A Dynamic Approach to the Theory of Effective Demand

by

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This paper attempts to resituate the theory of effective demand within a dynamic nonequilibrium context. Existing theories of effective demand, which derive from the works of Keynes and Kalecki, are generally posed in static equilibrium terms. That is to say, they serve to define a given level of output which corresponds to the equilibrium point between aggregate demand and supply. We propose to generalize this analysis in three ways. First, we will extend the analysis to encompass a dynamic (i.e. moving) short run path of output, rather than a merely static level. Second, we will show that this dynamic short run path need not imply an equilibrium analysis, since it can arise from either stochastically sustained cycles or deterministic limit cycles[1]. And third, we will prove that the preceding generalization of the theory of effective demand will allow us to solve a long standing problem in growth theory: namely, the puzzle surrounding the apparently intractable instability of warranted growth.

The issue of warranted growth has long been problematic. On the Keynesian side the question was originally taken up by Harrod and Domar, and on the Kaleckian side by Kalecki himself. All of them ended up concluding that the warranted path was highly unstable (Harrod, 1939; Domar, 1946; Kalecki, 1962). This conclusion has yet to be overthrown. We will show that the secret to this puzzle lies in the contradiction between the static short run level of output which results from the conventional formulation of effective demand theories, and the dynamic path of output which is the point of departure for considerations of warranted growth. This will allow us to show that the actual path of the economy does indeed gravitate around the warranted path in a cyclical sense.

We will also show that it is possible to derive two distinct types of growth cycles which follow quite natually from the short run and long run dynamics considered above: a fast growth cycle arising from the oscillations of growing aggregate supply around growing aggregate demand; and a slower growth cycle arising from the oscillations of the average supply path generated by the fast process around the corresponding growth path of capacity. These two intrinsic growth cycles appear to provide a natural foundation for the observed 3-5 yr. inventory cycle (since imbalances in aggregate demand and supply will show up as inventory fluctuations), and for the observed 7-11 yr. fixed capital cycle (van Duijn, 1983).

I. Fast and Slow Macrodynamics

Modern macrodynamics has traditionally focused on two quite different adjustment processes, each operating at its own characteristic range of speeds (Kaldor, 1960, 31-33): so-called short run adjustments in aggregate demand and supply in the face of excess demand or supply; and so-called long run adjustments in aggregate supply (output) and capacity in response to under- or overutilization of existing capacity.

The fairly fast adjustments in aggregate demand and supply are the most familiar ones. If these process are stable, in the sense that demand and supply end up gravitating around some balance point, one may assume that the two are roughly equal over some appropriate period of time. Such an assumption is implicit in the basic Keynesian and Kaleckian notions that aggregate demand and supply are equated by some "short run" (i.e. relatively fast) process. But this does not imply that aggregate demand and supply need ever be in some state of "equilibrium", because their average equality achieved over some interval of time is perfectly consistent with a process of perpetual oscillation (limit cycling) around a balance point[2]. Nor does it exclude the general possibility that this average equality defines a dynamic (i.e. growth) path rather than a mere static level of output and employment (Hicks, 105-106). Both of these points will play an important role in what follows.

The relatively fast process described above creates a rough equality between average aggregate demand and average supply, and hence between average aggregate investment and savings. But that portion of aggregate investment which is made up of fixed investment serves to expand the stock of fixed capital and hence to augment the (normal economic) capacity to produce[3]. It is natural, therefore, to ask how fixed investment responds to discrepancies between the average aggregate demand/supply generated over the fast process and the corresponding average level of aggregate capacity. Notice that this new adjustment process is implicitly slower, because it operates on the average result of the fast process. Moreover, the issue itself is intrinsically dynamic because capacity is continually being expanded by ongoing net investment. This is the second major adjustment process which has traditionally occupied macroeconomic theory.

The relatively slow adjustment process between the path of average output and the path of average capacity was the principal focus of the seminal contributions by Harrod and Domar. But their analysis of this second adjustment process produced one of the most enduring puzzles of modern macrodynamics. In effect, they came to the "rather astonishing" conclusion (Baumol 1959, p.44) that the normal feedback of the market would cause the actual growth rate to fly away from the particular growth rate needed to maintain a balance between capacity and actual production. What Harrod calls the "warranted" path and Domar the "required" path will in general be knife-edge unstable (Kregel, 1987, Vol 3, pp. 601-602). This unsettling result has continues to fascinate and frustrate economists to the present day (Sen, 1970, pp. 23, 227-230; Goodwin, 1986).

The central issue at hand is whether or not a long run **disequilibrium** adjustment process will either converge to the warranted path or oscillate around it, so that average aggregate output will roughly equal average aggregate capacity.

If such an average equality does hold, capacity utilization will fluctuate around its normal level, the actual profit rate will fluctuate around the normal (potential) profit rate, and the associated growth will be **internally driven,** in the sense that it arises from the reinvestment of profits even when there is no technical change (or population growth, since normal capacity growth does not imply the full employment of labor). Moreover, since the normal rate of profit and the wage share are inversely related for a given state of technology, the understanding of this latter relation becomes crucial to the analysis of the long term growth patterns of capitalist growth[4]. This is precisely why the inverse relation between wages and profits has always played such a crucial role in growth theory, in neoclassical and neoricardian economics, and in their classical and marxian antecedents[5]. It should be noted, however, that an average equality between output and production capacity does not imply that **labor** is fully employed, since the normal capacity of capital need not be adequate to the full employment of labor. Indeed, Goodwin (1967) has most elegantly shown that capitalist long run dynamics are perfectly consistent with a persistent unemployment[6].

On the other hand, if normal capacity utilization is not attainable, then it seems reasonable to displace the regulating role of profitability by the influence of other factors such as expectations, government intervention, population growth and technical change. This is exactly the direction taken by the bulk of growth theory, in the face of the apparently impossibility of normal capacity growth.

By far the most prevalent response to the Harrod-Domar problem of knife edge instability has been to try and spirit it away by simply assuming that the actual growth rate equals the warranted rate. Attention is then either shifted to the properties of this assumed path, or to the relation between this path and the natural rate of growth defined by population growth and the rate of growth of productivity. The Solow-Swan models are of this class (Sen, 1970, Introduction, Ch 10). So too is the famous ceiling/floor growth-cycle model of Hicks (1950) and the elegant nonlinear growth-cycle model by Goodwin (1967)[7].

The second most prevalent response to the Harrod-Domar paradox has been to treat growth as an "exogeneous trend" and concentrate instead on cyclical fluctuations around this given trend. The basic Lucas Rational Expectation models and Nordhaus Political Business Cycle models fall into this category (Mullineaux, 1984, Ch 3), as do the nonlinear cycle models from Kaldor (1940), Hicks (1950), and Goodwin (1951) (Mullineaux, 1984, Ch 2)[8]. The various versions of Kalecki's model also fall into this camp, though he does indicate that his provisional recourse to an exogenously given growth trend awaits a more satisfactory solution to the problem of growth (Kalecki, 1968, pp. 165-166; Steindl, 1981).

Multiplier-accelerator models form the third major branch of macroeconomic modelling since Harrod. Here, over certain parameter ranges one can get damped oscillations around a stationary path, and over other ranges one can get growth asymptotic to some non-warranted rate (still other plausible ranges yield explosive oscillations). But warranted growth is generally not possible in either the basic models or in more complex ones in which price, wage, money supply, and technology effects are added onto the multiplier-accelerator relation[9].

To sum up. Warranted growth is implicit in many approaches to macrodynamics. Yet such growth appears difficult to justify because of the apparently intractable instability of the warranted path. This difficulty has had a major effect on the growth and cycle literature, and has even convinced many theorists "that the warranted growth path is one place the economy will never be" (Goodwin, 1986, p. 209). The aim of this paper is to show that such a conclusion is, so to speak, quite unwarranted. The problem of warranted growth arises from the attempt to move beyond the short run considerations of the theory of effective demand to the long run considerations of output and capacity growth. We will try and show that the difficulty in explaining warranted growth has its roots in a contradiction between the static focus of conventional theories of effective demand and the dynamic focus inherent in the question of warranted growth. Harrod had hoped to create a 'new branch of economics' which would replace the static approach of Keynesian theory with a new approach formulated from the start in 'dynamic terms' (Harrod, cited in Kregel, 1980, pp. 101-102). Yet his famous instability result actually ended up inhibiting the study of dynamics. It is our contention that this ironic result came to pass because Harrod did not take his dynamic approach far enough. That is to say, that he did not begin from a dynamic analysis of the short run.

III. A Dynamic Approach to the Theory of Effective Demand.

The theory of effective demand centers around the (relatively fast) reactions of aggregate demand and supply to any imbalances between the two. If we define excess demand E as the (positive or negative) difference between aggregate demand and supply, then we may express this as the corresponding difference between aggregate investment demand I and aggregate savings S. Following Kalecki and Kaldor, we adopt a classical savings function (though this is not critical to the results), so that S

= sP where s = the propensity to save out of profits and P = aggregate profit on produced output. As defined here, produced profit P is profit net of interest-equivalent on capital advanced -- i.e. what Marx calls profit-of-enterprise[1]. This means that we must include the interest-equivalent as part of costs. Next, we write total investment as I = Ic + Iv + If, where Ic = investment in working capital (i.e. in raw materials and goods-in-progress), Iv = the change in the desired level of finished goods inventories (not to be confused with actual change in finished goods inventory levels), and If = investment in fixed capital. This division of total investment into several components is standard, although not all authors interpret it in the same way[10]. Iv represents the portion of final goods which would be desired as additions to final goods inventories even when demand and supply are balanced (E=0). When E=0, actual inventory levels will equal desired levels (the latter depending on the particular specification of Iv). On the other hand, when demand and supply are not balanced, actual final goods inventory levels will depart from the desired levels, production plans will be revised in response to the discrepancy, and input levels will therefore also adjust. It is this latter reaction in the use of circulating capital that is captured in Ic. Taken together, Ic and Iv represent the "inventory adjustment" portion of total investment.

1. $E = I - S = Ic + Iv + If - sP$

We now turn to the **effects** of Ic, Iv, and If on other variables. The **determinants** of these same investment components will be treated later.

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[1]If r = the rate of profit, i = the interest rate, and K = the money value of capital advanced, then re = r-i = the rate of profit-of-enterprise and P = re K = (r - i)K = the mass of profit-of-enterprise.

Investment in fixed capital results in a change in aggregate capacity, since changing the stock of fixed capital also serves to change the capacity to produce (i.e. to potential output). This link was at the heart of the issues addressed by Harrod and Domar. In the same way, investment in circulating capital leads to a change in the level of production, because any planned change in the level of production will require a corresponding change in the use of raw materials and labor power required. If purchases of these additional circulating inputs are strongly connected to their use, then investment in circulating capital will be linked to the change in the level of production. This is an empirically sound assumption, **and is in fact the basis of Leontief's input-output analysis** (since the observed input-output coefficients are the ratios of purchased inputs to outputs).

Notice that there is an exact parallel here between the Harrodian assumption that fixed investment purchases lead to an increase in the capacity to produce and the Ricardo-Marx-Leontief assumption that circulating investment purchases lead to an increase in the level of production. Moreover, just as the former does not imply that the capacity will actually be utilized, so too the latter does not imply that the output will be actually sold. Indeed, equation 1 above tells us that aggregate output and demand generally do not balance. Finally, it should be noted that whereas the link between circulating capital and output is algebraically similar to some formulations of an "accelerator relation", it is conceptually quite different. This is because our input-to-output relation implies that the change in output depends on the level of circulating investment, whereas an accelerator relation implies that the level of investment depends on the (past or future) change in output[11]. We will turn to the question of investment functions in the next section.

Investment in final goods inventories is different from the above two, because it represents a virtual (benchmark) flow rather than a real one. As we noted earlier, some allowance has to be made for changes in the desired inventory level even when demand and supply balance. For example, if the ratio of desired inventories is proportional to sales, then in a growing economy some portion of output corresponds merely to this desired additions to stocks, and this must be allowed for either as a nominal "investment demand", or as a deduction from total product so as to arrive at the effectively available supply. Either way, it will show up as one of the determinants of excess demand E.

Let us now formalize the effects of fixed and circulating capital investments. Let the notation P' stand for the change in P, etc. We can then express the effect of circulating capital investment Ic on aggregate output Q and (through the profit margin) on aggregate produced profit-of-enterprise P. Let C = total circulating capital, Q = aggregate output, Ic =C'

2'. $Q' = (1/k)C' = (1/k)Ic$

2. $P' = m \cdot C' = m \cdot Ic$, $1+m = 1/k$

where m = the profit margin on prime costs (circulating capital, including the interest-equivalent of capital advanced), and k = prime costs per unit output (average variable cost)[12]. The profit-margin m will play an important role at a later point.

Next, consider the effect of fixed capital investment on capacity. Let Kf = stock of fixed capital, N = aggregate capacity, If = Kf.'

3. $N' = q \cdot Kf' = q \cdot If$

where q = the capacity-capital ratio[13].

Lastly, we define capacity utilization u as the ratio of output Q to capacity N, so that u=1 corresponds to normal capacity utilization. Then over- or under-utilization of capacity corresponds to positive or negative levels, respectively, of excess utilization X.

4. $X \equiv u-1 \equiv (Q-N)/N$, where $u = Q/N$ = capacity utilization rate

Equations 1-2 above represent the core of the fast adjustment ("short run") process centering around on the interactions of aggregate demand and supply. Equations 3-4 in turn represent the core of the slow adjustment ("long run") process centering around the interactions of aggregate supply and capacity. In order to proceed any further, we need to now consider the determinants (as opposed to the effects) of each of the three investment components, first in the short run and then in the long run.

1. The Fast Adjustment Process

1. $E = I - S \equiv Ic + Iv + If - sP$
2. $P' = m \cdot Kc' = m \cdot Ic$

To fill out the picture of the fast adjustment process, we must supplement the core equations 1-2 with specifications of the "short run" determinants of Ic, Iv, and If. **It is here that the question of a dynamic versus a static specification becomes crucial.** A dynamic specification is one in which allowance is made for the possibility that variables may be moving over time, so that all adjustments take place relative to any trends in these variables. Such relative adjustments must therefore either be in terms of changes in ratios of variables, or in terms of changes in growth rates.

By contrast, static specifications tend to focus on the level, rather than the path, of the main variable, so that adjustments are posed in terms of changes in absolute levels rather that relative ones[14]. Not surprisingly, static specifications tend to yield static results.

Conventional formulations of the theory of effective demand yield static results because they are implicitly specified in static terms. To show this, we will derive the standard Kaleckian/Keynesian short run equilibrium by closing our core equations in a static way. Fixed investment will be assumed to be constant in the short run, on the usual grounds. Desired final goods inventory levels will be assumed constant in the short run, so that ex ante inventory investment (which represents the change in the desired levels) will be zero.

5. If = constant
6. Iv = 0

Now consider possible reactions of the system to a positive or negative level of excess demand. The basic Kaleckian and Keynesian approach is to assume that production levels will adjust whenever aggregate demand and supply do not balance. This is because realized profits P+E will differ from produced profits when $E \neq 0$, and if the margin of produced profit on costs (the degree of "markup")[15] does not vary with excess demand (because the relation of costs to prices does not change), produced profit will equal the normal profit, so that positive or negative excess demand will be a measure of positive or negative excess profits. On this basis, Q' = F(E). But from equation 2' above, Q' = (1/k)Ic, since any change in production requires a prior (positive or negative) investment in circulating capital. Therefore, Ic = f(E). We will assume f(E) to be linear.

7. $Ic = h \cdot E, \; 0<h<1$

Substituting equations 5-7 into equation 1, and then substituting P' for Ic from equation 2, we get

$$Ic/h = Ic + If - sP$$
$$P'/mh = P'/m + If - sP$$

8. $P' = [smh/(1-h)]\cdot[If/s - P]$

The first term in brackets is positive because s, m, and h are all positive, and h<1. The term If/s is constant in the short run, which means that whenever P is greater than this term, P' will be negative and P will fall back, while whenever P is smaller than this term P' will be positive and P will rise towards it. This is a monotonic process which converges to the familiar short run equilibrium level of profit in the Kaleckian and Keynesian model (with the usual "multiplier" = 1/s).

9. $P^* = If/s$

Since P* is constant in the short run, P*' = 0, which from equation 2 implies that Ic* = 0, which in turn from equation 7 implies E* = 0. Actual inventory levels will also be constant in equilibrium, since E* = 0.

10. $E^* = 0$ and $Ic^* = 0$

We see therefore that the familiar static results of Kaleckian/Keynesian economics are merely the consequences of having implicitly specified the adjustment process in static terms. **Growth then appears as something external to the "short run"**[16].

It was Harrod's intention to supplant this traditional static approach with a new one formulated from the start in 'dynamic terms'. In order to do so, he begins by translating the short run condition that investment = savings into a long run statement about the relation between the actual rate of growth and the warranted rate, only to find that the apparently stable short run equilibrium implies an apparently unstable long run equilibrium.

A central contention of this paper is that Harrod did not take his dynamic approach far enough. Or, more precisely, he did not move to a dynamic framework early enough in his analysis early. Harrod begins from the short run equilibrium of Keynesian economics. But, as we have seen, this short run equilibrium is inherently static. Thus his "new" dynamic formulation is in fact an inconsistent mixture of short run statics and long run dynamics. This suggests that in order to formulate a consistent dynamic approach, we must reformulate the theory of effective demand itself. Hicks has pointed out, for instance, that the general solution to the equations of short run balance involves a time path in output, employment, and profits (Hicks, 1965, Ch X, pp. 105-106). This can be seen by noting that when $E=0$ in equation 1, total investment $I = Ic+Iv+If$ = total savings S, so that if $Ic > 0$ then from equation 2 $P' = m \cdot Ic > 0$, which means that produced profit and hence output is growing over time. Conversely, only if $Ic = 0$ do we get a static solution.

Kalecki and Keynes implicitly select the static solution to the general time path defined by short run equilibrium. But if, in the spirit of Harrod, we are to dynamize the short run theory of effective demand, then like Harrod we must do two things: show that a short run dynamic path exists; and show that it is stable.

The first step in this proposed reformulation is to recall that a dynamic specification requires that adjustments be posed in trend-relative terms, that is, as changes in either ratios of variables or in their growth rates. Let us therefore begin by first expressing all variables relative the level of produced profit P.

Let $e \equiv E/P$, $ac \equiv Ic/P$, $av \equiv Iv/P$, and $af \equiv If/P$, where the latter three terms can be interpreted as the average aggregate "propensities to invest" in, or "accumulation ratios" of, the corresponding three types of ex ante investments. Our fast adjustment core equations 1-2 then become

11. $e = ac + av + af - s$

12. $P'/P = m \cdot ac$

The next step is to write dynamic analogues to the previously derived static investment functions. Where static theory takes the level of fixed investment If as constant in the short run, we will take the corresponding accumulation ratio af to be approximately fixed, on the grounds that it is a slowly changing variable in the short run. Where static theory takes the desired level of final goods inventories to be fixed, we will take the corresponding ratio v of desired inventories to circulating capital C to be fixed. Since inventory investment is the change in desired inventories, $Iv = v \cdot C' = v \cdot Ic$, so that $av \equiv Iv/P = v \cdot Ic/P = v \cdot ac$.

13. $af = \text{constant}$

14. $av = v \cdot ac$

The dynamic specification of our circulating capital reaction function requires a bit more work. Recall that in the static model it was assumed that the level of circulating capital investment changes in response to the level of excess profit, and that the level of the latter is measured by the level of excess demand E if the margin of produced profit over costs (the "markup") does not vary with E. A dynamic equivalent of these connections would be to assume that the accumulation ratio of (the propensity to invest in) circulating capital changes in in response to **the excess profit margin μ** (the excess of the realized profit margin on prime costs C over the normal margin). This amounts to assuming that the **trend** of planned production changes when demand and supply do not balance. Thus $ac' = f(\mu)$.

15. $ac' = h \cdot \mu$, $h>0$

Equations 12-15 form a dynamic analogue to the static model of effective demand. The properties of the resulting system will then depend on how we specify the determinants of the excess profit margin μ.

Suppose we retain our earlier assumption that the ratio of costs to prices does not vary with excess demand, so that the profit margin does not vary over the cycle (see equation 7 above). Then excess profit is the same as excess demand, and the excess profit margin $\mu = E/C = (E/P) \cdot (P/C) = e \cdot m$.

16. $\mu = m \cdot e$ when the markup m is constant

Equation 16 completes our short run dynamic system. Substituting equations 13-14 into equation 11, we get $e = ac(1+v) + af - s$, and since ak and s are constant in the short run, $e' = ac'(1+v)$. Substituting equation 15 into this gives

17. $e' = H \cdot \mu$, where $H=h(1+v)$

and combining equations 16-17 gives

18. $e' = Hm \cdot e$, $H > 0$.

Equation 18 is a linear first order differential equation which describes a system with a short run positive feedback loop between the level of relative excess demand e and its rate of change e'. It is exactly analogous to the Harrod-Domar long run positive feedback loop between the level of capacity utilization and its rate of change. **And like the latter, the former is also knife-edge unstable around its corresponding short run dynamic** balance path. A rise of e above zero (excess demand) will make e'>0, so that e will rise still further, and so on. Similarly, a fall in e below zero (excess supply) will reduce it still further, etc.

In the light of the apparent instability of short run equilibrium growth, it is natural to ask whether other factors might alter this result. In an earlier paper, I began from the premise that the basic accumulation reaction function in equation 15 should be modified to allow for the negative effects of debt service commitments. On this basis I was able to show that while an excess of investment over savings showed up in the commodity market as a growth accelerating excess demand, the corresponding debt service on the borrowing which fueled this excess demand showed up as a growth decelerating decline in the liquidity of firms. The net result was to stabilize accumulation around a dynamic short run path defined by e = 0 and characterized by a constant rate of growth of output. When subject to random perturbations, this model yielded a stochastically sustained cycle in which the system perpetually cycled around the balance path (Shaikh, 1988).

In this paper I show that there exists an alternate mechanism by which the apparent instability of short run equilibrium growth may be contained. This apparent instability was derived on the assumption of a cyclically constant profit margin. But it is a well established empirical fact the profit margin varies systematically over the business cycle. In the early stages of a boom, prices rise faster than costs and the profit margin rises. However, as the boom proceeds, costs begin to accelerate and eventually overtake prices, thus reducing profit margins. The opposite pattern holds in the bust (Klein and Moore, 1981). To quote Wesley Clair Mitchell,

The very conditions that make business profitable

> gradually evolve conditions that threaten a reduction of profits. When the increase in business ... taxes the productive capacity of the existing industrial equipment, the early decline of supplementary costs per unit of output comes gradually to a standstill. Meanwhile, ... active bidding among business enterprises for materials, labor, and loans funds ... sends up their prices. At the same time the poorer parts of the industrial equipment are brought back into use, the efficiency of labor declines, and the incidental wastes of management rise. Thus the prime costs of doing business become heavier. After these processes have been running cumulatively for awhile, it becomes difficult to advance selling prices fast enough to avoid a reduction of profits by the encroachment of costs (Mitchell, 1913, cited in Klein and Moore, 1981, p. 56).

To formalize the idea of changing ratios of costs to prices, we need to replace equation 16 (which was predicated on a constant cost/price ratio) with a more general formulation.

We will take the price level of output to be the numeraire, so that all quantities are in real terms. Then real aggregate excess demand is $E = D - Q$, where D = real aggregate demand and Q = real output. Similarly, real realized aggregate profit $PR = D - pC$, where C = real inputs, and p = input costs relative to output prices. Now let us define pn = some normal level of relative input costs (corresponding to $E = 0$). Then real realized profits PR may be written as

$$PR \equiv D - pC = (D - Q) + (Q - pn \cdot C) + (pn - p)C$$

$PR = E + P + (pn - p)C$, where $P \equiv Q - pn \cdot C$ = <u>normal</u> produced profit

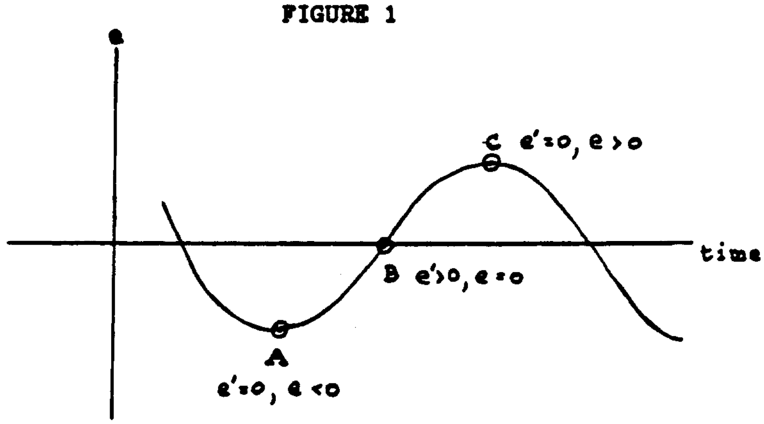
$$\text{Excess Profits} \equiv PR - P = E + (pn - p)C$$

μ = excess profit margin $\equiv (PR - P)/C = (E/P)(P/C) + (pn - p)$

19. $\mu = e \cdot m + (pn - p)$, where $m \equiv P/C$ = <u>normal</u> profit margin

It now remains to model the behavior of relative input costs p over the various phases of the fast cycle. According to our formulation, these phases will consist of alternating episodes of positive and negative excess demand. At the beginning of an upturn, costs will still be falling relative to prices. But as the recovery turns into a boom, costs will overtake prices so that relative costs will begin to rise. Consider the upturn phase of the stylized cycle in Figure 1 below: point A marks the beginning of the recovery, at a point which the cycle has bottomed out ($e'=0$) but there is still excess supply ($e<0$). Relative costs are falling here, so that $p'<0$ at this point. Point B marks the point at which the cycle passes through the transitory point at which aggregate demand and supply balance ($e=0$) and hence $p'=0$. And point C marks the top of the boom, at which the cycle has peaked ($e'=0$) but there is still excess demand ($e>0$). Here, relative costs are rising so that $p'>0$. A similar partition can obviously be constructed for the downturn phase.

FIGURE 1



It is evident that the phases of the stylized cycle are characterized by varying levels of e and e'. According, we may generally consider a relative cost reaction function of the form $p' = f(e, e')$, subject to the requirements delineated above.

One simple function which satisfies the above conditions is

20. $p' = ae + b(e)\cdot e'$, where $b(e) = b\cdot e^2$

The coefficient b(e) is made an increasing function of the <u>size</u> of excess demand[17] to capture the idea that the influence of the rate of change of excess demand itself depends on the tightness of the market: when e is small, the rate of change of e is of no great consequence; but when e is large, then the impact of the rate of change of e is correspondingly more serious. It is easily shown that equation 21 satisfies the requirements for p' at the various phases of the cycle.

Equations 12-15 from our previous system, and equations 19-20 (which replace the previous equation 16) form a new dynamical system. As we noted previously, equations 11, 13-15 can be combined to derive $e' = H\mu$ (equation 17 above), so that

21. $e'' = H\mu' = H(m\cdot e' - p')$ from equation 19
$= Hme' - Hae - H(be^2)\cdot e'$ from equation 20

22. $e'' + H(be^2 - m)\cdot e' + Hae = 0$

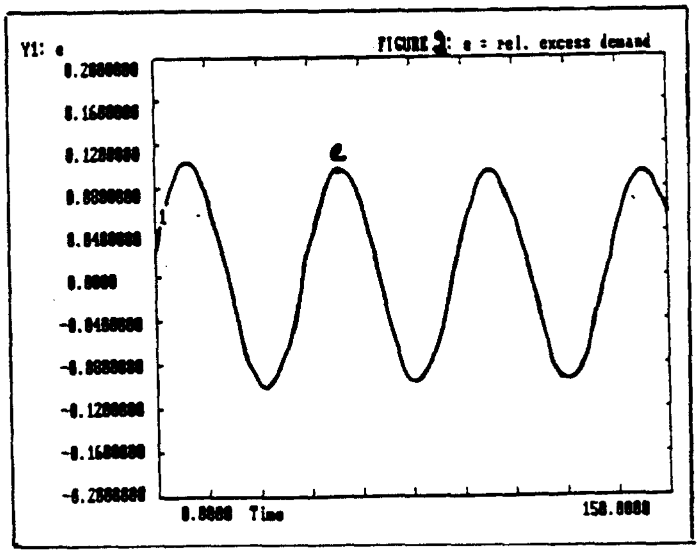
Equation 22 is the reduced form of our new dynamical system. It can be shown that it is also a particular expression of a general second order nonlinear differential equation known as the Lienard Equation (see the Appendix for the proof), so that <u>it has a unique stable limit cycle around the critical point e = 0</u> (Lakin and Sanchez, 1970, section 4.4). That is to say, the system perpetually cycles around the point at which aggregate demand and supply balance, alternately overshooting and undershooting it. The system never settles into a "short run equilibrium". And yet, aggregate demand and supply balance on average, precisely because they are subject to mutually offsetting errors. The order in the system is expressed in-and-through its disorder.

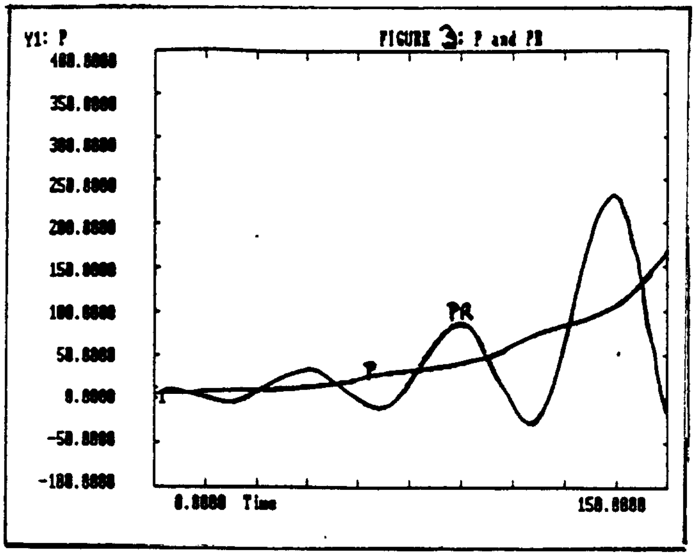
The fact that the system cycles around $e=0$ implies investment approximately equals savings, over an average cycle.

23. $I \approx S \dashrightarrow ac(1+v) + af \approx s$ (from equations 11, 13, 14)

Secondly, $e \approx 0$ implies $\mu \approx 0$, so that the actual profit margin $m+\mu$ fluctuates around the normal profit margin m, rising in the boom and falling in the bust. And thirdly, since $ac \approx (s - af)/(1+v)$ from equation 23, and $P'/P = mac$ from equation 12, we get the result that the gravitational path around which realized and produced profit perpetually oscillate is a <u>endogeneously generated growth path</u>, provided the propensity to invest in fixed capital af < average aggregate propensity to save s (because then ac >0). Lastly, $e \approx 0$ implies that the actual inventory/sales ratio will fluctuates the desired ratio v.

Figures 2-3 below show the simulation results of the model for the indicated values of the parameters. Figures 2 depicts the pure limit cycle in e, while Figure 3 shows the corresponding path of realized and produced profits.





The above approach opens up a new dynamical perspective on the theory of effective demand. Its properties provide an interesting contrast to those of the Kaleckian and Keynesian theories of effective demand. For instance, these latter theories predict that a rise in the propensity to consume (a fall in the propensity to save) is beneficial in the short run because it stimulates aggregate demand and hence output and employment. Yet within our new dynamic model, a rise in in the propensity to consume has two contradictory effects which operate at different speeds. It initially raises excess demand by raising consumption demand, which at first raises the average level of output and employment above its trend level. This is the "Keynesian" effect. But since a rise in the propensity to consume is a drop in the propensity to save s, it lowers the short run trend rate of growth $P^{*\prime}/P^{*} = m \cdot ac^{*} = (af - s)/(1+v)$. This is the Classical effect. Since the system ends up gravitating around a new lower rate of growth, the eventual effect is to lower the level of output below what it would otherwise have been. A rise in the proportion of government deficit spending has the same effect, other things being equal, because it is equivalent to a rise in the average propensity to consume[18].

2. The Slow Adjustment Process

Perhaps the most remarkable thing about a dynamic solution to the fast adjustment process is that it opens up a host of natural solutions to the famous puzzle of the Harrod-Domar knife edge. To see how this works, let us first reproduce some of our previously derived equations.

3. $N' = q \cdot Kf' = q \cdot If$

where Q = aggregate output, C = prime costs, Ic = C' = investment in circulating capital, N = aggregate capacity, Kf = stock of fixed capital, If = Kf'= investment in fixed capital, and q = N/Kf = the (constant) capacity-capital ratio.

4. $X \equiv u-1 \equiv (Q-N)/N$

where u = Q/N = the actual capacity utilization rate, and the normal rate is defined as 1. Thus X is the positive or negative degree of overutilization of capacity.

12. $P'/P = m \cdot ac$

Finally, since over the average result of the fast adjustment process is $e \approx 0$, we can write from equations 11 and 14

23. $ac(1+v) + af \approx s$ (average result in the short run)

Combining equations 3-4,

24. $N'/N = (q/N) \cdot If = If/Kf = (If/P) \cdot (P/Kf) = af \cdot r = af \cdot rn \cdot u$

where r = P/Kf = the actual rate of profit on fixed capital, rn = r/u = the normal capacity rate of profit on fixed capital (which we will take as constant over the long run, since we are not considering technical change and long run distributional variations here).

We have already noted that over an average fast adjustment cycle the excess profit margin $\mu \approx 0$, so that the actual profit margin $m+\mu \approx m$ = the short run normal profit margin, which we took to be given in the short run. Then since $m \equiv P/C$ and $Q \equiv P + C$, a constant m implies a constant profit share P/Q so that $P'/P = Q'/Q$. Thus equation 12 becomes

25. $Q'/Q = m \cdot ac$

In the fast adjustment process, the average propensity to invest in fixed capital af was taken to be approximately constant, on the grounds that it was a slow variable. Now, over the slow adjustment process, af is a variable, and it seems plausible that it would react to $X \equiv u-1$, the positive or negative degree of overutilization of capacity. With this, we can show that the secret to the apparent dynamic instability of the long run warranted path actually lies in hidden in the analysis of the short run. Harrod began from the **static** solution to the short run problem, and found that the long run dynamic path is then knife edge unstable. We can show, on the other hand, that if we begin from a **dynamic** solution to short run balance, then the long run path is stable.

Equations 23-25 enable us to see why a **dynamic** solution to the short run adjustment process unlocks the secret of the warranted path puzzle. In effect, any dynamic short run path in which $e \approx 0$ implies that total investment = total savings, which in turn implies that the propensities to invest in circulating capital, inventories, and fixed capital must all sum to the given propensity to save. But $av = v \cdot ac$, so that the short run restriction on the sum of investment propensities really implies the circulating and fixed investment propensities are inversely related, as is indicated by equation 23 above. But equation 24 tells us that the growth rate of capacity is positively related to fixed capital propensity, while equation 25 tells us that the growth rate of output is proportional is positively related to circulating capital propensity. This means that any long run adjustment process which raises the fixed capital propensity af (say because capacity utilization is above normal) will also lower the circulating capital propensity ac. The former effect will raise the growth rate of capacity, while the latter will lower the growth rate of output, and these two acting in concert will serve to lower the level of capacity utilization back toward normal. The opposite movement would occur if the capacity utilization was initially below normal. The end result is a process which is stable around the warranted path.

Let us now formalize the above argument. The fixed investment propensity af is assumed to react to the degree of over- or under-utilization of capacity.

25. $af' = k \cdot X = k \cdot (u-1)$

To complete the picture, we need to supplement the above fixed capital accumulation reaction function with an expression for X'. From $u \equiv Q/N$,

$$u'/u = Q'/Q - N'/N = Q'/Q - af \cdot rn \cdot u, \text{ from equation 24.}$$
$$u'/u = Q'/Q - af \cdot rn \cdot u = P'/P - af \cdot rn \cdot u = mac - af \cdot rn \cdot u$$

since $P'/P = mac$ from equation 2. Substituting for ac from equation 23, and recalling that $X \equiv u-1$

$$u'/u = X'/(1+X) = (s-af)/(1+v) - af \cdot rn \cdot u$$

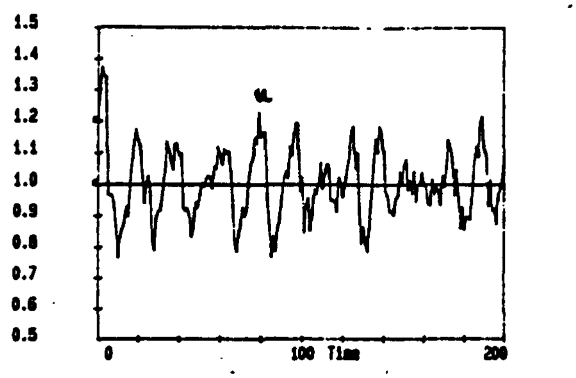
26. $X' = [(s-af)/(1+v)] \cdot (1+X) - af \cdot rn \cdot (1+X)^2$

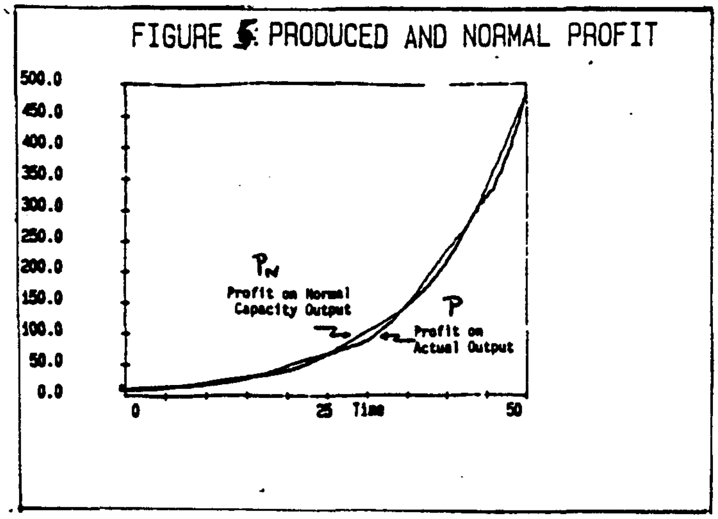
Equations 25-26 form a nonlinear dynamical system which is stable around u = 1. In other words, it is stable around the

Harrodian warranted path. It can be shown that for all plausible values of the reaction coefficient k, the stability is oscillatory as long as the system is at all profitable. Moreover, when subject to random shocks, actual capacity utilization u oscillates endlessly around the point u = 1, alternately overshooting and undershooting this point but never settling down to it. Finally, the corresponding critical value of the fixed capital investment propensity af is af* = ms/(m+rn) >0, which along with the fact that u ≈ 1, implies from equation 24 that the system follows a growth path (as we already know from fact that it is stable around the warranted path). **The end result is a slow fixed capital cycle which complements the fast inventory cycle previously derived in section III.1.**[19]

Figure 4 below shows the simulation results for the path of capacity utilization u, and Figure 5 shows the corresponding paths of actual produced profit and normal produced profit, both with with random noise added to the system.

FIGURE 4:CAPACITY UTILIZATION





3. Summary and Conclusions

This paper is an attempt to wed Kalecki's analyis of the business cycle to Harrod's analysis of dynamic paths. Kalecki argued that growth had "no independent entity" from cycles, and that the proper way to proceed was to formulate the problem "in such a way as to yield the trend cum business-cycle". Yet in spite of his repeated attempts to extend his cycle analysis to the issue of growth, he never quite found a formulation which he considered satisfactory (Kalecki, 1968B, p. 78). From the other side, Harrod tried to extend his analysis of growth to encompass the theory of cycles, but he too remained frustrated (Kregel, 1980, pp. 99-102). In the end, a satisfactory synthesis of the theories of growth and cycles seemed to elude them both.

It has been the aim of this paper to show that the above synthesis is possible, and that it can be achieved precisely by integrating Kalecki's treatment of endogeneous cycles with Harrod's treatment of endogeneous growth. To this end, we have shown that one can formulate a nonequilibrium theory of effective demand in which aggregate demand and supply trace out a dynamic "short run" growth path as they perpetually cycle around each other, and in which the resulting average output and capacity themselves trace out a dynamic "warranted" as they cycle around each other. The combined dynamic consists of a fast cycle marked by mutually offsetting imbalances of demand and supply (which will be therefore reflected in corresponding inventory fluctuations), and a slower medium cycle consisting of mutually offsetting imbalances of output and capacity (reflected in corresponding fluctuations in capacity utilization). Most interestingly, a rise in a factor such as the proportion of government deficit spending can be shown to have an <u>initial</u> Keynesian "pumping" effect on the <u>level</u> of output and employment, attended by a corresponding Classical "drag" effect on the <u>rate of growth</u> of output and employment, so that the <u>eventual</u> effect is to lower the level of output and employment below what it would otherwise have been.

APPENDIX

The nonlinear dynamical system in equation 22 can be written in the form

22.' $e'' + f(e)e' + g(e) = 0$
where $g(e) = Hae$, with constants $H, a > 0$
$f(e) = H(be^2 - m)$, with constants $b, m > 0$

Lakin and Sanchez (1970) list six conditions which ensure a unique limit cycle for such a (Lienard) equation.
i. $g(e) = - g(e)$ ii. $eg(e) > 0$ for $x =/ 0$
iii. $f(e) = f(-e)$ iv. $f(0) < 0$
v. $f(u)du = F(u) \dashrightarrow \infty$ as $e \dashrightarrow \infty$
vi. $F(e) = 0$ has a unique positive root $e = n$

Conditions i-iv are easily verified. Condition vi is also easily verified, since $f(e)$ has roots $\pm(m/b)^2$, so that it has a unique positive root $n = (m/b)^2$. This leaves condition v, which is also satisfied since

$$H(bu^2 - m)du = H(b^u 3/3 - mu)| = H(be^3/3 - me)$$
$$= He(e^2/3 - m) = F(e) \dashrightarrow \infty \text{ as } e \dashrightarrow \infty$$

It follows that the equation system 22' has a <u>unique stable limit cycle</u> (Lakin and Sanchez, 1970, pp. 92-93).

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FOOTNOTES

1.Deterministic limit cycles arise from local instability which is reversed by bounding forces. Stochastically sustained cycles can arise from (generally nonlinear) stable oscillatory solutions which are kept alive by random pertubations representing the turbulence inherent in an uncertain and fluctuating economic environment.

2.Goodwin's famous Lotka-Volterra limit cycle model of the relation between the wage share and the unemployment rate yields constant **average** values for these variable even though their **actual** levels perpetually fluctuate around these average levels (Goodwin, 1986, p.207).

3.Production capacity as defined here refers to economic, not engineering, capacity.

4.The investment-savings equality brought about in the fast process may be expressed as a relation between the rate of growth of fixed capital, the capacity utilization, and the normal rate of profit. Let $I = S = s \cdot P$, where s = the propensity to save out of profits, and P = aggregate profits. Since actual profits $P = u \cdot Pn$, where u = the rate of capacity utilization and Pn = the normal capacity level of profit, then by dividing through by the aggregate capital stock K, we get $g \equiv I/K = s \cdot u \cdot (Pn/K) = s \cdot u \cdot rn$, where g = the rate of growth of capital and rn = the normal rate of profit. It is evident then that if some process results in an average u = 1, then the resulting long run rate of accumulation $g^* = s \cdot rn$ is regulated by the wage share and technology which lie behind the normal rate of profit rn.

5.Smith, Ricardo and Marx typically abstract from supply/demand and supply/capacity variations in order to focus on the long term patterns produced by the effects of factors such as technical change, population growth, and fertility of land, on the relation between real wages and the normal rate of profit. Sraffa's inverse relation between the wage share and the uniform rate of profit is a direct extension of Ricardo's problematic, and is predicated on the implicit assumption that the so-called uniform rate of profit expressed a normal rate of capacity utilization (if it did not, then the increased effective demand consequent to a rise in the wage share might conceivably raise the rate of capacity utilization u more than the increased wage costs served to lower the normal rate of profit rn, so that the actual rate of profit $r = rn \cdot u$ would actually rise). See Garegnani, (1978), p.183.

6.Goodwin (1967) has shown that the interaction between the growth of real wages and the level of unemployment is perfectly capable of producing perpetual oscillations around a stable level of unemployment. Thus the notion that supply and demand balance over a fast process, and that supply and capacity balance over a slow process, need not carry with it any notion that labor is ever fully employed, even in the longest of runs.

7.Goodwin (1967) assumes a constant capital-"output" ratio because of Harrod-Neutral technical change. But such technical change only yields a constant ratio of capital to potential output (capacity), since it tells us nothing about the use of this capacity. Thus Goodwin implicitly assumes that output is equal to capacity, which is equivalent to assuming that the actual growth rate is equal to the warranted rate. This warranted rate is made flexible linking it to a tradeoff between the unemployment rate and the growth rate of real wages (Gandolfo, 1985, pp. 474-481). The end result is that the warranted rate ends up fluctuating around the exogeneously given natural growth rate in such a way that the two are equal over any one complete cycle. To derive this last result, note that Goodwin assumes that all profits P are invested, so that the actual (and warranted) rate of growth of capital $\equiv g =$ the rate of profit $\equiv r = P/K$. The natural growth rate, on the other hand, is $gn \equiv \alpha+\beta$, where $\alpha =$ the growth rate of productivity, and $\beta =$ the growth rate of labor supply. But $r \equiv P/K= (P/Y)\cdot(K/Y)= (1 - W/Y)\cdot(K/Y)= (1-u)k$, where $u \equiv W/Y =$ the wage share and $k =$ the given capital-output ratio. Substituting the average value of u over one complete cycle (Gandolfo, 1985, pp. 481, 478) yields $r = \alpha + \beta$, which is the same thing as $g = gn$.

8.Hicks (1950) bounds the unstable parameter range of a multiplier-accelerator model with exogenously given ceilings and floors which grow at some exogeneously given growth rate. The model then fluctuate around this externally given growth trend (which seems to be the Harrodian natural rate of growth gn since Hicks' abstracts from productivity growth and suggests that the ceiling is a full employment ceiling) (Mullineaux, 1984, pp. 16-18).

9.R.G.D Allen exhaustively analyzes the structure of multiplier-accelerator models (Allen, 1968, Ch 17). Stable growth itself requires a particular range of parameters, and even this limited possibility is does not yield normal capacity utilization because the warranted growth rate s/v is generally inconsistent with the characteristic equation of the system. This result is not altered by models such as those by Phillips or Bergstrom, which embed the multiplier-accelerator relation in a more general set involving prices, wages and the rate of interest (Allen, 1968, Ch 20).

10.For instance, Keynes says that total investment "consists of fixed, working capital or liquid capital" investment, where by liquid capital he means inventories of finished goods (Keynes, 1936, Ch 7, p. 75). Kalecki distinguishes between "fixed capital investment" and "investment in inventories", where by in the latter categories he apparently lumps investment in both working capital and final goods (Kalecki, 1971, Ch 10, pp. 121-123). Harrod divides investment into "circulating and fixed capital" (Harrod, 1948, pp. 17-18); Hicks divides it into fixed and "working capital" (Hicks, 1965, Ch X, p. 105), and Joan Robinson divides it into investment in "capital goods, including equipment, work-in-progress, technically necessary stocks of materials, etc." (Robinson, 1966, p. 65). Similar distinctions play a vital role in the classical and marxian traditions, as well as in input-output analysis and sraffian economics.

11.For instance, Kalecki has circulating investment depending on past changes in output, "with a certain time lag" (Kalecki, 1971, Ch 10, p. 122), while Hicks has circulating capital investment depending on the expected change in (future) output (Hicks, 1965, Ch X, pp. 105-106).

12.Unit costs and profit margins are given for any one production period, so that the planned changes in output and produced profit are linked to the corresponding change in circulating inputs via that period's unit costs and margins. This does not preclude the possibility that costs and margins can vary through time from one production period to another.

13.The capital-capacity ratio q is also taken to be given for any one production period (see the previous footnote), but can be variable across periods.

14.Keynes was so used to thinking in static terms, in which output change appears as a "once over" change in the level, that he initially found it difficult to grasp Harrod's notion of a steady advance inherent in a dynamic path (Kregel, 1980, p. 99, footnote 5).

15.The fact that the profit margin measures the "markup" over costs does not imply that this profit margin is a reflection of monopoly power. A given normal competitive rate of return will also imply a particular "markup".

16.Keynes writes to Harrod that "growth [is] a long-period conception" (cited in Kregel, 1980, pp. 100).

17. An alternate formulation would be $b(e) = b \cdot |e|$

18.With government taxes T and spending G, equation 1 becomes $I + G = S + T$, which can be written as $I = S - GD$, where $GD \equiv G-T$ is the government deficit. A rise in the ratio of the government deficit to profits would then be equivalent to a drop in the combined savings rate $s^* \equiv s - gd = S/P - GD/P$.

19.The proofs of the properties of our slow adjustment process are presented in Shaikh (1989).