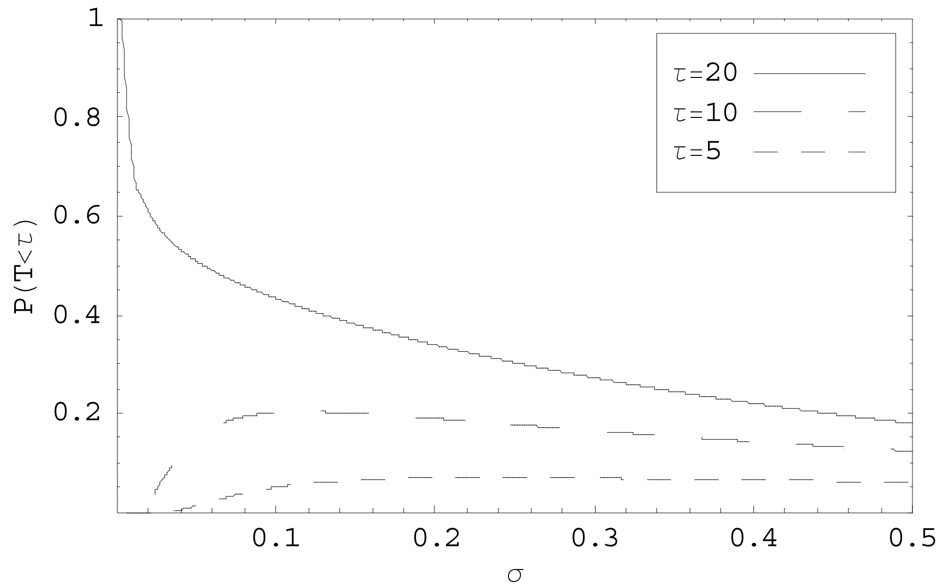

Figure 3.3: The cumulative probability of reaching the optimal follower replacement threshold as a function of demand uncertainty for a set of parameter values: $A = 4$, $r = 0.05$, $\alpha = 0.015$, $K = 3$, $k = 0$ and $I = 60$.

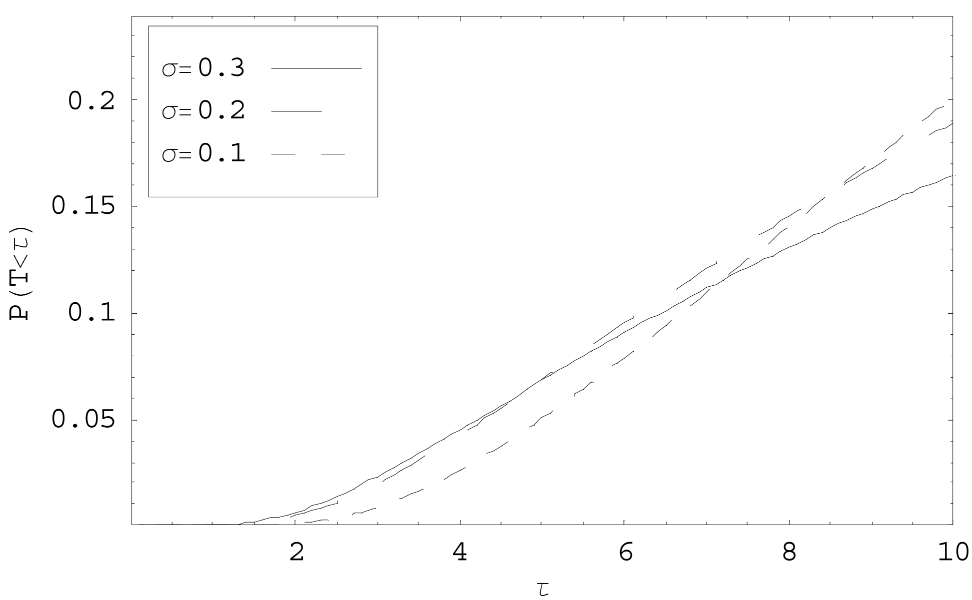

Figure 3.4: The cumulative probability of reaching the optimal follower replacement threshold as a function of time horizon for the set of parameter values: $A = 4$, $r = 0.05$, $\alpha = 0.015$, $K = 3$, $k = 0$ and $I = 60$.

the probability mass of the first passage time density function moves to the right (cf. (3.50)) and longer times of reaching the demand level triggering replacement become more likely. Moreover, the trigger itself is increasing with σ.

For low values of τ the probability of reaching the replacement threshold first increases and then decreases. For $\sigma = 0$ the probability of reaching the threshold within a certain time interval is zero when the optimal replacement time lies outside this interval. Increasing σ results in a spread of the probability mass, so that the probability of reaching the demand threshold becomes positive for a strictly positive σ. A larger spread is initially equivalent to a higher probability of hitting the optimal replacement threshold. However, when volatility continues to rise, at a certain moment the effect of the probability mass shifting to the right starts to dominate the effect of the spread. As a consequence, the cumulative probability of reaching the threshold becomes smaller again.

Figure 3.4 indicates that the probability of reaching the follower threshold always increases with the time interval, which is of course trivial. The relevant observation is that this relationship is more pronounced for low levels of market uncertainty. This results from the fact that in the absence of uncertainty the optimal investment trigger is reached at a specified point in time with probability 1 and the corresponding cumulative density function is a heaviside step function. Increasing volatility spreads the probability mass around the point corresponding to the deterministic case. This leads to an increased cumulative chance of reaching the trigger at points in time situated to the left of this specified point in time, while the reverse is true for the point situated to the right. This influences the shape of the cumulative distribution function whose slope decreases with uncertainty.

Figure 3.5 allows for a closer inspection of the relationship between the timing of asset replacement and uncertainty. It can be concluded that, irrespective from the length of the time interval, there exists a level of uncertainty beyond which a further increase in uncertainty always reduces the probability of the optimal asset replacement. The relationship between this level and the length of the time interval is inverse, i.e. the longer the time interval, the lower level of uncertainty for which a further uncertainty increase reduces the probability of the optimal replacement. For example, using the parameters from Figure 3.5 we can conclude that for $\tau = 5$ this critical value of uncertainty, σ, is 0.234, for $\tau = 10$ it is only 0.118, whereas for $\tau = 20$ increased uncertainty always reduces the cumulative probability of optimal investment.

Figure 3.6 indicates that the probability of the optimal replacement increases in uncertainty for a sufficiently short time interval and decreases for a sufficiently long

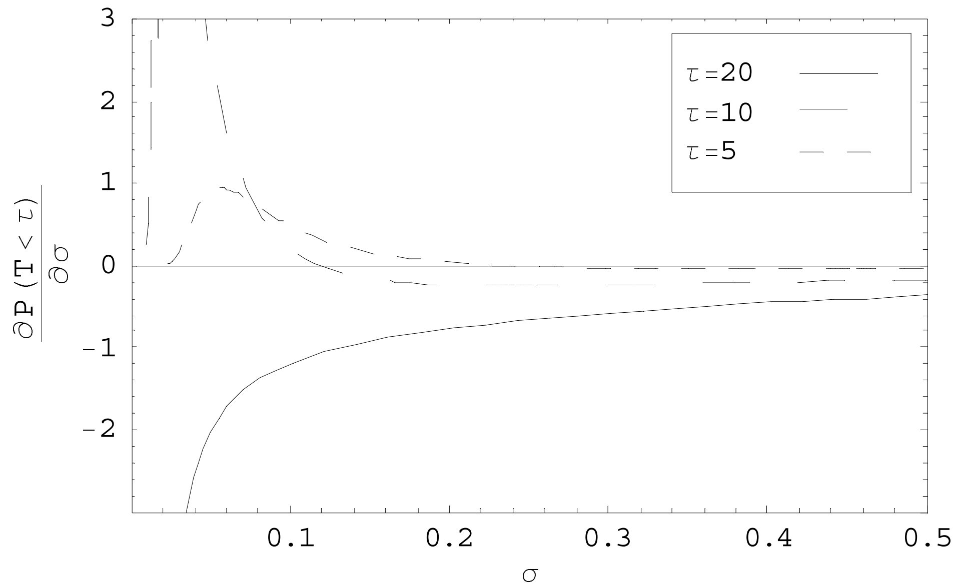

Figure 3.5: The derivative with respect to market uncertainty of the cumulative probability of reaching the optimal follower threshold as a function of uncertainty for the set of parameter values: $A = 4$, $r = 0.05$, $\alpha = 0.015$, $K = 3$, $k = 0$ and $I = 60$.

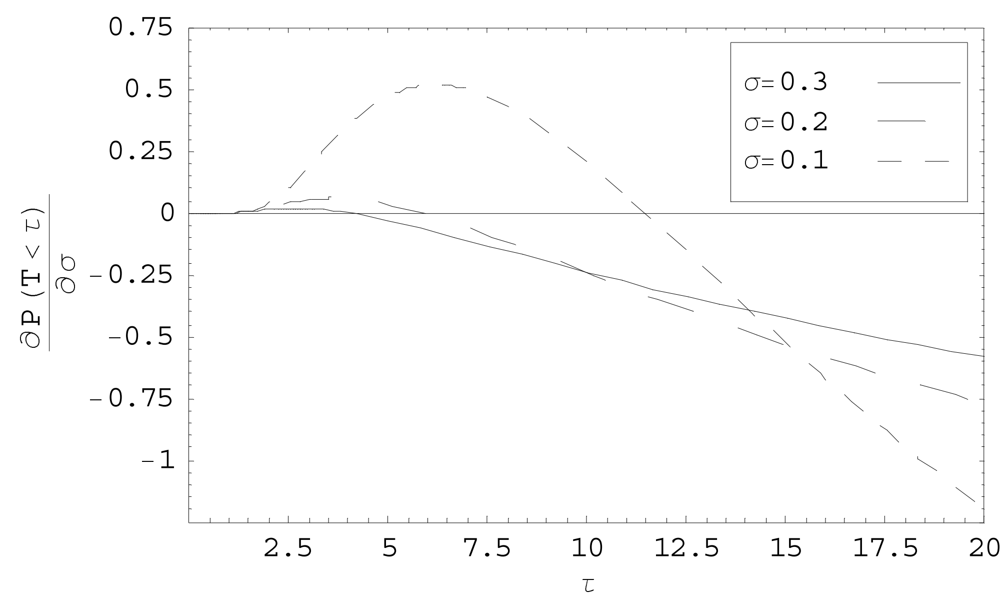

Figure 3.6: The derivative with respect to market uncertainty of the cumulative probability of reaching the optimal follower threshold as a function of time horizon for the set of parameter values: $A = 4$, $r = 0.05$, $\alpha = 0.015$, $K = 3$, $k = 0$ and $I = 60$.

horizon. Moreover, the derivative of the probability of reaching the optimal threshold changes its sign only once. Finally, Figure 3.6 allows for the conclusion that the length of time interval beyond which uncertainty negatively affects the probability of optimal replacement is, again, negatively related to the uncertainty level. For $\sigma = 0.1$ the interval length which separates the areas of a positive and a negative relationship equals 11.46 years, for $\sigma = 0.2$ it equals 5.87 years, while for $\sigma = 0.3$ it drops to 4.06 years.

Despite the presence of strategic effects, the probability of asset replacement of the leader within a given time interval responds to changes in uncertainty and the length of the interval in a similar way as the corresponding probabilities of the follower. For low σ's the probability of investing increases more rapidly with the length of the time interval than for high σ's. Moreover, for high τ's the probability of replacing the existing asset is always decreasing with uncertainty, while for low τ's the probability behaves in a non-monotonic way.

The relationship between uncertainty, first passage time and probabilities of reaching the leader threshold is illustrated in Table 3.1 below.

σ	$\tau = 1$	$\tau = 2$	$\tau = 5$	$\tau = 10$	$\tau = 15$	$\tau = 20$
0.05	0.06	2.39	24.11	54.32	71.17	80.97
0.10	0.61	5.93	26.79	47.94	59.70	67.24
0.20	0.62	5.14	21.00	36.47	45.10	50.72
0.30	0.46	3.97	16.50	28.66	35.31	39.55
0.40	0.39	3.30	13.57	23.22	28.29	31.39
0.50	0.36	2.93	11.55	19.23	23.02	25.21

Table 3.1: The cumulative probability (in percentages) of reaching the optimal leader replacement threshold as a function of demand uncertainty for the set parameter values: $A = 2$, $r = 0.05$, $\alpha = 0.015$, $k = 0$, $K = 3$ and $I = 60$.

The relationship between the investment probability of the leader and uncertainty is analogous to the corresponding relationship of the follower. The probability that the leader replaces its production facility within a given time interval decreases with uncertainty when the length of this interval is sufficiently large. In a situation where the relevant interval is sufficiently short, there are two contradictory effects. On the one hand, the investment probability increases because higher volatility enhances the chance of reaching a particular threshold early. On the other hand, this probability eventually declines with uncertainty because then the effect of the probability mass shifting to the right begins to dominate.

Now, we formulate the following proposition, which extends Sarkar (2000) by defining the time interval lengths separating a monotonic and non-monotonic relationship between uncertainty and the investment probability .

Proposition 3.4 *Define*

$$\tau^* \equiv \frac{1}{\alpha} \ln \frac{A^*}{A}, \ \alpha > 0, \tag{3.52}$$

as the point in time at which the replacement threshold A^* is reached in the deterministic case. Then it holds that for $\tau < \tau^*$ the probability of reaching the investment threshold A^* before τ increases with uncertainty at a relatively low level of uncertainty and decreases for a relatively high level, whereas for $\tau > \tau^*$ the probability of reaching the optimal threshold before τ always decreases with uncertainty.

Proof. See the Appendix. ■

On the basis of Proposition 3.4 it may be concluded that the replacement horizon being equal to the optimal timing of replacement in the deterministic case separates the regions of monotonic and non-monotonic relationship between uncertainty and the probability of replacement. In Table 3.1, the parameters are chosen in such a way that the optimal timing of replacement in the deterministic case equals $\tau^* = 9.36$. Therefore the investment-uncertainty relationship in columns 2-4 is non-monotonic, while it is negative in columns 5-7.

In order to determine τ^* for the leader, we need to determine its replacement timing in the deterministic case. It holds that the optimal investment timing of the leader in the model without uncertainty is equal to the rent equalization point in the preemption game of Fudenberg and Tirole (1985). In the case of the follower, τ^* is equal to the point of time at which the incremental flow from operations, $\pi^{11} - \pi^{01}$, reaches the flow associated with the replacement cost, Ir. Hence, τ^* corresponds to the optimal Jorgensonian trigger, which equalizes the flow costs and revenues of the project. The optimal simultaneous replacement closely resembles the case of the follower trigger. The only difference is that now, the incremental profit flow equals $\pi^{11} - \pi^{00}$.

Finally, we would like to point out that our analysis also extends to the situation where $\alpha \leq 0$. In such a case and without uncertainty the firms would face now-or-never decisions. Therefore, it holds that $\tau^* \in \{0, \infty\}$, so that the relationship between the investment probability within a given time interval and uncertainty will be either non-monotonic or decreasing for all time horizons. This implies that in mature industries (i.e. those with non-positive growth rate), the probability of launching existing positive NPV projects always decreases with uncertainty. As far as initially negative NPV projects are concerned, the probability of their optimal execution is initially increasing with uncertainty. When uncertainty becomes sufficiently high, the replacement probability starts to fall.

3.8 Conclusions

The purpose of this chapter is to analyze the firm's decision to replace an existing production facility with a technologically superior one. In order to capture the effect of strategic interactions among the firms operating in an imperfectly competitive and uncertain environment we model the product market as a Cournot duopoly with a stochastic demand parameter. Such a formulation results in the payoff functions being convex in the stochastic demand parameter.

We determine the types of equilibria of the real option game played by the firms. We show that it is optimal for the firms to replace their production facilities sequentially when the associated cost is relatively low and simultaneously otherwise.

Furthermore, we find that the direct effect of uncertainty (related to the *waiting* option) on the replacement threshold of the leader is always larger than the indirect effect (*strategic* option) resulting from the delay in the follower decision to replace its production facility. Consequently, irrespective from the type of equilibrium, increasing uncertainty always raises the level of demand triggering the optimal replacement. This result also holds in case of the decision to start production rather than to replace the existing asset.

Moreover, it can be concluded that the expected timing of replacement increases with uncertainty. This result supports the view that uncertainty delays the implementation of the new technology, even in the presence of strategic interactions combined with a convex profit function. Moreover, it shows that the result of Kulatilaka and Perotti (1998) that uncertainty can stimulate investment due to strategic interactions does not carry over from a two period model to a continuous time setting.

We also determine the probability of replacing the production facility within a certain time interval. Here, the point in time at which replacement is made optimally in the deterministic case plays a crucial role. For an interval that contains this point in time, the probability of optimal replacement within this time interval decreases with uncertainty. However, if this time interval is that short that the optimal replacement time in the deterministic case lies outside this interval, then the replacement probability goes up with uncertainty when uncertainty is low while it goes down otherwise.

Finally, we would like to discuss the limitations of our approach. In order to ensure analytical tractability and to make our results comparable to Kulatilaka and Perotti (1998), we used a linear demand specification and parallel shifts in demand. Such a model specification allowed us to show that in a continuous-time framework the convexity of payoff functions does not result in investment occurring at lower states of demand when uncertainty is higher, as it does in a two-period model. Our setting enabled us to show that uncertainty enhancing convex payoffs and therefore stimulating investment in a two-period case does not accelerate strategic replacement when there is flexibility in timing the replacement. Of course, our predictions do not automatically carry over to other forms of product market uncertainty. In other words, we do not formally define the classes of demand functions for which the analysis holds. With respect to robustness of our results, it is also important to relax other assumptions like constant marginal costs and Cournot competition.

3.9 Appendix

Proof of existence and uniqueness of A^P. The outline of the proof follows Grenadier (1996). First, we establish the existence of a root of $\xi(A)$ on the interval $(0, A^F)$. Evaluating $\xi(A)$ at $A = 0$ gives

$$\begin{aligned} \xi(0) &= -I + \frac{(K-2k)^2}{9r} - \frac{(2K-k)^2}{9r} \\ &= -I - \frac{K^2 - k^2}{3r} < 0. \end{aligned} \tag{3.53}$$

Similarly, evaluating $\xi(A)$ at $A = A^F$ yields $\xi\left(A^F\right) = 0$. Finally, calculating the left limit of the first-order derivative of $\xi(A)$ for $A \to A^F$ gives

$$\lim_{A \uparrow A^F} \xi'(A) = -\frac{4(\beta_1 - 1)(K-k)^3}{3\delta\left[9Ir + 4K(K-k)\right]} < 0. \tag{3.54}$$

The signs of $\xi(A)$ at the ends of interval $(0, A^F)$ and the sign of (3.54) implies that $\xi(A)$ has at least one root in the relevant interval.

The uniqueness is proved by showing strict concavity of $\xi(A)$ over interval

$(0, A^F)$. It holds that

$$\xi''(A) = -\frac{\beta_1(\beta_1-1)}{A^2}\left[\left(\frac{3\beta_1}{2(\beta_1-1)}-1\right)9I+\frac{\beta_1}{\beta_1-1}\frac{6K(K-k)}{r}-\frac{3(K^2-k^2)}{r}\right]\left(\frac{A}{A^F}\right)^{\beta_1} < 0. \tag{3.55}$$

The last inequality results from the fact that the sign of the sum of the two last components of the expression in the square brackets is positive for $\beta_1 \to \infty$ and the sum is decreasing with β_1. Consequently, the root is unique. ■

Proof of Proposition 3.1. First, let us define

$$\zeta(A) \equiv V^S(A) - V^L(A). \tag{3.56}$$

Here, we assume $k = 0$. After substituting (3.20) and (3.24) into (3.56) we get[20]

$$\zeta(A) = -\frac{4}{9}\frac{KA}{\delta} + I + \left(\frac{\frac{1}{\beta_1-1}\left(I+\frac{K^2}{9r}\right)}{(A^S)^{\beta_1}} + \frac{\frac{1}{2}\frac{\beta_1}{\beta_1-1}\left(I+\frac{4K^2}{9r}\right)+\frac{K^2}{9r}}{(A^F)^{\beta_1}}\right)A^{\beta_1} \tag{3.57}$$

for $A \leq A^F$. From (3.33) it follows that if on the interval $[A^P, A^F]$ the minimum of $\zeta(A)$ is smaller than zero, a preemptive equilibrium occurs. Otherwise, the firms replace their production facilities simultaneously.[21] The existence of a negative minimum of $\zeta(A)$ depends on the value of the input parameters. The minimum of $\zeta(A)$ occurs for

$$A^{**} = \left(\frac{\frac{4}{9\beta_1}\frac{K}{r-\alpha}\left(A^S A^F\right)^{\beta_1}}{\frac{1}{\beta_1-1}\left(I+\frac{K^2}{9r}\right)(A^F)^{\beta_1}+\left(\frac{1}{2}\frac{\beta_1}{\beta_1-1}\left(I+\frac{4K^2}{9r}\right)+\frac{K^2}{9r}\right)(A^S)^{\beta_1}}\right)^{\frac{1}{\beta_1-1}}. \tag{3.58}$$

It is sufficient to show that

$$\left.\frac{d\zeta(A)}{dI}\right|_{A=A^{**}} = \frac{\partial\zeta(A)}{\partial I} + \left.\frac{\partial\zeta(A)}{\partial A}\right|_{A=A^{**}}\frac{dA^{**}}{dI} > 0. \tag{3.59}$$

Using the fact that

$$\left.\frac{\partial\zeta(A)}{\partial A}\right|_{A=A^{**}} = 0, \tag{3.60}$$

and differentiating (3.57), we obtain that

$$\frac{d\zeta(A)}{dI} = 1 - \left(\frac{A}{A^S}\right)^{\beta_1} - \beta_1\frac{\frac{1}{2}I+\frac{K^2}{3r}}{I+\frac{4}{9}\frac{K^2}{r}}\left(\frac{A}{A^F}\right)^{\beta_1}. \tag{3.61}$$

[20]The proof for general k goes along the same lines and is skipped for the sake of brevity.

[21]Strictly speaking, the preemptive equilibrium still exists in this case but is Pareto-dominated by the simultaneous replacement equilibrium (cf. Fudenberg and Tirole, 1985).

Subsequently, we substitute for A in (3.61) the expression (3.58) for A^{**}. Complexity of the resulting expression yields the necessity to use a numerical procedure. A geometric grid search indicates that $\left.\frac{d\zeta(A_t)}{dI}\right|_{A=A^{**}}$ is positive for $\beta_1 \in [1, \frac{r}{\alpha})$, and $\alpha \in (0,\infty)$, $\beta_1 \in [1,\infty)$, and $\alpha \in (-\infty, 0]$, and for other parameters falling into intervals: $r \in (\alpha, \infty)$, $K \in (0,\infty)$ and $I \in \left(\frac{4}{9}\frac{K^2}{r}, \infty\right)$.[22] ■

Proof of Proposition 3.2. Differentiating (3.25) (for $A \leq A^F$) yields

$$\frac{d\xi(A)}{d\beta_1} = \frac{\left(\frac{\beta_1}{\beta_1-1}\left(\frac{3}{2}I + \frac{2}{3}\frac{K^2}{r}\right) - \frac{K^2}{3r} - I\right)\ln\left(\frac{A^F}{A}\right) - \frac{1}{\beta_1-1}\left(\frac{1}{2}I + \frac{1}{3}\frac{K^2}{r}\right)}{\left(\frac{A^F}{A}\right)^{\beta_1}}. \tag{3.62}$$

Since the threshold of the leader is equal to A^P, and A^P is the smallest root of the concave function $\xi(A)$, we know that

$$\left.\frac{\partial\xi(A)}{\partial A}\right|_{A=A^P} > 0. \tag{3.63}$$

Consequently, by differentiating (3.25) totally, we conclude that it is sufficient to show that

$$\left.\frac{d\xi(A)}{d\beta_1}\right|_{A=A^P} > 0 \tag{3.64}$$

to conclude that the replacement threshold of the leader is increasing with uncertainty (decreasing with β_1). Moreover, upon analyzing (3.62) we know that $\frac{d\xi(A)}{d\beta_1}$ changes its sign only once and the corresponding realization of A to the zero value of the derivative is

$$A^* = A^F e^{-\frac{\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}}{\beta_1\left(\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}\right)+\frac{K^2}{3r}+I}}. \tag{3.65}$$

Therefore

$$\frac{d\xi(A)}{d\beta_1} > 0 \text{ iff } A < A^*. \tag{3.66}$$

Consequently, $\xi(A^*) > 0$ would imply that $A^* > A^P$ and $\left.\frac{d\xi(A)}{d\beta_1}\right|_{A=A^P} > 0$. In order to prove that $\xi(A^*) > 0$, we plug (3.65) into (3.25) to obtain

$$\begin{aligned}\xi(A^*) &= \frac{\beta_1}{\beta_1-1}\left(\frac{3}{2}I + \frac{2}{3}\frac{K^2}{r}\right)e^{-\frac{\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}}{\beta_1\left(\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}\right)+\frac{K^2}{3r}+I}} - \frac{K^2}{3r} - I \\ &\quad -\left(\frac{\beta_1}{\beta_1-1}\left(\frac{3}{2}I + \frac{2}{3}\frac{K^2}{r}\right) - \frac{K^2}{3r} - I\right)e^{-\beta_1\frac{\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}}{\beta_1\left(\frac{1}{2}I+\frac{1}{3}\frac{K^2}{r}\right)+\frac{K^2}{3r}+I}}.\end{aligned} \tag{3.67}$$

[22] The domain of I results from the fact that for this range of values of K, thresholds A^F and A^S are decreasing in K, which constitutes the economically relevant case.

An analytical proof is again not possible but numerically it can be shown that $\xi(A^*)$ is positive for $\beta_1 \in [1, \frac{r}{\alpha})$, and $\alpha \in (0, \infty)$, $\beta_1 \in [1, \infty)$, and $\alpha \in (-\infty, 0]$, and for other parameters falling into intervals: $r \in (\alpha, \infty)$, $K \in (0, \infty)$ and $I \in \left(\frac{4}{9}\frac{K^2}{r}, \infty\right)$. ■

Proof of Proposition 3.3. We rewrite the derivative (3.47) as

$$\frac{d\xi^N(A)}{d(\sigma^2)} = \frac{1}{4}\frac{A^2}{\varrho^2} - \frac{\frac{5}{4}I}{\beta_1 - 2}\left(\frac{A}{A^{FN}}\right)^{\beta_1} \times \left[\frac{\partial \beta_1}{\partial(\sigma^2)} + \left(\beta_1 + \frac{8}{5}\right)\left(\frac{\beta_1}{2\varrho} + \ln\left(\frac{A}{A^{FN}}\right)\frac{\partial \beta_1}{\partial(\sigma^2)}\right)\right]. \tag{3.68}$$

Denote the smallest solution of $\xi^N(A) = 0$ by A^{PN}. Since A^{PN} cannot be explicitly derived, we proceed as follows. First, we consider a particular point $\overline{A} > A^{PN}$. Second, we show that $\frac{d\xi^N(A)}{d(\sigma^2)}$ is negative for all $A \in (\underline{A}, \overline{A})$, where $\underline{A}$ is a realization of A such that $\underline{A} < A^{PN}$. Let us define

$$\overline{A} \equiv 2\sqrt{\frac{\beta_1}{\beta_1 - 2}I\varrho}. \tag{3.69}$$

First, we show that $\xi^N\left(\overline{A}\right) > 0$, which would imply that $\overline{A} > A^{PN}$. After substituting (3.69) into (3.46) we obtain that

$$\xi^N\left(\overline{A}, \beta_1\right) = \frac{2I}{\beta_1 - 2}\left(1 - \left(\frac{5}{8}\beta_1 + 1\right)\left(\frac{2}{3}\right)^{\beta_1}\right) = \frac{2I}{\beta_1 - 2}\phi(\beta_1). \tag{3.70}$$

Since $\beta_1 > 2$ (recall that for $\beta_1 \leq 2$ no firm is willing to enter), we know that $\frac{2I}{\beta_1 - 2}$ is always positive. Therefore we are interested only in the sign of $\phi(\beta_1)$. For $\beta_1 \downarrow 2$ we obtain that

$$\lim_{\beta_1 \downarrow 2} \phi(\beta_1) = 0. \tag{3.71}$$

Then we establish that

$$\frac{\partial \phi(\beta_1)}{\partial \beta_1} = -\left(\frac{2}{3}\right)^{\beta_1}\left(\frac{5}{8} + \left(\frac{5}{8}\beta_1 + 1\right)\ln\left(\frac{2}{3}\right)\right) > 0 \tag{3.72}$$

for $\beta_1 \in (2, \infty)$. This implies that $\xi^N\left(\overline{A}\right)$ is positive so that $\overline{A} > A^{PN}$. We proceed with proving that expression (3.68) changes signs twice, i.e. it is positive for $A \in (0, \underline{A}) \cup \left(\overline{\overline{A}}, A^{FN}\right)$, where $\overline{\overline{A}}$ is some realization of A such that $\overline{\overline{A}} > \overline{A}$, and negative otherwise. First, we express (3.68) as

$$\frac{d\xi^N(A)}{d(\sigma^2)} = A^2\left[KA^{\beta_1 - 2} + LA^{\beta_1 - 2}\ln\left(\frac{A}{A^{FN}}\right) + M\right], \tag{3.73}$$

where

$$K = -\frac{\frac{5}{4}I}{\beta_1 - 2}\left(A^{FN}\right)^{-\beta_1}\left(\frac{\partial\beta_1}{\partial(\sigma^2)} + \frac{\beta_1\left(\beta_1 + \frac{8}{5}\right)}{2\varrho}\right), \tag{3.74}$$

$$L = -\frac{\frac{5}{4}I}{\beta_1 - 2}\left(A^{FN}\right)^{-\beta_1}\left(\beta_1 + \frac{8}{5}\right)\frac{\partial\beta_1}{\partial(\sigma^2)} > 0, \text{ and} \tag{3.75}$$

$$M = \frac{1}{4\varrho^2} > 0. \tag{3.76}$$

From (3.73)-(3.76) it can be derived that[23]

$$\lim_{A\downarrow 0} KA^{\beta_1-2} + LA^{\beta_1-2}\ln\left(\frac{A}{A^{FN}}\right) + M = M, \text{ and} \tag{3.77}$$

$$\lim_{A\to\infty} KA^{\beta_1-2} + LA^{\beta_1-2}\ln\left(\frac{A}{A^{FN}}\right) + M = \infty. \tag{3.78}$$

Moreover

$$\begin{aligned}&\frac{\partial}{\partial A}\left(KA^{\beta_1-2} + LA^{\beta_1-2}\ln\left(\frac{A}{A^{FN}}\right) + M\right)\\ = \;& A^{\beta_1-3}\left((\beta_1-2)K + (\beta_1-2)L\ln\left(\frac{A}{A^{FN}}\right) + L\right),\end{aligned}$$

which implies that there exists only one extremum of $\frac{d\xi^N(A)}{d(\sigma^2)}$ that is different from zero. This result, combined with (3.77) and (3.78), implies that $\frac{d\xi^N(A)}{d(\sigma^2)}$ is negative at most in only one interval. Substituting $\overline{A}$ into (3.68) yields

$$\begin{aligned}\left.\frac{d\xi^N(A)}{d(\sigma^2)}\right|_{A=\overline{A}} &= \frac{2I}{(\beta_1-2)\varrho} - \frac{\frac{5}{4}I}{\beta_1-2}\left(\frac{2}{3}\right)^{\beta_1}\\ &\times\left[\frac{\partial\beta_1}{\partial(\sigma^2)} + \left(\beta_1+\frac{8}{5}\right)\left(\frac{\beta_1}{2\varrho} + \ln\left(\frac{2}{3}\right)\frac{\partial\beta_1}{\partial(\sigma^2)}\right)\right]\end{aligned} \tag{3.79}$$

Numerically it can be shown that $\left.\frac{\partial\xi^N(A)}{\partial(\sigma^2)}\right|_{A=\overline{A}}$ is negative for $\beta_1 \in [1, \frac{r}{\alpha})$, and $\alpha \in (0,\infty)$, $\beta_1 \in [1,\infty)$, and $\alpha \in (-\infty, 0]$, $\alpha \in \mathbb{R}$, $r \in (\alpha,\infty)$ and $I \in (0,\infty)$. Therefore the only remaining part of the proof is to show that $\underline{A} < A^{PN}$ for any vector of input parameters. Since the explicit analytical forms of $\underline{A}$ and A^{PN} do not exist, we use a numerical procedure. Using a grid search technique (for the domains of input parameters as in the proofs of Propositions 3.1 and 3.2), we calculate the differences $A^{PN} - \underline{A}$ and it turns out that they are always positive. Given that $\left.\frac{d\xi^N(A)}{d(\sigma^2)}\right|_{A\in(\underline{A},\overline{A}]} < 0$, $A^{PN} \in (\underline{A}, \overline{A})$ and $\xi^N\left(\overline{A}\right) > 0$, we conclude that $\frac{dA^{PN}}{d(\sigma^2)} > 0$, i.e. the investment threshold of the leader increases with uncertainty. ■

[23] The result (3.77) has been derived using l'Hôpital's rule.

Proof of Proposition 3.4. First, we show that τ^* is the time to reach the replacement threshold A^* in the deterministic case. After observing that $x = \alpha t$ is the solution to $dx = \alpha dt$ with initial condition $x(0) = 0$, and substituting $x^* = \ln \frac{A^*}{A}$, we obtain that

$$\ln \frac{A^*}{A} = \alpha \tau^*, \tag{3.80}$$

so τ^* in (3.52) is the time to reach the threshold A^*. Now, we consider the density function $\varphi(\tau; \mu(\sigma), \sigma^2)$ being the density function of the first passage time for a geometric Brownian motion, which has a mean $\mu(\sigma)$ and variance σ^2. For the moment we assume that $\mu = \tau^*$ irrespective from σ. Then, raising the variance σ^2 is equivalent to performing a mean preserving spread. Consequently, in such a case

$$\frac{\partial}{\partial \sigma}\left(\int_0^\tau \varphi(s)\, ds\right) \quad (\tau - \tau^*) \leq 0, \tag{3.81}$$

with equality holding iff $\tau = \tau^*$. The expectation of the first passage time, $E[\tau]$, associated with hitting the replacement threshold A^*, is increasing with σ (cf. (3.50)) and A^* is increasing with σ, too. For $\tau > \tau^*$, an increase in uncertainty not only reduces the probability mass to the left of τ via the mean preserving spread but also because of the mean itself moving to the right. Therefore the effect of uncertainty on the probability of the replacement decision is unambiguous in this region and negative. For $\sigma \to \infty$ the probability of investing before τ decreases to zero. The latter conclusion is true since from (3.51) it is obtained that

$$\begin{aligned}
\lim_{\sigma\to\infty} P(T < \tau) &= \lim_{\sigma\to\infty} \Phi\left(\frac{-\ln\frac{A^*}{A} + \left(\alpha - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right) \\
&\quad + \lim_{\sigma\to\infty}\left[\Phi\left(\frac{-\ln\frac{A^*}{A} - \left(\alpha - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right)\left(\frac{A^*}{A}\right)^{\frac{2\alpha}{\sigma^2}-1}\right] \\
&= \lim_{\sigma\to\infty}\left[\Phi\left(\frac{-\ln\frac{A^*}{A} - \left(\alpha - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right)\left(\frac{A}{A^*}\right)\right].
\end{aligned} \tag{3.82}$$

Hence, if

$$\lim_{\sigma\to\infty} A^* = \infty \tag{3.83}$$

it holds that

$$\lim_{\sigma\to\infty} P(T < \tau) = 0. \tag{3.84}$$

We will show later that (3.84) holds for all relevant thresholds.

For $\tau < \tau^*$, the two effects work in opposite directions. As in the previous case, the mean $E[\tau]$ is increasing with uncertainty. Without a change in the volatility, an increase in the mean would then decrease the probability of replacing the existing production facility. However, increasing uncertainty results in a greater probability mass being present in the left tail of $\varphi(\tau)$. Therefore, the total effect of increasing uncertainty is ambiguous in this region. However, we are able to conclude that the probability of investing at a given τ behaves in a certain non-monotonic way. For $\sigma = 0$, there is no probability mass on the interval $[0, \tau^*)$, since the investment takes place at $\tau < \tau^*$ with probability 1. Therefore an increase in uncertainty initially leads to an increased probability of investment. For relatively large σ the effect of moving the mean of the distribution to the right starts to dominate and the probability of asset replacement falls. For $\sigma \to \infty$ the probability of replacing the existing asset before a given time τ decreases to zero.

Finally, we show that all the thresholds increase with uncertainty monotonically and unboundedly. We already know (from Sections 3.5 and 3.6) that the optimal replacement thresholds increase with uncertainty monotonically. So now we only have to prove that the thresholds grow in uncertainty unboundedly. For the thresholds of the follower and in case of simultaneous replacement it is easy to observe that $\frac{\beta_1}{\beta_1 - 1}$ tends to infinity when $\sigma \to \infty$.[24] The replacement threshold of the leader requires slightly more attention.[25] We already know that the leader replaces its asset as soon as the stochastic variable reaches the smallest root of the following equation (cf. (3.25))

$$0 = \frac{2}{3}\frac{KA}{\delta} - \frac{K^2}{3r} - I - \left(\frac{2}{3}\frac{KA^F}{\delta} - \frac{K^2}{3r} - I\right)\left(\frac{A}{A^F}\right)^{\beta_1}. \tag{3.85}$$

After substituting (3.16) into (3.85) and rearranging, we obtain that

$$\begin{aligned} 0 \;=\; & \left(1 - \left(\frac{\frac{4}{9}KA}{\frac{\beta_1}{\beta_1-1}\left(I + \frac{4K^2}{9r}\right)2I}\right)^{\beta_1 - 1} + \frac{\left(\frac{K^2}{3r} + I\right)A^{\beta_1 - 1}}{\frac{2}{3}\frac{K}{\delta}\left(\frac{\beta_1}{\beta_1 - 1}\frac{I + \frac{4K^2}{9r}}{\frac{4}{9}KA}\delta\right)^{\beta_1}}\right) \\ & \times \frac{2KA}{3\delta} - \frac{K^2}{3r} - I. \end{aligned} \tag{3.86}$$

It holds that

$$\lim_{\beta_1 \downarrow 1}\left(1 - \left(\frac{\frac{4}{9}KA}{\frac{\beta_1}{\beta_1-1}\left(I + \frac{4K^2}{9r}\right)\delta}\right)^{\beta_1 - 1} + \frac{\left(\frac{K^2}{3r} + I\right)A^{\beta_1 - 1}}{\frac{2K}{3\delta}\left(\frac{\beta_1}{\beta_1 - 1}\frac{I + \frac{4K^2}{9r}}{\frac{4}{9}KA}\delta\right)^{\beta_1}}\right) = 0. \tag{3.87}$$

[24]For a new market model a similar conclusion can be drawn after the substitution of parameters in the original geometric Brownian motion.

[25]The unboundedness of the leader threshold in the new market entry can be proven in a similar way as in the presented case of asset replacement.

Since

$$\lim_{\beta_1 \downarrow 1} \frac{\partial}{\partial \beta_1} \left[1 - \left(\frac{\frac{4}{9}KA}{\frac{\beta_1}{\beta_1 - 1}\left(I + \frac{4K^2}{9r}\right)\delta} \right)^{\beta_1 - 1} + \frac{\left(\frac{K^2}{3r} + I\right) A^{\beta_1 - 1}}{\frac{2K}{3\delta}\left(\frac{\beta_1}{\beta_1 - 1}\frac{I + \frac{4K^2}{9r}}{\frac{4}{9}KA}\delta\right)^{\beta_1}} \right] > 0, \tag{3.88}$$

the LHS of (3.87) approaches zero from above. To shorten the notation we rewrite (3.86) into

$$0 = M(A)A - N. \tag{3.89}$$

Now, we are looking for the solution of (3.89). From (3.88) it can be seen that $m(A)$ is tending to zero from above $\forall A \in \mathbb{R}^{++}$ when uncertainty is increasing. Consequently, any solution (so the smallest one as well) of (3.89) is tending to infinity. This is equivalent to

$$\lim_{\beta_1 \downarrow 1} A^P = \infty, \tag{3.90}$$

which completes the proof. ■

Limiting value of the optimal follower threshold to start production. We are interested in the following limit

$$\begin{aligned} &\lim_{\sigma^2 \to r - 2\alpha} 3\sqrt{\frac{\beta_1}{\beta_1 - 2} I \varrho} \\ = \; & 3\sqrt{I \lim_{\sigma^2 \to r - 2\alpha} \frac{\beta_1}{\beta_1 - 2} \varrho}. \end{aligned} \tag{3.91}$$

Furthermore, we have

$$\begin{aligned} &\lim_{\sigma^2 \to r - 2\alpha} \frac{\beta_1}{\beta_1 - 2} \varrho \\ = \; & \lim_{\sigma^2 \to r - 2\alpha} \frac{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right)(r - 2\alpha - \sigma^2)}{-\frac{3}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}} \\ = \; & 2(r - 2\alpha) \lim_{\sigma^2 \to r - 2\alpha} \frac{r - 2\alpha - \sigma^2}{-\frac{3}{2}\sigma^2 - \alpha + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}}. \end{aligned}$$

Applying l'Hôpital's rule yields

$$\begin{aligned}
&2(r-2\alpha)\lim_{\sigma^2\to r-2\alpha}\frac{-1}{-\frac{3}{2}+\frac{-\alpha+\frac{1}{2}\sigma^2+2r}{2\sqrt{\left(\alpha-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}}}\\
=\;&2(r-2\alpha)\frac{-1}{-\frac{3}{2}+\frac{-\alpha+\frac{1}{2}(r-2\alpha)+2r}{2\sqrt{\left(\alpha-\frac{1}{2}(r-2\alpha)\right)^2+2r(r-2\alpha)}}}\\
=\;&\frac{-4(r-2\alpha)\sqrt{\left(\alpha-\frac{1}{2}(r-2\alpha)\right)^2+2r(r-2\alpha)}}{-3\sqrt{\left(\alpha-\frac{1}{2}(r-2\alpha)\right)^2+2r(r-2\alpha)}-\alpha+\frac{1}{2}(r-2\alpha)+2r}\\
=\;&\frac{-4(r-2\alpha)\left|2\alpha-\frac{3}{2}r\right|}{-3\left|2\alpha-\frac{3}{2}r\right|-\alpha+\frac{1}{2}(r-2\alpha)+2r}.
\end{aligned}$$

Since $2\alpha<r$, this is equal to

$$\begin{aligned}
&\frac{4(r-2\alpha)\left(2\alpha-\frac{3}{2}r\right)}{3\left(2\alpha-\frac{3}{2}r\right)-\alpha+\frac{1}{2}(r-2\alpha)+2r}\\
=\;&\frac{4(r-2\alpha)\left(2\alpha-\frac{3}{2}r\right)}{-2(r-2\alpha)}\\
=\;&3r-4\alpha. \qquad (3.92)
\end{aligned}$$

Substituting (3.92) into (3.91) yields the desired result. ■

Limiting value of the optimal leader threshold to start production. To obtain the leader's limiting threshold, we are interested in the form of function ξ^N when σ^2 tends to $r-2\alpha$. For any $A\in\left(0,A^{FN}\right)$ we have (cf. (3.44), (3.45) and $\xi^N=V^{LN}-V^{FN}$)

$$\begin{aligned}
&\lim_{\sigma^2\to r-2\alpha}\left[\frac{1}{4}\frac{A^2}{r-2\alpha-\sigma^2}-I-\left(\frac{1}{4}\frac{\left(A^{FN}\right)^2}{r-2\alpha-\sigma^2}-I\right)\left(\frac{A}{A^{FN}}\right)^{\beta_1}\right] \qquad (3.93)\\
=\;&\lim_{\sigma^2\to r-2\alpha}\left[\frac{1}{4}\frac{A^2}{r-2\alpha-\sigma^2}-I-\left(\frac{1}{4}\frac{9(3r-4\alpha)I}{r-2\alpha-\sigma^2}-I\right)\left(\frac{A}{3\sqrt{(3r-4\alpha)I}}\right)^{\beta_1}\right],
\end{aligned}$$

by using the limit of (3.41). Consequently, we rearrange (3.93) to get[26]

$$\begin{aligned}
&\lim_{\sigma^2\to r-2\alpha}\left[\frac{1}{4}\frac{A^2}{r-2\alpha-\sigma^2}-I+\frac{A^2}{9(3r-4\alpha)}-\frac{1}{4}\frac{9(3r-4\alpha)I}{r-2\alpha-\sigma^2}\left(\frac{A}{3\sqrt{(3r-4\alpha)I}}\right)^{\beta_1}\right]\\
=\;&\frac{A^2}{9(3r-4\alpha)}-I+\frac{A^2}{4}\lim_{\sigma^2\to r-2\alpha}\frac{1-\left(\frac{A}{3\sqrt{(3r-4\alpha)I}}\right)^{\beta_1-2}}{r-2\alpha-\sigma^2}. \qquad (3.94)
\end{aligned}$$

[26] We do so by observing that $\lim_{\sigma^2\to r-2\alpha}\beta=2$.

The limit in the last component can be calculated as follows:

$$\lim_{\sigma^2\to r-2\alpha}\frac{1-\left(\frac{A}{3\sqrt{(3r-4\alpha)I}}\right)^{\beta_1-2}}{r-2\alpha-\sigma^2}=\lim_{\sigma^2\to r-2\alpha}\frac{1-\left(\frac{A}{3\sqrt{(3r-4\alpha)I}}\right)^{-\frac{3}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}}}{r-2\alpha-\sigma^2}.$$

Application of l'Hôpital's rule yields

$$\lim_{\sigma^2\to r-2\alpha}\frac{\partial}{\partial\left(\sigma^2\right)}\left(\left(\frac{A}{3\sqrt{(3r-4\alpha)\,I}}\right)^{-\frac{3}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}}\right) \tag{3.96}$$

$$\begin{aligned}
&= \lim_{\sigma^2\to r-2\alpha}\ln\left(\frac{A}{3\sqrt{(3r-4\alpha)\,I}}\right)\left(\frac{A}{3\sqrt{(3r-4\alpha)\,I}}\right)^{-\frac{3}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}}\\
&\quad\times\left(\frac{\alpha}{\sigma^4}-\frac{\frac{\alpha}{\sigma^4}\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)+\frac{r}{\sigma^4}}{\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}}\right)\\
&= \ln\left(\frac{A}{3\sqrt{(3r-4\alpha)\,I}}\right)\left(\frac{\alpha}{(r-2\alpha)^2}-\frac{\frac{\alpha}{r-2\alpha}\left(\frac{\alpha}{r-2\alpha}-\frac{1}{2}\right)+\frac{r}{r-2\alpha}}{\frac{3}{2}r-2\alpha}\right)\\
&= \ln\left(\frac{A}{3\sqrt{(3r-4\alpha)\,I}}\right)\left(-\frac{1}{\frac{3}{2}r-2\alpha}\right).
\end{aligned} \tag{3.97}$$

Consequently, after substituting (3.97) into (3.94), we obtain the formula for the limiting case of ξ^N :

$$\left(\left(\frac{A}{A^{FN}}\right)^2-1\right)I-\frac{A^2}{2\,(3r-4\alpha)}\ln\left(\frac{A}{A^{FN}}\right). \tag{3.98}$$

■

Chapter 4

Profit Uncertainty and Asymmetric Firms

4.1 Introduction

The aim of this chapter is to study the effects of imperfect competition on the optimal real option exercise strategies in a situation where the costs of exercising options differ among firms. Such a framework, which relaxes the restrictive assumption that the duopolistic rivals are identical, is motivated by the existence of many sources of potential cost asymmetry.[1] First, investment cost asymmetry is present when the firms have different access to the capital markets. In such a case, the cost of capital of a liquidity-constrained firm is higher than of its counterpart having access to a credit line or with substantial cash reserves (cf. Lensink et al., 2001). Consequently, the investment cost of the firm facing capital market imperfections is higher.

Moreover, cost asymmetry occurs when the firms exhibit a different degree of organizational flexibility at implementing a new production technology. This flexibility, known as absorptive capacity (cf. Cohen and Levintal, 1994), measures the firm's ability to adopt external technologies, to assimilate to a changing economic environment, and to commercialize newly invented products. A higher absorptive capacity is therefore equivalent to a lower cost associated with an investment project.

Differing real options embedded in the existing assets of the firms due to past decisions are another source of possible investment cost asymmetry. After the arrival

[1]Alternatively, we could introduce asymmetry by introducing firm-specific profit functions or parameters of the stochastic process. However, we expect that other forms of asymmetry lead to similar results (cf. also Huisman, 2001, Ch. 8, and Joaquin and Butler, 2000, who analyze different forms of asymmetry in a new market model).

of a new invention it may appear that one of the existing technologies is more easily extendable than the other. For instance, Kaplan (1986) reports that in the 1970s some manufacturing firms invested in electronically controlled production facilities. This investment did not bring significant improvements to the firms' profits. However, after the arrival of microprocessor-based technology in the 1980s, the firms that invested in electronically controlled facilities were able to adopt the new technology more quickly and at a lower cost.

Finally, the difference in investment costs is often a consequence of purely exogenous factors, resulting, among others, from the intervention of the authority. For instance, the effective investment cost of the firms is reduced after obtaining governmental credit guarantees, which result in a lower cost of capital (see the evidence by Kleimeier and Megginson, 2000).

As in Chapter 3, we consider the optimal real option exercise strategy of duopolistic firms already competing in a product market.[2] Both firms have an investment opportunity enhancing *ceteris paribus* the profit flow. If one firm invests, the other firm's payoff is reduced.[3] This is, for example, the case when the investment gives the firm the possibility to produce more efficiently and thus cheaper, which leads to a higher market share. The firms differ ex ante only with respect to the required sunk cost associated with the investment. Our framework most directly generalizes Smets (1991) and Grenadier (1996), who restrict the analysis to a game between symmetric firms, and Huisman (2001), Ch. 8, who considers a new market entry of asymmetric firms. This generalization results in the presence of three different equilibrium strategies. First, when the asymmetry among firms is relatively small and so is the first-mover advantage, the firms invest at the same time. When the first-mover advantage is sufficiently large, the lower-cost firm preempts the higher-cost firm. In the situation where both the first-mover advantage and asymmetry between firms are significant, the firms exercise their investment options sequentially and their investment timing do not affect each other directly. The two latter equilibria are also present in Perotti and Rossetto (2000), in which the problem of cross-market entry is considered.[4] The model presented in this chapter is also closely related to Boyer et al. (2002), Section 5, in

[2]Contrary to Chapter 3, we do not model the structural form of the product market competition. Instead, we impose a reduced form of profit functions.

[3]Mason and Weeds (2003) allow for positive network externalities among the firms, which results in a higher profit of the incumbent after the investment of the entrant.

[4]It is never optimal for firms to invest simultaneously in the framework of Perotti and Rosetto (2000) since the instantaneous profits of firm competing in both the market segments are lower than monopolistic profits realized in the firms' own market segments.

which asymmetry across firms has a form of differing initial capacities.

Furthermore, we analyze the impact of uncertainty on the optimal investment thresholds. We find that the value of the waiting option increases with the profit volatility despite the presence of strategic interactions.

Finally, we determine the firms' values and present welfare implications of the strategic option exercise. We find that, when an increase in the investment expenditure of the higher-cost firm results in a switch from joint investment to preemption equilibrium, the value of *both* firms decrease. Moreover, in the preemption equilibrium, an increase in the higher-cost firm's investment expenditure results in the appreciation of this firm's value. After a cost increase of the competitor, the low cost firm knows that it itself could delay the investment without bearing the risk of being preempted. This investment delay raises the value of the higher cost firm. Using an example of a duopoly in which after the investment the firms can offer a good with a higher quality, we derive the relationship between the type of equilibrium and the level of consumer surplus. This analysis indicates that an equal access of competitors to a new technology (or a new market) may not be socially optimal.

This chapter is organized as follows. In Section 4.2 we present the model. Section 4.3 contains the derivation of value functions and optimal investment thresholds. The discussion of the resulting equilibrium strategies is presented in Section 4.4 and the analysis of the impact of uncertainty on the timing of investment is included in Section 4.5. In Section 4.6 we analyze the impact of strategic interactions on the value of the firms whereas Section 4.7 discusses the relationship between the firms' investment strategies and social welfare. Section 4.8 concludes.

4.2 Framework of the Model

In this chapter, the framework of Dixit and Pindyck (1996), Ch. 6, is adapted here, with the difference that we consider two firms rather than one. The two risk-neutral firms compete in the product market, and realize a non-negative stochastic profit flow. The uncertainty in each of the firms' profits is introduced via a geometric Brownian motion process:

$$dx(t) = \alpha x(t)\,dt + \sigma x(t)\,dw(t)\,, \tag{4.1}$$

where α and σ are constants corresponding to the instantaneous drift and to the instantaneous standard deviation, respectively, dt is the time increment and $dw(t)$ is the Wiener increment. Let r be the deterministic instantaneous riskless interest rate.

It is assumed that the drift rate, α, exhibits a shortfall δ below the riskless rate, i.e. $\alpha = r - \delta$. The uncertainty in the profit function is included in a multiplicative way. The instantaneous profit of Firm i can be expressed as

$$\pi_{N_i N_j}(t) = x(t) D_{N_i N_j}, \tag{4.2}$$

where, for $k \in \{i, j\}$, :

$$N_k = \begin{cases} 0 & \text{if firm } k \text{ has not invested,} \\ 1 & \text{if firm } k \text{ has invested.} \end{cases}$$

$D_{N_i N_j}$ stands for the deterministic contribution to the profit function, and it holds that

$$\begin{array}{ccc} D_{10} & > & D_{00} \\ \vee & & \vee \\ D_{11} & > & D_{01}. \end{array} \tag{4.3}$$

$D_{10} > D_{00}$ implies that the profit of the firm that invests as first exceeds *ceteris paribus* the initial (symmetric) profit. Moreover, this investment leads to a deterioration of the profit of the firm that did not undertake the project yet, i.e. $D_{00} > D_{01}$. Finally, the 'catch-up' investment made by the lagging firm enhances its profit, so $D_{11} > D_{01}$, but, at the same time, it reduces the profit of the first mover, so that $D_{11} < D_{10}$. The last inequality implies that there are negative network externalities among the firms.[5] Such a general formulation embraces, for instance, Cournot or Stackelberg quantity competition.

The investment opportunity is assumed to last forever and the structure of the associated payoff can only change as a result of the competitor's action. Therefore, the opportunity can be modeled as a perpetual American option with a payoff determined endogenously. Consequently, we denote the investment cost of Firm i, $i \in \{1, 2\}$ by I_i. Without loss of generality I_1 is normalized to I, which is the investment cost of the low-cost firm, and I_2 is set equal to κI, where $\kappa \in [1, \infty)$.

Finally, we assume that the initial realization of the process underlying both firms' profits, $x(0)$, is low enough, so that an immediate investment is not optimal.[6]

[5] Mason and Weeds (2003) allow for $D_{11} > D_{10}$ to reflect the positive network externalities on the supply side that can arise among the competitors. In our setting (firms already compete in a product market) such an assumption would be more difficult to justify. Moreover, $D_{10} > D_{11}$ does not preclude the presence of positive network externalities among the firms' customers (for example, the profits generated by Microsoft in the office software segment are not likely to be positively affected by technological improvements made by Corel).

[6] Immediate investment is optimal in case of a sufficiently high initial realization of the stochastic process. The mixed strategies equilibria occurring then are discussed (for identical firms) in Chapter 3 (see also Thijssen et al., 2002).

4.3 Value Functions and Investment Thresholds

As in Chapter 3, there are three possibilities concerning the relative timing of firms investment. First, Firm i may become the leader. Alternatively, Firm j may invest sooner which results in Firm i becoming the follower. Finally, firms may invest simultaneously.

In this section we establish the payoffs associated with the three situations described above. As in the standard approach used to solve dynamic games, we analyze the problem backwards in time. First, we derive the optimal strategy of the follower, who takes the strategy of the leader as given. Subsequently, we analyze the decision of the leader. Finally, the case of joint investment is discussed.

4.3.1 Follower

Consider the investment decision of the follower (Firm i) at time t, where t is the leader's (Firm j's) investment timing. Firm i will undertake the investment when profits are sufficiently large, i.e. when x exceeds a certain threshold level denoted by x_i^F. Determining x_i^F is equivalent to finding the optimal option exercise strategy. At $x(t)$, the value of Firm i as the follower equals

$$\begin{aligned} V_i^F(t) &= E\left[\int_t^{T_i^F} x(s) D_{01} e^{-r(s-t)} ds\right] \\ &+ E\left[e^{-r\left(T_i^F - t\right)} \left(\int_{T_i^F}^{\infty} x(s) D_{11} e^{-r\left(s - T_i^F\right)} ds - I_i\right)\right], \end{aligned} \tag{4.4}$$

where

$$T_i^F = \inf\left(t | x \geq x_i^F\right). \tag{4.5}$$

The realization x_i^F corresponds to the follower's optimal investment threshold

$$x_i^F = \frac{\beta_1}{\beta_1 - 1} \frac{I_i \delta}{D_{11} - D_{01}}, \tag{4.6}$$

and β_1 is the larger root of the quadratic equation

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - r = 0. \tag{4.7}$$

The first integral in (4.4) corresponds to the present value of profits obtained before the investment is undertaken. The second part of (4.4) reflects the present value of profits after the investment is made minus the associated sunk cost.

The value of the firm as well as the optimal investment threshold can be calculated explicitly by applying the standard dynamic programming methodology (see Section 1.6). By solving the differential equation describing the dynamics of Firm i's value with corresponding value-matching, smooth-pasting and no-bubbles conditions, we arrive at the following expression for the value of Firm i as the follower at t:

$$V_i^F(x) = \begin{cases} \frac{xD_{01}}{\delta} + \left(\frac{x_i^F(D_{11}-D_{01})}{\delta} - I\right)\left(\frac{x}{x_i^F}\right)^{\beta_1} & \text{if} \quad x \leq x_i^F, \\ \frac{xD_{11}}{\delta} - I_i & \text{if} \quad x > x_i^F. \end{cases} \tag{4.8}$$

The interpretation of (4.8) is as follows. The first row is the present value of profits when the follower does not invest immediately. The first term is the payoff in case the follower refrains from investing forever, whereas the second term is the value of the option to invest. The second row corresponds to the present value of enhanced cash flows resulting from immediate investment minus its cost.

4.3.2 Leader

Following a similar reasoning as in the previous subsection, we determine the payoff of Firm i when it invests first, thus Firm i is the leader. Then the value function of Firm i, evaluated at the moment of investing, t, equals

$$V_i^L(t) = E\left[\int_t^{T_j^F} x(s)\, D_{10} e^{-r(s-t)} ds - I_i + \int_{T_j^F}^{\infty} x(s)\, D_{11} e^{-r(s-t)} ds\right]. \tag{4.9}$$

The first two components of (4.9) correspond to the present value of the leader's profits realized until the moment of the follower's investment net of the leader's sunk cost. The second integral corresponds to the discounted perpetual stream of profits obtained after the investment of the follower.

Using the results of the follower problem, we can express the time-t value of Firm i as the leader in the following way

$$V_i^L(x) = \begin{cases} \frac{xD_{10}}{\delta} - I_i - \frac{x_j^F(D_{10}-D_{11})}{\delta}\left(\frac{x}{x_j^F}\right)^{\beta_1} & \text{if} \quad x \leq x_j^F, \\ \frac{xD_{11}}{\delta} - I_i & \text{if} \quad x > x_j^F. \end{cases} \tag{4.10}$$

The first row of (4.10) is the net present value of profits before the follower made the investment minus the present value of future profits lost due to the follower's investment. The second row corresponds to the net present value of profits in a situation where it is optimal for the follower to invest immediately.

4.3.3 Simultaneous Investment

It is possible that the firms, despite the asymmetry in the investment cost, decide to invest simultaneously. The value function of Firm i investing at its optimal threshold simultaneously with Firm j, calculated for $t \leq T_i^S$, is

$$\begin{aligned} V_i^S(x) &= E\left[\int_t^{T_i^S} x_s D_{00} e^{-r(s-t)} ds\right] + \\ & E\left[\int_{T_i^S}^{\infty} x_s D_{11} e^{-r(s-T_i^S)} ds - I_i e^{-rT_i^S}\right], \end{aligned} \tag{4.11}$$

where

$$T_i^S = \inf\left(t | x(t) \geq x_i^S\right) \tag{4.12}$$

and

$$x_i^S = \frac{\beta_1}{\beta_1 - 1} \frac{I_i \delta}{D_{11} - D_{00}}. \tag{4.13}$$

Expression (4.11) is interpreted analogously to (4.4) and (4.9). The simultaneous investment threshold exists as long as D_{11} is larger than D_{00}. Otherwise, it is optimal for the firms to abstain from investing.[7] Consequently, the time-t value of Firm i when the investment is made simultaneously equals

$$V_i^S(x) = \begin{cases} \frac{x_t D_{00}}{\delta} + \left(\frac{x_i^S (D_{11} - D_{00})}{\delta} - I_i\right)\left(\frac{x_t}{x_i^S}\right)^{\beta_1} & \text{if} \quad x \leq x_i^S, \\ \frac{x_t D_{11}}{\delta} - I_i & \text{if} \quad x > x_i^S, \end{cases} \tag{4.14}$$

The second row equals the value of Firm i when the simultaneous investment is made immediately. In such a case, we denote the value of Firm i by $V_i^J(x)$. Hence, the difference with $V_i^S(x)$ is that $V_i^J(x)$ represents the value of simultaneous *immediate* investment, while $V_i^S(x)$ is the value of *optimal* simultaneous investment. From (4.13) it can be seen that x_i^S differs among the firms. As it is shown in the next section, this divergence does not preclude the simultaneous investment strategy.

4.4 Equilibria

There are three types of equilibria that can occur in the choice of strategies, namely the preemptive, sequential and simultaneous equilibrium. In this section we discuss the characteristics of each type of equilibrium and present the conditions under which each of them occurs.

[7] The lack of the simultaneous equilibrium in Perotti and Rossetto (2000) is exactly due to the violation of this condition.

4.4.1 Preemptive Equilibrium

The first type of equilibrium we consider is the preemptive equilibrium (cf. Subsection 3.4.1). It occurs in the situation in which both firms have an incentive to become the leader, i.e. when the cost disadvantage of Firm 2 is relatively small. Therefore, Firm 1 has to take into account the fact that Firm 2 will aim at preempting Firm 1 as soon as a certain threshold is reached. This threshold, denoted by x_{21}^P, is the lowest realization of the process x for which Firm 2 is indifferent between being the leader and the follower. Formally, x_{21}^P is the smallest solution to

$$\xi_2(x) = 0, \tag{4.15}$$

where $\xi_i(x)$ is defined as

$$\xi_i(x) \equiv V_i^L(x) - V_i^F(x), \tag{4.16}$$

in which V_i^F and V_i^L are given by (4.8) and (4.10), respectively. As a consequence, Firm 1 invests at

$$\min\left(x_{21}^P, x_1^L\right),$$

where x_1^L is Firm 1's optimal leader threshold equal to[8]

$$x_1^L = \frac{\beta_1}{\beta_1 - 1} \frac{I\delta}{D_{10} - D_{00}}. \tag{4.17}$$

Figures 4.1 and 4.2 illustrate the firms' payoffs associated with being the leader, follower, both investing at Firm 1's optimal simultaneous investment threshold and both investing immediately. Firm 1 invests as soon as the process reaches the smaller of two values: x_{21}^P at which Firm 2 is indifferent between being the leader and the follower, and x_1^L at which it is optimal for Firm 1 to invest given that Firm 2 does not invest until x_1^L is reached. Figure 4.1 illustrates the case where $x_{21}^P < x_1^L$. Consequently, in the preemption equilibrium the payoff of Firm 1 as a leader is higher than the payoff obtained if Firm 1 was second to invest. It can be seen in Figure 4.2 that to the left of x_{21}^P the value of Firm 2 being the leader is lower than the value being the follower, while to the right the opposite is true. Firm 1 uses the fact that Firm 2 has no incentive to invest before x_{21}^P and preempts it by just an instant. For κ tending to 1, i.e. when firms become symmetric, x_{21}^P gets closer to Firm 1's preemption point, x_1^P, at which Firm 1 itself is indifferent between being the leader and the follower.

[8] At first sight it may look surprising that the optimal threshold x_1^L does not depend on Firm 2's investment timing. This is due to the fact that Firm 2's investment affects equally the value of Firm 1's investment opportunity and the present value of its project after the investment is made.

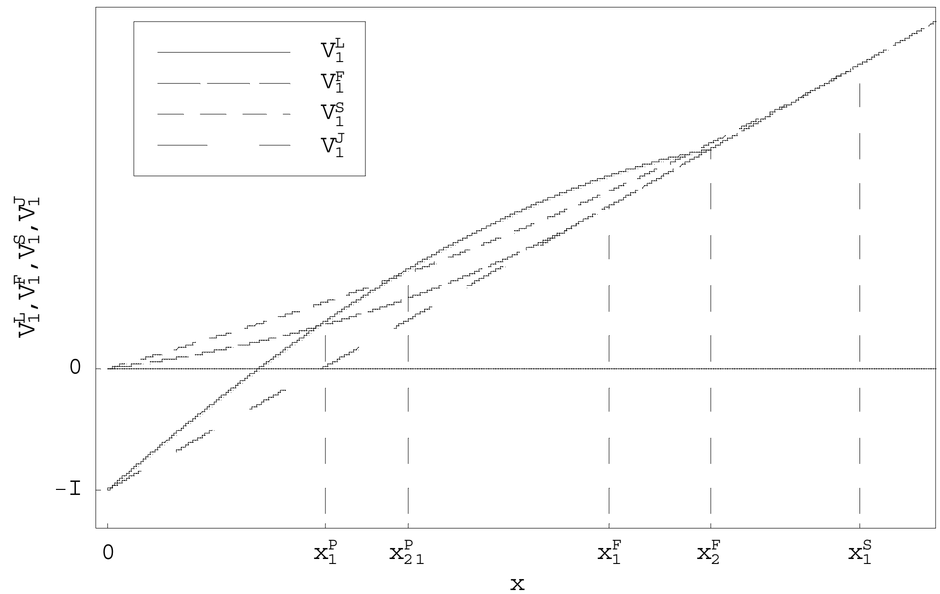

Figure 4.1: Firm 1's value functions when the resulting equilibrium is of the preemptive type.

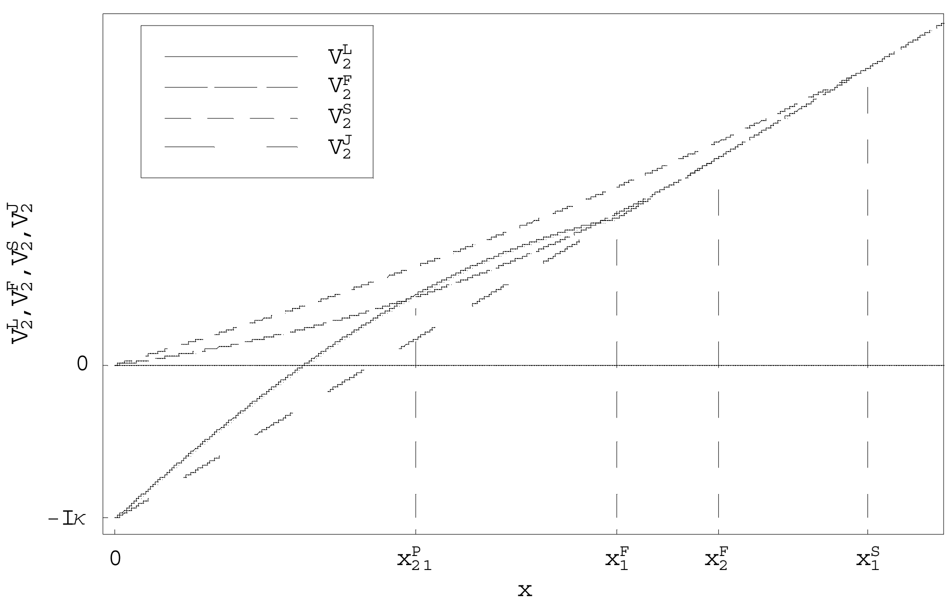

Figure 4.2: Firm 2's value functions when the resulting equilibrium is of the preemptive type.

Consequently, the presence of cost asymmetry implies the following corollary.

Corollary 4.1 *Firm 1 extracts a relative surplus from becoming the leader vs. being the follower, i.e.*

$$\xi_1\left(\min\left(x_{21}^P, x_1^L\right)\right) = V_1^L\left(\min\left(x_{21}^P, x_1^L\right)\right) - V_1^F\left(\min\left(x_{21}^P, x_1^L\right)\right) > 0. \tag{4.18}$$

Proof. The proof directly follows from the definition of the preemption point and the observation that $x_1^P < \min\left(x_{21}^P, x_1^L\right)$. ■

4.4.2 Sequential Equilibrium

The sequential equilibrium occurs when Firm 2 has no incentive to become the leader, i.e. when equation (4.15) does not have a solution. In this case, Firm 1 simply maximizes the value of the investment opportunity, which always leads to investment at the optimal threshold x_1^L. In other words, Firm 1 acts as if it had exclusive rights to invest in a profit-enhancing project but, of course, Firm 2's investment still affects Firm 1's payoffs.

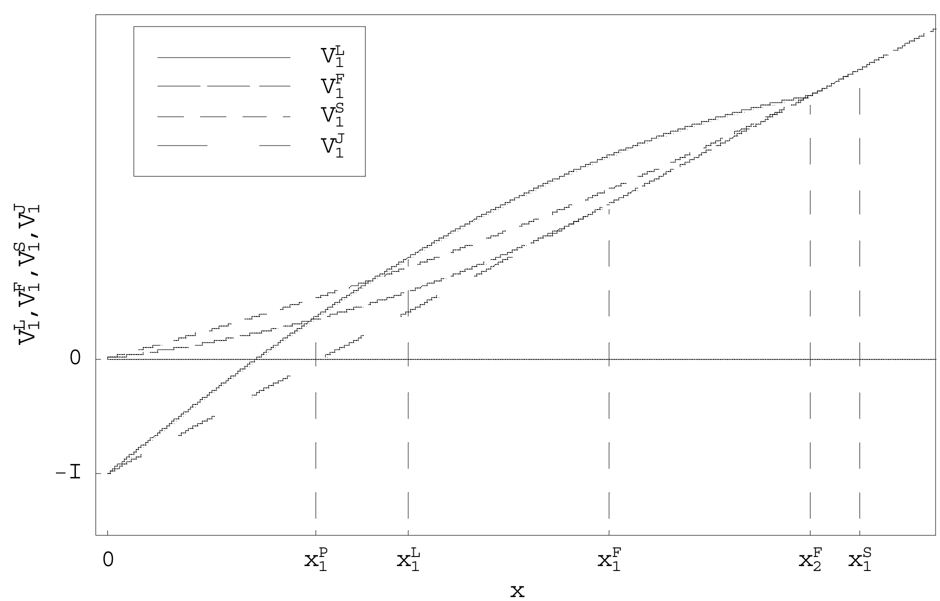

Figure 4.3: Firm 1's value functions when the resulting equilibrium is of the sequential type.

Figures 4.3 and 4.4 illustrate the firms' payoffs associated with the sequential investment equilibrium. From Figure 4.4 it can be concluded that Firm 2 is never better off by becoming the leader compared to being the follower. Therefore Firm 1

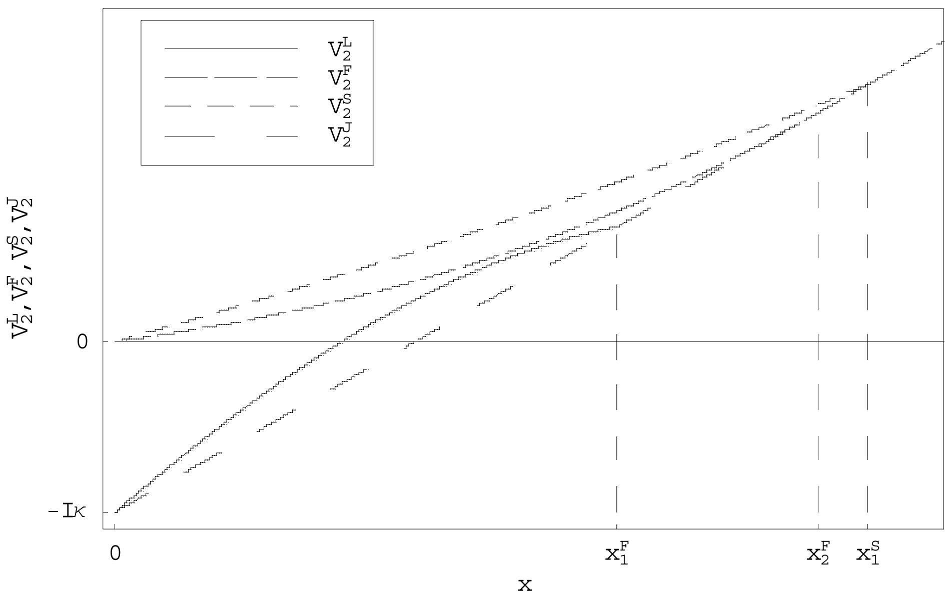

Figure 4.4: Firm 2's value functions when the resulting equilibrium is of the sequential type.

does not need to take into account the possibility of being preempted by Firm 2. As a result, Firm 1 is able to invest at its unconditional threshold, x_1^L (see Figure 4.3). At x_1^L the value of the investment opportunity smooth-pastes to the net present value of incremental benefits from making the investment (cf. Dixit and Pindyck, 1996). As in the previous case, Firm 2 invests at its follower threshold x_2^F.

Proposition 4.1 *There exists a unique value of $\kappa > 1$, denoted by κ^*, which is equal to*

$$\kappa^* = \frac{1}{D_{11} - D_{01}} \left(\frac{(D_{10} - D_{01})^{\beta_1} - (D_{11} - D_{01})^{\beta_1}}{\beta_1 (D_{10} - D_{11})} \right)^{\frac{1}{\beta_1 - 1}}, \tag{4.19}$$

that separates the regions of the preemptive and the sequential equilibrium. For $\kappa < \kappa^*$ Firm 1 needs to take into account possible preemption by Firm 2, whereas $\kappa \geq \kappa^*$ implies that firms always invest sequentially at their optimal thresholds.

Proof. See the Appendix. ■

Intuitively, Proposition 4.1 states that there is a cut-off level for the cost disadvantage of Firm 2 above which Firm 1 can act as a monopolist in exercising its investment option.

4.4.3 Simultaneous Equilibrium

Another type of equilibrium is the simultaneous (or joint investment) equilibrium (cf. Subsection 3.4.2). In this case the firms invest at the same point in time. In the simultaneous investment equilibrium one of the firms has to adopt a strategy that does not optimize its payoff unconditionally (note that the optimal joint investment thresholds differ). Since the optimal threshold of Firm 1 is lower than that of Firm 2, the only candidate for a simultaneous investment threshold is x_1^S, defined by (4.13). For simultaneous investment to occur, the payoff of Firm 1 associated with being the leader has to be lower than the payoff resulting from simultaneous investment at x_1^S. Otherwise, Firm 1 will invest either at x_1^L or at x_2^P (depending on the level of cost asymmetry). Moreover, Firm 2's follower threshold must be lower than x_1^S. In other words, Firm 2 has to find it more profitable to respond to Firm 1's investment at x_1^S immediately than to wait. Otherwise, Firm 2 would invest as the follower at x_2^F. It turns out that wherever it is optimal for Firm 1 to invest simultaneously, Firm 2 prefers simultaneous investment to being the follower (see the proof of Proposition 4.2 below).

Figures 4.5 and 4.6 depict both firms' payoffs associated with the simultaneous equilibrium.

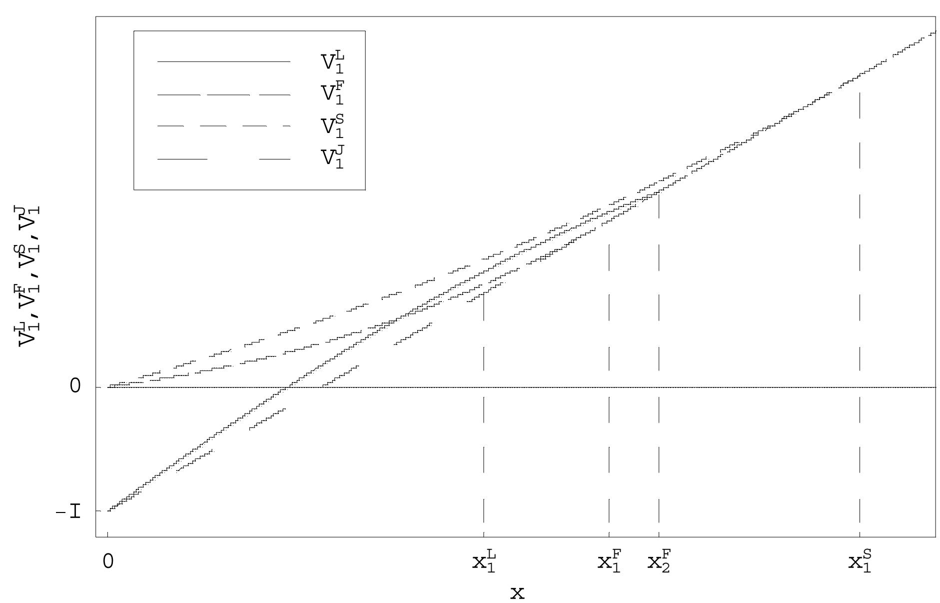

Figure 4.5: Firm 1's value functions when the resulting equilibrium is of the simultaneous type.

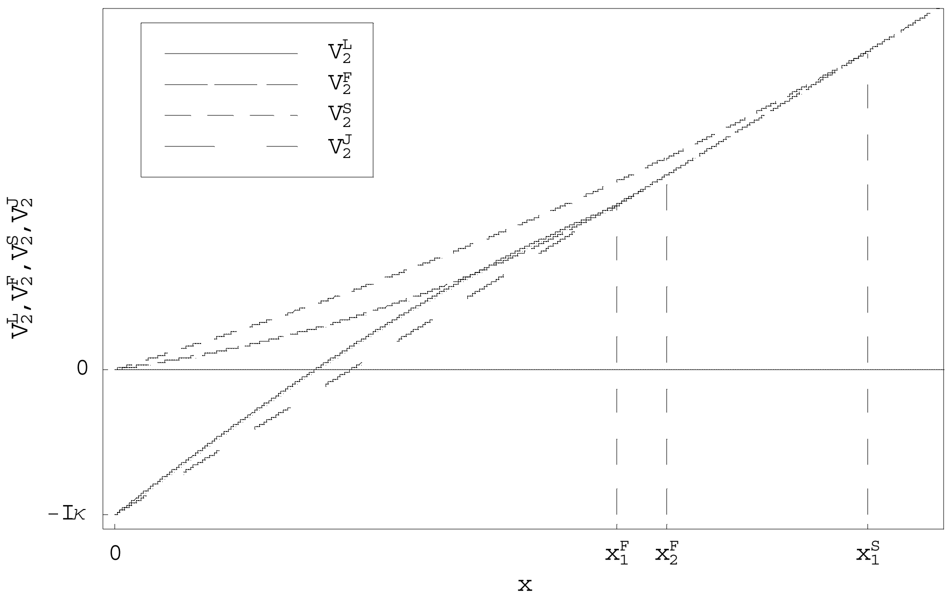

Figure 4.6: Firm 2's value functions when the resulting equilibrium is of the simultaneous type.

4.4.4 Conditions for Equilibria

The occurrence of a particular type of equilibrium is determined by the relationship between the relative payoffs, which in turn depend on the level of cost asymmetry, first-mover advantage and market parameters such as volatility, the growth rate and the interest rate. From Proposition 4.1 we already know the cut-off value of the cost asymmetry parameter κ that separates the preemptive and the sequential equilibrium. Now, we concentrate on determining the region in which the simultaneous equilibrium occurs. In order to do so, let us define

$$\zeta_i(x) \equiv V_i^S(x) - V_i^L(x). \tag{4.20}$$

$\zeta_i(x)$ can be interpreted as the change in Firm i's value associated with refraining from an immediate investment as the leader in favor of the simultaneous investment strategy. If the minimum of $\zeta_1(x)$ on the interval $[x(0), x_1^F]$ is larger than zero, the change is positive, and thus a simultaneous equilibrium occurs. In other words, the simultaneous equilibrium requires that Firm 1 is always better off by investing jointly at its optimal threshold x_1^S compared to becoming the leader.[9] Otherwise, either the sequential or the preemption equilibrium occurs.

[9]Strictly speaking, the equilibrium with sequential/preemptive investment still exists in this case but is Pareto-dominated by the simultaneous entry equilibrium (cf. Fudenberg and Tirole, 1985).

Proposition 4.2 *There exists a unique value of $\kappa \geq 1$, denoted by κ^{**}, which is equal to*

$$\kappa^{**} = \max\left((D_{11} - D_{01}) \left(\frac{\beta_1 (D_{10} - D_{11})}{(D_{10} - D_{00})^{\beta_1} - (D_{11} - D_{00})^{\beta_1}} \right)^{\frac{1}{\beta_1 - 1}}, 1 \right), \qquad (4.21)$$

that determines the regions of the simultaneous and the sequential/preemptive investment equilibria. For $\kappa < \kappa^{**}$ the resulting equilibrium is of the joint investment type, whereas for $\kappa \geq \kappa^{**}$ the sequential/preemptive investment equilibrium occurs.

Proof. See the Appendix. ■

Proposition 4.2 implies that for a relatively high degree of asymmetry between firms (for a given set of D_{ij}s and β_1), simultaneous investment is not optimal and either a sequential or preemption equilibrium occurs. Moreover, there exists a set of parameter values for which simultaneous investment is not optimal even when the firms are symmetric. In this case κ^{**} is equal to 1. We present an illustration of when the resulting equilibria occur in a two-dimensional graph. In Figure 4.7 we depict the investment strategies as a function of the first-mover advantage, D_{10}/D_{11}, and the investment cost asymmetry, κ.

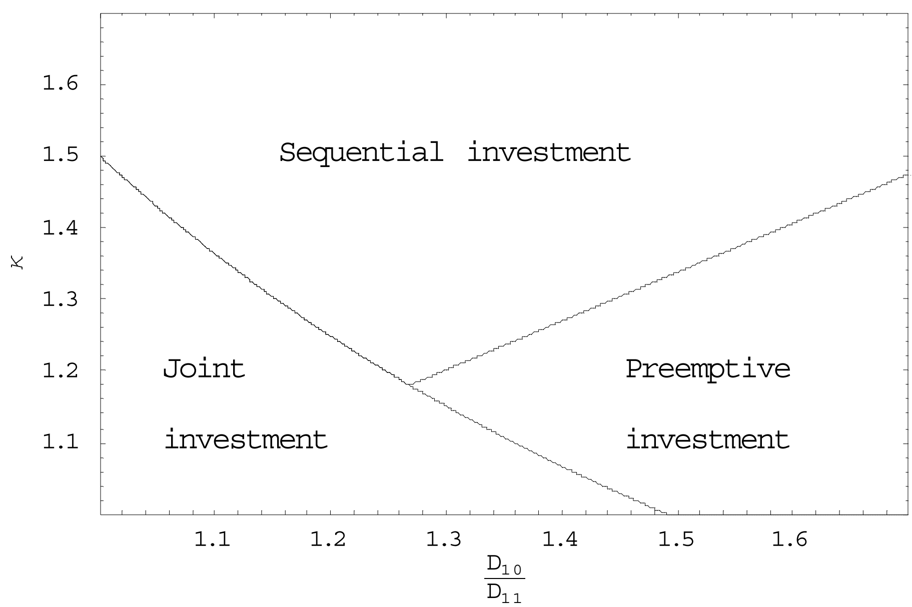

Figure 4.7: Regions of sequential, preemptive and joint investment equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, and $D_{11} = 1$.

When the investment cost asymmetry is relatively small and there is no significant first-mover advantage, the firms invest jointly (a triangular area in the south-west).

When the first-mover advantage becomes significant, Firm 1 prefers being the leader to investing simultaneously. This results in the preemption equilibrium (area in the south-east). Finally, if the asymmetry between firms is significant (for the set of parameter values in the upper part of Figure 4.7), the firms invest sequentially and Firm 1 can act as a sole holder of the investment opportunity.

4.5 Uncertainty and Investment Thresholds

From the real option literature it is known that in a non-strategic framework increasing uncertainty leads to a higher optimal investment threshold. As we show below, this observation also holds in strategic models as long as the firms' investment thresholds are solutions to the optimization problem. The follower's threshold, the leader's threshold in the sequential equilibrium, and the critical value triggering simultaneous investment satisfy this condition. Conversely, in the preemptive equilibrium the leader (Firm 1) does not always invest at the threshold that solves its optimization problem, but instead, for certain parameter values it invests at the follower's (Firm 2's) preemption point.

From (4.6), (4.13) and (4.17) it is concluded that the optimal thresholds can be expressed as

$$x_i^{opt} = \frac{\beta_1}{\beta_1 - 1} \frac{I_i \delta}{D_{after} - D_{before}}, \tag{4.22}$$

where D_{after} and D_{before} are the deterministic contributions to the profit function corresponding to a given threshold (cf. (4.3)). Consequently

$$\frac{\partial x^{opt}}{\partial (\sigma^2)} = -\frac{\delta}{(\beta_1 - 1)^2} \frac{I_i}{D_{after} - D_{before}} \frac{\partial \beta_1}{\partial (\sigma^2)} > 0, \tag{4.23}$$

i.e. the optimal follower threshold, optimal leader threshold and the critical value corresponding to simultaneous investment increase with uncertainty.

The impact of volatility on Firm 2's preemption point, x_{21}^P, at which Firm 1 invests, requires slightly more attention. Let us recall that x_{21}^P is the smallest root of $\xi_2(x) = 0$. Consequently, we calculate the derivative of $\xi_2(x)$ with respect to the profit uncertainty. The change of (4.16), calculated for Firm 2, resulting from a marginal increase in σ^2 can be decomposed as follows:

$$\frac{d\xi_2(x)}{d(\sigma^2)} = \left(\frac{\partial \xi_2(x)}{\partial \beta_1} + \frac{\partial \xi_2(x)}{\partial x_1^F} \frac{dx_1^F}{d\beta_1} \right) \frac{\partial \beta_1}{\partial (\sigma^2)}. \tag{4.24}$$

The derivative $\frac{\partial \xi_2(x)}{\partial \beta_1} \frac{\partial \beta_1}{\partial (\sigma^2)}$ measures the direct influence of uncertainty on the net benefit of being the leader. The product $\frac{\partial \xi_2(x)}{\partial x_1^F} \frac{dx_1^F}{d\beta_1} \frac{\partial \beta_1}{\partial (\sigma^2)}$ reflects the impact on the net benefit of being the leader of the fact that the follower investment threshold increases with uncertainty.

It can be shown that

$$\frac{\partial \xi_2(x)}{\partial \beta_1}\frac{\partial \beta_1}{\partial(\sigma^2)} < 0, \tag{4.25}$$

$$\frac{\partial \xi_2(x)}{\partial x_1^F}\frac{dx_1^F}{d\beta_1}\frac{\partial \beta_1}{\partial(\sigma^2)} > 0. \tag{4.26}$$

Apparently, the joint impact of both effects is ambiguous. The first effect is (4.25), which represents the simple value of waiting argument: if uncertainty is large, it is more valuable to wait for new information before undertaking the investment. As we have just seen, this also holds for the follower. The implication for the leader of the follower investing later is that the leader has a cost advantage for a longer time. This makes an earlier investment of the leader more beneficial. This effect is captured by (4.26), which can thus be interpreted as an increment in the strategic value of becoming the leader vs. the follower resulting from the delay in the follower's investment.

However, it is possible to show that the direct effect captured by (4.25) dominates, irrespective of the values of the input parameters.

Proposition 4.3 *When uncertainty of the product market increases, the leader investment threshold increases as well.*

Proof. See the Appendix. ■

In addition to the results obtained in (4.23) and Proposition 4.3, we perform extensive numerical experiments aiming at determining the impact of uncertainty on the boundaries of the equilibrium type regions. These simulations indicate that κ^{**} increases and κ^{*} decreases with σ, which implies that the preemption region reduces with σ. This fact contributes to the positive impact of uncertainty on the firms' investment thresholds. Our numerical results are thus consistent with Boyer et al. (2002), who show an increase in uncertainty may result in a switch from preemptive to the joint investment equilibrium.

Our conclusions concerning the relationship between the investment timing and uncertainty are consistent with recent empirical evidence. The negative investment-uncertainty relationship for firms operating in an imperfectly competitive environment is documented, for example, by Guiso and Parigi (1999).

4.6 Cost Asymmetry and Value of the Firm

In this section we discuss the impact of the degree of investment cost asymmetry on the value of each firm and, in particular, on the present value of the investment opportunities. We show that, in the presence of strategic interactions, the relationship between the magnitude of the investment cost asymmetry and the value of the firm can be, in general, discontinuous and non-monotonic.

In the absence of strategic interactions among the firms the value-asymmetry relationship is relatively straightforward. An increase in the investment cost of Firm 2 affects its value via *i*) a higher present value of the investment expenditure that has to be incurred and *ii*) a delay in the optimal timing of investment which results in postponing the moment of the profit flow increase. Consequently, the value of Firm 2 decreases monotonically with κ. Conversely, the value of Firm 1 remains unaffected by a change in κ since the firms do not interact with each other.

Introducing competition changes the way the asymmetry affects the values of both firms. In such a case, the value of Firm 2 is affected not only by an increase in its investment cost but also by the fact that Firm 1 moves along its reaction curve in response to the changing characteristics of Firm 2. Consequently, the value of Firm 2 will also be affected by the change of Firm 1's investment timing influencing the cash flow of the former. We illustrate the impact of strategic interactions with an example in which parameter values are chosen in such a way that for different values of the cost asymmetry parameter all three types of equilibria are possible (cf. Figure 4.7). The firms' values resulting from their optimal strategies are depicted in Figure 4.8.

The lowest degree of asymmetry between the firms corresponds to the simultaneous investment equilibrium. In the simultaneous equilibrium the outcome closely resembles the case where strategic interactions are absent, in the sense that a marginal increase in κ does not affect the value of Firm 1 and has a negative impact on the value of Firm 2.

As κ increases, sequential investment becomes more attractive for Firm 1 because of the increasing Firm 2's follower threshold. This means that Firm 2 will invest later so that Firm 1's sequential investment profit goes up. Consequently, for κ exceeding κ^{**}, Firm 1 would optimally invest at its leader threshold x_1^L. However, Firm 2 anticipates this and, since its leader value at x_1^L is larger than its follower value, it is willing to invest an instant before Firm 1 does. Again, Firm 1 reacts on this and, as explained in Section 4.1, invests at Firm's 2 preemption point $x_{21}^P < x_1^L$. In such a situation the shift in Firm 1's reaction curve is discontinuous and a preemption

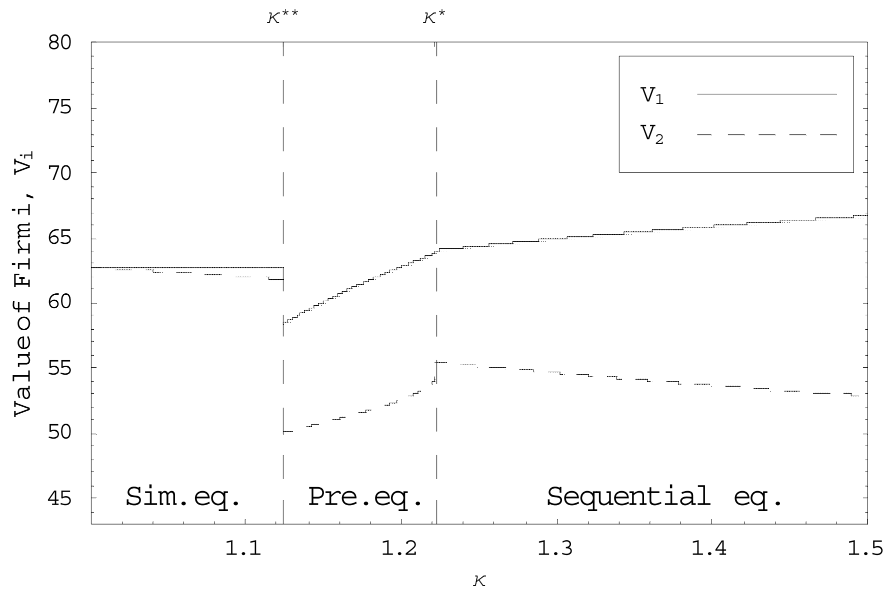

Figure 4.8: The value of Firm i (V_i) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.33$, $D_{11} = 1$, $I = 100$ and $x = 4$.

equilibrium resulting in lower values of both firms occurs. The implication is that a marginal increase in the investment cost of Firm 2 that changes the equilibrium from simultaneous to preemptive, results in both firms' payoffs jumping downward.

Once the firms are in the preemption region, the value of both increases with κ. The at first sight surprising positive relationship between Firm 2's investment cost and its value is caused by the fact that increasing κ makes Firm 2 a 'weaker' competitor. This implies that the preemption threat of Firm 2 declines in the investment cost asymmetry, so that x_{21}^P increases with κ. Therefore, Firm 1 invests later, and this is beneficial for the cash flow of Firm 2 since it can enjoy a higher cash flow for a longer period. In this case, the non-strategic, i.e. increasing investment cost for Firm 2, and strategic effects work in the opposite direction and the latter dominates. As far as Firm 1 is concerned, its value increases because its investment threshold moves closer to x_1^L. Moreover, it benefits from the delayed investment of Firm 2.

When the asymmetry between the firms reaches the critical level κ^*, above which it is not optimal anymore for Firm 2 to become the leader, the sequential equilibrium occurs. Upon the switch to the sequential equilibrium the values of both firms move upward. In both cases this is caused by the discontinuous change, from x_{21}^P to x_1^L, of Firm 1's investment threshold. By investing at x_1^L Firm 1 maximizes its value, and lets Firm 2 enjoy a higher cash flow for a longer period.

In the sequential equilibrium region the changes in the firms' values result entirely from the sunk cost asymmetry and its impact on Firm 2's investment timing. Firm 1 benefits from the delayed investment of Firm 2 and the value of the latter decreases for the same reason as in the non-strategic case.

In order to provide better intuition about the nature of the non-monotonic relationship between the value of the firm, V_i, and the investment cost asymmetry, κ, we decompose V_i into three components. First, we calculate the expected value of discounted future profits in case no investment is made, which reflects the value of assets in place, A/P_i. Further, we derive the value of the firm's own investment opportunity given that the other firm does not invest, $PVGO_i^O$. Finally, the magnitude of the impact of the competitor's investment on the firm's profits, $PVGO_i^C$ is determined. The sum of $PVGO_i^O$ and $PVGO_i^C$ can be interpreted as the strategic NPV of the investment opportunity of Firm i.

Table 4.1 presents the decomposition of Firm 1's value for different levels of the cost-asymmetry.

κ	1.1	1.15	1.2	1.25	1.33	1.5
A/P_1	57.14	57.14	57.14	57.14	57.14	57.14
$PVGO_1^O$	14.19	15.14	17.16	18.15	18.15	18.15
$PVGO_1^C$	−8.58	−12.17	−11.51	−10.91	−10.05	−8.57
V_1	62.75	60.12	62.80	64.39	65.24	66.72
				$\kappa^* = 1.222$		$\kappa^{**} = 1.124$

Table 4.1: Decomposition of Firm 1's value into the expected present value of the perpetual cash flow from assets in place, A/P_1, the option to invest, $PVGO_1^O$, short the competitor's option to invest, and the value reduction due to the competitor's investment, $PVGO_1^C$, for the set of parameter values $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.33$, $D_{11} = 1$, and $I = 100$. The value of the firm, V_1, equals $A/P_1 + PVGO_1^O + PVGO_1^C$.

From Table 4.1 a number of conclusions can be drawn. First, we notice that the value attributed to assets in place does not change with the investment cost asymmetry. This is understandable since the existing production assets of the firms are identical. Second, the value of Firm 1's investment opportunity rises with κ. This reflects the fact that the growing competitive advantage allows Firm 1 to keep its investment strategy closer to the unconditional optimum, x_1^L (at which the value of $PVGO_1^O$ in the example equals 18.15). Consequently, the only source of non-monotonicity is the interaction of Firm 2's investment decision with Firm 1's profit (see $PVGO_1^C$ in Table 4.1). When the cost-asymmetry becomes larger, i.e. when $\kappa \geq \kappa^{**} = 1.124$, then Firm 1 has no longer an incentive to wait until the optimal simultaneous threshold is reached and is aiming at preempting Firm 2. As discussed above, the resulting preemption game deteriorates both firm's payoffs and, as a direct consequence, their values.

Table 4.2 contains an analogous decomposition of the value of Firm 2.

κ	1.1	1.15	1.2	1.25	1.33	1.5
A/P_2	57.14	57.14	57.14	57.14	57.14	57.14
$PVGO_2^O$	13.45	11.94	11.29	10.70	9.80	7.88
$PVGO_2^C$	−8.58	−18.25	−15.85	−12.67	−12.67	−12.67
V_2	62.01	50.83	52.59	55.18	54.61	52.36
				$\kappa^* = 1.222$		$\kappa^{**} = 1.124$

Table 4.2: Decomposition of Firm 2's value into the expected present value of the perpetual cash flow from assets in place, A/P_2, the option to invest, $PVGO_2^O$, short the competitor's option to invest and recapture the part of the market share, $PVGO_2^C$, for the set of parameter values $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.33$, $D_{11} = 1$, and $I = 100$. The value of the firm, V_2, equals $A/P_2 + PVGO_2^O + PVGO_2^C$.

Upon analyzing Table 4.2 it can be concluded that increasing investment cost asymmetry has two effects on the value of Firm 2. First, it results in the reduction of the value of Firm 2's investment opportunity, $PVGO_2^O$. This relationship is monotonic irrespective from the type of the prevailing equilibrium and results from the increase in the investment expenditure that has to be incurred. Second, it influences the way the competitor's option to invest, $PVGO_2^C$, affects the value of the firm. In the region of the preemptive equilibrium, i.e. for $\kappa \in [1.124, 1.222]$, the value of Firm 2 lost due to the exercise of the investment opportunity by Firm 1, $PVGO_2^C$, is inversely related to the investment cost asymmetry. In other words, when Firm 2's cost becomes higher, the investment of its competitor has a smaller negative impact on its value since the competitor invests later. This is the strategic effect of the marginal increase in investment cost (Firm 2 becomes a "weaker competitor"), which dominates the direct effect of the increase in κ on the net present value of the project, $PVGO_2^O$.[10]

[10] Using a static framework, Gelman and Salop (1983) show that the profit of a smaller entrant may be positively related to its competitive disadvantage interpreted as a capacity constraint.

So far, we considered the impact of a difference in the investment cost on the value of the firms. We have shown that there exists a non-monotonic and discontinuous relationship between the cost asymmetry and the firms' values resulting from the switches among the different types of equilibrium strategies. In the next section we discuss the impact of κ on social welfare by showing how particular types of strategies affect the consumer surplus.

4.7 Welfare Analysis

In order to assess the desirability of policies influencing the firms' access to new market segments and technologies, we investigate how investment cost asymmetry affects social welfare. The investment cost that has to be incurred by the firm can be influenced by the regulator, for instance, via fiscal measures and governmental guarantees resulting in a lower cost of capital. Kleimeier and Megginson (2000) provide empirical evidence that the presence of a third party guarantee lowers the cost of capital. Moreover, the firms' access to new markets and technologies can be equalized via knowledge spillovers. Stoneman and Diederen (1994) analyze the actual diffusion policies of the governments and their implications for the firms' behavior.

The desirability of a policy can be measured by the way it affects social welfare, which is the sum of the consumer surplus and the firms' values.[11] Since in the previous sections we already established the firms' payoffs, here we begin the analysis with deriving the consumer surplus. Subsequently, we discuss how this surplus is influenced by the firms investment strategies. After having done this, we are ready to present the relationship between the investment strategies and social welfare. Finally, we provide some conclusions.

In order to derive the consumer surplus, we specify the way investment is beneficial to the consumers. To do so, we introduce a simple setting in which after the investment Firm i is offering a product of quality $b_1 > b_0$, where b_0 denotes the initial quality of the product. As long as the firms offer the same quality b_k, $k \in \{0, 1\}$, they compete à la Cournot, whereas after making the investment first, Firm 1 achieves a Stackelberg advantage in the differentiated product market. The Cournot outcome is restored after Firm 2 has invested. Then both firms compete in the market with a higher quality.

The market we consider has a continuum of consumers with an instantaneous

[11] Tirole (1988), Ch. 5-8, provides an extensive introduction to oligopoly theory.

utility function

$$U_i(t) = \theta_i b - p(t), \tag{4.27}$$

where θ_i is a consumer-specific parameter that is uniformly distributed over the interval $[0, A(t)]$, b is the quality of the product and $p(t)$ is its price at time t. The parameter $A(t)$ reflecting consumers' valuations follows the geometric Brownian motion

$$dA(t) = \frac{1}{2}\left(\alpha - \frac{1}{4}\sigma^2\right) A(t)\, dt + \frac{1}{2}\sigma A(t)\, dw(t), \tag{4.28}$$

where α, σ and $dw(t)$ are the same as in (4.1). It is useful to observe (by applying Itô's lemma) that A^2 can be replaced by x since it exactly follows process (4.1).

If both firms offer the same quality, the instantaneous demand function corresponding to utility function (4.27) can be expressed as

$$p(t) = (A - q_1(t) - q_2(t))\, b, \tag{4.29}$$

where $q_i(t)$ denotes the quantity offered by Firm i at time t.

Let us now derive the expressions for the instantaneous consumer surplus, denoted by $cs_{kl}(t)$, where k and l relate to the quality offered by the firms. In order to analyze the complete structure of the game, we consider three cases. In the first case only quality b_0 is provided. In the second case one firm provides quality b_0 and the other b_1 ($> b_0$) and, finally, both firms offer b_1. In the first and third case, maximizing the firm's instantaneous profits, calculating social welfare, and the residual surplus given (4.29), yields (for a derivation see the Appendix)

$$cs_{kk}(t) = \frac{2}{9} b_k x(t). \tag{4.30}$$

The formulation of cs_{10} (the second mentioned case) is slightly more involved and it corresponds to a Stackelberg equilibrium with second degree price discrimination. Consequently, cs_{10} consists of two components: the surplus of consumers purchasing the good of quality b_1 and the surplus of those who choose b_0. Solving the Stackelberg game yields the instantaneous consumer surplus (see the Appendix)

$$cs_{10} = \frac{4b_1 + 5b_0}{32} x. \tag{4.31}$$

To find out in what way the consumer surplus is related to the firms' investment strategies, we analyze the changes in the consumer surplus across the equilibria. If the resulting equilibrium is of the simultaneous type the consumer surplus, $CS^S(t)$, equals

$$CS^S(t) = E\left[\int_t^{T^{S_1}} e^{-r(s-t)} cs_{00}(s)\, ds + \int_{T^{S_1}}^{\infty} e^{-r(s-t)} cs_{11}(s)\, ds\right], \tag{4.32}$$

where T_1^S is given by (4.12). When the resulting equilibrium is of the preemption type, the consumer surplus, $CS^P(t)$, amounts to

$$CS^P(t) = E\left[\int_t^{\min\left(T_{21}^P, T_1^L\right)} e^{-r(s-t)} cs_{00}(s)\, ds\right] \tag{4.33}$$
$$+E\left[\int_{\min\left(T_{21}^P, T_1^L\right)}^{T_2^F} e^{-r(s-t)} cs_{10}(s)\, ds + \int_{T_2^F}^{\infty} e^{-r(s-t)} cs_{11}(s)\, ds\right],$$

where

$$T_{21}^P = \inf\left(t \mid x \geq x_{21}^P\right), \tag{4.34}$$
$$T_1^L = \inf\left(t \mid x \geq x_1^L\right), \tag{4.35}$$

and T_2^F is defined by (4.5). The consumer surplus in the sequential equilibrium is the same as (4.33), with the exception that $\min\left(T_{21}^P, T_1^L\right)$ is replaced by T_1^L.

After taking into account that the firms invest later in the simultaneous equilibrium, a comparison of (4.32) and (4.33) enables us to formulate the following proposition.

Proposition 4.4 *Under the preemptive/sequential equilibrium the consumer surplus is always larger than in the joint investment equilibrium.*

Proof. See the Appendix. ■

Consequently, from the consumers' viewpoint, the situation in which the firms invest simultaneously is undesirable. This is easy to understand since in this case the firms invest later so that during a longer period of time the product with a higher quality is not available.

Now, let us investigate social welfare, which equals, as mentioned earlier, the consumer surplus plus the value of the firms. In order to relate the latter to the analyzed market, we can make the following substitution, where the expressions at the RHS of each equality result from the maximization of the firms' profits (see the Appendix):

$$D_{00} \equiv \frac{b_0}{9} \qquad D_{01} \equiv \frac{b_0}{16} \qquad D_{10} \equiv \frac{2b_1 - b_0}{8} \qquad D_{11} \equiv \frac{b_1}{9}$$

For a particular example, the consumer surplus and the firms' values are depicted as functions of the asymmetry in the investment cost in Figure 4.9. From this figure it can be concluded that low asymmetry in the investment costs results in a

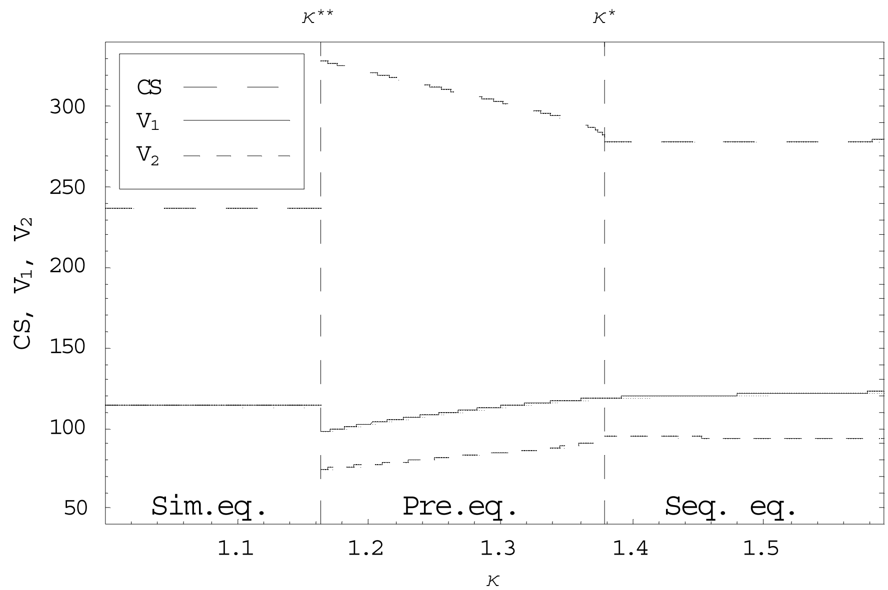

Figure 4.9: Value of Firm i (V_i) and consumer surplus (CS) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $b_0 = 5$, $b_1 = 7$, $I = 100$, and $x = 7$.

relatively low consumer surplus and higher values of the firms. Increasing the asymmetry among the firms, such that the simultaneous equilibrium is superseded by the preemption equilibrium, leads to a downward jump in the firms' values and, at the same time, to an upward jump in the consumer surplus. As seen before, the decline in the firms' values mainly results from the need to incur the investment expenditure, I, earlier. The increase in the consumer surplus is the consequence of an earlier provision of the higher quality product. When the investment cost is large compared to the increase in the consumer surplus associated with higher quality, it is optimal from a welfare perspective to postpone the investment. Therefore, in such a case an increase in κ leading to a switch from simultaneous to preemption equilibrium has a detrimental effect on welfare. Conversely, when the required sunk cost is relatively small, the resulting preemption equilibrium is socially desirable.

The impact of increasing κ on social welfare is summarized in the following corollary.

Corollary 4.2 *There exists a critical level of investment expenditure below which social welfare is always larger in the preemptive/sequential equilibrium than in the joint investment equilibrium.*

Consequently, if the investment expenditure is small relative to the consumer surplus, social welfare is highest under the preemption equilibrium. In this case, the loss in the firms' values resulting from the preemption game is outweighed by the effect on the consumer surplus of an earlier provision of the high quality product. This implies that in the case of a relatively low investment expenditure, a relative cost disadvantage of one of the competitors results in strategies yielding a socially preferred outcome.

Conversely, a relatively high investment expenditure implies the social optimality of the simultaneous equilibrium. This results from the fact that the simultaneous equilibrium is associated with the investment outlay occurring later. Since the increase in consumer surplus resulting from providing a higher quality product earlier is not sufficient to fully compensate for the higher present value of an early investment, postponing the investment is socially desirable. Therefore, in the presence of a high sunk cost of the project, investment strategies resulting in the simultaneous equilibrium maximize social welfare. This, in turn, implies that the cost asymmetry is not desirable. Corollary 4.3 summarizes these findings.

Corollary 4.3 A *socially desirable outcome is more likely to occur when investment that requires a high sunk cost is associated with a low asymmetry across firms and when a low sunk cost investment is to be made by highly asymmetric firms.*

Corollaries 4.2 and 4.3 are closely associated with the impact of uncertainty on the social welfare in equilibrium. Other things equal, higher profit volatility discourages investment and may result in a switch from the preemptive to the joint investment equilibrium. Therefore, if entering a new market segment is associated with a significant investment cost, it is possible that higher uncertainty in this segment can positively influence social welfare. Conversely, if the investment cost is relatively low, so the preemptive equilibrium is socially optimal, uncertainty will be negatively related to the social welfare.

We conclude that an equal access of two firms to a new market segment does not maximize consumer surplus. Moreover, after taking into account the values of the firms, it is not always socially desirable. If the firms' investment costs are not excessively high, the presence of asymmetry among them yields a socially more desirable outcome.

However, it is important to notice that these conclusions do not carry over to the case where the first-mover advantage is large, which would occur when the product quality difference is higher. Then, as illustrated in Figure 4.7 the preemption equilibrium prevails even if firms are symmetric. Consequently, from a welfare perspective, asymmetry is not desirable even if the investment is associated with a relatively low sunk cost.

4.8 Conclusions

In this chapter, the impact on the firms' optimal investment strategies of a difference in the costs associated with their profit-enhancing investments is analyzed. Since the firms operate in an imperfectly competitive duopolistic market, the profitability of each firm's project is affected by the other firm's decision to invest. We show that when the asymmetry among firms is relatively small and so is the first-mover advantage, the firms invest jointly. When the first-mover advantage is significant, the lower-cost firm preempts the higher-cost firm. In the situation where the asymmetry between firms becomes sufficiently large, the firms exercise their investment options sequentially and their mutual decisions do not affect each other directly.

Subsequently, we analyze the impact of uncertainty on optimal investment timing. Despite the presence of strategic interactions, increasing uncertainty always results in a higher investment threshold. This holds not only for the optimal investment thresholds but also for the case when the lower-cost firm faces the threat of being preempted by its higher-cost opponent.

Furthermore, the effects of investment cost asymmetry on the values the two firms are explored. It is shown that the relationship between the firm's value and the cost asymmetry is non-monotonic and discontinuous. We obtain a number of counter-intuitive results. For reasonable parameter values, deepening the firm's competitive disadvantage due to a marginal rise in its irreversible cost may reduce the value of its competitor. This situation results when a switch from simultaneous to preemptive equilibrium occurs upon the marginal change in the cost asymmetry. Another interesting effect of strategic interactions is present when the firms are engaged in a preemption game. Then increasing the extent to which the firms is set at cost-disadvantage leads to an appreciation of its value due to the strategic effect on the competitor's investment timing.

Finally, we discuss the welfare effects of strategic interactions between the firms. In an example where the investment increases product quality, we show that the relationship between cost asymmetry and social welfare depends on the cost of investment. If it is relatively high and the first-mover advantage is not too large, social welfare is maximized when none of the firms suffers from competitive disadvantage. However, if the investment cost is low, an increase of the consumer surplus resulting from the early investment in the preemption equilibrium exceeds the loss of the firms' joint value associated with such an investment. Therefore, the preemption equilibrium, occurring when the sufficiently costs differ, is in this case desirable. This observation allows for the conclusion that an equal access of competitors to a new technology or market segment may not be socially optimal.

4.9 Appendix

Proof of Proposition 4.1. The sequential equilibrium occurs when Firm 2 has no incentive to invest as the leader. Formally, this requires that $\xi_2(x)$ is negative for all $x \in [x(0), x_2^F)$. Therefore, in order to determine the domain of κ-values where the sequential equilibrium prevails, we are interested in finding a pair $(x^*;\kappa^*)$ that satisfies the following system of equations

$$\begin{cases} \xi_2(x^*;\kappa^*) = 0 \\ \frac{\partial \xi_2(x^*;\kappa^*)}{\partial x}\Big|_{x=x^*} = 0. \end{cases} \tag{4.36}$$

In other words, we are interested in a point $(x^*;\kappa^*)$ at which Firm 2's leader function is tangent to the follower function. After substituting (4.8) and (4.10) into (4.16), all defined for Firm 2 for $x \le x_2^F$, and rearranging we obtain that (4.36) becomes:

$$\begin{cases} \frac{x^*(D_{10}-D_{01})}{\delta} - I\kappa^* + \frac{x_1^F(D_{11}-D_{10})}{\delta}\left(\frac{x^*}{x_1^F}\right)^{\beta_1} - \frac{x_2^F(\kappa^*)(D_{11}-D_{01})}{\beta_1\delta}\left(\frac{x^*}{x_2^F(\kappa^*)}\right)^{\beta_1} = 0 \\ \frac{D_{10}-D_{01}}{\delta} + \beta_1\frac{D_{11}-D_{10}}{\delta}\left(\frac{x^*}{x_1^F}\right)^{\beta_1-1} - \frac{D_{11}-D_{01}}{\delta}\left(\frac{x^*}{x_2^F(\kappa^*)}\right)^{\beta_1-1} = 0. \end{cases} \tag{4.37}$$

After multiplying both sides of the second equation in (4.37) by $\frac{x^*}{\beta_1}$, subtracting it from the first equation, and rearranging, we obtain

$$x^* = \frac{\beta_1}{\beta_1 - 1}\frac{I\kappa^*\delta}{D_{10}-D_{01}}. \tag{4.38}$$

Substituting (4.38) into the first equation in (4.37) and (4.6) for x_1^F yields

$$\frac{\beta_1}{\beta_1-1}I\kappa^* - I\kappa^* + \left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\kappa^*\right)^{\beta_1}\frac{\beta_1}{\beta_1-1}\frac{D_{11}-D_{10}}{D_{11}-D_{01}}I - \left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\right)^{\beta_1}\frac{I\kappa^*}{\beta_1-1} = 0. \tag{4.39}$$

Rearranging (4.39) leads to the expression (4.19).

In the remaining part of the proof, we demonstrate that $\kappa^* > 1$. It holds that

$$\kappa^* > 1 \Longleftrightarrow \frac{(D_{10}-D_{01})^{\beta_1} - (D_{11}-D_{01})^{\beta_1}}{\beta_1(D_{10}-D_{11})} - (D_{11}-D_{01})^{\beta_1-1} > 0, \tag{4.40}$$

which can be rewritten into

$$\begin{aligned} & \frac{(D_{10}-D_{01})^{\beta_1}-(D_{11}-D_{01})^{\beta_1}}{\beta_1(D_{10}-D_{11})}-(D_{11}-D_{01})^{\beta_1-1} \\ = & \frac{(D_{10}-D_{01})^{\beta_1}-(D_{11}-D_{01})^{\beta_1}-\beta_1(D_{10}-D_{11})(D_{11}-D_{01})^{\beta_1-1}}{\beta_1(D_{10}-D_{11})}>0. \end{aligned} \tag{4.41}$$

By substituting

$$a = D_{11}-D_{01}, \tag{4.42}$$

$$b = D_{10}-D_{01}, \tag{4.43}$$

and rearranging, we conclude that (4.41) is equivalent to

$$\frac{a^{\beta_1}}{\beta_1(b-a)}\left(\left(\frac{b}{a}\right)^{\beta_1}-1-\beta_1\frac{b}{a}+\beta_1\right). \tag{4.44}$$

After observing that $b>a$ and $\frac{a^{\beta_1}}{\beta_1(b-a)}>0$, we have to prove that the second factor of (4.44) is positive. Let us denote $w=\frac{b}{a}$ and $g(w)=w^{\beta_1}-1-\beta_1 w+\beta_1$. Consequently, we have

$$g(1) = 0, \text{ and} \tag{4.45}$$

$$\frac{\partial g(w)}{\partial w} = \beta_1 w^{\beta_1-1}-\beta_1>0,\ \forall\beta_1, w>1. \tag{4.46}$$

This completes the proof. ■

Proof of Proposition 4.2. Firm 1 prefers simultaneous investment unless for some x its leader payoff, $V_1^L(x)$, exceeds the optimal joint investment payoff, $V_1^S(x)$. Formally, the simultaneous equilibrium occurs only if $\zeta_1(x)$ is positive for all $x\in\left(x_1^P,x_2^F\right)$. Therefore, in order to determine the domain of κ-values for which the simultaneous equilibrium prevails, we are interested in finding a pair $(x^{**};\kappa^{**})$ that satisfies the following system of equations

$$\begin{cases} \zeta_1(x^{**};\kappa^{**})=0 \\ \left.\frac{\partial\zeta_1(x^*;\kappa^*)}{\partial x}\right|_{x=x^{**}}=0. \end{cases} \tag{4.47}$$

In other words, we are interested in a point $(x^{**};\kappa^{**})$ in which Firm 1's simultaneous investment function is tangent to its leader function. After substituting (4.10) and (4.14) into (4.20), all defined for Firm 1 for $x\le x_1^S$, and rearranging, we obtain

$$\begin{cases} \frac{x^{**}(D_{00}-D_{10})}{\delta}+I+\frac{x_1^S(D_{11}-D_{00})}{\beta_1\delta}\left(\frac{x^{**}}{x_1^S}\right)^{\beta_1}-\frac{x_2^F(\kappa^{**})(D_{11}-D_{10})}{\delta}\left(\frac{x^{**}}{x_2^F(\kappa^{**})}\right)^{\beta_1}=0 \\ \frac{D_{00}-D_{10}}{\delta}+\frac{D_{11}-D_{00}}{\delta}\left(\frac{x^{**}}{x_1^S}\right)^{\beta_1-1}-\beta_1\frac{D_{11}-D_{10}}{\delta}\left(\frac{x^{**}}{x_2^F(\kappa^{**})}\right)^{\beta_1-1}=0. \end{cases} \tag{4.48}$$

After multiplying the second equation in (4.48) by $\frac{x^{**}}{\beta_1}$, subtracting it from the first equation, and rearranging, we obtain

$$x^{**} = \frac{\beta_1}{\beta_1 - 1} \frac{I\delta}{D_{10} - D_{00}}. \tag{4.49}$$

Substituting (4.49) into the first equation in (4.48) and (4.13) and (4.6) for x_1^S and x_2^F, respectively, yields

$$-\frac{I}{\beta_1 - 1} + \left(\frac{D_{11} - D_{00}}{D_{10} - D_{00}}\right)^{\beta_1} \frac{I}{\beta_1 - 1} - \left(\frac{D_{11} - D_{01}}{(D_{10} - D_{00})\,\kappa^{**}}\right)^{\beta_1} \frac{\beta_1 I \kappa^{**}}{\beta_1 - 1} \frac{D_{11} - D_{10}}{D_{11} - D_{01}} = 0. \tag{4.50}$$

Given that we only consider the case that $\kappa^{**} \geq 1$, rearranging (4.50) leads to the expression (4.21).

In the remaining part of the proof we show that the optimality of the simultaneous investment for Firm 1 implies that Firm 2 is better off by investing simultaneously as well. Consequently, we prove that as long as it is optimal for Firm 1 to invest simultaneously, Firm 2's follower threshold is always smaller than Firm 1's optimal joint investment threshold (since if this is true, then it is always optimal for Firm 2 to invest immediately when Firm 1 invests). First, we determine $\widehat{\kappa}$ which solves

$$x_2^F(\widehat{\kappa}) = x_1^S(\widehat{\kappa}). \tag{4.51}$$

For $\kappa < \widehat{\kappa}$ it holds that $x_2^F(\widehat{\kappa}) < x_1^S(\widehat{\kappa})$. After substituting (4.6) for Firm 2 and (4.13) for Firm 1 into (4.51), and rearranging, we obtain

$$\widehat{\kappa} = \frac{D_{11} - D_{01}}{D_{11} - D_{00}}. \tag{4.52}$$

Now, we show that $\widehat{\kappa} > \kappa^{**}$, i.e. that

$$\frac{D_{11} - D_{01}}{D_{11} - D_{00}} - (D_{11} - D_{01}) \left(\frac{\beta_1 (D_{10} - D_{11})}{(D_{10} - D_{00})^{\beta_1} - (D_{11} - D_{00})^{\beta_1}}\right)^{\frac{1}{\beta_1 - 1}} > 0 \tag{4.53}$$

holds. After substituting

$$\begin{aligned} c &= D_{11} - D_{00}, \\ d &= D_{10} - D_{00}, \end{aligned}$$

and rearranging, we obtain that condition (4.53) is equivalent to

$$\frac{1}{c} - \left(\frac{\beta_1 (d - c)}{d^{\beta_1} - c^{\beta_1}}\right)^{\frac{1}{\beta_1 - 1}} > 0. \tag{4.54}$$

This implies

$$\left(\frac{d}{c}\right)^{\beta_1} - 1 - \beta_1 \left(\frac{d}{c} - 1\right) > 0.$$

Let us denote $z = \frac{d}{c}$ and $h(z) = z^{\beta_1} - 1 - \beta_1 (z-1)$. Consequently, we have

$$h(1) = 0, \text{ and} \tag{4.55}$$

$$\frac{\partial h(z)}{\partial z} = \beta_1 z^{\beta_1 - 1} - \beta_1 > 0, \tag{4.56}$$

since $z > 1$ and $\beta_1 > 1$. This completes the proof. ■

Proof of Proposition 4.3. The difference of Firm 2's payoffs as the leader and the follower for $x \leq x_2^F$, can be expressed as (cf. (4.8) and (4.10))

$$\xi_2(x) = \frac{x(D_{10} - D_{01})}{\delta} - I\kappa + \tag{4.57}$$

$$+\frac{I}{D_{11} - D_{01}} \frac{\left(x \frac{\beta_1 - 1}{\beta_1} \frac{D_{11} - D_{01}}{I\delta}\right)^{\beta_1}}{\beta_1 - 1} \left(\beta_1 (D_{11} - D_{10}) - \frac{D_{11} - D_{01}}{\kappa^{\beta_1 - 1}}\right).$$

We are interested in the direction in which uncertainty affects x_{21}^P, i.e. the smallest root of (4.57). The derivative of (4.57) with respect to β_1 equals

$$\frac{\partial \xi_2(x)}{\partial \beta_1} = \frac{I}{D_{11} - D_{01}} \frac{\left(x \frac{\beta_1 - 1}{\beta_1} \frac{D_{11} - D_{01}}{I\delta}\right)^{\beta_1}}{\beta_1 - 1} \times \tag{4.58}$$

$$\times \left(\ln \left(x \frac{\beta_1 - 1}{\beta_1} \frac{D_{11} - D_{01}}{I\delta} \right) \left(\beta_1 (D_{11} - D_{10}) - \frac{D_{11} - D_{01}}{\kappa^{\beta_1 - 1}} \right) \right.$$

$$\left. + D_{11} - D_{10} + \frac{D_{11} - D_{01}}{\kappa^{\beta_1 - 1}} \ln \kappa \right).$$

It is straightforward to observe that for sufficiently small x (4.58) is positive. This can be generalized into the statement that there exists $\overline{x}$ satisfying

$$\operatorname{sgn} \frac{\partial \xi_2(x)}{\partial \beta_1} = \begin{cases} 1, & x \in (0, \overline{x}), \\ 0, & x = \overline{x}, \\ -1, & x \in \left(\overline{x}, x_2^F\right). \end{cases} \tag{4.59}$$

Since (in general) it is not possible to obtain an analytical formula for x_{21}^P, we evaluate the sign of the derivative (4.58) at such a realization of x for which the corresponding sign is the same as at x_{21}^P. Consequently, we are interested in the realization of x that satisfies the following two properties

$$\xi_2(x^*) < 0 \Longrightarrow \xi_2(x) < 0 \ \forall x, \text{ and} \tag{4.60}$$

$$\exists x_{21}^P \Longrightarrow x_{21}^P < x^*. \tag{4.61}$$

Properties (4.59) and (4.61) imply that

$$\left.\frac{\partial\xi_2(x)}{\partial\beta_1}\right|_{x=x^*}>0\Longrightarrow\left.\frac{\partial\xi_2(x)}{\partial\beta_1}\right|_{x=x_{21}^P}>0. \tag{4.62}$$

The realization x^* equal to (cf. (4.38))

$$x^*=\frac{\beta_1}{\beta_1-1}\frac{I\kappa\delta}{D_{10}-D_{01}} \tag{4.63}$$

satisfies (4.60) and (4.61). Property (4.60) can be verified by examining the definition of x^* (cf. (4.36)) and by observing that

$$\frac{\partial\xi_2(x)}{\partial\kappa}<0.$$

Property (4.61) follows directly. Subsequently, we determine the sign of the derivative (4.58) at x^* :

$$\begin{aligned}\left.\frac{\partial\xi_2(x)}{\partial\beta_1}\right|_{x=x^*} &= \frac{I}{D_{11}-D_{01}}\frac{\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\kappa\right)^{\beta_1}}{\beta_1-1}\times \\ &\times\left(\ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\kappa\right)\left(\beta_1(D_{11}-D_{10})-\frac{D_{11}-D_{01}}{\kappa^{\beta_1-1}}\right)\right. \\ &\left.+D_{11}-D_{10}+\frac{D_{11}-D_{01}}{\kappa^{\beta_1-1}}\ln\kappa\right).\end{aligned} \tag{4.64}$$

Let us denote

$$\begin{aligned}\varphi(\kappa) &= \ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\kappa\right)\left(\beta_1(D_{11}-D_{10})-\frac{D_{11}-D_{01}}{\kappa^{\beta_1-1}}\right) \\ &+D_{11}-D_{10}+\frac{D_{11}-D_{01}}{\kappa^{\beta_1-1}}\ln\kappa.\end{aligned} \tag{4.65}$$

Positive $\varphi(\kappa)$ for $\forall\kappa\in[1,\kappa^*]$, where κ^* is defined by (4.19), would imply the positive relationship between uncertainty and the leader threshold. First, we show that $\varphi(\kappa^*)$ is positive. Subsequently, we prove that

$$\varphi(\kappa^*)>0\Longrightarrow\varphi(\kappa)>0\ \forall\kappa\in[1,\kappa^*]. \tag{4.66}$$

The proof that $\varphi(\kappa^*)>0$ consists of three steps. First, we change the variables and factorize the function $\varphi(\kappa^*)$, which yields the product of two factors: one with negative and one with unknown sign. Second, we show that the factor with the unknown sign is increasing with the relevant variable. Finally, we show that the value of the factor with a priori unknown sign approaches zero when the underlying variable approaches the upper limit of its domain. The last two steps imply that the sign of the analyzed factor is negative, which is equivalent to $\varphi(\kappa^*)$ having a positive sign.

Consequently, we substitute (4.19) into (4.65) and obtain

$$\begin{aligned}
&\varphi(\kappa^*) = \ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\right)\times \\
&\times\left(\beta_1(D_{11}-D_{10}) - (D_{11}-D_{01})^{\beta_1}\frac{\beta_1(D_{10}-D_{11})}{(D_{10}-D_{01})^{\beta_1}-(D_{11}-D_{01})^{\beta_1}}\right) \\
&+\ln\left(\frac{1}{D_{11}-D_{01}}\left(\frac{(D_{10}-D_{01})^{\beta_1}-(D_{11}-D_{01})^{\beta_1}}{\beta_1(D_{10}-D_{11})}\right)^{\frac{1}{\beta_1-1}}\right)\beta_1(D_{11}-D_{10}) \\
&+D_{11}-D_{10}.
\end{aligned} \tag{4.67}$$

Now, we change the variables in order to simplify the expression for $\varphi(\kappa^*)$. Substitution of (4.42) and (4.43) into (4.67) yields

$$\begin{aligned}
\varphi(\kappa^*) &= \ln\left(\frac{a}{b}\right)\left(\beta_1(a-b) - a^{\beta_1}\frac{\beta_1(b-a)}{b^{\beta_1}-a^{\beta_1}}\right) + \\
&+\ln\left(\frac{1}{a}\left(\frac{b^{\beta_1}-a^{\beta_1}}{\beta_1(b-a)}\right)^{\frac{1}{\beta_1-1}}\right)\beta_1(a-b)+a-b.
\end{aligned} \tag{4.68}$$

We proceed by dividing (4.68) by b, and defining

$$p = \frac{a}{b}. \tag{4.69}$$

As an immediate result we get

$$\begin{aligned}
\frac{\varphi(\kappa^*)}{b} &= \ln p\left(\beta_1(p-1) - p^{\beta_1}\frac{\beta_1(1-p)}{1-p^{\beta_1}}\right) \\
&+\ln\left(\left(\frac{1-p^{\beta_1}}{\beta_1(p^{\beta_1-1}-p^{\beta_1})}\right)^{\frac{1}{\beta_1-1}}\right)\beta_1(p-1)+p-1.
\end{aligned} \tag{4.70}$$

Factorization of (4.70) yields

$$\begin{aligned}
\frac{\varphi(\kappa^*)}{b} &= -\frac{(1-p)}{(\beta_1-1)(1-p^{\beta_1})}\times \\
&\times\left[(\beta_1-1)\left(1-p^{\beta_1}+\ln p^{\beta_1}\right) - \left(1-p^{\beta_1}\right)\beta_1\ln\left(\beta_1\frac{p^{\beta_1-1}(1-p)}{1-p^{\beta_1}}\right)\right].
\end{aligned} \tag{4.71}$$

Since it always holds that

$$-\frac{(1-p)}{(\beta_1-1)(1-p^{\beta_1})} < 0, \tag{4.72}$$

we are interested in the sign of the second factor of (4.71). Therefore, we define

$$\widetilde{\varphi}(p,\beta_1) \equiv (\beta_1 - 1)\left(1 - p^{\beta_1} + \ln p^{\beta_1}\right) - \left(1 - p^{\beta_1}\right)\beta_1 \ln\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\right) \tag{4.73}$$

Now, we determine the sign of the derivative of (4.73) with respect to p:

$$\frac{\widetilde{\varphi}(p,\beta_1)}{\partial p} = -\frac{\beta_1}{p(1-p)}\left(\beta_1 p^{\beta_1}(1-p)\left(1 - \ln\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\right)\right) - p\left(1 - p^{\beta_1}\right)\right). \tag{4.74}$$

This can be expressed as

$$\frac{\widetilde{\varphi}(p,\beta_1)}{\partial p} = -\frac{\beta_1\left(1 - p^{\beta_1}\right)}{1-p}\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\left(1 - \ln\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\right)\right) - 1\right). \tag{4.75}$$

The first factor of (4.75) is always negative. After the following substitution

$$z = \beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}, \tag{4.76}$$

the second factor of (4.75) can be expressed as

$$-z\left(\frac{1}{z} - 1 - \ln\frac{1}{z}\right), \tag{4.77}$$

which is negative for every $z \in \mathbb{R}_{++}$. This implies that

$$\frac{\widetilde{\varphi}(p,\beta_1)}{\partial p} > 0. \tag{4.78}$$

In the last step we show that $\lim_{p\uparrow 1}\widetilde{\varphi}(p,\beta_1) = 0 \;\forall \beta_1$. The limit of (4.73) can be decomposed as

$$\lim_{p\uparrow 1}(\beta_1 - 1)\left(1 - p^{\beta_1} + \ln p^{\beta_1}\right) - \lim_{p\uparrow 1}\left(1 - p^{\beta_1}\right)\beta_1 \ln\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\right). \tag{4.79}$$

The first part can be determined directly

$$\lim_{p\uparrow 1}(\beta_1 - 1)\left(1 - p^{\beta_1} + \ln p^{\beta_1}\right) = 0. \tag{4.80}$$

The second part requires a slightly closer examination

$$\begin{aligned} &\lim_{p\uparrow 1}\left[-\left(1 - p^{\beta_1}\right)\beta_1 \ln\left(\beta_1 \frac{p^{\beta_1 - 1}(1-p)}{1 - p^{\beta_1}}\right)\right] = \\ &= \lim_{p\uparrow 1}\left[-\left(1 - p^{\beta_1}\right)\beta_1 \ln\left(\beta_1 \frac{1-p}{1 - p^{\beta_1}}\right) - \left(1 - p^{\beta_1}\right)\beta_1 \ln\left(p^{\beta_1 - 1}\right)\right] = \\ &= \left[\lim_{p\uparrow 1} - \left(1 - p^{\beta_1}\right)\beta_1 \ln\left(\beta_1 \frac{1-p}{1 - p^{\beta_1}}\right)\right] = 0 \end{aligned} \tag{4.81}$$

The last equality holds since

$$\lim_{p\uparrow 1} \ln\left(\beta_1 \frac{1-p}{1-p^{\beta_1}}\right) = 0. \tag{4.82}$$

Substitution of (4.80) and (4.81) into (4.79) yields

$$\lim_{p\uparrow 1} \widetilde{\varphi}\left(p, \beta_1\right) = 0. \tag{4.83}$$

(4.83) together with (4.78) imply that (4.73) is negative and, as a consequence, (4.67) is positive.

Having proven the positive sign of $\varphi\left(\kappa^{*}\right)$, now we show that (4.66) holds. Differentiating (4.65) with respect to κ yields

$$\frac{\partial \varphi\left(\kappa\right)}{\partial \kappa} = \frac{1}{\kappa}\left(\beta_1\left(a-b\right) - \frac{a}{\kappa^{\beta_1 - 1}}\right) + \ln\left(\frac{a}{b}\kappa\right)\frac{\left(\beta_1 - 1\right)a}{\kappa^{\beta_1}} - \left(\left(\beta_1 - 1\right)\ln \kappa - 1\right)\frac{a}{\kappa^{\beta_1}}, \tag{4.84}$$

where a and b are defined by (4.42) and (4.43). Defining

$$\widetilde{\widetilde{\varphi}}\left(\kappa\right) \equiv \frac{\varphi\left(\kappa\right)}{b}, \tag{4.85}$$

and substitution of (4.69) result in

$$\frac{\partial \widetilde{\widetilde{\varphi}}\left(\kappa\right)}{\partial \kappa} = \frac{\beta_1\left(p-1\right)}{\kappa} + \ln\left(p\kappa\right)\frac{\left(\beta_1 - 1\right)p}{\kappa^{\beta_1}} - \ln \kappa \frac{\left(\beta_1 - 1\right)p}{\kappa^{\beta_1}} < 0. \tag{4.86}$$

This completes the proof. ■

Derivation of the consumer surplus and profit functions. When both firms offer a product of the same quality, the resulting equilibrium is symmetric. The prices and quantities are equal to

$$p_{kk} = \frac{b_k A}{3} \qquad q_{kk} = \frac{A}{3},$$

which yields the instantaneous profit

$$\pi_{kk} = \frac{b_k A^2}{9}.$$

As it can be seen from Figure 4.10, the consumer surplus equals

$$cs_{kk} = \frac{1}{2}\left(b_k A - \frac{b_k A}{3}\right)\frac{2A}{3} = \frac{2b_k}{9}A^2.$$

After Firm 1 achieves a Stackelberg advantage by investing, the prices and quantities obtained by solving the firms' maximization problem equal

$$p_{10} = \frac{2b_1 - b_0}{4}A \qquad p_{01} = \frac{b_0}{4}A \qquad q_{10} = \frac{A}{2} \qquad q_{01} = \frac{A}{4}.$$

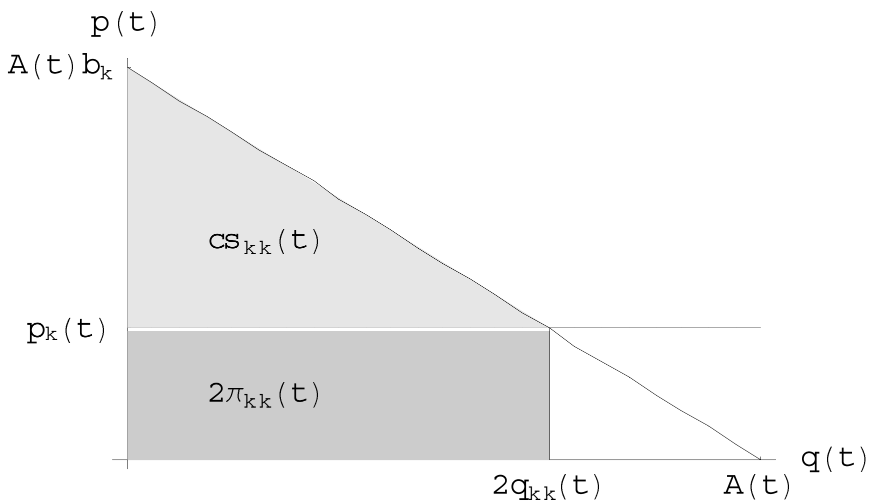

Figure 4.10: Firm's profits, π_{kk}, and the instantaneous consumer surplus, cs_{kk}, in a market where firms compete with an identical product quality b_k.

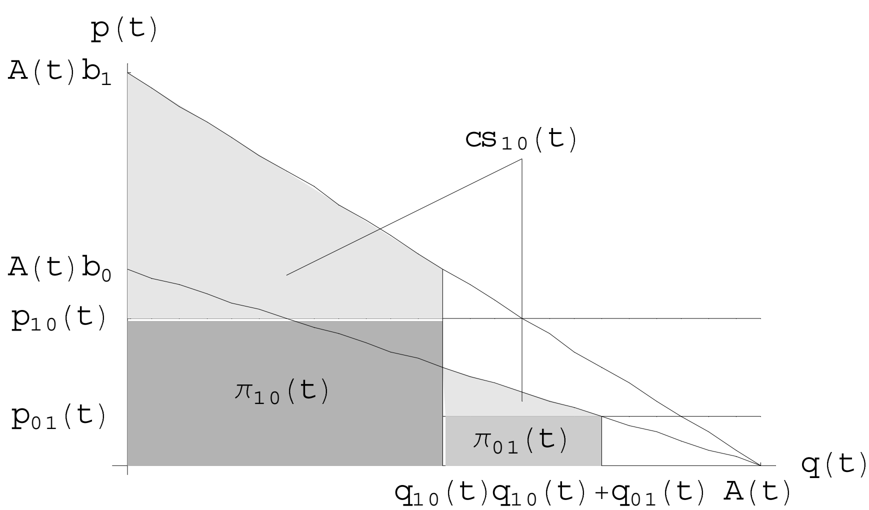

Figure 4.11: Firm's profits, π_{10} and π_{01}, and the instantaneous consumer surplus, cs_{10}, in a market where firms compete with product qualities, respectively, b_1 and b_0.

The instantaneous profits are therefore equal to

$$\pi_{10} = \frac{2b_1 - b_0}{8} A^2 \qquad \pi_{10} = \frac{b_0}{16} A^2,$$

and the consumer surplus is (see Figure 4.11)

$$cs_{10} = \frac{1}{2}\left(b_1 A - \frac{2b_1 - b_0}{4} A\right)\frac{A}{2} + \frac{1}{2}\left(b_0 A - \frac{b_0}{4} A\right)\frac{A}{4} = \frac{4b_1 + 5b_0}{32} A^2.$$

The observation that $A^2 = x$ allows for an immediate calculation of the consumer surplus in terms of (6.1) and for an identification of the deterministic contributions of the profit functions. ■

Proof of Proposition 4.4. Since $T_{21}^P < T_2^F < T_1^S$ for each $\omega \in \Omega$ (see Section 1.6), subtracting the value of consumer surplus in the joint investment equilibrium from the value corresponding to the preemptive investment yields

$$\Delta CS^{P-S}(t) =$$
$$E_t\left[\int_{T_{21}^P}^{T_2^F} e^{-r(s-t)}\left(cs_{10}(s) - cs_{00}(s)\right) ds + \int_{T_2^F}^{T_1^S} e^{-r(s-t)}\left(cs_{11}(s) - cs_{00}(s)\right) ds\right] > 0.$$

An identical reasoning can be applied while comparing the simultaneous equilibrium with the sequential exercise strategy. ■

Chapter 5

Entry and Strategic Quality Choice

5.1 Introduction

An uncertain economic environment results in firms managing their investment opportunities not only by choosing the timing of market entry but also by selecting product characteristics, such as quality. Higher quality is associated with higher costs but allows for capturing the benefits of good states of demand. On the contrary, bad states of demand can lead to lower quality since the cost of possible quality improvement outweighs benefits from a moderate increase of the consumers' interest in the product. For example, the options available to the subscribers of a Japanese operator NTT DoCoMo via the *i*-mode and related third generation (3G) services have been scaled down comparing to the initial plans since demand, in relation to the associated costs, turned out to be lower than expected. Consequently, at the time of launching the new product, the subscribers did not have the possibility of videoconferencing or receiving video clips, and what remains in the package offered to them is accessing e-mail, downloading news and weather reports, and calling up location-specific information. Adding new services was planned to be considered if the future demand was sufficiently high.[1] The case of the Japanese operator illustrates that firms face a trade-off between the costs of quality and foregone revenues resulting from offering limited functionality of the product. In this chapter we analyze the impact of demand uncertainty and competition on the optimal choice of the firm's strategic variables, such as investment timing and the product quality, as well as on its valuation.

We apply the real options approach which allows to determine the value of flexibility concerning the investment timing and the quality of the offered product/service.

[1]See *The Economist*, October 13-19, 2001, The Mobile Internet: A Survey.

The implementation of option-based techniques requires taking into consideration two major differences between financial and real options. First, in most cases real options are not exclusive, i.e. exercising a given option by one party results in the termination of corresponding options held by other parties (cf. the analysis of Chapters 3 and 4). For example, an option to lay down a fiber-optic cable between an internet backbone and a residential area is alive only until a competitive firm does so. Second, the firm can influence both the value of the underlying asset as well as the exercise price of the corresponding option. In many situations there exists a positive relationship between the amount of the sunk cost and the revenue of the project (i.e. via the level of automatization of the production process or via the product quality). Consequently, the firm is often faced with a menu of mutually exclusive real options with different exercise prices and payoff structures.

Both these aspects of real options have been incorporated into this chapter and are applied to investigate the investment decision in a market with stochastic demand, positive network externalities and competitive entry threat. We develop a strategic model in which a firm chooses the timing of irreversible investment and the quality of the product. Competitive entry occurs as a result of the optimal investment decision of a second firm. We compare the cases of fixed and flexible quality in order to determine the additional value of flexibility in quality choice. Flexible quality, which can be adjusted over time, requires sufficient know-how within the firm, the use of a more advanced technology or contractual flexibility (e.g. via a flexible agreement with content providers in the case of a 3G mobile operator). Fixed and flexible quality can also be interpreted as resulting from a licensed and internally developed technology, respectively. In the fixed quality case, once chosen quality cannot be changed. For instance, it may not be possible for an internet infrastructure provider to save on quality reduction (equivalent to narrowing bandwidth capacity) since fiber-optic cable cannot be easily resold or hired to another party during market downturn. Adding capacity when the demand is high can also be prohibitively costly, especially if the high state of demand results from its high volatility. In case of flexible quality, the firm is able to change it at a low cost in response to demand fluctuations and/or competitive entry. In practice, flexible quality is often associated with higher up-front costs. We show that these higher costs are especially justified in competitive environments with large demand uncertainty where the value of flexible quality more than doubles compared to the monopoly case.

Consequently, we aim at unifying two streams of literature: strategic real options and industrial organization-based endogenous quality choice.[2] As far as the real option framework is concerned, our model builds up upon such contributions as Smets (1991), Grenadier (1996), Perotti and Rossetto (2000), Huisman (2001), Nielsen (2002), Lambrecht and Perraudin (2003), and Mason and Weeds (2003), which all have in common that they analyze the effects of both competition and uncertainty on investment timing.

Introducing quality choice as a strategic variable results in the extension of the existing continuous-time strategic real options framework to a class of models in which firms are equipped with two control variables. Besides choosing the timing of investment, the firms now also have to decide about the optimal quality of the product they are going to offer. The implication is that some of the classic real options results cease to hold. For example, in the fixed quality case, the optimal investment timing of the second firm is no longer irrelevant for the investment decision of the leader in the open-loop strategies (cf. Huisman, 2001). This is due to the fact that the entry decision of the follower interacts with the second control variable of the leader (quality), which, in turn, influences the leader's optimal investment timing. With flexible quality the follower's investment decision becomes again irrelevant since the leader can change quality instantaneously. As a consequence, until the follower's entry it can act as a monopolist, thus without being influenced by the entry threat.

In this chapter it is shown that, due to strategic interaction between the leader and follower, the value of the investment option of the former can decrease with uncertainty if the fixed-quality technology is used. Moreover, the value of the leader is lower than the one of the follower. This latter result is due to the strategic disadvantage of the first mover in a Stackelberg game in which firms compete in strategic complements. Once the leader has invested it cannot change its quality. Hence, the follower is in the comfortable position where it can optimally adjust its quality level to the leader's choice. The situation reverses under the flexible-quality technology of the leader. Now, the value of the follower, which still has a fixed quality choice, can decrease with uncertainty since its project's value becomes concave in the realizations of random demand. This is caused by the fact that now the leader can change its quality level after the follower has made its choice.

Furthermore, we show that in the flexible-quality case the leader can drive its competitor out of the market in high states of the demand. This is caused by the fact that the leader can afford investing in high quality when demand is high. This

[2] In a recent paper, Pennings (2002) analyzes the optimal quality choice in a real options framework using a different model set-up.

reduces the demand for the product offered by the follower to zero for states of demand exceeding a certain trigger. Flexible quality can thus serve as an entry deterrent control. In this case the quality level need not be set higher than monopoly level since the leader's ability to raise quality instantly after a potential entry is sufficient to prevent such an entry from occurring.

We also discuss the impact of network externalities on the optimal investment timing, quality choice and firms' valuations. Since, from the point of view of a consumer, an increase of the degree of network externalities can compensate the decrease in quality, the optimal quality choice of firms is inversely related to network externalities. Moreover, firms invest sooner and their valuations are higher when the product market exhibits strong network externalities.

As far as the literature on strategic quality choice is concerned, our model is related to the contributions by Motta (1993), Aoki and Prusa (1996), Foros and Hansen (2001), Dubey and Wu (2002), Hoppe and Lehmann-Grube (2001) and Banker et al. (1998). In general, it can be remarked that we generalize this stream of research by analyzing a dynamic, continuous-time framework while taking into account economic uncertainty.

Motta (1993) considers a two-stage duopoly model with either fixed or variable costs of quality (i.e. independent from or proportional to the scale of improvement). Fixed costs can be associated with R&D or advertising activities. Variable costs, that correspond to our framework, reflect more skilled labor and more expensive raw materials and inputs. The result of the paper is that firms differentiate qualities, which is possible due to setting different prices. In a similar framework Aoki and Prusa (1996) analyze optimal sequential and simultaneous quality choice. Again, due to the fact that the authors assume only vertical product differentiation and price competition, there exists a first-mover advantage in the quality choice game. In our case, products are differentiated also horizontally, so the firms set different qualities even if the cost of the good to consumer is equal. As a consequence, qualities become strategic complements, reaction curves are continuous, and the profit of the second mover is higher.

Foros and Hansen (2001) apply a two-stage model extended to allow for horizontal differentiation and network externalities to the market of Internet Service Providers. They find that the optimal choice of quality is positively related to network externalities. Their result differs from ours due to the fact that in Foros and Hansen (2001) the substitution effect between quality and network externalities is dominated by the impact of lower competitive pressure resulting from higher network externalities.

Dubey and Wu (2002) investigate firms' incentives to invest in product innovation, which ultimately leads to a quality increase. They show that the relationship between the number of firms and the propensity to innovate is bell-shaped. In other words, if the number of firms is "too large" or "too small" the innovation process does not occur. The results of Dubey and Wu (2002) are consistent with our model that predicts that the possibility of entry increases the quality provided by the otherwise monopolistic firm. Using a different analytical framework Banker et al. (1998) conclude that in the absence of synergies among the firms in the quality cost, an increasing number of firms leads to decreasing quality. This finding coincides with the argument of Dubey and Wu (2002) for a "too large" number of firms and is caused by the fact that improving quality is assumed to be sufficiently costly.

An alternative dynamic model of strategic quality choice is developed by Hoppe and Lehmann-Grube (2001). In their framework, the firms chose the optimal timing of entry, given that the available quality is a deterministic function of time. Prior to the investment, firms are assumed to pay R&D costs which are proportional to time until investing. The authors show that, depending on the cost of R&D, there can be either rent equalization (cf. Fudenberg and Tirole, 1985) or a second-mover advantage in the quality choice game. The assumption made by Hoppe and Lehmann-Grube (2001) that the costs of higher quality are incurred prior to investment differs from ours in which the costs of quality occur after the investment is made (similar to the notion of variable quality costs in Motta, 1993). As a consequence, contrary to Hoppe and Lehmann-Grube (2001), we do not observe the first-mover advantage (corresponding to payoff equalization without exogenous firms' roles) in the fixed-quality case in our model.

This chapter is organized as follows. In Section 5.2 we present the model of a monopolistic firm with a fixed-quality technology. Section 5.3 extends the model to a duopolistic environment. The discussion of the monopolistic model with a flexible quality choice is presented in Section 5.4 and the analysis of its duopolistic extension is included in Section 5.5. In Section 5.6 we compare the impact of fixed and flexible quality on the value of the firm. Section 5.7 concludes.

5.2 Non-Strategic Model with Fixed Quality

Consider a situation in which a risk-neutral firm has an investment opportunity to launch a product/service in an uncertain market. It chooses the optimal investment timing and quality of the product. In this section we assume that once chosen quality cannot be changed. The idea of the fixed quality choice is therefore similar to Ueng

(1997), who considers an infinitely repeated oligopoly game in which the qualities are chosen before the first period. It is realistic to assume that the revenue per customer is not constant but evolves stochastically over time.[3] The instantaneous revenue per customer at time t is equal to $x(t)$, where x follows the geometric Brownian motion

$$dx(t) = \alpha x(t)\, dt + \sigma x(t)\, dw(t). \tag{5.1}$$

Here α denotes the deterministic drift rate and σ is the instantaneous volatility of the process. In the analysis we assume that the initial realization of (5.1), $x(0)$, is sufficiently low, so that in all possible cases the market is too small for immediate investment to be optimal.

There is a continuum of heterogenous consumers with valuations ω_i distributed uniformly over the interval $[0, 1]$. A consumer derives utility not only from the stand-alone good but also from the number of other consumers using it. A utility function satisfying these characteristics is[4]

$$U_i = \omega_i q + an - k, \tag{5.2}$$

where $q \in \mathbb{R}_+$ is the quality of the good, $k \in \mathbb{R}_+$ is the cost the consumer has to bear to acquire the good, and $a \in \mathbb{R}_+$ is a parameter that measures the intensity of the network externalities.[5] Consequently, ω_i can be interpreted as the marginal rate of substitution between income and quality, so that a higher ω_i reflects a lower marginal utility of income and, as a consequence, a higher income (see also Tirole, 1988, p. 98). Large a implies that the consumer's utility grows fast with the number of other users. In the opposite case, when a tends to zero, the number of users of the same good does not affect the utility of the consumer.[6] The size of the network, $n \in [0, 1]$, is interpreted as the fraction of the total market that has bought a given product. Without loss of generality, we normalize the absolute size of the total market to 1.

[3]For instance, the revenue per customer of a mobile telephone network depends on the intensity of voice traffic, competitive pressure, and arrival of new services that can be offered to the customer against an additional fee. It is natural to assume that the evolution of these economic variables over time contains an unpredictable component.

[4]Heterogeneity of consumers with respect to the value attached to the quality of the stand-alone good and their homogeneity with respect to the degree of network externalities is a common assumption in the economics of network literature (cf. Mason, 2000, and references therein).

[5]Parameter k should not be associated with a price that the consumer has to pay for the product. It can be interpreted as a non-monetary cost associated with the effort and time used for searching the good with suitable characteristics.

[6]Of course, there are examples of negative a as well. For instance, the utility from having a Rolls-Royce is decreasing in the number of other owners of this brand in the neighborhood.

Network externalities are thus present if the number of other consumers using the same product influences the utility of a given consumer. Positive (negative) network externalities imply that the utility of the consumer increases (decreases) with the number of other users. An example of a good a demand for which exhibits positive network externalities is an access to the web via a given Internet Service Provider, a computer operating system, an audio recorder using a particular standard (DCC, MD, or CD-R), or a mobile phone (GSM vs. CDMA). We analyze a good for which the consumer's utility depends both on the network size and the quality (MacOS vs. Windows). The purchase decision is determined mainly by these two parameters, so that we do not incorporate a pricing strategy. This choice of modeling approach follows recent empirical evidence. In an analysis of the on-line book retail market Latcovich and Smith (2001) claim that "consumers do not respond much to significant price differences between sellers [...]. But they [...] care about vertical characteristics such as reliability, security, and ease of use". This supports the idea of the quality-oriented market analyzed in our paper. Also Shapiro and Varian (1998) point out that the price is an insignificant determinant of the purchase decision for many network goods, such as software. Referring to the market for spreadsheets they claim that "the purchase price of the software is minor in comparison with the cost of deployment, training and support. Corporate purchasers, and even individual customers, were much more worried about picking the winner of the spreadsheet wars than they were about whether their spreadsheet costs \$49.95 or \$99.95" (Shapiro and Varian, 1998, p. 288).

On the basis of the consumers' utility function, we can determine the size of the network as a function of the quality chosen by the firm. Define the consumer of type $\overline{\omega}$ to be indifferent between acquiring the good or not. Consequently, it holds that

$$\overline{\omega}q + an - k = 0. \tag{5.3}$$

By setting $a < k < q$, which is to ensure an interior solution for the size of the network (we waive these restrictions later), and observing that the size of the network, n, equals $1 - \overline{\omega}$, we obtain that

$$n(q) = \frac{q-k}{q-a}. \tag{5.4}$$

We further assume a constant value per customer, constant economies of scale on the supply side, and that the unit cost of operation, $c(q)$, satisfies $c'(q) > 0$ and $c''(q) \geq 0$. The firm chooses quality q so as to maximize the value of the investment opportunity. In order to determine the value of the investment opportunity, we begin with calculating the value of the project after the investment decision is made. The value of the project is found by integrating over time the discounted difference between the instantaneous value of the installed base of consumers, $xn(q)$, and the operating costs $c(q)\, n(q)$.[7] Therefore, if we denote the project value at time t by $V(t)$, it holds that

$$\begin{aligned} V(t) &= E\left[\int_t^\infty (x(s) - c(q))\, n(q)\, e^{-r(s-t)} ds\right] && (5.5) \\ &= \frac{n(q)\, x}{\delta} - \frac{c(q)\, n(q)}{r} \\ &\equiv R(q)\, x - C(q) && (5.6) \end{aligned}$$

where r is the risk-free rate and δ, defined as

$$\delta \equiv r - \alpha, \tag{5.7}$$

is the return shortfall of the demand process x. The firm has to incur a sunk investment cost, $I \in \mathbb{R}_{++}$. Although I does not depend on the choice of quality, the cost associated with pursuing the project increases with quality due to a higher present value of operating costs.[8] The decision of the firm is to choose the optimal quality, q, and timing of entry, x^*, in order to maximize the value of the investment opportunity.

To find the optimal investment threshold and product quality we proceed in two steps. First, we solve the optimal stopping problem for an arbitrary level of q. As an intermediate result we obtain the optimal investment threshold and the value of the investment opportunity as a function of q. Second, we maximize the value of the investment opportunity with respect to q.

The threshold $x^*(q)$, being the lowest value of x at which the firm enters the market, is

$$x^*(q) = \frac{\beta_1}{\beta_1 - 1} \frac{I + C(q)}{R(q)}, \tag{5.8}$$

where

$$\beta_1 = -\frac{\alpha}{\sigma^2} + \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{5.9}$$

The value of the investment opportunity, $F(q, x)$, equals

$$F(q, x) = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \frac{R(q)^{\beta_1} x^{\beta_1}}{(I + C(q))^{\beta_1 - 1}}. \tag{5.10}$$

[7] The instantaneous value of the installed base of consumers can be obtained by integrating the utilities of participating consumers over their types, $\int_{\bar{\omega}}^{1} (\omega q + an - k)\, d\omega$. This equals $0.5n^2 a + 0.5(q - k)\, n$ which is convex in n. However, here we assume that the firm does not price discriminate so that it does not extract the whole consumer surplus. Instead, we impose linearity in n of the firm's profit.

[8] An alternative interpretation of the cost structure is that the initial investment outlay equals $I + c(q)\, n(q)/r$, and the marginal production cost is zero for all levels of q.

The derivation of (5.8), (5.9) and (5.10) follows directly from Section 1.6. Subsequently, we maximize the value of the investment opportunity with respect to q, given the optimal investment rule, $x^* (q)$. In order to ensure that our solution is a maximum, we introduce the following assumption.

Assumption 5.1 *Let q^* be the solution to $\partial F(q, x^*)/\partial q = 0$. Then it holds that*[9]

$$\left(\beta_1 (C + I) R_{qq} + C_q R_q - (\beta_1 - 1) C_{qq} R\right)\big|_{q=q^*} < 0. \tag{5.11}$$

The solution to the problem of quality choice is given in the following proposition.

Proposition 5.1 *Under Assumption 5.1 the optimal quality of the product, q^*, is implicitly given by the following equation*

$$C_q = x^* R_q. \tag{5.12}$$

Proof. See the Appendix. ■

From Proposition 5.1 it is obtained that the value of the investment opportunity is maximized if at the optimal investment threshold the marginal cost of increasing the quality is equal to the expected marginal benefit. (5.12) implies that in the optimum the ratio of elasticities of functions $C(q) + I$ and $R(q)$ equals the wedge occurring in the threshold value $x^* (q)$ (cf. (5.8)), i.e.

$$\left.\frac{\varepsilon_{C+I,q}}{\varepsilon_{R,q}}\right|_{q=q^*} = \frac{\beta_1}{\beta_1 - 1}, \tag{5.13}$$

where $\varepsilon_{f,x} \equiv \frac{x f_x}{f}$.

In order to provide more insight into the obtained result, we analyze the relationship between market uncertainty, intensity of the network externalities, size of the network and the optimal quality. Proposition 5.2 provides part of the results.

Proposition 5.2 *The quality of the product increases with revenue uncertainty and its growth rate, i.e.*

$$\frac{dq^*}{d\sigma} > 0, \textit{ and} \tag{5.14}$$

$$\frac{dq^*}{d\alpha} > 0. \tag{5.15}$$

Proof. See the Appendix. ■

[9] When it does not yield ambiguity, subscripts denote partial derivatives.

The fact that higher uncertainty concerning the demand side of the market influences the quality choice of the firm positively results from the option-like structure of the project value and upside potential from higher quality investment. Furthermore, a higher growth rate of the market also implies a higher quality choice since the firm prefers to incur additional cost to increase quality when revenue is expected to grow faster.[10]

Furthermore, numerical simulations indicate that the impact of network externalities on the optimal quality choice is negative. The latter relationship results from the fact that the level of quality and the degree of network externalities act as substitutes in the marginal consumer's utility function. Since a higher quality is equivalent to a larger consumer base (cf. (5.4)), the size of the network in optimum, n^*, also rises with σ and α.

Market uncertainty and intensity of network externalities also have an impact on the optimal investment threshold. Since both factors affect the optimal investment threshold directly and indirectly (via the change of the optimal quality), the total impact is determined by calculating the following derivative:

$$\frac{dx^*(q)}{d\theta} = \frac{\partial x^*(q)}{\partial \theta} + \frac{\partial x^*(q)}{\partial q}\frac{dq}{d\theta}, \quad \theta \in \{a, \sigma\}. \tag{5.16}$$

In the Appendix we prove the following proposition:

Proposition 5.3 *It holds that*

$$\frac{dx^*(q)}{d\sigma} > 0. \tag{5.17}$$

Hence, the relationship between uncertainty and the optimal investment threshold is positive. Therefore we conclude that the flexibility in the quality choice does not change the classical result of real option theory (cf. Dixit and Pindyck, 1996).

Extensive numerical simulations show that the optimal investment threshold decreases with a magnitude of network externalities. This is associated with the fact that a higher magnitude of network externalities makes the product market more valuable for the firm. This results in a higher value of the investment project (other things equal) and, thus, a lower value of x suffices to achieve the required profitability ratio, $\beta_1/(\beta_1 - 1)$, of the project at the time of investing.

[10]The positive sign of the derivative with respect to α is equivalent to the negative derivative with respect to the return shortfall δ of the demand process x.

5.3 Strategic Model with Fixed Quality

Here we introduce the possibility of competitive entry by a second firm (Firm 2). In order to focus on the incumbent-entrant problem, we impose that Firm 2 can only enter after Firm 1 has already done so (i.e. firms play the timing game in the open-loop strategies as in Reinganum, 1981). After entering the market, Firm 2 starts offering the good having a quality q_2. In general, q_2 will differ from q_1, i.e. from the quality choice made by Firm 1. The fact that the firms do not compete in prices implies that for the consumers the cost of accessing each network is equal across the networks. Consequently, if the products were perfect substitutes, consumers would always choose the product with a higher quality and the resulting market outcome would always be a monopoly.[11] In case of imperfect substitution this does not hold any longer. Denote the degree of substitution by $\rho \in (0,1)$. For ρ close to unity, the goods are close substitutes, whereas a very small ρ implies that the firms operate in virtually separated markets.

In order to analyze the impact of entry on the valuation of the first firm in the market (Firm 1), we adopt a simple structure for the market with differentiated goods (as in, e.g., Spence, 1976) and allow for the presence of network externalities as in Section 5.2. The system of inverse demand functions is given by

$$\begin{cases} k = (1 - n_1)\, q_1 - \rho n_2 q_2 + a\,(n_1 + \rho n_2) \text{ for Firm 1's network, while} \\ k = (1 - n_2)\, q_2 - \rho n_1 q_1 + a\,(n_2 + \rho n_1) \text{ for Firm 2's network,} \end{cases} \tag{5.18}$$

and n_i, $n \in \{1, 2\}$, is the size of Firm i's network. Each of the inverse demand functions can be interpreted as follows. The LHS represents the instantaneous cost (utility loss) of accessing the network. The RHS corresponds to the linear demand schedule that decreases with the offered quantities, n_i and n_j, while its negative slope is reduced by the presence of a component $a\,(n_1 + \rho n_2)$ which reflects network externalities. The impact of the quantity offered by Firm j on Firm i's demand, and the network externalities among its consumers is scaled down by factor ρ reflecting imperfect substitution among the goods. It can be easily noticed that for n_j equal to zero, (5.18) reduces to the monopolistic demand function of Section 5.2 (cf. equation (5.4)).

The size of the network of Firm i obtained by solving (5.18), subject to $n_i \in$

[11] Provided that the trivial case of equal qualities is excluded.

$[0,1]$, equals

$$n_i(q_i) = \begin{cases} 0 & q_i < \underline{q}_i, \\ \frac{1}{1-\rho^2}\frac{q_i-\underline{q}_i}{q_i-a} & q_i \in \left[\underline{q}_i, \overline{q}_i\right], \\ \frac{q_i-k}{q_i-a} & q_i > \overline{q}_i, \end{cases} \tag{5.19}$$

where

$$\underline{q}_i = k(1-\rho) + \rho \max[k, q_j], \tag{5.20}$$

$$\overline{q}_i = \frac{\max[k, q_j] - k(1-\rho)}{\rho}, \tag{5.21}$$

and $i, j \in \{1,2\}$, $i \neq j$. Depending on the quality offered, Firm i competes with Firm j for moderate values of q_i, it is a monopolist for high q_i, or has no customer base if q_i is low. Both qualities $\underline{q}_i$ and $\overline{q}_i$ depend positively on quality q_j offered by the competitor. Moreover, higher substitutability of the goods, captured by ρ, results in shrinking the range of qualities in which firms compete. This is intuitive since the closer substitutes the goods are, the less they can differ in qualities for both firms to be present in the product market. Since the once chosen qualities remain fixed and neither $\underline{q}_i$ nor $\overline{q}_i$ depends on x, both firms being active implies that $q_i \in \left[\underline{q}_i, \overline{q}_i\right]$ for $i \in \{1,2\}$. Otherwise, one of the firms would be better off by not entering.

For analytical convenience, we impose the following linear specification of the cost function:

$$c(q_i) = c_0(q_i - a), \; c_0 \in \mathbb{R}_{++}, \; q_i \in [a, \infty), \tag{5.22}$$

where c_0 can be interpreted as an efficiency parameter. Consequently, higher values of c_0 correspond to industries that are less efficient in R&D. Setting a quality equal to $a\,(< k)$ is equivalent to the firm producing no output and incurring no cost (since $n_i(a) = c(a) = 0$ in this case). The instantaneous profit function corresponding to (5.22) is

$$\pi_i = (x - c_0(q_i - a))\, n_i. \tag{5.23}$$

We solve the problem backwards in time. First, the optimal investment threshold and quality choice of Firm 2 is determined. The value of Firm 2's investment opportunity at $t \leq T_2$ equals

$$F_2^*(t) = E\left[\int_{T_2}^{\infty} (x(s) - c_0(q_2^* - a))\, n_2(q_1^*, q_2^*) e^{-r(s-t)} ds - I e^{-r(T_2-t)}\right], \tag{5.24}$$

where T_2 denotes the random stopping time associated with x reaching Firm 2's optimal investment threshold. A well-known procedure (cf. Section 1.6) allows for deriving

Firm 2's optimal threshold, x_2^*, and the value of its investment opportunity:

$$x_2^* = \frac{\beta_1}{\beta_1 - 1}\left(\frac{I\left(1-\rho^2\right)}{q_2 - \underline{q}_2} + \frac{c_0}{r}\right)(q_2 - a)\,\delta, \tag{5.25}$$

$$F_2^* = \max_{q_2} \frac{\left(q_2 - \underline{q}_2\right)x_2^*}{\beta_1\left(1-\rho^2\right)\delta\left(q_2 - a\right)}\left(\frac{x}{x_2^*}\right)^{\beta_1}. \tag{5.26}$$

From (5.26) it follows that the quality maximizing the value of Firm 2's investment opportunity, q_2^*, is

$$q_2^* = \frac{1}{2\left(\beta_1 - 1\right)} \times \tag{5.27}$$
$$\left[\left(2\beta_1 - 1\right)\underline{q}_2 - a + \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + 4\beta_1 Ir\left(\beta - 1\right)\left(1-\rho^2\right)c_0^{-1}}\right].$$

Upon analyzing (5.27) it can be concluded that the qualities chosen by the firms are strategic complements. Since $\underline{q}_2$ is an increasing function of q_1 (see (5.20)) and q_2^* rises with $\underline{q}_2$, the quality chosen by Firm 2 is positively related to the quality choice made by Firm 1.

This relationship, in combination with a closer inspection of (5.25), leads to the following proposition.

Proposition 5.4 *Firm 2 responds optimally to an increased quality of Firm 1 not only by raising its own quality but also by delaying its timing of entry, i.e. the following inequalities hold*

$$\frac{dq_2^*}{dq_1} > 0, \text{ and}$$
$$\frac{dx_2^*}{dq_1} > 0.$$

Proof. See the Appendix. ■

Consequently, it can be concluded from Proposition 5.4 that the choice of higher q_1 is equivalent to entry-deterrent behavior of Firm 1.

Having calculated the optimal investment threshold of Firm 2, we are in position to analyze the investment decision of Firm 1. First, we note that the value of Firm 1's investment project at the time of investing, t, is given by

$$V_1(t) = E\left[\int_t^{T_2}\left(x(s) - c_0\left(q_1^* - a\right)\right)n(q_1^*)e^{-r(s-t)}ds - I\right]$$
$$+E\left[\int_{T_2}^{\infty}\left(x(s) - c_0\left(q_1^* - a\right)\right)n_1(q_1^*, q_2^*)e^{-r(s-t)}ds\right]. \tag{5.28}$$

Working out the expectations yields

$$V_1 = \underbrace{\frac{q_1^* - k}{q_1^* - a}\left(\frac{x}{\delta} - \frac{c_0\left(q_1^* - a\right)}{r}\right) - I}_{\text{Monopolistic value}} + \tag{5.29}$$

$$\underbrace{\left(\frac{1}{1-\rho^2}\frac{q_1^* - \underline{q}_1}{q_1^* - a} - \frac{q_1^* - k}{q_1^* - a}\right)\left(\frac{x_2^*}{\delta} - \frac{c_0\left(q_1^* - a\right)}{r}\right)\left(\frac{x}{x_2^*}\right)^{\beta_1}}_{\text{Value lost due to the competitive entry}}.$$

Again, an application of the well-known procedure yields the optimal threshold, x_1^*, and the value of investment opportunity, F_1^*, of Firm 1

$$x_1^* = \frac{\beta_1}{\beta_1 - 1}\frac{I + C\left(q_1\right)}{R\left(q_1\right)}, \tag{5.30}$$

$$F_1^* = \tag{5.31}$$

$$\max_{q_1}\frac{\left(q_1 - k\right)\frac{x_1^*}{r-\alpha} + \beta_1\left(\frac{q_1 - \underline{q}_1}{1-\rho^2} - q_1 + k\right)\left(\frac{x_2^*}{r-\alpha} - \frac{c_0(q_1 - a)}{r}\right)\left(\frac{x_1^*}{x_2^*}\right)^{\beta_1}}{\beta_1\left(q_1 - a\right)}\left(\frac{x}{x_1^*}\right)^{\beta_1}.$$

It is worthwhile noticing that the optimal investment timing of Firm 1 does not *explicitly* depend on the action taken by Firm 2. This outcome results from the fact that the roles of the firms (leader vs. follower) are exogenously determined. However, this result still differs from the classical result from the real options theory (see, e.g., Huisman, 2001, p. 170) concerning the irrelevance of the follower's investment timing for the decision of the leader. The reason is that Firm 1's timing decision is affected by the choice of quality, q_1, and, according to (5.31), q_1 depends on Firm 2's threshold x_2^* and on the threshold quality $\underline{q}_1$, which is a function of q_2 (cf. (5.20)).

The resulting dependence of Firm 1's investment threshold on the behavior of Firm 2 is caused by the fact that in our model firms have two control variables (investment timing and quality) as opposite to a single variable in classic real option models. It still holds that introducing the competitor does not change the optimal *ceteris paribus* choice of the timing variable. However, competitive entry changes the optimal choice of quality (the second control variable). This makes the monopolistic choice of timing no longer optimal and, as a consequence, it holds that $x_1^* \neq x^*$.

As far as the value of the investment opportunity is concerned, it can be determined by maximizing the argument of the RHS of (5.31). The derivative of F_1^* with respect to q_1 can be computed since x_1^*, x_2^* and q_2 are known functions of q_1. Due to complexity of the resulting relationship, the unique (in the relevant interval) root of the derivative has to be determined numerically.

5.3.1 Comparative Statics: Valuation of Firms

We are interested in the sensitivity of the value of the firms with respect to changes of market parameters. Figures 5.1 and 5.2 depict the relationship between the market volatility and the value of the investment opportunity of Firm 1 and Firm 2, respectively, for different magnitudes of network externalities. On the basis of both figures two observations can be made. First, the value of Firm 1's investment opportunity is lower than the one of Firm 2. The first phenomenon results from the strategic disadvantage of the first mover in a game in which the firms compete in strategic complements. As it can be shown in a simple Stackelberg setting, the follower's payoff is higher than the payoff of the leader if the control variables are strategic complements (cf. Tirole, 1988, p. 331, footnote 53). Despite the fact that Firm 1 enjoys profit from investment for a longer period (it invests as first), its value is still lower than the one of Firm 2.

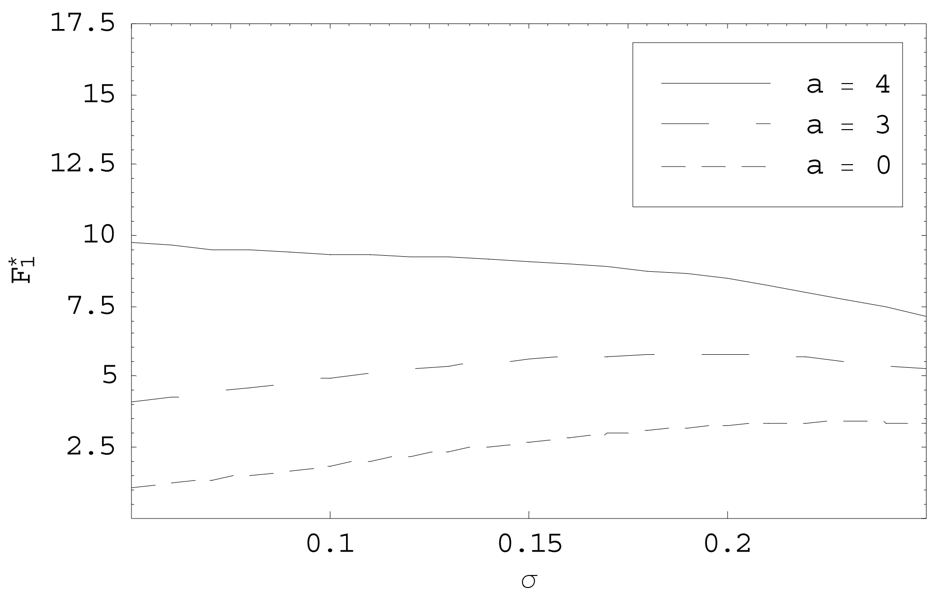

Figure 5.1: The value of the investment opportunity of Firm 1 for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

Second, the sign of the relationship between the value of Firm 1's project and uncertainty crucially depends on the magnitude of network externalities, a. The standard option argument indicates that the sign of this relationship is positive. However, under higher uncertainty Firm 2 sets its quality more "aggressively" (cf. (5.14)), which negatively influences the value of Firm 1. The sign of the joint effect is ambiguous and reflects the above mentioned trade-off. As illustrated in Figure 5.1, the presence of

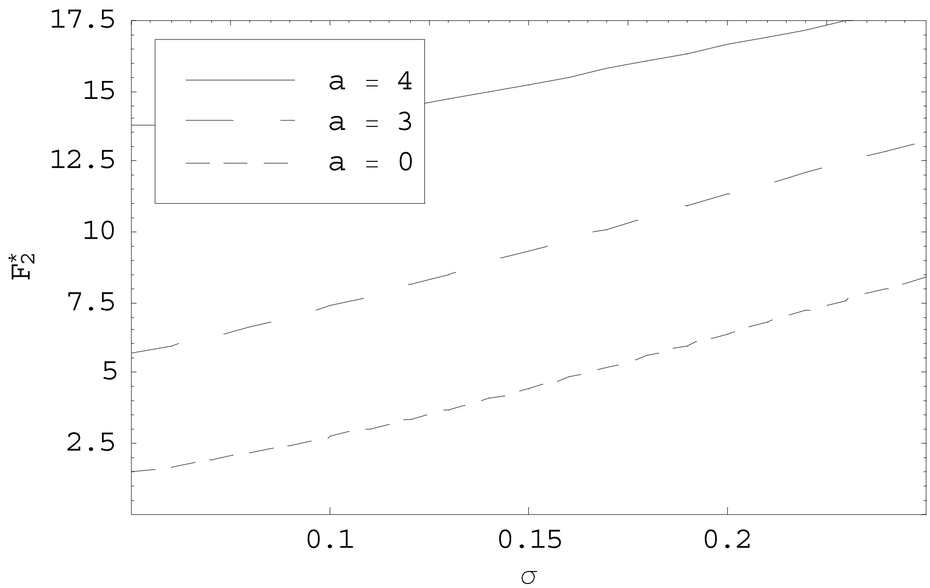

Figure 5.2: The value of the investment opportunity of Firm 2 for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

strong network externalities amplifies the latter (strategic) effect.

Finally, it can be seen that the presence of the network externalities significantly enhances the value of the investment opportunities of both firms. The rate of increase is most dramatic when the degree of network externalities approaches the cost of joining the network (i.e. when the marginal consumer's valuation of the stand alone-good is equal to zero). Therefore, for the set of parameters as in Figures 5.1 and 5.2, the change in the value of the firms' investment opportunities following an increase in a from 3 to 4 ($k = 5$) is higher than the analogous change associated with an increase in a from 0 to 3.

5.3.2 Comparative Statics: Firm 1's Strategic Choice of Variables

Finally, we compare the non-strategic and strategic case with respect to Firm 1's optimal choice of strategic variables. Figures 5.3 and 5.4 illustrate the optimal investment threshold, whereas Figures 5.5 and 5.6 depict the optimal quality choice. From Figures 5.3 and 5.4 it follows that the optimal investment threshold is higher if a subsequent competitive entry threat exists. This contradicts the result known from the strategic real option literature that the optimal investment threshold of the market leader is not influenced by the entry threat if the roles of the firms are predetermined. As we already concluded from (5.30), Firm 1's investment threshold depends on the investment timing and quality decision of its competitor.

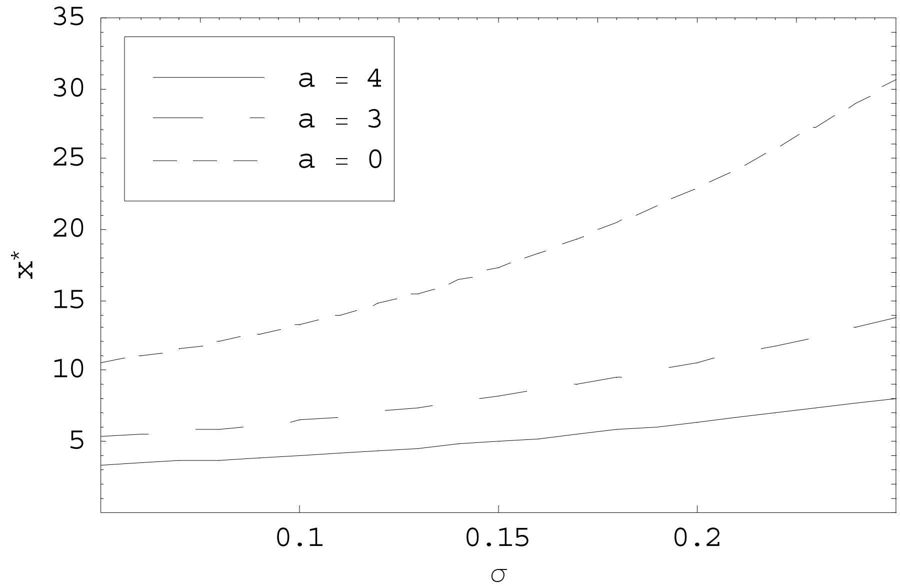

Figure 5.3: The optimal investment threshold of Firm 1 in the non-strategic case for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

On the basis of Figures 5.5 and 5.6 we conclude that the presence of a (potential) competitor increases the quality provision of Firm 1. Higher quality (as shown in Section 5.2), as well as the fact that, from the timing of the second firm onwards, the market must be shared with the competitor, results in the optimality of a higher - than in the non-strategic case - investment threshold which, in turn, leads to the outcome depicted in Figure 5.4.

This result and the one concerning the project's value contradict the findings of Foros and Hansen (2001), who analyze a duopoly model of Internet Service Providers. In a modified Hotelling framework they show that profits decrease and the offered quality increases with the degree of network externalities. The reason why this differs from our results is the following. Here, in a non-strategic framework, network externalities can act as a substitute of quality in a consumer's utility function. Consequently, a firm can have less incentive to invest in (costly) quality when network externalities are present. This effect also takes place in a strategic framework if the increase of quality occurs for a single product. In case of Foros and Hansen (2001), the increase of interconnection quality affects both products so that the substitution effect is dominated

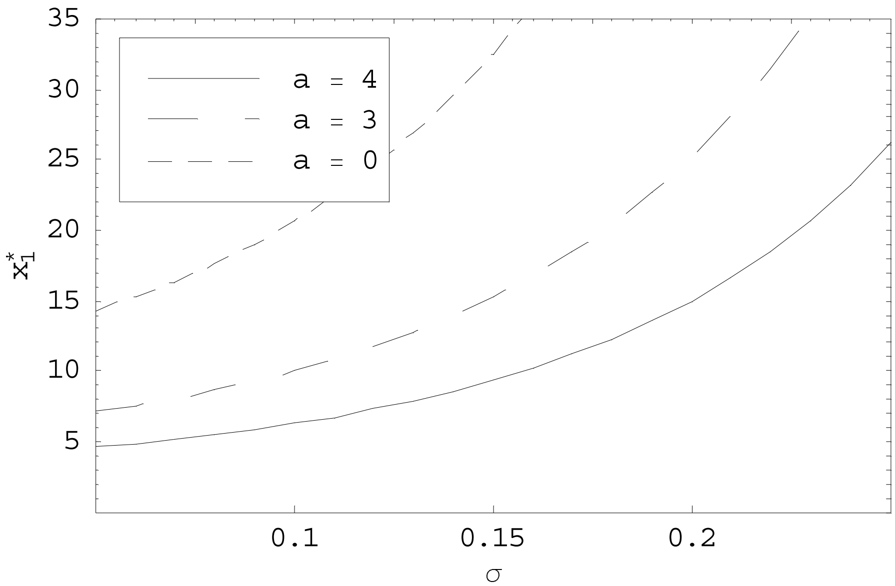

Figure 5.4: The optimal investment threshold of Firm 1 in the strategic case for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

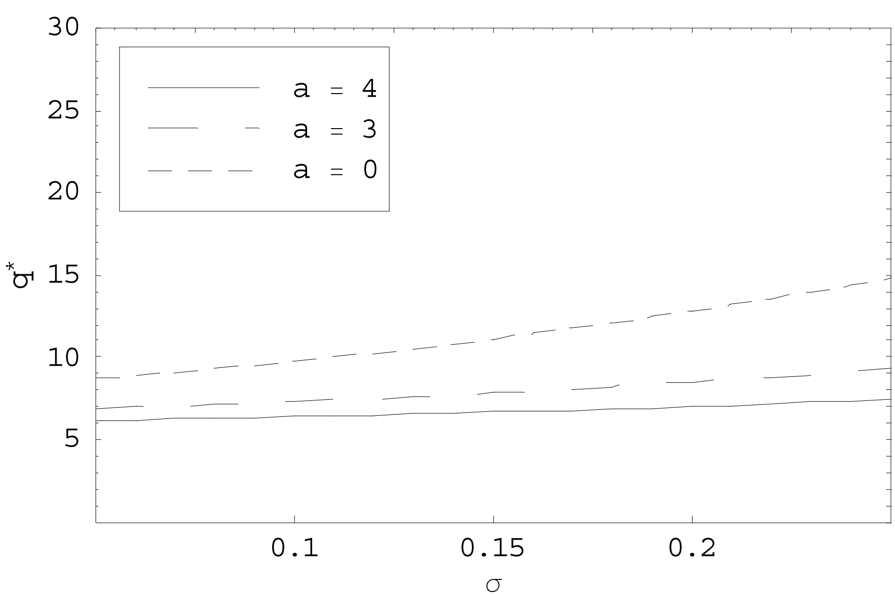

Figure 5.5: The optimal quality choice of Firm 1 in the non-strategic case for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

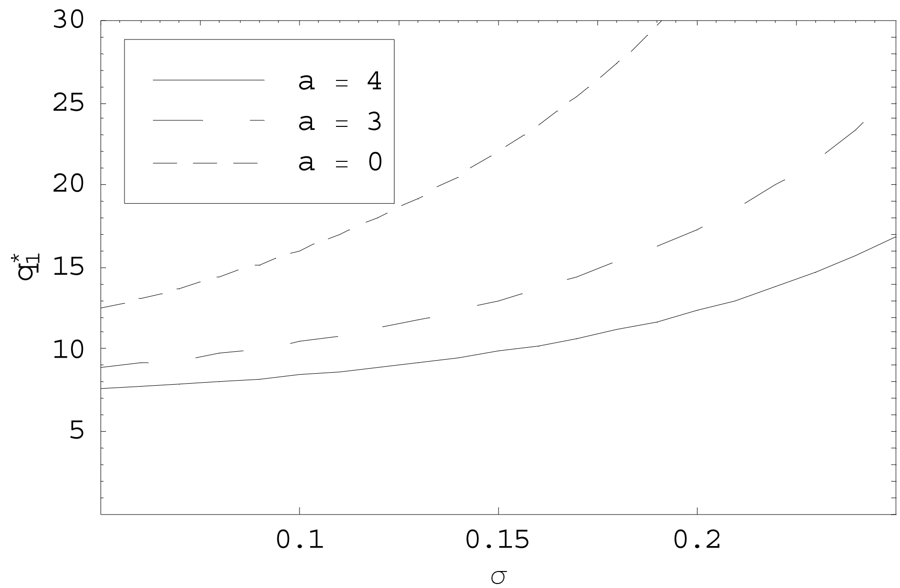

Figure 5.6: The optimal quality choice of Firm 1 in the strategic case for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

by lower competitive pressure resulting from higher network externalities.

5.4 Non-strategic Model with Flexible Quality

Here, it is assumed that within the firm sufficient know-how is present for adjusting quality, which can be valuable in case of changing demand characteristics. The fact that the firm can change quality could be caused for instance by the fact that its technology is the result of its own R&D process. Such an interpretation implies that in the previous section quality was fixed because the production technology was provided by an external vendor.

Once the entry threshold, x^{**}, is reached, production commences. The marginal cost, $c(q(x))$, is a function of the instantaneously chosen product/service quality. This quality is chosen in such a way that the value of the firm is maximized. In this section we assume that no competitive entry threat exists.

Consequently, at each point in time the firm chooses quality $q(t)$ such that[12]

$$q^{**}(x) = \arg\max_{q} \left[(x - c_0 (q-a)) \, n(q) \right]. \tag{5.32}$$

From this the present value of the firm's expected cash flow at time t can be determined

$$V(t) = E\left[\int_t^{\infty} (x(s) - c_0 (q^{**}(x(s)) - a)) \, n(q(x(s))) e^{-r(s-t)} ds \right]. \tag{5.33}$$

Since in general we allow for $q < k$, let us redefine $n(q)$ (cf. (5.4)) as

$$n(q) = \max\left[0, \frac{q-k}{q-a} \right]. \tag{5.34}$$

Maximizing (5.32) with cost specification (5.22) leads to the optimal quality choice

$$q^{**}(x) = a + \sqrt{\frac{(k-a)\,x}{c_0}} \mathbf{1}_{\{x>\eta\}}, \tag{5.35}$$

where

$$\eta = c_0 (k-a)$$

and $\mathbf{1}_B$ is an indicator function. (5.35) implies that for low states of demand (i.e. for $x < \eta$) the optimal choice of quality is a ($< k$), which corresponds to the situation in which the market is not served and the firm incurs no cost (see (5.22)). As soon as x hits η from below, quality jumps to k and, subsequently, adjusts continuously to changes in x. When x hits η from above, the quality drops to a and the firm again becomes idle without incurring variable costs.

Define the instantaneous profit function, π, to be equal to the expression under the arg max operator in (5.32). Substituting q^{**} into the instantaneous profit function yields

$$\pi = \left(\sqrt{x} - \sqrt{\eta} \right)^2 \mathbf{1}_{\{x>\eta\}}. \tag{5.36}$$

Solving the Bellman equation[13]

$$0.5\sigma^2 x^2 V'' + \alpha x V' + \pi = rV \tag{5.37}$$

[12] Our formulation differs from the optimal control models of quality as, e.g., presented by El Ouardighi and Tapiero (1998, see also references therein) since these authors consider a deterministic setting in which they include elements absent here such as pricing strategy and learning effects.

[13] The value of the firm, V, (cf. (5.33)) still satisfies the differential equation (5.37) since q is an $\mathcal{F}$-previsible process. Consequently,

$$dV = V_x dx + 0.5 V_{xx} (dx)^2 + V_q dq = V_x dx + 0.5 V_{xx} (dx)^2,$$

which, after the substitution of (5.1), yields the LHS of (5.37).

for appropriate value-matching and smooth-pasting conditions yields:

$$V = \begin{cases} B_{M2}x^{\beta_1} & \text{for} \quad x < \eta, \\ B_{M1}x^{\beta_2} + C_0 + C_1x^{0.5} + C_2x & \text{for} \quad x > \eta, \end{cases} \tag{5.38}$$

where constants B_{M1}, B_{M2}, C_0, C_1, and C_2 are given by equations (5.76)-(5.80) in the Appendix, β_1 is given by (5.9), and β_2 is given by

$$\beta_2 = -\frac{\alpha}{\sigma^2} + \frac{1}{2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{5.39}$$

The value functions in the two regimes of the stopping region are the solutions of the standard ODE (5.37) with the non-homogeneity term defined by (5.36). Under the regime $x < \eta$ demand is too low and no service/product is offered. Consequently, the value of the firm consists entirely of the option value to relaunch the activities should the market turn out to be favorable. For $x > \eta$ the firm offers the service and makes positive profit. Now, the value of the firm consists of two parts: the perpetuity value of the current instantaneous profit and the option-like component reflecting the possibility of ceasing the operations if x falls below η. The perpetuity value of the instantaneous profit has the structure of a portfolio of continuously paid dividends proportional to various powers of the GBM (5.1).

The optimal investment threshold and the value of the investment opportunity are found by applying the standard procedure for the optimal exercise of an American option when the value of the investment project in the stopping region is described by (5.38). It should just be noticed that it is never optimal to exercise the investment option for $x < \eta$ since by waiting an increment dt the present value of investment cost diminishes by $Irdt$, whereas the expected present value of the cash flow remains unchanged. The value-matching and smooth-pasting conditions regarding the expression for V when $x > \eta$ in (5.38) are

$$A_M x^{\beta_1} = B_{M1}x^{\beta_2} + C_0 + C_1x^{0.5} + C_2x - I, \tag{5.40}$$

$$\beta_1 A_M x^{\beta_1 - 1} = \beta_2 B_{M1}x^{\beta_2 - 1} + 0.5C_1x^{-0.5} + C_2. \tag{5.41}$$

From (5.40) and (5.41) the following implicit equation for the optimal investment threshold, x^{**}, can be obtained

$$(\beta_1 - \beta_2) B_{M1} (x^{**})^{\beta_2} + \beta_1 (C_0 - I) + (\beta_1 - 0.5) C_1 (x^{**})^{0.5} + (\beta_1 - 1) C_2 x^{**} = 0. \tag{5.42}$$

The value of the investment opportunity equals

$$F = \left(V\left(x^{**}\right) - I\right)\left(\frac{x}{x^{**}}\right)^{\beta_1} \equiv A_M x^{\beta_1}, \tag{5.43}$$

where $V\left(x^{**}\right)$ is given by the first row in (5.38).

Here, we would like to make an additional remark concerning the implications of the flexible quality choice on the cost structure. Compared with the fixed-quality case, the effective sunk cost in the current case equals I, as opposed to $I + C$ in the former. Consequently, the choice of flexible quality not only allows for optimizing the product parameter when demand changes but also for avoiding commitment to fixed production costs in the future.

5.5 Strategic Model with Flexible Quality

In this section we introduce the possibility of entry of a second firm (Firm 2). As in the fixed quality case, such an entry threat is going to influence both the optimal investment timing and the value of the investment opportunity of Firm 1. We proceed as follows. First, we discuss possible market outcomes dependent on the realization of the stochastic variable, x. Subsequently, we determine the value of Firm 1 in the situation where both firms have already invested. Then, we move backwards and calculate the value of Firm 1 after it entered the market but before Firm 2 invested. Finally, we determine the value of Firm 1's investment opportunity and its optimal investment threshold, and provide some comparative statics.

As in Section 5.3, Firm 2 is assumed to have the fixed-quality technology. Profit maximization of Firm 1 yields the following optimal quality schedule

$$q_1^{**} = \begin{cases} a, & \text{when Firm 1 is idle,} \\ a + \sqrt{\frac{\left(\underline{q}_1 - a\right)x}{c_0}}, & \text{when Firm 1 is a duopolist,} \\ a + \sqrt{\frac{(k-a)x}{c_0}}, & \text{when Firm 1 is a monopolist.} \end{cases} \tag{5.44}$$

The first (idle) and the third (monopoly) case have already been derived in Section 5.4. The result for the duopoly case can be obtained by maximizing the profit function (5.23) with respect to q_i, $i = 1$, and using the observation that n_1 is in this case defined by the second equation in (5.19). Before we derive Firm 1's profit as a function of x, we formulate the following proposition.

Proposition 5.5 *There are three regimes of the product market structure when the quality of Firm 1's product is flexible. For low realizations of x the market is served* *only by the entrant (Firm 1 stays idle), intermediate realizations of x correspond to the duopoly outcome, whereas under high realizations of x Firm 1 is a monopolist. The three regimes correspond to the following intervals*

$$\begin{aligned} x &\in (0, \varphi), \\ x &\in (\varphi, \varkappa), \textit{ and} \\ x &\in (\varkappa, \infty), \end{aligned}$$

where

$$\varphi \equiv c_0 \left(\underline{q}_1 - a \right), \tag{5.45}$$

$$\varkappa \equiv \frac{c_0^2 \psi^2}{\rho^2 \varphi}, \tag{5.46}$$

where $\psi \equiv \rho (\overline{q}_1 - a)$, and $\underline{q}_1$ and $\overline{q}_1$ are given by (5.20) and (5.21).

Proof. See the Appendix. ■

The existence of three regimes of quality choice result from the fact that now Firm 1 is able to adjust its quality, q_1, as x evolves. Since from (5.20) and (5.21) we learn that $\underline{q}_2$ and $\overline{q}_2$ explicitly depend on q_1, it follows that $\underline{q}_2$ and $\overline{q}_2$ become functions of x. Consequently, for low realizations of x (lower than φ) Firm 1 remains idle (in order to avoid operating loss), whereas for intermediate values of x it competes against Firm 2. If x becomes large (larger than $\varkappa$), Firm 1 can afford to choose quality that is high enough to prevent Firm 2 (with a fixed quality q_2) from serving the market. Consequently, the quality choice (5.44) reflects the optimal response in the state of inaction, duopoly and monopoly, respectively. This relationship is illustrated in Figure 5.7.

We denote the value of Firm 1, provided that Firm 2 has already entered the market, by V_1^d. V_1^d satisfies the following Bellman equation

$$0.5\sigma^2 x^2 \frac{\partial^2 V_1^d}{\partial x^2} + \alpha x \frac{\partial V_1^d}{\partial x} + \pi_1 = r V_1^d, \tag{5.47}$$

where

$$\pi_1 = \begin{cases} 0 & \text{for} \quad x < \varphi, \\ \frac{1}{1-\rho^2} \left(\sqrt{x} - \sqrt{\varphi} \right)^2 & \text{for} \quad \varphi < x < \varkappa, \\ \left(\sqrt{x} - \sqrt{\eta} \right)^2 & \text{for} \quad x > \varkappa. \end{cases} \tag{5.48}$$

For $x < \varphi$ Firm 1 is idle, for $x > \varkappa$ it earns monopoly profit, whereas for $x \in (\varphi, \varkappa)$ it has a duopoly profit. The latter can be calculated by substituting the intermediate

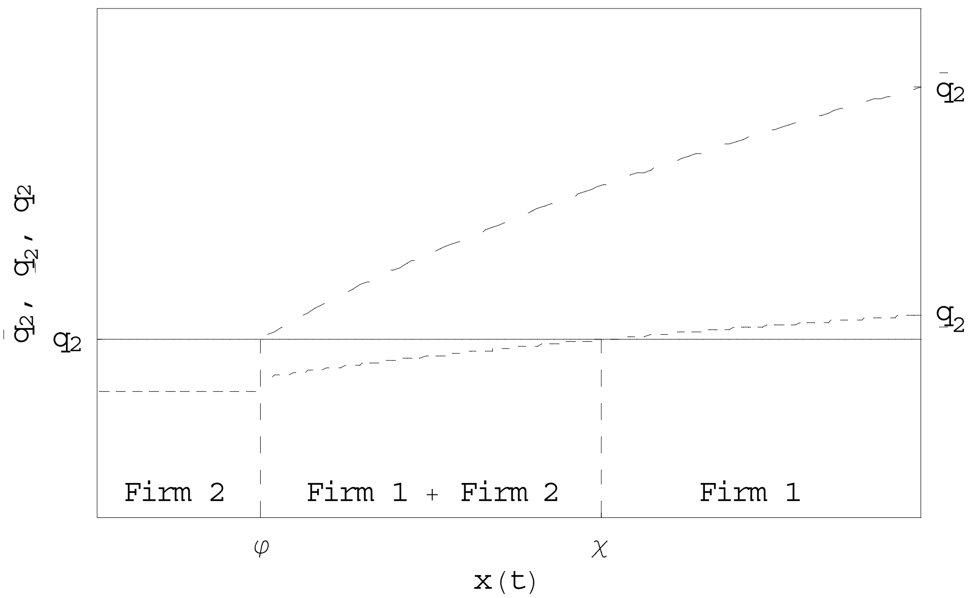

Figure 5.7: Trigger qualities $\underline{q}_2$ (short-dotted line), $\overline{q}_2$ (long-dotted line) as a function of x, for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$, $c_0 = 1$, and $q_2 = 7.5$ (solid line). For low realizations of x (below φ) only Firm 2 is active in the market whereas for high realizations (above $\varkappa$) Firm 1 becomes a monopolist - the quality of Firm 2 is too low. For intermediate values of x both firms serve the market since q_2 remains within the bounds determined by $\underline{q}_2$ and $\overline{q}_2$.

cases of (5.19) and (5.44) into (5.23). Solving (5.47) with the value matching and smooth pasting conditions satisfied for realizations φ and $\varkappa$ yields

$$V_1^d = \begin{cases} (D_2 + D_4)\, x^{\beta_1} & \text{for} \quad x < \varphi, \\ D_1 x^{\beta_2} + D_2 x^{\beta_1} + E_0 + E_1 x^{0.5} + E_2 x & \text{for} \quad \varphi < x < \varkappa, \\ (D_1 + D_3)\, x^{\beta_2} + C_0 + C_1 x^{0.5} + C_2 x & \text{for} \quad x > \varkappa, \end{cases} \tag{5.49}$$

where (see the Appendix) C_0, C_1, and C_2 are defined by (5.78)-(5.80), whereas coefficients D_1, D_2, D_3, D_4, E_0, E_1, and E_2 are defined by (5.81)-(5.87). Again, it can be seen that the value of Firm 1 consists of the present value of the expected cash flow and the option-like components reflecting possible switches across regimes. Parameters E_k and C_k, $k \in \{1, 2, 3\}$, correspond to the duopolistic and monopolistic profit function, respectively. Components of the form $D_l x^{\beta_2}$, $l \in \{1, 2, 3, 4\}$, reflect the possibility of switching to the regime corresponding to lower than current realizations of x, whereas the opposite is true for components $D_l x^{\beta_1}$.

Equipped with the valuation formula for Firm 1 when both firms are already present in the market, we are ready to derive the value of Firm 1, V_1^m, prior to Firm 2's entry

$$V_1^m = V + \left(V_1^d\left(x_2^{**}\right) - V\left(x_2^{**}\right)\right)\left(\frac{x}{x_2^{**}}\right)^{\beta_1}, \tag{5.50}$$

where V is defined by (5.38) and x_2^{**} denotes Firm 2's entry threshold (derived in the Appendix). V_1^m equals the monopolistic value of Firm 1 (as defined by (5.38)) adjusted for the component reflecting competitive entry. The latter component equals the value loss from switching from monopoly to duopoly multiplied by the probability-weighted discount factor corresponding to the random time of Firm 2's entry.

In the last step, we determine the value of Firm 1's investment opportunity. We already know that the valuation formulae for V differ across the two regimes (cf. (5.38)) and that it is never optimal for Firm 1 to invest in the first regime. Consequently, when applying the value-matching and smooth-pasting conditions to (5.50), we substitute for V the expression corresponding to the second row in (5.38). A simple manipulation of the value-matching and smooth pasting conditions (cf. (5.40) and (5.41)) yields the following implicit formula for the optimal investment threshold of Firm 1, x_1^{**}

$$\begin{aligned}&\left(\beta_1-\beta_2\right)B_{M1}\left(x_1^{**}\right)^{\beta_2}+\beta_1\left(C_0-I\right)+\left(\beta_1-0.5\right)C_1\left(x_1^{**}\right)^{0.5}\\&+\left(\beta_1-1\right)C_2x_1^{**}=0.\end{aligned} \tag{5.51}$$

A comparison of (5.51) with (5.42) leads to the observation that $x_1^{**} = x^{**}$. This is in line with the classic strategic real option models in which the roles of the firms (leader vs. follower) are determined exogenously and where the firms have a single control variable (investment timing). This finding can be explained by the fact that in our case the decision problem of the Firm 1 with one discrete control variable (timing) and with one continuous control variable (quality) can be transformed into the problem of a single discrete variable whereas the relevant payoff functions are at each moment optimized with respect to the continuous variable. Consequently, the value of Firm 1 is no longer a function of quality since this is chosen optimally given the realization of x_t and the choice of exogenous parameters.

The value of the investment opportunity of Firm 1, F_1, equals

$$\begin{aligned}F_1 &= \left(V\left(x_1^{**}\right)+\left(V_1^d\left(x_2^{**}\right)-V\left(x_2^{**}\right)\right)\left(\frac{x_1^{**}}{x_2^{**}}\right)^{\beta_1}-I\right)\left(\frac{x}{x_1^{**}}\right)^{\beta_1}\\&\equiv A_1x^{\beta_1}.\end{aligned} \tag{5.52}$$

It can immediately be noticed that $F_1 < F$ (cf. (5.43)) because of the present value of future revenues lost due to competitive entry, which is equal to

$$\left(V_1^d\left(x_2^{**}\right) - V\left(x_2^{**}\right)\right)\left(\frac{x_1^{**}}{x_2^{**}}\right)^{\beta_1}.$$

As soon as competitive entry becomes very remote, i.e. when $x_2^{**} \to \infty$, it holds that the problem reduces to the valuation of a monopolistic firm and $F_1 = F$.

5.5.1 Comparative Statics: Valuation of Firms

Analogous to Section 5.3, we are interested in the sensitivity of the firms' value with respect to changes of market parameters. Figures 5.8 and 5.9 depict the relationship between the market volatility and the value of the investment opportunities of both firms for different magnitudes of network externalities. Inspection of both the figures leads to two main conclusions First, contrary to the fixed quality case, the value of Firm 1's investment opportunity is higher than the one of Firm 2. Second, the value of Firm 2's project is non-monotonic in uncertainty (like the value of Firm 1 in the previous case). The first result is implied by the fact that Firm 1 is a leader in the investment game but, thanks to its flexibility with regard to quality choice, acts as a follower in the Stackelberg quality game. Consequently, Firm 1 not only receives cash flow from the project over a longer period but also is able to adjust its quality optimally to the fixed the quality choice of Firm 2, q_2, and the realization of the demand, x.

The non-monotonicity of Firm 2's value in uncertainty results from the fact that Firm 1 can exploit to a (relatively) larger extent the changes in the demand by changing its quality when uncertainty is high. Therefore, higher uncertainty affects the effective discount rates of the components of Firm 2's value that are concave in x (see (5.90)). Consequently, the presence of such concavities leads to a lower valuation in a more uncertain environment. A positive relationship between Firm 2's value and uncertainty at the low levels of uncertainty can be explained by the traditional option argument that, in this case, dominates the strategic effects.

As far as the relationship between the degree of network externalities and the value of the firms is concerned, it resembles the picture of the fixed quality case. Again, the presence of the network externalities leads to an increase in the value of the investment opportunities of both firms and the rate of this increase is high when network externalities are relatively strong.

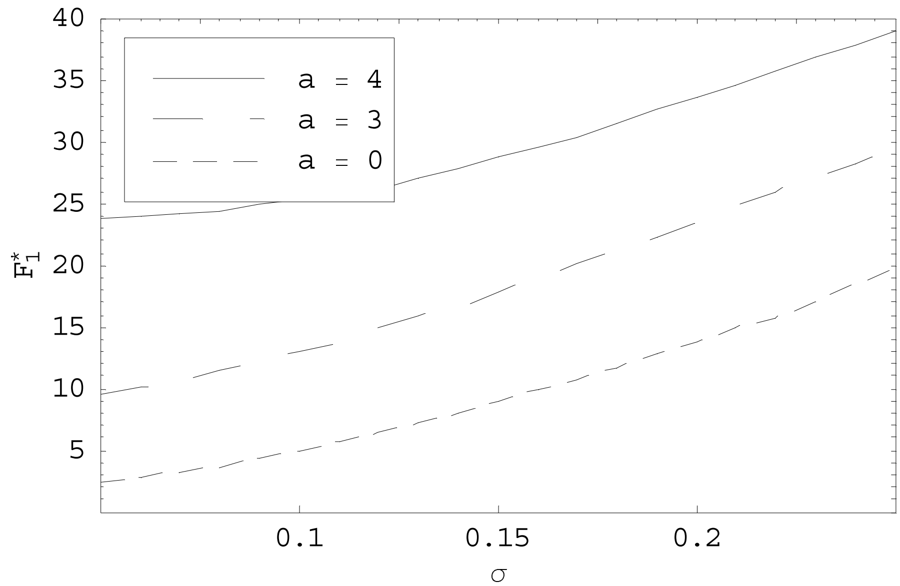

Figure 5.8: The value of the investment opportunity of Firm 1 for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

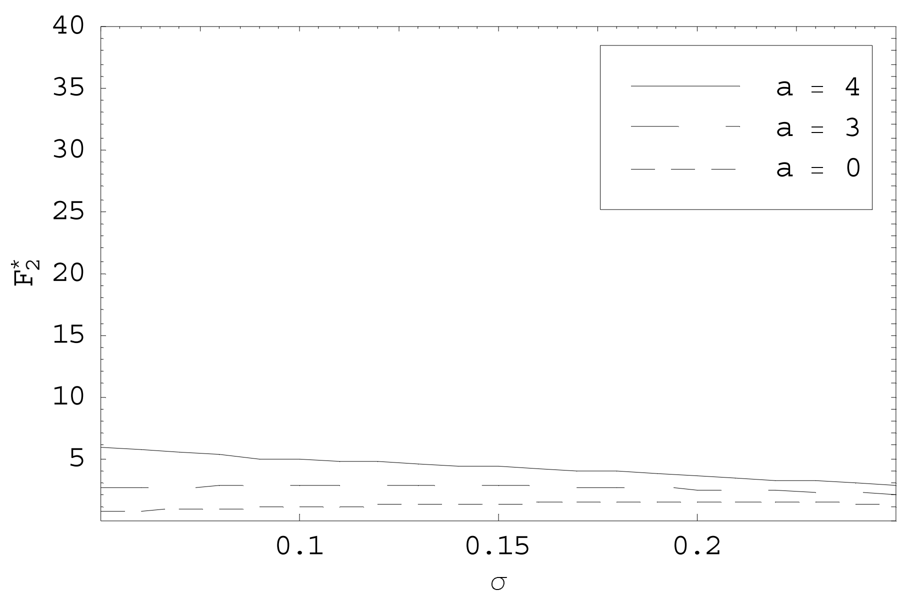

Figure 5.9: The value of the investment opportunity of Firm 2 for the parameter values $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

5.5.2 Comparative Statics: Firm 1's Strategic Choice of Variables

In the case in which quality is flexible, the following observations can be made. First, the optimal investment threshold in the presence of entry threat is identical to the level of x triggering the investment of the monopolist. This is due to the well-known fact that if the roles of the firms are predetermined and the only choice variable of the leader is the investment timing, future entry of the follower does not impact the investment timing of the leader (cf. Huisman, 2001). Second, upon examining (5.44), we can conclude that the quality chosen by Firm 1 does not change in a continuous way. In the following subsection, we present a short discussion of the properties of $q_1^{**}(x)$.

Properties of $q_1^{**}(x)$

The optimal quality choice, q_1^{**}, piecewise (weakly) increases with the state of the market, x. At φ and $\varkappa$ the quality exhibits discontinuities. Calculating the relevant limits yields (cf. (5.44))

$$\lim_{x\downarrow\varphi} q_1^{**}(x) - \lim_{x\uparrow\varphi} q_1^{**}(x) = \rho q_2 + (1-\rho)k - a > 0, \tag{5.53}$$

$$\lim_{x\downarrow\varkappa} q_1^{**}(x) - \lim_{x\uparrow\varkappa} q_1^{**}(x) = (\overline{q}_1 - a)\left(\sqrt{\frac{k-a}{\underline{q}_1 - a}} - 1\right) < 0. \tag{5.54}$$

Realizations φ and $\varkappa$ are reversible switch points in which the functional form of the optimal quality changes. As pointed out by Mella-Barral and Perraudin (1997), the function describing the optimal choice of a control variable is in general discontinuous in the switch points (see also Dumas, 1991). Continuity is implied if the switch points are chosen optimally so as to maximize the value of the firm. Here, the switch points are not chosen optimally by Firm 1 but, instead, they result from the change of the product market structure. From (5.44) it can be seen that for low x Firm 1 ceases operations as the revenues do not cover the operating costs. When x reaches φ from below, Firm 1 resumes operations and the resulting outcome is duopolistic. Finally, when x reaches $\varkappa$ Firm 1 covers the entire market and the monopoly prevails. Consequently, the discontinuity of $q_1^{**}(x)$ occurs at both φ and $\varkappa$.

The positive sign of (5.53) results from the fact that the quality of the idle firm equals a (cf. (5.44)), whereas resuming the operations requires the quality exceeding $k\,(> a)$. The negative sign of (5.54) can be explained as follows. At the moment x equals $\varkappa$ (cf. Figure 5.4), quality chosen by Firm 1 is that high that Firm 1 captures all customers. Hence, Firm 2 leaves the market after which Firm 1 reduces quality. It can do so since Firm 2 will not re-enter (unless x falls below $\varkappa$). Firm 2 knows that if it re-entered, Firm 1 would immediately raise quality to the optimal duopoly level. We conclude that flexible quality serves as an entry deterrent control here, while still it can be set at the optimal monopoly level.

5.6 Valuation Effects of Flexible vs. Fixed Quality

In this section we analyze the effects on the valuation of the flexible vs. fixed technology choice made by Firm 1. We address the following two related questions: *i*) what is the relationship between the loss in value due to the expected competitive entry (in comparison with monopoly) and the fixed or flexible quality choice, and *ii*) what is the impact of flexibility on the valuation with and without competitive entry threat.

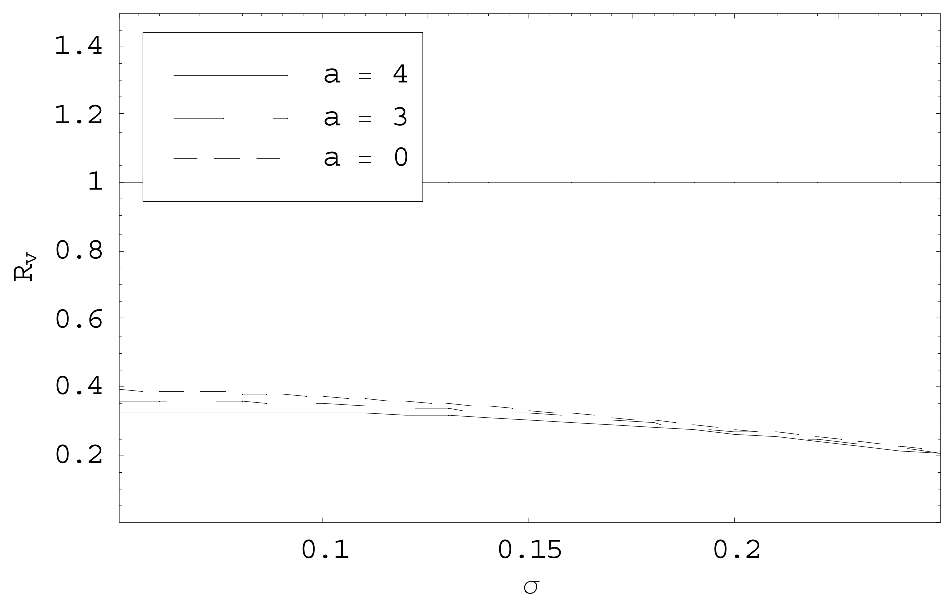

Figure 5.10: The relationship between the ratio of Firm 1's duopolistic to monopolistic value and uncertainty under fixed quality choice for $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

Figures 5.10-5.13 contain a comparison of the ratio of Firm 1's value in the monopoly vs. duopoly case for flexible and fixed quality choice. On the basis of Figures 5.10 and 5.11 it can be concluded that the value lost due to competitive entry is much lower when quality is flexible (as opposed to fixed quality). This results from the fact

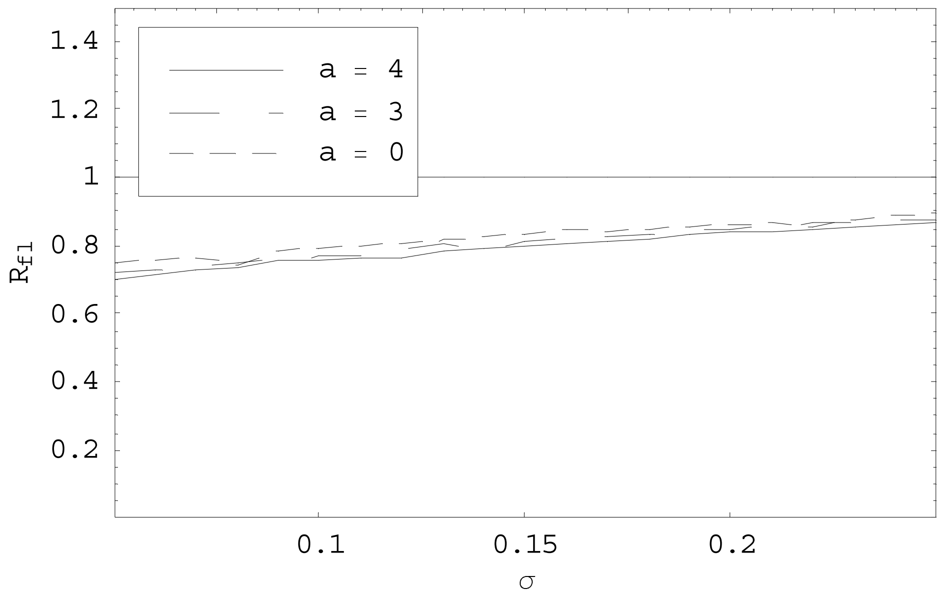

Figure 5.11: The relationship between the ratio of Firm 1's duopolistic to monopolistic value and uncertainty under flexible quality choice for $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

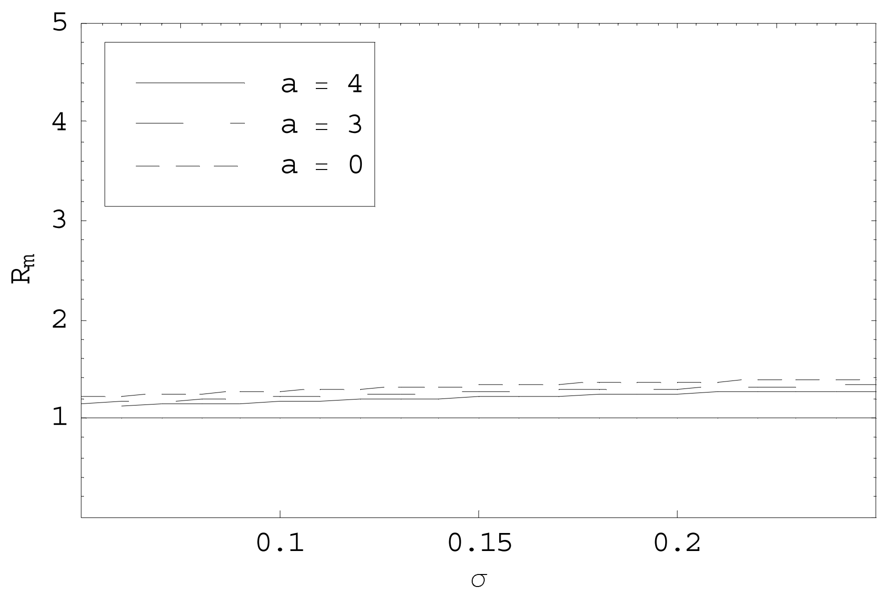

Figure 5.12: The relationship between the ratio of Firm 1's flexible to fixed technology non-strategic value and uncertainty for $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

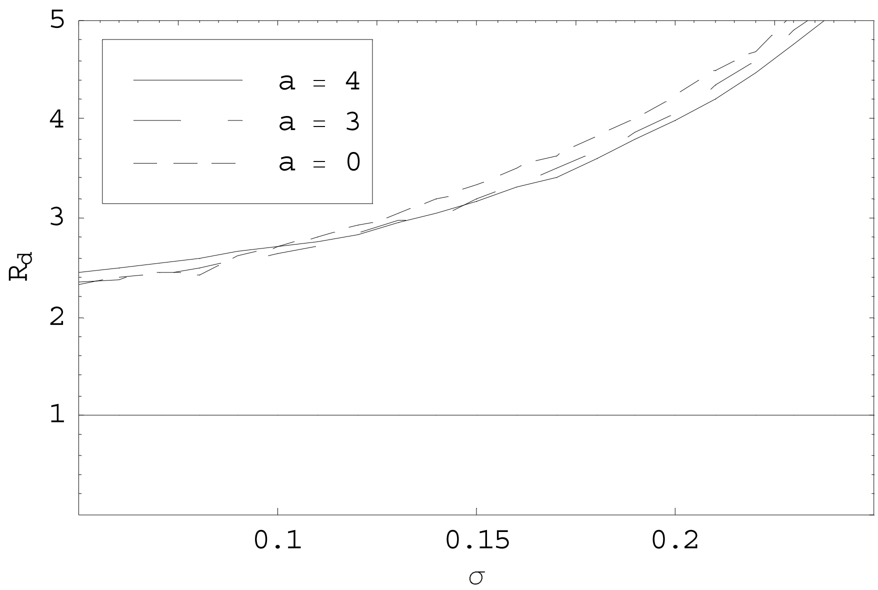

Figure 5.13: The relationship between the ratio of Firm 1's flexible to fixed technology strategic value and uncertainty for $\rho = 0.5$, $k = 5$, $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

that the flexible quality choice is associated with Firm 1's follower's role in the quality game played by the firms at each instant. The second-mover advantage in setting the quality by the incumbent is stronger when the demand uncertainty is higher. Therefore, when uncertainty is high, the gap between the monopolistic and the strategic value of Firm 1 is almost closing (cf. Figure 5.11). Finally, we can observe that the degree of network externalities have little effect on the firms' relative valuation until they become very high in the fixed quality case. Then the fraction of Firm 1's value lost due to the competitive entry as compared to monopoly is even higher (cf. Figure 5.10).

Moreover, we analyze the impact of flexible quality choice on the firms' valuation from a slightly different angle. Instead of looking at the value lost due to competitive entry, we investigate the value impact of a switch from the fixed- to flexible-quality technology. Figures 5.12 and 5.13 illustrate this effect as a function of demand uncertainty for different magnitudes of network externalities. The following conclusions can be drawn. First, the incremental value of the flexible technology over the fixed-quality technology is higher in a strategic than in a monopolistic framework. Moreover, the strategic impact of flexibility is increasing with demand uncertainty (cf. Figure 5.13). Whereas in the monopolistic framework the value gain occurring due to the flexible technology is moderate and does not increase sharply in σ, both the value gain and its sensitivity towards growing uncertainty are much more dramatic. Like previously, the value impact of network externalities is relatively small and affects the advantage of the flexible technology adversely.

5.7 Conclusions

In this chapter we determine advantages of flexibility in quality choice of a firm considering an uncertain product market sector exhibiting network externalities. The firm is able to adjust quality over time when it, for instance, possesses sufficient know-how, invented the technology itself, or adopted a more advanced technology. In general, this requires larger sunk costs and the aim of this paper is to determine in which cases it is particularly justified to incur these larger costs.

First, we derive the optimal investment threshold and the quality choice of the firm using the fixed-quality technology in both the monopolistic and duopolistic framework. Second, we repeat the analysis for the flexible technology choice. Finally, we perform a comparison of outcomes resulting from applying the two alternative technologies.

We show that the qualities chosen by the firms in the fixed-quality framework are strategic complements. This implies that a higher quality chosen by the market leader is associated with a higher quality provided by the second firm to enter. Moreover, the market leader uses the quality as a means to deter entry since its level of quality chosen under competitive entry threat is higher than in an isolated monopolistic market. Finally, since the firms play a version of a Stackelberg game in strategic complements, the value of the second firm to enter exceeds the one of the leader.

We also extend general results of strategic real options theory. From this theory it is known that if the roles of the firms are exogenous or when they sufficiently differ in characteristics, the follower's investment timing is irrelevant for the decision of the leader. However, due to the addition of a second control in the form of quality choice, the investment timing of the first investor is influenced by the decision of the other firm.

If the market leader is able to adjust quality over time, its optimal investment strategy is identical to the monopolistic case. This observation results from the fact that the loss due to the competitive entry equally affects the value of its investment opportunity before investing and the value of the project once the sunk cost is incurred. Moreover, the flexible quality choice of the leader implies three different market structures as functions of the underlying demand. When demand is low, only the second firm is active, moderate demand is associated with both firms serving the market, whereas high demand implies that the entire market is served by the leader.

A comparison of firms' values under two alternative technologies leads to further conclusions. It appears that the relative value of the flexible (as opposed to fixed) technology is much higher in the duopolistic case than in an isolated monopoly. A related observation is that the value loss from a competitive entry is much lower when the quality is flexible. Second, the value of flexible quality choice increases with uncertainty since an immediate quality adjustment to the changes in stochastic demand is possible. Moreover, the case of flexibility also allows for achieving the second-mover advantage in the Stackelberg game after the competitive entry. The latter result is amplified if the market uncertainty is high.

5.8 Appendix

Proof of Proposition 5.1. The optimal quality level is calculated by maximizing (5.10) with respect to q. The corresponding first-order condition is (dependence on q is dropped for the sake of transparency)

$$0 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \frac{x^{\beta_1}}{(C+I)^{2\beta_1 - 2}} \times \left(\beta_1 R^{\beta_1 - 1} (C+I)^{\beta_1 - 1} R_q - (\beta_1 - 1) R^{\beta_1} (C+I)^{\beta_1 - 2} C_q\right), \tag{5.55}$$

from which it follows that

$$\beta_1 (C+I) R_q - (\beta_1 - 1) R C_q = 0. \tag{5.56}$$

Dividing by $(\beta_1 - 1) x^* R$ and observing that $\frac{\beta_1}{\beta_1 - 1} \frac{C+I}{x^* R} = 1$ yields the desired result. The corresponding second-order condition is

$$\left(\beta_1 (C+I) R_{qq} + C_q R_q - (\beta_1 - 1) C_{qq} R\right)\big|_{q=q^*} < 0. \tag{5.57}$$

This is a necessary and sufficient condition for the relevant functions which ensures that q^* corresponds to a local maximum. This is formulated as Assumption 5.1. If (5.56) has multiple solutions satisfying (5.57), then the one corresponding to the highest value of (5.10) is chosen. ■

Proof of Proposition 5.2. We begin by defining (cf. (5.56))

$$H(q) = \beta_1 (C(q) + I) R_q(q) - (\beta_1 - 1) C_q(q) R(q). \tag{5.58}$$

For q^* it holds that $H(q^*;\cdot) = 0$. Therefore, the impact of a change in $\theta \in \{a, \sigma\}$ can be determined by calculating the total derivative of H:

$$\frac{dq^*}{d\theta} = -\frac{H_\theta}{H_q}. \tag{5.59}$$

By Assumption 5.1 we know that

$$\left.\frac{\partial H(q)}{\partial q}\right|_{q=q^*} < 0. \tag{5.60}$$

Consequently, from (5.59) and (5.60) it follows that (we drop the dependence of variables on q)

$$\operatorname{sgn}\left.\frac{\partial H}{\partial \theta}\right|_{q=q^*} = \operatorname{sgn}\left.\frac{dq}{d\theta}\right|_{q=q^*} \quad \text{for } \theta \in \{a, \sigma\}. \tag{5.61}$$

We have

$$\left.\frac{\partial H}{\partial \sigma}\right|_{q=q^*} = \frac{\partial \beta_1}{\partial \sigma}\left((C+I)R_q - C_q R\right) > 0, \tag{5.62}$$

$$\begin{aligned} \left.\frac{\partial H}{\partial \alpha}\right|_{q=q^*} &= \frac{\partial \beta_1}{\partial \alpha}\left((C+I)R_q - C_q R\right) \\ &\quad + \beta_1 (C+I) R_{q\alpha} - (\beta_1 - 1) C_q R_\alpha \\ &= \frac{\partial \beta_1}{\partial \alpha}\left((C+I)R_q - C_q R\right) > 0. \end{aligned} \tag{5.63}$$

■

Proof of Proposition 5.3. Repeating (5.16) for σ, we have

$$\frac{dx^*(q)}{d\sigma} = \frac{\partial x^*(q)}{\partial \sigma} + \frac{\partial x^*(q)}{\partial q}\frac{dq}{d\sigma}. \tag{5.64}$$

We are interested in the signs of the components of (5.16). It holds that

$$\frac{\partial x^*(q)}{\partial q} = \frac{\beta_1}{\beta_1 - 1}\frac{C_q R - (C+I) R_q}{R^2}. \tag{5.65}$$

Consequently, in the optimum

$$\begin{aligned} \left.\frac{\partial x^*(q)}{\partial q}\right|_{q=q^*} &= \frac{\beta_1}{\beta_1 - 1}\frac{C_q R - (C+I) R_q}{R^2} \\ &> \frac{\beta_1}{(\beta_1 - 1)^2}\frac{(\beta_1 - 1) C_q R - \beta_1 (C+I) R_q}{R^2} \\ &= 0. \end{aligned} \tag{5.66}$$

The last equality directly results from (5.56). Moreover, by differentiating (5.8), we immediately obtain that

$$\frac{\partial x^*(q)}{\partial \sigma} > 0. \tag{5.67}$$

Furthermore, using the results of Proposition 5.2 we obtain that

$$\frac{dq^*}{d\sigma} > 0. \tag{5.68}$$

This completes the proof. ■

Proof of Proposition 5.4. The sign of derivative dq_2^*/dq_1^* immediately follows from (5.27) and the argument thereafter. In order to determine the sign of dx_2^*/dq_1^*, we first express x_2^* as (cf. (5.25))

$$x_2^* = \frac{\beta_1}{\beta_1 - 1} \frac{I(1-\rho^2)r + c_0\left(q_2 - \underline{q}_2\right)}{r} \frac{q_2 - a}{q_2 - \underline{q}_2}\delta. \tag{5.69}$$

Since we already know that $\underline{q}_2$ is an increasing function of q_1 (cf. (5.20)), the desired result is obtained if we can show that the two last factors of (5.69) increase with $\underline{q}_2$. We first derive expressions for $q_2 - \underline{q}_2$ and $q_2 - a$ on the basis of (5.27):

$$q_2 - \underline{q}_2 = \frac{1}{2(\beta_1 - 1)} \times \tag{5.70}$$
$$\left[\underline{q}_2 - a + \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + 4\beta_1 Ir(\beta-1)(1-\rho^2)c_0^{-1}}\right],$$
$$q_2 - a = \frac{1}{2(\beta_1 - 1)} \times \tag{5.71}$$
$$\left[(2\beta - 1)\left(\underline{q}_2 - a\right) + \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + 4\beta_1 Ir(\beta-1)(1-\rho^2)c_0^{-1}}\right].$$

By inspecting (5.70) we immediately conclude that the second factor of (5.69) is increasing with $\underline{q}_2$. Now, we concentrate on the derivative of the ratio $\frac{q_2 - a}{q_2 - \underline{q}_2}$. It can be written (using (5.70) and (5.71)) as

$$\frac{d}{d\underline{q}_2}\left(\frac{q_2 - a}{q_2 - \underline{q}_2}\right) = \frac{d}{d\underline{q}_2}\left(\frac{(2\beta_1 - 1)\left(\underline{q}_2 - a\right) + f\left(\underline{q}_2\right)}{\underline{q}_2 - a + f\left(\underline{q}_2\right)}\right), \tag{5.72}$$

where

$$f\left(\underline{q}_2\right) = \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + K}.$$

and $K = 4\beta_1 Ir(\beta_1 - 1)(1-\rho^2)c_0^{-1} > 0$. Now, (5.72) can be expressed as

$$\frac{d}{d\underline{q}_2}\left(\frac{q_2 - a}{q_2 - \underline{q}_2}\right) = \frac{2(\beta_1 - 1)\left(f\left(\underline{q}_2\right) - f'\left(\underline{q}_2\right)\left(\underline{q}_2 - a\right)\right)}{\left(\underline{q}_2 - a + f\left(\underline{q}_2\right)\right)^2}.$$

In the final step, we determine the sign of the second factor in the numerator

$$
\begin{aligned}
& f\left(\underline{q}_2\right) - f'\left(\underline{q}_2\right)\left(\underline{q}_2 - a\right) = \\
& = \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + K} - \frac{\left(2\underline{q}_2 - 2a + K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} = \\
& = \frac{\left(2\underline{q}_2 - 2a + 2K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} - \frac{\left(2\underline{q}_2 - 2a + K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} > 0.
\end{aligned}
$$

This completes the proof. ■

Proof of Proposition 5.5. The proposition can be proven by analyzing the profit functions of the firms in a duopoly and two cases of a monopoly. Profit maximization based on the system of demands (5.18) with the optimal quality schedule of Firm 1 (5.44) yields the following Stackelberg profits of Firm 1 and Firm 2, denoted by π_1 and π_2, respectively:

$$\pi_1 = \frac{1}{1-\rho^2}\left(\sqrt{x} - \sqrt{\varphi}\right)^2, \tag{5.73}$$

$$\pi_2 = \frac{1}{1-\rho^2}\left(\frac{-\rho\sqrt{\varphi}}{\kappa}x^{1.5} + \frac{c_0\psi}{\kappa}x + \rho\sqrt{\varphi}x^{0.5} - c_0\psi\right), \tag{5.74}$$

and

$$\kappa = c_0\left(q_2 - a\right). \tag{5.75}$$

Here, φ, ψ and κ are functions of q_2, which is chosen at the beginning of the game (the quality chosen by Firm 2 is fixed at the moment of undertaking investment). Since π_2 is concave and decreasing for sufficiently large x (but smaller than $\varkappa$), we get that for $x > \varkappa$ it holds that $\pi_2 = n_2 = 0$ (since Firm 2 will cease the production in the region $x \in (\varkappa, \infty)$, where the attainable profit is negative). In the same fashion in can be shown that $\pi_1 = 0$ for $x < \varphi$. What remains to be proved is that $\varphi < \varkappa$. It can be seen upon manipulating (5.45) and (5.46) that

$$\varkappa - \varphi = c_0\frac{\left(\overline{q}_1 - a\right)^2 - \left(\underline{q}_1 - a\right)^2}{\left(\underline{q}_1 - a\right)} > 0 \Leftrightarrow \overline{q}_1 - \underline{q}_1 > 0.$$

The latter inequality is proven directly by observing that

$$
\begin{aligned}
\overline{q}_i - \underline{q}_i = 0 \text{ for } \rho &= 1, \text{ and} \\
\frac{\partial\left(\overline{q}_i - \underline{q}_i\right)}{\partial\rho} &< 0.
\end{aligned}
$$

This completes the proof. ■

Value function coefficients with flexible quality: non-strategic case. The following coefficients are derived on the basis of Bellman equation (5.37) with value-matching and smooth-pasting conditions applied to V for $x = \eta$ and with no-bubble conditions for $x \to 0$ and $x \to \infty$:

$$\begin{aligned} B_{M1} &\equiv C_0 \frac{\eta^{-\beta_2}\beta_1}{\beta_2-\beta_1} + C_1 \frac{\eta^{0.5-\beta_2}(\beta_1-0.5)}{\beta_2-\beta_1} + C_2 \frac{\eta^{1-\beta_2}(\beta_1-1)}{\beta_2-\beta_1}, && (5.76)\\ B_{M2} &\equiv C_0 \frac{\eta^{-\beta_1}\beta_2}{\beta_2-\beta_1} + C_1 \frac{\eta^{0.5-\beta_1}(\beta_2-0.5)}{\beta_2-\beta_1} + C_2 \frac{\eta^{1-\beta_1}(\beta_2-1)}{\beta_2-\beta_1}, && (5.77)\\ C_0 &\equiv \frac{\eta}{r}, && (5.78)\\ C_1 &\equiv \frac{-2\sqrt{\eta}}{r-0.5\alpha+0.125\sigma^2}, && (5.79)\\ C_2 &\equiv \frac{1}{\delta}. && (5.80) \end{aligned}$$

By either solving the Bellman equation of type (5.37) with a non-homogeneity term being proportional to the n-th power of x, or by calculating the drift coefficient in the GBM for $y \equiv x^n$ using Itô's lemma, it can be shown that the effective discount rate corresponding to the n-th power has a form $r - n\alpha - 0.5n(n-1)\sigma^2$ (cf. Dixit, 1993, p. 13). Of course, this puts a restriction on the pairs (α, σ^2) if finite valuations are to be obtained. ■

Value function coefficients with flexible quality: strategic case. The following coefficients are derived on the basis of Bellman equation (5.47) with value-matching and smooth-pasting conditions applied to V_1^d for $x = \varphi$ and $x = \varkappa$, and with no-bubble conditions for $x \to 0$ and $x \to \infty$:

$$\begin{aligned} D_1 &\equiv E_2 \frac{\varphi^{1-\beta_2}(\beta_1-1)}{\beta_2-\beta_1} + E_1 \frac{\varphi^{0.5-\beta_2}(\beta_1-0.5)}{\beta_2-\beta_1} + E_0 \frac{\varphi^{-\beta_2}\beta_1}{\beta_2-\beta_1}, && (5.81)\\ D_2 &\equiv -E_2 \frac{\varkappa^{1-\beta_1}(\beta_2-1)\rho^2}{\beta_2-\beta_1} - E_1 \frac{\varkappa^{0.5-\beta_1}(\beta_2-0.5)}{\beta_2-\beta_1}\frac{(1-\rho^2)\varphi-\eta}{\varphi} \\ &\quad -E_0^d \frac{\varkappa^{-\beta_1}\beta_2\rho}{\beta_2-\beta_1}, && (5.82)\\ D_3 &\equiv -E_2 \frac{\varkappa^{1-\beta_2}(\beta_1-1)\rho^2}{\beta_2-\beta_1} - E_1 \frac{\varkappa^{0.5-\beta_2}(\beta_1-0.5)}{\beta_2-\beta_1}\frac{(1-\rho^2)\varphi-\eta}{\varphi} \\ &\quad -E_0^d \frac{\varkappa^{-\beta_2}\beta_1\rho}{\beta_2-\beta_1}, && (5.83)\\ D_4 &\equiv E_2 \frac{\varphi^{1-\beta_1}(\beta_2-1)}{\beta_2-\beta_1} + E_1 \frac{\varphi^{0.5-\beta_1}(\beta_2-0.5)}{\beta_2-\beta_1} + E_0 \frac{\varphi^{-\beta_1}\beta_2}{\beta_2-\beta_1}, && (5.84) \end{aligned}$$

$$E_0 \equiv \frac{1}{1-\rho^2}\frac{\varphi}{r}, \quad (5.85)$$

$$E_1 \equiv \frac{1}{1-\rho^2}\frac{-2\sqrt{\varphi}}{r-0.5\alpha+0.125\sigma^2}, \quad (5.86)$$

$$E_2 \equiv \frac{1}{1-\rho^2}\frac{1}{r-\alpha}. \quad (5.87)$$

■

Derivation of Firm 2's optimal investment threshold. First, we derive the value of the Firm 2. Denote the value of the Firm 2 after entering the market by V_2. V_2 satisfies the following Bellman equation

$$0.5\sigma^2 x_t^2 V_2'' + \alpha x V_2' + \pi_2 = rV_2, \quad (5.88)$$

where

$$\pi_2 = \begin{cases} \frac{c_0\psi-\rho\varphi}{(1-\rho^2)\kappa}(x-\kappa) & \text{for} \quad x<\varphi, \\ \frac{1}{1-\rho^2}\left(\frac{-\rho\sqrt{\varphi}}{\kappa}x^{1.5}+\frac{c_0\psi}{\kappa}x+\rho\sqrt{\varphi}x^{0.5}-c_0\psi\right) & \text{for} \quad \varphi<x<\varkappa, \\ 0 & \text{for} \quad x>\varkappa. \end{cases} \quad (5.89)$$

In (5.89) the value of π_2 for $x>\varkappa$ is zero since for high demand, Firm 1 captures the entire market share (cf. Lemma 5). For $\varphi<x<\varkappa$ the result corresponds to (5.74), whereas for $x<\varphi$ Firm 2 achieves monopoly profit (cf. (5.23)) since Firm 1 remains idle. Solving (5.88) with the value matching and smooth pasting conditions satisfied for realizations φ and $\varkappa$ yields

$$V_2 = \begin{cases} (B_2+B_4)x^{\beta_1}+C_0^M+C_2^M x & \text{for} \quad x<\varphi, \\ B_1 x^{\beta_2}+B_2 x^{\beta_1}+C_0^D+C_1^D x^{0.5}+C_2^D x+C_3^D x^{1.5} & \text{for} \quad \varphi<x<\varkappa, \\ (B_1+B_3)x^{\beta_2} & \text{for} \quad x>\varkappa, \end{cases} \quad (5.90)$$

where

$$B_1 \equiv C_3^D\frac{\varphi^{1.5-\beta_2}(\beta_1-1.5)}{\beta_2-\beta_1}+C_2^D\frac{\varphi^{1-\beta_2}(\beta_1-1)}{\beta_2-\beta_1}\frac{\rho\varphi}{c_0\psi} +C_1^D\frac{\varphi^{0.5-\beta_2}(\beta_1-0.5)}{\beta_2-\beta_1}+C_0^D\frac{\varphi^{-\beta_2}\beta_1}{\beta_2-\beta_1}\frac{\rho\varphi}{c_0\psi}, \quad (5.91)$$

$$B_2 \equiv -C_3^D\frac{\varkappa^{1.5-\beta_1}(\beta_2-1.5)}{\beta_2-\beta_1}-C_2^D\frac{\varkappa^{1-\beta_1}(\beta_2-1)}{\beta_2-\beta_1} -C_1^D\frac{\varkappa^{0.5-\beta_1}(\beta_2-0.5)}{\beta_2-\beta_1}-C_0^D\frac{\varphi^{-\beta_1}\beta_2}{\beta_2-\beta_1}, \quad (5.92)$$

$$B_3 \equiv -C_3^D\frac{\varkappa^{1.5-\beta_2}(\beta_1-1.5)}{\beta_2-\beta_1}-C_2^D\frac{\varkappa^{1-\beta_2}(\beta_1-1)}{\beta_2-\beta_1} -C_1^D\frac{\varkappa^{0.5-\beta_2}(\beta_1-0.5)}{\beta_2-\beta_1}-C_0^D\frac{\varkappa^{-\beta_2}\beta_1}{\beta_2-\beta_1}, \quad (5.93)$$

$$
\begin{aligned}
B_4 &\equiv C_3^D \frac{\varphi^{1.5-\beta_1}(\beta_2-1.5)}{\beta_2-\beta_1} + C_2^D \frac{\varphi^{1-\beta_1}(\beta_2-1)}{\beta_2-\beta_1}\frac{\rho\varphi}{c_0\psi} \\
&\quad + C_1^D \frac{\varphi^{0.5-\beta_1}(\beta_2-0.5)}{\beta_2-\beta_1} + C_0^D \frac{\varphi^{-\beta_1}\beta_2}{\beta_2-\beta_1}\frac{\rho\varphi}{c_0\psi}, && (5.94) \\
C_0^M &\equiv \frac{-(c_0\psi-\rho\varphi)\,c_0 a}{r}, && (5.95) \\
C_2^M &\equiv \frac{c_0\psi-\rho\varphi}{\delta}, && (5.96) \\
C_0^D &\equiv \frac{-1}{1-\rho^2}\frac{c_0\psi}{r}, && (5.97) \\
C_1^D &\equiv \frac{1}{1-\rho^2}\frac{\alpha\sqrt{\varphi}}{r-0.5\alpha+0.125\sigma^2}, && (5.98) \\
C_2^D &\equiv \frac{1}{\kappa(1-\rho^2)}\frac{c_0\psi}{\delta}, && (5.99) \\
C_3^D &\equiv \frac{-1}{\kappa(1-\rho^2)}\frac{\alpha\sqrt{\varphi}}{r-1.5\alpha-0.375\sigma^2}. && (5.100)
\end{aligned}
$$

Despite the fact that the expressions for the value of Firm 2 differ across the regimes, calculating the option value of the investment opportunity of Firm 2 represents no additional difficulty comparing to the traditional analysis. It can be shown that the value is negative under the first regime ($x < \varphi$), reaches a peak under the second regime ($\varphi < x < \varkappa$), and tends asymptotically to zero under the third regime ($x > \varkappa$). Therefore, it cannot be optimal for Firm 2 to invest under regimes one and three. Consequently, the value of Firm 2's option to invest can be calculated on the basis of the value-matching and smooth-pasting conditions corresponding to the second regime

$$
\begin{aligned}
A_2 x^{\beta_1} &= B_1 x^{\beta_2} + B_2 x^{\beta_1} + C_0^D + C_1^D x^{0.5} + C_2^D x + C_3^D x^{1.5} - I \\
\beta_1 A_2 x^{\beta_1-1} &= \beta_2 B_1 x^{\beta_2-1} + \beta_1 B_2 x^{\beta_1-1} + 0.5 C_1^D x^{-0.5} + C_2^D + 1.5 C_3^D x^{0.5}.
\end{aligned}
$$

From this it is obtained that the optimal investment threshold of Firm 2, x_2^{**}, is implicitly defined by

$$
\begin{aligned}
&(\beta_1-\beta_2) B_1 (x_2^{**})^{\beta_2} + \beta_1 \left(C_0^D - I\right) + (\beta_1-0.5) C_1^D (x_2^{**})^{0.5} \\
&+ (\beta_1-0.5) C_2^D x_2^{**} x + (\beta_1-0.5) C_3^D (x_2^{**})^{1.5} = 0
\end{aligned}
$$

Furthermore, the value of the investment opportunity of Firm 2 is

$$F_2 = A_2 x^{\beta_1},$$

where

$$A_2 \equiv \max_{q_2} \frac{B_1 (x_2^{**})^{\beta_2} + B_2 (x_2^{**})^{\beta_1} + C_0^D + C_1^D (x_2^{**})^{0.5} + C_2^D x_2^{**} + C_3^D (x_2^{**})^{1.5} - I}{(x_2^{**})^{\beta_1}}.$$

■

Chapter 6

Investment and Debt Renegotiation

6.1 Introduction

One of the consequences of debt financing is its influence on the firm's investment policy. As it is known from Myers (1977), the presence of a risky debt in the company's books leads to underinvestment, i.e. a situation in which some positive NPV projects are foregone. Although the impact of the agency costs of debt on the firm's investment policy has been widely discussed in the literature in qualitative terms, relatively little has been done to analyze the magnitude of these costs. Moreover, the existing contributions yield differing predictions concerning the influence of the renegotiability of debt on the investment policy (cf. Mella-Barral and Perraudin, 1997, and Mauer and Ott, 2000). This chapter uses the contingent claims approach to examine the firm's optimal investment and liquidation policy in the presence of debt financing and the equityholders' option to default and renegotiate the original debt contract.

The main objective of this chapter is to investigate the impact of the renegotiation option, the distribution of bargaining power, and indirect bankruptcy costs on the optimal investment and liquidation policy of the firm. In particular, we are interested in the impact of those debt characteristics on the magnitude of underinvestment problem. Furthermore, the impact of a growth opportunity on the optimal bankruptcy and renegotiation timing is analyzed. In this way it can be investigated whether firms operating in sectors with significant growth opportunities are less likely to file for debt restructuring than their counterparts in more mature industries.

The motivation for this chapter arises also from the ongoing debate on the differences in bankruptcy codes between the European Union and the United States, and the implications of the EU countries bankruptcy law for the firms' operating decisions.[1] Under Chapter 11 of the US bankruptcy law, financially distressed firms suspend their coupon payments and a reorganization plan including writing new debt contracts is implemented. The operations of a firm entering Chapter 11 reorganization usually remain unaffected by the negotiations process, which makes it relatively easy to remain in business if the financial restructuring is successful. In Europe, however, a distressed firm most likely goes under court administration, and its operations are suspended. As a result, the reputation of the firm deteriorates and there is a high chance that liquidation occurs.

In our model debt renegotiation constitutes a good approximation of a private work-out. Under the work-out the initial debt contract is changed so that the equityholders, as the first-best users of assets, are better off running the company than declaring bankruptcy. Moreover, the creditors benefit from the fact that the modified debt contract reduces the probability of bankruptcy. The case of bankruptcy better resembles the European system. A firm that defaults on its debt obligations goes bankrupt and its assets are foreclosed by the creditors. Such foreclosure leads in many cases to inefficiently early liquidation since the value of the assets to the creditors is lower than their value to the original owners. US Chapter 11 remains between these two cases as far as the time allowed for renegotiation is concerned, but it is more shareholder-friendly from the point of view of coupon suspension.

Our analysis also provides insight into the differences between the impact of a bank credit and diffusely held debt on the firm's operating policy. Bank credit is mostly associated with the possibility of debt renegotiation upon financial distress, whereas diffusely held debt makes renegotiation less likely (cf. Bolton and Scharfstein, 1996). The outcome of renegotiating the bank debt depends on the bargaining power of the equityholders vis-à-vis the bank and on both parties' outside options. Usually, the bargaining power of the bank is large, in particular when the firm is relatively small and uses a portfolio of its services. Consequently, the share of the renegotiation surplus received by the bank may be substantial (cf. Hackbarth et al., 2002). When corporate debt is held by dispersed bondholders, the bargaining power of the creditors is usually small and such is the surplus from renegotiation that accrues to the creditors (cf. Hege and Mella-Barral, 2002).

The model is based on the following assumptions. The firm has an investment opportunity to scale up its activities upon incurring an irreversible cost. The cash flow

[1] See, e.g. *The Economist*, 23rd March 2002, 'Up from the ashes', and 7th September 2002, The firms that can't stop falling: Bankruptcy in America.

of the firm follows a random process and the firm has to pay an instantaneous coupon on its debt. Failure to pay the coupon triggers bankruptcy. Following Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000), we assume that the coupon payment can be renegotiated so that bankruptcy is avoided and the surplus is split among the equityholders and creditors.

A number of other models known from the literature can be nested in our framework. Setting the coupon level equal to zero leads to the basic model of Dixit and Pindyck (1996) with the firm scaling up its activities. Excluding the renegotiation possibility reduces our model to Mauer and Ott (2000). By setting the investment cost to infinity and liquidation value to zero, we arrive at Fan and Sundaresan (2000), whereas imposing prohibitively high investment cost in combination with take-it or leave-it offers and no taxes reduces our model to Mella-Barral and Perraudin (1997).

Consequently, this chapter builds upon Mauer and Ott (2000), who analyze the interaction between the leverage and investment option when renegotiation is not allowed for, and both Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000), who focus on strategic debt service.[2] Bankruptcy and renegotiation concepts used in our paper coincide with two polar cases analyzed by Morellec and Francois (2001), who model US Chapter 11 as costly reorganization with a limited duration. The extreme cases in which the renegotiation is not allowed for (duration equal to zero) and can last infinitely long, are analyzed by Leland (1994) and Mella-Barral and Perraudin (1997), respectively.

In this chapter it is shown that the presence of the renegotiation option exacerbates the underinvestment problem. This is due to the fact that the wealth transfer to the debtholders, which occurs upon investment, is higher if the shareholders can default strategically on their original debt contract. The additional underinvestment does not occur if all the bargaining power is given to the creditors. Another implication of the renegotiability of the debt contract is that the problem of inefficient early liquidation can be reduced. This results from the fact that firm remains in the hands of the original shareholders, who can run it most efficiently. However, it cannot be avoided fully, due to the impact of the suboptimal investment policy on the choice of

[2]A far from complete list of references includes Vercammen (2000), analyzing how bankruptcy, triggered by the assets value falling below the face value of the debt, influences investment, Leland and Toft (1996), considering a finite maturity debt with a stationary structure, Anderson and Sundaresan (1996), Mella-Barral (1999), Acharya et al. (2002), and Hackbarth et al. (2002), analyzing debt renegotiation. Related work is presented by Mauer and Triantis (1994), Fischer et al. (1989), and Dangl and Zechner (2001), who focus on the optimal recapitalization policy.

liquidation trigger.[3]

The firm's operating decisions partially influence its optimal debt restructuring policy. The presence of a positive NPV project, in combination with a high debtors' bargaining power, may result in an earlier timing of debt reorganization. However, the firm's liquidation policy determined, among others, by the magnitude of its tangible collateral, does not affect its optimal debt reorganization policy. This finding may be to some extent counterintuitive since the magnitude of collateral influences both the creditors' outside option and the value of the firm. It appears that these two effects cancel out when the debt renegotiation decision is made.

The chapter is organized as follows. In Section 6.2 the basic model of the firm is described, whereas in Section 6.3 debt renegotiation is introduced. Comparative statics and some empirical implications are presented in Section 6.4. Section 6.5 concludes.

6.2 The Basic Model

As a starting point, consider a firm that generates a random cash flow $x(t)$, where $x(t)$ is the time-t realization of a stochastic process. The firm has an option to make an irreversible investment, I, after which it will be entitled to a cash flow, $\theta x(t)$, where $\theta > 1$. Randomness of the cash flow is incorporated in our model by letting x follow the stochastic differential equation

$$dx(t) = \alpha x(t)\,dt + \sigma x(t)\,dw(t), \tag{6.1}$$

where α and σ are constants corresponding to the instantaneous growth rate and the volatility of the project's cash flow, respectively, and $w(t)$ denotes a standard Brownian motion.[4] Let r be the deterministic instantaneous riskless interest rate. It is assumed that all the agents are risk neutral and the drift rate of the cash flow, α, exhibits a shortfall δ below the riskless rate, i.e. $\alpha = r - \delta$.

We begin the analysis with the simple case of an all-equity financed firm. In Subsection 6.2.1 the optimal liquidation and investment decisions of the unlevered firm are investigated. Subsequently, we introduce a mixed capital structure. The presence

[3] These results show the limitations of the two-period model of Myers (1977). In his case, the investment and the liquidation decisions are made simultaneously so that the possibility of renegotiation enhances investment and reduces liquidation. In the continuous-time framework of the present model, renegotiation reduces inefficient liquidation in bad states of nature but (anticipated by the shareholders in good states of nature) also impairs the investment activity.

[4] We do not impose a constant positive marginal cost to avoid the need of tackling the issue of limited liability of the creditors in some states of nature.

of debt results in a positive probability of bankruptcy and the shareholders' option to default. The optimal bankruptcy trigger and the impact of bankruptcy on the investment decision are analyzed in Subsection 6.2.2.

6.2.1 All-Equity Financing

The cash flow of the firm is subject to taxation and the corporate tax rate is τ. No other taxes are assumed. The firm may always decide to sell its assets and liquidate. Define an indicator $i \in \{0, 1\}$ to be equal to 0 if the investment has not yet been made, and 1 in the opposite case. Liquidation entails receiving a lump sum payment, γ_i, in return for the present value of the firm's expected future cash flow.

The standard no-arbitrage argument (cf. Dixit and Pindyck, 1996) implies that any claim, F, contingent on the process x and having an instantaneous payoff $Bx + C$, where $B, C \in \mathbb{R}$, satisfies the ordinary differential equation

$$rF = (r - \delta)\, x \frac{\partial F}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} + Bx + C. \tag{6.2}$$

For the value of the unlevered firm, V_i, parameters B and C are $\theta^i (1 - \tau)$ and zero, respectively. The general solution to (6.2) is of the form

$$F = \frac{B}{\delta} + \frac{C}{r} + M_1 x^{\beta_1} + M_2 x^{\beta_2}, \tag{6.3}$$

where β_1 (β_2) is the positive (negative) root of the characteristic equation

$$\frac{1}{2}\sigma^2 \beta (\beta - 1) + (r - \delta)\beta - r = 0, \tag{6.4}$$

and M_1 and M_2 are constants determined from boundary conditions specific to the type of the contingent claim.

Let us first consider the value of the firm after the investment has been made. The only decision that is to be made by the shareholders at each instant is whether to continue running the firm or to liquidate it. The value of the firm after the investment, V_1, equals

$$V_1 = \begin{cases} \gamma_1 & x < x_1^L, \\ \frac{x\theta(1-\tau)}{\delta} + \left(\gamma_1 - \frac{x_1^L \theta(1-\tau)}{\delta}\right)\left(\frac{x}{x_1^L}\right)^{\beta_2} & x \geq x_1^L, \end{cases} \tag{6.5}$$

where x_1^L is the optimal liquidation threshold. The value of the firm prior to liquidation equals the present value of earnings in perpetuity and the value of the option to liquidate. Analogous to, e.g., Dixit and Pindyck (1996), Ch. 6, the solution to the liquidation problem equals

$$x_1^L = \frac{-\beta_2}{1 - \beta_2} \frac{\gamma_1 \delta}{\theta (1 - \tau)}. \tag{6.6}$$

Before the investment, the strategy space of the firm consists of the three following elements

$$\{continue, liquidate, invest\}.$$

Liquidation occurs when earnings fall below a certain trigger, whereas investment takes place when earnings are sufficiently high. This results in a double-barrier problem where the optimal investment threshold and liquidation trigger before the investment have to be found simultaneously. The optimal investment and liquidation policies are found by solving ODE (6.2) for V_0 subject to

$$V_0(x^*) = V_1(x^*) - I, \tag{6.7}$$

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=x^*} = \left.\frac{\partial V_1}{\partial x}\right|_{x=x^*}, \tag{6.8}$$

$$V_0\left(x_0^L\right) = \gamma_0, \tag{6.9}$$

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=x_0^L} = 0, \tag{6.10}$$

where x_0^L denotes the before-investment liquidation trigger and x^* is the optimal investment threshold.

6.2.2 Debt and Equity Financing

Now, let us assume that the firm is partially financed with debt. The debt contract is associated with a perpetual coupon stream b, which is tax deductible. The par value of debt is assumed to equal b/r. Because of the limited liability of equityholders, in some states of nature it is optimal for them to default on debt obligations. A failure to pay the contracted coupon results in bankruptcy upon which creditors take over the firm. We impose the absolute priority rule (APR) so the equityholders receive nothing in the event of bankruptcy as long as the claim of debtholders is not fully satisfied.[5]

Since we are interested in the optimal debt restructuring policy, we assume an *endogenous bankruptcy* procedure. Such a procedure stipulates that equityholders declare bankruptcy so to maximize the value of equity. In such a case it is possible that for low cash flow realizations, the equityholders may actually inject cash to the firm. This modeling approach is consistent with, for instance, Leland (1994), Mella-Barral

[5]Evidence presented by Franks and Torous (1989) indicates significant departures from the absolute priority rule in many bankruptcy settlements. Our assumption has been introduced for simplicity. Waiving this assumption would result in bankruptcy occuring for higher realizations of cash flow than with APR.

and Perraudin (1997), and Acharya and Carpenter (2002). It differs from the models of *exogenous bankruptcy,* which is triggered by the asset value falling below a prespecified level. For instance, in Merton (1974) bankruptcy occurs when the terminal value of assets is lower than the debt principal, whereas in Black and Cox (1976) it is triggered when the level of assets hits a deterministic barrier.[6] Yet another approach is taken by Kim, Ramaswamy and Sundaresan (1993), who assume that bankruptcy is triggered by illiquidity, i.e. when net profits fall negative.

The value of the firm operated by the creditors after bankruptcy is a function of the cash flow from output, denoted by $R_i(x)$. Following Fan and Sundaresan (2000), we abstain from analyzing the issue of dynamic recapitalization. As a consequence, the firm run by the creditors remains all-equity financed for ever, and the tax shield is irreversibly lost upon bankruptcy. Moreover, if bankruptcy occurs prior to the investment, the growth option expires unexercised. Finally, as in Mella-Barral and Perraudin (1997), it is assumed that the debtholders will run the firm less efficiently, so that the cash flow generated by the firm in the hands of the creditors equals $\rho\theta^i(1-\tau)x$, where $\rho \in (0,1)$.[7] The latter assumption reflects, among others, superior ability of existing management to run the firm and distraction of management upon bankruptcy, combined with impaired ability to contract and suboptimal investment in firm-specific human capital (see Hackbarth et al., 2002).

Since the value of the firm, V_i, its equity, E_i, debt, D_i, and creditors' reservation value, R_i, are securities contingent on the earnings process, x, they all satisfy ODE (6.2). The values of constants B and C defining their instantaneous payoffs are depicted in Table 6.1.

	V_i	E_i	D_i	R_i
B	$\theta^i(1-\tau)$	$\theta^i(1-\tau)$	–	$\rho\theta^i(1-\tau)$
C	$b\tau$	$-b(1-\tau)$	b	–

Table 6.1: Instantaneous payoffs associated with the value of the firm, V_i, equity, E_i, debt, D_i, and the creditors' outside option, R_i.

First, we determine the value of the firm run by the creditors, R_i. It is obtained by solving (6.2) with value-matching and smooth-pasting conditions reflecting the fact

[6]See Bielecki and Rutkowski (2002) for a detailed reference list concerning related safety covenants.

[7]Hege and Mella-Barral (2000) develop a model in which the firm in the hands of new owners has exactly the same set options concerning new debt issues and subsequent reorganizations as under the management of incumbents. The assumption about proportional reduction of cash flow upon bankruptcy remains unchanged.

that the only option available to the firm run by the creditors is to liquidate. It holds that R_i is equal to

$$R_i = \begin{cases} \gamma_i & x < x_1^{LR}, \\ \frac{\rho x \theta^i (1-\tau)}{\delta} + \left(\gamma_i - \frac{\rho x_i^{LR} \theta^i (1-\tau)}{\delta}\right) \left(\frac{x}{x_i^{LR}}\right)^{\beta_2} & x \geq x_1^{LR}, \end{cases} \tag{6.11}$$

where

$$x_i^{LR} = \frac{-\beta_2}{1-\beta_2} \frac{\gamma_i \delta}{\rho \theta^i (1-\tau)} \tag{6.12}$$

is the optimal liquidation trigger of the creditors running the firm.

We determine the value of the firm and the optimal investment threshold by first considering the case in which the firm has already invested. We solve (6.2) for the firm's equity, E_1, and debt, D_1 with value-matching conditions at the bankruptcy trigger that correspond to the absolute priority rule. The value of the firm's equity, E_1, and debt, D_1, after the investment is made, can be described as follows

$$E_1 = \begin{cases} 0 & x < x_1^B, \\ (1-\tau)\left[\left(\frac{x\theta}{\delta} - \frac{b}{r}\right) - \left(\frac{x_1^B \theta}{\delta} - \frac{b}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2}\right] & x \geq x_1^B, \end{cases} , \tag{6.13}$$

and

$$D_1 = \begin{cases} R_1(x) & x < x_1^B, \\ \frac{b}{r} + \left(R_1\left(x_1^B\right) - \frac{b}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2} & x \geq x_1^B. \end{cases} , \tag{6.14}$$

The optimal equityholders' bankruptcy trigger is determined using the smooth-pasting condition for the equity value upon bankruptcy and equals

$$x_1^B = \frac{-\beta_2}{1-\beta_2} \frac{b\delta}{r\theta}. \tag{6.15}$$

The value of the firm equals

$$V_1 = E_1 + D_1 = \tag{6.16}$$
$$= \begin{cases} R_1(x) & x < x_1^B, \\ \frac{x\theta(1-\tau)}{\delta} + \frac{b\tau}{r} + \left(R_1\left(x_1^B\right) - \frac{x_1^B \theta (1-\tau)}{\delta} - \frac{b\tau}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2} & x \geq x_1^B, \end{cases} .$$

Figure 6.1 depicts the value of the firm and the claims written on it after the investment is made. The value of the firm approaches the present value of earnings increased by the tax shield for a high earnings level. For lower realizations of the earnings process, the concavity of the firm's value increases, which reflects the value of the equityholders' option to default. At the bankruptcy trigger, x_1^B, the firm's value function exhibits a kink which reflects the fact that bankruptcy is neither optimal nor reversible as seen from the perspective of the firm value maximization.[8] The value of equity approaches the present value of earnings minus the after-tax coupon payment. For lower realizations of earnings, its convexity increases due to the limited liability effect. At the equityholders' optimal bankruptcy trigger, the value of equity smooth-pastes to zero. Finally, the value of debt tends to its riskless valuation for high realizations of the earnings process, and equals the firm's value at the bankruptcy trigger.

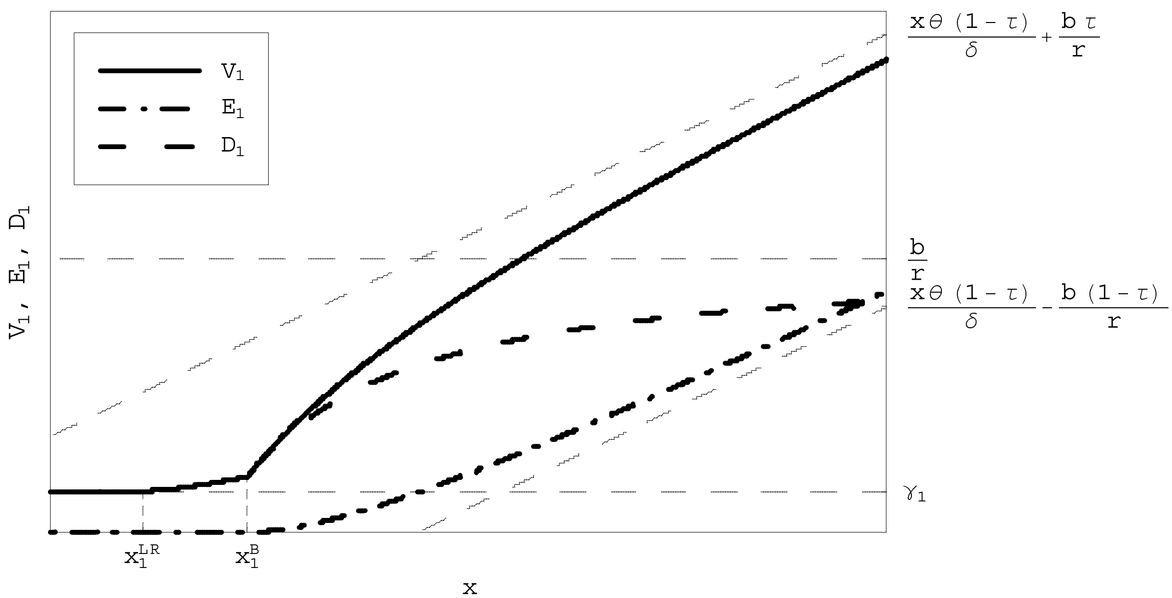

Figure 6.1: Valuation of the firm, V_1, its debt, D_1, and equity, E_1, with bankruptcy occurring upon default.

Equipped with the value of the firm after the investment has been made, we are ready to determine the optimal exercise policy of the investment option. We calculate both the firm value-maximizing and the equity value-maximizing investment thresholds. Here, we use the framework of Mauer and Ott (2000) and correct two of their boundary conditions[9]. We start by observing that the value of the firm as well as

[8] If bankruptcy was optimal then the value function would be differentiable at x_1^B as a result of the smooth-pasting condition. Reversibility would imply the continuity of the first derivative of the value function at x_1^B due to the no-arbitrage condition (for details see Dumas, 1991).

[9] First, we replace the investment bankruptcy trigger in condition (9.20a) on p. 159 of Mauer and Ott (2000) that ignores the impact of the investment opportunity, by the one determined optimally. Second, we add a smooth-pasting condition necessary for calculating the optimal trigger in the presence of the investment opportunity.

its equity and debt before the investment, V_0, E_0, and, D_0, respectively, satisfy ODE (6.2). Therefore the corresponding values can be written as

$$V_0 = \frac{x(1-\tau)}{r-\alpha} + \frac{b\tau}{r} + K_0 x^{\beta_1} + B_0 x^{\beta_2}, \tag{6.17}$$

$$E_0 = \frac{x(1-\tau)}{r-\alpha} - \frac{b(1-\tau)}{r} + A_{01} x^{\beta_1} + A_{02} x^{\beta_2}, \tag{6.18}$$

$$D_0 = V_0 - E_0. \tag{6.19}$$

The component $K_0 x^{\beta_1}$ is the value of the growth option and $B_0 x^{\beta_2}$ reflects the value lost due to the potential future bankruptcy. $A_{01} x^{\beta_1}$ is the fraction of the value of the investment option that accrues to the equityholders and $A_{01} x^{\beta_2}$ is the equityholders' option to default. The constants K_0, B_0, A_{01}, A_{02}, the optimal bankruptcy trigger x_0^B and the firm value-maximizing investment threshold, x^*, are uniquely determined by the system of equations

$$V_0(x^*) = V_1(x^*) - I, \tag{6.20}$$

$$E_0(x^*) = E_1(x^*) - I, \tag{6.21}$$

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=x^*} = \left.\frac{\partial V_1}{\partial x}\right|_{x=x^*}, \tag{6.22}$$

$$E_0\left(x_0^B\right) = 0, \tag{6.23}$$

$$\left.\frac{\partial E_0}{\partial x}\right|_{x=x_0^B} = 0, \tag{6.24}$$

$$V_0\left(x_0^B\right) = R_0\left(x_0^B\right). \tag{6.25}$$

Equations (6.20) and (6.21) are the value-matching conditions ensuring the continuity of the value of the firm as well as of its equity and debt (by $D_0 = V_0 - E_0$) at the optimal investment threshold. (6.22) is the smooth-pasting condition associated with the firm value-maximizing property of the investment threshold. (6.23) and (6.24) are the value matching and smooth-pasting conditions for the equityholders at the bankruptcy trigger. (6.24) ensures that the bankruptcy trigger is chosen such that the value of equity is maximized. (6.25) is the value-matching condition for the firm at the bankruptcy trigger. Its RHS implies that the investment option expires upon bankruptcy.

It holds that x_1^B is lower than x_0^B. This is due to the fact that the cash flow is higher after the investment has been undertaken and the present value of the incremental cash flow from investment is worth more than the option to acquire it.

The debtholders benefit from undertaking the investment project in two ways. First, the probability of bankruptcy decreases so that the present value of the expected coupon stream is higher. Second, the outside option of the debtholders becomes more valuable. After bankruptcy is declared, the debtholders will run a firm that generates a higher cash flow than prior to the investment.

Since in most cases it is impossible to implement an investment schedule that maximizes the value of the firm, we compare the first-best solution with the second-best that maximizes the value of the equity.[10] The investment decision associated with maximizing the value of the equity requires replacing (6.22) by

$$\left.\frac{\partial E_0}{\partial x}\right|_{x=x^*} = \left.\frac{\partial E_1}{\partial x}\right|_{x=x^*} \tag{6.26}$$

Constants K_0, B_0, A_{01}, A_{02}, and triggers x_0^B and x^* are completely described by the system of equations (6.20)-(6.25) with (6.22) replaced by (6.26). The optimal investment threshold is in this case higher since the wealth transfer to debtholders occurring upon investment causes that the equityholders invest later than the first-best solution would indicate. Furthermore, the optimal bankruptcy trigger is lower under the second-best investment rule than under the first-best policy. The reason for such a relationship is that under the second-best investment rule the value of the investment opportunity for the equityholders is higher than under the first-best policy. Therefore, at any x the continuation value is higher under the second-best that under first-best. As a consequence, the continuation value under the second-best smooth-pastes to the stopping value (equal to zero) at a lower x than under the first-best.[11]

6.3 Debt Renegotiation

The divergence between the optimal liquidation trigger of the firm and the equityholders' endogenous bankruptcy trigger implies that there is a scope for debt renegotiation. The scope for renegotiation stems from the fact that upon bankruptcy the three following components of the firm's value are irreversibly lost. First, the investment opportunity ceases to exist when the creditors take over the company. Second, upon bankruptcy the firm forgoes the present value of the tax shield. Finally, creditors run

[10] In general, it is not in the interest of shareholders to align perfectly the incentives of the managers with their own in the presence of debt (cf. Brander and Poitevin, 1992, and John and John, 1993). The optimal compensation scheme should be constructed in such a way that the combined agency costs of equity and debt are minimized. However, in this paper's framework with a single owner-manager the first-best solution is not achievable.

[11] Mauer and Ott (2000) fail to incorporate this relationship in their model.

the firm less efficiently so the instantaneous earnings of the firm are reduced by fraction $(1-\rho)$ of the current cash flow.

In this section we analyze the impact of debt renegotiation on the investment policy and the value of the firm. We assume that the renegotiation process has a form of Nash bargaining in which the bargaining power is split between the two types of the firm's stakeholders (cf. Perraudin and Psillaki, 1999, and Fan and Sundaresan, 2000). The distribution of the bargaining power is given exogenously and is described by parameter $\eta \in [0,1]$, where a high η is associated with high bargaining power of the shareholders. The take-it or leave-it offers made either by the shareholders or by the creditors (as in Mella-Barral and Perraudin, 1997) are limiting cases of the Nash bargaining solution. Consequently, they correspond to the cases where $\eta = 1$ and $\eta = 0$, respectively. The former situation can be related to large corporations that are likely to be aggressive in negotiations, whereas the latter corresponds to small and young firms that use a portfolio of the bank's services.

The remainder of this section consists of two parts. In Subsection 6.3.1, we calculate the value of the firm as a function of the equityholders' renegotiation trigger and determine the optimal sharing rule. In Subsection 6.3.2 we simultaneously derive the values of debt and equity, and determine the optimal equityholders' renegotiation and investment policies and the firm's optimal liquidation rule.

6.3.1 Nash Bargaining Solution

Debt renegotiation has a form of a strategic debt service, i.e. it is associated with a lower than contractual coupon payment. The new coupon payment schedule has to satisfy both the shareholders' and debtholders' participation constraints associated with the renegotiation process. We follow Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) in assuming that the coupon is a function of the current cash flow. Such an approach allows for avoiding path-dependency, which leads to analytical intractability.[12] Repeated renegotiation is possible and occurs in equilibrium with positive probability.

It is assumed that bargaining power is distributed among the shareholders and the creditors which results in the surplus from renegotiation being distributed with a certain proportion among the two groups. Moreover, we impose the assumption made by Fan and Sundaresan (2000) that during the renegotiation process the tax shield is temporarily suspended. As soon as the cash flow from operation recovers and

[12] Hege and Mella-Barral (2000) assume that a once reduced coupon cannot be increased.

debtholders are receiving coupon b again, the tax shield is restored.[13,14] Finally, it is assumed that γ_i, $i \in \{0,1\}$, satisfies

$$\gamma_i < \frac{b}{r}\rho\,(1-\tau)\,. \tag{6.27}$$

Condition (6.27) implies that the liquidation value is small enough so that it will not be optimal for the creditors to liquidate the firm immediately after the original debt contract is infringed.[15]

First, we determine the value of the firm, V_i^{NB}, as a function of the optimal renegotiation trigger. Since the present value of the tax shield depends on the moment of commencing the debt renegotiation, the value of the firm as a whole depends on the renegotiation trigger. V_i^{NB} can be expressed as the sum of the present value of cash flow, tax shield, TS_i, growth option (for $i=0$), $K_0 x^{\beta_1}$, and liquidation option, $L_i x^{\beta_2}$:

$$V_i^{NB} = \frac{\theta^i x\,(1-\tau)}{\delta} + TS_i + (1-i)\,K_0 x^{\beta_1} + L_i x^{\beta_2}. \tag{6.28}$$

In the Appendix we show that for a given choice of the renegotiation trigger, x_i^{NB}, the tax shield, TS_i, equals

$$TS_i = \begin{cases} \frac{b\tau}{r}\frac{-\beta_2}{\beta_1-\beta_2}\left(\frac{x}{x_i^{NB}}\right)^{\beta_1} & x \le x_i^{NB}, \\ \frac{b\tau}{r}\left(1-\frac{\beta_1}{\beta_1-\beta_2}\left(\frac{x}{x_i^{NB}}\right)^{\beta_2}\right) & x > x_i^{NB}. \end{cases} \tag{6.29}$$

The expressions on the right-hand side have an immediate interpretation. They are the products of the present value of the perpetual tax shield, $\frac{b\tau}{r}$, a stochastic discount factor associated with hitting the renegotiation boundary, $\left(x/x_i^{NB}\right)^{\beta_2}$, and a fraction $\frac{\beta_1}{\beta_1-\beta_2}$ that reflects the fact that the tax shield operates only in the renegotiation region.

The constants K_0 and L_0 will be determined later, i.e. at the time of solving the firm's investment problem. The constant L_1 is given by

$$L_1 = \left(\gamma_1 - \frac{\theta\,(1-\tau)\,x_1^{LN}}{\delta} - \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(x_1^{LN}\right)^{-\beta_2}, \tag{6.30}$$

[13] According to Fan and Sundaresan (2000), p. 1072, the fact of temporary tax shield suspension in the renegotiation region "may be interpreted as debtholders agree to forgive some debt and the Internal Revenue Service (IRS) suspends tax benefits until contractual payments are resumed." An alternative approach is proposed by Hege and Mella-Barral (2000), and Hackbarth et al. (2002), who assume that the magnitude of the tax shield corresponds to the prevailing coupon payment.

[14] Fan and Sundaresan (2000) claim (footnote 12, p. 1073) that the optimal renegotiation trigger is lower when the tax benefits accrue during the strategic debt service. In fact, the optimal renegotiation trigger is higher when the tax shield is not suspended since the value of starting renegotiation is higher in such a situation. Therefore, our main results would be even stronger if we did not impose suspension of the tax shield. See also footnote 17.

[15] Since $\rho\,(1-\tau) < 1$, condition (6.27) also implies that the debt is risky.

where x_1^{LN} is the after-investment liquidation trigger. The latter is implicitly given by

$$\frac{1-\beta_2}{-\beta_2}\frac{x_1^{LN}\theta(1-\tau)}{\delta}+\frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}=\gamma_1 \tag{6.31}$$

(for derivation of (6.30) and (6.31) see the Appendix). It can be directly seen that in the absence of taxes, (6.31) reduces to (6.6) with $\tau = 0$. Upon comparing (6.31) with (6.6) it can be concluded that x_1^{LN} is lower than x_1^L as long as x_1^{NB} is finite. Consequently, in the presence of taxes the liquidation option is exercised later when the firm is partially financed with debt and renegotiation is possible.

Having determined the value of the firm, we are ready to calculate the solution to the bargaining game. Let φ_i^* be the outcome of the Nash bargaining process being equal to the fraction of the firm received by the shareholders. Given that the value of the firm is described by (6.28), the shareholders receive $\varphi_1^* V_i^{NB}$ and the debtholders get $(1-\varphi_i^*)\,V_i^{NB}$. The outside options (the off-renegotiation payoffs) of equityholders and debtholders are zero and R_i, respectively. Consequently, the solution to the bargaining game can be written as follows:[16]

$$\begin{aligned}\varphi_i^* &= \arg\max_{\varphi}\left[\left(\varphi V_i^{NB}\right)^{\eta}\left((1-\varphi)\,V_i^{NB}-R_i\right)^{1-\eta}\right]\\ &= \eta\frac{V_i^{NB}-R_i}{V_i^{NB}}.\end{aligned} \tag{6.32}$$

From (6.32) it can be concluded that the fraction of the firm received by the equityholders in the renegotiation process critically depends on the creditors' outside option, R_i. If the creditors' outside option equals zero (i.e. if $\gamma_i = \rho = 0$), shareholders receive the fraction of the firm equal to their bargaining power coefficient. In the opposite case, i.e. when creditors outside option equals the value of the firm ($\rho = 1$, $\tau = 0$, and $i = 1$), shareholders receive nothing in the renegotiation process. Moreover, the optimal sharing rule again depends on the amount of the current cash flow, x.

6.3.2 Equity Valuation and Optimal Renegotiation Policy

Having calculated the value of the firm and the optimal sharing rule given the shareholders' renegotiation trigger, we now derive the optimal renegotiation policy.

[16]In the formulation of bargaining problem we follow Perraudin and Psillaki (1999), and Fan and Sundaresan (2000), (where for $\eta = 0.5$ the game is the one of Rubinstein, 1982, with $\Delta t \to 0$) who impose this multiplicative form of the objective function. The drawback of an alternative, additive formulation is that it yields bang-bang solutions.

We begin by deriving the formulae for the securities values. Subsequently, we simultaneously determine the optimal renegotiation and investment policy by maximizing the value of the equity and the optimal liquidation policy by maximizing the value of the firm.

Given the value of the firm as a function of the underlying cash flow, we are ready to determine the after-investment value of equity, E_1^{NB}, and to find the optimal renegotiation trigger. The value of equity is determined by solving ODE (6.2) with an appropriate value-matching condition associated with the renegotiation trigger, x_1^{NB}. Consequently, E_1^{NB} equals

$$E_1^{NB} = \begin{cases} \eta\left(V_1^{NB}(x) - R_1(x)\right) \qquad \left(= \varphi^* V_1^{NB}(x)\right) & x \le x_1^{NB}, \\ \frac{\theta x(1-\tau)}{\delta} - \frac{b(1-\tau)}{r} + \left(\frac{x}{x_1^{NB}}\right)^{\beta_2} \times & \\ \left(\eta\left(V_1^{NB}\left(x_1^{NB}\right) - R_1\left(x_1^{NB}\right)\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right) & x > x_1^{NB}. \end{cases} \tag{6.33}$$

Applying the smooth-pasting condition allows for finding the optimal renegotiation trigger, x_1^{NB} (cf. the Appendix), which is equal to

$$x_1^{NB} = \frac{-\beta_2}{1-\beta_2} \frac{b\left(1-\tau+\eta\tau\right)\delta}{\left(1-\eta\left(1-\rho\right)\right)\theta\left(1-\tau\right)r}, \tag{6.34}$$

It is straightforward to notice that the trigger x_1^{NB} increases with taxes. This is because the effect of taxes on the cash flow that accrues to the firm's shareholders dominates the effect of a temporarily suspended tax shield. Therefore, despite the fact that the tax shield is suspended under renegotiation, the shareholders prefer an earlier debt reorganization.[17]

From (6.34) it can be seen that the renegotiation trigger is independent from taxes only if η is equal to zero. This is equivalent with the creditors holding the entire bargaining power. In such a case the optimal renegotiation trigger equals the optimal bankruptcy trigger and the latter has already been shown to be independent of taxes (cf. (6.15)). Moreover, the optimal renegotiation trigger does not depend on the liquidation value γ_1. This results from the fact that the change of the instantaneous

[17] The impact of taxes on cash flow is not taken into account while analyzing the optimal bankuptcy trigger in Leland (1994) (see footnote 22 therein concerning the *ceteris paribus* assumption) and the renegotiation trigger in Fan and Sundaresan (2000) (see Assumption (6), p. 1061 therein). Consequently, the optimal renegotiation trigger in Fan and Sundaresan (2000) is reported to decrease in taxes since only the effect of the increasing tax shield is taken into account. Moreover, contrary to the result of Fan and Sundaresan (2000) obtained without the liquidation option, introducing taxes does not always imply that shareholders receive a higher fraction of the firm in the renegotiation process.

payoff when the renegotiation commences is not influenced by the collateral.[18]

The after-investment value of the firm, V_1^{NB}, can be determined now by substituting (6.29) and (6.30) into (6.28). Having also calculated the value of equity, E_1^{NB}, and knowing the value of R_1 (see (6.11)), we are able to provide the value of its debt, D_1^{NB}. It holds that

$$D_1^{NB} = \begin{cases} (1-\eta) V_1^{NB} + \eta R_1 & x \leq x_1^{NB}, \\ \frac{b}{r} + \left((1-\eta) V_1^{NB} + \eta R_1 - \frac{b}{r}\right)\left(\frac{x}{x_1^{NB}}\right)^{\beta_2} & x > x_1^{NB}. \end{cases} \tag{6.35}$$

Figure 6.2 depicts the value of the firm and the claims written on it after the investment is made and there exists a possibility of renegotiation. The value of the firm remains within the band bounded from below by the present value of the cash flow and from above by the present value of the cash flow and of the perpetual tax shield. The value of the equity behaves as in the case without renegotiation with the only difference being that the option to default is replaced by a more highly valued option to renegotiate. The value of debt tends to its riskless valuation for high levels of cash flow as in the previous case, and it equals a fraction of the firm value, $(1-\varphi^*) V_1^{NB}$, for its low levels.

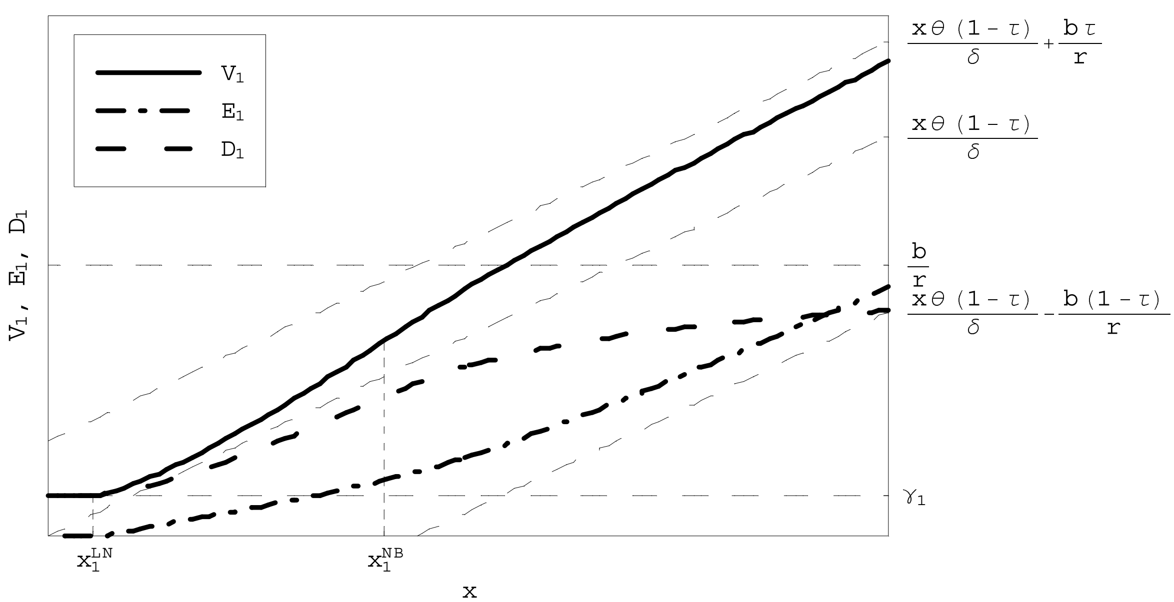

Figure 6.2: Valuation of the firm, V_1, its debt, D_1, and equity, E_1, with the shareholder's option to renegotiate the debt.

[18]If γ_1 was high enough so that $R_1\left(x_1^{NB}\right) = \gamma_1$, then the renegotiation trigger would depend on γ_1. However, this is ruled out by assumption (6.27).

At the optimal equityholders' renegotiation trigger, the value of all the claims remain differentiable. For the equity it is the result of the smooth-pasting condition that guarantees optimality of the trigger. For the value of the firm and its debt it is a no-arbitrage condition. Since the renegotiation process is reversible, i.e. the equityholders will restore the original coupon flow, b, as soon as the earnings process again exceeds the critical threshold x_1^{NB}, the first-order derivative of the value of all the claims must be continuous. As a consequence, the value of the firm and of its debt does not exhibit kinks at the renegotiation trigger, x_1^{NB}.

In order to determine the optimal investment, renegotiation and liquidation triggers and the value of the corporate securities, we first observe that the value of equity before investment, E_0^{NB}, can be expressed as

$$E_0^{NB} = \frac{x\left(1-\tau\right)}{\delta} - \frac{b\left(1-\tau\right)}{r} + A_{01}x^{\beta_1} + A_{02}x^{\beta_2}. \tag{6.36}$$

$A_{01}x^{\beta_1}$ and $A_{02}x^{\beta_2}$ are the components of the value of equity associated with the investment and debt renegotiation option, respectively. Using equation (6.36) for E_0^{NB}, (6.33) for E_1^{NB}, (6.28) with $i = 0$ and $i = 1$ for V_0^{NB} and V_1^{NB}, respectively, and (6.11) for R_0, we obtain the optimal triggers and securities' values by solving the following system of equations

$$V_0^{NB}\left(x^*\right) = V_1^{NB}\left(x^*\right) - I, \tag{6.37}$$

$$E_0^{NB}\left(x^*\right) = E_1^{NB}\left(x^*\right) - I, \tag{6.38}$$

$$\left.\frac{\partial V_0^{NB}}{\partial x}\right|_{x=x^*} = \left.\frac{\partial V_1^{NB}}{\partial x}\right|_{x=x^*}, \tag{6.39}$$

$$E_0^{NB}\left(x_0^{NB}\right) = \eta\left(V_0^{NB}\left(x_0^{NB}\right) - R_0\left(x_0^{NB}\right)\right), \tag{6.40}$$

$$\left.\frac{\partial E_0^{NB}}{\partial x}\right|_{x=x_0^{NB}} = \eta\left.\frac{\partial\left(V_0^{NB} - R_0\right)}{\partial x}\right|_{x=x_0^{NB}}, \tag{6.41}$$

$$V_0^{NB}\left(x_0^{LN}\right) = \gamma_0, \tag{6.42}$$

$$\left.\frac{\partial V_0^{NB}}{\partial x}\right|_{x=x_0^{LN}} = 0. \tag{6.43}$$

Equations (6.37) and (6.38) are the value-matching conditions required for the value of the firm and equity to be continuous at the optimal investment threshold, x^*. The smooth-pasting condition (6.39) guarantees the optimality of the investment threshold, x^*. Conditions (6.40) and (6.41) are the value-matching and smooth-pasting conditions associated with the optimal renegotiation trigger chosen by the equityholders, respectively. The RHS of (6.40) is the share of the value of the firm received by the shareholders upon renegotiation. (6.42) is the value matching condition reflecting the value of the firm at the liquidation trigger. Finally, (6.43) is the smooth-pasting condition for the value of the firm at the closure point.

Now, we are ready to state the following proposition.

Proposition 6.1 *The optimal investment threshold, x^*, renegotiation trigger, x_0^{NB}, and liquidation trigger, x_0^{LN}, can be obtained by simultaneously solving the following equations*

$$\frac{(\theta-1)(1-\tau)}{\delta} - \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{rx^*}\left(\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - \left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2}\right)$$
$$+\beta_2 (L_1 - L_0)(x^*)^{\beta_2-1} - \beta_1 K_0 (x^*)^{\beta_1-1} = 0, \tag{6.44}$$
$$\frac{1-\beta_2}{-\beta_2}\frac{x_0^{NB}(1-\tau)(1-\eta(1-\rho))}{\delta} - \frac{b}{r}(1-\tau+\eta\tau)$$
$$-\frac{1-\beta_2}{-\beta_2}\left(\eta K_0\left(x_0^{NB}\right) - A_{01}\left(x_0^{NB}\right)\right) = 0, \tag{6.45}$$
$$\frac{1-\tau}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{rx_0^{LN}}\left(\frac{x_0^{LN}}{x_0^{NB}}\right)^{\beta_1} + \beta_1 K_0\left(x_0^{LN}\right)^{\beta_1-1} = 0. \tag{6.46}$$

The constants K_0, L_0, A_{01}, and A_{02} are defined by equations (6.65) and (6.66) in the Appendix.

Proof. See the Appendix. ■

Unfortunately, an analytical solution to the above system of equations cannot be obtained. Therefore, we rely on numerical methods. Figure 6.3 depicts the values of the firm, its debt and its equity, in the presence of the investment and renegotiation options.

The boundary conditions for the equity value-maximizing investment policy are the same as for the firm value-maximizing policy, except for that

$$\left.\frac{\partial E_0^{NB}}{\partial x}\right|_{x=x^*} = \left.\frac{\partial E_1^{NB}}{\partial x}\right|_{x=x^*} \tag{6.47}$$

replaces (6.39). This leads to the following proposition.

Proposition 6.2 *The shareholders' value-maximizing investment threshold is obtained by solving simultaneously equations (6.45), (6.46), and*

$$\frac{(\theta-1)(1-\tau)}{\delta} + \beta_2 (A_{12} - A_{02})(x^*)^{\beta_2-1} - \beta_1 A_{01} (x^*)^{\beta_1-1} = 0. \tag{6.48}$$

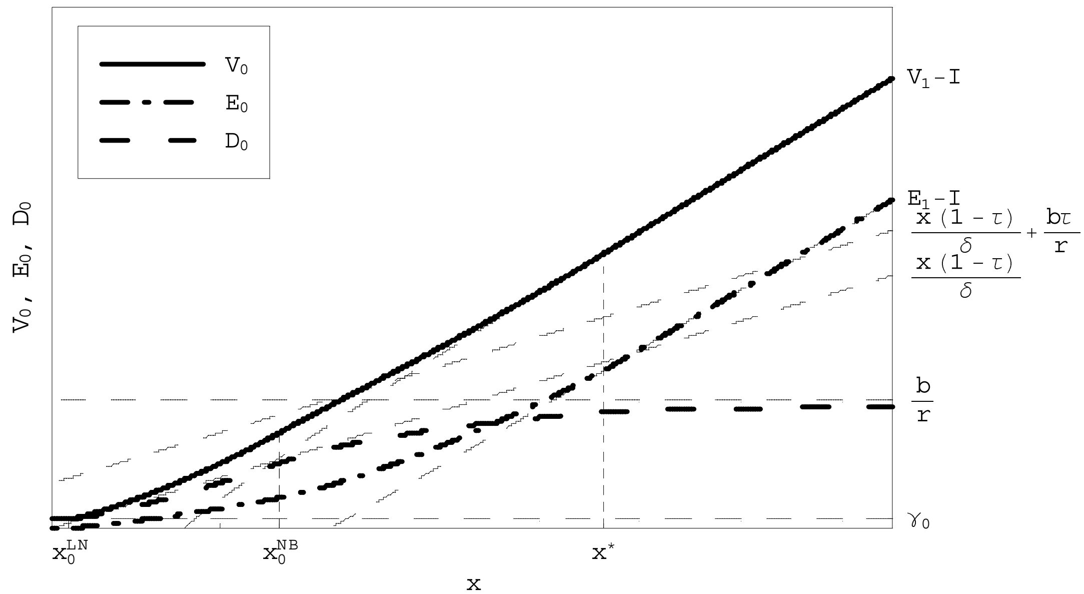

Figure 6.3: Valuation of the firm, V_0, its equity, E_0, and debt, D_0, with the shareholder's option to renegotiate the debt and the option to invest exercised at the firm value-maximizing level of earnings.

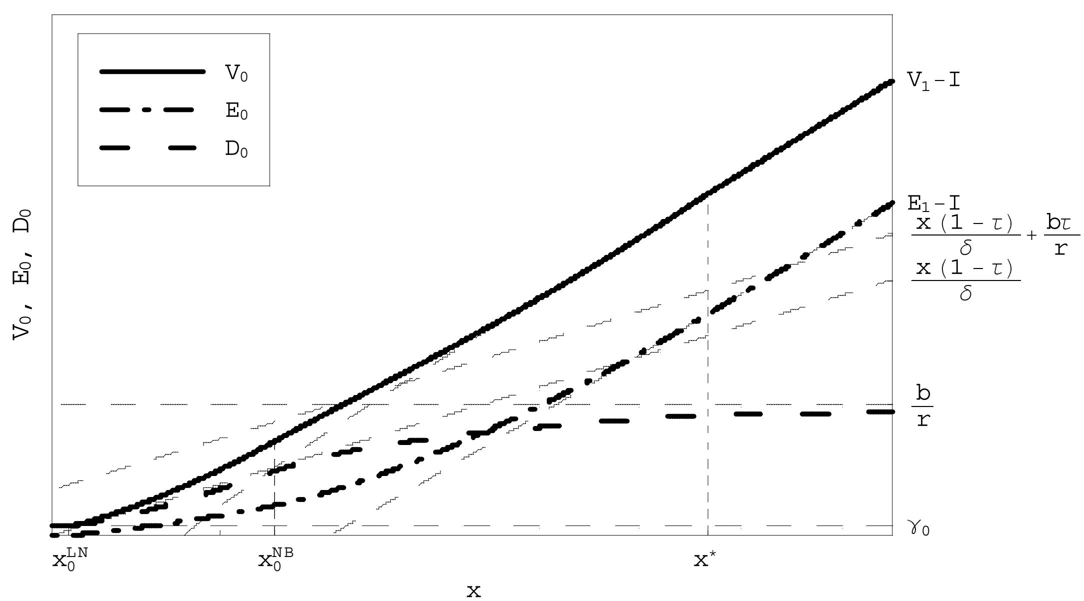

Figure 6.4: Valuation of the firm, V_0, its equity, E_0, and debt, D_0, with the shareholder's option to renegotiate the debt and the option to invest exercised at the equity value-maximizing level of earnings.

Constants A_{01}, A_{02} and A_{12} are defined by equations (6.66) and (6.67) in the Appendix.

Proof. See the Appendix. ■

Figure 6.4 depicts the value of the firm, and its debt and equity in the presence of the investment and renegotiation options when the second-best investment policy is implemented. Now, it is the value of equity that is differentiable at x^* (cf. (6.47)).

Finally, we are able to present the optimal debt service prior to and after exercising the growth option. The coupon stream resulting from renegotiating the original debt contract, c_i^{NB}, can be expressed as follows

$$c_i^{NB} = \begin{cases} (1-\eta)\, x\theta^i (1-\tau) + \eta r \gamma_i & x \in \left(x_i^{LN}, x_i^{LR}\right], \\ (1-\eta(1-\rho))\, x\theta^i (1-\tau) & x \in \left(x_i^{LR}, x_i^{NB}\right], \\ b & x > x_i^{NB}. \end{cases} \tag{6.49}$$

The first regime in the strategic debt service corresponds to earnings remaining between the firm's optimal liquidation trigger, x_i^{LN}, and the level triggering liquidation if the firm was run by the creditors, x_i^{LR}. In this case the creditors receive a weighted average of cash flow from holding the collateral, $r\gamma_i$, and operating the firm as the first-best users, $x\theta^i(1-\tau)$. These streams are weighted with the shareholders' bargaining power coefficient, η. For the earnings level above x_i^{LR}, but still in the renegotiation region, the creditors receive a weighted average of the cash flow from operating the company as the second-best , $x\rho\theta^i(1-\tau)$, and as the first-best users, $x\theta^i(1-\tau)$. Outside the renegotiation region, the contractual coupon b is paid.

Note that for $\tau = 0$ and $\eta \in \{0, 1\}$ the coupon schedule corresponds to the outcome of the take-it or leave-it offers in Mella-Barral and Perraudin (1997), whereas setting γ_i to zero reduces the solution to the payment scheme of Fan and Sundaresan (2000).

On the basis of (6.49) it can be concluded that the presence of the growth opportunity does not change the coupon flow to the creditors within given regimes. This results from the following fact. In the bargaining process both groups of stakeholders receive the following portfolio: a fraction of the firm's value, V_i^{NB}, and the fraction of the creditors' outside option, R_i. Strategic debt service reflects cash flows to which these portfolios of securities are entitled. Since the investment opportunity that constitutes a part of the firm's value is not associated with any payment stream, the strategic debt service within a given regime is not influenced by its presence.

Although the growth option does not influence cash flows from the firm's securities, it affects, via its impact on optimal triggers, the regimes determining the structure of payoff under the strategic debt service. Let us observe that the following relationships hold

$$\frac{x_1^{LN}}{x_0^{LN}} > \frac{\gamma_1}{\theta\gamma_0} = \frac{x_1^{LR}}{x_0^{LR}} \lesseqgtr \frac{x_1^{NB}}{x_0^{NB}}. \tag{6.50}$$

The first inequality is implied by the positive value of the growth option. Without the growth option, the liquidation trigger x_0^{LN} would be equal to $\theta\gamma_0 x_1^{LN}/\gamma_1$. However, the presence of growth option raises the opportunity cost of liquidating the firm. As a consequence, the firm is liquidated optimally at a cash flow level lower than $\theta\gamma_0 x_1^{LN}/\gamma_1$. The equality in the middle follows from the solution to the creditors' liquidation problem when the value of the firm run by the creditors is given by (6.11). The remaining relationship reflects an ambiguous sign of the impact of the growth opportunity on the renegotiation policy.

All the above relationships directly translate into the changes in the strategic debt service resulting from the presence of the growth option. First, the inequality on the left reflects the effect of the investment opportunity on the liquidation trigger. It implies that in the presence of the growth option, the debt will be strategically serviced for a longer period before the ultimate decision to abandon the firm. Furthermore, the boundary between the regimes delineated by the trigger x_1^{LR} is unaffected by the presence of the investment opportunity. After all, the creditors running the company after the bankruptcy do not hold the growth option anymore. Finally, the impact of the investment opportunity on the cash flow level that triggers the renegotiation is ambiguous. On the one hand, since the value of equity contains an additional component reflecting the value of the option to invest, the equityholders' value of the outside option increases, which makes renegotiation *ceteris paribus* less attractive. However, the value of the firm is also higher when the investment opportunity exists. Therefore, the value of renegotiation increases as well. Since these two effects work in the opposite directions, the presence of the investment opportunity can, in general, either raise or reduce the renegotiation trigger.

Proposition 6.3 *The optimal renegotiation threshold in the presence of the investment opportunity can either be lower or higher than the corresponding threshold in a situation where there is no such opportunity. The condition*

$$\eta K_0 > A_{01} \tag{6.51}$$

determines the range of η in which the presence of investment opportunity results in earlier renegotiation.

Proof. See the Appendix. ■

From Proposition 6.3 we conclude that it is possible to determine the critical level of the shareholders' bargaining power, η, that demarcates the two cases. It holds that under both the first-best and second-best solution, the optimal renegotiation trigger exceeds the one without the investment opportunity if and only if $K_0 - A_{01} > \frac{1-\eta}{\eta} A_{01}$. This condition describes the case where the present value of the wealth transfer to the creditors occurring upon investment exceeds the value of the option to invest that accrues to the shareholders by more than a factor $\frac{1-\eta}{\eta}$. This means that if the bargaining power of the shareholders is high enough, it is optimal for them to begin the renegotiation process earlier in the presence of an investment opportunity. By doing so, the shareholders forgo the component of the value of equity associated with the investment option $A_{01}x^{\beta_1}$, but they are more than compensated by receiving a fraction (dependent on η) of the firm's value including the firm's growth option $K_0 x^{\beta_1}$.

Introducing the option to renegotiate the debt may adversely affect the value of the debt itself. This happens in a situation where the renegotiation trigger is close, but the bankruptcy trigger (in the absence of renegotiation) lies much below the renegotiation trigger, i.e. when the shareholders' bargaining power, η, is sufficiently high and the efficiency of creditors as the would-be managers, ρ, is low. Naturally, for x close enough to the bankruptcy trigger, allowing for renegotiation increases the debt value since the creditors' renegotiation payoff is higher than the one received after the bankruptcy.

6.4 Numerical Results and Testable Implications

This section presents comparative statics concerning the firm's optimal investment, liquidation, and debt restructuring policies, the first passage time probabilities and securities' values. Moreover, it presents some testable implications of the model. The input parameters used for graphical illustrations as follows: risk-free rate $r = 0.05$, drift rate of the earnings process $\alpha = 0.015$, volatility of earnings $\sigma = 0.2$, effective tax rate $\tau = 0.05$, instantaneous coupon $b = 0.66$, efficiency of the creditors as the second-best users of the firm's assets $\rho = 0.5$, bargaining power of the shareholders $\eta = 0.5$, liquidation value before investment $\gamma_0 = 1$, investment cost $I = 10$, earnings multiplier resulting from exercising the growth option $\theta = 2$, liquidation value after investment $\gamma_1 = 2$. In Subsection 6.4.1 we analyze the optimal policies, whereas in Subsection 6.4.2 we look at the first passage time probabilities. Subsection 6.4.3 discusses securities' valuation and Subsection 6.4.4 provides empirical implications.

6.4.1 Optimal Policies

The comparative statics for optimal investment, debt restructuring, and liquidation triggers are depicted in Table 6.2 below.

	σ	α,δ	r,δ	r,α	b	ρ	η	I,θ^{-1}	τ	γ_0	γ_1
x^*	+	−	+	(i)	(ii)	(iii)	(iv)	+	+	(v)	(v)
x_0^{NB}	−	−	+	−	+	−	+	+	+	−	+
x_0^B	−	−	+	(i)	+	(vi)	0	+	+	(vi)	(vi)
x_0^{LN}	−	−	+	+	+	−	+	+	+	+	−

Table 6.2: Comparative statics concerning the optimal investment, x^*, renegotiation, x_0^{NB}, bankruptcy, x_0^B, and liquidation, x_0^{LN}, thresholds. "+" ("−") denotes a positive (negative) derivative with respect to a given parameter. The numbers in brackets refer to the explanatory notes in the text.

The signs of first derivatives for both the first-best and second-best policy are included in Table 6.2. Below, we provide a discussion of those results that differ from the well-known results from the real options and corporate finance literature.

(i) From real options theory it is known that under all-equity financing the relationship between the optimal investment threshold and the risk-free *interest rate*, r, given constant return shortfall, δ, is increasing.[19] Such a relationship holds because the wedge between the Marshallian and optimal investment threshold increases with r, whereas the present value of the project does not change. Debt financing introduces another effect, which works in the opposite direction. Given that the coupon b is fixed, a higher r is associated with a lower debt value, and thus with a lower magnitude of the underinvestment problem. Consequently, a higher r can stimulate earlier investment since it is associated with a lower wealth transfer from shareholders to debtholders. The latter effect dominates if cash flow uncertainty is low. For low levels of uncertainty the optimal investment threshold is low, and this implies a relatively high leverage at the moment of undertaking the project. In such a case the impact of the change in r on the value of wealth transfer to debtholders is high and the wealth transfer effect dominates the waiting option effect. As a result, for low cash flow uncertainty the relationship between interest rate and optimal investment threshold is U-shaped (cf. Figure 6.5).[20]

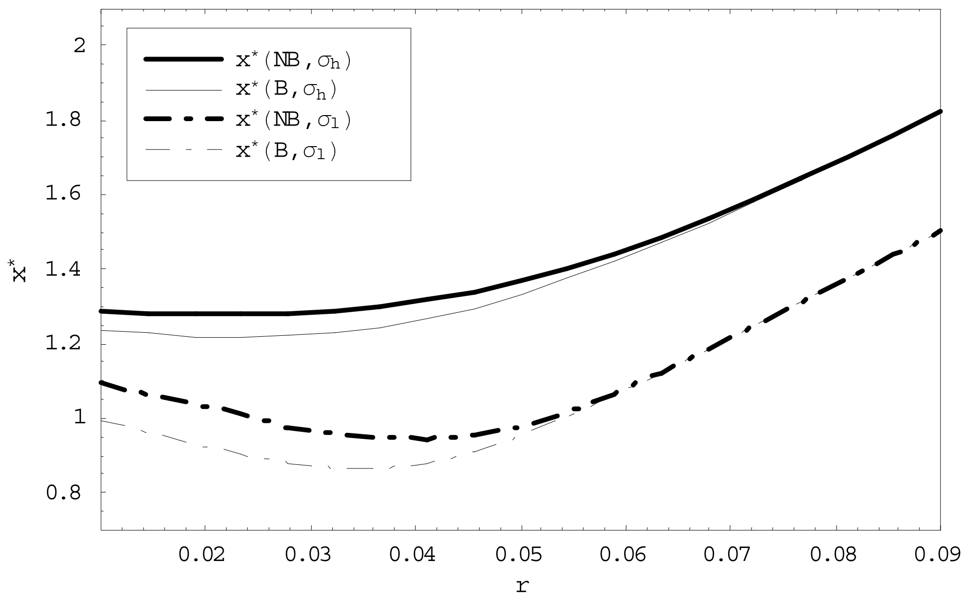

Figure 6.5: Equity value maximizing investment threshold in the presence of renegotiation option, $x^*(NB, \cdot)$, and without renegotiation, $x^*(B, \cdot)$, for $\sigma_l = 0.1$, $\sigma_h = 0.2$ and varying interest rate with the return shortfall rate, δ, kept constant at the 0.035 level.

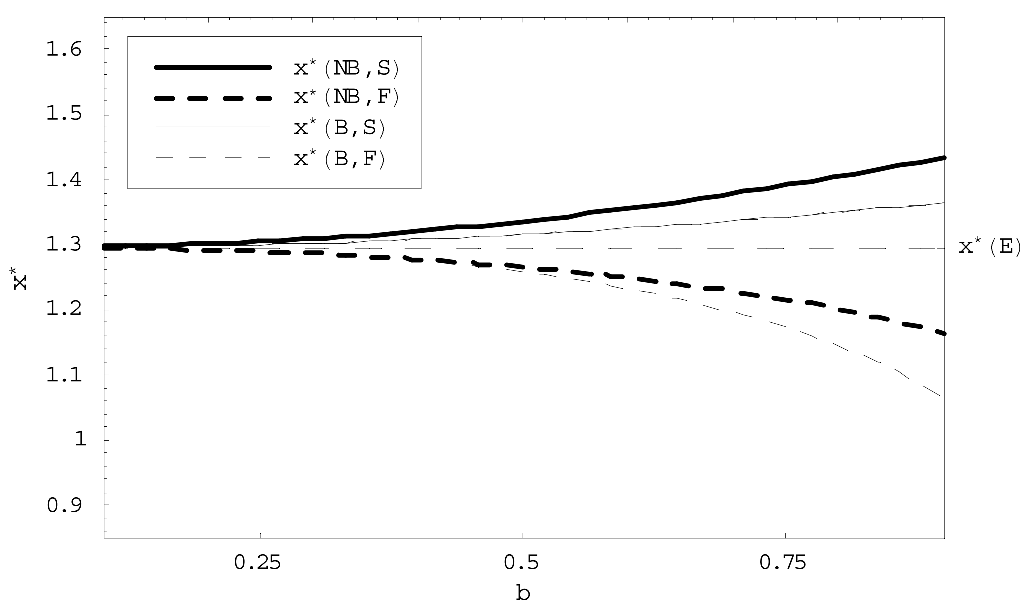

Figure 6.6: First-best, $x^*(NB, F)$, and second-best, $x^*(NB, S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B, F)$, and second-best, $x^*(NB, F)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying leverage (coupon rate), b.

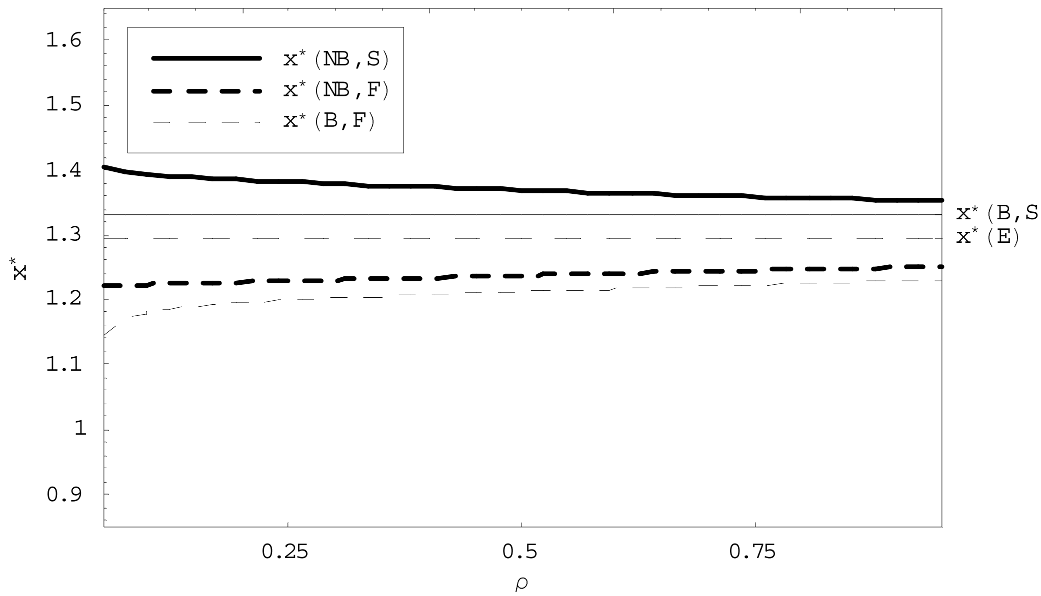

Figure 6.7: First-best, $x^*(NB, F)$, and second-best, $x^*(NB, S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B, F)$, and second-best, $x^*(B, S)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying magnitude of the creditors outside option, ρ.

(*ii*) The impact of *leverage*, b, on the optimal investment threshold for the first-best and second-best solutions differs (see Figure 6.6). If the investment is made so as to maximize the value of the firm, the optimal investment threshold decreases with leverage. The latter relationship results from a higher increase in the present value of the tax shield upon completing the investment. The opposite is true in the situation where the investment threshold is chosen so to maximize the value of equity. In this case the optimal investment threshold increases with leverage. This can be explained by the wealth transfer from the equityholders to the debtholders, positively related to the level of leverage. The wealth transfer occurs since after undertaking the project the renegotiation trigger is lower than before the investment has been made.

(*iii*) The outside option of the debtholders, ρ, influences the optimal investment threshold either by delaying investment, if the threshold is chosen so that the value of the firm is maximized, or by accelerating it, if the shareholders choose

[19] See Dixit and Pindyck (1996), Ch. 6.

[20] When the first-best solution is applied, the wealth transfer to the debtholders does not directly influence the investment policy so that the optimal investment threshold always increases in r.

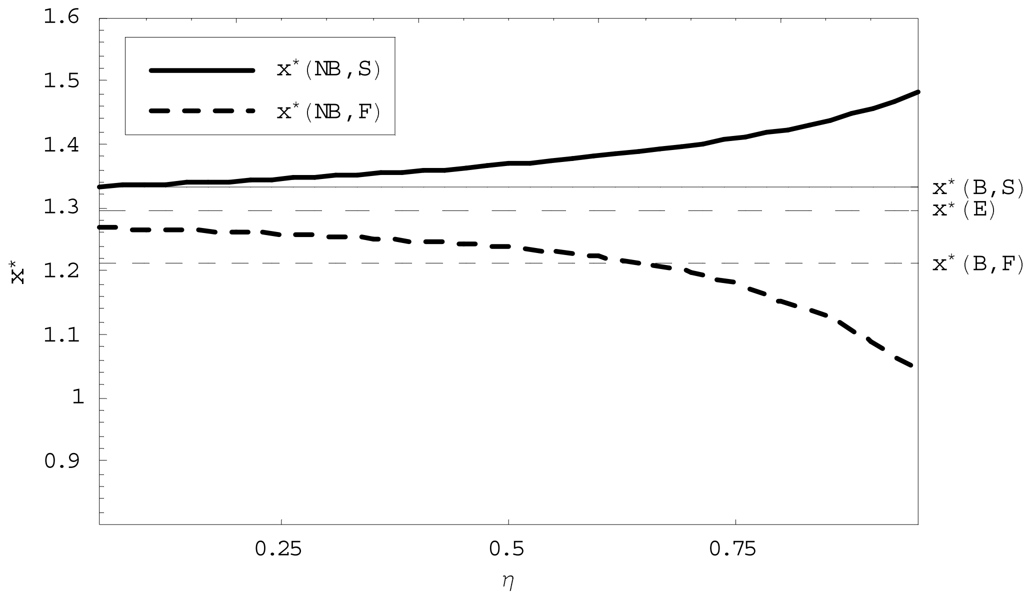

Figure 6.8: First-best, $x^*(NB, F)$, and second-best, $x^*(NB, S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B, F)$, and second-best, $x^*(NB, F)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying distribution of bargaining power, η.

the investment timing (cf. Figure 6.7). The reason for which the first-best investment threshold increases with ρ is that the optimal renegotiation trigger decreases with ρ. Consequently, since a lower renegotiation trigger is equivalent to a lower increase of the PV of the tax shield, the value of the project decreases with ρ and the investment is undertaken later. In the special case of $\tau = 0$, the tax shield argument is no longer present and the threshold is equal to the 100% equity one. Conversely, if the value of equity is maximized, a lower wealth transfer associated with high ρ (thus low x_0^{NB}) moves the investment threshold closer to the all-equity case. When the second-best solution is applied, the wealth transfer from debtors to creditors always occurs upon investment so that even in case of $\tau = 0$ the equity value-maximizing investment rule differs from the one given by the optimal all-equity threshold.

(*iv*) The shareholders' bargaining power, η, affects the optimal investment threshold in an opposite way than ρ (cf. Figure 6.8). If the timing of investment is chosen optimally so as to maximize the value of the firm, the optimal investment threshold decreases with η. This results from the fact that the value of the investment

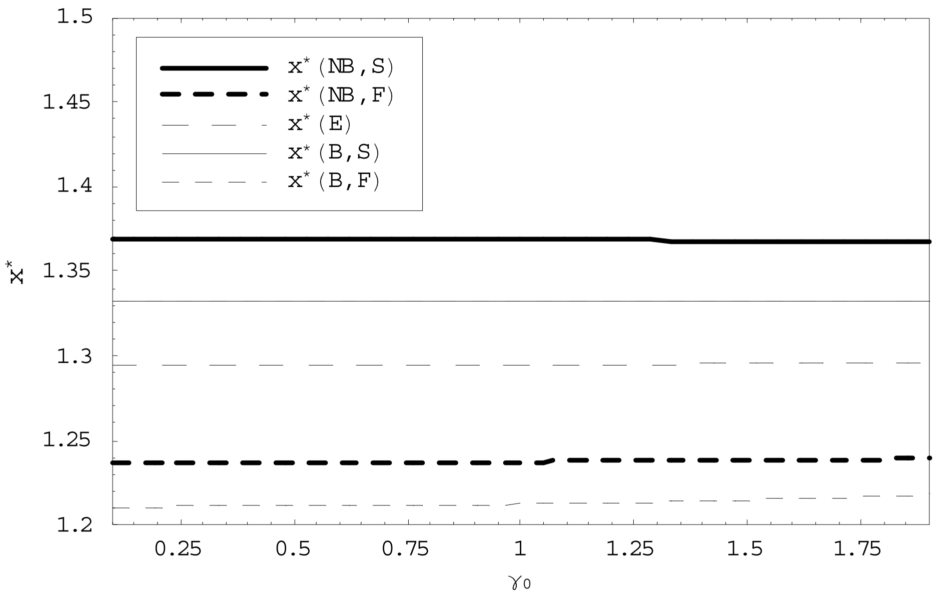

Figure 6.9: First-best, $x^*(NB, F)$, and second-best, $x^*(NB, S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B, F)$, and second-best, $x^*(NB, F)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for different liquidation values, γ_0.

opportunity to the firm increases with η, since the present value of the additional tax shield (due to investment) increases. For an analogous reason as in (*iii*), the first-best investment threshold is insensitive to changes in η in the absence of taxes. However, if the timing of investment is chosen by the equityholders so that the value of equity is maximized, the optimal investment threshold increases with η. This is due to the fact that the renegotiation trigger is positively related to η. Since the renegotiation trigger decreases upon investment, debtholders benefit most from investment when the initial trigger is high. A higher wealth transfer that accrues to the debtholders upon undertaking the project results in a later investment.

(*v*) The impact of the liquidation value of the firm on the optimal investment policy depends on the presence of the renegotiation option and on the fact whether the first-best solution can be implemented (cf. Figure 6.9). When the investment threshold is chosen as to maximize the value of the firm, the investment is always undertaken later (thus closer to the all-equity trigger) when the liquidation value γ_0 (γ_1) is higher (lower). This results from the fact that investment becomes less attractive if it is associated with a lower increase in the liquidation value. This effect is reversed if in the presence of the renegotiation option the choice of the investment trigger maximizes the equity value. Since a higher initial liquidation value negatively influences the probability of strategic default, the wealth transfer to the debtholders, which occurs at the moment of investment, is lower. This results in an earlier investment. The same argument can be applied to analyze the impact of γ_1. Finally, when renegotiation is not allowed for and the second-best solution is implemented, the investment trigger does not depend on the firm's liquidation value.

(vi) The bankruptcy trigger, x_0^B, is influenced neither by the firm's liquidation value nor by the efficiency of the creditors as the second-best users of the firm's assets as long as the investment threshold is chosen as to maximize the equity value. In a situation where the first-best solution can be implemented, the optimal bankruptcy threshold is positively related to the liquidation value γ_1 and negatively related to the creditor's efficiency and liquidation value γ_0. A positive change in a liquidation value and low creditor's efficiency make investment particularly attractive since it lowers the present value of the economic cost of bankruptcy. Consequently, investment occurs too early comparing with the case when the effect of the change of economic costs of bankruptcy is absent. This results in a lower value of the firm's claims as a going-concern and lower opportunity cost of bankruptcy.

6.4.2 First Passage Time Probabilities

Interactions between the options to scale up the operations and to reorganize debt can already be observed by analyzing the relevant optimal triggers. However, since equityholders face a double-barrier control problem, there is no one to one correspondence between the optimal triggers and the first passage time probabilities. Therefore, we extend the analysis and calculate the first passage time probabilities associated with the optimal renegotiation trigger and with the optimal investment threshold.

In order to evaluate the influence of a given option, or parameter, on the relevant decision trigger, we calculate the probabilities of reaching the trigger within a time interval of length T. For example, the probability of strategic debt restructuring is equivalent to the probability of the cash flow process hitting, either the renegotiation trigger, x_0^{NB}, or, first, the investment threshold x^* and then the renegotiation trigger, x_1^{NB}. Conversely, the probability of investment equals the probability of hitting the investment threshold, x^*, conditionally on not hitting the liquidation trigger, x_0^{LN}. The derivation of the relevant probabilities, based on solving a partial differential equation (PDE), is presented in the Appendix.

In Table 6.3 we present the comparative statics concerning the first passage time probabilities. The presented results have been obtained by numerical calculation of the relevant probabilities for an extensive range of input parameters.

	σ	α, δ	r, δ	r, α	b	ρ	η	I, θ^{-1}	τ	T	γ_0	γ_1
p^*	(vii)	+	−	$(viii)$	(ii)	(iv)	(iv)	−	−	+	(v)	(v)
p^{NB}	(vii)	−	+	$-^*$	+	−	+	+	+	+	$-^*$	$+^*$
p^B	(vii)	−	+	$-^*$	+	(vi)	0	+	+	+	(vi)	(vi)

Table 6.3: Comparative statics concerning the first passage time probabilities associated with investment, p^*, debt renegotiation, p^{NB}, and bankruptcy, p^B.* relationship can be reversed when $x^* - x$ is very small. The numbers in brackets refer to the explanatory notes in the text.

(vii) Non-monotonicity of the investment-uncertainty relationship has been already pointed out by Sarkar (2000) and analyzed further in Chapter 3 of this thesis. It crucially depends on the relationship between the horizon T and the time to reach the deterministic Jorgensonian threshold. From Chapter 3 it is obtained that if the horizon T is relatively short, the investment-uncertainty relationship is humped, while for high T it is negative. Another factor that influences the probability of investment in this double-barrier problem of the firm is the probability of bankruptcy (or of liquidation when renegotiation is possible) which is also sensitive to the changes in uncertainty. On the basis of Figure 6.10 one can conclude that higher uncertainty results in a lower probability of investment when cash flow is high. However, for lower levels of cash flow, uncertainty raises the probability of investment since bankruptcy becomes less likely. The latter holds since the bankruptcy threshold decreases with σ.

The presence of renegotiation option affects the probability of investment twofold. First, it raises the optimal investment threshold. Second, it allows to preserve the investment opportunity for the levels of cash flow lower than the bankruptcy trigger. The smaller than one ratios of the probabilities with and without renegotiation illustrate that the effect of an increased investment threshold in the presence of

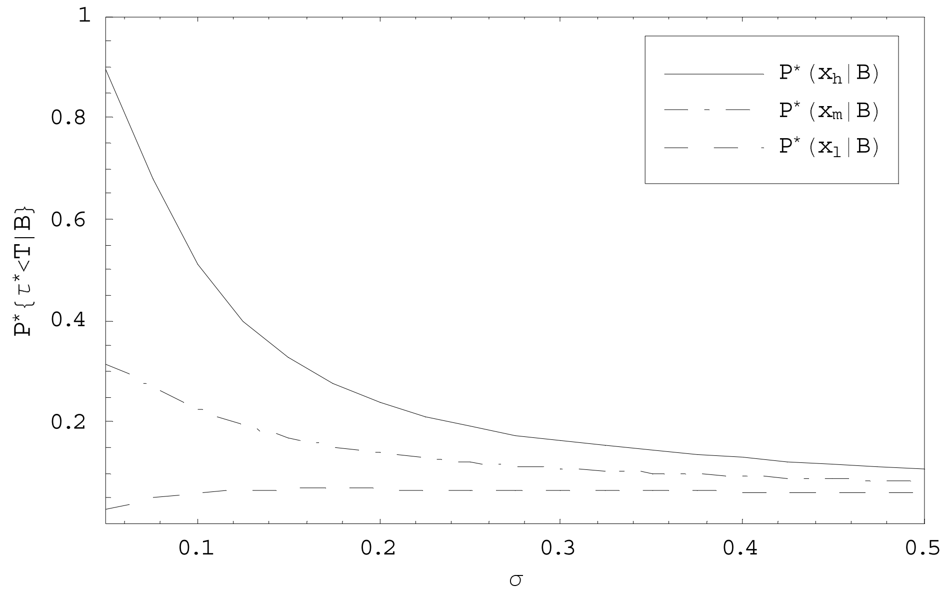

Figure 6.10: The probability of investment when the renegotiation is not possible for $x_l = 0.6$, $x_m = 0.7$, and $x_h = 0.8$, as a function of cash flow volatility, σ.

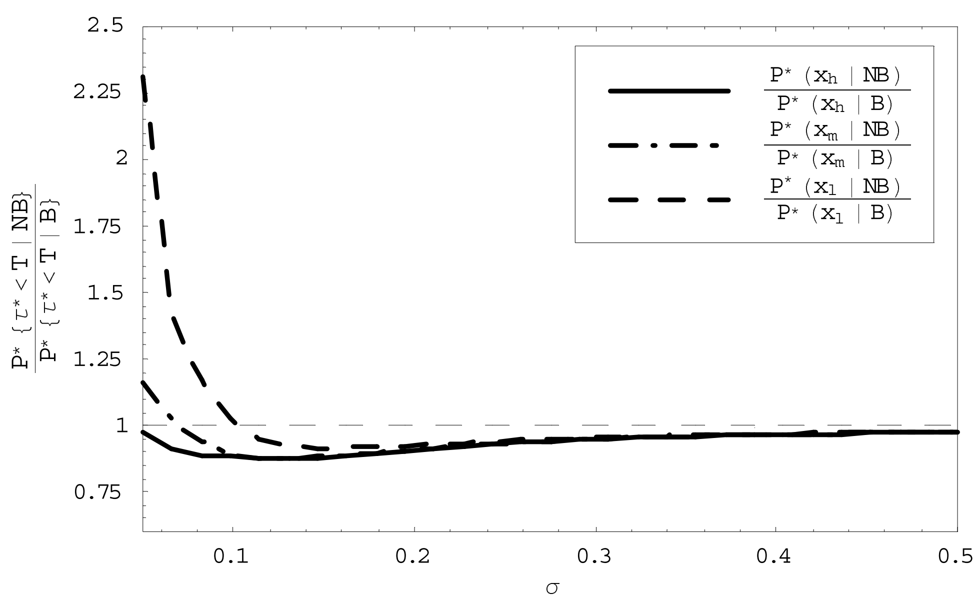

Figure 6.11: The ratio of probabilities of investment when the renegotiation is and is not possible for $x_l = 0.6$, $x_m = 0.7$, and $x_h = 0.8$, as a function of cash flow volatility, σ.

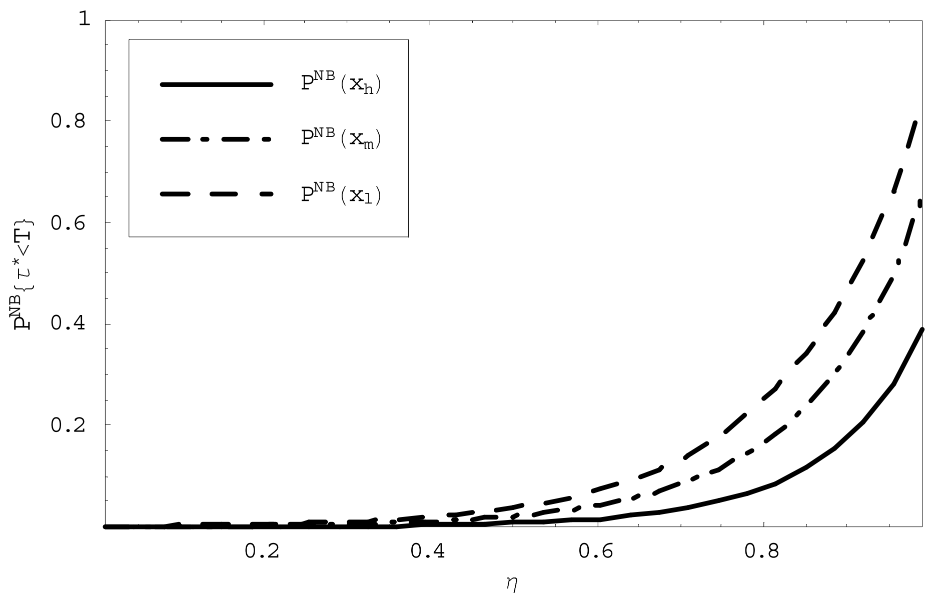

Figure 6.12: The probability of renegotiation for $x_l = 1.0$, $x_m = 1.1$, and $x_h = 1.25$, as a function of shareholders' bargaining power, η.

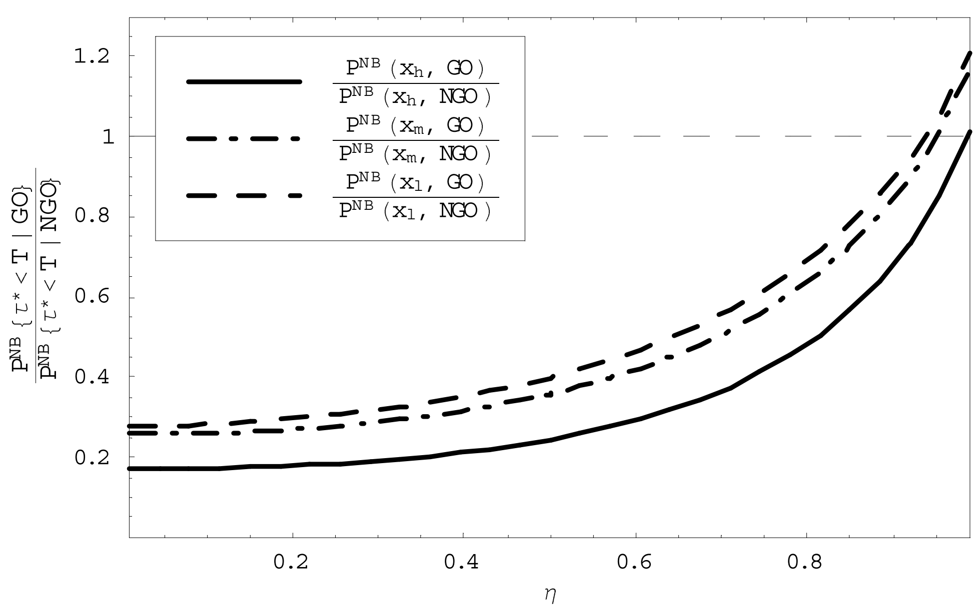

Figure 6.13: The ratio of probabilities of debt renegotiation with, $p^{NB}(\cdot, GO)$, and without, $p^{NB}(\cdot, NGO)$, the growth option for $x_l = 1.0$, $x_m = 1.1$, and $x_h = 1.25$, as a function of shareholders' bargaining power, η.

renegotiation more than offsets the impact of losing the investment opportunity upon bankruptcy (see Figure 6.11).

(*viii*) An increase in the interest rate, r, when the return shortfall, δ, is kept constant, can change the probability of investment in both directions. If the investment threshold decreases with r (see (*i*)), then the probability of investment always increases with r. However, when the investment threshold is positively related to r (also see (*i*)), the sign of the investment-interest rate relationship is ambiguous. This results from the fact that such increase in the investment threshold is counterbalanced by the increase in the cash flow drift rate as well as by a decrease in the bankruptcy trigger (see (6.15)). The sign of the joint effect depends on the specific choice of model parameters.

What remains to be considered is the relationship between the presence of the growth option and the probability of strategic debt restructuring. Renegotiation is more likely when the debtors are given more bargaining power (cf. Figure 6.12). However, the magnitude of the influence of bargaining power on the renegotiation probability highly depends on whether the firm holds positive NPV growth opportunities. Such a comparison is illustrated in Figure 6.13. It appears that in the presence of a positive NPV project, the probability of debt renegotiation can be *higher* than without the investment option. Such a situation occurs when the actual renegotiation trigger x_0^{NB} exceeds the renegotiation trigger without the investment opportunity (equal to $x_1^{NB}\theta$), and the current cash flow is not excessively high.[21] This situation occurs when the shareholders' bargaining power, η, is large (cf. Proposition 6.3). This effect is magnified for moderate levels of uncertainty (high uncertainty relatively increases the shareholders' value of the investment option which makes renegotiation less likely).

6.4.3 Valuation of Securities

In this section the comparative statics concerning the valuation of the firm's securities are presented. Since the signs of the relevant relationships does not depend on the presence of the renegotiation option, the existence of such an option is assumed here. Table 6.4 depicts the direction of the impact of model parameters on the valuation of equity, debt and the entire firm.

[21] In the absence of the renegotiation option the bankruptcy triggers are the relevant ones. Since it holds that x_0^B is always lower than $x_1^B\theta$, the presence of the investment opportunity always reduces the default probability when there is no option to renegotiate.

	σ	α, δ	r, δ	r, α	b	ρ	η	I, θ^{-1}	τ	γ_0	γ_1
E_0^{NB}	$+$	$+$	$-$	$+$	$-+$	$-$	$+$	$-$	$-$	$-$	$-$
D_0^{NB}	(ix)	$+$	$-$	(xi)	$+-$	$+$	$-$	$-$	$-$	$+$	$+$
V_0^{NB}	(x)	$+$	$-$	(xii)	$+-$	$+$	$-$	$-$	$-$	$+$	$+$

Table 6.4: Comparative statics concerning the valuation of the firm, its debt and equity. "$+$" ("$-$") denotes a positive (negative) derivative with respect to a given parameter, and "$+-$" ("$-+$") indicates a humped (U-shaped) relationship. The numbers in brackets refer to the explanatory notes in the text.

Since changes in the valuation of the claims resulting from the changes in input parameters are mostly consistent with those reported in the dynamic capital structure literature (e.g. Leland, 1994), we mainly discuss the results that are directly influenced by the interactions between the option to invest and to restructure the debt.

(ix) The relationship between the *cash flow volatility*, σ, and the value of debt, D_0^{NB}, depends on the current level of the earnings process, x. When this level is high, the value of the debt decreases with volatility since higher volatility makes renegotiation, other things equal, more likely. However, for realizations of x sufficiently close to x_0^{NB}, two other effects result in a positive relationship between the value of the debt and uncertainty. First, for low x, the impact of x_0^{NB} decreasing with σ is stronger than the impact of a higher probability of hitting any *fixed* trigger lower than x.[22] Second, the renegotiation value of debt rises with σ. The latter relationship results from the fact that the value of the firm rises with σ, because of the included investment opportunity component.

(x) The relationship between the *cash flow volatility*, σ, and the value of the firm, V_0^{NB}, results from the impact of the volatility on the value of debt and equity. For a given σ and varying x, the value function is first convex (which mainly reflects the option value of the tax shield after the contractual debt service is restored), then it becomes concave (as a result of a short option on the tax shield once contractual service is restored), becomes once again convex (when the option component associated with the investment opportunity starts to dominate) and, eventually, becomes and remains concave (when it value-matches to $V_1^{NB} - I$, see (6.37)). Consequently, the effect of changes in σ is only unambiguous when the firm either is financially distressed (positive relationship) or close to the optimal exercise of its growth option (negative relationship).

[22]Using a similar reasoning Leland (1994) explains the behavior of junk bonds.

(xi) The sign of the relationship between the value of debt and the risk-free *interest rate*, r, given constant return shortfall, δ, is in general ambiguous. The relationship is hump-shaped for low uncertainty combined with a high convenience yield, and decreasing otherwise. If the firm's debt was riskless, its value would always decrease with r, irrespective of the drift rate, α, and return shortfall, δ. Here, the positive probability of renegotiation makes it risky. A very low interest rate, in combination with a positive return shortfall, is associated with a negative drift rate. If the uncertainty is small, the stochastic discount factor associated with renegotiation is high. Therefore, for low levels of uncertainty the value of the debt may benefit from an increasing interest rate if the latter is sufficiently low.

(xii) The relationship between the value of the firm and the risk-free *interest rate*, r, given constant return shortfall reflects the impact of r on the value of equity, E_0^{NB}, and debt, D_0^{NB}. Since the value of equity increases monotonically with r, the impact of the interest rate on the value of the firm depends on the relative slope of the debt value function comparing to equity. Since the former can both increase and decrease with r (see (xi)), the value of the firm is in general hump-shaped or increasing with r. For a very high r, the value of the firm levels off since the impact of changes in leverage becomes negligible (as $b/r \to 0$).

The comparative statics results from the last two columns of Table 6.4 coincide with the findings in the recent dynamic capital structure literature (cf. Flor, 2002, and references therein). It appears that ex post (i.e. when the capital structure is already fixed) the value of the firm's equity decreases with the asset resale value, γ_i. This results from the fact that the asset resale value increases the bargaining position of the creditors (who can always seize the assets upon the violation of the original debt contract by the equityholders), who are granted bigger concessions in the renegotiation process.

6.4.4 Empirical Implications

Testing empirical predictions of our model requires identifying proxy variables that can capture the effects of different costs of renegotiation (in the model we consider only two polar cases: zero costs and costs offsetting entire benefits from renegotiation), equityholders' bargaining power η, and creditors' outside option, ρ. The costs of renegotiation (cf. Bolton and Scharfstein, 1996) are expected to be low when the firm is financed with a bank debt or, in general, when the number of its creditors is small.

The distribution of bargaining power (cf. Hackbarth et al., 2002) crucially depends on the firm's size, age, and degree of diversification. Moreover, it is also influenced by the country's legal system (US Bankruptcy Code of 1978 is more shareholder-friendly than the codes in most continental European countries). Finally, creditors' efficiency as managers of the firm is expected to be higher when the brand recognition is low (cf. Mella-Barral, 1999) and in the sectors with low intensity of R&D.

In this section, we first analyze the sensitivity of investment to the firm's cash flow. Subsequently the stock price behavior and credit spreads are discussed. Finally, some social welfare results are presented.

Investment-cash flow sensitivity. The set-up of this paper's model stipulates that investment is triggered by a sufficiently high level of cash flow from operations. This implies that a higher magnitude of Myers' (1977) underinvestment makes the investment *ceteris paribus* less likely to be triggered by an incremental cash flow increase. As a consequence, the presence of the renegotiation option and high shareholders' bargaining power, which both result in higher underinvestment, is likely to decrease the sensitivity of investment to the firm's cash flow. Therefore, our model provides an alternative explanation of the empirical evidence that small and young firms exhibit relatively higher investment-cash flow sensitivity (cf. Lensink et al., 2001, Ch. 3, and references therein). Since small firms usually have a limited bargaining power in the debt renegotiation with banks, the magnitude of the additional underinvestment resulting from the renegotiation option will be in the most cases insignificant. This relatively lower magnitude of underinvestment implies that their investment-cash flow sensitivity is likely to remain high. The same argument can be used to claim that the capital investment of big and mature firms with dispersed bond market debt will be on average more sensitive to cash flow than investment of similar firms with a mixture of bank and bond market debt and with bank debt only (cf. Moyen, 2002).

Stock price behavior. Asymmetric returns are inherent to the equity of firms that hold a substantial portfolio of real options. As Bernardo and Chowdhry (2002) point out (cf. also Berk et al., 1997, and Pope and Stark, 1997), positive earnings surprises have a stronger effect on the prices of equity than negative ones. This is because the presence of a real option makes the payoff to equityholders convex in the stochastic variable that underlies the firm's cash flow. In the current model, the equity value function consists of two convex components, options to invest and to restructure the debt/declare bankruptcy, and one linear, present value of cash flow. Therefore, it is itself convex. As a consequence, the stock price returns exhibit right-skewness.

The presence of an investment and a renegotiation option has also implications for the responsiveness of the stock price to the earnings surprises. Upon introducing the renegotiation option alone, one can observe that the stock price becomes less responsive to the earnings surprises. This is associated with a decrease of the first derivative of the equity value function with respect to the process x. The reason for that is that the renegotiation option has a relatively higher value in the adverse states of nature (i.e. for low realizations of x). Consequently, any variation in x results in less drastic changes in E_0 in the presence of renegotiation option. The responsiveness of the stock price to the earnings surprises is magnified by introducing the growth option. This results from the fact that higher realizations of x not only give rise to the present value of cash flow but also enhance the value of the growth option. As a consequence, the derivative $\frac{\partial E_0}{\partial x}$ increases and so does the responsiveness to the earnings surprises.

Credit Spreads. The riskiness of debt reflected by the credit spread is highly influenced by the presence of both an investment and a renegotiation option. On the basis of the formula for the credit spread (in bps), SPR, where

$$SPR = \left(\frac{b}{D_0} - r \right) * 100, \tag{6.52}$$

it can be concluded that for a given coupon and a riskless rate, the credit spread is inversely monotonic in the market value of debt. Consequently, the results of the analysis of Section 6.3 can be translated into implications for the credit spreads.

The first theoretical prediction is that the presence of growth options reduces *ceteris paribus* credit spreads. Anticipated future exercise of such options is associated with the prospect of lowering both the bankruptcy and renegotiation thresholds, which negatively affects the riskiness of the debt. In the absence of a renegotiation option, introducing the growth option the results not only in a decreasing the after-investment bankruptcy threshold but also in lowering the initial bankruptcy threshold. The latter holds since the opportunity cost of declaring bankruptcy is higher in the presence of the growth option. Consequently, in the absence of the renegotiation option, the impact of the investment opportunity on credit spreads is substantial.

When the renegotiation option is allowed for, a lower renegotiation threshold, which arises after completing the investment, reduces the riskiness of the debt even before the investment project is undertaken. However, there is a second effect that can increase the firm's credit risk. Contrary to the bankruptcy case, the impact of the growth option does not have to make the debt restructuring less likely. In the situation described in Proposition 6.3, the presence of the growth option increases the renegotiation trigger. This can lead to a higher riskiness of the debt, resulting in a higher credit spread. The magnitude of both opposing effects highly depends on the shareholders' bargaining power and the creditors' outside option. Higher shareholders' bargaining power results in a higher magnitude of the latter effect, whereas a higher creditors' outside option has an opposite effect. In general, for an extensive grid of the model parameters' values, the presence of the growth option reduces credit spreads even in the presence of strategic debt restructuring.

The impact of the market parameters such as interest rate, return shortfall and earnings volatility, as well as of the indirect bankruptcy costs is consistent with the literature on firm-value based models of credit risk (cf. Anderson and Sundaresan, 2000).

Social Value of the Firm. According to Hege and Mella-Barral (2000), the social value of the firm is not affected by the distribution of the bargaining power among the debtors and the creditors. The reason is that any loss of the tax shield, which is associated with premature renegotiation due to a higher bargaining power of the debtors, is just a transfer to the government. Contrary to that observation, in the current model the distribution of the bargaining power has an externality on the investment and the liquidation decision. Despite the fact that the changes in the present value of the tax shield do not directly influence the social value of the firm (they merely change the redistribution of wealth), they do affect the investment and liquidation policy. Consequently, in order to assess the impact of the distribution of bargaining power on the social value of the firm, one has to compare the first-best investment and liquidation thresholds calculated under all-equity financing assumption with the ones determined in the presence of a mixed capital structure.

In our set-up debt distorts the optimal investment and liquidation policies. As it can be seen from Figure 6.6, the optimal equityholders' investment threshold is higher than in the all-equity case. Moreover, the optimal investment threshold increases with the shareholders' bargaining power coefficient. Consequently, a high shareholders' bargaining power exacerbates the underinvestment problem, in this case the inefficiently late exercise of the option to expand (i.e. beyond the point at which the marginal cost of investing equalizes with the marginal revenue from expansion taking into account irreversibility and uncertainty).

Allowing for the possibility of renegotiating the original debt contract results in the liquidation trigger being a function of the shareholders' relative bargaining power. This is because the liquidation trigger is determined so as to maximize the value of the firm. The latter quantity is endogenous and depends on the renegotiation trigger that in turn is affected by the distribution of bargaining power. As it can be concluded on the basis of Table 6.2 the optimal liquidation threshold is an increasing function of η. The optimal liquidation threshold in the presence of debt financing and renegotiation lies between the all-equity liquidation threshold in the world without taxes and the all-equity threshold with when corporate tax is sufficiently high. Therefore, reducing the shareholders' relative bargaining power mitigates the negative externality of debt on the optimal liquidation decision.

We conclude that there are two negative welfare effects of a high bargaining power of the debtors. The first is associated with an excessively delayed investment, and the other with a too early liquidation.

6.5 Conclusions

The investment policy of the firm is affected by its capital structure. Introducing debt financing results in an inefficient delay in exercising the growth option. We show that eliminating costly bankruptcy by introducing the possibility of debt restructuring does not solve this problem. In fact, underinvestment is higher if the renegotiation option exists.

The departure from the all-equity financing affects the firm's liquidation policy. If renegotiation is not allowed for, the decision to liquidate the firm is made by the creditors who become the owners of the firm upon the bankruptcy. This results in an ex ante inefficient liquidation and this inefficiency constitutes part of the indirect bankruptcy costs. The introduction of a mixed capital structure combined with a renegotiation option influences the optimal liquidation policy twofold. First, the presence of the tax shield delays liquidation since *ceteris paribus* it enhances the value of the firm. Second, partial debt financing leads to the departure from the first-best investment policy, which results in the value of the firm being deteriorated and in the opportunity cost of its liquidation being lowered. For sufficiently high taxes the former effect dominates, thus liquidation occurs later than under all-equity financing but not as late as under the optimal liquidation all-equity financing in the world without taxes. Since there exists a positive relationship between the liquidation trigger and the shareholders' bargaining power, reducing this power brings the liquidation policy closer to the optimum.

Furthermore, we show that the debt restructuring policy is affected by the presence of the growth option. The growth option positively influences the renegotiation trigger if a high shareholders' bargaining power is combined with a substantial wealth transfer to the creditors occurring upon investment. In the opposite situation, this is when the creditors possess higher bargaining power and if they do not gain much upon investment, the renegotiation trigger falls.

Finally, we would like to indicate several extensions that may potentially constitute interesting research areas. A more realistic setting would include constructing a model with multiple investment opportunities (cf. Morellec, 2001). The model can also be extended to provide a pricing framework for a renegotiable debt with finite maturity where the coupon flow is a function of the underlying state variable (cf. Shackleton and Wojakowski, 2001). Moreover, the current analysis can be modified to incorporate the impact of product market interactions on the firm's investment behavior (the area pioneered by Fries et al., 1997, and Lambrecht, 2001). Another extension would include investigating the impact of Chapter 11 regulation on the intra-industry bankruptcy intensity. Current anecdotal evidence often indicates that artificially sustained capacity results in a lower sector profitability and, as a consequence, a higher chance of exit of other players.[23] The choice of the second-best solution in the current modeling set-up calls for an introduction of an executive compensation scheme that would allow for aligning the incentives of the self-interested managers with the value of the firm. Such alignment may prove to be ex post optimal from the equityholders' point of view. Finally, the divergence of the stakeholders' objectives may lead to an asset substitution problem, which will influence the equityholders' investment policy (cf. Leland, 1998, and Subramanian, 2002, in an agency, and Dangl and Lehar, 2002, in a banking regulation application).

6.6 Appendix

Derivation of (6.29). The value of the tax shield, TS_i, satisfies ODE (6.2) with the following instantaneous payoffs coefficients

$$(B, C) = \begin{cases} (0, 0) & x < x_i^{NB}, \\ (0, b\tau) & x \geq x_i^{NB}. \end{cases}$$

Consequently TS_i can be written as

$$TS_i = \begin{cases} M_1 x^{\beta_1} + M_2 x^{\beta_2} & x < x_i^{NB}, \\ \frac{b\tau}{r} + M_3 x^{\beta_1} + M_4 x^{\beta_2}. & x \geq x_i^{NB}. \end{cases} \tag{6.53}$$

[23]Cf. *The Economist*, 7th September 2002, The firms that can't stop falling: Bankruptcy in America, and 14th December 2002, Testing the limits of Chapter 11.

Since

$$\lim_{x\uparrow\infty} TS_i = \frac{b\tau}{r}, \text{ and} \quad (6.54)$$

$$\lim_{x\downarrow 0} TS_i = 0, \quad (6.55)$$

it holds that $M_2 = M_3 = 0$. The only remaining unknown constants are M_1 and M_4. They can be determined by applying the value-matching and smooth-pasting conditions at x_i^{NB}

$$\lim_{x\uparrow x_i^{NB}} TS_i = \lim_{x\downarrow x_i^{NB}} TS_i, \quad (6.56)$$

$$\left.\frac{\partial TS_i}{\partial x}\right|_{x\uparrow x_i^{NB}} = \left.\frac{\partial TS_i}{\partial x}\right|_{x\downarrow x_i^{NB}}, \quad (6.57)$$

which results in

$$M_1 = \frac{b\tau}{r}\frac{-\beta_2}{\beta_1-\beta_2}\left(x_i^{NB}\right)^{-\beta_1}, \text{ and} \quad (6.58)$$

$$M_4 = \frac{b\tau}{r}\frac{-\beta_1}{\beta_1-\beta_2}\left(x_i^{NB}\right)^{-\beta_2}. \quad (6.59)$$

■

Derivation of (6.31). The value of the firm at the optimal liquidation trigger satisfies the Bellman equation (6.2) with $B = \theta(1-\tau)$ and $C = 0$, subject to the following value-matching and smooth-pasting conditions

$$V_1^{NB}\left(x_1^{LN}\right) = \quad (6.60)$$

$$\frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1} + L_1\left(x_1^{LN}\right)^{\beta_2} = \gamma_1,$$

$$\left.\frac{\partial V_1^{NB}}{\partial x}\right|_{x=x_1^{LN}} = \quad (6.61)$$

$$\frac{\theta(1-\tau)}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{x_1^{LN}r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1} + \beta_2 L_1\left(x_1^{LN}\right)^{\beta_2-1} = 0.$$

The constant L_1 can be directly calculated from (6.60). Multiplying both sides of (6.60) by $\beta_2 x_1^{LN}$ and subtracting it from (6.61) yields the implicit formula for x_1^{LN}. ■

Derivation of (6.34). When the shareholders' optimal renegotiation trigger is approached from above, the value of equity satisfies the Bellman equation (6.2) with $B = \theta(1-\tau)$ and $C = -b(1-\tau)$, subject to the following value-matching and smooth-pasting conditions

$$\lim_{x \downarrow x_1^{NB}} E_1^{NB} = \frac{x_1^{NB}\theta(1-\tau)}{\delta} - \frac{b(1-\tau)}{r} + A_{12}\left(x_1^{NB}\right)^{\beta_2}, \tag{6.62}$$

$$\begin{aligned}
&\lim_{x \uparrow x_1^{NB}} E_1^{NB} = \eta\left(V_1^{NB} - R_1\right) \\
&= \eta\left[\frac{x_1^{NB}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\right. \\
&+ \left(\gamma_1 - \frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(\frac{x_1^{NB}}{x_1^{LN}}\right)^{\beta_2} \\
&\left.- \left(\gamma_1 - \frac{x_1^{LR}\rho\theta(1-\tau)}{\delta}\right)\left(\frac{x_1^{NB}}{x_1^{LR}}\right)^{\beta_2}\right],
\end{aligned} \tag{6.63}$$

$$\begin{aligned}
&\lim_{x \uparrow x_1^{NB}} \frac{\partial E_1^{NB}}{\partial x} = \eta\left[\frac{\theta(1-\tau)}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r x_1^{NB}}\right. \\
&+ \frac{\beta_2}{x_1^{NB}}\left(\gamma_1 - \frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(\frac{x_1^{NB}}{x_1^{LN}}\right)^{\beta_2} \\
&\left.- \frac{\beta_2}{x_1^{NB}}\left(\gamma_1 - \frac{x_1^{LR}\rho\theta(1-\tau)}{\delta}\right)\left(\frac{x_1^{NB}}{x_1^{LR}}\right)^{\beta_2}\right].
\end{aligned} \tag{6.64}$$

Calculating the derivative of (6.62), and applying value matching and smooth pasting at x_1^{NB} yields the formula for x_1^{NB}. ■

Proof of Proposition 6.1. First, on the basis of (6.28), (6.33), (6.36)-(6.38), (6.40), and (6.42), we determine the constants K_0, L_0, A_{01}, and A_{02}:

$$\begin{aligned}
&\begin{bmatrix} K_0 \\ L_0 \end{bmatrix} = \\
&\frac{1}{(x^*)^{\beta_1}\left(x_0^{LN}\right)^{\beta_2} - (x^*)^{\beta_2}\left(x_0^{LN}\right)^{\beta_1}}\begin{bmatrix} \left(x_0^{LN}\right)^{\beta_2} & -(x^*)^{\beta_2} \\ -\left(x_0^{LN}\right)^{\beta_1} & (x^*)^{\beta_1} \end{bmatrix} \times \\
&\begin{bmatrix} \frac{(\theta-1)x^*(1-\tau)}{\delta} - \frac{\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - \left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2}\right) - I + L_1(x^*)^{\beta_2} \\ \gamma_0 - \frac{x_0^{LN}(1-\tau)}{\delta} - \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_0^{LN}}{x_0^{NB}}\right)^{\beta_1} \end{bmatrix},
\end{aligned} \tag{6.65}$$

and

$$\begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix} = \tag{6.66}$$

$$\frac{1}{(x^*)^{\beta_1}\left(x_0^{NB}\right)^{\beta_2} - (x^*)^{\beta_2}\left(x_0^{NB}\right)^{\beta_1}} \begin{bmatrix} \left(x_0^{NB}\right)^{\beta_2} & -(x^*)^{\beta_2} \\ -\left(x_0^{NB}\right)^{\beta_1} & (x^*)^{\beta_1} \end{bmatrix} \times$$

$$\begin{bmatrix} \frac{(\theta-1)x^*(1-\tau)}{\delta} + \left(\eta\left(V_1^{NB}\left(x_1^{NB}\right) - R_1\left(x_1^{NB}\right)\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right)\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - I \\ \eta\left(V_0\left(x_0^{NB}\right) - R_0\left(x_0^{NB}\right)\right)\left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2} - \frac{x_0^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r} \end{bmatrix}.$$

Moreover, on the basis of (6.48) we define

$$A_{12} \equiv \left(x_1^{NB}\right)^{-\beta_2}\left(\eta\left(V_1^{NB} - R_1\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right), \tag{6.67}$$

so that $A_{12}x^{\beta_2}$ is the equityholders' value of the option to renegotiate. The implicit formulae for the optimal investment threshold, x^*, optimal renegotiation trigger, x_0^{NB}, and liquidation trigger, x_0^{LN}, are obtained by rearranging equations (6.39), (6.41) and (6.43). ■

Proof of Proposition 6.2. Proposition 2 directly results from replacing equation (6.39) by (6.47) in the system of equations (6.37)-(6.43). ■

Proof of Proposition 6.3. The optimal renegotiation trigger can be calculated on the basis of equations (6.40) and (6.41). After multiplying (6.40) by β_2 and subtracting (6.40) from (6.41) we obtain that

$$\begin{aligned} & (1-\beta_2)\frac{x_0^{NB}(1-\tau)(1-\eta(1-\rho))}{\delta} + \beta_2\frac{b}{r}(1-\tau+\eta\tau) \\ = \; & (\beta_1-\beta_2)(\eta K_0 - A_{01})\left(x_0^{NB}\right)^{\beta_1}. \end{aligned} \tag{6.68}$$

This yields

$$\begin{aligned} x_0^{NB} \; = \; & \frac{-\beta_2}{1-\beta_2}\frac{b(1-\tau+\eta\tau)\delta}{(1-\eta(1-\rho))(1-\tau)r} \\ & + \frac{\beta_1-\beta_2}{1-\beta_2}\frac{\delta(\eta K_0 - A_{01})\left(x_0^{NB}\right)^{\beta_2}}{(1-\eta(1-\rho))(1-\tau)}. \end{aligned} \tag{6.69}$$

The first row in (6.69) equals the optimal renegotiation trigger in the absence of the investment opportunity (cf. (6.34)). Consequently, x_0^{NB} is higher than such a trigger if and only if $\eta K_0 - A_{01}$ is positive. ■

Derivation of the First Passage Time Probabilities. In general, the probability that an event (i.e. bankruptcy, renegotiation or investment) will occur within the time interval of length T, denoted by $p(x,T)$, satisfies the following partial differential equation (PDE)

$$-(r-\delta)\,x\frac{\partial p}{\partial x}+\frac{1}{2}\sigma^2x^2\frac{\partial^2 p}{\partial x^2}=-\frac{\partial p}{\partial T}, \tag{6.70}$$

subject to the following boundary conditions

$$p(\underline{x},T) = a, \tag{6.71}$$

$$p(\overline{x},T) = b, \tag{6.72}$$

$$p(x,0) = 0. \tag{6.73}$$

where the lower bound, $\underline{x}$, upper bound, $\overline{x}$, and parameters a and b are given in the following matrix.

$\underline{x},\overline{x};a,b$	Probability of investment	Probability of debt restructuring
Growth option present		
Renegotiation possible	$x_0^{LN},x^*;0,1$	$x_0^{NB},x^*;1,q\left(x^*,x_1^{NB}\right)$
Bankruptcy upon default	$x_0^{B},x^*;0,1$	$x_0^{B},x^*;1,q\left(x^*,x_1^{B}\right)$
No growth option		
Renegotiation possible	-	$x_0^{NB},\infty;1,0$

The function $q(x,y)$ denotes the the probability of reaching the lower trigger y before time T conditional on starting at x. It can be obtained by applying a change of variables to Corollary B.3.4 in Musiela and Rutkowski (1998), p. 470. Consequently, it holds that

$$\begin{aligned} q(x,y) &= 1+\left(\frac{x}{y}\right)^{-\frac{2\alpha}{\sigma^2}+1}\mathbf{\Phi}\left(\frac{-\ln\frac{x}{y}+\left(\alpha-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) \\ &\quad -\mathbf{\Phi}\left(\frac{\ln\frac{x}{y}+\left(\alpha-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right), \end{aligned} \tag{6.74}$$

where $\mathbf{\Phi}(\cdot)$ denotes the standard normal cumulative density function.

As an example, let us interpret the boundary conditions for the probability of debt renegotiation in the presence of the growth option. Condition (6.71) implies that the renegotiation is certain if the level of cash flow hits the boundary x_0^{NB}. Equation (6.72) means that upon reaching the investment threshold, x^*, the renegotiation trigger switches to x_1^{NB} and the probability of renegotiation is described by (6.74). Finally, when the length of the time interval tends to zero, the probability of renegotiation approaches zero as well.

Since an analytical solution to the PDE (6.70) with boundaries (6.71)-(6.73) has not been found, a numerical procedure has to be applied. To calculate the relevant probabilities, the explicit finite difference method is used (cf. Brennan and Schwartz, 1978). ■

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Samenvatting

Investeringsmogelijkheden kunnen gezien worden als (reële) opties om kapitaalgoederen te verwerven. Een juiste identificatie van de optimale uitoefeningsstrategieën van reële opties speelt een cruciale rol in het bepalen van het optimale investeringsgedrag en in het maximaliseren van de waarde van een onderneming.

De meeste standaard leerboeken in de financiering beschrijven de netto contante waarde (Net Present Value of NPV) regel als het criterium voor het waarderen van investeringsprojecten. Volgens deze regel moet de contante waarde van de verwachte kasstroom, die gegenereerd wordt door een nieuwe fabriek of een productielijn, geschat worden. Vervolgens moeten de uitgaven, die noodzakelijk zijn om de fabriek of de nieuwe productlijn te lanceren, van deze kasstroom afgetrokken worden. Een positief verschil (een positieve NPV) impliceert dat het project uitgevoerd zou moeten worden.

Zoals aangegeven door Dixit en Pindyck (1996) leidt het NPV-criterium alleen tot optimaal investeringsgedrag wanneer een van de volgende cruciale en dikwijls genegeerde aannames gelden: de investering is ofwel helemaal omkeerbaar (in dit geval kan het geïnvesteerde geld teruggekregen worden indien de toestand van de markt ex post slechter is dan verwacht), of de investering is een nu-of-nooit beslissing. In de meeste gevallen wordt aan geen van de bovenvermelde voorwaarden voldaan. In feite geldt voor de meeste investeringsprojecten dat ze onomkeerbaar zijn, dat de resulterende kasstroom onzeker is en dat het mogelijk is de investering uit te stellen.

Onomkeerbaarheid betekent dat de investeringskosten verzonken kosten zijn. Dit houdt in dat het onmogelijk is om de investeringskosten terug te krijgen nadat de investering is gedaan. Daarom is de investeringsuitgave equivalent aan de uitoefeningsprijs van een financiële optie. Onomkeerbaarheid is een gevolg van tenminste een van de volgende drie factoren: het investeringsproject is alleen van belang voor het betreffende bedrijf of industrie, of er is sprake van adverse selection op de tweedehands markt voor deze goederen.

In de meeste situaties is de kasstroom die voortvloeit uit een investeringsproject onzeker. In veel gevallen worden de opbrengsten van de investering in een nieuw product beïnvloed door onzekerheid in de productmarkt. De waarde van een investeringsproject in de oliesector is een functie van de variërende olieprijs op de internationale markt. In de literatuur wordt aangenomen dat de ontwikkeling van de huidige waarde van deze kasstroom beschreven kan worden door een stochastisch proces. Daarom spelen bij het bepalen van de waarde van een investeringsmogelijkheid dezelfde soort effecten een rol als bij een financiële optie.

Voor de meeste projecten bestaat er een mogelijkheid om de investering uit te stellen. Uitstel is in principe kostbaar omdat de onderneming tot het tijdsstip van de investering geen opbrengsten heeft. Het voordeel van wachten met investeren is echter dat de onderneming meer informatie over de waarde van het project kan vergaren alvorens tot investeren over te gaan. Een soortgelijke trade-off speelt ook een rol bij de optimale uitoefeningsbeslissing van een Amerikaanse optie.

De noodzaak van het ontwikkelen van waarderingsmodellen die investeringskenmerken als onomkeerbaarheid, onzekerheid, timing en flexibiliteit in het beslissingsproces opnemen, heeft geresulteerd in een groot aantal publicaties op het gebied van reële opties en investeren onder onzekerheid (zie o.a. Myers, 1977, Brennan en Schwartz, 1985, McDonald en Siegel, 1986, Dixit, 1989, en een gedetailleerd overzicht door Dixit en Pindyck, 1996).

Reële optiemodellen kunnen niet alleen gebruikt worden om de waarde van een investeringsproject te berekenen, maar ook om het optimale investeringsbeleid van een onderneming te bepalen. In veel situaties moeten de modellen bekend van de financiële optieliteratuur uitgebreid en aangepast worden om rekening te houden met de economische omgeving. Hierbij valt te denken aan exogene discrete veranderingen in de economische omgeving, strategische interacties tussen ondernemingen of de financieringsaspecten van een investeringsproject.

De huidige literatuur verschaft betrekkelijk weinig inzicht in de invloed van structurele veranderingen van de economische omgeving op het investeringsgedrag van de onderneming. Bestaande artikelen analyseren meestal continue veranderingen in de waarde van een relevante economische variabele. Evenwel is het vaak realistischer om de economische variabele te modelleren als een proces dat op bepaalde tijdstippen discrete sprongen maakt. In zulke gevallen wordt er gebruik gemaakt van een Poisson (sprong) proces. Dit gebeurt bijvoorbeeld in Hassett en Metcalf (1999), waarin de invloed van een verwachte reductie van een investeringssubsidie geanalyseerd wordt.

Het uitgebreide proces van deregulatie en de golf van fusies en overnames die plaatsvonden in het vorige decennium hebben geresulteerd in een oligopolistische marktstructuur in een groot aantal sectoren. Imperfecte concurrentie in de productmarkt impliceert dat de onderneming rekening moet houden met strategische interacties met andere marktparticipanten. Het opnemen van imperfecte concurrentie in reële optiemodellen vereist dat het resulterende model gebaseerd moet zijn op bijdragen op het gebied van timing spelen in de niet-coöperatieve speltheorie (zie o.a. Reinganum, 1981, en Fudenberg en Tirole, 1985).

In geval een investeringsproject wordt gefinancierd met vreemd vermogen zouden er twee soorten agencyproblemen kunnen ontstaan, die suboptimaal investeringsgedrag tot gevolg kunnen hebben. Ten eerste leidt financiering met vreemd vermogen tot een keuze van riskantere projecten, hetgeen het welzijn van de houders van vreemd vermogen reduceert (Jensen en Meckling (1976)). Een ander effect van financiering met vreemd vermogen op het investeringsgedrag van de onderneming is beschreven door Myers (1977). Hij heeft aangetoond dat de investering ondernomen door de houders van eigen vermogen gepaard gaat met een welzijnstransfer aan de houders van vreemd vermogen. Deze transfer impliceert dat sommige goede investeringsprojecten (waarvoor geldt dat de NPV niet opweegt tegen de welzijnstransfer) niet uitgevoerd worden.

In dit proefschrift worden de drie bovenvermelde aspecten van reële opties geanalyseerd.

In Hoofdstuk 2 ontwikkelen we een niet-strategisch model waarin de invloed van een plotselinge beleidsverandering op het investeringsgedrag geanalyseerd wordt. Voorbeelden van zo'n beleidsverandering betreffen het opheffen van een investeringssubsidie of een verandering in de voorkeursbehandeling van een buitenlandse investeerder. In het model leidt de beleidsverandering tot een opwaartse sprong in de effectieve investeringskosten (zie Hassett en Metcalf, 1999). De sprong vindt plaats op het moment dat de waarde van het project een bovengrens bereikt. De onderneming heeft incomplete informatie over de drempelwaarde van het proces waar de sprong plaatsvindt, en actualiseert haar schatting betreffende die drempelwaarde middels de regel van Bayes. De invloed van de onzekerheid aangaande het moment van de beleidsverandering kan geanalyseerd worden door het effect te bepalen van een verandering in de variantie van de onderliggende waarschijnlijkheidsverdeling. In dit hoofdstuk wordt de optimale investeringsdrempel die de waarde van de onderneming maximaliseert afgeleid. Verder wordt aangetoond dat de drempel een niet monotone functie is van de mate van beleidsonzekerheid.

Hoofdstuk 3 analyseert de beslissing van de onderneming om een bestaande technologie te vervangen door een nieuwe, kostenefficiëntere versie. Kulatilaka en Perotti (1998) leiden af dat, binnen een tweeperioden model, een stijgende productmarkt onzekerheid de onderneming kan aanmoedigen om eerder strategisch te investeren in een nieuwe technologie. We breiden hun raamwerk uit tot een model in continue tijd en tonen aan dat, in tegenstelling tot het tweeperioden model, meer onzekerheid impliceert dat de onderneming naar verwachting later investeert. Daarnaast wordt aangetoond dat onder stijgende onzekerheid de waarschijnlijkheid van een optimale vervanging van het productiegoed binnen een bepaalde periode altijd daalt, indien de betreffende periode het optimale deterministische vervangingstijdstip omvat. Voor kortere periodes zijn er tegenovergestelde effecten in werking, die bewerkstelligen dat de verhouding tussen onzekerheid en de investeringswaarschijnlijkheid een omgekeerd U-vorm heeft (zie ook Sarkar, 2000).

In Hoofdstuk 4 wordt een model bekeken met twee ondernemingen die verschillende investeringskosten hebben. We analyseren de invloed van de onderlinge verschillen in die investeringskosten op de ondernemingswaarde en op de optimale investeringstijdstippen. Beide ondernemingen hebben de mogelijkheid om te investeren in een project dat ceteris paribus de kasstroom verbetert. We tonen aan dat drie soorten evenwichten bestaan. Bovendien bepalen we de kritische niveaus van de kostenasymmetrie die de bestaansregio's van de evenwichten begrenzen. De aanwezigheid van strategische interacties leidt tot contra-intuïtieve resultaten. Ten eerste kan een marginale toename in de investeringskosten van de onderneming met het kostennadeel een toename in de waarde van deze onderneming veroorzaken. Ten tweede kan zo'n kostenstijging leiden tot een daling van de marktwaarde van de concurrent. Vervolgens bespreken we de welzijnsimplicaties van het optimale investeringsgedrag en tonen aan dat kostenasymmetrie kan leiden tot een sociaal meer gewenste uitkomst. Tenslotte bewijzen we dat winstonzekerheid altijd leidt tot uitstel van de investering. Dit laatste geldt zelfs in een situatie waarin het zeer gewenst is om eerder te investeren dan de andere onderneming.

In Hoofdstuk 5 wordt de waarde van flexibiliteit in strategische kwaliteitskeuze bekeken. Ondernemingen beslissen over de kwaliteit van hun producten op het moment dat ze een productmarkt betreden. Flexibiliteit in kwaliteitskeuze impliceert ceteris paribus dat eerder investeren optimaal is. Verder wordt afgeleid dat de waarde van flexibele kwaliteit toeneemt als er sprake is van meer onzekerheid in de vraag en/of bij aanwezigheid van een potentiële concurrent. We tonen aan dat flexibele kwaliteit ook dienst kan doen als afschrikking van potentiële concurrentie, waarbij het niet eens nodig is om het kwaliteitsniveau af te laten wijken van het optimale monopolistische niveau. In bestaande reële optiemodellen is het bepalen van het optimale investeringsmoment vaak de enige beslissing die genomen moet worden. In het onderhavige model komt daar de kwaliteitsbeslissing bij. Wij tonen aan dat dit impliceert dat de timing van investeren van de tweede investeerder van invloed is op het investeringstijdstip van de leider, hetgeen normaal gesproken niet het geval is in reële optiemodellen waarin de rollen van eerste en tweede investeerder vastliggen. Dit betekent een uitbreiding van de theorie van strategische reële opties. Tenslotte tonen we aan dat als de vraag groot is de leider de mogelijkheid heeft om door een "agressieve" kwaliteitskeuze de volger uit de markt te stoten.

Hoofdstuk 6 analyseert het optimale investerings- en liquidatiebeleid van de onderneming wanneer financiering met vreemd vermogen en heronderhandelen van het oorspronkelijke schuldcontract mogelijk zijn. We tonen aan dat de aanwezigheid van de optie tot heronderhandelen ("zachte schuld") het onderinvesteringsprobleem beschreven door Myers (1977) versterkt. De nadelige invloed van de optie tot heronderhandelen op het investeringsbeleid wordt veroorzaakt door het feit dat op het moment van de investering deze optie de welzijnstransfer aan de houders van vreemd vermogen verhoogt. Dit is het gevolg van een significante reductie in de waarschijnlijkheid van een strategische wanbetaling die voorkomt op het moment van de investering. Bovendien vinden we dat, als financiering met vreemd vermogen mogelijk is, het liquidatiebeleid verschilt van het optimale liquidatiebeleid onder volledige financiering met eigen vermogen. Zelfs als we belasting wegdenken, hetgeen effecten van het belastingsschild elimineert, wordt het liquidatiebeleid beïnvloed door het second-best investeringsbeleid. Dit impliceert dat liquidatie te vroeg zal plaatsvinden. Ook wordt de invloed van de groei optie op de optimale timing van faillissement en heronderhandelingen geanalyseerd. Aangetoond wordt dat een combinatie van hoge onderhandelingsmacht van de houders van eigen vermogen met de aanwezigheid van groei opties kan leiden tot een grotere waarschijnlijkheid van strategische wanbetaling.

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