Properties of Flexible Functional Forms for Modeling Bilateral Export Supply and Import Demand in Multi-Country Agri-Food Models

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PROPERTIES OF FLEXIBLE FUNCTIONAL FORMS FOR MODELING BILATERAL EXPORT SUPPLY AND IMPORT DEMAND IN MULTI-COUNTRY AGRI-FOOD MODELS

Abstract

This paper illustrates the opportunities of incorporating more advanced functional forms into multi-country trade policy models. It suggests the use of flexible forms such as the Symmetric Generalized McFadden Function (SGMF) or the Normalized Quadratic-Quadratic Expenditure System (NQQES). Particularly if issues namely trade in differentiated products, preferential trade and effects of standards and traceability on bilateral trade are considered the NQQES offers attractive properties since it allows estimating variety specific expenditure elasticities which might compensate for example increased certification costs. A second aim of the paper refers to the critique on the handling of model parameters in calibrated policy models. In general, employed elasticities violate the theoretical conditions. We describe the calibration procedure developed to obtain model parameters consistent with economic theory.

Key Words: Bilateral Trade Modelling, Flexible Functions, Calibration, JEL: B41, C61, F13, Q18, Q17

1 Introduction

Advances over the last decades with regard to methodology for analyzing international trade are substantial. The concurrent development of new functional forms and rigorous application of duality theory allows for more structural complexity with simultaneously adhering to the theoretical conditions. This holds for the analyses of supply as well as demand in both general and partial equilibrium models. At the same time model builders are confronted with numerous problems if carrying out relevant policy analysis in a changing agri-food sector. Policy makers' attention is increasingly consumer driven focusing on food safety and traceability, product attributes and differentiation, the importance of vertical market linkages and market power. As a result trade policy analysis gains relevance because trade is considered to become a more and more important part of agri-food demand, especially trade in higher-value differentiated commodities (Rude and Meilke, 2004). The question is how to adjust existing agri-food models designed for trade policy analysis to comply with these new issues. In this paper we will focus on the potential improvement of assessing bilateral trade flows for policy making by applying more advanced functional forms. Besides functional representations other aspects are of equal importance, such as depicting more explicit the relevant policies, improving data and considering imperfect competition, multi-stage production as well as dual labour markets. Recently, these advancements have been considered by Bouët et al. (2004) showing contrasting fortunes of developing countries in the Doha Round with the enhanced MIRAGE general equilibrium model. While cross-commodity effects between homogenous products and commodity-specific farm programs lose in importance, relevant models have to explain interactions between different levels of the supply chain. In an illustrative multi-country model with multi-stage production and country-specific trade costs Markusen and Venables (2004) demonstrate how the ability to fragment production combined with changes in countries' trade costs might alter common trade patterns. The success of the gravity equation has an outcome on trade theory and empirical investigations as well. Bilateral trade patterns are explained by "home market" effects, the level of product differentiation, distances, boarders and income differences (Schumacher 2003, Davis and Weinstein 2003, Feenstra et. al., 2001). Bergstrand (1989) shows, that the gravity equation is a reduced form of a general equilibrium model of bilateral trade among many countries with differentiated products, assuming a Constant Elasticity of Transformation (CET) function. The gravity approach underlines the continuing dominance of the CET functional form.

For decades, the Cobb-Douglas, the Constant Elasticity of Substitution (CES) and, with some limitation, also the Leontief functional forms dominate quantitative assessments of policies and technical change on supply, demand and trade as well as providing examples for text books on applications in microeconomic theory. The Armington specification is one of the most popular methods to handle trade in products differentiated by country of origin but at the same time it is highly restrictive (Carter and Alston, 1990). CES and CET functions include a number of maintained hypotheses which limit their applicability. To those who use the model results it is usually rather difficult to distinguish the models based on their algebraic functional form, and the maintained hypotheses hidden in these functions cannot easily be detected, especially by those not familiar with modelling (Frohberg 2001).

A rather severe limitation is the restricted substitutability among the arguments of the functions mentioned. The Leontief form does not allow for substitution, the Cobb Douglas function maintains the substitution elasticity to be unity and the CES/CET ones allow a sufficient range for substitution only if there are not more than two arguments. Due to this single constant elasticity of substitution or transformation they do not have the flexibility needed to reflect adequately second-order effects if three or more independent variables are included in the analysis.

The debate on the gains from preferential trade liberalization and "new regionalism" as well as the discussion on the tools employed for trade policy analysis offer new critical arguments on trade modelling in general and especially concerning the Armington approach. Schiff and Winters (2003) criticize CGE models because of their typically ad hoc estimates of behavioural parameters and moreover they argue that CGE models overestimate the terms of trade benefits to the members of regional trade agreements because they question the monopoly power of small countries implicitly supposed by the Armington approach.

Regarding the degree of substitutability, Hillberry et al. (2001) as well as McDaniel and Balistreri (2002) postulate that too small numerical values for the substitution parameters are presumed in multi-country CGE models for quantifying the degree to which agents respond to differences in bilateral trade costs. Instead most of the buying pattern is explained by the so called taste (Armington efficiency) parameter. The authors find that Armington taste parameters in CGE models are like error terms in econometric models, in that they contain the unexplained variance in the dependent variable. A low substitutability limits the model's response to changes in trade policies caused by changes in relative bilateral trade costs. This explains the widely-held belief in policymaking circles that empirical analyses based among others on the CES/CET functions understate the effects of policy changes on the economy. Moreover, these authors demonstrate that not only the quantitative effects of liberalization but also the qualitative ones i.e. losses or benefits, are sensitive to the choice of the CES substitution parameter. Similar problems arise in models depicting export possibilities of a country in a mirror like fashion to those of imports and using for this purpose the CET function. Besides limiting substitution possibilities in their conventional form, the CES and CET functions also imply a homothetic structure. Using them for differentiating products in trade analysis implies that changes in the total quantities of imports or exports leaves their respective country shares unaltered.

To summarize the debate on the influence of CGE models on policy and further the effect of functional forms on simulation results, it is important to distinguish small stylized models and complex applied models. Stylized models developed for example by Panagariya and Duttagupta (2001) indicate how sensibly models react to the selection of CES/CET parameters and how switching of substitution/transformation elasticities may turn a loss of welfare into a gain and vice versa, but they may be misused when pushed beyond their domain of applicability (Devarajan and Robinson, 2002). Large applied models reviewed by Robinson and Thierfelder (1999) on the other hand, have been criticized for their ad hoc parameterization and their black box character (Dawkins et al., 2001). Hence, the limitations of the functional forms usually employed in both kinds of models should be kept in mind when interpreting simulation result. Empirical models may gain from the experimentation with more a advanced functional form since they have advantages.

In the following, we want to highlight the effects resulting from different specifications of bilateral trade. We suppose 5 illustrative trading partners for a single traded good all having different trade shares but initially identical import and export prices. We specify second stage trade functions applying the CES/CET functions and compare their results with those obtained by using flexible forms. For the latter, the SGMF (Symmetric Generalized McFadden) developed by Diewert and Wales (1987) is employed for both bilateral exports and imports. In an additional comparison, domestic demand and bilateral imports are modelled using the NQQES (Normalized Quadratic-Quadratic Expenditure System) by Ryan and Wales (1996, 1999). Furthermore, we describe the procedure for calibrating the parameters for the NQQES consistent with economic theory.

The paper is organized as follows. In chapter 2 the alternatively used functional forms are discussed (section 2.1) and the calibration procedure is explained (section2.2). In chapter 3 the advantages of the SGMF and NQQES for modelling bilateral trade flows are explained using stylized data. The paper ends with the conclusions.

2 Model specification

To illustrate the advantages of flexible functional forms for modelling bilateral trade flows, we use the standard CGE country model developed at IFPRI (Löfgren et al., 2001) with a complete and consistent data set for Mozambique.[1] The IFPRI model is based on a Social Accounting Matrix (SAM), implemented in GAMS and solved as a mixed complementarity program (MCP). The GAMS code is written in a manner that provides the necessary flexibility to change single parts of the model and gives the analyst the possibility to supplement additional economic accounts being of special interest. The main focus of such efforts is a numerical implementation of theoretical structures to

[1] The flexible SGMF and NQQES supply and demand functions are usually implemented in the partial agricultural equilibrium models developed at IAMO, in particular, flexible bilateral trade systems were introduced in a recent model of Croatia's agricultural and food sector to analyse the effects of the country's various trade agreements on agriculture and food processing (Frohberg and Winter, 2003, Winter and Frohberg, 2004).

provide insights about the effects of policy or other changes given theory is maintained rather than to test it. As a consequence, model results depend on the functional form employed and strongly rely on the set of parameters either specified entirely by the modeller or partly endogenously determined. Especially substitution parameters are key elements in policy-oriented models and thus crucial in determining the quantitative and sometimes also the qualitative results (McDaniel and Balistreri, 2002). The standard IFPRI country model distinguishes domestic and foreign goods and aggregates them by using the CES function for imports; .i.e. the Armington approach and the CET function for exports. The model, however, does not cover bilateral trade flows, neither for imports nor for exports.

2.1 Functional Forms

In the following we describe those functions which we linked into the standard IFPRI CGE country model for analyzing bilateral trade.

Bilateral trade

For any commodity, say good i, at the first stage of trading the disappearance of domestic production is split into domestic sales and exports employing the standard CET transformation, and, following Armington, domestic demand is combined as a composite of imported and domestically produced goods by the CES.

At the second stage of trading the good i, the substitutability among foreign goods is represented. In other words, exporters and importers differentiate between countries of destination and origin, respectively. Equation **Error! Reference source not found.** shows the aggregation of bilateral export quantities to the composite export quantity QE_i and of bilateral export prices to the composite export price PE_i of good i. The latter is equal to the marginal revenue of the composite export commodity.

$$QE_i = \Omega_i \cdot \left(\sum_r \upsilon_{i,r} \cdot QER_{i,r}{}^{\tau_i} \right)^{\frac{1}{\tau_i}} \quad and \quad PE_i = \frac{1}{\Omega_i} \cdot \left(\sum_r \upsilon_{i,r}{}^{\frac{1}{1-\tau_i}} \cdot PER_{i,r}{}^{\frac{-\tau_i}{1-\tau_i}} \right)^{\frac{\tau_i - 1}{\tau_i}} \tag{1}$$

Where

i : commodity index

r : country index

QE_i : composite export quantity of Mozambique of commodity i

$QER_{i,r}$: bilateral export of commodity i to country r

PE_i : composite export price of Mozambique of commodity i

$PER_{i,r}$: price of exports of commodity i to country r

Ω_i : efficiency parameter of the CET of commodity i

τ_i : technical substitution parameter of the CET of commodity i

ν_i : share parameter of the CET of commodity i

Equation **Error! Reference source not found.** defines the CES aggregation of bilateral import quantities to the composite import quantity QM_i and of the bilateral import prices to the composite import price PM_i of good i. The latter equals the marginal cost of the composite import. Bilateral import quantities are given in equation

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$$QM_i = \Psi_i \cdot \left(\sum_r \varphi_{i,r} \cdot QMR_{i,r}^{-\rho_i} \right)^{\frac{-1}{\rho_i}} \quad and \quad PM_i = \frac{1}{\Psi_i} \cdot \left[\sum_r \varphi_{i,r}^{\frac{1}{\rho_i+1}} \cdot PMR_{i,r}^{\frac{\rho_i}{\rho_i+1}} \right]^{\frac{\rho_i+1}{\rho_i}}$$

(2)

$$QMR_{i,r} = \frac{QM_i}{\Psi_i} \cdot PMR_{i,r}^{\frac{-1}{\rho_i+1}} \cdot \varphi_{i,r}^{\frac{1}{\rho_i+1}} \cdot \left(\sum_s \varphi_s^{\frac{1}{\rho_i+1}} \cdot PMR_{i,s}^{\frac{\rho_i}{\rho_i+1}} \right)^{\frac{1}{\rho_i}}$$

(3)

Where the meaning of indices, variables and parameters not yet defined is as follows:

s : country index

QM_i : composite import quantity of Mozambique of commodity i

$QMR_{i,r}$: bilateral import of commodity i from country r

PM_i : composite import price of Mozambique of commodity I

$PMR_{i,r}$: price of import of commodity i from country r

Ψ_i : bilateral "taste" parameter of the CES of commodity i

ρ_i : technical substitution parameter of the CES of commodity i

φ_i : share parameter of the CES of commodity i

For all cases in our example, bilateral exports and imports involve more than two countries. Hence, flexibility of the CES and CET functions when they are used for bilateral trade (second stage of trade modelling) is not given. However, at the first stage involving only two arguments, domestic supply and aggregate imports, flexibility does hold.

Alternatively, the bilateral trade flows are modelled by employing the SGMF. For bilateral imports, this function represents the costs of importing the composite quantity QM_i as shown in equation (4). The first derivative of this cost function with respect to QM_i yields the price of this composite import, equation (5) and with respect a bilateral import price in the bilateral import demand equations, equation (6).[2] For bilateral exports, the SGMF represents the revenue of exporting the composite quantity QE_i (not shown as an equation). The derivative of this revenue function with regard to QE_i provides the price of this composite quantity and with respect to any bilateral export price yields corresponding bilateral export supply (also not shown).[3] According to economic theory, the matrix of second derivatives of the import cost version of the SGMF must be negative semi-definite and those of the export revenue version positive semi-definite. The matrix σ_{irs} is a symmetric negative semi-definite matrix of substitution parameters, α_{ir} and β_{ir} are predetermined parameters of the McFadden cost function.

[2] For ease of presentation, this equation and the following ones do not include the commodity index. The calibration has to be carried out separately for each traded product considered in the analysis.

[3] Bilateral export supply is obtained as first derivatives of the revenue version of the SGMF; these equations are similar to those for bilateral imports and are not listed. Besides the variables included in these equations the only other difference regards the signs of parameters which must be consistent with economic theory.

$$C = \sum_r \beta_{\mathrm{i},r} PMR_{\mathrm{i},r}\, QM_i + 0.5 \frac{\sum_r \sum_s \sigma_{\mathrm{i},rs} PMR_{\mathrm{i},r} PMR_{i,s}}{\sum_r \alpha_{i,r} PMR_{i,r}} QM_i$$

(4)

Where the meaning of variables and parameters not yet defined is as follows:

C : costs of importing the composite quantity QM_i of commodity i

$\alpha_{i,r}$: parameter for normalising the cost function of commodity i

$\beta_{i,r}$: parameter of the linear response of costs to import price changes of commodity i in country r

$\sigma_{i,rs}$: element of a symmetric negative semi-definite substitution matrix

$$\frac{\partial C}{\partial QM_i} = PM_i = \sum_r \beta_{i,r} PMR_{i,r} + 0.5 \frac{\sum_r \sum_s \sigma_{i,rs} PMR_{i,r} PMR_{i,s}}{\sum_r \alpha_{i,r} PMR_{i,r}}$$

(5)

$$\frac{\partial C}{\partial PMR_{i,r}} = QMR_{i,r} = \beta_{i,r} QM_i + \frac{\sum_s \sigma_{i,rs} PMR_{i,s}}{\sum_s \alpha_{i,s} PMR_{i,s}} QM_i - 0.5 \frac{\alpha_{i,r} \sum_s \sum_t \sigma_{i,st} PMR_{i,s} PMR_{i,t}}{\left(\sum_s \alpha_{i,s} PMR_{i,s} \right)^2} QM_i$$

(6)

Where t is a country index, all other indices, variables and parameters are as above.

Though the focus of the analysis concerns the impact of using different functional forms for representing bilateral trade relations of good i of a specific country it should not be overlooked that the trading partners are also more or less affected by these exchanges of goods. All too often, their production and demand conditions are not explicitly considered in such models. In our approach, we use linear export supply and import demand functions as a rather simplified depiction of these countries' ability to fulfil imports by and absorb exports from the specific country. For any specific commodity, equation **Error! Reference source not found.** shows the export function and equation **Error! Reference source not found.** the one for imports.

$$QE^{f}_{i,r} = ce_{i,r} + se_{i,r} pwe_{i,r} \left(1 + te_{i,r}\right)$$

(7)

$$QM^{f}_{i,r} = cm_{i,r} + sm_{i,r}\, pwm_{i,r}\left(1 + tm_{i,r}\right) \quad (8)$$

Where the meaning of the variables is as follows:

$QE^{f}_{i,r}$: quantity of commodity i exported by trading partner r to Mozambique

$pwe_{i,r}$: fob export price of country r and commodity i for exporting to Mozambique

$te_{i,r}$: subsidy of country r paid for exporting commodity i to Mozambique

$QM^{f}_{i,r}$: quantity of commodity i imported from Mozambique by country r

$pwm_{i,r}$: cif import price of country r and commodity i for importing from Mozambique

$tm_{i,r}$: tariff of country r levied on imports of commodity i originating from Mozambique

$ce_{i,r}$, $se_{i,r}$, $cm_{i,r}$, $sm_{i,r}$: parameters of linear export supply and import demand functions.

Consumer Demand and Bilateral Import Demand derived from the Normalized Quadratic-Quadratic Expenditure System:

To indicate all the changes made in the standard IFPRI CGE country model it is to be mentioned that its Linear Expenditure System (LES) is replaced by the NQQES. There are several characteristics of the NQQES which make it to be superior to the LES such as flexibility of second-order with regard to prices and third-order with regard to income. The latter property has relevance for analyzing consumption under uncertainty. The third derivative of the utility function plays a critical role in comparative static analyses and a functional form with even third-order flexibility should be chosen because there is the need to know not only the level of the elasticity but also its rate of change. The second feature of the NQQES concerns derived Engel curves which are quadratic in income with linearity as a special case.[4] Of course, this demand system is also capable of adhering to all theoretical demand conditions such as adding-up and homogeneity of degree zero in prices and total expenditure when a linear budget constraint is specified. Its compensated price responses are symmetric and form a negative semi-definite matrix which is the consequence of consistent preferences and of concavity of the expenditure function (Deaton and Muellbauer, 1992).

We will not go into more details regarding this specification rather discuss using the NQQES also for modelling bilateral imports. This approach differs from the one employing the SGMF or CES/CET functions by having bilateral imports not only depend on bilateral import prices but also on expenditure for the particular composite imported good. Due to policy changes like EU regulations on traceability or new technical restrictions on imports like sanitary and phyto-sanitary measures these total outlays might increase if imports as a whole should become more safety or more regulated (Jaffe and Henson, 2004). Bilateral import flows adjust with altered trade costs but also with growing incomes and changing preferences of domestic consumers. If we think of a differentiated good distinguished by quality attributes consumers may prefer a higher

[4] Engel curves are derived by assigning the utility-maximizing commodity bundle to each point on the income expansion path, holding prices constant. While linear Engel curves, which are very often employed, assume a proportional increase in demand for each consumption item, non-linear Engel curves are more in line with empirical evidence, which suggests that growing income modifies consumption patterns and expenditure shares disproportionally and thus, functional forms involving non-linear Engel curves are more suitable for empirical analysis.

quality standard if more income is spent on the composite imported good. However, quality standards or attributes differ due to the origin of the differentiated product. Using the NQQES, this information can be considered by the specification of expenditure elasticities for country specific varieties of an imported product. A high expenditure elasticity value is related to high quality varieties, thus a price increase caused for example by increased certification costs may be compensated by an increasing willingness to pay for these varieties because consumers put certification value on things as health, food safety, sustainable production or traceability of a product from the farm to the table. In many models standards have simply cost-raising effects captured by a price-wedge. The results of these models fit with results on "tariff wars" meaning a small country cannot win a "standards war" (Ganslandt and Markusen, 2000). The effects of standards and technical regulations on bilateral trade surely cannot be explained by a change in relative prices alone. Standards may also change the substitutability between domestic and foreign products and by this having a direct impact on the utility of domestic consumers. Moreover a price increase is not a necessary consequence of a standard.[5]

In their paper (2000) Ganslandt and Markusen discuss different methods to account for these impacts on the utility of different agents considered in a CGE trade model. Here we take up three cases which will be further analyzed for illustrative trade flows in section 3.2 using the NQQES. In the first case the import price goes up for a single country's variety. This is due to the increased costs carried by the exporting country to meet the quality standard of the importing country. At the same time it is supposed that the importing country can realize a constant level of utility by a reduced aggregate quantity of the composite imported good, keeping total expenditure for this good at the initial level. This increased willingness to pay may be explained by an increased average quality of the aggregate imported good. But the composition of the imported varieties will change due to changed relative prices. In the second case the initial expenditure share of the imported commodity gets higher, this means utility obtained from imported varieties improves at the expense of the domestic variety. In a third case, both the import price and total expenditure on the composite imported good grow. In addition substitutability might be adjusted by twisting the indifference curves for country specific varieties if product attributes change. The alternative modifications of utility by shifting and twisting the indifference curves have different impacts on the welfare of particular agents considered in the model. However, following the arguments of Ganslandt and Markusen it cannot be decided which method to account for an increased willingness to pay is the most adequate. The modeller should keep this in mind. The recalibration of the CES at the first stage of trade where domestic consumers choose between domestic and imported varieties is necessary to yield consistent results on both levels of utility maximization. This issue will not be further pursued here. Instead, we continue with the description of bilateral NQQES import demand functions.

Using the NQQES the modeller has sufficient parameters to account for price and income effects in bilateral trade equations. The system of bilateral import functions derived from the NQQES is given in equation **Error! Reference source not found.** employing some

[5] It is important for model results, which agent bears the costs caused by regulations, mostly they are treated as additional component of variable costs, but often it is more appropriate to specify adjustment costs which might sometimes have public good character in this case the price may be stable.

auxiliary functions f, g, h and their first partial derivatives with respect to prices described by f ', g', and h' for making the structure more obvious as shown in equation (10).

$$QMR_{i,r}(PMR_{i,r},Y_i)=\frac{h'}{g}(Y_i-f)^2+\frac{g'}{g}(Y_i-f)+f' \qquad (9)$$

$$g=\sum_r b_{i,r}PMR_{i,r}+\tfrac{1}{2}\left(\frac{\sum_{r,s}B_{i,rs}PMR_{i,r}PMR_{i,s}}{\sum_r \alpha_{i,r}PMR_{i,r}}\right),\ g'=\frac{\partial g}{\partial PMR_{i,r}}=b_{i,r}+\frac{\sum_s B_{i,rs}PMR_{i,s}}{\sum_s \alpha_{i,s}PMR_{i,s}}-\tfrac{1}{2}\frac{\alpha_i\sum_{s,t}B_{i,st}PMR_{i,s}PMR_{i,t}}{\left(\sum_s \alpha_{i,s}PMR_{i,s}\right)^2}$$

$$f=\sum_r PMR_{i,r}d_{i,r}\ ,\qquad f'=\frac{\partial f}{\partial PMR_{i,r}}=d_{i,r}$$

$$h=\sum_r e_{i,r}\log(PMR_{i,r})\ ,\ \sum_r e_{i,r}=0\ ,\ h'=\frac{\partial h}{\partial PMR_{i,r}}=\frac{e_{i,r}}{PMR_{i,r}} \qquad (10)$$

Where the meaning of variables and parameters not yet defined is as follows:

Y_i : total expenditure for obtaining composite import quantity QM_i

b_i, d_i, e_i,α_I : vectors of parameters of the NQQES for commodity i

B_i : matrix of parameters of the NQQES for commodity i

The NQQES is an alternative specification to the CES and SGMF for representing bilateral imports. The calibration of its parameters is very similar to that of the parameters of the SGMF. The calibrating procedure is described next.

2.2 Calibration of model parameters

Flexible functional forms of the second-order type can be calibrated with relatively little effort. This is briefly explained for import demand derived from the NQQES and defined in equation **Error! Reference source not found.** using the auxiliary functions f, h and g and their first order derivatives f ', g', and h' with respect to price p and consumption expenditure y which are shown in equation **Error! Reference source not found.**. A set of constraints for normalization is listed in equation (11). Again, B_i, b_i, d_i and e_i are a matrix and vectors of parameters of the NQQES.

$$\sum_s e_{i,s}=0,\ \ \sum_s b_{i,s}=1,\ \ \sum_s d_{i,s}PMR_{i,s}=0,\ \ \sum_s B_{i,rs}PMR_{i,s}=0,\ \ with\ B_{i,rs}=B_{i,sr} \qquad (11)$$

In an initial step, a set of realistic bilateral import elasticities with regard to prices, $\varepsilon^0_{i,rs}$, and expenditure, ε^{y0}_{ir}, for the country of interest is to be determined. In general, the initial price and expenditure elasticities violate theoretical conditions and must be adjusted in order to comply with them; i.e. symmetry, homogeneity, the budget and the curvature conditions. In a first step, for obtaining theoretically consistent elasticities, $\varepsilon_{i,rs}$ and $\varepsilon^{y}_{i,r}$, function Z0 shown in (12) is solved. Based on weights, $w^{0}_{i,rs}$ and $w^{y0}_{i,r}$, this minimizes the squared deviation between the elasticities initially set, $\varepsilon^{0}_{i,rs}$ and $\varepsilon^{y0}_{i,r}$, and those to be found for meeting the requirements of demand theory listed in equations (13) and (14).[6] Adherence to symmetry, homogeneity, adding up and the budget constraint by the NQQES is assured if it meets the conditions listed in (11). However, concavity of the substitution matrix is not inherent and must be imposed. A procedure for achieving this, shown in equation (13), was suggested by Diewert and Wales (1987) and Ryan and Wales (1999). We impose the concavity by using the Cholesky decomposition for the Slutsky matrix, where L is a lower triangular matrix and L^T its transpose.

$$Z^{0}(\varepsilon,\varepsilon^{y}) = \min_{\varepsilon,\varepsilon^{y}}\left[\sum_{rs} w^{0}_{i,rs}\ (\varepsilon_{i,rs}-\varepsilon^{0}_{i,rs})^{2} + \sum_{r} w^{y0}_{i,r}\,(\varepsilon^{y}_{i,r}-\varepsilon^{y0}_{i,r})^{2}\right] \tag{12}$$

$$Symmetry: S_{i,rs} = \varepsilon_{i,rs}\frac{QMR_{i,r}}{PMR_{i,s}} + \varepsilon^{y}_{i,r}\frac{QMR_{i}QMR_{j}}{Y_i} = S_{i,sr},\ \ Curvature: S_i = -LL^{T} \tag{13}$$

$$Adding\ up: \sum_{r}\varepsilon^{y}_{i,r}\frac{PMR_{i,r}QMR_{i,r}}{Y_i} = 1,\ \ Homogeneity: \sum_{s}\varepsilon_{i,rs} = -\varepsilon^{y}_{i,r},\ Budget: \sum_{r} PMR_{i,r}QMR_{i,r} = Y_i \tag{14}$$

In a second step, values for the parameters of the NQQES (b_i, d_i, e_i B_i) are determined by solving again an objective function, ZF in equation **Error! Reference source not found.**, which minimises the squared deviation of the elasticities to be finally used ($\varepsilon^{F}_{i,rs}$ and $\varepsilon^{yF}_{i,r}$) and those which were arrived at in the first step of calibration given the observed bilateral import defined in equation **Error! Reference source not found.**,subject to the conditions shown in **Error! Reference source not found.** and again subject to the constraints given in equations **Error! Reference source not found.** as well as (14).

$$Z^{F}(b,d,e,B) = \min_{bi,di,ei,Bi}\left[\sum_{rs} w_{i,rs}(\varepsilon_{i,rs}-\varepsilon^{F}_{i,rs})^{2} + \sum_{r} w^{y}_{i,r}(\varepsilon^{y}_{i,r}-\varepsilon^{yF}_{i,r})^{2}\right] \tag{15}$$

Because the NQQES in its restricted form is flexible the value of ZF must be zero since it entails minimizing the differences of second order effects of a theoretical consistent set of values and those effects will be implicitly represented by a flexible function. In other words, the elasticities εi,rs and $\varepsilon^{y}_{i,r}$ obtained in the first step of the procedure are perfectly matched by the elasticities $\varepsilon^{F}_{i,rs}$ and ε^{yF}_{ir}, respectively. Hence, the important step in the calibration procedure is step one; i.e. to find a set of elasticities which fulfils all theoretical conditions. The same calibration procedure is applied to fit the parameters of the bilateral import and export functions, derived from the SGMF revenue and cost functions respectively. Despite the fact that calibrating rather than estimating models has some deficiencies this procedure is widely used for empirical economic investigation.

[6] During step 1, additional constraints like bounds may be placed on single elasticities to not allow implausible values. S_{ij} is the Slutsky substitution matrix and LL^T is a lower triangular matrix and L^T its transpose.

The method developed leads to a better practice in this process requiring the analyst to verify at least consistency between model parameters specified and economic theory.

3 Empirical details

3.1 SGMF and CES bilateral import demand systems

To compare the functional forms specified, first we show the properties of the SGMF and CES for modelling bilateral trade flows. Though we will demonstrate this by discussing bilateral import systems the arguments forwarded for the SGMF and CES hold equally for bilateral export systems. Assigning values to parameters of the bilateral CES functions and in particular setting the substitution parameter ρ_i equal to -0.5 a set of own- and cross-price elasticities was derived in an initial step using equation (16).

$$\varepsilon_{i,rs}^{import} = \frac{\partial QMR_{i,r}}{\partial PMR_{i,s}} \cdot \frac{PMR_{i,s}}{QMR_{i,r}} \tag{16}$$

To keep the exposition relatively simple, illustrative values for only one imported good and five trading partners are reported. We assume country “C1” delivers 50 % of total imports, 25% come from country “C2” while “C3” and “C4” each ship 10 % whereas the rest of 5 % is imported from country “C5”.

These (point) elasticities are taken as starting values for calibrating the parameters of the SGMF import demand functions by following the procedure described in section 2.2. Since the CES bilateral import functions fulfil all required theoretical conditions they yield a set of theoretically consistent elasticities. Being flexible of second-order the SGMF can represent precisely the same price elasticities as implied by the CES specification. In other words, the calibration procedure assigns values to the parameters of this function which lead to the same price elasticities as the initial ones (see Table 1).

Table 1: CES price elasticities given the substitution parameter ρ_i is 0.5 and computed SGMF elasticities[1),2)]

	CES specification					SGMF specification				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
C1	-1.00	0.50	0.20	0.20	0.10	-1.00	0.50	0.20	0.20	0.10
C2	1.00	-1.50	0.20	0.20	0.10	1.00	-1.50	0.20	0.20	0.10
C3	1.00	0.50	-1.80	0.20	0.10	1.00	0.50	-1.80	0.20	0.10
C4	1.00	0.50	0.20	-1.80	0.10	1.00	0.50	0.20	-1.80	0.10
C5	1.00	0.50	0.20	0.20	-1.90	1.00	0.50	0.20	0.20	-1.90

1) The numbers refer to bilateral imports of an illustrative good by Mozambique
2) The column and row indices C1 to C5 indicate trading partners of Mozambique
Source: own calculations

However, we will demonstrate that the SGMF function can represent very different bilateral trade relations. To do this we imposed some condition while calibrating the SGMF. We assumed that a single initial elasticity, the own-price elasticity of country „C1”, is modified and reaches the value of -2.0. This number is fixed during the calibration procedure without imposing any further constraint on other values but maintaining again all theoretical conditions. As is to be expected the SGMF yields a set of new elasticities depicted in the left half of Table 2. After changing a single elasticity theoretical conditions are not anymore met if the remaining ones of the initial set are not adjusted either. Hence, the remaining ones have to be altered keeping the value of the changed one fixed.[7]

The outcome of the calibration of the SGMF function is depicted in the left half of Table 2. The numbers are arranged in such a way that columns represent the country specific price changes and rows the countries' quantity responses for the respective commodity. Own price elasticities are to be interpreted as the response of imports by Mozambique from the corresponding country due to a change in the import price of the same country. For example, the value of the own price elasticity for the country "C2" is -2.38. Hence, a one percentage increase in the country's price for the good imported by Mozambique will lead to a decline in Mozambique's imports from this country by 2.38 %. Imports from other countries increase due to this price change in "C2"; e.g. in "C1" by 1.14 %. As theory requires all own-price elasticities are negative. Though the SMGF can depict complementary relations here all cross-price elasticities are supposed to be positive. Since countries "C3" and "C4" have by assumption the same share in total imports they respond to price changes alike.

It is assumed that the own-price elasticity of country "C1" is -2.0, thus, we calibrated the CES functions so to yield the same value. For this purpose the substitution parameter ρi had to be set to -0.75. The right half of Table 2 depicts the own- and cross-price elasticities as calculated from the CES with this substitution parameter.

The CES specification implies that cross-price elasticities for all countries are the same; i.e. a price change in one country triggers percentage wise the same response in all other countries as can be seen for example from the column headed "C2" in Table 2. If the price in country "C2" rises by one percent imports from all other countries go up by one percent. This is a very strong (maintained) hypothesis of this functional form; a fact which is all too often overlooked when employing the Armington approach.

As mentioned above all three bilateral trade systems discussed (CES, the SGMF and the NQQES) adhere to the homogeneity, symmetry and curvature conditions. The latter condition implies positive values for own price elasticities of imports and the first leads to having at least one cross price elasticity to be negative. For the CES this means if a single one needs to be negative all others must be too. In other words, the relative import growth from a country following a price decline is offset by decreased buying from all other trading partners. Complementarity between two or more countries of origin cannot be depicted using the CES. This is another restrictive maintained proposition of the Armington approach.

Table 2: SGMF price elasticities assuming the own-price elasticity of "C1" is -2.0 (left half) and computed CES price elasticities given the substitution parameter of bilateral imports[1)] ρ_i is -0.75 (right half)

	SGMF specification					CES specification				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
C1	-2.00	1.14	0.36	0.36	0.14	-2.00	1.00	0.40	0.40	0.20
C2	2.27	-2.38	0.02	0.02	0.06	2.00	-3.00	0.40	0.40	0.20

[7] Putting particular weighting factors on single elasticities will change the whole set again, thus the modeller has the possibility to influence the outcome of the calibration procedure.

C3	1.80	0.06	-1.96	0.04	0.05	2.00	1.00	-3.60	0.40	0.20
C4	1.80	0.06	0.04	-1.96	0.05	2.00	1.00	0.40	-3.60	0.20
C5	1.45	0.29	0.10	0.10	-1.94	2.00	1.00	0.40	0.40	-3.80

1) For an explanation of the variables see Table 1
Source: own calculations

3.2 NQQES bilateral import demand

We turn now to the second flexible functional form, the NQQES, which has several very suitable properties to represent bilateral import flows. As mentioned above NQQES bilateral import quantities are derived from utility maximization. They depend on bilateral import prices, total expenditure on the composite import commodity and/or changes in preferences of domestic consumers as well as on new technical restrictions on imports. Table 3 shows the numbers for own- and cross price elasticities derived from the NQQES given an initial elasticity matrix of price elasticities, an initial vector of expenditure elasticities, benchmark bilateral import quantities and prices and applying again the calibration procedure described in section 2.2. The elasticity matrix for the illustrative example indicates a high quality variety originating from country "C1". Declining prices for other varieties do not cause an increase in imports of less quality products delivered by other regions since these are poor substitutes. In addition increased costs due to quality improvement are compensated by an increased willingness to pay for varieties from region "C1". This characteristic is captured by the high elasticity of expenditure for products imported from „C1". In the example depicted in Table 3 the respective number is 1.47 and as required by homogeneity this number is equal to the negative sum over own- and cross-price elasticities.

Table 3: NQQES price and expenditure elasticities for varieties from countries "C1" to "C5"[1)]

	Price elasticity					Expenditure Elasticity
	C1	C2	C3	C4	C5	
C1	-1,47	0,00	0,00	0,00	0,00	1,47
C2	0,22	-1,61	0,15	0,15	0,06	1,02
C3	0,70	0,62	-1,78	0,25	0,15	0,06
C4	0,71	0,62	025	-1,78	0,15	0,05
C5	0,72	0,54	0,3	0,03	-1,89	0,02

1) For an explanation of the variables see Table 1
Source: own calculations

As analysed by Ganslandt and Markusen (2000), standards can alter both the substitutability and the complementarity between products. While an increased substitutability generally is pro-competitive, increased complementarity allows products to move together. Such effects cannot be modelled in a multi-country case using the CES. However, it can be done using the NQQES and also the SGMF. An example for the former is depicted in Table 4. We suppose a complementary relationship between imports from country "C1" and country "C2". Two tea varieties to be blended to a new brand is an example.
The calibration results depicted in Table 4 were obtained by restricting the cross-price elasticity between "C1" and "C2" to be negative for the recalibration of the NQQES

bilateral import system. As can be seen at the shaded area in the left upper corner of that table the cross-price elasticities for varieties imported from „C1" and "C2" have negative signs indicating complementary relations. The own-price effects became smaller compared to values shown in Table 3. In addition, the expenditure elasticities for imported varieties from countries "C3", "C4" and "C5" become negative. This indicates inferiority of the differentiated imports from these three countries.

Table 4: NQQES price and expenditure elasticities of bilateral imports assuming complementarity of imports from "C1" and "C2" 1)

	Price elasticity					Expenditure Elasticity
	C1	C2	C3	C4	C5	
C1	-1.20	-0.29	0.01	0.00	0.00	1.47
C2	-0.40	-1.37	0.25	0.25	0.15	1.12
C3	0.80	0.93	-1.83	0.14	0.04	-0.08
C4	0.80	0.93	0.14	-1.83	0.04	-0.08
C5	0.76	1.04	0.08	0.08	-0.91	-0.04

1) For an explanation of the variables see Table 1
Source: own calculations

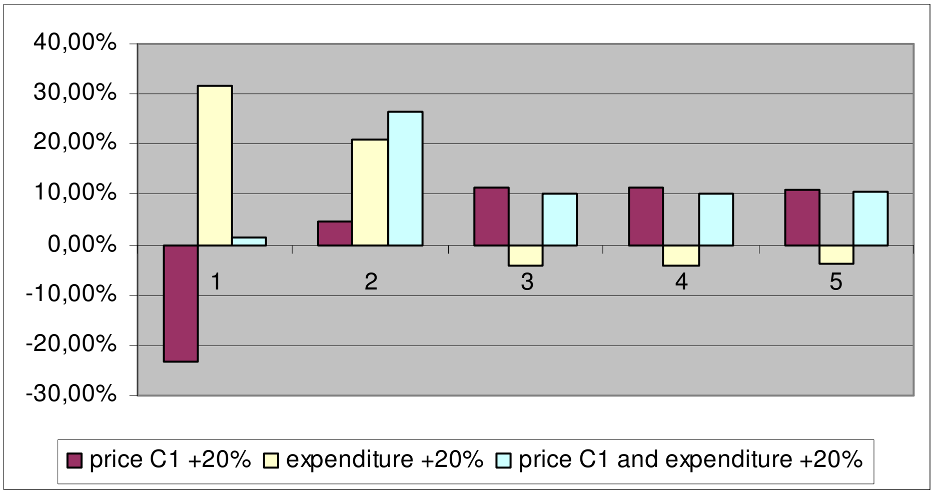
As long as the substitution between the domestic variety and the composite of foreign varieties is not modified at the first stage of trading the aggregate result for the importing country remains the same, irrespectively of the functional form selected to model the second stage of import demand. However, the composition of the aggregate and of course the effects on the export performance and welfare of single trading partners are quite different.

To obtain insight into the magnitude of price and income effects, Figure 1 and Figure 2 show the relative change in quantities imported from the five countries according to three scenarios:

- i) the price of the product supplied by "C1" rises by 20% due to the costs to comply with the standard but total expenditure for the aggregate import good remain constant
- ii) the expenditure for the aggregate import product increases by 20%
- iii) both changes occur simultaneously

These scenarios correspond to the alternatives to model an increased willingness to pay discussed in section 2.1. However, if the standard has only a prise rising effect without impacting the willingness to pay in the importing country, aggregate imports will decline and the consumption of the domestic variety will grow. The effect may be compared with an increase in the ad valorem tariff rate for products delivered by a country which improved product quality. As it is well known from trade theory, this pure price increase causes a decline in the aggregate welfare.

Figure 1: Quantity response due to a 20% price increase for the „C1" variety, an expenditure increase by 20% for the composite product and both changes occurring simultaneously with substitutability between varieties from "C1" and "C2", see Table 3 (in %)



Source: own calculations

As can be expected, the price increase of the differentiated good delivered by "C1" causes a significant reduction in import quantity from this country. At constant relative prices higher total expenditure on the composite import good significantly modifies the bilateral import flows, imports from "C1" and "C2" develop at the expense of the other three countries. Regarding a 20% price increase of the "C1" type with a simultaneous shift in consumer preferences for the composite commodity the effect leads to a small welfare improvement of "C1". The winner of this third scenario is country "C2".
The results look somewhat different if complementarity between the varieties "C1" and "C2" is assumed as shown in Figure 2. As it may be expected, the negative impacts due to an increase in its own price are diminished for country "C1" at the expense of imports originating from "C2" which now also decline. On the other hand, if scale effects were to reduce costs in "C1" this would also have positive impacts on products originating "C2" as well. Again imports of varieties from "C3", "C4" and "C5" decline with growing expenditure due to their (assumed) inferiority.

Figure 2: Quantity response due to a 20% price increase for the "C1"variety, an expenditure increase by 20% for the composite product and both changes occurring simultaneously with complementarity between varieties from "C1" and "C2", see Table 4 (in %)

Source: own calculations

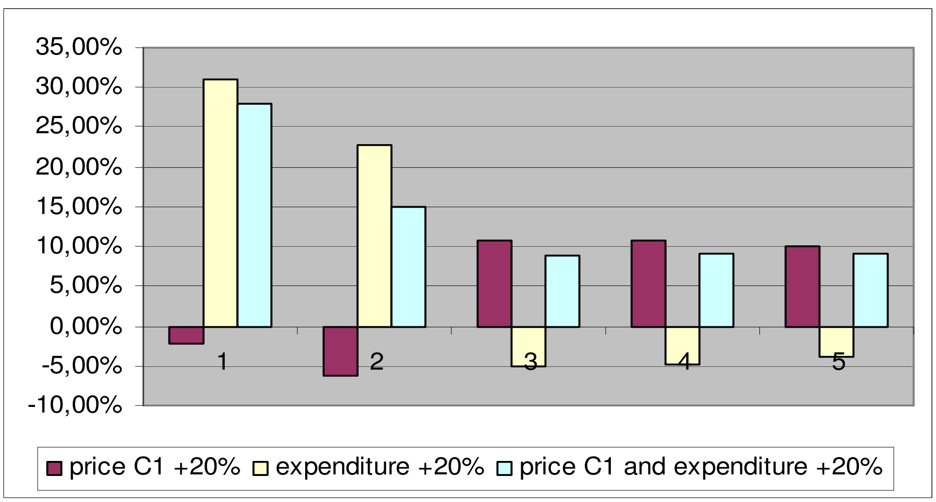
The significant reduction of the quantity imported from country “C1” is party compensated by an increased willingness to pay for the improved product quality supplied by this country. Figure 3 and Figure 4 show the changes of bilateral export revenue for the three scenarios and the five exporting counties. The results shown in Figure 3 refer to the scenario where the varieties supplied by country “C1” and country “C2” are substitutes. Results shown in Figure 4 refer to the scenario supposing a complementary relation between varieties imported from country “C1” and country “C2”. Comparing Figure 1 and Figure 3 and looking at the results for country “C1” in scenario 3, where a combined price and expenditure increase is supposed, it can be seen, that the import quantity slightly increased but export earnings develop significantly due to income effects and modified preferences in the importing country. NQQES bilateral import functions allow the modeller to consider not only price effects but also income effects to explain fluctuations in bilateral trade flows. However the extent of these changes depends on the modeller’s specification of the willingness to pay for imported varieties compared with domestic varieties. Accordingly, the calculated impacts on welfare will be different.

Though these impacts are calculated using stylised data they also are affected by the elasticities implied in the function. As mentioned above, the latter depend to some degree on the initial values set by the modeller for calibration. Thus the modeller is not released from providing plausible default values which means that some prior knowledge about sign and magnitude of the elasticities is indispensable

Figure 3: Change in export earning due to a 20% price increase for the “C1“variety, an expenditure increase by 20% for the composite product and both changes occurring simultaneously with complementarity between varieties from “C1” and “C2”, see Table 3 (in %)

Source: own calculations

Figure 4: Change in export earning due to a 20% price increase for the "C1"variety, an expenditure increase by 20% for the composite product and both changes occurring simultaneously with complementarity between varieties from "C1" and "C2", see Table 4 (in %)



Source: own calculations

4 Conclusion

In this paper the theoretical framework of computable trade models for policy analysis is discussed. Advances in both theory and methodology suggest using improved functional forms in modelling bilateral trade for policy analysis. It is shown that frequently applied functional forms such as the Cobb-Douglas and CES/CET maintain strong assumptions which also affect and even may dominate the results. Recent literature on policy modelling pays much attention to these aspects. Model structure and functional forms employed matter. Concerning model parameters, a calibration procedure leading to parameters consistent with microeconomic theory is explained by employing the NQQES. The CES/CET as included in the standard IFPRI model also keeps these theoretical properties but at the same time it is not flexible to account for differences in elasticities of substitution and/or transformation if more than two countries are represented in the bilateral trade matrix. This also holds for complementarity relations between product varieties which cannot be represented by these functions while the NQQES and the SGMF are capable of doing so. The NQQES has, in addition, sufficient parameters to account also for changes in bilateral imports due to adjustments of expenditure on the composite import. Expanding the model structure by multi-stage decision making on supply and demand with subsequent international fragmentation can provide important insights in trade responses. A bilateral import system based on the NQQES might further accentuate the suitability of this flexible functional form for modelling multi-country bilateral trade.

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