A Theory of Advocates: Trading Advice for Influence

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Abstract

An advocate for a special interest provides information to an uninformed planner for her to consider in making a sequence of important decisions. Although the advocate may have valuable information for the planner, it is is also known that the advocate is biased and will distort her advice if necessary to influence the planner's decision. Each time she repeats the problem, however, the planner learns about the accuracy of the advocate's recommendation, mitigating some of the advocate's incentive to act in a self-serving manner. We propose a theory of advocacy to explain why planners do sometimes rely on information provided by advocates in making decisions. The interaction takes place in two stages, a cheap talk recommendation from the advocate, followed by decisions and learning by the planner. The theory predicts conditions under which an advocate's advice will be ignored and when it will influence a planner's decision, when planners will prefer the advice of an advocate to the advice of a neutral adviser and, finally, how an advocate gains influence with a decision maker by making his preferences for action unpredictable. Applications of our theory are used to explain why regulated enterprises are sometimes delegated authority to determine how they are monitored and why some consumers of financial services give financial advisors who benefit from their business such great latitude in managing their investments and finances.

1 Introduction

What do the following situations share in common? A financial advisor suggests to a client that she invest more money in the advisor's mutual fund. A drug company assures the FDA that its antidepressant medication is safe and effective for the treatment of a variety of ailments, beyond those for which it is currently approved. A representative of a polluting firm gives testimony to congress about the economic and environmental impact of stricter emissions standards. An advocate for children petitions the mayor for greater funding of head start and school nutrition programs, claiming that these programs improve academic performance and behavior.

In each of these examples a decision maker or planner solicits a recommendation from an informed advisor. In these settings the advisor may be an advocate for his own self interest, as in the case of the financial advisor[1] or drug company, or, like the child advocate, he may be a supporter of a particular cause. In each instance the advisor is known to have strong preferences for actions that may conflict with the planner's best interest: the financial advisor benefits when the consumer invests more money, whether or not this is in the interest of the consumer; the child advocate seeks to maximize the budget for the head start program, to the possible detriment of other deserving social programs, a polluting firm benefits financially when emissions standards are relaxed, regardless of the larger environmental impact. Furthermore, in each of these scenarios it is not possible for the planner to directly observe whether the advocate's advice is based on his superior information, or if it is simply self serving. However, in each of these settings, the planner could face a sequence of decisions, and therefore may have an opportunity to learn something about the accuracy of the advocate's advice from her own experience.

Each of these settings is an example of a planner "relying on information from interested parties." This peculiar practice has been much studied by economists and political scientists, but is still not completely understood. Many analyses of these situations have assumed that, while the advocate may withhold damaging information, he can not make false representations of what he knows.[2] The inability to make false claims renders harm-

[1] For the most part, we ignore the possibility that financial advisors have fiduciary responsibilities, and treat financial advisors as self-interested agents. Section 5, deals with what may happen if a certain portion of advisors take their fiduciary obligations seriously.

[2] For example, Dewatripont and Tirole (1999), Milgrom (1981), Cotton (2009, 2010), Milgrom and Roberts (1986), Bull and Watson (2007, 2004).

less the misalignment of preferences between the advocate and planner and may even make the advocate a preferred type of advisor. Despite his incentive to offer biased information, the advocate can always make a complete and truthful disclosure. If the advocate does not provide evidence for her most favored state of the world, the planner can safely conclude it doesn't exist, otherwise he would have exposed it. Because of this unravelling, the advocate has no recourse but to disclose the state that actually occurs. Furthermore, because advocates benefit if they can prove that their preferred state exists, they have greater incentives to acquire information than impartial advisors.[3]

This rationale for relying on the information of interested parties appears much less compelling, however, in settings where planners are unable to independently verify the information or advice provided by advocates. If information is soft, the planner must evaluate the advocate's motives; here opportunities for mutually beneficial exchange of information are diminished. Other analyses of influential communication of soft information have identified situations in which influential communication can take place in equilibrium. In the delegation and single dimensional cheap talk literature,[4] the advisor's ideal action is higher than the decision maker's by a finite bias; the advisor therefore has an incentive to exaggerate the state, but only to a point. The bounded incentive to exaggerate means that the planner and advocate have some common interest, which allows some, but not all information to be conveyed. Influential communication also can take place in repeated settings in which.the decision maker has some uncertainty about the "agenda" of the advisor (whether he is representing her interests or his own). In this environment both types of advisors have a reputational incentive to appear unbiased. Influential communication can happen in equilibrium both because the advisor may truly represent the interests of the decision maker, and because a self-interested advisor would like to appear unbiased (at least initially).[5] In our analysis, we rule out these channels for influential communication by focusing on settings in which it is common knowledge that the advocate always benefits

[3] See Aghion and Tirole (1997) who argue that advocates who are delegated decision making authority are more easily induced to gather information. A complementary explanation for the use of (competing) advocates in judicial proceedings is provided by Dewatripont and Tirole(1994) who demonstrate the benefits that biased advocates derive from gathering information. Che and Kartik (2009) show decision makers may prefer to consult with advisers who are unbiased but have different opinions about the likely state of the world.

[4] Crawford and Sobel (1982), Dessein (2002), Ambrus and Egorov (2009)

[5] Morris (2002) and Sobel (1985), similar ideas appear in the carrer concerns literature, for example Prendergast (1993), Ottaviani and Sorenson (2006), Prat (2005). Although communication is influential, these settings have their own set of difficulties. See the previous references for more information.

from increased actions.[6]

Despite their differences, each party has something valuable to offer the other: the advocate can offer the planner valuable advice, and the planner has the authority to select actions that are important to the advocate. Is it possible for the parties to somehow arrange a mutually beneficial exchange of advice for influence even with common knowledge of the extreme misalignment of preferences? If so, how? Our goal in this paper is develop a theory of advocacy to address these questions.

Our investigation centers on a cheap talk model that captures features of different advocacy relationships that we observe in practice and discuss above. In a repeated setting, a planner adjusts her action in each period to match an unknown state of the environment. The state may represent the returns on investment or the comparative benefits of child assistance in a particular period; the planner's action may be the amount of money she allocates to her financial advisor, or the resources she devotes to social services for children. The state in each period is stochastic, and is publicly revealed after the planner has selected an action.[7] The planner is uncertain about the distribution of the state. The financial advisor's investment strategies may generate returns with a higher mean than the market, or they may be worse on average than the market as a whole. While the impact of child assistance programs certainly varies over time, effective programs are more likely to produce substantial improvements than ineffective programs. An advisor knows the distribution from which the states are drawn, either ϕ_H or ϕ_L, but like the planner is unable to observe the state until after the planner selects an action. The advisor is known to be an advocate; he always prefers that the planner increase her action, independent of the true state. The advocate is biased either because he personally benefits from higher actions, as in the case of the financial planner, drug company, or polluting firm, or because he cares about a particular cause, like the advocate for children.

The theory we propose addresses the fundamental problem that advocates pose for

[6] The multidimensional cheap talk literature (See for example, Chakraborty and Harbaugh (2009).) has also identified conditions under which the capability to trade off different aspects of a decision allows influential communication to take place in equilibrium. Although our setting appears to be similar, with future decisions playing the role of multiple "aspects" of a single problem, we rule out these types of effects by assuming that the planner and advocate have the same discount factor, so that each "aspect" of the decision is given the same weight. Consult the extended appendix for more information.

[7] For simplicity we also assume that the planner's ability to observe the true state after she selects her action does not depend on the action that she chooses. This simplifying assumption may not be realistic in certain situations. Relaxing this assumption introduces an additional tradeoff into the problem, which we leave for future work.

planners. The planner clearly benefits from knowing the state distribution at the beginning of the interaction, but without access to payments (contingent on the advocate's recommendation), it is very difficult to align the advocate's incentives with the planner's.[8] With common knowledge of the advocate's unyielding preference for high action and his associated incentive to manipulate the planner, it may seem that there is little or no scope for meaningful communication between the parties. Because the planner can learn about the state distribution through her own experience (that is, from the history of observed states) she may prefer to ignore the advocate's advice and act on her own. Does the planner's ability to learn also allow the parties to arrange beneficial exchanges of advice for influence? Our analysis of this setting leads to a theory of advocacy that has the following predictions

1) *An advocate can only influence the planner in a formal long term arrangement*

Given the planner's choice between learning on her own or learning from the advocate, our theory characterizes settings in which the advocate's advice is ignored entirely. This occurs whenever (i) the planner makes a one-time decision, or (ii) planner makes a sequence of decisions but she can not commit to act based on the advocate's advice. In the first case if the planner changes her action in response to the advocate's advice, he will always manipulate her into choosing the higher action. Absent the ability to fine the advocate for bad advice, or a future in which to punish him, the planner has no recourse other than to ignore his advice. In the second case, the problem is very similar. If the advocate's recommendation suggests to the planner that the low distribution is more likely, the planner will choose a lower action in each period, hurting the advocate's payoff. Without the ability to commit, the planner can not assure the advocate that he won't be penalized for revealing the low distribution Without this assurance, we show that no informative communication equilibrium exists.[9] These no-trade results are rather disappointing, but

[8]There are two rationale for assuming that report-contingent payments are not possible. The first is that payments for information are difficult if not impossible to implement without the ability to objectively verify the accuracy of the information that is reported. The second is that monetary transfers between public officials and private interests may facilitate corrupt or illegal exchanges of favors or services, and is therefore prohibited.

If monetary or non monetary utility transfers can be conditioned on the advocates report, then it is possible to induce the advocate to reveal information. See Ambrus and Egorov (2009 a,b) in the context of a theory of bureaucracy and Inderst and Ottaviani (2009) in the context of commercial transactions.

[9]In contrast Morris (2001) and Sobel (1985) find informative communication may exist in repeated settings without commitment. However they assume (1) that the advisor's preferences may be identical to those of the decision maker and (2) that the state changes and the advisor learns the new state in each

predictable, given the disparate preferences the parties and the limited agreements that are available for governing the planner's behavior in response to the advocate's advice.

2) *Advocates who appear to be credible are influential*

Our theory predicts conditions under which beneficial exchanges of information for influence can be arranged; we also characterize the optimal implementation of these agreements. In a repeated setting we show it is possible for the parties to arrange for the advocate to advise the planner in exchange for the planner's commitment to select certain actions. Under the optimal arrangement, when the advocate reports ϕ_L, the unfavorable distribution, the planner commits to a fixed long term action that exceeds the action she finds optimal under the low distribution. We interpret this as a *compromise* the planner makes in return for the advocate's disclosure of unfavorable news. When the advocate reports that the distribution is ϕ_H, the planner commits to a sequence of actions that is contingent on the history of observed states. The actions converge towards the planner's optimal high action when the history of state observations supports the advocate's claim that the distribution is ϕ_H. However, if the history of observed states contradicts the advocate's claim, the planner's actions are progressively reduced. We interpret this as the planner's commitment to trust the advocate's advice, as long as his advice appears *valid* given her experience.

Our theory of how advocates exert influence also suggests an explanation for many of the formal and implicit advisory agreements we observe in practice. For instance drug regulators who adjust the scope in which certain medicines are used depending on the outcomes employ a form of the credible oversight agreement predicted by the theory. Similarly, an agreement to follow the advice of a financial planner as long as his performance is consistent with his claims is consistent with our theory.

3) *Biased advocates may be preferred to impartial advisors*

Our theory provides some clues as to the origins of advocacy by describing situations in which advocates are preferred to impartial advisors. Most analyses of advocacy presume that planners rely on advocates for advice because advocates must be informed about issues that they care so much about. While this is often true it doesn't explain why planners don't prefer to consult other informed but potentially less biased sources for advice.

period. By issuing truthful recommendations, biased advisors may establish a reputation for credibility, which they may profitably exploit in later periods.

One rationale for preferring advocates to impartial advisers, implied by our theory, is that it is easier to motivate advocates to acquire information than impartial advisors. Suppose the planner may consult a biased advocate or an objective adviser for information on what action to take. The advocate and impartial advisor must both expend resources of $c > 0$ to become informed. Once informed, the impartial advisor truthfully reports his information, whereas the advocate requires the planner to adopt certain costly actions in return for revealing his information truthfully. We show, surprisingly, that it can be less costly for the planner to obtain information from the advocate than from the impartial adviser. Because the advocate cares about the planner's actions while the impartial advisor doesn't, the advocate can be motivated to learn through the planner's actions, while the impartial advisor will only learn if his cost is fully reimbursed. When the advocate has a strong incentive to correctly communicate the state distribution he subsidizes learning; the distorted actions necessary to induce learning generate a smaller payoff cost to the planner than the cost of learning, c. Ironically our theory shows that it is the advocate's extreme preference for high actions that sometimes makes him a more attractive source of information than an objective advisor.

4) *When the advisor might be impartial, advocates exert greater influence*

Our theory predicts that an advocate with a known preferences for high actions has no impact on the planner's expected action choice; the planner chooses the same average action whether or not she is advised by the advocate. Although the advocate's advice influences the planner to make better decisions to the *planner's* benefit, the advocate is unable to induce the planner to select higher actions on average. In contrast, when the planner is uncertain whether the advisor is an advocate or impartial, our theory shows that the advocate commands a rent to reveal his conflict of interest, leading to a higher average action.

To illustrate, suppose the planner has $D \geq 2$ distinct types of decisions to make. She consults with an advocate who has extreme preferences for one unknown type of decision, but is impartial with respect to the other $D - 1$ decisions. For any given decision, the planner will not know if the advisor has an incentive to offer impartial or biased advice. An advocate thus has an incentive to appear impartial in order to give a manipulative report undeserved credibility. To make this possibility unattractive, the planner commits to a concession, choosing actions that are, on average more aligned with the advocate's preferences when he reveals his conflict of interest.

This finding has a paradoxical quality to it. The most effective advocate is a *stealth* advocate, one who is not known for supporting a particular cause or special interest group Advocates are severely constrained by the causes that they are identified with; it is ironic, that once an advocate becomes a known supporter of a particular issue, he relinquishes his ability to impact the planners choice of action on that issue. By hiding his causes from the planner, the advocate is able to advance his agenda more effectively.

This paper is an application of the literature on strategic information transmission that began with Crawford and Sobel (1982), Grossman (81) and Milgrom (81) who provide the first analyses of decision makers relying on the information of interested parties. Dewatripont and Tirole (1999) and Morris (2001) were the first to model the role of biased advocates in advising planners about what actions to take. Our results shed light on the planner's choice of adviser and predict when an advocate is preferred to an objective adviser. Like Dewatripont and Tirole we find that advocates are sometime preferred because they can be induced to acquire information at a lower cost.[10]

The overarching theme of our theory is that advocates are severely handicapped by their inability to certify their advice. This exposes an important qualification for the use of advocates in the fact finding settings analyzed by Dewatripont and Tirole(1999), Shin(1998) and Krishna and Morgan(1998) who require advocates to present hard evidence of their claims. In contrast, in the cheap talk setting of this paper, advocates with known biases can have only a limited impact on the planners' behavior. Nonetheless planners continue to rely on advocates for advice, in situations where the *validity* of the advocate's advice can be assessed. This is consistent with the results of Sobel (1985) and Morris (2002) although the channels for implementing meaningful communication are quite different across the three models. It also shares elements of Strausz's (2005) theory of interim information in employment contracts, and Cooper and Hayes (1987) analysis of price discrimination in long term insurance contracts.

The process for eliciting useful information from biased parties in cheap talk settings has spawned a large literature in economics, organization theory and political science. Krishna

[10] Che and Kartik(2009) come to similar conclusion regarding the decision maker's preference for dealing with advisers who are known to have a different opinion about the state of the world. Ambrus, Azevedo, Kamada, Takagi (2010) analyze a setting in which biased advisors can be preferred to impartial ones, which does not have costly information acquisition.

and Morgan (2008) and Sobel (2008) are recent surveys of some of the more influential applications of the theory that include, (in addition to the papers discussed above) Alonso, Dessein and Matouschek (2008) and Rantakari (2008) in organization theory, Chakraborty and Harbaugh (2007, 2010) and Battaglini (2002) in economics and Austen-Smith (1994) and Grossman and Helpman (2002) in political Science. This literature encompasses a wide range of topics and settings, (including opposing advocates, and multidimensional issues) that the current paper does not address. Our theory focuses instead on explaining the role of advocates in facilitating the exchange of information for influence in long term economic and political decision processes.

The plan for the rest of the paper is as follows. Our model of advocacy is presented in Section 2. Section 3 demonstrates conditions under which informative communication between the advocate and planner is impossible. In section 4 we characterize when informative communication is possible and how it can be implemented. Section 5 extends our analysis to two settings. In the first setting advocates must expend effort to learn; we also permit the planner to choose whether to consult an impartial advisor or a known advocate for advice. The second setting allows the advocate to hide his preferences for action by choosing the set of issues on which he offers advice. The combined analyses of the two settings permits us predict the type environment in which advocacy relationships are likely to exist. Section 6 concludes with a summary of results and directions for further research. Proofs of formal results are contained in the appendix.

2 The Model

A planner (e.g. lawmaker, CEO, regulator etc.) faces a decision or sequence of decisions. In each period, the planner's choice, called the action, is represented by a value $q \in R$. The planner's payoff in each period depends on the action she chooses and an unknown state of the world $x \in R$. Given state x and action q, her payoff is a quadratic loss function:

$$u_p(q, x) = -(q - x)^2$$

The planner has no inherent preferences for action; she cares only about choosing the action that is most appropriate given the state of the environment.

In each period, the true state is independently and identically distributed, but the exact distribution of the state in all periods is unknown to the planner. There are two possible distributions for the state, a high distribution $F(x|\phi_H)$ and a low distribution $F(x|\phi_L)$,.with common support X.[11] The expected value of the state under the high distribution, μ_H, is larger than the expected value of the state under the low distribution, μ_L.

$$\begin{aligned} E[x|\phi_H] &= \mu_H \\ E[x|\phi_L] &= \mu_L \\ &\mu_H \geq \mu_L \end{aligned}$$

Given her information at the beginning of the interaction, the planner believes the distribution of the state to be the low distribution with probability $\gamma \in (0,1)$.

Faced with a choice of action in each period, the planner would clearly benefit from knowing the true distribution at the beginning of the interaction. With quadratic preferences, her ideal action in every period is the mean of the true distribution. However, if doesn't know the true distribution, her action in any period will reflect all available information, but it will never be her ideal action. We refer to the situation in which the planner knows the true distribution as the *first best*, and her expected payoff (prior to learning the true distribution) as the *first best payoff,* denoted $\overline{V}_N$.[12]

Fortunately, an advisor knows the true distribution of the state(though he has no additional information about the realized state in each period); unfortunately the advisor is an advocate whose payoff differs from the planner's:[13]

$$u_a(q,x) = q$$

Unlike the planner, who has no inherent preferences for actions, the advocate always benefits when the planner increases her action, regardless of the true state of the world. The advocate therefore has an incentive to suggest to the planner that the distribution is ϕ_H, to manipulate her into choosing higher actions. The preference misalignment between the planner and advocate is more extreme than the one frequently analyzed in the cheap talk and delegation literature;[14] the extreme misalignment of preferences acts as a significant

[11] The assumption of common support can be easily relaxed, and leads to a similar results.

[12] It is not difficult to verify that $\overline{V}_N = -\left(\gamma\sigma_L^2 + (1-\gamma)\sigma_H^2\right)\frac{1-\delta^N}{1-\delta}$.

[13] More general payoff functions of the form $u_a(q,x) = g_1(x)q + g_2(x)$ lead to identical results, provided for both possible distributions $D \in \{H, L\}$, $E[g_1(x)|\phi_D] > 0$.

[14] See Ambrus and Egorov, Alonzo and Matouschek, etc. A notable exception is Chakraborty and Harbaugh (2009). According to the usual assumptions of this literature, both the expert and decision maker have single-peaked preferences, but the expert's most preferred point is shifted relative to the

barrier to influential communication.

Payments between the planner and the advocate are either explicitly ruled out, or are constrained to be independent of the advocate's message. These restrictions may arise for a variety of legal or institutional reasons; payments between planners and advocates open the door for a variety of corrupt behaviors. Without payments, the only means by which the planner can reward or punish the advocate is through her choice of action.

The challenge for the planner is formidable: she would like the advocate to recommend the best course of action given what he knows, but because the planner can not pay him for his advice, it is very difficult for the planner to overcome the advocate's incentive to issue manipulative recommendations. Nonetheless, the advocate does have information that is valuable to the planner, and the planner has the authority to select actions that benefit the advocate. There are potential gains from an exchange of advice for influence, if only the parties could figure out a way to exploit their common interests without money.

2.1 Advice for Influence Game

We model the interaction between planner and advocate as a game in which advice is exchanged for influence. The game takes place over periods $k = \{0, 1, .., N\}$ according to the following sequence:

0. The advocate observes the true state distribution, ϕ_L or ϕ_H, and issues a recommendation (or sends a message) $M \in \{H, L\}$.[15]

k=1:N. At the beginning of period k, the planner selects an action $q_k \in R$. Once the action is selected, the true state x_k is revealed, and both planner and advocate realize their period k payoffs.

The advocate's strategy is a pair of probabilities $(r_H, r_L) \in [0, 1]^2$; r_X represents the probability that the advocate reports H when the true distribution is ϕ_X. In each period, the planner can base her choice of action in any period on the message she receives from

decision maker by his bias. In our case, the advocate always prefers the the decision maker increase her action and is infinitely biased.

[15] We have assumed that the message space contains only two messages. In the case of full commitment the Revelation Principle guarantees that this assumption is without loss of generality. Without commitment, this assumption can be easily relaxed with no impact on the results. We maintain this assumption to simplify the exposition.

the advocate and on the history of past states that she has observed; if $X_{k-1} = \left(\times_{i=1}^{k-1} X\right)$, represents the set of possible histories at time $k-1$, and $h_{k-1} = (x_1, ..., x_{k-1}) \in X_{k-1}$ represents a particular history of past states,[16] then the planner's strategy is a family of functions, $\{q_M(h_{k-1})\}_{k=1}^N$, each of which maps the Cartesian Product of M and X_{k-1} into an action $q_k \in R$:[17]

$$q_M(h_{k-1}) : M \times H_{k-1} \rightarrow R$$

Both planner and advocate have a common discount factor δ.

The equilibrium that we focus on for this game will depend on the setting which best describes the relationship between the advocate and planner. In particular in situations where the planner is unable to commit ex ante to a strategy, we analyze Perfect Bayesian equilibrium, PBE, in which the planner and advocate select strategies that are sequentially rational given the strategy of the other player, and the planner's beliefs are Bayesian updates of her prior based on the observed history and the advocate's strategy. In situations where the planner can commit to her strategy, we focus on Bayesian incentive compatible mechanisms, whereby the planner chooses her action selection to maximize her expected payoff, subject to inducing truthful disclosure from the advocate.

The information structure of the advice for influence game differs from the standard information structure in price discrimination and employment settings in one key respect. In standard settings the agent's private information is about some attribute of the *agent* (e.g. his cost for executing a task, or his valuation for a good); the only way for the principal to learn this information is to provide the agent with incentives to reveal it. In contrast, the advocate's private information is about an exogenous feature of the *planner's problem.* Each time she repeats the problem, the planner acquires additional information about this feature, and in the long run, she can learn the advocate's private information on her own.

[16] By convention, h_0 is the null set and $f(h_0|\phi) = 1$.

[17] Throughout the paper we avoid discussing issues related to variations in the decision maker's strategy on sets of measure zero without loss of substance. Readers uncomfortable with this can assume that X is discrete with no substantive changes.

3 Settings Without Influential Communication

With common knowledge of the severe misalignment in preferences, and the lack of available instruments for aligning incentives, we would expect that in a variety of settings, the planner would ignore the advocate's advice. The following propositions illustrate two such settings, shedding light on conditions necessary for communication to be influential. Formally, we say that communication is *influential* if both messages are sent in equilibrium, and for some history (or non-negligible set of histories) different messages induce the planner to choose different actions. In the alternative, the planner only ever chooses one sequence of actions on the equilibrium path.

Imagine that the planner and advocate interact for only one period. Because the state is only learned after the decision has been made, the only information that the planner could use to help her choose an action is the report of the advocate, but because he will always try to manipulate the planner, the planner has no choice but to ignore his recommendation. In this case, even with the ability to commit to her strategy, the planner is unable to elicit influential information from the advocate.

Proposition 1.1 *In a single period interaction with commitment, in the only incentive compatible mechanism communication is non-influential.*

Proposition 1 is in stark contrast to the single encounter cheap talk literature, exemplified by Crawford and Sobel (1982), and Dessein (2002) who demonstrate the possibility of influential communication in one time cheap talk encounters even if the planner can not commit to an action. The inability of the advocate to influence the planner in our settings follows directly from the extreme misalignment of preferences. To illustrate, suppose that the planner commits to choose action q_X if the advocate reports $X \in \{H, L\}$. If $q_H \neq q_L$, then the advocate will optimally issue the recommendation that induces the higher action. In order for the recommendation to be incentive compatible, the planner must commit to the same action for the two reports, so that $q_H = q_L$, and the advocate's report can not influence the planner choice of action.

The planner's situation seems different in a setting with multiple periods, because she can learn about the true distribution from the observed history. However, if the planner is unable to commit to her strategy, her actions must be sequentially rational, and only one action is sequentially rational for any combination of history and message. Although the planner will adjust her equilibrium actions to account for her updated beliefs about the state distribution, she is unable to elicit influential information from the advocate.

Proposition 1.2 *In every Perfect Bayesian Equilibrium of the multiperiod game without commitment the advocate's reporting strategy is independent of his information. There is no influential communication in equilibrium.*

With quadratic preferences, the sequentially rational action following a given history is the conditional expected value of the state

$$q\left(\widehat{\gamma}, h_{k-1}\right) = \gamma\left(h_{k-1}\right)\mu_L + \left(1 - \gamma\left(h_{k-1}\right)\right)\mu_H$$

where $\gamma\left(h_{k-1}\right)$ represents the updated belief that the distribution is low:

$$\gamma\left(h_{k-1}\right) = \Pr\left(\phi = \phi_L | h_{k-1}\right) = \frac{\widehat{\gamma} f(h_{k-1}|\phi_L)}{\widehat{\gamma} f(h_{k-1}|\phi_L) + (1-\widehat{\gamma}) f(h_{k-1}|\phi_H)}$$

The planner's updated belief $\gamma\left(h_{k-1}\right)$ is monotone in $\widehat{\gamma}$, her updated belief that the distribution is low following the advocate's report. By sending a message that induces a lower belief that the distribution is ϕ_L the advocate increases the planner's action following any history of states, improving his payoff. Therefore, the advocate always sends the message associated with the lowest possible posterior belief. When $\widehat{\gamma}_H \neq \widehat{\gamma}_L$, he sends only one message in equilibrium, and the posterior belief associated with this message is equal to the prior belief. If, $\widehat{\gamma}_H = \widehat{\gamma}_L$ then both messages may be sent in equilibrium, but, if the posteriors are equal, the probability of sending either messages can't depend on the true distribution; therefore, both posteriors are equal to the prior. In both cases, no influential communication takes place in equilibrium.

The findings of Proposition 1.2 provide an interesting contrast to Morris (2001) and Sobel (1985) who demonstrate the possibility of influential communication in repeated settings where the planner is unable to commit. In environments where an advisor wishes to establish reputation for sharing the planner's preferences, there exists some common interest between the planner and the advisor that permits the advisor to provide useful information in exchange for greater influence in the future. This opportunity does not exist, however, when the advisor is an advocate with known preferences for extreme actions (the "bad type" in Morris).

Propositions 1.1 and 1.2 make clear that advocates can have no influence whatsoever, unless the planner repeats her problem and can commit to an action policy that depends on the advocate's report and on the observed history. This is the setting that we next turn to in section 4.[18]

4 Influential Communication With Commitment

In this section we consider possibility that the planner can commit to a strategy prior to receiving the advocate's report, relaxing the requirement that the planner's strategy must be sequentially rational. Once she selects an action, the planner observes the true state; each time she repeats the problem, she learns about the true distribution. By committing to actions, the planner can leverage the additional information revealed by the history of states to dissuade the advocate from issuing self-serving recommendations.[19]

The planner commits at the beginning of the interaction to her strategy, or mechanism, m, that specifies an action in each period, conditioned on the advocate's recommendation and the observed history:

$$m \equiv \{q_L(h_{k-1}), q_H(h_{k-1})\}_{k=1}^{N}$$

The advocate privately observes the true distribution and issues a recommendation. Because the advocate only has one piece of private information, according to the Revelation Principle there is no loss of generality in restricting attention to mechanisms that induce the advocate to issue a truthful recommendation; incentive compatibility requires that the advocate can not induce a higher discounted sum of expected actions by misreporting the distribution of the state:

$$\begin{aligned}\textstyle\sum_{k=1}^{N}\delta^{k-1}E[q_L(h_{k-1})|\phi_L] &\geq \textstyle\sum_{k=1}^{N}\delta^{k-1}E[q_H(h_{k-1})|\phi_L] \quad \text{(ICL)}\\ \textstyle\sum_{k=1}^{N}\delta^{k-1}E[q_H(h_{k-1})|\phi_H] &\geq \textstyle\sum_{k=1}^{N}\delta^{k-1}E[q_L(h_{k-1})|\phi_H] \quad \text{(ICH)}\end{aligned}$$

Because the mechanism is incentive compatible, the planner knows that the advocate's report is truthful; her expected payoff from offering m can be written:

$$P(m) = -\gamma\sum_{k=1}^{N}\delta^{k-1}E\left[(q_L(h_{k-1})-x)^2|\phi_L\right] - (1-\gamma)\sum_{k=1}^{N}\delta^{k-1}E\left[(q_H(h_{k-1})-x)^2|\phi_H\right]$$

[18] We revisit the possibility that the advocate is unbiased in a later section.

[19] Throughout the history of states is assumed to be verifiable. Even if the history were not verifiable but some signal correlated with each period's state were verifiable the mechanism would exhibit similar features.

Finally, to ensure that the advocate will participate in the mechanism, it is enough that the planner commit to interpret the absence of a message as a message in support of distribution ϕ_L; constraint (ICH) would then ensure that the advocate (weakly) prefers to send a message than no message at all.[20]

The optimal mechanism maximizes (P) subject to (ICL) and (ICH). In the absence of incentive constraints, the planner would like to choose the first-best actions (μ_H or μ_L), but these actions are obviously not incentive compatible. At least one of the incentive constraints must therefore be binding. For a clue as to which constraint binds, note that an advocate who learns that the true distribution is ϕ_H has favorable information and would have no incentive to report L; on the other hand, an advocate who knows that the true distribution is ϕ_L, could try to manipulate the planner by reporting H instead of L. In light of this intuition, in the appendix we formulate and solve a simple relaxed problem in which we impose only constraint (ICL).[21] We then show that the solution to the relaxed problem always satisfies constraint (ICH), and therefore characterizes the optimal mechanism.[22]

The optimal mechanism depends on three key variables that we introduce now in anticipation of the formal results to follow. First, the history of past states h_{k-1} influences the planner's actions through its likelihood ratio:.

$$\Lambda\left(h_{k-1}\right) = \prod_{i=1}^{k-1} \frac{f(x_i|\phi_L)}{f(x_i|\phi_H)}$$

The value of the likelihood ratio indicates which of the two possible distributions are more consistent with the observed history. High values of $\Lambda\left(h_{k-1}\right)$ support the inference that the true distribution is ϕ_L, while low values of $\Lambda\left(h_{k-1}\right)$ support the inference that the true

[20] We will discover that the advocate's payoff under the optimal mechanism is equal to his payoff in the absence of influential communication. A participation constraint that requires that the advocate's payoff is no less than he would get in the absence of influential communication is also satisfied.

[21] The resulting relaxed problem has a concave payoff function and a linear constraint. The solution to the relaxed problem is the unique stationary point of the Lagrangian.

[22] Clearly, any mechanism in which there is no influential communication (because $q_H\left(h_{k-1}\right) = q_L\left(h_{k-1}\right)$ for all histories) always satisfies these constraints, and is therefore feasible; if it is not chosen, it must be sub-optimal.

distribution is ϕ_H.[23] Second, the optimal mechanism also depends on parameter

$$\begin{aligned} \alpha &= \int_X \frac{f(x|\phi_L)^2}{f(x|\phi_H)} dx \\ &= E[\Lambda(x)|\phi_L] \end{aligned}$$

Parameter α is an important measure of the "similarity" between distribution ϕ_L and ϕ_H that plays a significant role in a variety of statistical settings; this value is always larger than one, and is only equal to one if the distributions are the same. Larger values of α indicate that the distributions are less similar.[24] We will see that, in the optimal mechanism, α is a measure of the strength of the incentives that the planner can provide without payments. Finally, the solution also depends on parameter

$$\omega_N = \frac{(1-\alpha\delta)\left(1-\delta^N\right)}{\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)(1-\alpha\delta)\left(1-\delta^N\right)} (\mu_H - \mu_L)$$

This parameter affects the optimal distortion from the first best actions, required to induce the advocate to make truthful recommendations. We refer to this parameter as the *magnitude* of the distortions. We are now ready to state our first main result.

Proposition 2 *The mechanism which maximizes (P) subject to (ICL) and (ICH) is characterized as follows:*

(a) *If the advocate reports L, the planner's action is a constant, greater than her first best action μ_L.*

$$q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_N$$

(b) *If the advocate reports H, the planner's action depends on the observed history and is always less than her first best action μ_H*

$$q_H(h_{k-1}) = \mu_H - \gamma\omega_N\Lambda(h_{k-1})$$

(c) *The planner's and advocate's payoffs are given by*

$$V_N = \overline{V}_N - \frac{1-\delta^N}{1-\delta}\omega_N\left(\gamma-\gamma^2\right)(\mu_H-\mu_L)$$
$$U_N = (\gamma\mu_L + (1-\gamma)\mu_H)\frac{1-\delta^N}{1-\delta}$$

[23] By convention, $\Lambda(h_0) = 1$.

[24] Throughout we assume that $\alpha < \infty$. By the end of this section, it will become clear that if $\alpha = \infty$, then the planner can approximate her first-best payoff even in a finite-period problem.

Without access to payments, the planner provides incentives for truthful reporting by committing to vary her actions in response to history. Truth-telling is optimally induced by rewarding the advocate when he issues recommendations that he personally finds unfavorable and punishing the advocate when he reports personally favorable information that seems to be inconsistent with the observed history of states. When the advocate reports L, the distortion from the efficient action is a positive constant, $(1-\gamma)\,\omega_N$. By choosing an action higher than the first best action μ_L, the planner makes a report of L more attractive for the advocate, which helps to provide the advocate with incentives to report truthfully. We interpret the fixed reward that follows the advocate revealing personally unfavorable information as a commitment to *compromise.*

The planner commits to treat seemingly self-serving advice based on her assessment of its *validity.* When the advocate reports H, the planner commits to a history-dependent distortion that depends on the likelihood ratio, $\gamma\omega_N\Lambda\,(h_{k-1})$. If the likelihood ratio is small, the history of states supports the inference that the distribution is ϕ_H. The advocate's advice therefore appears valid, and the planner's action is close to the first best action μ_H. If the history of states suggests that the true distribution is ϕ_L, the advice appears invalid, and the planner reduces her action; this punishes the advocate but also hurts her payoff. To optimally provide incentives, the planner acts "as if" she is skeptical of the advocate's advice, cross-checking his recommendation against the observed history, even though she knows that the advocate's report is truthful. This commitment to update is a common feature of incentive compatible mechanisms in a variety of settings.[25]

The less similar the distributions (as measured by α), the more the planner benefits under the mechanism. To understand why, observe that if the advocate reports H when the true distribution is ϕ_L, he experiences a negative distortion from action μ_H that is proportional to the likelihood ratio. The expected value of the period k distortion is therefore $-\gamma\omega_N\alpha^{k-1}$. Holding ω_N fixed, increases in α lead to greater punishments for an advocate who tries to manipulate the planner. If the advocate expects a greater punishment for manipulation, his incentive to do so is reduced; the planner can therefore reduce the magnitude of the distortions, ω_N, without violating incentive compatibility. If ω_N is smaller, then the distortions from the first-best actions are smaller for every combination of action and history, improving the planner's payoff. Parameter α therefore represents the strength

[25] See for example the seminal paper of Holmstorm (1977).

of the incentives that the planner can provide the advocate by distorting her actions.

Although the planner benefits under this arrangement, the advocate's expected payoff is identical to his expected payoff in the absence of influential communication.[26] This is a curious result, in light of the fact, that standard incentive theory teaches us that privately informed agents must be paid information rents in order to divulge their information. This result points out an interesting difference between our setting and standard settings; in our setting incentive compatibility does not guarantee each type of advocate a certain share of social surplus. In our setting the planner, in some sense, uses the advocate's information against him, leveraging the fact that the advocate knows the true distribution, and the fact she has the capability to learn, to provide the advocate with incentives to report honestly. However, in a setting with costly learning, the absence of rent offered to the advocate would appear to undermine the advocate's incentive to acquire information. We turn to this issue in the next section. Before doing so, we consider the properties of the optimal mechanism with a large number of periods.

4.1 Properties of the Mechanism in Long Interactions

In a long interaction ($N \to \infty$), the planner is able to learn the true distribution (with virtual certainty) from the observed history of states. This raises two interesting issues. First, to what extent does the planner's ability to learn mitigate the information asymmetry that exists at the beginning of the interaction? Second, if the planner can learn the true distribution, in the long run does the advocate's recommendation impact the planner's action? Both of these issues are addressed by the following corollary.

Corollary *(High Power Incentives) When $\frac{1}{\delta} \leq \alpha < \infty$, as $N \to \infty$, $\omega_N \to 0$; the optimal mechanism approaches the first best mechanism, and the planner's payoff approaches the first best payoff, $V_N \to \overline{V}_\infty$*

(Low Power Incentives) When $\alpha < \frac{1}{\delta}$, as $N \to \infty$, $\omega_N \to \omega_\infty \in (0,1)$. The optimal mechanism converges to an incentive compatible mechanism. The planner's payoff is bounded away from the first best payoff. When $N = \infty$ the sequence of actions $q_H(h_{k-1}) \xrightarrow{P} \mu_H$

To understand this corollary, recall that if the advocate reports H, but the true dis-

[26] In the absence of influential communication the planner's sequentially rational action given a history is just $\gamma(h_{k-1})\mu_L + (1-\gamma(h_{k-1}))\mu_H$. By the law of iterated expectation, $E[\gamma(h_{k-1})] = \gamma$.

tribution is ϕ_L, he expects a penalty in the future that grows at rate $\alpha - 1$. However the long term growth of this penalty may not be enough to completely eliminate his incentive to manipulate the planner; the advocate's lifetime penalty for manipulating the planner is the *discounted sum* of the expected distortions, which may either converge or diverge. If the incentives are sufficiently high powered, this discounted sum diverges. In the limit, the powerful incentives completely eliminate the advocate's incentive to manipulate the planner, the distortions vanish, and the optimal mechanism approaches the first best.[27] Thus when incentives have high power, in a long interaction, there is virtually no difference between information provided by an advocate, and information that the planner acquires herself.[28] If the incentives have low power, the expected penalty explodes too slowly relative to discounting, and the discounted sum of expected distortions converges. The planner can not shrink the magnitude of the distortions to zero without violating incentive compatibility and, the planner's payoffs are bounded away from the first best.

When the true distribution is ϕ_H, the planner's action converges in probability to the first best action μ_H;[29] therefore, the damaging distortions associated with a report of H are inherently a short run phenomenon. However, even in the absence of influential communication, the planner's action would converge to μ_H in probability; thus, when the true distribution is ϕ_H, the impact of the advocate's recommendation vanishes in the long run. In contrast, when the true distribution is ϕ_L, the advocate's impact on the planner's action never diminishes. Paradoxically, when the advocate makes the seemingly more believable report, the distortion from the first best action μ_L persists forever; when the advocate reports in a seemingly biased way, the distortion from the first best action μ_H is likely to be small after a large number of periods. In the long run, the advocate's recommendation only impacts the planner's behavior when he reports against his bias.

[27] Although the first best is not incentive compatible, by choosing an arbitrarily large T, and repeating the optimal T period mechanism an infinite number of times, the planner can construct an infinitely repeated mechanism that is incentive compatible and approximates her payoff under the first best with arbitrary degree of accuracy.

[28] This result does not merely hold in the limit as $\delta \to 1$, but is valid for any value of $\delta \geq \frac{1}{\alpha}$.

[29] The asymptotic behavior of the distortions is driven by the asymptotic behavior of the likelihood ratio under distribution ϕ_H, which converges in probability to zero. Therefore, $\lim_{N\to\infty} \Pr\left(\left|q_H\left(h_{N-1}\right) - \mu_H\right| \leq \varepsilon\right) = 1$, for any ε, no matter how small.

5 Motivating Learning, Discovering Hidden Agendas

The theory of advocacy we have developed so far explores how a planner can elicit a truthful recommendations from an informed, but extremely biased advocate. In analyzing this question, we assumed that the advocate was already informed before encountering the planner.[30] However, if he is uninformed (initially), learning is costly, and the planner has an opportunity to exert effort to become informed, the advocate seems almost irrelevant. Like the planner, he is uninformed, and if he does become informed (which the planner can not independently verify), he has an incentive to issue manipulative recommendations. Why bother? Why would planners consult advocates as opposed to unbiased advisors? From the advocate's perspective, the situation does not appear to be much better. His recommendation influences the planner's behavior, but on average advising the planner does not benefit his cause. Why would he exert effort to learn? How can planners optimally motivate advocates to acquire information? We turn to these issues in the subsection 5.1.

To this point we have also assumed that the extreme conflict of interest between the planner and advocate is common knowledge. In subsection 5.2, we analyze the impact of impartial advisors on the advisory relationship. For example, if some portion of financial advisors takes its fiduciary responsibilities seriously and offers advice in the best interest of the client, how can the client separate the impartial advisor from the self-interested advisors? Who benefits under these modified arrangements?

5.1 Motivating a Known Advocate to Learn

In certain settings, the advocate may need to expend resources to become informed. For instance, a pharmaceutical company spends significant resources running clinical trials in order to determine the likely impact of its drug. Similarly, in order to truly determine the likely academic impact of school nutrition programs the child advocate may spend time and effort collecting and analyzing large quantities of data. In these situations, the planner's reliance on the advocate seems puzzling; if the advocate is uninformed initially and has an obvious incentive to manipulate his findings[31] why wouldn't the planner learn

[30] or could become informed at zero cost

[31] Short of outright falsification of its findings, the pharmaceutical company may manipulate the criteria by which it selects patients for trials, how it determines ongoing eligibility, and how it defines the side effects. It also may be virtually impossible for the Welfare Commission to audit the data acquired by the child advocate; even if he discloses this data it would be difficult to independently verify that this data is

from an unbiased party or expend effort to learn herself? Furthermore, if (as in the previous section) he expects to have no impact on the planner the advocate may have weak incentives to acquire information; how can the planner motivate him to learn? From a social perspective, is it a good idea for the planner to acquire information from the advocate?

In light of these issues, we consider how the advocate might be motivated to gather the information that the planner requires to make decisions. In order to acquire information, the advocate must expend $c \geq 0$ resources, in the form of money, time and effort; throughout the section, we focus on the case of relatively small values for c.[32] Crucially, the advocate's decision to become informed can not be observed by the planner. The advocate not only has the ability to lie when issuing an informed recommendation, he can issue a recommendation without exerting effort to acquire any information at all. We start by deriving the optimal incentive compatible mechanism that provides incentives for the advocate to acquire information. This will help us understand how the planner motivates the advocate to acquire information. We will then compare the optimal mechanism to consulting an impartial advisor or exerting effort, to help us understand why an advocate may be preferred to an impartial advisor.

If the planner would like to motivate the advocate to become informed, two additional constraints appear in the mechanism design problem. If an advocate decides to remain uninformed, he could just issue an uninformed recommendation, either H or L.[33] On the other hand, if the advocate chooses to acquire information, he anticipates that with probability γ he will learn that the distribution is ϕ_L, and with probability $1-\gamma$ he will learn that the distribution is ϕ_H; whatever, he learns, (ICH) and (ICL) ensure that he will report truthfully. Therefore, for the advocate to choose to learn, exerting effort and reporting truthfully should be preferred by an uninformed advocate to not exerting effort and either reporting H all the time (AICH) or reporting L all the time (AICL).

accurate.

[32] This is the most relevant case for the issues that we analyze. Although running clinical trials can be expensive, the cost can be small in comparison to the gains and losses inherent in the outcome of the FDA's decision. The cutoff between small and large costs, $\widetilde{c}$, is defined in the appendix. While we discuss the mechanism that induces learning with high costs in a later footnote, we point out that if the cost is high enough, inducing learning is not in the planner's interest.

[33] Unless the uninformed advocate is indifferent between reporting H all the time and reporting L all the time, he would never choose to randomize. In the cases we consider in the body of the paper, an uninformed advocate always strictly prefers to report H.

Formulating and simplifying these constraints leads to the following conditions:

$$\sum_{k=1}^{N} \delta^{k-1} E\left[q_H\left(h_{k-1}\right) - q_L\left(h_{k-1}\right) | \phi_L\right] \leq -\frac{c}{\gamma} \qquad \text{(AICH)}$$
$$\sum_{k=1}^{N} \delta^{k-1} E\left[q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right) | \phi_H\right] \leq -\frac{c}{1-\gamma} \qquad \text{(AICL)}$$

Of course, the optimal mechanism must also satisfy the constraints for truthful reporting:

$$\sum_{k=1}^{N} \delta^{k-1} E\left[q_H\left(h_{k-1}\right) - q_L\left(h_{k-1}\right) | \phi_L\right] \leq 0 \qquad \text{(ICL)}$$
$$\sum_{k=1}^{N} \delta^{k-1} E\left[q_H\left(h_{k-1}\right) - q_L\left(h_{k-1}\right) | \phi_H\right] \leq 0 \qquad \text{(ICH)}$$

Writing the constraints in this way makes it apparent that if $c = 0$, the constraints for reporting truthfully and the constraints for acquiring it are identical. Furthermore, because the optimal mechanism of Proposition 3 satisfies (ICL) as an equality, it violates constraint (AICH); if information acquisition is costly and subject to moral hazard, then confronted with the mechanism of proposition 3, an advocate would never acquire information and would always report H. Finally, it is clear that the constraints for incentive compatible information acquisition imply the incentive constraints for truthful reporting; constraints (ICL) and (ICH) can therefore be ignored in the planner's problem. As before, participation at both interim and ex ante stages is ensured by the planner's commitment to interpret the absence of a message as a report of L. If the planner makes such a commitment, constraint (AICL) would ensure that the advocate prefers to learn rather than report nothing to the planner.[34]

We derive the optimal mechanism that induces information acquisition by maximizing (P) subject to (AICH) and (AICL).

Proposition 3 *For small costs, the mechanism that maximizes (P) subject to (AICH) and (AICL) is identical to the optimal mechanism of proposition 2 with an increased magnitude of distortions:*

$$\widetilde{\omega}_N = \frac{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)+c(1-\delta)}{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)}\omega_N$$

(a) *The planner's actions are*

$$\begin{aligned} q_L\left(h_{k-1}\right) &= \mu_L + (1-\gamma)\,\widetilde{\omega}_N \\ q_H\left(h_{k-1}\right) &= \mu_H - \gamma\widetilde{\omega}_N \Lambda\left(h_{k-1}\right) \end{aligned}$$

[34] This may be a strong assumption in certain settings. See the extended appendix for a discussion of the case in which the planner must act in a sequentially rational manner following the advocate's refusal of the mechanism at the ex ante stage. There are some interesting details in this discussion, but the results are qualitatively similar.

(b) *The planner's and advocate's payoffs are given by*

$$V_N^c = \overline{V}_N - \left(\frac{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)+c(1-\delta)}{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)}\right)^2 \frac{1-\delta^N}{1-\delta}\omega_N\left(\gamma-\gamma^2\right)\left(\mu_H-\mu_L\right)$$
$$U_N^c = U_N - c$$

(c) *If ω_N is not too large, the planner prefers to offer an uninformed advocate the optimal mechanism to acquire information and report truthfully, rather than pay an impartial advisor to acquire the information or exert effort to acquire the information herself*

Proposition three indicates that when the cost of acquiring information is strictly positive the planner "raises the stakes" of the mechanism, rewarding the advocate more when he reports personally unfavorable information and proportionally increasing the downward distortion (for every history) when the advocate reports a personally favorable distribution. Intuitively, there are two types that could profitably deviate by reporting the high distribution: the type that learned that the true distribution is ϕ_L, and the uninformed advocate, who is pretending to be informed. Because a report of the low distribution can still be believed, the structure of the mechanism is similar to the zero-cost case. To motivate the advocate to learn, the magnitude of the distortions needs to be increased relative to the zero-cost case. With zero cost of information, the goal was to prevent an advocate from deliberately misleading the planner; here the mechanism also needs to motivate the advocate to exert effort to learn, rather than "gamble" by reporting H without learning anything. This result is similar to Szalay (2005) and Lewis and Sappington (1997) who find that increasing the variation in payoffs for privately informed agents is the optimal way to induce the agents to gather information.[35]

In the optimal mechanism the social cost of information acquisition exceeds c. Although the advocate bears the full cost c himself, in order to motivate the advocate to learn, the planner must increase the magnitude of the distortions inherent in the mechanism, hurting her own payoff. If the planner can acquire the information herself at cost c (or hire an impartial advisor to do so on her behalf), it is socially wasteful for the planner to rely on

[35] In the case of large costs the optimal mechanism is quite different. When the cost of becoming informed is very high, not becoming informed at all becomes tempting for the advocate. Although no advocate would ever report L *to manipulate the decision maker,* if the cost of acquiring information is very high, the advocate may report L as a way *to avoid learning the true distribution.* The decision maker must cross-check *both* reports against the history in order to provide the advocate with incentives to exert effort in learning the true distribution.

the advocate.

Although it is socially inefficient, the planner may prefer to deal with the advocate rather than pay an impartial agent to acquire the information or exert effort c to acquire the information herself. The planner cares only about her own payoff, so she would prefer to interact with the advocate if the cost of direct information acquisition c, were greater than her payoff cost of acquiring an informed and truthful recommendation from the advocate:

$$c \geq \left(\frac{\gamma\left(1-\delta^{N}\right)(\mu_H-\mu_L)+c(1-\delta)}{\gamma\left(1-\delta^{N}\right)(\mu_H-\mu_L)} \right)^2 \left(\frac{1-\delta^N}{1-\delta}\right) \omega_N \left(\gamma - \gamma^2\right) \left(\mu_H - \mu_L\right)$$

A small value of ω_N ensures that this inequality is satisfied;[36] it also guarantees that the planner prefers the mechanism that induces learning to going it alone.[37]

This result may seem quite surprising, but the rationale for the planners' preference to consult an advocate for advice is quite compelling. Imagine that the planner has access to the same information acquisition technology as the advocate. If she exerts effort to learn,.the planner doesn't need to worry about truthful revelation of information, but she bears the full cost of information acquisition. On the other hand, because of his extreme bias, when dealing with an advocate, the planner bears a cost of inducing the advocate to truthfully report his information; however, because he cares about her actions, the planner can use her actions to motivate the advocate to acquire information. By increasing the magnitude of the distortions, the planner passes the direct cost of information acquisition to the advocate, hurting her own expected payoff in the process .However, when the advocate expects a large lifetime penalty from reporting H when the true distribution is ϕ_L, either deliberately or because he is uninformed, the planner does not need to increase the magnitude very much (in absolute terms) in order to induce learning. In this case the total loss of payoff to the planner from acquiring truthful information using the optimal mechanism is smaller than the cost of direct information acquisition.[38]

[36] As discussed in the previous section, if $\alpha \geq \frac{1}{\delta}$, then as $N \to \infty$, $\omega_N \to 0$. Alternatively, keeping N fixed, if α is large then ω_N is close to zero.

[37] It is worth pointing out that for sufficiently small costs, the planner prefers the mechanism that induces learning to the case of no influential communication, regardless of ω_N. Because the mechanism with $c = 0$ is always preferred by the planner to the case without influential communication, by continuity, for c small, the mechanism is also be preferred to the case without influential communication.

[38] This result does not rely on the assumption that the planner commits to interpret the absence of message as L. If the planner were unable to make such a commitment, she would act in a sequentially rational manner following rejection of the agreement by the advocate at the ex ante stage, leaving him with an outside option of U_N. With this outside option, the mechanism of proposition 3 would be rejected; to induce the advocate to accept, the planner will optimally offer the same mechanism as in proposition

This result suggests an important explanation for the prominence of advocates in many regulatory, policy making and personal decision making processes, including some of the ones that we've highlighted in the introduction. In contrast to impartial advisors, advocates represent a particular point of view or interest that is affected by the planner's decision. Because of this, the planner has some power over the advocate; she can use her decisions to motivate the advocate to acquire information. As a result it can be easier and less costly for a planner to rely on a biased advocate rather than an objective adviser to acquire the information that is needed for decision making. Dewatripont and Tirole (1999) make a similar argument in their explanation of why an advocacy system may be preferred to a tribunal in fact finding processes, though in their setting no manipulation is possible. Che and Kartik (2009) point to a similar rationale to explain why planners may prefer to consult with advisers who share their preferences but have different prior beliefs about the best action to take.

5.2 Discovering Hidden Agendas

Known advocates can not escape from the fact that they have no inherent credibility; any influence which they have on decision making is exclusively a consequence of the planner's design. Even if the advocate for children has acquired excellent information about the likely benefits of the head start program, because it is common knowledge that he supports this cause, only the planner can provide the advocate with incentives to reveal his information truthfully, thereby giving the advice credibility and influence. Is this inevitable? Can the advocate exert a greater impact for her cause by being less transparent to the commission about the issues that he supports?

Imagine that the planner is crafting new legislation that defines the goals and responsibilities for a future health and welfare initiative.[39] The legislation prioritizes support for a variety of social programs, including education, health and human services, and childhood nutrition. The planner requires the advice of a knowledgeable advisor on how to draft the

3, with an additional constant distortion of $c\left(\frac{1-\delta}{1-\delta^N}\right)$ added to every action, independent of the report or the history. Taken together, these additional distortions exactly compensate the advocate for the cost of learning, leaving him an ex ante payoff of U_N. In this circumstance, the planner implicitly commits to cover the cost of learning, increasing her payoff cost by $c^2\left(\frac{1-\delta}{1-\delta^N}\right)$. If c is sufficiently small, $c \geq c^2\left(\frac{1-\delta}{1-\delta^N}\right)$, and the corollary continues to hold. See the extended appendix for more information.

[39] A similar example is in Morris (2001).

legislation to be most effective. The advisor's motives for advising the planner may be unknown; he may have an allegiance to a special interest group whose agenda he would like to promote, or he may have no agenda at all, issuing truthful recommendations without concern for their impact on the legislation. Alternatively, imagine that a financial advisor is offering a client investment advice and services. In offering advice, he may be pursuing his own bottom line, or he may feel bound by certain oaths or obligations to place the interests of the client ahead of his own.

To account for these possibilities, we admit the possibility that the advisor is impartial into the model. An impartial advisor is indifferent over actions and is always willing to report his information truthfully. If the planner could observe that her advisor is impartial, she would optimally delegate her choice of action to him with no restrictions. However, if the advisor's conflict of interest is privately known, an advocate would have an incentive to misrepresent himself as impartial in order to give a manipulative report of H the appearance of credibility. We therefore augment the mechanism to provide incentives for an advisor to voluntarily reveal a hidden agenda, if it exists.

In analyzing this issue, we assume that the planner can not prevent the advisor from learning both the true distribution and his preferences simultaneously. If it were possible for the planner to control the advisor's access to information, the planner would require that the advisor disclose a conflict of interest before allowing him to learn the true distribution. We do not allow this type of restriction to avoid discussing the issue of whether this type of control over information is possible, how the planner could implement this type of restriction and whether such a restriction is consistent with zero cost of learning, which we assume throughout this section.[40]

With uncertainty about the advisor's preferences, the agreement between the planner and the advisor is governed by a mechanism that is more elaborate than in earlier sections. With a hidden agenda, there are two possible types of advisors, advocates (with a hidden agenda) and impartial advisors. The advisor's private information consists of the true distribution and his agenda, and could be of four possible types. We assume that the probability that the advisor is an advocate is a; we also assume that the advisor's agenda is statistically independent of whether the distribution is ϕ_H or ϕ_L. The mechanism

[40] In the extended appendix we analyze the optimal mechanism assuming that the planner can restrict access to information in this way. We will discuss some of the results in a later footnote. Interested readers should consult the extended appendix for more information.

offered by the planner is therefore a family of four functions, one function for each possible combination of interest conflict and information:

$$\left\{q_H^i\left(h_{k-1}\right), q_L^i\left(h_{k-1}\right), q_H^a\left(h_{k-1}\right), q_L^a\left(h_{k-1}\right)\right\}_{k=1}^N$$

If the advocate reports that he has preferences $t \in \{i, a\}$ (where i denotes the impartial type), and that the true distribution is ϕ_Z, the planner commits to implement actions $q_Z^t\left(h_{k-1}\right)$. The planner's expected payoff from an incentive compatible mechanism is just a weighted average of her payoffs from the "sub-mechanism" intended for the impartial advisor, $m^i = \left(q_H^i\left(h_{k-1}\right), q_L^i\left(h_{k-1}\right)\right)$,and the "sub-mechanism" intended for the advocate $m^a = \left(q_H^a\left(h_{k-1}\right), q_L^a\left(h_{k-1}\right)\right)$;

$$P^d\left(m^i, m^a\right) = (1-a)\, P\left(m^i\right) + aP\left(m^a\right)$$

In order for the mechanism to be incentive compatible, the advisor should be willing to disclose both the distribution and his conflict of interest truthfully. Because the impartial advisor is indifferent over all possible outcomes, he is always willing to do so; however, the advocate type needs incentives to report truthfully. This leads to a set of six incentive constraints, denoted by (ICD).

$$\begin{array}{cc} \sum_{k=1}^N \delta^{k-1} E\left[q_Z^t\left(h_{k-1}\right) - q_L^a\left(h_{k-1}\right) | \phi_L\right] \leq 0 & \text{(ICL-Z,t)} \\ \sum_{k=1}^N \delta^{k-1} E\left[q_Z^t\left(h_{k-1}\right) - q_H^a\left(h_{k-1}\right) | \phi_H\right] \leq 0 & \text{(ICH-Z,t)} \\ Z \in \{H, L\}, t \in \{i, a\} & \end{array}$$

The optimal mechanism maximizes (PD) subject to the system (ICD). Intuitively, we would expect that several of these constraints would not be binding in the optimal mechanism.[41] In the appendix we formulate a relaxed problem in which we impose only constraints (ICL-H,a), (ICL-H,i). We then prove that the solution to the relaxed problem satisfies the remaining constraints, and therefore characterizes the optimal incentive

[41] The advocate has an incentive to claim to be impartial in order to give a manipulative report of H additional credibility. There is no reason for the advocate to claim to be impartial, while issuing a report of L. Furthermore, following the reasoning in section 3, we would also suspect that (ICH-L,a) is non-binding. There is no simple intuition that suggests that either of the remaining constraints are non-binding: an advocate who knows the distribution is ϕ_L could potentially benefit by reporting H (ICL-H,a), he could also potentially benefit by claiming to be impartial and reporting H (ICL-H,i). Furthermore, the advocate who knows that the true distribution is ϕ_H could try to gain additional credibility by claiming to be impartial and reporting H (ICH-H,i). Although there is no good reason to eliminate (ICH-H,i), it turns out to be active in the optimal mechanism, but with a zero Lagrange multiplier; it is active but not binding.

compatible mechanism. Before stating the proposition, we introduce a parameter:

$$\theta = \frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}\left(\mu_H - \mu_L\right)$$

This parameter is very similar to the magnitude introduced in section 3; in fact, $a = 1$ implies $\theta = \omega_N$. Furthermore, parameter θ plays a very similar role to ω_N in the optimal mechanism, but unlike ω_N which enters the planner's promised actions in a "symmetric" way in propositions 2 and 3, θ does not enter in a symmetric way in proposition 4.

Proposition 4 *The mechanism that maximizes (PD) subject to (ICD) is characterized as follows:*

(a) *If the advisor claims to be impartial and reports L, the planner's action is equal to the first best action*

$$q_L^i\left(h_{k-1}\right) = \mu_L$$

(b) *If the advisor claims to be an advocate and reports L, the planner's action is a constant, greater than her first best action μ_L.*

$$q_L\left(h_{k-1}\right) = \mu_L + (1-\gamma)\,\theta$$

(c) *If the advisor reports H, regardless of his claim about his preferences, the planner's action depends on the observed history and is always less than her first best action μ_H*

$$q_H^i\left(h_{k-1}\right) = q_H^a\left(h_{k-1}\right) = \mu_H - \gamma\Lambda\left(h_{k-1}\right)(a\theta)$$

(d) *If the advisor is an advocate his expected payoff is given by*

$$U_N + \tfrac{1-\delta^N}{1-\delta}\gamma\,(1-\gamma)\,(1-a)\,\theta.$$

Proposition 4 illustrates the differences that arise when the advocate's preferences on a specific issue are not common knowledge. Because the advocate would have no reason to claim to be impartial and then report L, a report of L from an advisor who claims to be impartial can be believed at face value. A report of H is treated identically, whether the advisor is an advocate or claims to be impartial; however, the magnitude of the distortion associated with a self serving report, $a\theta_N$, is *smaller* the greater the probability that the advisor is impartial. The possibility that the advisor is impartial allows the planner to treat reports of H with greater credibility. On the other hand, the constant distortion induced when the advisor reveals a conflict of interest, but reports *against* his bias is larger when there is a possibility that the advisor may be impartial. Because the planner gives greater credibility to reports of H, these reports are more attractive to an advocate who knows that the distribution is actually ϕ_L.This increases the advocate's incentive to issue manipulative advice, and the planner must compensate the advocate for reporting L with an additional distortion.[42]

The advocate's world changes for the better when he becomes less transparent and his allegiance to special interests are more difficult to predict. When there is a possibility that the advocate *is not* an advocate but rather an impartial advisor, the planner benefits from a greater compromise with the *true* advocate. The advocate earns a rent for disclosing his agenda, and his ex ante payoff from the mechanism rises above U_N, his expected payoff if he were not consulted and his advice was totally ignored. Finally, the advocate is able to have a positive impact on the welfare of the special interests that he supports. It is ironic, however, that in order to help his cause the advocate must disavow and conceal his support of the cause that he cares so deeply about! In effect, just as Morris (2001) demonstrates that "bad advisors" can gain a reputation for being impartial, and Sobel finds that "unfriendly" advisors can establish a reputation for acting in the best interests of the decision maker, we find that if the advocate's ties to special interests are hidden, the advocate can credibly claim to be impartial, and is therefore better off.

6 Conclusion

In this paper we propose a theory of advocacy that attempts to explain why planners take advice from an advocate with a known allegiance to a special interest or cause.

We show that advocates can only influence a policy maker's decision, when she commits

[42] Similar results hold if the planner can prevent the advocate from learning the true distribution until he has disclosed his interest conflict. In that setting, the planner is able to treat the advice of an impartial advisor who reports H differently from the advice of an advocate who reports H. Both of these sequences of messages lead to stochastic distortions that depend of $\Lambda(h_{k-1})$, but the impartial type generates smaller distortions on average; however the advocate does not capitalize on this because he earns a rent. See the extended appendix for more information.

to act on the advice from an advocate that proves to be credible. The commitment to action can be implemented by the planner delegating or regulating the actions that the advocate takes based on how accurate her predictions turn out to be. Provisional agreements likes these to delegate decision making authority to a trusted adviser are common in settings were elected representatives cede important decisions to better informed agency staff or where clients entrust the management of their private wealth to a respected financial adviser. Moreover we show that planners may prefer the advice of a biased advocate to the recommendation of an indifferent adviser in instances where information about the planner's problem is costly to acquire. Finally our theory predicts that advocates are better served to conceal their ties to special interest as this permits them to have greater sway over the planners that they advise.

Our theory of advocacy is special in a number of respects and therefore the predictions of our model should be interpreted with some care. For instance, we assume that there is just one advocate who can advise the planner. In many applications, including judicial and policy making deliberations, it is common for decision makers to consult two or more advocates representing opposing interests. One fruitful direction for future research would be to extend our analysis to consider how "dueling" advocates representing disparate points of view may be most effectively paired to provide informative advice to a policy maker. More generally, the optimal use of advocates with known allegiances might be analyzed as an optimal fact finding mechanisms in which informed parties with differing views are assigned to gather information and make recommendations to common decision maker who may commit to a course of action conditional on the recommendations of the advocates as well as their relative "track" records in predicting the history of states occurring.

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7 Appendix

In the appendix we prove the results discussed in the body of the text. A more complete appendix with proofs of assertions in footnotes is available upon request.

Proposition 1.1 *In a single period interaction with commitment, the only incentive compatible mechanism is non-influential:* $q_H = q_L$.

Proof 1.1 Discussed in text

Lemma 1: *If the decision maker begins with belief* $\widehat{\gamma}$ *that the true distribution is* ϕ_L*, the unique sequentially rational action in period* k *following history* h_{k-1} *is the expected value of the state, updated based on the observed history:*

$$q(\widehat{\gamma}, h_{k-1}) = \gamma(h_{k-1})\,\mu_L + (1 - \gamma(h_{k-1}))\,\mu_H$$

$$\gamma(h_{k-1}) = \Pr(\phi = \phi_L | h_{k-1}) = \frac{\widehat{\gamma} f(h_{k-1}|\phi_L)}{\widehat{\gamma} f(h_{k-1}|\phi_L) + (1-\widehat{\gamma})\, f(h_{k-1}|\phi_H)}$$

Proof: If she begins her sequence of decisions with belief $\Pr(\phi = \phi_L) = \widehat{\gamma}$ the sequentially rational sequence of decisions is characterized by the following optimization problem:

$$\begin{aligned} \max_{q(h_{k-1})} \quad & -\widehat{\gamma} \quad \textstyle\sum_{k=1}^{N} \delta^{k-1} \displaystyle\int\limits_{X_{k-1}} f(h_{k-1}|\phi_L) \left[\int\limits_{X} f(x|\phi_L)\,(q(h_{k-1}) - x)^2\, dx \right] dh_{k-1} \\ & -(1-\widehat{\gamma}) \quad \textstyle\sum_{k=1}^{N} \delta^{k-1} \displaystyle\int\limits_{X_{k-1}} f(h_{k-1}|\phi_H) \left[\int\limits_{X} f(x|\phi_H)\,(q(h_{k-1}) - x)^2\, dx \right] dh_{k-1} \end{aligned}$$

This objective function can be simply rewritten

$$\begin{aligned} & \widehat{\gamma} \quad \textstyle\sum_{k=1}^{N} \delta^{k-1} \displaystyle\int\limits_{X_{k-1}} f(h_{k-1}|\phi_L) \left(-q(h_{k-1})^2 + 2\mu_L q(h_{k-1}) - \eta_L \right) dh_{k-1} \\ + & (1-\widehat{\gamma}) \quad \textstyle\sum_{k=1}^{N} \delta^{k-1} \displaystyle\int\limits_{X_{k-1}} f(h_{k-1}|\phi_H) \left(-q(h_{k-1})^2 + 2\mu_H q(h_{k-1}) - \eta_H \right) dh_{k-1} \end{aligned}$$

by introducing a variable to denote the decision maker's belief that the distribution is ϕ_L given the observed history of states, $\gamma(h_{k-1})$, and the conditional mean, $\mu(h_{k-1})$ the objective function can be simplified even further

$$\begin{aligned}
\gamma\left(h_{k-1}\right) &= \Pr\left(\phi=\phi_L | h_{k-1}\right)=\frac{\widehat{\gamma} f\left(h_{k-1}|\phi_L\right)}{\widehat{\gamma} f\left(h_{k-1}|\phi_L\right)+\left(1-\widehat{\gamma}\right) f\left(h_{k-1}|\phi_H\right)} \\
\mu\left(h_{k-1}\right) &= \gamma\left(h_{k-1}\right)\mu_L+\left(1-\gamma\left(h_{k-1}\right)\right)\mu_H \\
\eta\left(h_{k-1}\right) &= \gamma\left(h_{k-1}\right)\eta_L+\left(1-\gamma\left(h_{k-1}\right)\right)\eta_H
\end{aligned}$$

Substituting this into the objective function gives

$$\sum_{k=1}^{N}\delta^{k-1}\int_{X_{k-1}} f\left(h_{k-1}|\phi_0\right)\left(-q\left(h_{k-1}\right)^2+2\mu\left(h_{k-1}\right)q\left(h_{k-1}\right)-\eta\left(h_{k-1}\right)\right)dh_{k-1}$$

The first and second order conditions imply that the unique sequentially rational decision following history h_{k-1} is given by

$$q\left(h_{k-1}\right)=\gamma\left(h_{k-1}\right)\mu_L+\left(1-\gamma\left(h_{k-1}\right)\right)\mu_H$$

Therefore, given an initial belief $\widehat{\gamma}$, following observed history h_{k-1} the unique sequentially rational actions is

$$\widetilde{q}\left(\widehat{\gamma},h_{k-1}\right)=\gamma\left(h_{k-1}\right)\mu_L+\left(1-\gamma\left(h_{k-1}\right)\right)\mu_H$$

QED

Proposition 1.2 *In every Perfect Bayesian Equilibrium of the multiperiod game without commitment the advocate's reporting strategy is independent of his information. There is no influential communication in equilibrium.*

Proof: We prove this result by constructing the PBE of this game. Once the planner receives a message from the advocate, she Bayesian updates her belief that the distribution is ϕ_L to either $\widehat{\gamma}_H$ or $\widehat{\gamma}_L$. She then faces a repeated single-player decision problem. According to Lemma 1, the planner's unique sequentially rational action following history

h_{k-1}, given that her initial belief is $\widehat{\gamma}$ is the conditional expected value of the state.

$$q\left(\widehat{\gamma}, h_{k-1}\right) = \gamma\left(h_{k-1}\right)\mu_L + \left(1 - \gamma\left(h_{k-1}\right)\right)\mu_H$$

$$\gamma\left(h_{k-1}\right) = \Pr\left(\phi = \phi_L | h_{k-1}\right) = \frac{\widehat{\gamma} f\left(h_{k-1}|\phi_L\right)}{\widehat{\gamma} f\left(h_{k-1}|\phi_L\right) + \left(1-\widehat{\gamma}\right) f\left(h_{k-1}|\phi_H\right)}$$

Because $\gamma\left(h_{k-1}\right)$ is monotone decreasing with respect to $\widehat{\gamma}$, inducing a smaller value of $\widehat{\gamma}$ increases the planner's action *following every history*, benefitting the advocate. Recall that the advocate's strategy is just $\left(r_H, r_L\right) \in [0,1]^2$, where r_X represents the probability that the advocate reports H when the true distribution is ϕ_X. The advocate's sequentially rational strategy is therefore

$$\begin{aligned}
\widehat{\gamma}_H &> \widehat{\gamma}_L \rightarrow r_H = 0,\ r_L = 0 \\
\widehat{\gamma}_H &< \widehat{\gamma}_L \rightarrow r_H = 1,\ r_L = 1 \\
\widehat{\gamma}_H &= \widehat{\gamma}_L \rightarrow r_H \in [0,1],\ r_L \in [0,1]
\end{aligned}$$

Consider first the case in which $\widehat{\gamma}_H > \widehat{\gamma}_L$ (the reverse case is identical, replacing H and L). In this case, the advocate always reports L. Upon observing L, the planner's Bayesian update is equal to the prior, $\widehat{\gamma}_L = \gamma$. The off-the path-belief associated with a report of H is assigned to be any value $\widehat{\gamma}_H > \gamma$. The planner's actions are $q\left(\gamma, h_{k-1}\right)$ if the advocate reports L and the planner observes history h_{k-1}, and $q\left(\widehat{\gamma}_H, h_{k-1}\right)$ if the planner receives message H and observes history h_{k-1} (off the equilibrium path).These strategies and beliefs together constitute a PBE.

The remaining possibility is that $\widehat{\gamma}_H = \widehat{\gamma}_L$. In this case, the advocate can send either message with any probability, as the advocate is indifferent between the messages. If only one message is sent in equilibrium the PBE is identical to the one described previously. If both messages are sent in equilibrium, then according to Bayes Rule,

$$\begin{aligned}
\widehat{\gamma}_H &= \frac{\gamma r_L}{\gamma r_L + (1-\gamma) r_H} \\
\widehat{\gamma}_L &= \frac{\gamma\left(1 - r_L\right)}{\gamma\left(1 - r_L\right) + (1-\gamma)\left(1 - r_H\right)}
\end{aligned}$$

It a matter if calculation to verify that

$$\widehat{\gamma}_H = \widehat{\gamma}_L \rightarrow r_H = r_L$$

$$r_H = r_L \rightarrow \widehat{\gamma}_H = \widehat{\gamma}_L = \gamma$$

Thus, the only other possible PBE has $r_H = r_L = r \in (0,1)$, $\widehat{\gamma}_H = \widehat{\gamma}_L = \gamma$, and an action conditional on history $q(\gamma, h_{k-1})$, regardless of the message sent. In both cases, the Bayesian update on the equilibrium path is equal to the prior, and on the equilibrium path, the planner's actions are $q(\gamma, h_{k-1})$. There is no influential communication in equilibrium.

QED

Lemma 2 *A mechanism* $q_L(h_{k-1}) = \mu_L + (1-\gamma)w$, $q_H(h_{k-1}) = \mu_H - \gamma w \Lambda(h_{k-1})$ *satisfies constraint (ICH) if and only if* $w \leq \mu_H - \mu_L$.

Proof:

$$\sum_{k=1}^{N} \delta^{k-1} E\left[q_H(h_{k-1}) | \phi_H\right] \geq \sum_{k=1}^{N} \delta^{k-1} E\left[q_L(h_{k-1}) | \phi_H\right] \tag{ICH}$$

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) q_H(h_{k-1}) dh_{k-1} \geq \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H) q_L(h_{k-1}) dh_{k-1}$$

$$\Leftrightarrow$$

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)(\mu_H - \gamma w \Lambda(h_{k-1})) dh_{k-1} \geq$$
$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)(\mu_L + (1-\gamma)w) dh_{k-1}$$

Because $f(h_{k-1}|\phi_H)\Lambda(h_{k-1}) = f(h_{k-1}|\phi_L)$, this line simplifies

$$(\mu_H - \gamma w)\tfrac{1-\delta^N}{1-\delta} \geq (\mu_L + (1-\gamma)w)\tfrac{1-\delta^N}{1-\delta}$$
$$\Leftrightarrow$$
$$w \leq \mu_H - \mu_L$$

QED

Lemma 3 *The planner' s payoff under mechanism* $q_L(h_{k-1}) = \mu_L + (1-\gamma)w$, $q_H(h_{k-1}) =$

$\mu_H - \gamma w \Lambda(h_{k-1})$ *is given by.*

$$\overline{V}_N - \gamma(1-\gamma) w^2 \left(\frac{1-\delta^N}{1-\delta}(1-\gamma) + \gamma \frac{1-(\alpha\delta)^N}{1-\alpha\delta} \right)$$

Proof:

$$\begin{aligned}
\gamma \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L)\left(-q_L(h_{k-1})^2 + 2\mu_L q_L(h_{k-1}) - \eta_L\right) dh_{k-1} \\
+(1-\gamma) \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)\left(-q_H(h_{k-1})^2 + 2\mu_H q_H(h_{k-1}) - \eta_H\right) dh_{k-1} = \\
\gamma \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \left(-(\mu_L + (1-\gamma)w)^2 + 2\mu_L(\mu_L + (1-\gamma)w) - \eta_L\right) \\
+(1-\gamma) \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)\left(-(\mu_H - \gamma w \Lambda(h_{k-1}))^2 + 2\mu_H(\mu_H - \gamma w \Lambda(h_{k-1})) - \eta_H\right) dh_{k-1} = \\
\gamma \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \left(-\sigma_L^2 - (1-\gamma)^2 w^2\right) \\
+(1-\gamma) \quad & \textstyle\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)\left(-\sigma_H^2 - \gamma^2 w^2 \Lambda^2(h_{k-1})\right) dh_{k-1} = \\
& \overline{V}_N - \gamma(1-\gamma) w^2 \left(\tfrac{1-\delta^N}{1-\delta}(1-\gamma) + \gamma \tfrac{1-(\alpha\delta)^N}{1-\alpha\delta} \right) =
\end{aligned}$$

QED

Proposition 2 *The mechanism which maximizes (P) subject to (ICL) and (ICH) is characterized by*

$$q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_N$$
$$q_H(h_{k-1}) = \mu_H - \gamma \omega_N \Lambda(h_{k-1})$$

$$\alpha = \int_X \tfrac{f(x|\phi_L)^2}{f(x|\phi_H)} dx$$

$$\omega_N = \tfrac{(1-\alpha\delta)\left(1-\delta^N\right)}{\gamma(1-\delta)\left(1-(\alpha\delta)^N\right) + (1-\gamma)(1-\alpha\delta)\left(1-\delta^N\right)} (\mu_H - \mu_L)$$

provided α is finite. The planner"s payoff V_N, and the advocate's payoff U_N are given by

$$V_N = \overline{V}_N - \tfrac{1-\delta^N}{1-\delta}\omega_N\left(\gamma-\gamma^2\right)\left(\mu_H-\mu_L\right)$$
$$U_N = \left(\gamma\mu_L + \left(1-\gamma\right)\mu_H\right)\tfrac{1-\delta^N}{1-\delta}$$

Proof: We first derive the optimal mechanism in a relaxed problem, imposing only constraint (ICL). Using Lemma 2, we show that the solution satisfies constraint (ICH). Finally we compute the planner's and advocate's payoffs. Therefore, consider the following relaxed problem

$$\begin{aligned}\max_{q(h_{k-1})} \quad & -\widehat{\gamma} \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)\left[\int\limits_{X} f\left(x|\phi_L\right)\left(q_L\left(h_{k-1}\right)-x\right)^2 dx\right] dh_{k-1} \\ & -\left(1-\widehat{\gamma}\right) \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_H\right)\left[\int\limits_{X} f\left(x|\phi_H\right)\left(q_L\left(h_{k-1}\right)-x\right)^2 dx\right] dh_{k-1}\end{aligned}$$

subject to

$$\textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)q_L\left(h_{k-1}\right)dh_{k-1} \quad = \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)q_H\left(h_{k-1}\right)dh_{k-1} \quad \text{(ICL)}$$

The objective problem is strictly concave, and the constraint is linear. The solution is therefore the unique stationary point of the Lagrangian. Simplifying the planner's objective function slightly, leads to the following Lagrangian:

$$\begin{aligned} & \gamma \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)\left(-q_L\left(h_{k-1}\right)^2 + 2\mu_L q_L\left(h_{k-1}\right) - \eta_L\right)dh_{k-1} \\ & +\left(1-\gamma\right) \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_H\right)\left(-q_H\left(h_{k-1}\right)^2 + 2\mu_H q_H\left(h_{k-1}\right) - \eta_H\right)dh_{k-1} \\ & +\lambda \quad \textstyle\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)\left[q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right)\right]dh_{k-1}\end{aligned}$$

The stationarity conditions are therefore,

$$q_L(h_{k-1}): \qquad \delta^{k-1} f(h_{k-1}|\phi_L)\{2\gamma(\mu_L - q_L(h_{k-1})) + \lambda\} = 0$$

$$q_H(h_{k-1}): \quad \delta^{k-1} f(h_{k-1}|\phi_H)\left\{2(1-\gamma)(\mu_H - q_H(h_{k-1})) - \lambda \tfrac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)}\right\} = 0$$

$$\lambda: \qquad \textstyle\sum_{k=1}^{N} \delta^{k-1} \displaystyle\int_{X_{k-1}} f(h_{k-1}|\phi_L)(q_L(h_{k-1}) - q_H(h_{k-1}))\, dh_{k-1} = 0$$

These first order conditions imply that:

$$q_L(h_{k-1}) = \mu_L + \tfrac{\lambda}{2\gamma}$$

$$q_H(h_{k-1}) = \mu_H - \tfrac{\lambda}{2(1-\gamma)} \Lambda(h_{k-1})$$

Let

$$\omega_N = \frac{\lambda}{2\gamma(1-\gamma)}$$

Then the first order conditions simplify to

$$q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_N$$

$$q_H(h_{k-1}) = \mu_H - \gamma \omega_N \Lambda(h_{k-1})$$

To find the optimal value of ω_N, we solve the remaining condition:

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L)(q_L(h_{k-1}) - q_H(h_{k-1}))\, dh_{k-1} = 0$$

Substituting

$$\frac{q_L\left(1-\delta^N\right)}{1-\delta}=\sum_{k=1}^{N}\delta^{k-1}\int_{X_{k-1}}f\left(h_{k-1}|\phi_L\right)q_H\left(h_{k-1}\right)dh_{k-1}$$

$$\begin{aligned}\frac{q_L\left(1-\delta^N\right)}{1-\delta}&=\mu_H\frac{1-\delta^N}{1-\delta}-\gamma\omega_N\sum_{k=1}^{N}\left(\alpha\delta\right)^{k-1}\\&=\mu_H\frac{1-\delta^N}{1-\delta}-\gamma\omega_N\frac{1-\left(\alpha\delta\right)^N}{1-\alpha\delta}\end{aligned}$$

$$\omega_N=\frac{\left(1-\alpha\delta\right)\left(1-\delta^N\right)}{\gamma\left(1-\delta\right)\left(1-\left(\alpha\delta\right)^N\right)+\left(1-\gamma\right)\left(1-\alpha\delta\right)\left(1-\delta^N\right)}\left(\mu_H-\mu_L\right)$$

By Lemma 2, to establish that this mechanism satisfies (ICH) it is enough to verify that $\omega_N\leq\left(\mu_H-\mu_L\right)$.

$$\begin{aligned}&\omega_N\leq\mu_H-\mu_L &&\Leftrightarrow\\&\tfrac{\left(1-\alpha\delta\right)\left(1-\delta^N\right)}{\gamma\left(1-\delta\right)\left(1-\left(\alpha\delta\right)^N\right)+\left(1-\gamma\right)\left(1-\alpha\delta\right)\left(1-\delta^N\right)}\leq 1 &&\Leftrightarrow\\&\gamma\left(1-\alpha\delta\right)\left(1-\delta^N\right)\leq\gamma\left(1-\delta\right)\left(1-\left(\alpha\delta\right)^N\right) &&\Leftrightarrow\\&\tfrac{1-\delta^N}{1-\delta}\leq\tfrac{1-\left(\alpha\delta\right)^N}{1-\alpha\delta} &&\Leftrightarrow\\&\textstyle\sum_{k=1}^{N}\delta^{k-1}\leq\sum_{k=1}^{N}\left(\alpha\delta\right)^{k-1}\end{aligned}$$

where the last line follows because $\alpha\geq 1$. Finally, we calculate the expected payoffs. For the planner apply Lemma 3:

$$\begin{aligned}&\overline{V}_N-\gamma\left(1-\gamma\right)\omega_N^2\left(\tfrac{1-\delta^N}{1-\delta}\left(1-\gamma\right)+\gamma\tfrac{1-\left(\alpha\delta\right)^N}{1-\alpha\delta}\right)=\\&\overline{V}_N-\tfrac{1-\delta^N}{1-\delta}\left(\gamma-\gamma^2\right)\omega_N\left(\mu_H-\mu_L\right)\end{aligned}$$

For the advocate:

$$u_H = \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) q_H\left(h_{k-1}\right) dh_{k-1}$$
$$= \left(\mu_H + (1-\gamma)\,\omega\right) \frac{1-\delta^N}{1-\delta}$$

$$u_L = \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_L\right) q_L\left(h_{k-1}\right) dh_{k-1}$$
$$= \left(\mu_L - \gamma\omega\right) \frac{1-\delta^N}{1-\delta}$$

$$U_N = \gamma u_L + (1-\gamma)\, u_H$$
$$= \left(\gamma\mu_L + (1-\gamma)\,\mu_H\right) \left(\frac{1-\delta^N}{1-\delta}\right)$$

QED

Proposition 3 *For small costs, the mechanism that maximizes (P) subject to (AICH) and (AICL) is given by*

$$q_L\left(h_{k-1}\right) = \mu_L + (1-\gamma)\,\widetilde{\omega}_N$$
$$q_H\left(h_{k-1}\right) = \mu_H - \gamma\widetilde{\omega}_N \Lambda\left(h_{k-1}\right)$$

$$\widetilde{\omega}_N = \frac{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)+c(1-\delta)}{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)}\omega_N$$

Constraint (AICH) is active at the optimum, and constraint (AICL) is slack. The planner's payoff under this contract is given by

$$V_N^c = \overline{V}_N - \left(\frac{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)+c(1-\delta)}{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)}\right)^2 \left(\frac{1-\delta^N}{1-\delta}\right) \omega_N \left(\gamma-\gamma^2\right)\left(\mu_H-\mu_L\right)$$

while the advocate's payoff is $U_N - c$.

Proof: The optimal mechanism is characterized by the solution to the following optimization problem:

$$\max_{q_L(.),q_H(.)} \quad \gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_L\right) \left(-q_L\left(h_{k-1}\right)^2 + 2\mu_L q_L\left(h_{k-1}\right) - \eta_L\right) dh_{k-1}$$
$$+ (1-\gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) \left(-q_H\left(h_{k-1}\right)^2 + 2\mu_H q_H\left(h_{k-1}\right) - \eta_H\right) dh_{k-1}$$

subject to

$$\sum_{h_k}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_L\right) \left(q_H\left(h_{k-1}\right) - q_L\left(h_{k-1}\right)\right) dh_{k-1} \le -\tfrac{c}{\gamma} \qquad \text{(AICH)}$$
$$\sum_{h_k}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) \left(q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right)\right) dh_{k-1} \le -\tfrac{c}{1-\gamma} \qquad \text{(AICL)}$$

To formulate these conditions, consider the Lagrangian

$$\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_L\right) \left(-q_L\left(h_{k-1}\right)^2 + 2\mu_L q_L\left(h_{k-1}\right) - \eta_L\right) dh_{k-1}$$
$$+ (1-\gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) \left(-q_H\left(h_{k-1}\right)^2 + 2\mu_H q_H\left(h_{k-1}\right) - \eta_H\right) dh_{k-1}$$
$$-\lambda_H \left[\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_L\right) \left(q_H\left(h_{k-1}\right) - q_L\left(h_{k-1}\right)\right) dh_{k-1} + \tfrac{c}{\gamma}\right]$$
$$-\lambda_L \left[\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) \left(q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right)\right) dh_{k-1} + \tfrac{c}{1-\gamma}\right]$$

The stationarity conditions conditions are:

$$q_L\left(h_{k-1}\right) = \mu_L + \tfrac{\lambda_H}{2\gamma} - \tfrac{\lambda_L}{2\gamma} \tfrac{f(h_{k-1}|\phi_H)}{f(h_{k-1}|\phi_L)}$$

$$q_H\left(h_{k-1}\right) = \mu_H - \tfrac{\lambda_H}{2(1-\gamma)} \tfrac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} + \tfrac{\lambda_L}{2(1-\gamma)}$$

Making our favorite substitution:

$$\lambda_H = 2\gamma(1-\gamma)\omega_H$$
$$\lambda_L = 2\gamma(1-\gamma)\omega_L$$

The complete set of Kuhn Tucker conditions can be written:

$$\text{Stat:} \quad q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_H - (1-\gamma)\omega_L \tfrac{f(h_{k-1}|\phi_H)}{f(h_{k-1}|\phi_L)}$$

$$q_H(h_{k-1}) = \mu_H - \gamma\omega_H \tfrac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} + \gamma\omega_L$$

$$\text{CS:} \quad \omega_H \left[\textstyle\sum_{k=1}^N \delta^{k-1} \displaystyle\int_{X_{k-1}} f(h_{k-1}|\phi_L)(q_H(h_{k-1}) - q_L(h_{k-1}))\, dh_{k-1} + \tfrac{c}{\gamma}\right] = 0$$

$$\omega_L \left[\textstyle\sum_{k=1}^N \delta^{k-1} \displaystyle\int_{X_{k-1}} f(h_{k-1}|\phi_H)(q_L(h_{k-1}) - q_H(h_{k-1}))\, dh_{k-1} + \tfrac{c}{1-\gamma}\right] = 0$$

$$\text{PF:} \quad \textstyle\sum_{k=1}^N \delta^{k-1} \displaystyle\int_{X_{k-1}} f(h_{k-1}|\phi_L)(q_H(h_{k-1}) - q_L(h_{k-1}))\, dh_{k-1} \leq -\tfrac{c}{\gamma}$$

$$\textstyle\sum_{k=1}^N \delta^{k-1} \displaystyle\int_{X_{k-1}} f(h_{k-1}|\phi_H)(q_L(h_{k-1}) - q_H(h_{k-1}))\, dh_{k-1} \leq -\tfrac{c}{1-\gamma}$$

$$\text{DF} \quad \omega_H \geq 0, \;\; \omega_L \geq 0$$

Consider first the $\omega_H > 0$, $\omega_L = 0$ (AICH Active, AICL Slack). In this case, the most pressing deviation that must be prevented is that of an uninformed advocate who represents himself as informed, and learning that the true distribution is ϕ_H. This case is most similar to the zero-cost case. Under this assumption, the KT conditions become

$$\text{Stat:} \quad q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_H$$
$$q_H(h_{k-1}) = \mu_H - \gamma\omega_H \tfrac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)}$$

$$\text{CS:} \quad \sum_{h_k}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L)(q_H(h_{k-1}) - q_L(h_{k-1}))\, dh_{k-1} = -\tfrac{c}{\gamma}$$

$$\text{PF:} \quad \sum_{h_k}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right)\left(q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right)\right) dh_{k-1} \leq -\tfrac{c}{1-\gamma}$$

$$\text{DF} \quad \omega_H \geq 0$$

Condition (PF) reduces as follows:

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right)\left(q_L\left(h_{k-1}\right) - q_H\left(h_{k-1}\right)\right) dh_{k-1} \leq -\frac{c}{1-\gamma}$$

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f\left(h_{k-1}|\phi_H\right)\left(\mu_L + (1-\gamma)\,\omega_H - \mu_H + \gamma\omega_H \frac{f\left(h_{k-1}|\phi_L\right)}{f\left(h_{k-1}|\phi_H\right)}\right) dh_{k-1} \leq -\frac{c}{1-\gamma}$$

$$\sum_{k=1}^{N} \delta^{k-1}\left(\omega_H - \left(\mu_H - \mu_L\right)\right) \leq -\frac{c}{1-\gamma}$$

$$\omega_H \leq \left(\mu_H - \mu_L\right) - \frac{c}{1-\gamma}\left(\frac{1-\delta}{1-\delta^N}\right)$$

Condition (CS) reduces as follows:

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)\left(q_H\left(h_{k-1}\right)-q_L\left(h_{k-1}\right)\right)dh_{k-1}=-\frac{c}{\gamma}$$

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_L\right)\left(\mu_H-\gamma\omega_H\frac{f\left(h_{k-1}|\phi_L\right)}{f\left(h_{k-1}|\phi_H\right)}-\left(\mu_L+\left(1-\gamma\right)\omega_H\right)\right)dh_{k-1}=-\frac{c}{\gamma}$$

$$\sum_{k=1}^{N}\delta^{k-1}\left(\mu_H-\gamma\omega_H\alpha^{k-1}-\mu_L-\left(1-\gamma\right)\omega_H\right)=-\frac{c}{\gamma}$$

$$\left(\mu_H-\mu_L\right)\frac{1-\delta^N}{1-\delta}-\omega_H\left(\gamma\frac{1-\left(\alpha\delta\right)^N}{1-\left(\alpha\delta\right)}+\left(1-\gamma\right)\frac{1-\left(\delta\right)^N}{1-\left(\delta\right)}\right)=-\frac{c}{\gamma}$$

$$\omega_H=\frac{\left(\mu_H-\mu_L\right)\frac{1-\delta^N}{1-\delta}+\frac{c}{\gamma}}{\left(\gamma\frac{1-\left(\alpha\delta\right)^N}{1-\left(\alpha\delta\right)}+\left(1-\gamma\right)\frac{1-\left(\delta\right)^N}{1-\left(\delta\right)}\right)}$$

$$\omega_H=\omega_N\left(\frac{\gamma\left(1-\delta^N\right)\left(\mu_H-\mu_L\right)+c\left(1-\delta\right)}{\gamma\left(1-\delta^N\right)\left(\mu_H-\mu_L\right)}\right)$$

Obviously condition (DF) holds. Therefore, provided that

$$\omega_N\left(\frac{\gamma\left(1-\delta^N\right)\left(\mu_H-\mu_L\right)+c\left(1-\delta\right)}{\gamma\left(1-\delta^N\right)\left(\mu_H-\mu_L\right)}\right)\leq\left(\mu_H-\mu_L\right)-\frac{c}{1-\gamma}\left(\frac{1-\delta}{1-\delta^N}\right)$$

we have a solution. As established previously, when $c=0$, this condition is satisfied. The left hand side of the inequality grows with c, while the right hand side shrinks. Both sides are linear in c, and therefore there is only one intersection. Call the intersection $\widetilde{c}$. For $c\leq\widetilde{c}$ the solution presented holds. Applying Lemma 3 yields:

$$
\begin{aligned}
V_N^C &= \overline{V}_N - \left(\gamma - \gamma^2\right)\omega_H^2\left(\frac{1-\delta^N}{1-\delta}(1-\gamma) + \gamma\frac{1-(\alpha\delta)^N}{1-\alpha\delta}\right) \\
&= \overline{V}_N - \left(\frac{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L) + c(1-\delta)}{\gamma\left(1-\delta^N\right)(\mu_H-\mu_L)}\right)^2 \frac{1-\delta^N}{1-\delta}\left(\gamma-\gamma^2\right)\omega_N(\mu_H-\mu_L)
\end{aligned}
$$

This proves Proposition 3

QED

Proposition 4 *The mechanism that maximizes (PD) subject to (ICD) is given by*

$$
\begin{gathered}
q_L^a = \mu_L + (1-\gamma)\theta_N \\
q_H^a(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})(a\theta_N) \\
q_L^i(h_{k-1}) = \mu_L \\
q_H^i(h_{k-1}) = q_H^a(h_{k-1})
\end{gathered}
$$

$$
\theta_N = \frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H-\mu_L)
$$

If the advisor is an advocate, his expected payoff under this mechanism is given by $U_N + \frac{1-\delta^N}{1-\delta}\gamma(1-\gamma)(1-a)\theta_N$.

Proof: We first consider a relaxed problem in which we impose only the following constraints:

$$
\begin{gathered}
\sum_{k=1}^N \delta^{k-1}\int_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^i(h_{k-1}) - q_L^a(h_{k-1})\right)dh_{k-1} \leq 0 \\
\sum_{k=1}^N \delta^{k-1}\int_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^a(h_{k-1}) - q_L^a(h_{k-1})\right)dh_{k-1} \leq 0
\end{gathered}
$$

This problem has a concave objective function and linear constraints. The KT conditions therefore characterize the solution. Making substitutions

$$
\begin{aligned}
\frac{\lambda_1}{2(1-a)a(1-\gamma)\gamma} &= \omega_1 \\
\frac{\lambda_2}{2a\gamma(1-\gamma)} &= \omega_2
\end{aligned}
$$

and assuming both constraints are binding, the KT conditions reduce to

$$q_L^i(h_{k-1}) = \mu_L$$
$$q_H^i(h_{k-1}) = \mu_H - a\left(\gamma\omega_1 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)}\right)$$
$$q_L^a(h_{k-1}) = \mu_L + (1-\gamma)\,\omega_2 + (1-\gamma)(1-a)\,\omega_1$$
$$q_H^a(h_{k-1}) = \mu_H - \gamma\omega_2 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)}$$

$$\omega_1, \omega_2 > 0$$

$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^i(h_{k-1}) - q_L^a(h_{k-1})\right) dh_{k-1} = 0$$
$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^a(h_{k-1}) - q_L^a(h_{k-1})\right) dh_{k-1} = 0$$

Simplifying constraint 1)

$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^i(h_{k-1}) - q_L^a(h_{k-1})\right) dh_{k-1} = 0$$
$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(\mu_H - a\gamma\omega_1 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} - \mu_L - (1-\gamma)\,\omega_2 - (1-\gamma)(1-a)\,\omega_1\right) dh_{k-1} = 0$$
$$\sum_{k=1}^{N} \delta^{k-1}\left(\mu_H - a\gamma\omega_1\alpha^{k-1} - \mu_L - (1-\gamma)\,\omega_2 - (1-\gamma)(1-a)\,\omega_1\right) = 0$$
$$\left(-(1-\gamma)(1-a)\frac{1-\delta^N}{1-\delta} - a\gamma\frac{1-(\alpha\delta)^N}{1-\alpha\delta}\right)\omega_1 - (1-\gamma)\frac{1-\delta^N}{1-\delta}\omega_2 + \frac{1-\delta^N}{1-\delta}(\mu_H - \mu_L) = 0$$

Simplifying constraint 2)

$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^a(h_{k-1}) - q_L^a(h_{k-1})\right) dh_{k-1} = 0$$
$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(\mu_H - \gamma\omega_2 \frac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} - \left(\mu_L + (1-\gamma)\,\omega_2 + (1-\gamma)(1-a)\,\omega_1\right)\right) dh_{k-1} = 0$$
$$\sum_{k=1}^{N} \delta^{k-1}\left(\mu_H - \gamma\omega_2\alpha^{k-1} + (1-a)\,\omega_3 - \mu_L - (1-\gamma)\,\omega_2 - (1-\gamma)(1-a)\,\omega_1\right) = 0$$
$$-\frac{1-\delta^N}{1-\delta}(1-\gamma)(1-a)\,\omega_1 - \left((1-\gamma)\frac{1-\delta^N}{1-\delta} + \gamma\frac{1-(\alpha\delta)^N}{1-\alpha\delta}\right)\omega_2 + \frac{1-\delta^N}{1-\delta}(\mu_H - \mu_L) = 0$$

It is straightforward to check that:

$$\omega_1 = \frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right) + (1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H - \mu_L)$$
$$\omega_2 = a\frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right) + (1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H - \mu_L)$$

and both of these values are positive. The solution to the relaxed problem is therefore:

$$q_L^i(h_{k-1}) = \mu_L$$
$$q_H^i(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})\left(a\frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H-\mu_L)\right)$$
$$q_L^a(h_{k-1}) = \mu_L + (1-\gamma)\frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H-\mu_L)$$
$$q_H^a(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})\,a\frac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}(\mu_H-\mu_L)$$

As in the proposition: The other constraints need to be verified. Because (H,i) is treated the same as a report of (H,a) a ϕ_H-advocate has no gain from reporting (H,i). For the same reason, the incentive constraint

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H(h_{k-1}) - q_L^a(h_{k-1})\right)dh_{k-1} = 0$$

Also ensures that

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f(h_{k-1}|\phi_L)\left(q_H^i(h_{k-1}) - q_L^a(h_{k-1})\right)dh_{k-1} = 0$$

If a ϕ_H advocate were to report L, he would certainly prefer to report (L,a) than (L,i). Therefore, if

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f(h_{k-1}|\phi_H)\left(q_L^a - q_H^a(h_{k-1})\right)dh_{k-1} \leq 0$$

then it also follows that

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f(h_{k-1}|\phi_H)\left(q_L^i - q_H^a(h_{k-1})\right)dh_{k-1} \leq 0$$

What remains to check, then is

$$\sum_{k=1}^{N}\delta^{k-1}\int\limits_{X_{k-1}} f(h_{k-1}|\phi_H)\left(q_L^a - q_H^a(h_{k-1})\right)dh_{k-1} \leq 0$$

To verify this condition, note

$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) q_H^a\left(h_{k-1}\right) dh_{k-1} =$$

$$\sum_{k=1}^{N} \delta^{k-1} \int\limits_{X_{k-1}} f\left(h_{k-1}|\phi_H\right) \left(\mu_H - \gamma \tfrac{f(h_{k-1}|\phi_L)}{f(h_{k-1}|\phi_H)} \left(a \tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)} \left(\mu_H - \mu_L\right)\right)\right) dh_{k-1} =$$

$$\sum_{k=1}^{N} \delta^{k-1} \left(\mu_H - \gamma \left(a \tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)} \left(\mu_H - \mu_L\right)\right)\right)$$

For the inequality to be valid,

$$\mu_H - \gamma \left(a \tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)} \left(\mu_H - \mu_L\right)\right) \geq$$
$$\mu_L + (1-\gamma) \tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)} \left(\mu_H - \mu_L\right)$$

$$1 \geq (1-\gamma) \tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)} + \gamma a \left(\tfrac{\left(1-\delta^N\right)(1-\alpha\delta)}{a\gamma(1-\delta)\left(1-(\alpha\delta)^N\right)+(1-\gamma)\left(1-\delta^N\right)(1-\alpha\delta)}\right)$$

$$1 \geq \frac{\frac{1-\delta^N}{1-\delta}(a\gamma+(1-\gamma))}{a\gamma\frac{1-(\alpha\delta)^N}{1-\alpha\delta}+(1-\gamma)\frac{1-\delta^N}{1-\delta}}$$

This holds because

$$\alpha \geq 1 \rightarrow \tfrac{1-(\alpha\delta)^N}{1-\alpha\delta} \geq \tfrac{1-\delta^N}{1-\delta}$$

To verify the advocate's payoff, note that the advocate is offered mechanism:

$$q_L^a = \mu_L + (1-\gamma) a\theta + (1-\gamma)(1-a)\theta$$
$$q_H^a\left(h_{k-1}\right) = \mu_H - \gamma\Lambda\left(h_{k-1}\right)(a\theta)$$

As previously established, a mechanism

$$q_L^a = \mu_L + (1-\gamma) a\theta$$
$$q_H^a\left(h_{k-1}\right) = \mu_H - \gamma\Lambda\left(h_{k-1}\right)(a\theta)$$

leaves the advocate with payoff U_N. Therefore, if the advisor is an advocate, and draws L, (probability γ) the planner selects a higher action in each period than under the no-rent mechanism. The advocate's expected payoff is therefore

$$U_N + \left(\frac{1-\delta^N}{1-\delta d}\right)\gamma(1-\gamma)(1-a)\theta$$

QED