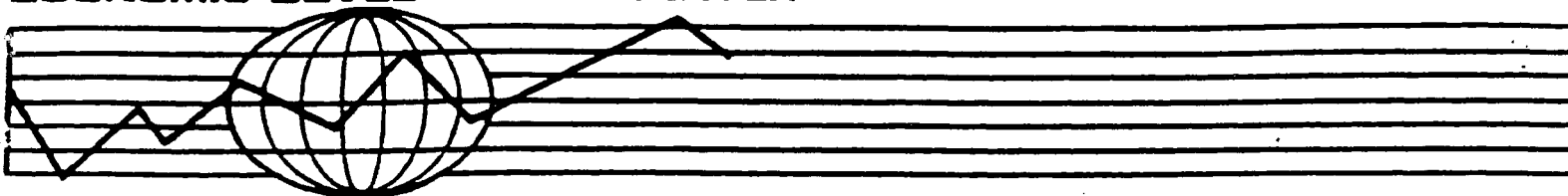# ECONOMIC DEVELOPMENT CENTER

A Dynamic CGE Model: An Application of R&D-Based Endogenous Growth Model Theory

X. Diao, E.H. Elbasha, T.L. Roe, and E. Yeldan

ECONOMIC DEVELOPMENT CENTER
Department of Economics, Minneapolis
Department of Applied Economics, St. Paul

UNIVERSITY OF MINNESOTA

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Abstract

An R&D based endogenous growth - applied general equilibrium model is developed from an underlying analytical model which combines Romer's capital variety with Grossman and Helpman's multi-sector open economy model. The transitional dynamics of the analytical model are derived. For numerical implementation, a time discrete empirical model, with an Armington structure, is fit to East Asian data of the social accounting matrix variety. Simulations of trade reform are performed and their static and dynamic effects compared. The transition paths of the state variables are found to have a half-life of five to six periods. A solution of the Social Planner's problem, and interventions which seek to obtain this outcome from the decentralized model are also obtained[1].

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Contact: Terry L. Roe
Dept. of Applied Economics
University of Minnesota
1994 Buford Ave.
St. Paul, MN 55108
Phone: 612-625-6706
E-Mail: troe@dept.agecon.umn.edu

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[1]Diao is a Research Associate, University of Minnesota and Stationed with the Economic Research Service, Washington D.C. Elbasha is a Research Associate, and Roe is Professor of Applied Economics, University of Minnesta. Yeldan is Professor of Economics, Bilkent Univeristy, Turkey.

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I. Introduction

Contributors to the new growth theory have produced various analytical models which together account for some of what Kaldor (1961) and others refer to as the "stylized" facts of economic growth[1]. While effort, e.g., Backus, Kehoe, Kehoe (1992) has been made to confront some of the models to economic time series, the analytical models remain fairly far removed from application in structural form to country level data and removed from use as empirically based policy models. At one extreme, many applied general equilibrium models tend to be either based on single period optimization assumptions or to use this structure in a discrete sequential manner to model dynamic processes. An earlier attempt of the latter is Adelman and Robinson's (1978) model of the South Korean economy. Recent renditions, albeit with some enhancement, are those of Bourguignon et al (1992) and Yeldan et al (1995). This approach often leads to models designed for policy analyses that is parsimonious in structure and computation but prone to providing misleading insights into the adjustment and growth process since, as Devarajan and Go (1995) note, "the same agent behaves rationally for one set of decisions but irrationally for

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[1]Contributors include Romer (1986, 1990), Lucas (1988), Grossman and Helpman (1991), Rebelo (1991) and others. Part of this literature is surveyed by Sala-i-Martin (1990), Helpman (1992), and Lucas (1993). See also Grossman and Helpman (1994), Romer (1994), and Solow (1994) for a review. The book by Barro and Sala-i-Martin (1995) is another excellent reference.

another - a characterization which is difficult to defend." Dynamic applied general equilibrium models have been developed by Mercenier (1993, 1995), Jorgenson and Wilcoxen (1990), Ho (1989), Mckibbin (1993), Devarajan and Go (1995), and Keuschnigg and Kohler (1994,1995) where the discipline of steady state dynamics via exogenous specification of the rate of growth is imposed. Private households are assumed to solve an inter temporal optimization problem to maximize discounted utility of an inter temporal sequence of consumption given their discounted stream of net income. Investment decisions are carried out in a similar fashion by forward-looking firms (or by the consumer as in the case of Mercenier, 1995, or with the intermediation of a "bank" as in Wilcoxen, 1988, and Ho, 1989)[2].

While the latter contributions resolve the aforementioned inconsistency problems of intra and inter-temporal optimization, they rely on the exogenous specification of technological change and other effects that can affect the rate of economic growth. Consequently, they fall short of providing a link between economic structure and policy effects on growth rates, i.e., they are removed from applying the structure suggested by the endogenous growth models of the genre of Romer (1990) and Grossman and Helpman (1991, 1994). The purpose of this paper is to bridge the gap between the dynamic applied general equilibrium models which treat growth rates exogenously and an approach which allows for endogenous growth. In so doing, the paper contributes to theory and to the application of the new growth theory to country level data.

The analytical model developed in the following section is a continuous-time infinitely lived agent model which extends, modestly, the genre of models developed by Grossman and Helpman (1991) and Romer (1990). We model an open economy with multiple production sectors, as in Grossman and Helpman, and specify the production of differentiated capital which can be employed in final good production, as in both Grossman and Helpman (1991) and Romer (1990), but which can also be accumulated as in Romer (1990). Then, we derive the local and global stability properties of the steady state and the transition path for the continuous time model.

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[2]Keuschnigg and Kohler (1995) view the implications of trade reform to be largely distributional in nature which they demonstrate using an overlapping generations model.

As Mulligan and Sala-i-Martin (1993) note, this tends to be difficult for multisectoral models of endogenous growth. The second contribution is to show how this model can, in principle, be cast into a discrete time empirical general equilibrium model based on country level data of the social accounting matrix variety, and solved with the software used for static CGE models. We chose data from East Asia for this purpose. We also employ the familiar Armington and CET aggregator functions to account for intra industry trade and other stylized facts of real economies. Solutions are shown to yield the standard level-type effects of static CGE models, and the dynamic effects of endogenous growth. Then, we derive empirically the discrete time transition path to the steady state, and find a half-life for most simulations to be within the five - six period range. The empirical section is concluded by contrasting the results of the decentralized solutions of the model to those of the Social Planner and interventions to attain the optimal Planner outcomes.

II. The Model

The economy is small in the sense that it faces perfectly elastic demand for final goods in world markets and trades at exogenously given prices. There are two primary factors, L and B. Their levels are constant over time, mobile among sectors, but immobile internationally. Producers undertake three distinct activities. Producers in the R&D sector choose two primary inputs, given the existing stock of knowledge M, to produce new designs. The accumulated designs are proportional to accumulated knowledge M. Hence, accumulated knowledge is non-rival in the sense that its use by one as an input into a new design does not diminish the amount available to another, and non-excludable in the sense that it is common to all. In the capital-goods sector, new firms obtain rights to the new designs and employ foregone outputs to produce differentiated capital. Two final goods are produced using two primary inputs, and the set of differentiated capital from pre-existing and new firms.

The concept of capital departs from the concept used in most growth models. Following

Romer (1990), capital is differentiated, and assumed to not depreciate. The rights to a new design or patent must be obtained by a firm before it can produce a type of differentiated capital. A firm can only have the rights to one design. Once a type of capital is produced, it can be used as an input in the production of final goods forever. The cost of purchasing a design or patent for the production of new differentiated capital by a new firm only needs to be borne once. Thus, a type of capital based on the new design can generate a stream of income for the producer for ever. Since each new design is owned by a single firm, the firm has monopoly power over the rental rate of capital to final good producers. The remaining markets are perfectly competitive.

The household owns the primary factors and the profit making firms. Households maximize utility over an infinite horizon by allocating income to consumption and investment. The investment decision includes savings for new capital formation and the purchases of new designs. Sources of household income include factor rental and interest earned on assets (capital and the profit making firms).

Final output sectors. The two final output sectors each consist of a large number of identical firms which, at the beginning of each period, rent primary inputs (L) and (B) from consumers, and a set of differentiated capital from the capital goods sector. Technology of the two traded goods are given by

$$Y = A_y L_y^{\alpha_1} B_y^{\alpha_2} H_y^{\alpha_3} \tag{1}$$

$$Z = A_z L_z^{\beta_1} B_z^{\beta_2} H_z^{\beta_3}, \tag{2}$$

where $A_j > 0$, $\sum_{i=1}^{3} \alpha_i = 1$, $\sum_{i=1}^{3} \beta_i = 1, \alpha_i, \beta_i > 0$, and $\alpha_3 = \beta_3$ is assumed to assure the existence of a balanced growth path. The differentiated capital index H is given by

$$H_y + H_z = H = \left(\int_0^M k(s)^\delta ds \right)^{1/\delta}, \ 0 < \delta < 1, \tag{3}$$

where M denotes the measure (number) of differentiated capital available in the market at time t, and $k(s)$ is the amount of differentiated capital of variety $s : s \in [0, M(t)]$.

Capital goods sector. Once a firm acquires the property rights to a new design, it can produce

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differentiated capital using Y and Z given the technology:

$$k(s) = A_k Y_{k(s)}^{\eta} Z_{k(s)}^{1-\eta}. \qquad (4)$$

The R&D sector. Firms producing new designs choose levels of primary factors L and B, given a stock of common knowledge using the technology:

$$\dot{M} = A_m L_m^{\theta} B_m^{1-\theta} M, \quad A_m > 0, 1 > \theta > 0, M > 0. \qquad (5)$$

The stock of knowledge is assumed proportional to the number of existing designs M. M is a public good which is available to all firms.

Consumers. The infinitely-lived representative consumer's momentary utility function is given by:

$$u = \begin{cases} \dfrac{1}{1-\sigma} \left[\left(C_y^{\phi} C_z^{1-\phi} \right)^{1-\sigma} - 1 \right], & \text{for } \sigma \neq 1 \\[2mm] \phi \log C_y + (1-\phi) \log C_z, & \text{for } \sigma = 1 \end{cases} \qquad (6)$$

where C_j, $j = y, z$, denotes consumption at time t. At each time period, consumers choose the levels of consumption and savings subject to their income from endowments and accumulated assets. Let E denote the total expenditure on consumption goods, $\dot{a}$ the savings allocated to investment in capital and purchases of new designs (to be defined more precisely later), "a" the value of accumulated assets, and r the return (interest rate) on these assets. Then, consumers' momentary budget constraint can be expressed as:

$$E + \dot{a} = ra + W_L L + W_B B \qquad (7)$$

where W_L and W_B are primary factor rental rates.

Profit maximization in the final goods sector requires equating unit costs to prices:

$$P_y = W_L^{\alpha_1} W_B^{\alpha_2} P_H^{\alpha_3} \qquad (8)$$

$$P_z = W_L^{\beta_1} W_B^{\beta_2} P_H^{\alpha_3} \qquad (9)$$

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where Y and Z are normalized such that $A_y^{-1} = \alpha_1^{\alpha_1} \alpha_2^{\alpha_2} \alpha_3^{\alpha_3}$ and $A_z^{-1} = \beta_1^{\beta_1} \beta_2^{\beta_2} \beta_3^{\beta_3}$. P_y and P_z are final output prices, and P_H is the index of the rental rate of capital variety:

$$P_H = \left(\int_0^M P_k(s)^{\frac{\delta}{\delta-1}} ds \right)^{\frac{\delta-1}{\delta}}. \tag{10}$$

This expression is obtained assuming producers choose $k(s)$ to obtain the least cost combination of differentiated capital $k(s)$, $\int_0^M P_k(s)k(s)ds$, subject to (3). Profit maximization in the R&D sector requires equating the price of a design to its unit cost:

$$P_m = \frac{1}{M} W_L^\theta W_B^{1-\theta} \tag{11}$$

where output M is normalized such that $A_m^{-1} = \theta^\theta (1-\theta)^{1-\theta}$.

Applying Shephard's lemma to the unit cost function in (10) yields the derived demand for capital variety $k(s)$:

$$k(s) = H \left(\int_0^M P_k(s)^{\frac{\delta}{\delta-1}} ds \right)^{\frac{\delta-1}{\delta}-1} P_k(s)^{\frac{\delta}{\delta-1}-1}. \tag{12}$$

Equation (12) is the demand for the s-th type of differentiated capital facing a differentiated capital good producer, a monopolist. As a precondition to production, the monopolist must incur the fixed cost of purchasing a new design or patent. Then, the monopolist is assumed to maximize profit by choosing the monopoly rental rate $P_{k(s)}$ taking into account the demand function in equation (12), i.e.,

$$\max_{P_{k(s)}} \pi(s) = P_{k(s)}k(s) - rMC_k k(s)$$
$$\text{s.t. (12)},$$

where $P_{k(s)}k(s)$ is the flow of rental income, $MC_k k(s)$ is the cost of producing $k(s)$ and, hence, $rMC_k k(s)$ is the interest cost on $MC_k k(s)$. Minimizing $P_y Y_{k(s)} + P_z Z_{k(s)}$ subject to (4) and $k(s) = 1$ yields unit cost, MC_k, as a function of world prices. Consequently, unit cost is identical to all firms:

$$MC_k = P_y^\eta P_z^{1-\eta}, \tag{13}$$

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where k is normalized such that $A_k^{-1} = \eta^\eta (1 - \eta)^{1-\eta}$.

We assume that capital is putty-putty, so that the firm can solve this problem at every point in time. Given patent rights, this implies that the firm can convert units of the differentiated capital good into another asset and avoid the interest cost if the demand for capital is less than in the previous period.

Maximization of profits yields the mark-up rental rate of capital,

$$P_{k(s)} = \frac{rMC_k}{\delta}. \tag{14}$$

Hence, the price of capital, the level of k, and profits are the same for all firms. Accordingly, we omit the "s" index. Using (14) profits for any firm in the intermediate sector is given by

$$\pi = P_k k - rMC_k k = (1 - \delta)P_k k. \tag{15}$$

Since all firms producing k do so at the same level, aggregate capital K is kM. Primary factor rental rates can be derived from the cost functions (8), (9) and (14). The solutions yield

$$W_L = P_y^{\frac{1}{1-\alpha_3}} \left(\frac{P_z}{P_y}\right)^{\frac{\alpha_1 + \alpha_3 - 1}{(\alpha_1 - \beta_1)(1 - \alpha_3)}} \left(\frac{\delta}{rMC_k}\right)^{\frac{\alpha_3}{1-\alpha_3}} M^{\frac{\alpha_3(1-\delta)}{\delta(1-\alpha_3)}} \tag{16}$$

$$W_B = P_y^{\frac{1}{1-\alpha_3}} \left(\frac{P_z}{P_y}\right)^{\frac{\alpha_1}{(\alpha_1 - \beta_1)(1 - \alpha_3)}} \left(\frac{\delta}{rMC_k}\right)^{\frac{\alpha_3}{1-\alpha_3}} M^{\frac{\alpha_3(1-\delta)}{\delta(1-\alpha_3)}}. \tag{17}$$

Together, (16) and (17) imply that primary factor rental rates evolve over time at the same rate:

$$\frac{\dot{W}_L}{W_L} = \frac{\dot{W}_B}{W_B} = \frac{\alpha_3(1-\delta)}{\delta(1-\alpha_3)} g - \frac{\alpha_3}{1 - \alpha_3} \frac{\dot{r}}{r}, \tag{18}$$

where $g = \dot{M}/M$.

The evolution of P_m is given by substituting (18) into (11)

$$\frac{\dot{P}_m}{P_m} = \frac{\dot{W}_L}{W_L} - \frac{\dot{M}}{M} = \left(\frac{\alpha_3(1-\delta)}{\delta(1-\alpha_3)} - 1\right) g - \frac{\alpha_3}{1 - \alpha_3} \frac{\dot{r}}{r}. \tag{19}$$

In equilibrium, the first-order conditions (FOCs) for maximizing profits for final output producers imply $\alpha_3 P_y Y / H_y = P_H = \alpha_3 P_z Y / H_z$. Then, from the identities $H_y + H_z = H$ and

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$P_H H = P_k k M$, we obtain

$$P_H H = P_k k M = \alpha_3 (P_y Y + P_z Z). \qquad (20)$$

Similarly, the demand functions for the primary factors are obtained from the cost functions as

$$L_y = \alpha_1 \frac{P_y Y}{W_L}, \; L_z = \beta_1 \frac{P_z Z}{W_L}, \; L_m = \theta \frac{P_m \dot{M}}{W_L}, \; B_y = \alpha_2 \frac{P_y Y}{W_B}, \; B_z = \beta_2 \frac{P_z Z}{W_B}, \; B_m = (1 - \theta) \frac{P_m \dot{M}}{W_B}.$$

Substituting these into the market clearing conditions for primary resources L and B and adding the resulting equations yields

$$(1 - \alpha_3)(P_y Y + P_z Z) + P_m \dot{M} = W_L L + W_B B \qquad (21)$$

Balance of payments. We abstract from international capital movements and assume no borrowing from and lending to the rest of the world[3]. Each period final output is allocated to consumption, exports and to capital formation. The latter has two components: producing the newly designed capital (by employing Y_k and Z_k) and updating the old capital (by using $Y_{\dot{k}}$ and $Z_{\dot{k}}$). Therefore, the balance of payments can be written as

$$P_y Y + P_z Z = E + P_y Y_k + P_z Z_k + P_y Y_{\dot{k}} + P_z Z_{\dot{k}}.$$

Using the cost functions in equation (13) we obtain

$$P_y Y_k = \eta M C_k k \dot{M}, \; P_z Z_k = (1 - \eta) M C_k k \dot{M}, \; P_y Y_{\dot{k}} = \eta M C_k \dot{k} M, \; P_z Z_{\dot{k}} = (1 - \eta) M C_k \dot{k} M.$$

Substituting these values into the balance of payments equation, rearranging and noting that $M\dot{k} + k\dot{M} = \dot{K}$, we arrive at

$$P_y Y + P_z Z = E + M C_k \dot{K}. \qquad (22)$$

Capital Market Equilibrium. Since firms are allowed to freely enter and exit the R&D sector, in equilibrium, the price of a new design is equivalent to the value of a firm in the capital goods

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[3] Abstracting from international capital markets results in an endogenous domestic interest rate. This issue is discussed further in the calibration section of the paper.

sector. To maintain asset market equilibrium, the rate of return from holding equities, (dividends plus changes in the value of the firms divided by the value of the firm) should be equal to the interest rate on a one period loan on some riskless asset. Thus, in equilibrium, the following no-arbitrage condition should be satisfied,

$$\frac{\pi}{P_m} + \frac{\dot{P}_m}{P_m} = r, \tag{23}$$

where π is defined in (15).

The consumer's optimization problem yields the following condition

$$\frac{\dot{E}}{E} = \frac{1}{\sigma}(r - \rho). \tag{24}$$

III. The Dynamic Properties of Equilibrium

There are many reasons to study the transitional dynamics of this model. First, we want to know if, starting from an arbitrary initial capital stock, the economy will converge to a steady state; and if it does, what are the economic forces that lead the economy to that state? Second, how many periods does it take the economy to reach that state? Third, one may be interested in comparing the behavior of some variables along the transition path to actual behavior. Finally, note that the seminal Grossman and Helpman model has no transition dynamics, while Romer did not investigate the dynamics of his model.

To facilitate the analysis of the dynamic properties of the model, it is useful to convert the time variant variables into what is called *state-like* and *control-like* variables, variables that are constant along the steady state path. To make the analysis simple we restrict our discussion to the case where $\alpha_3 = \delta$. This is a variant of the model analyzed by Romer (1990). As we will be shown later, k is constant along the steady state path. Thus we choose k as our state-like variable. We will also show that the rate of change in r is zero along the steady state path. Hence, r can be used as a control-like variable. The other control-like variable involves a transformation of

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E. From equation (24) E grows at a constant rate in the steady state since r is constant. We make the following change of variable

$$\bar{E} = \frac{E}{M}.$$

By substituting equations (11), (14-17), and (19) into equation (23), we obtain the following

$$\dot{r} = r \left(\frac{(1-\alpha_3)^2}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k - \frac{(1-\alpha_3)}{\alpha_3} r \right), \tag{25}$$

where $\hat{P}_m = P_y^{1/(1-\alpha_3)} (P_z/P_y)^{[\theta\alpha_1 + (1-\theta)\alpha_2]/[(\alpha_1-\beta_1)(1-\alpha_3)]} (\alpha_3/MC_k)^{\alpha_3/(1-\alpha_3)}$.

By substituting equations (14) and (20) into (22) we obtain

$$\dot{k} = \left(\frac{r}{\alpha_3^2} - g \right) k - \frac{\bar{E}}{MC_k}. \tag{26}$$

Substituting (14) and (20) into (21) yields

$$g = \bar{V} - \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k, \tag{27}$$

where $\bar{V} = (W_L L + W_B B)/M P_m$ is a function of world prices only.

Using equation (27) into (26) gives

$$\dot{k} = \left(\frac{r}{\alpha_3^2} - \bar{V} + \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k \right) k - \frac{\bar{E}}{MC_k}. \tag{28}$$

From the consumer optimality condition (24), equation (27), and the definition of $\bar{E}$ we obtain

$$\dot{\bar{E}} = \bar{E} \left(\frac{1}{\sigma} (r - \rho) - \bar{V} + \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{p}_m} r^{1/(1-\alpha_3)} k \right). \tag{29}$$

Thus, we have succeeded in reducing the competitive equilibrium to a system of three differential, equations, (25), (28) and (29) with one initial condition ($k(0) = k_0$) and two boundary conditions (transversality conditions).

We begin by studying the properties of the steady state (r^*, k^*, g^*) of this system. Letting $\dot{r} = 0$ gives

$$\frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{*1/(1-\alpha_3)} k^* = \frac{r^*}{\alpha_3}. \tag{30}$$

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Using this result in equation (29) and equating $\dot{\bar{E}}$ to zero yields a unique solution for r^*:

$$r^* = \frac{\alpha_3 \left(\sigma \bar{V} + \rho \right)}{\alpha_3 + \sigma}.$$

(31)

If we substitute equation (31) into equation (30), we will be able to get a unique value for k^* and by setting $\dot{k} = 0$ and plugging the values of r^* and k^* in equation (28) we obtain a unique value for $\bar{E}^*$.

Combining equations (27), (30) and (32) we obtain the steady state rate of economic growth

$$g^* = \frac{\alpha_3 \bar{V} - \rho}{\alpha_3 + \sigma}.$$

(32)

The growth rate increases with the elasticity of inter temporal substitution $(1/\sigma)$, the value of endowment $(\bar{V})$, the share of differentiated capital in total output (α_3), and decreases with the discount rate (ρ). While the interest rate increases with $\bar{V}$ and α_3, it decreases with $1/\sigma$ and ρ.

It is easily shown that if $\alpha_3 \neq \delta$, then similar results obtain:

$$g^* = \frac{\dfrac{\alpha_3(1 - \delta)}{1 - \alpha_3}\bar{V} - \rho}{\dfrac{(\sigma - 1)\alpha_3(1 - \delta)}{\delta(1 - \alpha_3)} + \dfrac{1 - \alpha_3\delta}{1 - \alpha_3}}.$$

(33)

For the parameters specification (see below) we were able to plot in Figure 1 the three differential equations when they take the value zero. When $\dot{r} = 0$ and $\dot{\bar{E}} = 0$ we get each equation being independent of $\bar{E}$. This is not true for $\dot{k} = 0$. Instead, we obtain a contour. The level curve shown in Figure 1 is obtained by equating $\bar{E}$ to its steady state value. The steady state for this economy is given by the intersection of the three curves. A more straight forward approach would be plotting the three differential equations when they are equal to zero as they are, i.e. in a three dimensional graph. This is done in Figure 2. The steady state is given by the intersection of the three planes. At that point $r = 0.1$, $k = 361.625$, and $\bar{E} = 966.21$.

In the Appendix I, we prove the following three propositions:

Proposition 1 *The steady state equilibrium is locally saddle-path stable.*

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Proposition 2 *Given the parameter specification, the competitive equilibrium is globally saddle-path stable, and*

Proposition 3 *The transition from low capital involves low interest rate, high productivity-adjusted expenditure, high growth rate, high productivity-adjusted wages, high prices for new designs, and high profits.*

IV. The CGE-Endogenous Growth Model

To easily and clearly illustrate the calibration of the analytical model to data in the tradition of typical CGE modeling, we choose data from the so called *archetypal* economy of East Asia. Construction of a data base for archetypal (or proto-typical) economies is a fairly standard approach in policy analysis situations where it is too costly and time-consuming to fit several country level CGE models to data for a region or category of country "types". The East Asian archetype data used throughout the rest of the analysis is an aggregation of the data compiled by Hazell and Associates (1994), and reported in Yeldan, et al (1995). We first aggregated these data into a social accounting matrix (SAM) of two final goods, two primary factors, and a single household. To capture the production of differentiated capital, and R&D activity, the SAM is further modified in a manner discussed below.

IV.1 Model Structure

There are two tradable good production sectors: agriculture (Y) and non-agriculture (Z), which employ two primary inputs, labor (L) and other durables (B), one accumulated differentiated capital input (referred to as capital for the remainder of this section), and other intermediate inputs. The R&D sector, which is non traded, uses only labor and other durables as inputs. Capital is produced using the two final goods as inputs together with new designs produced by the R&D sector. By not permitting international capital flows, r is endogenous. All technologies are assumed Cobb-Douglas (CD), except in the final goods sector where the CD technologies

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are augmented by a Leontief relationship among intermediate inputs.

The representative consumer is assumed to maximize inter temporal utility of the same form as in the analytical model. All government tax revenues are transferred to consumers in lump sum. To stay within the tradition of CGE modeling, the empirical model departs from the analytical model by accounting for the possibility of imperfect substitution in foreign trade. From the producer's perspective, constant elasticity of transformation (CET) functions are used to account for substitution between a good sold in the domestic market and in the foreign market. Likewise, from the consumer's perspective, an Armington structure is used to capture the possibility for imperfect substitutes between domestic goods and imports of what would otherwise appear to be the same good. The related equations and variable notation are in the Appendix III.

IV.2 Calibration Strategy

As in static models where calibration begins with the assumption that data are obtained from an economy in equilibrium, we assume here that a growing economy is evolving along a balanced growth path. Hence, data given by the initial period SAM characterizes the economy in its "base run" steady state. As growth rates of the endogenous variables are constant, all variables can be made independent of time by normalization in the manner indicated in Section II. Parameters are then calibrated for this base run ensuring that the model will generate a steady state equilibrium solution with values that match the benchmark data of the 1990 SAM. However, some parameters, such as the elasticity of substitution among differentiated capital (δ), the value of the inverse elasticity of inter temporal substitution (σ) and the interest rate (r), and the growth rate (g), cannot be obtained from a static SAM. We consider $\delta = \alpha_3$ (the share parameter of capital in final production) and $g = 0.07$, which is the average annual rate of growth in GDP experienced by this region over the 1980-1990 period. Values of 1.3 for σ and 0.10 for r are also assumed.

The major difficulty of calibration arises from the fact that capital is a broadly defined category, and an R&D sector per se is not identified in the original data. This forces us to

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impute investments in R&D[4] and the stock of differentiated capital from the existing system of national accounts. In order to derive a steady state equilibrium and be consistent with the analytical model, capital is disaggregated into two categories: other durable and differentiated capital. Further, to guarantee a steady state equilibrium, the ratio of labor to other durables has to be constant. We assume that supplies of the two primary factors are time invariant[5] while only the differentiated capital is accumulated. As in the analytical model, we restrict the final good sectors to be equally intensive in their use of differentiated capital. The stock of knowledge is normalized to unity so that the relationship between the original data (as depicted by the left-hand side of the following equations) and the sectoral adjustment to accommodate an R&D sector and inputs is shown to be:

$$\text{(Value Added to } Y) + \text{(Value Added to } Z) \quad = \quad PVA_y\overline{Y} + PVA_z\overline{Z} + P_mg \qquad (34)$$

$$\text{(Value of Labor)} + \text{(Value of Capital)} \quad = \quad \overline{W}_LL + \overline{W}_BB + P_k\overline{K} \qquad (35)$$

From these two equations, with factor rentals of labor and capital set to unity, the original data can be easily adjusted given knowledge of P_m and $\overline{K}$. Investment in capital, $(MC_k\overline{K}g)$, plus the value of new designs, (P_mg), equals the total household savings (as $\dot{a}$ in equation (7)), and can be obtained directly from the data[6]. The problem is how to disaggregate savings data into these two different investment activities, i.e., how to derive the $\dot{a}$ such that

$$\dot{a} = MC_k\overline{K}g + P_mg \qquad (36)$$

By equation (14), (15) and (23), equation (36) can be further simplified to

$$\dot{a} = \frac{P_mg}{1 - \alpha_3} \qquad (37)$$

---

[4]See Romer's (1992) discussion of producing ideas and using ideas for a broader perspective on what we have simply referred to as R&D. R&D can be viewed to include product and process innovations whose production consumes resources, the efficiency gains from which is captured from the employment of differentiated capital.

[5]A steady state is also consistent with primary factor endowments change at the same exogenous rate.

[6]Recall that $\delta = \alpha_3$.

Thus, the problem reduces to determining α_3 and P_m from the data. From equation (20) and (21) we obtain:

$$\alpha_3(V - P_m g) = (1 - \alpha_3)P_k \bar{K},$$

where V is equal to the total value added as given by the data, i.e.,

$$V \equiv (\text{Value Added to } Y) + (\text{Value Added to } Z) = (\text{Value of Labor}) + (\text{Value of Capital}).$$

Using (15) and (23) again to substitute for $P_k \bar{K}$, yields:

$$\alpha_3(V - P_m g) = r P_m \tag{38}$$

Thus, (37) and (38) can be used to solve for α_3 and P_m. Knowing these values permits the calculation of $\bar{K}$ since P_k is normalized to unity. This operation permits the original data for capital to be disaggregated into other durables, B, and total differentiated capital, $\bar{K}$, while an R&D sector is obtained by reducing the value added of final outputs.

The data of total investment is about 21% of the country's GDP. The calibration exercise yields an estimate of investment in R&D of about 13.6% of GDP for this country. This estimate is based on the notion that industrial research is regarded as the primitive force behind much of the output growth that is often attributed to capital accumulation, and that "investments are made in response to improve technological conditions, because extra equipment was needed to produce newly invented goods" (Grossman and Helpman p.7).

IV.3 Existence of a Steady State Equilibrium

The steady state algebraic structure of the empirical CGE model is complicated relative to the analytical model due the Armington and the CET commodity specifications. In the analytical model, it is easy to compute the steady state and show that, in the steady state, all endogenous variables are either constant, or grow at constant rates over time. However, in the applied general equilibrium model, as the price of outputs sold in the domestic market (PD_i, $i = y, z$) are endogenously determined by the market equilibrium condition, the derivation of a closed

form solution for the price system as a function of accumulated knowledge is difficult. Thus, the existence of a steady state is questioned by the unknown behavior of PD_i.

For this reason, we adopt a "guess and verify" strategy to check the existence of the steady state and the property of PD_i at the steady state. First we assume constancy of PD_i, and then check to verify that the equilibrium solution derived from the CGE system shown in Appendix III is at steady state – i.e, whether the endogenous variables are constant or grow at constant rates.

The model entails 38 equations (see Appendix III) in 38 endogenous variables. We make use of the General Algebraic Modeling System (GAMS)[7] to solve the system. To check whether the steady state properties hold for the equilibrium solution, we start from equations (III.6) to (III.9), i.e., the CET and the Armington functions in their dual forms, respectively. As the exogenous export and import prices (PE_i and PM_i) are time invariant, if PD_i are constant, then the output prices, P_i, and absorption prices, PC_i are constant, as are the value added PVA_i (Equations III.14 and III.15).

Once PVA_i are constant, Equation (III.1) to (III.5), and can be treated as being equivalent to respective equations (8), (9), (11), (13), and (14). The summation of (III.30) and (III.31) is equivalent to (21), and (III.32) is equivalent to (20). Thus, the analysis derived in Section II and III can be used here to prove that the steady state equilibrium solution derived in the CGE would have the same properties as those derived for the analytical mode. This permits the verification that a steady state exists in the CGE model where the different prices including PD_i for commodities are all constant.

V. Simulation Exercises

The numerical exercises entail a number of policy simulations designed to provide insights into the numerical nature of the model. Most attention is placed on the comparisons among

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[7]See Brooke, Kendrick and Meeraus, 1988,

steady state equilibria of the various simulations. The transition paths associated with the trade liberalization simulations are derived by the time discrete version of the same model. The effects of trade policy liberalization are presented first. Then, we obtain a solution to the social planner's problem, and conclude by considering other interventions that might approach the social planner's equilibrium.

V.1 Static and Dynamic Effects of Trade Liberalization

The data suggest a tariff rate on agricultural and non-agricultural imports of 29%, and 38%, respectively, while agriculture alone bears a 0.9% production tax. The simulations entail the elimination of (i) agricultural tariffs and the production tax only; (ii) the tariff on non-agricultural imports only; and (iii) all taxes in the economy. For each simulation, we analyze both static and dynamic effects. The static effects are once and for all changes, while the dynamic effects are long-run changes that, in the context of this model, exist forever. Since the supply of primary factors are fixed over time, the static effects of resource reallocation can be distinguished from the dynamic effects by evaluating the result of a simulation with accumulated capital fixed at its level of the base run.

The results from simulations (i) - (iii) are presented in Table 1. If only agriculture is liberalized, its relative value added price increases, while if only the non-agriculture is liberalized, or if all taxes are eliminated from the economy, the opposite result obtains. Since agriculture is labor intensive, standard Stolper-Samuelson effects are observed, i.e., the relative factor price varies in proportion to the change in relative value added price. The change induced in the relative factor price causes resources to reallocate. With capital fixed, as agriculture is liberalized, its output rises and output falls in non-agricultural. When either non-agriculture or the whole economy are liberalized, supply rises in non-agriculture and falls in agriculture.

The steady state dynamic effects are also shown in Table 1, columns (2), (4) and (6). In this case, all variables are normalized by accumulated knowledge, M, which implies that capital accumulation caused by the increase in the production of new designs is eliminated, while quan-

tity adjustments for each differentiated capital can still occur. Unlike the static efficiency gains from liberalization tending to be biased towards one sector, the increase in use of differentiated capital, (which is one of the important dynamic effects) increases output in both sectors.

Note the response of saving and investment to changes in trade policy (Table 1). Eliminating taxes in any sector causes the ratio of saving to income to rise, and to reach its highest level when all tax distortions are removed from the economy. Removing taxes and tariffs lowers the cost of investment in differentiated capital which causes the monopoly capital rental price to fall. Thus the demand for each differentiated capital employed in final good production increases. When the increase in the quantity of capital exceeds the effect of the fall in capital rental price, monopoly profits rise. They reach their highest level when all taxes are eliminated. These profits of course provide the incentive for consumers to forego current consumption and invest.

Based on which sector's tariff is eliminated, trade liberalization has different effects on growth. If agriculture is liberalized, the growth rate falls, while if the non-agricultural sector is liberalized, or if taxes are eliminated completely, the growth rate rises. As equations (1), (2) and (5) suggest, these results depend critically on the share parameters in the final good and R&D production functions. In this economy, the agricultural sector is labor intensive, and the share parameter for labor in the R&D sector is greater than 0.5. Eliminating taxes in agriculture, causes its relative value added price to rise, increasing output and labor employment. This leads to a reduction of labor employed in the R&D sector. Even though the other durable factor employed in the R&D rises, as R&D is labor intensive, R&D production falls, which results in a decline in the growth rate.

When the tariff is eliminated in non-agriculture only, the production of agriculture falls and rises in non-agriculture. When all taxes are eliminated in the economy, production increases in both sectors, but the non-agriculture increases more. Under both situations, as non-agriculture is not labor intensive, aggregate demand for labor in final good production falls. More labor can be employed in the R&D activity resulting in an increase in the growth rate. These results show that trade liberalization enhances growth only if the sector which experiences an improvement in

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the terms of trade does not use a factor more intensively than the level of intensity with which the factor is used in the R&D sector. This effect is also shown by Grossman and Helpman.

Typical welfare analyses of trade liberalization in static models entails comparing the levels of aggregate social utility. At the steady state of this model, the dynamic utility function can be transformed into a static function (see Appendix IV). The comparison of the levels of this transformed utility is shown in the last row of Table 1 where it can be seen that trade liberalization does improve inter temporal social welfare, even though the growth rate has declined in simulation (i). When the growth rate rises due to trade reform, the improvements in social welfare exceed the former case. In general, we observe that the level of the welfare improvement is determined by the degree of liberalization.

The growth and interest rate paths to the 'new' steady state for each of the simulations (i)-(iii), are shown in Figure 3 and 4. The initial level of capital stock is based on the data comprising the SAM, an assumed steady state. Each simulation introduces a one time tariff or tax rate shock. We then derive the path that takes the economy from the 'calibrated steady state' to the new steady state. Paths in Figure 3 and 4 describe the growth and interest rate transition from the initial capital stock to the new steady state. These paths depart from those reported in Figures 11-14 for the analytical model. The paths associated with the analytical model are based on an arbitrarily chosen level of capital stock which is smaller than the level observed in the data. Interestingly, the results reported in Figures 3 and 4 indicate that more than one-half of the gap between the initial level of the growth and interest rates and their new steady state values are eliminated in five to six years. In the case of simulations (ii) and (iii), the half-life is six periods, and about five periods for simulation (i), the case where only agriculture is liberalized. Thus, the results suggests that the pay-offs to policy reform work through the economy in a relatively short period of time.

V.2   Optimal Growth and Policies to Attain It

As monopoly power exists in the market for differentiated capital, and accumulated knowledge

is an externality in the production of R&D, the optimal growth rate should depart from the competitive equilibrium rate[8]. To estimate the difference in the two rates, an optimal growth rate for the empirical model is obtained by solving the Social Planner's problem (See Appendix IV). As shown the Appendix II, the Social Planner's problem for this model is to maximize an inter temporal utility, subject to (1) the technologies in the final good production, R&D production, (2) resources constraints, and (3) Walras law. Since we are only interested in the discussion of the steady state optimal growth, the transformed static welfare function becomes the maximand instead of the dynamic form in the empirical model. This transformation is shown in Appendix IV.

The solution to the Social Planner problem shows the optimal growth rate to be more than a factor of three of the decentralized rate (Table 2, column (1)). Taken literally, this result suggests that interventions to resolve the markets failure to reward resources their full marginal product will yield far higher rates of growth than trade reform alone. The reason lies with the two market failures built into the model. First, the returns to R&D are not fully appropriated, and thus investment in R&D falls short of its optimal level. Since accumulated knowledge, M, is a public good employed in the production of new designs, the accumulated R&D good is non-rival and partially excludable in the sense that it is available to all as common knowledge, a key insight of Romer (1990, 1994). Consequently, the returns to R&D are not fully appropriable. Second, markets for differentiated capital clear at a price, P_k, higher than would prevail under perfect competition[9]. In contrast to the decentralized steady state solution with no policy distortions, the R&D sector increases its employment of labor by a factor of three, and that of the durable good by a factor of five.

Next, we investigate whether policy instruments can be used to obtain an equilibrium approximately equivalent to that of the Social Planner, the results of which are reported in Table 2, column 2. It seems plausible that a subsidy to R&D could be used to internalize the net

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[8]The steady state optimal growth rate derived from the analytical model is shown in Appendix II.

[9]See Romer (1990) and Grossman and Helpman (1991) for a more detailed discussion of this issue.

positive externality associated with product development. We begin with the decentralized -
steady state and otherwise distortion free solution. Then, taking the optimal growth rate of the
social planner's solution as a constraint, we solve the accompanying subsidy rate of the R&D
sector. Total subsidy payments are allocated in a lump sum from the aggregate incomes of the
private agents. To support an optimal growth rate, the subsidy rate on inputs of the R&D sector
is calculated to be about 89% of factor prices.

Similar to the social planner's problem, when the R&D sector is subsidized to support the
optimal growth rate, its employment of the other resource and labor increase. The higher
growth rate is also associated with an interest rate that is three times the otherwise distortion
free-decentralized rate. As investment becomes more profitable, consumers forestall current
consumption. While the R&D subsidy supports the optimal growth rate, and a level of welfare
that is higher than the otherwise distortion free-decentralized solution, the welfare level is below
that obtained by the social planner's solution.

To investigate the effect of a subsidy to purchasers of differentiated capital, we proceed as
above and solve for the subsidy to end users of differentiated capital subject to the growth rate
obtained by the social planner (Table 2, column 3). The subsidy lowers the cost of final good
production, the demand for differentiated capital rises, and the capital rental price increases
over three fold. However, the subsidy does not yield the optimal level of inter temporal utility,
although the level is higher than in the decentralized solution.

Next, both instruments are used, (Table 2, column 4). It can be seen that these results
correspond exactly to those obtained by the social planner.

Finally, we investigate whether an intervention in trade exists that can support the optimal
growth rate. It is known from the previous results that taxing agriculture and subsidizing non-
agriculture can increase the rate of growth. However, simulation results show that no trade
intervention exists which supports the optimal growth rate. By changing the relative value added
price, trade policy tends to rise the output of one sector at the cost of the other sector's production.
Hence, even though the fall in agricultural output releases labor and the other durable factor, the

output of non-agriculture rises, increasing its demand for additional resources. In equilibrium, the R&D sector is unable to attract levels of resources commensurate with the social planner's problem. Raising the tariff on agricultural imports to 464%, and subsidizing non-agricultural exports at a 80% rate only yields a growth rate 3.9% higher than the Walrasian equilibrium growth rate.

VI. Conclusions

This paper extends, modestly, the R&D based analytical models developed by Grossman and Helpman and Romer by allowing for capital accumulation, similar to Romer, but for a multiple sector economy and, perhaps more importantly, by deriving the transition path properties of the model. The second contribution is to show how an empirical discrete-time equivalent of the analytical model, augmented by the familiar Armington functions, can be specified and calibrated to country level data of the social accounting matrix variety, for which we chose East Asia as an example. The model is solved with software commonly used to solve static applied general equilibrium models[10]. Three simulations of the model are conducted, the first of which removes production taxes and tariffs on agricultural goods, the other which does the same for the non-agricultural sector, while the third liberalizes both sectors. Results are reported for static and dynamic effects. For each of these simulations, the transition paths to the steady-state are also obtained. Finally, we specify and solve the Social Planner's problem, and then perform experiments with subsidies to R&D and capital in order to determine whether policy could attain the Social Planner's outcomes.

We find the steady state equilibrium of the analytical model to be locally saddle-path stable and for a parameter set consistent with data from East Asia, we find the competitive equilibrium to be globally saddle-path stable. We illustrate a method for deriving observations on R&D and other variables from the social accounting matrix. The results from the simulations of the

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[10]The Transitional dynamics for the analytical model are obtained using the Mathematica software.

discrete-time equivalent of the analytical model, albeit augmented by the Armington functions, show that reform increases welfare in all three cases, but not necessarily economic growth. Since agriculture is labor intensive, as is the R&D sector, reform that increase the relative output prices of agriculture, increase labor costs to the R&D sector, thus decreasing the production of patents, hence causing the rate of growth to decline. Total reform increases growth by increasing profits of firms producing capital variety, interest rates, the demand for new patents, the price of new patents and savings. The half life of the transition path was found to be rather short, ranging from five to six periods, depending on the simulation. A solution to the Social Planner's problem yielded a growth rate that exceeded the decentralized growth rate by a factor of about 3.4, suggesting that, for this data set, interventions which correct for market failures may have more welfare increasing potential than trade reform. Subsidy to the R&D sector and to end users of differentiated capital were found which reproduced the Social Planner's equilibrium

While the paper establishes the feasibility of developing empirical R&D based growth models, it merely introduces this notion and shows on approach. Much obviously remains, including the recognition that while there is likely no single explanation of why countries grow or fail to grow, it may be impractical to identify and model the various sources of growth in a single model. Of obvious further consideration must be the role of international capital flows, the partial excludability of knowledge embodied in these flows, and thus the importance of foreign trade in variety capital. Efforts in this direction should help to further unlock insights into the sources of growth and the policies needed to bring them about.

Table 1. Static and dynamic effects of trade liberalization relative to the base year

| | (i) Lib. Agriculture | | (ii) Lib. Non-Ag | | (iii) Lib. All Sectors | |
| | Static Sol. | Dynamic Sol. | Static Sol. | Dynamic Sol. | Static Sol. | Dynamic Sol. |
	(1)	(2)	(3)	(4)	(5)	(6)
L_y	1.0061	1.0061	0.9205	0.9205	0.9264	0.9264
L_z	0.9961	0.9961	1.0514	1.0514	1.0477	1.0477
B_y	1.0086	1.0086	0.8918	0.8918	0.8995	0.8995
B_z	0.9985	0.9985	1.0186	1.0186	1.0173	1.0173
$\bar{K}_y$	1.0069	1.0244	0.9104	0.9104	0.9169	1.1652
$\bar{K}_y$	0.9969	1.0142	1.0399	1.0399	1.0370	1.3178
$\bar{Y}$	1.0066	1.0126	0.9139	0.9139	0.9201	1.0084
$\bar{Z}$	0.9972	1.0031	1.0367	1.0367	1.0341	1.1227
g		0.9990		1.0065		1.0055
r		0.9991		1.0117		1.0099
$\overline{SAV}$		1.0037		1.0534		1.0605
$\bar{K}$		1.0246		1.2464		1.2797
π		1.0038		1.0528		1.0600
P_m		1.0047		1.0467		1.0547
Utility		1.0001		1.0011		1.0012

For the static effects, total capital supply is fixed at the level of the base year.
Values reported are to the steady state equilibria of the base solution

Table 2. Contrast of the optimal solution to: subsidies to R&D only,
end users of capital only, and both, relative to the base year

	Optimal Solution (1)	Subsidy to R&D (2)	Subsidy to Capital (3)	Subsidy to Both (4)
g	3.3740	3.3740	3.3740	3.3740
r		3.1897	3.1897	3.8970
$\bar{K}$	0.6724	0.0847	2.6135	0.6724
Utility	1.0206	1.0177	1.0130	1.0206

Values reported are relative to steady state equilibria.
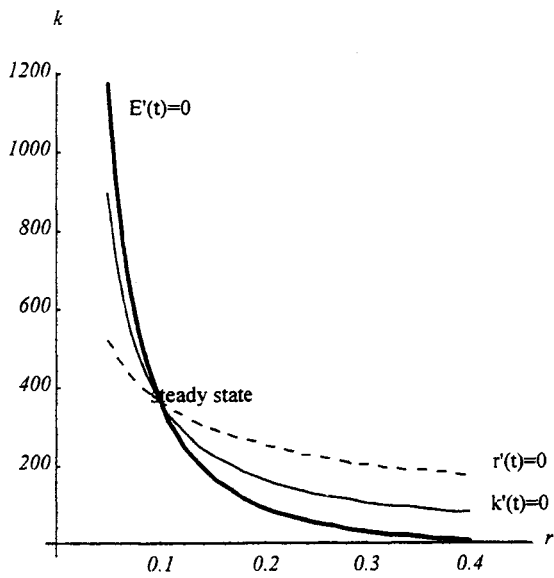
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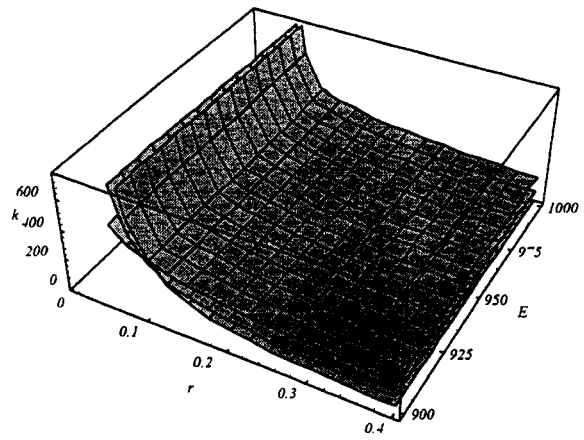
Figure 1: Zero-Motion Loci

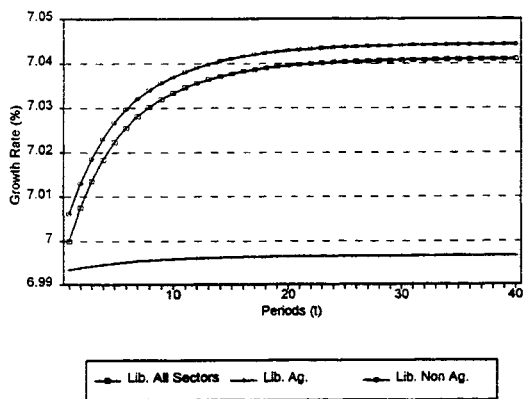
Figure 2: Zero-Motion Loci

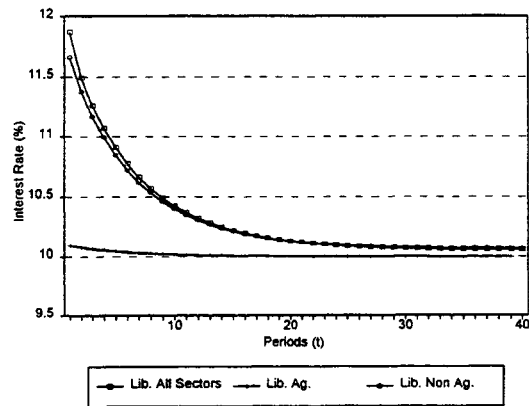
Figure 3: Growth Rate, Post Reform

Figure 4: Interest Rate, Post Reform

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Appendix I: Stability Properties of the Analytical Model

A. The local stability properties of the steady state

To investigate the local stability properties of the steady state we need to linearize the system give by equations (25), (28) and (29) around the steady state. This is done by evaluating the Jacobian of the system at the steady state. The competitive equilibrium is locally saddle-path stable (and also unique) if this Jacobian has two eigen values with positive real parts and one with negative real part. This is because we have only one predetermined value ($k(0)$) and the other two values ($\bar{E}(0)$ and $r(0)$) are free. the Jacobian evaluated at the steady state is given by

$$
J^* = \begin{bmatrix} \frac{\partial \dot{r}}{\partial r} & \frac{\partial \dot{r}}{\partial k} & \frac{\partial \dot{r}}{\partial E} \\ \frac{\partial \dot{k}}{\partial r} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial E} \\ \frac{\partial \dot{\bar{E}}}{\partial r} & \frac{\partial \dot{\bar{E}}}{\partial k} & \frac{\partial \dot{\bar{E}}}{\partial E} \end{bmatrix}_{|r^*,\bar{E}^*,k^*} = \begin{bmatrix} r^* & \frac{(1-\alpha_3)^2}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{*1+1/(1-\alpha_3)} k^* & 0 \\ \frac{k^*}{\alpha_3^2(1-\alpha_3)} & (\frac{2}{\alpha_3} + \frac{1}{\alpha_3^2})r^* - \bar{V} & -\frac{1}{MC_k} \\ [\frac{1}{\sigma} + \frac{1}{\alpha_3(1-\alpha_3)}]\bar{E}^* & \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{*1/(1-\alpha_3)} \bar{E}^* & 0 \end{bmatrix}.
$$

The characteristic equation for this matrix is given by

$$
\lambda^3 - TrJ^*\lambda^2 + BJ^*\lambda - DetJ^* = 0,
$$

where TrJ^* and $DetJ^*$ are the respective trace and determinant of J^* and BJ^* is an expression involving some of the principal minors of J^*. The solution to this equation gives the eigen values of matrix J^*. These roots satisfy

$$
\lambda_1 + \lambda_2 + \lambda_3 = TrJ^*
$$

$$
\lambda_1\lambda_2\lambda_3 = DetJ^*
$$

The determinant of J^* is given by

$$
DetJ^* = \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{*1+1/(1-\alpha_3)} \bar{E}^* \left(1 - \frac{(1-\alpha_3)}{\sigma} - \frac{1}{\alpha_3} \right) < 0
$$

since $\alpha_3 < 1$. The trace is given by

$$
TrJ^* = (1 + \frac{2}{\alpha_3} + \frac{1}{\alpha_3^2})r^* - \bar{V}.
$$

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By replacing r^* by its steady state value and after some manipulation we were able to arrive at

$$TrJ^* = \left\{ \frac{(1-\sigma)(\alpha_3\bar{V} - \rho)}{\alpha_3 + \sigma} + \rho \right\} + \frac{(1 + \frac{1}{\alpha_3})(\sigma\bar{V} + \rho)}{\alpha_3 + \sigma}.$$

The term between curly brackets is nothing but $(\sigma - 1)g^* + \rho$ which is always positive otherwise the utility function is not bounded in the steady state. Therefore, TrJ^* is always positive.

Since $DetJ^* < 0$ equation (8) suggest there is at least one negative root and the other two roots are of the same sign. But because $TrJ^* > 0$ and there is at least one negative root, equation (8) dictates at least one root has to be positive. Hence, we have two positive roots and one negative root. We have proved the following proposition.

Proposition 1 *The steady state equilibrium is locally saddle-path stable.*

B. The global stability properties of the steady state

We have shown previously that the model is locally stable. We now show it is also globally saddle-path stable. Because we have a system of three nonlinear differential equations, we have to resort to numerical techniques to do that. We adopt the Time Elimination Method (TEM) of Mulligan (1991). The application of TEM proceeds as follows. First, using change rule of calculus we obtain the slope of the so called policy functions $r(k)$ and $\bar{E}(k)$ as

$$r'(k) = \frac{\dot{r}}{\dot{k}} = \frac{r\left(\frac{(1-\alpha_3)^2}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k - \frac{(1-\alpha_3)}{\alpha_3} r\right)}{\left(\frac{r}{\alpha_3^2} - \bar{V} + \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k\right) k - \frac{\bar{E}}{MC_k}}$$

$$\bar{E}'(k) = \frac{\dot{\bar{E}}}{\dot{k}} = \frac{\bar{E}\left(\frac{1}{\sigma}(r - \rho) - \bar{V} + \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k\right)}{\left(\frac{r}{\alpha_3^2} - \bar{V} + \frac{(1-\alpha_3)}{\alpha_3^2} \frac{MC_k}{\hat{P}_m} r^{1/(1-\alpha_3)} k\right) k - \frac{\bar{E}}{MC_k}}.$$

This is a system of two differential equations in k. Second, the initial conditions for this system are obtained by noticing that the steady state satisfies equations (25), (28) and (29). Hence, when k assumes the value k^*, r and $\bar{E}$ are given by r^* and $\bar{E}^*$, respectively. Third, because the slopes of the policy functions at the steady state is of the form $0/0$, we use L'Hopital rule to obtain the slopes. Once a numerical solution is obtained we can used it to obtain the initial values for r and $\bar{E}$ and solve the system (25), (28) and (29) in time[11].

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[11]Mathematica was used to obtain these numerical solutions. k*, r* and $\overline{E}^*$ are chosen to be consistent with the

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Proposition 2 *Given the parameter specification, the competitive equilibrium is globally saddle-path stable.*

Proof We applied TEM and we were able to compute the policy functions over a wide range for k: between a very low and an arbitrary large number. We were also able to trace the behavior of the variables over time. We found that starting from any level of capital, all variables converge to their steady state values. See Figures 5-14. This means that the model is globally saddle-path stable for the give parameters. ■

The Policy functions $r(k)$, $\bar{E}(k)$, and $g(k)$ are depicted in Figures 5-10. As it is obvious from these figures $r(k)$ is downward sloping while $\bar{E}(k)$ and $g(k)$ are upward sloping. We have the following proposition.

Proposition 3 *The transition from low capital involves low interest rate, high productivity-adjusted expenditure, high growth rate, high productivity-adjusted wages, high prices for new designs, and high profits.*

Proof See Figures 5-10. ■

We now provide economic interpretation why these functions look like that. Low values of k means the productivity of k is very high and hence the economy is highly productive. This is what is called the Solow-effect (Mulligan and Sala-i-Martin, 1992). Low values of k also means the rate of interest is high because the opportunity cost of capital is high (r is downward sloping). High interest rates means higher savings, higher wages, higher prices for new designs. The latter means high innovation and growth.

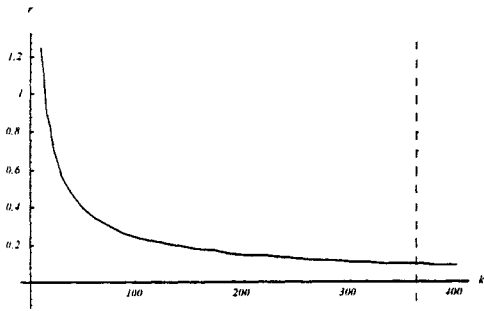
initial steady state of the model.
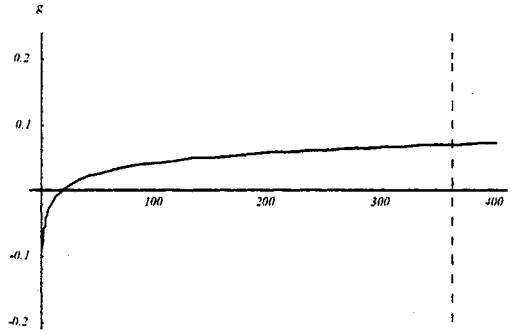
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Figure 5: Policy Function $r(k)$

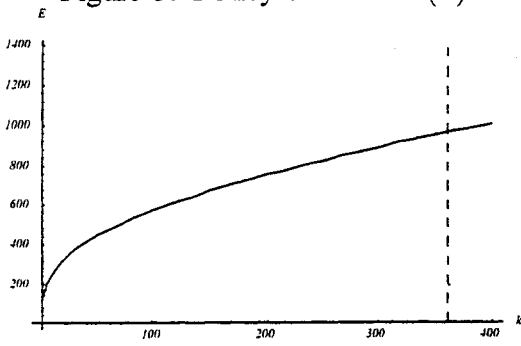Firgure 6: Policy Function $g(k)$

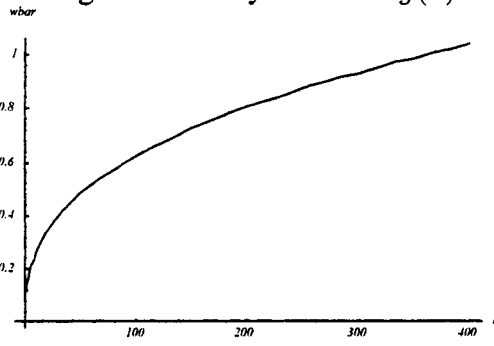Figure 7: Policy Function $E(k)$

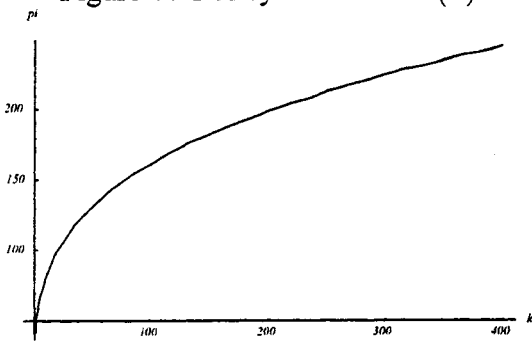Figure 8: Policy Function $\bar{W}(k)$
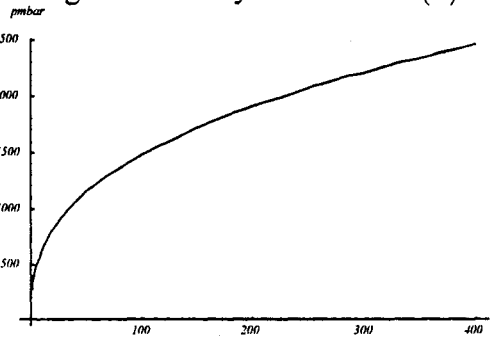

Figure 9: Policy Function $\pi(k)$

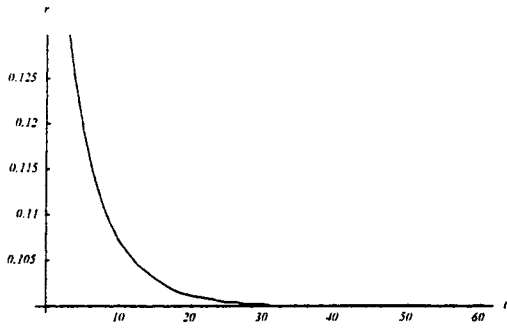Figure 10: Policy Function $\bar{P}_m(k)$
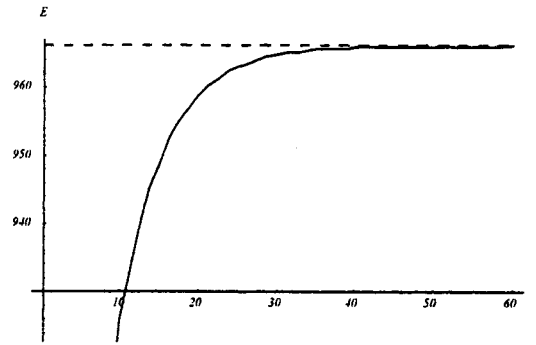
Figure 11: Transition Path $r(t)$
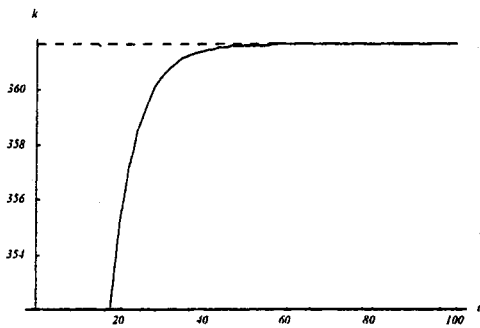

Figure 12: Transition Path $E(t)$

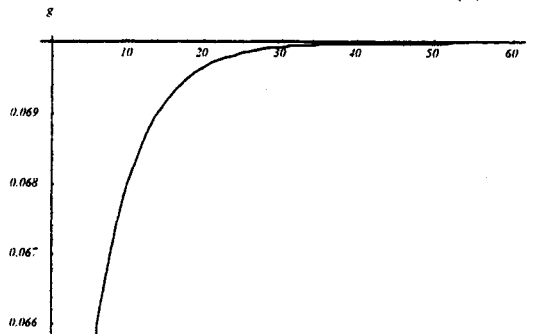Figure 13: Transition Path $k(t)$

Figure 14: Transition Path $g(t)$

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Appendix II: The Social Planner's Problem for the Analytical Model

The social planner's problem can be written as

$$\max \int_0^\infty e^{-\rho t} \frac{E^{1-\sigma} - 1}{1-\sigma} dt$$

subject to

$$\dot{K} = \frac{1}{MC_k} (P_y Y + P_z Z - E)$$

$$Y = A_y L_y^{\alpha_1} B_y^{\alpha_2} H_y^{\alpha_3}$$

$$Z = A_z L_z^{\beta_1} B_z^{\beta_2} H_z^{\alpha_3}$$

$$\dot{M} = A_m L_m^\theta B_m^{1-\theta} M$$

$$H = H_y + H_z$$

$$H = K M^{(1-\delta)/\delta}$$

$$L = L_y + L_z + L_m$$

$$B = B_y + B_z + B_m,$$

where MC_k is defined in equation (13). The current value Hamiltonian for this problem can be written as

$$
\begin{aligned}
H = {} & \frac{E^{1-\sigma} - 1}{1-\sigma} + \lambda \frac{1}{MC_k} (P_y Y + P_z Z - E) + r_y \left(Y - A_y L_y^{\alpha_1} B_y^{\alpha_2} H_y^{\alpha_3} \right) + \\
& r_z \left(Z - A_z L_z^{\beta_1} B_z^{\beta_2} H_z^{\alpha_3} \right) + p_h \left(H_y + H_z - K M^{(1-\delta)/\delta} \right) + \\
& \gamma A_m L_m^\theta B_m^{1-\theta} M + W_L \left(L - L_y - L_z - L_m \right) + W_B \left(B - B_y - B_z - B_m \right).
\end{aligned}
$$

where $\lambda, \gamma, \gamma_y, \gamma_z, P_h, W_L$, and W_B denote the shadow prices associated with the relevant constraints.

First-order conditions are:

$$\frac{\partial H}{\partial E} = E^{-\sigma} - \frac{\lambda P_z}{MC_k} = 0 \tag{II.1}$$

$$\frac{\partial H}{\partial Y} = \frac{\lambda P_y}{MC_k} + \gamma_y = 0 \tag{II.2}$$

$$\frac{\partial H}{\partial Z} = \frac{\lambda P_z}{MC_k} + \gamma_z = 0 \tag{II.3}$$

$$\frac{\partial H}{\partial H_y} = -\frac{\alpha_3 \gamma_y Y}{H_y} + P_h = 0 \tag{II.4}$$

$$\frac{\partial H}{\partial H_z} = -\frac{\alpha_3 \gamma_z Z}{H_z} + P_h = 0 \tag{II.5}$$

$$\frac{\partial H}{\partial L_y} = -\frac{\alpha_1 \gamma_y Y}{L_y} - W_L = 0 \tag{II.6}$$

$$\frac{\partial H}{\partial L_z} = -\frac{\beta_1 \gamma_z Z}{L_z} - W_L = 0 \tag{II.7}$$

$$\frac{\partial H}{\partial B_y} = -\frac{\alpha_2 \gamma_y Y}{B_y} - W_B = 0 \tag{II.8}$$

$$\frac{\partial H}{\partial B_z} = -\frac{\beta_2 \gamma_z Z}{B_z} - W_B = 0 \tag{II.9}$$

$$\frac{\partial H}{\partial L_m} = \frac{\theta \gamma \dot{M}}{L_m} - W_L = 0 \tag{II.10}$$

$$\frac{\partial H}{\partial B_m} = \frac{(1-\theta)\gamma \dot{M}}{B_m} - W_B = 0 \tag{II.11}$$

$$\dot{\lambda} = \rho\lambda - P_h M^{(1-\delta)/\delta} \tag{II.12}$$

$$\dot{\gamma} = \rho\gamma - \gamma g + \frac{1-\delta}{\delta}\frac{P_h(H_y + H_z)}{M} \tag{II.13}$$

Using equations (II.4) to (II.11) with the production functions give

$$\gamma_y = W_L^{\alpha_1} W_B^{\alpha_2} P_h^{\alpha_3} \tag{II.14}$$

$$\gamma_z = W_L^{\beta_1} W_B^{\beta_2} P_h^{\alpha_3} \tag{II.15}$$

$$\gamma = W_L^{\theta} W_B^{1-\theta}/M \tag{II.16}$$

Equations (II.14) and (II.15) gives $\gamma_y/\gamma_z = (W_L/W_B)^{\alpha_1 - \beta_1}$. Dividing(II.2) by (II.3) yields $\gamma_y/\gamma_z = P_y/P_z$. Hence, $W_L/W_B = (P_y/P_z)^{1/(\alpha_1 - \beta_1)}$. Since world prices are assumed to be constant we have $\dot{W}_L/W_L = \dot{W}_B/W_B$.

Equation (II.13) can be rewritten as

$$\frac{\dot{\gamma}}{\gamma} = \rho - g - \frac{1-\delta}{\delta}\frac{P_h(H_y + H_z)}{M\gamma}.$$

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Substituting equations(II.6) to (II.11) in the resources constraints and rearranging yields

$$\gamma_y Y + \gamma_z Z = -\frac{V - \gamma M g}{1 - \alpha_3}.$$

where $V \equiv W_L L + W_B B$. Hence,

$$\frac{\dot{\gamma}}{\gamma} = \rho - g - \frac{\alpha_3 (1 - \delta)}{\delta (1 - \alpha_3)} \frac{V - \gamma M g}{\gamma M}.$$

Equation (II.16) implies $\dot{\gamma}/\gamma = \dot{W}/W - g$, where $\dot{W}/W \equiv \dot{W}_L/W_L = \dot{W}_B/W_B$. From equations (II.1) and (II.2) we have $\dot{\gamma}_y/\gamma_y = \dot{\lambda}/\lambda = -\sigma g_e$, where g_e stands for the growth rate of output Z and Y and expenditure E. Equations (II.6) with the fact that L_y is constant in a steady state gives

$$\frac{\dot{W}}{W} = \frac{\dot{\gamma}_y}{\gamma_y} + \frac{\dot{Y}}{Y} = (1 - \sigma) g_e.$$

Combining these results we obtain

$$\frac{\dot{\gamma}}{\gamma} = (1 - \sigma) g_e - g.$$

We can use the production function for either Y or Z with the fact that H grows at $g_e + (1 - \delta) g / \delta$ to arrive at

$$g_e = \frac{\alpha_3 (1 - \delta)}{\delta (1 - \alpha_3)} g.$$

Thus equating the two formulas for $\dot{\gamma}/\gamma$ using the above result and rearranging yields a steady state optimal growth rate whcih is different from the competitive equilibrium rate in (33).

$$g = \frac{\dfrac{\alpha_3 (1 - \delta)}{\delta (1 - \alpha_3)} \left(\dfrac{W_L L + W_B B}{W_L^\theta W_B^{1-\theta}} \right) - \rho}{\dfrac{\alpha_3 (1 - \delta)}{\delta (1 - \alpha_3)} \sigma}.$$

Appendix III: The CGE Model Equations

Momentary utility at each time period

$$u_t = CY_c(t)^\gamma CZ_c(t)^{1-\gamma}$$

The time-discrete inter temporal utility

$$U_t = \sum_{t=0}^{\infty} (\frac{1}{1+\rho})^t \frac{u(t)^{1-\sigma} - 1}{1-\sigma}$$

The following equations are used to solve the steady state equilibrium in GAMS. In order to be consistent with the analytical model, we adopt the dual instead of the primal form for the equation system.

Value added:

$$PVA_y = \frac{1}{A_y \alpha_1^{\alpha_1} \alpha_2^{\alpha_2} \alpha_3^{\alpha_3}} \overline{W_L}^{\alpha_1} \overline{W_B}^{\alpha_2} P_k^{\alpha_3} \tag{III.1}$$

$$PVA_z = \frac{1}{A_z \beta_1^{\beta_1} \beta_2^{\beta_2} \alpha_3^{\alpha_3}} \overline{W_L}^{\beta_1} \overline{W_B}^{\beta_2} P_k^{\alpha_3} \tag{III.2}$$

The R&D sector

$$P_m = \frac{1}{A_m \theta^\theta (1-\theta)^{1-\theta}} \overline{W_L}^{\theta} \overline{W_B}^{1-\theta} \tag{III.3}$$

New capital formation

$$MC_k = \frac{1}{A_k \eta^\eta (1-\eta)^{1-\eta}} PC_y^\eta PC_z^{1-\eta} \tag{III.4}$$

Capital rental price

$$P_k = \frac{1}{\alpha_3} r MC_k \tag{III.5}$$

The CET

$$P_y = \Gamma_y^{-1} (\mu^{-\sigma_y} PE_y^{\sigma_y+1} + (1-\mu_y)^{-\sigma_y} PD_y^{\sigma_y+1})^{\frac{1}{\sigma_y+1}} \tag{III.6}$$

$$P_z = \Gamma_z^{-1} (\mu^{-\sigma_z} PE_z^{\sigma_z+1} + (1-\mu_z)^{-\sigma_z} PD_z^{\sigma_z+1})^{\frac{1}{\sigma_z+1}} \tag{III.7}$$

where $\sigma_j = \frac{1}{\rho_j - 1}$, $j = y, z$, and Armington functions

$$PC_y = \Lambda_y^{-1} (\nu^{\sigma_{my}} PM_y^{1-\sigma_{my}} + (1-\nu_y)^{\sigma_{my}} PD_y^{1-\sigma_{my}})^{\frac{1}{1-\sigma_{my}}} \tag{III.8}$$

$$PC_z = \Lambda_z^{-1} (\nu^{\sigma_{mz}} PM_z^{1-\sigma_{mz}} + (1-\nu_z)^{\sigma_{mz}} PD_z^{1-\sigma_{mz}})^{\frac{1}{1-\sigma_{mz}}} \tag{III.9}$$

where $\sigma_{mj} = \frac{1}{\varphi_j + 1}$, $j = y, z$.

Exogenous trade prices

$$PE_j = (1 - te_j)PWE_j \qquad \text{(III.10 - III.11)}$$

$$PM_j = (1 + tm_j)PWM_j \qquad \text{(III.12 - III.13)}$$

Value added prices

$$PVA_j = (1 - t_j)P_j - a_{yj}PC_y - a_{zj}PC_z \qquad \text{(III.14 - III.15)}$$

Demand system

$$\overline{DY} = (\frac{(1 - \mu_y)P_y}{PD_y})^{-\sigma_y}\Gamma_y^{-(\sigma_y+1)}\bar{Y} \qquad \text{(III.16)}$$

$$\overline{DZ} = (\frac{(1 - \mu_z)P_z}{PD_z})^{-\sigma_z}\Gamma_z^{-(\sigma_z+1)}\bar{Z} \qquad \text{(III.17)}$$

$$\overline{EY} = (\frac{\mu_y P_y}{PE_y})^{-\sigma_y}\Gamma_y^{-(\sigma_y+1)}\bar{Y} \qquad \text{(III.18)}$$

$$\overline{EZ} = (\frac{\mu_z PC_z}{PE_z})^{-\sigma_z}\Gamma_z^{-(\sigma_z+1)}\bar{Z} \qquad \text{(III.19)}$$

$$\overline{DY} = (\frac{(1 - \nu_y)PC_y}{PD_y})^{\sigma_{my}}\Lambda_y^{(\sigma_{my}+1)}\overline{CY} \qquad \text{(III.20)}$$

$$\overline{DZ} = (\frac{(1 - \nu_z)PC_z}{PD_z})^{\sigma_{mz}}\Lambda_z^{(\sigma_{mz}+1)}\overline{CZ} \qquad \text{(III.21)}$$

$$\overline{MY} = (\frac{\nu_y PC_y}{PM_y})^{\sigma_{my}}\Lambda_y^{(\sigma_{my}+1)}\overline{CY} \qquad \text{(III.22)}$$

$$\overline{MZ} = (\frac{\nu_z PC_z}{PM_z})^{\sigma_{mz}}\Lambda_z^{(\sigma_{mz}+1)}\overline{CZ} \qquad \text{(III.23)}$$

$$\overline{CY_c} = \frac{\gamma(\bar{I} - \overline{SAV})}{PC_y} \qquad \text{(III.24)}$$

$$\overline{CZ_c} = \frac{(1 - \gamma)(\bar{I} - \overline{SAV})}{PC_z} \qquad \text{(III.25)}$$

$$\overline{CY_i} = a_{iy}\bar{Y} \qquad \text{(III.26)}$$

$$\overline{CZ_i} = a_{iz}\bar{Z} \qquad \text{(III.27)}$$

$$\overline{CY_k} = \frac{\eta MC_k\bar{K}g}{PC_y} \qquad \text{(III.28)}$$

$$\overline{CZ_k} = \frac{(1 - \eta)MC_k\bar{K}g}{PC_z} \qquad \text{(III.29)}$$

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Factor market clearing

$$\alpha_1 PVA_y \overline{Y} + \beta_1 PVA_z \overline{Z} + \theta P_m g = \overline{W_L} L \qquad \text{(III.30)}$$

$$\alpha_2 PVA_y \overline{Y} + \beta_2 PVA_z \overline{Z} + (1 - \theta) P_m g = \overline{W_B} B \qquad \text{(III.31)}$$

$$\alpha_3 (PVA_y \overline{Y} + PVA_z \overline{Z}) = P_k \overline{K} \qquad \text{(III.32)}$$

Commodity market clearing

$$\overline{CY} = \overline{CY_c} + \overline{CY_y} + \overline{CY_z} + \overline{CY_k} \qquad \text{(III.33)}$$

$$\overline{CZ} = \overline{CZ_c} + \overline{CZ_y} + \overline{CZ_z} + \overline{CZ_k} \qquad \text{(III.34)}$$

Momentary income

$$\overline{I} = \overline{W_L} L + \overline{W_B} B + P_k \overline{K} + t_y P_y \overline{Y} + tm_y PWM_y \overline{MY} \qquad \text{(III.35)}$$

$$+ te_y PWE_y \overline{EY} + t_z P_z \overline{Z} + tm_z PWM_z \overline{MZ} + te_z PWE_z \overline{EZ}$$

Savings and investment balance

$$\overline{SAV} = PC_y \overline{CY_k} + PC_z \overline{CZ_k} + P_m g \qquad \text{(III.36)}$$

Growth and interest rate

$$g = \frac{r - \rho}{\sigma} \qquad \text{(III.37)}$$

$$r = \frac{(1 - \alpha_3) P_k \overline{K}}{P_m} \qquad \text{(III.38)}$$

Equations (III.1 to III.36) together with the following difference equations are used to solve the transitional equilibrium. As the empirical model is a time discrete version of the analytical model all variables denoted by a 'bar' are replaced by their corresponding variables without a 'bar' and divided by M(t). E.g., $\overline{W_L}$ is replaced by $\frac{W_L(t)}{M(t)}$.

The accumulation of designs

$$M(t + 1) = M(t) + NEWM(t) \qquad \text{(III.39)}$$

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where $M(1) = 1$.

The Euler condition derived from inter temporal utility

$$\frac{1+\rho}{1+r(t+1)}(\frac{u(t+1)}{u(t)})^\sigma = \frac{PC_y(t)^\gamma PC_z(t)^{1-\gamma}}{PC_y(t+1)^\gamma PC_z(t+1)^{1-\gamma}}$$ (III.40)

Non-arbitrage condition

$$(1 + r(t))P_m(t-1) = \frac{(1-\alpha_3)P_k(t)K(t)}{M(t)} + P_m(t)$$ (III.41)

Growth rate

$$g(t) = \frac{NEWM(t)}{M(t)}$$ (III.42)

At steady state (time period T), equation (III.40) is replaced by

$$\frac{M(T)}{M(T-1)} = (\frac{1+r(T)}{1+\rho})^{\frac{1}{\sigma}}$$ (III.43)

equation (III.41) is replaced by

$$r(T)P_m(T-1) = \frac{(1-\alpha_3)P_k(T)K(T)}{M(T)}$$ (III.44)

Definitions of Variables and Parameters

Definition of exogenous variables and parameters

σ : Inverse elasticity of inter temporal substitution in consumption

ρ : Rate of time preference in utility

γ : Share parameter in momentary utility function

α_i : Share of primary inputs and capital in the production function for Y, $i = 1, 2, 3$

β_i : Share of primary inputs and capital in the production function for Z, where $\beta_3 = \alpha_3$

a_{ij} : Input-output coefficient for intermediate input i in the production of good, j, $j = y, z$

θ : Share parameter for labor in the R&D production function

η : Share parameter for the composite good Y in the capital formation function

A_i : Shift parameter in technology function, $i = y, z, m, k$

σ_i : Elasticity of substitution between domestic and exportable goods in the CET function, $i = y, z$

σ_{mi} : Elasticity of substitution between domestic and imported good in the Armington function, $i = y, z$

μ_i : Share parameter for export goods in the CET function, $i = y, z$

ν_i : Share parameter for import goods in the Armington function, $i = y, z$

Γ_i : Shift parameter in the CET function, $i = y, z$

Λ_i : Shift parameter in the Armington function, $i = y, z$

PWE_i : Exogenous world export price for good i

PWM_i : Exogenous world import price for good i

te_i : Export subsidy rate for good i

tm_i : Tariff rate for good i

it_i : Indirect producer tax rate for good i

L : Labor endowment

B : The other durable input endowment

Definition of Endogenous Variables

$\overline{CY_c}$: Momentary consumer's demand the composite good Y

$\overline{CZ_c}$: Momentary consumer's demand for the composite good Z

u : Momentary utility

$\overline{Y}$: Output of good Y

$\overline{Z}$: Output of good Z

$\overline{CY_i}$: Intermediate demand for the composite good Y in the production of good i

$\overline{CZ_i}$: Intermediate demand for the composite good Z in the production of godd i

$\overline{CY_k}$: Investment demand for composite good Y

$\overline{CZ_k}$: Investment demand for composite good Z

$\overline{K}$: Capital stock

$\overline{DY}$: Output Y produced and consumed in the home country

$\overline{DZ}$: Output Z produced and consumed in the home country

$\overline{EY}$: Output of Y produced in the home country and exported

$\overline{EZ}$: Output of Z produced in the home country and exported

$\overline{CY}$: Composite good Y consumed (final and intermediate demand) in home country

$\overline{CZ}$: Composite good Z consumed (final and intermediate demand) in home country

$\overline{MY}$: Imported good Y

$\overline{MZ}$: Imported good Z

PE_i : Export price for good i

PM_i : Import price for good i

PC_i : Composite price for good i

PD_i : Price for good i produced and consumed in the home country

P_i : Producer's price for good i

PVA_i : Value added price for good i

$\overline{W_i}$: Factor prices, $i = L, B$

MC_k :Unit cost of capital formation

P_k : Monopoly capital rental rate

P_m : The price of new designs

$\overline{I}$: Total disposable income (inclusive of savings)

$\overline{SAV}$: Total savings

g : Growth rate

r : Interest rate

$M(t)$: Accumulated R&D outputs at time t

$NEWM(t)$: New R&D inputs at time t

$*$ The "bar" variables are normalized by $1/M$, and hence are time independent variables.

Appendix IV: Social Planner's Problem for the CGE Model

We first transform the dynamic social welfare function into a static form. The dynamic welfare function is

$$\frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} \left(u(t)^{1-\sigma} - 1 \right) dt$$

where $u(t) \equiv CY_c(t)^\gamma CZ_c(t)^{1-\gamma}$.

We analyze only the special case where $\alpha_3 = \delta$. Let $\overline{CY_c} \equiv CY_c(t)/M(t)$ and $\overline{CZ_c} \equiv CZ_c(t)/M(t)$. These variables are independent of time in the steady state. Given $M(0) > 0$, we have along the steady state path $M(t) = M(0)e^{gt}$. Hence, $u(t) = \bar{u}M(0)e^{gt}$, where $\bar{u} = \overline{CY_c}^\gamma \overline{CZ_c}^{1-\gamma}$.

We will assume that the utility function is bounded in the steady state equilibrium. A necessary condition for this is $(1-\sigma)g - \rho < 0$. We assume this condition is satisfied. This allows us to rewrite the social welfare function as

$$\frac{[\bar{u}M(0)]^{1-\sigma}}{(1-\sigma)[\rho - (1-\sigma)g]} - \frac{1}{(1-\sigma)\rho}.$$

We can now write the social planner's problem in the following transformed static form

$$\max \quad \frac{[\bar{u}M(0)]^{1-\sigma}}{(1-\sigma)[\rho - (1-\sigma)g]} - \frac{1}{(1-\sigma)\rho}$$

subject to

$$\bar{u} = \overline{CY_c}^\gamma \overline{CZ_c}^{1-\gamma}$$

$$\overline{Y} = A_y L_y^{\alpha_1} B_y^{\alpha_2} \overline{K}_y^{\alpha_3}$$

$$\overline{Z} = A_z L_z^{\beta_1} B_z^{\beta_2} \overline{K}_z^{\alpha_3}$$

$$\overline{Y} = \Gamma_y \left(\mu_y \overline{EY}^{\rho_y} + (1-\mu_y)\overline{DY}^{\rho_y} \right)^{1/\rho_y}$$

$$\overline{Z} = \Gamma_z \left(\mu_z \overline{EZ}^{\rho_z} + (1-\mu_z)\overline{DZ}^{\rho_z} \right)^{1/\rho_z}$$

$$\overline{CY} = \Lambda_y \left(\nu_y \overline{MY}^{\phi_y} + (1-\nu_y)\overline{DY}^{\phi_y} \right)^{1/\phi_y}$$

$$\overline{CZ} = \Lambda_z \left(\nu_z \overline{MZ}^{\phi_z} + (1-\nu_z)\overline{DZ}^{\phi_z} \right)^{1/\phi_z}$$

$$
\begin{aligned}
g &= A_m L_m^{\theta} B_m^{1-\theta} \\
\bar{K}g &= A_k \overline{Y}_k^{\eta} \overline{Z}_k^{1-\eta} \\
\overline{K} &= \overline{K}_y + \overline{K}_z \\
L &= L_y + L_z + L_m \\
B &= B_y + B_z + B_m \\
\overline{CY} &= a_{yy}\overline{Y} + a_{yz}\overline{Z} + \overline{CY_c} + \overline{CY_k} \\
\overline{CZ} &= a_{zy}\overline{Y} + a_{zz}\overline{Z} + \overline{CZ_c} + \overline{CZ_k},
\end{aligned}
$$

where all variables with a bar are productivity-adjusted variables.

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