Welfare Trade-offs between Transferable and Non-Transferable Lotteries

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Abstract

The Four Rivers lottery run by the National Forest Service distributes the opportunity to raft four sections of rivers in Idaho through a non-transferable lottery. The restriciton of trade and focus on equity in distribution creates a deadweight loss in total surplus compared with a market or auction system. If the NFS allowed the transfering of permits, then there exists a potential for rafters to gain surplus in trade. However, non-rafters have an incentive to enter the transferable lottery to make a profit from trade. Using the NFS lottery as a guide, this paper examines welfare under the two lottery system to understand how changes in transferability affect the welfare of users and non-users, and the revenues of the government. Since variables, such as number of permits, permit fees, and application fees, also impact welfare, we derive comparative statics for these variables to demonstrate how these government controls affect rafter welfare, non-rafter welfare, and government revenue differently under transferable and non-transferable lotteries. Our results show the welfare trade-offs rafters have between transferable and non-transferable lotteries.

1 Introduction

In 2003, the National Forest Service (NFS) had 205 million visitors to forest and grassland areas (NFS 2004). As the number of outdoor enthusiasts and recreationers increases so does damage to the environment. Conservation through limiting the number of visitors becomes a priority in order to minimize damage and overuse.

There exist several mechanisms to distribute resource access. Auctions distribute limited goods to the highest bidder, queues to those willing to wait the longest, and merit or preference programs to those most deserving of the good. Lotteries provide an allocation mechanism which allows everyone an equal chance at resource access regardless of age, experience, merit, or income. Examples of a lottery distribution include hunting permits, river rafting permits, and hiking permits.

The Four Rivers Lottery run by the NFS demonstrates an example of conservation and restriction of resource access through a lottery. The Selway, Snake-Hell's Canyon, the Middle Salmon, and Main Fork of the Salmon have become four popular rivers in the rafting community for multiday float trips. Known for their scenic views and fast waters, each of these sections of rivers wander through federally protected wilderness areas, set aside for conservation of wildlife. The government controls access to the river to protect the beaches and waterways from overuse and exploitation. Only about 60 trips a season raft down the Selway without costly damage to the area. For the Salmon, Middle Fork, and Snake sections, only about 300 to 400 trips travel downstream per season.

The people who value the resource the most may or may not obtain a permit. Loomis (1982) evaluates the pricing system against the lottery system to demonstrate the total benefits of each, and the loss of benefits from using a lottery. In a pricing market, those willing to pay the most get the good, while lotteries may allocate goods to consumers with a marginal willingness to pay (WTP) lower than the market value of the good. This creates a deadweight loss and inefficiency in allocation of resources in terms of total welfare. Loomis (1980) finds that deadweight causes a benefits loss of 43% because of the focus on equity rather than optimization.

Although the government uses a lottery to distribute permits for equity purposes, the agency further restricts the permit holder by requiring the winner to be present at the launch site on the issue date of the permit. In other words, the permit holder cannot use the permit on another day nor can he resell the permit to another rafter who might value the river trip more. Because of this restriction, the permits and therefore resource access are defined as non-transferable. This restriction keeps outside speculators from entering the lottery and reselling permits for a profit. However, rafters do have some benefit to having transferable permits. They can gain surplus through trade, and those with the highest value for the rafting experience end up with the permits.

Some papers have looked at the benefits of secondary markets and trading permits. Eichberger, Guth, and Muller (2003) compare, both theoretically and experimentally, the attitudes toward risk in a repeated lottery with and without the option to sell the good after the lottery has been won. Their subjects show little risk aversion, but put a high value on the option to sell in the second decision stage. Weitzman (1974) looks at welfare gains from trading permits, and examines the benefits of using prices or quantities to control pollution. Given this trade-off between the two methods, he suggests that a mixed strategy may optimize welfare in some cases. Sandrey, Buccola, and Brown (1983) examine the market for elk hunting permits in order to recommend better pricing policy strategies. They demonstrate the negative effects for hunters with a relatively low WTP. Boyce (1994) develops a model to compare auctions to transferable and non-transferable lotteries. He shows that rebate offers, which come from an auction or lottery proceeds, cause some participants to prefer a transferable lottery or auction over a non-transferable lottery.

Although previous research has shown the trade-off between transerability and non-transferability, these studies did not consider the additional impact of changes to the lottery system. Variables such as application fee, permit fee and number of available permits also impact welfare, the demand for permits, the user and non-user surplus, and government revenue. Increases in permit fees can cause applicants to drop out, while increases in the number of permits can increase demand, but cause damage to the environment.

Several studies have examined the impacts of policy changes on welfare. Nickerson (1990) measures how regulation in the management of big game hunting affects the amount of lottery applications. Creel and Loomis (1992) examine the demand for hunting when a policy on bag limits constrain the possible amount of hunting. They develop an econometric model that accounts for this bag limit and compare it against models that do not. Scrogin (2005) developed an individual model and empirically tested it using data from the New Mexico Department of Game and Fish for quota hunts on public lands for deer, elk , antelope, bighorn sheep, wild pig, bison, ibex, and oryx. He showed that changes in quality and quantity, due to policy adjustments, can affect an individual's WTP both adversely and favorably. Buschena, Anderson, and Leonard (2001) examine a lottery system where applicants compete by accumulating preference points.

Because of this unique allocation system, their study estimates the impact of different hunt characteristics on the value of permits. Scrogin, Berrens, and Bohara (2000) examine the effects of a change in a lottery program designed to increase participation, such as reduced participation fees and increased permit availability. They measure consumer welfare using the Marshallian surplus and a proposed measure which accounts for the probability of winning the lottery. Both measures show a significant increase in consumer welfare with the policy changes.

This paper compares transferable and non-transferable lotteries, and analyzes the impact on welfare from changes in the lottery system. We develop a measures for rafter and non-rafter welfare, similar to the measure used by Scrogin, Berrens, and Bohara (2000). We then compare welfare under transferability and non-transferability, and include an examination of government revenue, which previous research has not analyzed. The model of welfare demonstrates conditions for when users, non-users, and the government prefer a transferable lottery to a non-transferable lottery. Specifically, it shows the welfare trade-offs that rafters have between the two systems. Furthermore, we add to previous research by examining changes in control variables, such as application and permit fees, and the number of permits, to study how they affect welfare and revenue with and without the transfer restriction.

The paper proceeds as follows: Section 2 describes the four rivers lottery system. The model developed in section 3 measures rafter surplus and government revenue for a lottery when permits are non-transferable. Section 4 measures rafter and non-rafter surplus, and government revenue when the lottery allows permit transfers. Section 5 compares welfare under transferability and non-transferability to see when preferences change.

Additionally, we analyze comparative statics caused by changes in fees and permit availability. Finally, section 6 discusses the implications from these results and further possible research.

2 Overview of the Four Rivers Lottery

Although the requirement of trip permits exists year round on the Snake, Middle Fork, and Main Salmon, lottery permits control traffic during parts of the year with higher demand. For the Selway, lottery permits restrict access during May 15th to July 31st. The Snake and Middle Fork of the Salmon enforce lottery permits from late May through the middle of September. Finally, the Main Salmon requires lottery permits from June 20th through the middle of Septemeber. From this point further, we refer to lottery permits as simply permits.

In order to boat any one of the four rivers, rafters must apply to the same lottery, making the Four Rivers Lottery unique. Each year the application process starts December 1st, and ends January 31st. Applicants choose their top four picks of launch dates, and what river they prefer to boat for each launch date. For example, one rafter may choose June 1-4 as his top four choices for the Middle Fork. Popular dates and river combinations decrease the odds of winning, and an applicant may prefer to increase his odds of winning by choosing different rivers, such as June 1 or 2 for the Middle Fork and July 16 or 17 for the Main Salmon. Each river and launch date provides a different experience for rafters. Early season trips have high, fast flowing water that provides challenging and adventurous whitewater. However, other rafters prefer a more relaxing float with milder rapids and warm water provided by the late season. Each applicant states his preferences for rivers and dates, and if drawn, obtains a permit based on preferences and availability.

Table 1. Number of applications submitted, permit allocated, and percentage of winning for each river in 2006.

River Section	Submitted	Allocated	%
Main Salmon	3418	310	9.07
Middle Fork	10627	387	3.64
Snake	1058	324	30.62
Selway	1728	62	3.59
Total	16831	1083	6.43

Source: National Forest Service http://www.fs.fed.us/r4/sc/recreation/4rivers/stats.pdf (accessed April 19, 2007).

The lottery administration randomly selects winners and matches their choices with available permits. In the event that all choices of the selected winner have been filled, the lottery draws another winner. This selection process repeats until all permits have been distributed. The probability of winning a permit and the number of applicants for each section shows the differences in demand for each river. For instance, winning a permit on the Snake usually has the odds of 1 in 3, while winning a Middle Fork permit has odds around 1 in 27. Table 1 shows the summary statistics for the Four Rivers Lottery for the 2006 season. Allocated permits are considerably lower than the number of submitted applications. While each river has unique features, this table demonstrates that regardless, the number of available permits far exceeds the number of rafters who positively value them and submit applications.

The application process includes a non-refundable $6 fee, which covers the government's cost of administering the lottery. Furthermore, the Middle Fork and Main Salmon sections require a permit or boating use fee to boat on the waters for each person. The total cost of this fee varies on the group size of boaters. Having an annual National Parks pass also reduces the total cost of the boating fee. This money helps maintain facilities, and protect natural resources (NFS). The application fee and the boating fee generate the total cost to boating, not including personal and trip expenses.

The management plans for each river operate differently regarding cancellations, open dates, and waiting lists. However, there exists a no-show penalty or cost to rafters not present on launch dates. For the Selway and Snake, a one-year ban from the lottery penalizes rafters, while the Middle Fork and Main Salmon have a three year penalty. Park rangers verify that all rafting groups putting in at the launch site have a permit.

To model welfare, we consider three major agents; government, rafters, and non-rafters. The government determines the method of allocation for the permits, the number of permits, and the fees charged. Based on several different criteria, such as conservation of habitat, availability of beaches, and water flow, the government determines the optimal number of permits to allocate. Permits allow a person the right to raft the river. The rafters gain or lose surplus based on how the government allocates and charges fees for the permits. Non-rafters seek to exploit any possible rents with a high WTP by reselling the permit they win to a rafter.

The following section explains the model for rafter welfare and government revenue mathematically and graphically for the case of non-transferable permits. Building this initial case allows us to later extend the model to the transferable case and compare welfare comparative statics under both scenarios. From the two scenarios, we study the trade-offs of welfare that rafters face under different lottery systems.

3 Rafter Welfare and Government Revenue Measures under Non–transferable Permits

The current lottery system run by the NFS does not allow for the transfer of permits. This restriction creates welfare inefficiency, since those who value the rafting experience the most do not necessarily obtain a permit. However, disallowing transferring and trading of permits prevents outside speculators from entering the market. With a secondary market allowed, non-rafters have an incentive to apply for a permit with the intention of reselling to a rafter for a profit. An increase to the number of non-rafters entering the lottery, decreases a rafter's chance at winning as well as his expected surplus. By making the permits non-transferable, the government and rafters effectively deter the non-rafter's profit seeking behavior, but lose welfare by not being able to trade. To examine this trade-off, this section examines a model to analyze rafter welfare and government revenue under a lottery with permit transfers prohibited. In this scenario, rafters cannot sell or trade their permits to other rafters for a more preferred date.

In order to develop an estimate of welfare for the rafters, we begin by examining the rafter's value for permits, or in other words the rafter's value for the experience of a trip down the river. We define a rafter as any person willing to pay a positive price to obtain the right to raft one of the wild rivers. Let $v_i(q)$ determine the value of a permit for consumer i minus travel costs or the net value of a float trip, where q represents the number of permits used. Since each consumer knows he can only have one permit, the market demand for permits, $v(q)$, in a way, orders the rafters by their WTP for one permit. Ranking the rafters from highest to lowest by their WTP gives the downward sloping aggregate demand curve, $v(q)$. We assume the aggregate demand of the permits has the linear functional form:

$$v(q) = \alpha - \beta \cdot q - \gamma \cdot \bar{q} \tag{1}$$

where q notates the quantity demanded, α, $\beta > 0$ are constants, and $\bar{q}$ represents the number of permits. As the number of people on the river increases with the number of permits, the value of the river experience decreases and this is captured by the constant $\gamma > 0$ as a congestion parameter.

Given the uncertainty with a lottery, the probability of winning affects the expected value of the permit as well as rafter welfare. The expected value of a permit resembles the value except that the lottery creates uncertainty in obtaining the permit. In order to account for this uncertainty, expected value takes into consideration the probability of winning, which depends on $\bar{q}$ and the number of applicants, q_n. Then the probability of winning in the non-transferable lottery is notated as $\delta_{nt} = \frac{\bar{q}}{q_n}$. The expected aggregate value of the permit equals the probability of winning multiplied by the value of the permit and written as:

$$\delta_{nt} \cdot v(q) = \delta_{nt} \cdot (\alpha - \beta \cdot q - \gamma \cdot \bar{q}) \tag{2}$$

The cost of the application fee and uncertain permit fee affect the expected cost as well as rafter welfare. Let f_a denote the application fee and f_p notate the permit fee. The total cost to raft equals $f_a + f_p$. For simplification, we assume that the government always sets the application and permit fees such that market value exceeds total cost, $\alpha - \beta \cdot \bar{q} - \gamma \cdot \bar{q} > f_a + f_p$. With this assumption the lottery adds uncertainty, and the expected cost becomes $f_a + \delta_n t \cdot f_p$. This setup implies that number of applicants, q_n, becomes a function of government controlled variables, $\bar{q}$, f_p, and f_a.

$$q_n = g\left(\bar{q}, f_p, f_a\right) \tag{3}$$

Figure 1. Equilibrium Number of Rafters in a Non-Transferable Lottery

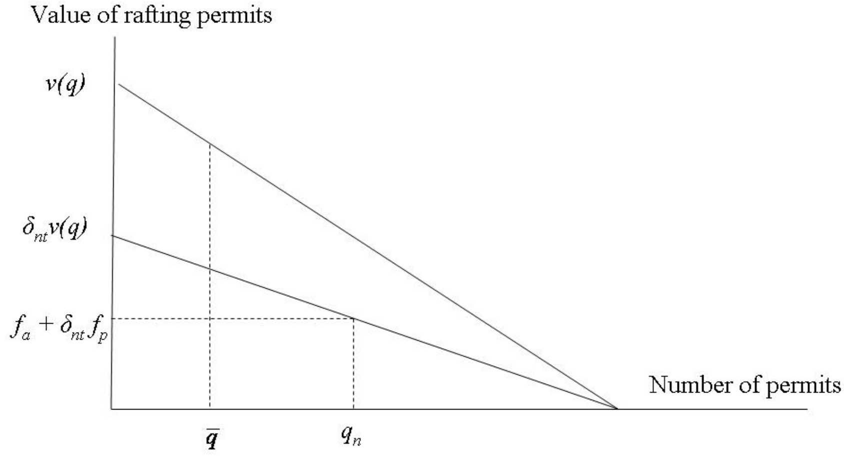


Figure 1 illustrates the equilibrium number of rafters in a non-transferable lottery. Again, the aggregate value of permits orders all of the rafters by the WTP from highest to lowest, which generates the downward slope of $v(q)$. Because of the uncertainty of obtaining a permit caused by the lottery, the expected aggregate value of permits takes into consideration the probability of winning which ranges between 0 and 1. Thus, the expected benefits from entering the lottery $\delta_{nt} \cdot v(q)$, must lie below $v(q)$. The expected cost, $f_a + \delta_{nt} \cdot f_p$, for the rafter includes the application fee, f_a, and the chance of paying the permit fee, f_p. The government determines the appropriate $\bar{q}$, that maintains the wilderness of the environment. Because so many people place a high value on rafting in the wilderness areas, the number of permits is set less than the number of applicants, $\bar{q} < q_n$.

The marginal rafter has a cost equal to his expected value, $\delta_{nt} \cdot v(q)$. Ordering the rafters by their WTP shows that rafters who apply have a WTP higher than the marginal rafter. Rafters with a WTP lower than the marginal rafter decline to enter the market. Thus, equating expected cost with expected value gives a solution for the number of applicants.

$$\delta_{nt} \cdot (\alpha - \beta \cdot q - \gamma \cdot \bar{q}) = f_a + \delta_{nt} \cdot f_p \tag{4}$$

Since $\delta_{nt} = \frac{\bar{q}}{q_n}$, we derive equation (5), the number of applicants, in terms of the number of permits, permit fee, application fee, and parameters α, β, γ from equation (4).

$$q_n = \frac{\bar{q} \cdot (\alpha - \gamma \cdot \bar{q} - f_p)}{\beta \cdot \bar{q} + f_a} \tag{5}$$

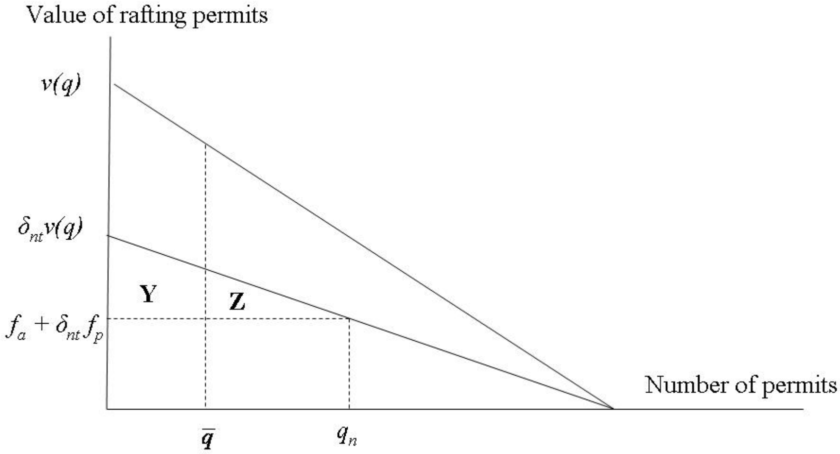
If every rafter obtained a permit, the maximum surplus possible would equal the area in figure 1 under $v(q)$ between 0 and q_n. Since the lottery randomly determines which applicants receive permits, we must calculate an expectation of rafter surplus rather than the actual total rafter surplus.

Measuring expected rafter surplus in the non-transferable lottery allows us to compare welfare in the transferable lottery. After the calculation of welfare, we can then determine how government control variables affect welfare under both scenarios. This comparison allows us to see the trade-off that rafters have between the two allocation mechanisms.

3.1 Welfare measures with non-transferable permits.

In this section, we calculate rafter welfare along with government revenue in the non-transferable lottery. To determine the expected rafter surplus before the drawing has taken place, expected surplus must take into consideration the probability of winning. In order to award applicants, the NFS contracts out the lottery assignment task to a statistical company, who randomly generates the winners. We assume that the lottery distributes permits randomly by using a uniform distribution for the probability of winning a permit, meaning each rafter has the same probability of winning.

Figure 2. Rafter Welfare under a Non-transferable Lottery



To measure expected rafter surplus (RS), we begin by calculating an individual's expected surplus, and then aggregate over all individuals. The expected value to rafter i is calculated from the expected value of winning minus the expected costs for the individual, and written as

$$E\left(v_i|\bar{q}, f_a, f_p\right) = \delta_{nt} \cdot v_i(q) - \left(f_a + \delta_{nt} \cdot f_p\right) \tag{6}$$

Figure 2 modifies figure 1 by including the area of expected surplus for rafters. The aggregation of individual surplus, equation (6), equals the area under the expected value curve, $\delta_{nt} \cdot v(q)$, between 0 and the number of applicants, q_n, minus the expected cost from the application and permit fees for each applicant. This rafter surplus estimate measures lottery-allocated welfare in a manner similar to the model developed by Scrogin, Berrens, and Bohara (2000), which takes into consideration the uncertainty of obtaining a permit. Area Y and Z in the above figure represent this measure of welfare. Plugging in the value curve, equation (1), the number of applicants, equation (5), probability, and integrating, we write RS in terms of those controlled factors, q_n, f_a, f_p, and the parameters α, β, γ.

$$\begin{aligned} RS_{nt} &= \sum_{i=0}^{n} E\left(v_i|\bar{q},\alpha,\beta\right) \\ &= \delta_{nt}\int_0^{q_n} v(q)\,dq - q_n\cdot(f_a+\delta_{nt}\cdot f_p) \\ &= \bar{q}\cdot\left[\frac{\gamma\bar{q}\cdot f_a+(\beta\bar{q}\cdot(\alpha-f_p))}{2\cdot(f_a+\beta\cdot\bar{q})}\right] \end{aligned} \quad (7)$$

While the government agency determines the allocation method, their objectives can have many dimensions. These include covering the cost of operation, equity in allocation, and river conservation. Although their objectives affect the mechanism used, we only examine revenues generated from the lottery.

The total revenue to the government agency comes from two sources: the application fees, and the permit fees. Revenue from the applications comes from everyone entering the lottery, while the revenue from permits only come from the $\bar{q}$ winners. Thus, the government revenue (GR) can be written in terms of the controlled variables.

$$\begin{aligned} GR_{nt} &= (q_n\cdot f_a)+(f_p\cdot\bar{q}) \\ &= \bar{q}\cdot\left(f_p+\frac{f_a\left(\alpha-\gamma\cdot\bar{q}-f_p\right)}{f_a+\beta\cdot\bar{q}}\right) \end{aligned} \quad (8)$$

The number of permits, $\bar{q}$, application fee, f_a, and permit fee, f_p affect the amount of welfare rafters and the government receives in the non-transferable case. From equation (8) and equation (10), we can derive comparative statics, which allow us to examine the impacts on welfare caused by changes in these control variables. These results can then be compared to the transferable case in order to understand the trade-offs between the two scenarios.

3.2 Effect of Government Controls on Welfare under non-transferability

The NFS can affect the total welfare by changing $\bar{q}$, f_a, and f_p, thereby changing rafter surplus and government revenue. As expected, RS decreases for increases in f_a and f_p. For any fee increase, expected cost increases $f_a + \delta_{nt} \cdot f_p$, decreasing the surplus for an individual. Although an increased fee causes the number of applicants to decrease and a higher probability of winning, the increase in cost does not offset the increase in the expected value.

$$\frac{\partial RS_{nt}}{\partial f_a} = -\frac{\beta \bar{q}^2 \cdot (\alpha - f_p - \gamma \bar{q})}{2\,(f_a + \beta \bar{q})^2} < 0 \tag{9}$$

$$\frac{\partial RS_{nt}}{\partial f_p} = -\frac{\beta \bar{q}^2}{2\,(f_a + \beta \bar{q})} < 0 \tag{10}$$

A positive increase in f_a or f_p causes an increase the government revenue in a non-transferable lottery. These results appear consistent with expectations, because increases in fees decrease the number of applicants but increase the probability of winning. As that probability increases, applicants re-enter until an equilibrium is reached, and revenue generated increases.

$$\frac{\partial GR_{nt}}{\partial f_a} = \frac{\beta \bar{q}^2\,(\alpha - \gamma \bar{q} - f_p)}{(f_a + \beta \bar{q})^2} > 0 \tag{11}$$

$$\frac{\partial GR_{nt}}{\partial f_p} = \bar{q} \cdot \left(1 - \frac{f_a}{(f_a + \beta \bar{q})}\right) > 0 \tag{12}$$

When the number of available permits increase, an increase inRS depends on the probability of winning, δ_{nt}. If the change in δ_{nt} is positive,

$$\frac{\partial \delta_{nt}}{\partial \bar{q}} = \frac{(\beta \bar{q} \cdot (\alpha - f_p) - \gamma \bar{q} \cdot f_a)}{(f_a + \beta \bar{q})^2} > 0$$

then RS will also increase. Expected value, $\delta_{nt} \cdot v(q)$, and the expected cost, $f_a + \delta_{nt} \cdot f_p$, shifts upwards and more applicants enter the lottery until expected value equals expected cost again. However, the expected value and expected cost functions do not shift proportionally, because cost includes the certainty of having to pay an application fee. Thus, expected value increases more, relative to the increase in expected cost, causing an increase in RS. A decrease in probability has the opposite affect.

$$\frac{\partial RS_{nt}}{\partial \bar{q}} = \frac{(2f_a + \beta\bar{q})\left[\beta\bar{q}\cdot(\alpha - f_p) - \gamma\bar{q}\cdot f_a\right]}{2\left(f_a + \beta\bar{q}\right)^2} \begin{matrix} > \\ - \\ < \end{matrix}\ 0 \tag{13}$$

An increase in the number of permits, $\bar{q}$, also has an uncertain affect on government revenue. If changes in $\bar{q}$ causes the probability of winning to increase more applicants enter, but an increase in $\bar{q}$ decreases the value of the permits through congestion, γ. These counteracting forces create the uncertain comparative static.

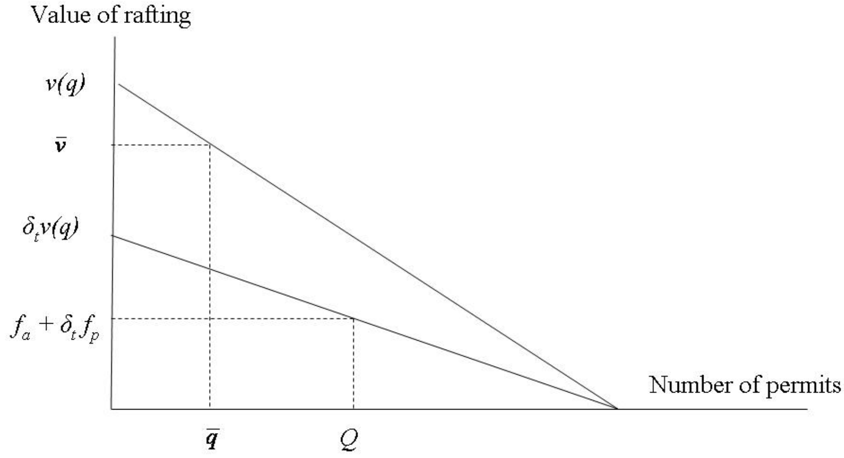
$$\frac{\partial GR_{nt}}{\partial \bar{q}} = f_p + \frac{f_a\left[f_a\left(\alpha - 2\cdot\gamma\bar{q} - f_p\right) - \beta\bar{q}\cdot\gamma\bar{q}\right]}{\left(f_a + \beta\bar{q}\right)^2} \begin{matrix} > \\ - \\ < \end{matrix} 0 \tag{14}$$

These comparative statics can then be compared with welfare measures and comparative statics under a transferable lottery. This analysis provides insight into the welfare trade-offs between the transferable and non-transferable lottery.

4 Rafter Welfare and Government Revenue Measures under Transferable Permits

Rafters have an incentive to keep non-rafters from entering a lottery for permits, because non-rafters decrease the probability of winning and the amount of potential welfare. However, in a transferable lottery, rafters gain surplus from trading as well as welfare efficiency. To see this trade-off, consider now the lottery scenario with transferable permits. The distribution of permits remains the same as before, but with transferable permits, rafters who win can trade the permit for a more preferred launch date, or they can sell the permit to another rafter who has a higher WTP.

Figure 3. Resale Value of a Permit



The ability to sell the permits also provides non-rafters with an incentive to enter the lottery for the purposes of resale. This section examines what happens to rafter welfare, non-rafter welfare, and government revenues if permits were made tradable.

Since non-rafters now have an incentive to enter the lottery, we define non-rafters as any person willing to enter the rafting market without any intention of rafting. They intend to gain rents from the lottery allocation and ability to transfer permits. The non-rafter resells his permit to a rafter with a high WTP and, hopefully, profits from it. Furthermore, rafters with a low WTP would rather resell their permits than raft, because their surplus is greater from selling than from rafting. They in essence become non-rafters due their low WTP and the ability to trade permits.

Figure 3 remains similar to the previous graphs, but now the probability of winning in the transferable lottery, $\delta_t = \bar{q}/(q^{nr} + q^{r})$, takes into consideration that both rafters, q^{r}, and non-rafters, q^{nr}, enter the market, or the total number of applicants, $Q = q^{nr} + q^{r}$. In the non-transferable scenario, the value for permits and the value for rafting were interchangeable. However, under transferability, the rafters' value for permits may change, but his value for rafting stays the same. For simplification we consider only, the value of rafting as $v(q)$.

Since $\bar{q}$ represents the number of permits, $v(\bar{q})$ represents the value of rafting for the $\bar{q}$th rafter. The resell price of a permit, denoted $\bar{v}$, becomes reasonable for rafters, non-rafters, buyers, and sellers. The first $\bar{q}$ number of rafters in order of WTP, who did not win a permit from the lottery, purchase permits at a price of $\bar{v}$. Any price higher would cause an excess supply. Rafters with a low WTP ($v_i(q) < \bar{v}$), or non-rafters who win, sell their permits for $\bar{v}$ because anything lower will cause excess demand.

Now consider the non-rafter who enters the lottery. He does so only if his expected value from entering the lottery and being able to resell the permit exceeds his expected cost. Thus, the marginal non-rafter equates expected value of resale equal to the expected cost.

$$\delta_t \cdot (\alpha - \beta\bar{q} - \gamma\bar{q}) = f_a + \delta_t \cdot f_p \tag{15}$$

Plugging δ_t into equation (15) and rewriting we solve for the total number of applicants in terms of the control variables.

$$Q = q^{nr} + q^r = \frac{\bar{q}}{f_a}(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p) \tag{16}$$

The rafters continue to enter the market until expected costs equal the expected benefits but now the probability of winning includes the non-rafters.

$$\delta_t \cdot (\alpha - \beta q^r - \gamma\bar{q}) = f_a + \delta_t \cdot f_p \tag{17}$$

Plugging in Q, the number of rafters simplifies to the number of available permits. Clearly, only increases in the number of permits cause increases to the number of rafters under transferability. This result makes logical sense. Only the first $\bar{q}$ rafters, with a WTP greater than the $\bar{q}$th rafter, actually raft.

$$q^r = \bar{q} \tag{18}$$

A rafter with a WTP lower than the $\bar{q}$th rafter sells his permit making him a non-rafter. Plugging (17) into the total number of applicants (16), we can find the number of non-rafters in the lottery.

$$q^{nr} = \frac{\bar{q}\left(\alpha - \beta\bar{q} - \gamma\bar{q} - f_a - f_p\right)}{f_a} \tag{19}$$

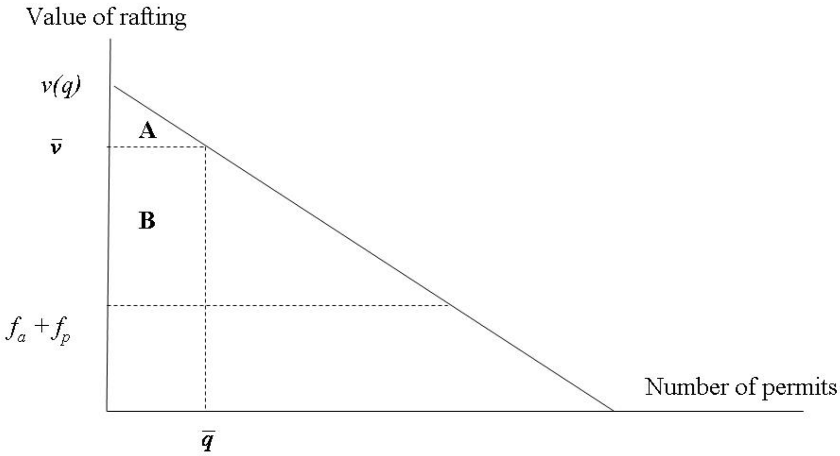
Determining the number of rafters q^r, and non-rafters, q^{nr}, allows us to derive rafter welfare, non-rafter welfare, and government revenue in the transferable lottery.

4.1 Welfare Measures under a Transferable Lottery Allocation

Calculating welfare under a transferable lottery allows for a comparison between the different lottery scenarios. When the lottery allows permit transfers, non-rafters may enter the lottery with hopes of gaining rents through resale. Rafters gain surplus by being able to trade, but also lose surplus through an increase in the number of applicants caused by non-rafter. Thus, rafter surplus (RS) depends on whether the winner of the lottery is a rafter or non-rafter.

Figure 4 shows the total applicant welfare gained from a transferable lottery by both rafters and non-rafters. Total applicant welfare equals the area under the value curve($v(q)$) from 0 to $\bar{q}$ minus the cost to the government for application and permit fees for each permit or areas A and B. Since the $\bar{q}$ rafters with the highest WTP obtain the permits either through winning or trade, rafters gain a share of B by winning, and

Figure 4. Total Applicant Welfare under a Transferable Lottery



additionally gain area A through trade because they never pay a price greater than $\bar{v}$.

Equation (20) calculates the area A, below the value curve minus the resale value.

$$\begin{aligned} A &= \bar{q} \cdot \frac{(\alpha - \bar{v})}{2} \\ &= \frac{\bar{q}\,(\beta \bar{q} + \gamma \bar{q})}{2} \end{aligned} \tag{20}$$

Equation (21) calculates the area B as the area between the resale value minus the total cost of the permits.

$$B = \bar{q} \cdot (\alpha - \beta \bar{q} - \gamma \bar{q} - f_a - f_p) \tag{21}$$

We assume that the proportion of rafters applying equals the proportion that wins, which follows from the uniformity assumption of winning. Thus, $\frac{q^r}{(q^{nr}+q^r)}$ represents the share of rafter winners. Plugging in the values for q^r and q^{nr} gives RS_t in a transferable lottery, which equals area A plus the rafter share of area B.

$$RS_t = \bar{q}\left[\frac{\beta}{2}\bar{q} + \frac{\gamma}{2}\bar{q} + f_a\frac{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_a - f_p)}{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p)}\right] \tag{22}$$

Even though non-rafters win a share of permits, they can gain surplus through exchange with rafters who have a high WTP. Again, we assume that the proportion of non-rafters applying equals the proportion that wins, and $\frac{q^{nr}}{(q^{nr}+q^r)}$ represents the share of non-rafter winners. Thus, the non-rafter proportion of area B represents the non-rafter surplus (NRS). Plugging in the values for q^r and q^{nr}, gives NRS_t under a tranferable lottery in terms of the control variables.

$$\begin{aligned} NRS_t &= \frac{q^{nr}}{q^{nr}+q^r} \cdot \bar{q}\,(\alpha - \beta\bar{q} - \gamma\bar{q} - f_a - f_p) \\ &= \frac{\bar{q}\cdot(\alpha - \beta\bar{q} - \gamma\bar{q} - f_a - f_p)^2}{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p)} \end{aligned} \tag{23}$$

Again, note that the ability to trade gives rafters with a low WTP, ie $v(\bar{q}) < \bar{v}$, an incentive to sell and permit the win and no incentive to purchase a permit. Thus, these rafters become non-rafters in this model. "True non-rafters", who have no intention of rafting under any market conditions, may also enter the lottery. In this model, non-rafter surplus (NRS) includes the surplus of both low WTP rafters, who trade away their permits, and "true non-rafters".

The total revenue to the government agency comes from two sources: the permit fees, and the application fees. Both rafters and non-rafters pay the fee for the application, while the revenue from permits only come from the $\bar{q}$ winners. Thus, the government revenue (GR) written in terms of the controlled variables this becomes equation (28).

$$GR_t = (Q \cdot f_a) + (\bar{q} \cdot f_p)$$

$$= \bar{q} \cdot (\alpha - \beta\bar{q} - \gamma\bar{q}) \tag{24}$$

All three government control variables affect RS_t and NRS_t, while only changes in the number of permits affect government revenue. These measures allow us to examine how changes in the control variables impact welfare and revenue.

4.2 Impacts of Government Controls on Welfare under Transferability.

Since $\bar{v} = \alpha - \beta\bar{q} - \gamma\bar{q}$, represents the price paid for the permit in the secondary market, and $f_a + f_p$ equals the total cost of the permit, then the "profit" or benefit from resale to the winning non-rafter is denoted $\pi = \alpha - \beta\bar{q} - \gamma\bar{q} - (f_a + f_p)$. Using this notation, allows us to simplify the comparative statics results for the transferable lottery.

Changes in the application fee have an uncertain impact on RS. If the benefit from resale $\pi > f_a$, then the RS_t increases. This relationship exists because some non-rafter will drop out of the lottery and increase the probability of winning for the rafters. When $\pi < f_a$, non-rafters do not enter the lottery, and thus, any increase in the application fee when $\pi < f_a$ implies that RS_t can only diminish.

$$\frac{\partial RS_t}{\partial f_a} = \frac{\bar{q} \cdot (\pi - f_a)}{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p)} \quad \lesseqgtr 0. \tag{25}$$

An increase in the permit fee decrease the number of non-rafters entering the market, which increases rafter share of area B. However, area B diminishes with an increase in the permit fee. This causes RS_t to decrease with certainty as shown in equation (26).

$$\frac{\partial RS_t}{\partial f_p} = \frac{-\bar{q} \cdot f_a^2}{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p)^2} < 0 \tag{26}$$

Increases in application and permit fees negatively impact non-rafter surplus, as shown by equations (27) and (28). Since cost to the non-rafter increases, his ability to gain from resale decreases. Hence, surplus decreases.

$$\frac{\partial NRS_t}{\partial f_a} = -\frac{2\bar{q}\left(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p - f_a\right)}{\left(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p\right)} \; < 0 \tag{27}$$

$$\frac{\partial NRS_t}{\partial f_p} = -\bar{q}\left[1 - \frac{f_a^2}{\left(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p\right)^2}\right] = -\bar{q}\left[1 - \delta_t^2\right] \; < 0 \tag{28}$$

Equations (29) and (30) shows that the effect of changes in the number of permits on RS_t and NRS_t. An increase in available permits causes area A and B to grow, but the value of permits shifts in due to more congestion, γ, on the river. Furthermore, an increase in $\bar{q}$ decreases the resale price $\bar{v}$, making it less profitable to trade. The impact on RS_t includes a surplus decrease caused by congestion, a surplus gain caused by an increase in rafters, and a surplus gain caused by a lower $\bar{v}$. Together, these factors create a positive impact on RS_t.

$$\begin{aligned} \frac{\partial RS_t}{\partial \bar{q}} &= \beta\bar{q} + \gamma\bar{q} + f_a - \frac{f_a^2 \cdot (\alpha - f_p)}{(\pi + f_a)^2} \\ &= \pi \cdot \left[\frac{\beta\bar{q} + \gamma\bar{q} + 2 \cdot f_a}{\pi + f_a}\right] > 0 \end{aligned} \tag{29}$$

The impact on NRS_t includes a loss in surplus caused by congestion, a gain in surplus caused by more $\bar{q}$, and a loss in surplus caused by a lower $\bar{v}$. Together, these factors create an uncertain impact on NRS_t

$$\frac{\partial NRS_t}{\partial \bar{q}} = -\beta\bar{q} - \gamma\bar{q} - f_a + \pi + \frac{f_a^2 \cdot (\alpha - f_p)}{(\pi + f_a)^2} \; \substack{> \\ <} \; 0 \tag{30}$$

Interestingly, application and permit fees have no affect on government revenue in a transferable lottery. Thus, changes in fees only affect rafters and non-rafters. Furthermore, increasing available permits, $\bar{q}$, has a positive impact on GR only if marginal value, $\alpha - 2\left(\beta\bar{q} + \gamma\bar{q}\right) > 0$.

These comparative statics show the effects of government control variables have on rafter surplus, non-rafter surplus, and government revenue. These results compared with the restriction of non-transferability shows how the two mechanisms differ, and where possible trade-offs exist.

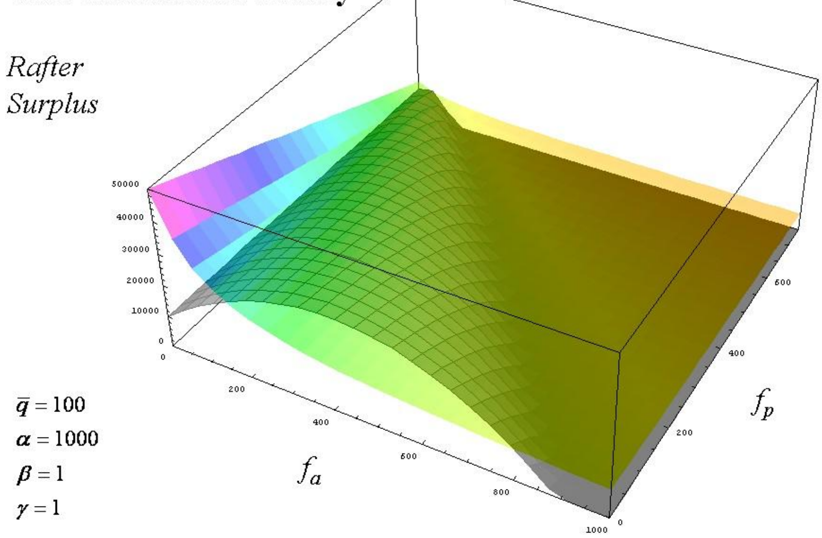
5 Results

In a non-transferable lottery, rafters gain surplus by not allowing non-rafters into the lottery. However, in the transferable lottery they gain surplus by being able to trade. Comparing the welfare measures from the two scenarios captures the preference tradeoff rafters have between a non-transferable and a transferable lottery. Comparative statics from each scenario show how the government control variables have different effects on this preference.

Examining NRS and GR under both scenarios shows that non-rafters and the government will always prefer the transferable lottery due to a greater amount of welfare. However, rafters experience both a gain and loss in surplus by allowing trade. They lose surplus from an increase in the number of applicants, ie non-rafters, but they gain surplus by being able to trade. RS under non-transferability and transferability are rewritten in equation (31) for non-transferability (NT) and equation (32) for transferability (T).

$$NT = \bar{q} \cdot \left[\frac{\gamma\bar{q} \cdot f_a + (\beta\bar{q} \cdot (\alpha - f_p))}{2 \cdot (f_a + \beta \cdot \bar{q})} \right] \tag{31}$$

Figure 5. Rafter Surplus for the Transferable and Non-Transferable Lottery



$$T = \bar{q}\left[\frac{\beta}{2}\bar{q} + \frac{\gamma}{2}\bar{q} + f_a\frac{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_a - f_p)}{(\alpha - \beta\bar{q} - \gamma\bar{q} - f_p)}\right] \tag{32}$$

where π notates the total profit from resale. In order for $NT < T$, requires that:

$$\left[\frac{\beta\bar{q}}{\beta\bar{q} + f_a} \cdot \frac{\pi}{f_a}\right] \cdot \pi < \left[\frac{f_a}{\beta\bar{q} + f_a}\right] \cdot \pi + \pi \tag{33}$$

The left hand side of equation (33) represents the loss of surplus caused by the increase in number of applicants, while the right hand side represents the gain in surplus from allowing trade. When $\pi = 0$, then $NT = T$, and rafters become indifferent between the two lottery mechanisms.

Figure 5 also represents this trade-off of surplus. The colored plain represents RS_{nt} under non-transferability and the black and white grid reprsents RS_t under transferability. Along the bottom axis, the application and permit fees change. The parameters are held constant at $\bar{q} = 1$, $\alpha = 1000$, $\beta = 1$, and $\gamma = 1$.

Table 2. Comparative Statics of Rafter and Non-Rafter Welfare and Government Revenue caused by Control Variables.

	Rafter		**Non-Rafter**		**Gov. Rev.**	
Control variables	*Non-transfer*	*Transfer*	*Non-transfer*	*Transfer*	*Non-transfer*	*Transfer*
f_a *app. fee*	(-)	$\gtreqless$	N/A	(-)	(+)	=0
f_p *permit fee*	(-)	(-)	N/A	(-)	(+)	=0
$\bar{q}$ *permits*	$\gtreqless$	(+)	N/A	$\gtreqless$	$\gtreqless$	$\gtreqless$

The figure shows that initially rafters prefer a non-transferable lottery, but as fees increase, the cost keeps non-rafters from entering the transferable lottery. Eventually, the fees become high enough that rafters will prefer a transferable lottery because they gain surplus from trading. If the fees continue to increase even further, then the cost will eventually become greater than the resale price, ie $f_a + f_p > \bar{v}$. When costs increase greater than $\bar{v}$, the number of applicants decreases to below the number of permits, $q_n < \bar{q}$, making the lottery unnecessary.

Table 2 shows the results for changes in welfare caused by changes in the controlled variables, f_a, f_p, and $\bar{q}$. To see the trade-off of welfare for rafters, examine the effects of changes in f_a under the tranferable lottery. An increase in f_a decreases the surplus of non-rafters. Because of this increased cost, their expected profit diminishes and some non-rafters will not apply. This increases the odds of winning for the rafters, and thereby increasing their surplus. However, the increasing cost also decreases their surplus. The counteracting forces cause the uncertain comparative static and demonstrate the welfare trade-offs for rafters.

The welfare effects from changes in $\bar{q}$, also demonstrate the changing perference of rafters. An increase in $\bar{q}$ has an uncertain impact on non-rafter surplus. However, in the transferable case, more permits means a lower selling price, and greater rafter surplus gained from trading, ie area A grow. If $\bar{q}$ is large enough to make trading the permits non-profitable for non-rafters, the rafters will prefer the transferable lottery.

6 Conclusion

Analysis of the Four Rivers Lottery provides a comparison of the welfare trade-offs between a transferable lottery and a non-transferable lottery. The NFS uses the lottery to distribute resource access, but disallows the transferring of permits among users. This restriction has the benefits of keeping non-rafters from applying, while the prohibition of trade creates an inefficient market in terms of total welfare. The tradeoffs between transferability and non-transferability provides many rent-seeking efforts by rafters and non-rafters alike to either maintain the status quo or seek changes to the lottery system. This paper evaluates that tradeoff from the viewpoint of rafters, non-rafters, and the government.

The model developed in this paper analyzes rafter and non-rafter welfare, and government revenue under both a non-transferable and a transferable lottery. The welfare measures from this model show that non-rafters and the government will always prefer a transferable lottery. However, depending on the fees and number of permits, rafters can prefer either lottery. Rafter welfare is greater under a transferable lottery when the benefits from trade exceed the reduction in surplus caused by entering non-rafters. Rafters prefer a non-transferable lottery only when low fees cause the profit from resale to be high. The high profit creates an incentive for non-rafters to enter the lottery, reducing rafter surplus. The non-transferable restriction keeps them from seeking that profit.

This study also examines the effects government control variables have on welfare and revenue measures. Changes in government controlled variables, such as permit fees, application fees, and the number of permits, can change the rafter's preference towards a transferable lottery. For a non-transferable lottery, increases to application and permit fees negatively affect rafter surplus, while increases to the number of permits have an uncertain affect on rafter surplus. Under transferability, only increases to permit fees negatively affect rafter surplus, while an increase to the number of permits positively affects rafter surplus. Finally, changes to the application fee have an uncertain effect on rafter surplus under transferability. The increased fee decreases rafter surplus, but also decreases the number of non-rafters entering. This result demonstrates how rafters face a trade-off between between transferable and non-transferable lotteries under changing control variables. Rafters only benefit from transferability when the gains from trading exceed the loss to additional non-rafters in the lottery.

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