

Accounting for the Decline in Private Sector
Unionization: Representation Elections,
Structural Change and Restructuring

by

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Abstract: During the 1980s several qualitative changes occurred in the union decline. First, net gains from certification (less decertification) elections fell to insignificant levels, tending to accelerate the union decline. On the other hand, union losses from the relative growth of nonunion services (structural change) also declined sharply as unionization rates became more homogeneous across sectors. As a consequence, virtually all changes in the unionization rate during the 1980s were caused by disproportional gains in nonunion employment within sectors (restructuring).

The erosion of private sector unionization during the past thirty-five years is by now a familiar fact among labor economists as are many of the reasons for the decline. Unions suffered losses from structural shifts as relative employment grew in nonunion sectors and from restructuring as unionization rates fell sharply within virtually every industry in the private sector. Of equal importance is the fact that unions have found it increasingly more difficult to recover these losses through certification elections.

Each of these three sources of decline, elections, structural change, and restructuring have been subjected to extensive analysis but seldom in a single study. Also, most of the studies on each of these topics focus on changes in

the 1960s and 1970s which are not exactly representative of what occurred during the 1980s. The unique contribution of this paper is that it combines estimates of union gains from elections with structural losses and restructuring in order to provide a comprehensive profile of changes in the U.S. unionization rate from 1962 to 1989. This approach illustrates how certain factors have continued to erode union membership during the 1980s while others have not. For example union gains through representation elections continued its historical decline while structural losses from the growth of nonunion services gradually tapered off. As a result, neither certification elections nor structural change had much effect on unionization rates by the end of the 1980s. Most of the recent decline has been caused by the disproportionate growth of nonunion employment in the private sector.

Unionization Rates: Totals and Changes

The trend of private sector unionization in the United States during the past thirty years is familiar and graphed in Figure 1.¹ Perhaps less obvious is the fact that the rate of change in unionization, illustrated in the second panel, has been extremely volatile. In both the early period, 1962 to 1976, and the later period, 1985 to 1989,

¹ The unionization rate is defined as total private sector union membership divided by the nonagricultural private sector labor force.

annual changes in the unionization rate ranged between $-.4$ and $-.6$ percentage points. However, union losses soared between $-.6$ and -1.2 percentage points during the important period from 1976 to 1985. This period was obviously a critical stage in the decline of unions as they lost a total of 13.3 percentage points in nine years compared to only 9.4 points during the remaining eighteen. Equally important is the fact that this collapse in union membership apparently subsided by 1985 as annual losses returned to their previous historical level. There are of course many possible explanations for this pattern (recessions, deregulation, oil crises, and trade deficits) but before considering them in detail it is useful to consider the effects of elections and structural change.

Representation Elections

What Figure 1 does not show is that the annual change in the unionization rate is the net result of gains from certification elections less gross losses from other sources. Throughout this entire period, 1962 to 1989, union gains from certification elections have steadily declined and losses from decertification elections have grown (but not enough to be particularly significant). An important study by Dickens and Leonard (1985) analysed the trend in union gains from certification elections using data from the National Labor Relations Board and the Bureau of Labor

Statistics for 1950 to 1980. They found that from representation elections alone the unionization rate would have increased by 1.7 percentage points in 1950. But because of declining success in elections, the unionization rate would have increased only .2 points in 1980.² Unfortunately their series ends in 1980, making it necessary to update their calculations to 1989 based on similar assumptions and methods.³

The results are presented in Figure 2 which shows the change in private sector unionization that would have occurred from representation elections alone, including relatively small losses from decertification elections. According to this graph, the contribution of elections began to diminish after 1965 and by 1980 elections had very little effect on the overall unionization rate. To illustrate this point, imagine that the only change in unionization was from elections. Based on the election gains made in 1989, it would take unions twenty-five years to recoup just one percentage point, equivalent to the actual amount lost every year from 1980 to 1985.

[Insert Figure 2 here]

² If we define U as total union membership and L as the labor force, then the values reported by Dickens and Leonard are $\Delta U/U$ which are converted to $\Delta U/L$ in this study.

³ These basic assumptions are as follows: 5% of voters eligible to vote in certification elections are already counted as union members, 12% of voters in elections won by unions fail to become union members, and 50% of the workers involved in decertification elections are not considered union members at the time of elections.

Why are unions performing so poorly in certification elections? This question has been extensively studied by other authors who have pointed to the important role of management opposition and anti-union consultants in defeating unions. Certainly unions have always been subjected to strong opposition but they appear particularly defenseless in the face of modern management practices designed to prevent certification elections or at the very least to defeat unions if an election is actually held. Although there are no comprehensive statistics on total expenditures for this type of activity, there is little doubt that it has been increasing. The proliferation of anti-union consulting firms in the 1970s and 80s attests to the success of this relatively new enterprise.

At the same time, unions have demonstrated little evidence of escalating their own organizing efforts to meet this growing challenge. Paula Voos has documented that real union organizing expenditures per union member have changed very little. In 1962 the average union member contributed a mere \$3.92 a year for union organizing and by 1974 this had increased to only \$4.16 (1967 dollars). Voos also found evidence that organizing outlays as a percentage of total union expenditures declined significantly from 1953 to 1977. It does not appear that union efforts to win elections have been any match for the offensive led by management consultants.

The positions are essentially reversed from the 1930s when unions were on the offensive. In 1936, John L. Lewis, president of the newly formed Congress of Industrial Organization, pledged \$500,000 to support the flagging effort to organize the steel industry. With this gesture Lewis initiated the "huge organizing drive financed by millions rather than hundreds of dollars."⁴ In the same year the International Ladies Garment Workers Union assessed their members \$1 each to finance the drive in steel. It is important to remember that the inspired organizing successes of the 1930s and 40s were backed by massive escalations in union expenditures.

Structural Change

If the only source of change in unionization was certification elections, then these two variables would move in parallel over time. But comparing the contribution of elections to the actual change in Figure 2 shows this is not the case. The union decline accelerated between 1976 and 1984 at a much faster rate than can be explained by elections alone and even more striking is the improvement since 1985, at a time when election gains remained extremely low.

The vertical distance between the two curves in Figure 2 is defined as "gross" losses since it represents changes

⁴ Bernstein, 1969, page 436.

from all sources other than representation elections. One explanation for these large annual losses is structural change, i.e. the autonomous shift in employment from groups with high levels of unionization to those with low levels. Perhaps the most familiar example of this is the growth of employment in lightly unionized services relative to more highly unionized goods production. But estimates of structural change have also included the increasing proportion of female workers, white collar workers, and workers in the South.⁵ Farber (1985) concluded that total structural change accounted for an annual decline in the unionization rate of .20 percentage points from the mid 1950s to 1977, Freeman and Medoff (1984) found a similar value of .27 from 1950 to 1977, and Dickens and Leonard (1985) found a significantly higher value of .63 for a shorter time period from 1974 to 1980.⁶

While these studies demonstrate a significant impact of structural change on unionization rates prior to 1980, there is one important qualification. Each of these studies relies on average values measured over relatively long

⁵ Structural change not only alters the proportion of union and nonunion jobs as measured here, but could also affect the results of representation elections. It is generally assumed that this effect is relatively small and can safely be ignored. See Fiorti and Maranto, 1987.

⁶ These estimates were derived from the authors' estimates of S , the percentage of total union decline accounted for by structural change. Estimates of S ranged from Farber's 41% to 68% for Dickens and Leonard, and 72% for Freeman and Medoff. The estimates reported in the text were converted by multiplying S by the total decline $\Delta(U/L)$ and dividing by the number of years in the period.

periods of time and therefore overlooks the strong cyclical nature of structural change. In order to illustrate this point, an index of structural change was constructed which is proportional to the changes in employment shares for eight major sectors from 1962 to 1989.⁷ Although this particular index does not show the specific effect of structural change on unions, it does indicate the general rate of change in relative employment between sectors. The results presented in Figure 3 show that structural change is obviously cyclical, rising to peaks during the major recessions of 1975 and 1980. This is consistent with the fact that goods producers tend to experience greater proportional decreases in employment during economic downturns.

[Insert Figure 3]

Since structural change is cyclical, one would expect losses in unionization associated with it to be cyclical as well. In addition, one should also expect structural losses to gradually diminish over time as unionization rates of expanding and contracting industries become more homogeneous. For example, in 1962 there was a difference of nearly 40 percentage points between unionization rates in

⁷ The sectors include: mining, construction, manufacturing durables, nondurables, transportation and public utilities, wholesale trade, retail, finance/insurance/and real estate, and services. The index is equal to $\sum (.5) |\Delta(L_i/L)|$ where $\Delta(L_i/L)$ is equal to the change in employment share for the i^{th} sector.

services and durable manufacturing but by 1989 the difference was only 17 points. As unionization rates gradually converged across the economy, shifting employment shares had less of an impact on unionization.

The effect of structural change on the total unionization rate was calculated for the same eight sectors in order to illustrate both its cyclical nature and long term trend. The area labeled "structural" in Figure 4 represents that portion of gross losses that can be attributed to structural change. These particular losses peaked during the recession of 1975 and clearly diminished after 1982. For a brief period between 1983 and 1984, the expansion in the economy even provided a slight structural gain. But on average from 1982 to 1989, structural losses amounted to .06 percentage points a year compared to .27 percentage points a year from 1962 to 1976. As mentioned earlier, the reason that structural losses subsided was only partly related to economic expansion. The primary reason was that by 1983 there was significantly less variation in unionization rates between industries.

[Insert Figure 4]

Estimates of structural losses have been criticized because they are based on the assumption that unionization rates remain constant within sectors (Freeman and Medoff 1984, p.287). Consequently, it is important to review what structural losses are and perhaps more importantly, what

they are not.⁸ Structural losses are essentially calculated by comparing the actual change in unionization to a counterfactual case which holds unionization rates constant within each sector.

To illustrate this consider an example where election gains are zero and employment gains within every sector are identical for both union and nonunion workplaces. While unionization rates within sectors remain unchanged, the overall rate can change if some sectors grow faster than others. In this particular situation we would reach the reasonable conclusion that all changes in overall unionization are caused by structural change. But in the more general and realistic case where unionization rates change within at least some sectors, structural change simply accounts for a relatively smaller fraction of the total change. The fact that unionization rates change within sectors doesn't invalidate the method, it only decreases the relative importance of structural change.

Structural change as an explanation of declining U.S. unionization rates has been challenged on the basis of international comparisons. Although services have expanded in other developed countries, like Canada and Europe,

⁸ According to Freeman and Medoff (1984, p.228) "A more realistic reading of the evidence is that structural factors increase or decrease the difficulty of organization but do not determine unionization." Dickens and Leonard (1986) conclude, "The structural approach is limited... This serves as an important warning that other factors are at work beyond those considered in structural models."

overall unionization rates have not declined.⁹ This is an important point because it demonstrates that structural change by itself is not sufficient to ensure a decrease in unionization. On the other hand it doesn't mean that structural losses were nonexistent in Canada or Europe, only that they must have been offset by gains from increases in unionization within sectors, related perhaps to representation elections.

An example closer to home is the twenty year period in the U.S. prior to 1955. At this time structural shifts towards services were very much in effect in the U.S. but this did not prevent unions from enjoying a spectacular expansion. Even though structural losses probably still occurred, especially during recessions, unions were able to overcome them through gains from certification elections. Today unions are not making those gains through elections but neither are they losing as much from structural change. The fact that unions are continuing to lose ground means we must look at one more source of change in unionization, restructuring.

Restructuring

When structural change explains a relatively small part of gross unionization losses then the difference must be attributed to changes in unionization rates within sectors,

⁹ Fiorito and Maranto (1987).

defined as restructuring.¹⁰ Consequently gross losses must by definition equal the sum of losses from structural change and restructuring. Their individual contributions are represented by the areas identified in Figure 4. The first point to notice is that the effect of restructuring has been consistently negative since 1962, implying a relatively slower growth rate of union workers compared to nonunion ones. Second, the pattern of union losses was primarily determined by restructuring in the 1980s when the effects of elections and structural change were almost insignificant. And finally, losses from restructuring exploded in 1980 and then abruptly receded after 1985.

Before considering some of explanations for the surge in restructuring losses during the early 1980s, it is useful to consider some specific examples. Restructuring occurs when industries, (and firms in those industries) alter the union percentage of their labor force. U.S. Steel underwent major restructuring as it pruned its union labor force from 65% in the late 1970s to 35% in the mid 1980s.¹¹ It was able to accomplish this reduction by disproportionately reducing union jobs as it slashed overall employment by 47 percent. To take another example, employment fell in the motor

¹⁰ For each sector we define the employment share as s_i and the unionization rate as u_i , then the total unionization rate (U) is given by $U = \sum u_i s_i$. The total increment in U (dU) is given by $dU = \sum s_i du_i + \sum u_i ds_i$ where the first term is restructuring and the second is structural. A complete decomposition including certification elections and employment changes are in a technical appendix available from the author.

¹¹ Nader and Taylor, 1986, page 17.

vehicle industry by 10% from 1974 to 1980 and unionization rates fell from 72% to 66%.¹² Within this same period, General Motors managed to open 14 new plants, 9 of them in right-to-work states and 1 in Mexico.¹³ Restructuring also includes the permanent replacement of union workers during unsuccessful strikes, the replacement of striking air traffic controllers (PATCO) in 1981 being the most prominent example. In these cases and dozens of others, restructuring by downsizing operations, closing plants, subcontracting previously unionized work, replacing striking workers, or relocating production, provided employers with the opportunity to deunionize their labor force.

While most labor economists acknowledge the important role of management opposition in representation elections there is much less recognition of how businesses actively undermine unions through restructuring. This is a serious omission since the systematic replacement of union workers with nonunion ones has become a prominent corporate strategy for eliminating collective bargaining. This point is contested by some economists who attribute union losses from restructuring to impartial market forces. They argue that deunionizing the workforce is inevitable as domestic and international competition weed out high cost producers.¹⁴ But this is an overly simplistic explanation which ignores the actual causes of restructuring.

¹² Kokkelenberg and Sockell, 1985.

¹³ Bluestone and Harrison, 1982, page 167.

¹⁴ See Linneman and Wachter, 1986.

Of the many developments that caused an acceleration of restructuring in the U.S. economy from the late 1970s to the mid 1980s, none of them can reasonably be blamed on unions. Leading up to this period was the deregulation of trucking and airlines, the second major oil crisis in 1979, and deregulation of oil prices in 1980. In addition, interest rates began their upward trajectory in the 1970s and reached their apex in 1981 with a prime rate in the neighborhood of 20%. To some degree, these dramatic events aggravated the consecutive recessions in 1980 and 1982 which eventually pushed unemployment to a post-Depression high of 10.6%.

None of these familiar events which accelerated the rapid transformation of America's productive capacity can reasonably be blamed on unions. Even the import explosion that rocked U.S. manufacturers in the early 1980s was primarily related to factors other than trade unions. In fact imports were not disproportionately concentrated among unionized industries but were broadly distributed among industries with a wide range of unionization rates (Karier, forthcoming).

In one of the few studies of restructuring, Bluestone and Harrison found that for 8.8 million net jobs created between 1969 and 1976, 35.5 million were destroyed and 44.3 million new jobs were created.¹⁵ In other words, for every five new jobs created during this period four existing jobs were destroyed. The small change in total employment

¹⁵ Bluestone and Harrison, 1982, page 30.

reported by government agencies belie the tumultuous transformations in the labor force. As businesses eliminated old jobs and created new ones, they discovered an unparalleled opportunity to reduce their union coverage by eliminating union workers, hiring nonunion ones, or both.

Although unions may not have been responsible for initiating this period of extensive restructuring, they nevertheless became a primary victim. Management has the obvious incentive to deunionize in order to cut labor costs and raise profits but there is at least one other reason why union work places would be disproportionately affected. The last great union organizing drive lasted from approximately 1934 to 1954 and union gains in the private sector have been less than impressive ever since. Consequently by the 1970s and 1980s many of the union work places were among the oldest in the country.

The Homestead Works of U.S. Steel, for example, was organized by the CIO in the 1930s and was closed in 1986. But the first steel rolled out of this mill in the early 1880s.¹⁶ The steel, auto, and rubber industries closed dozens of union plants during the 1970s and 1980s but many of these plants had already been in operation ten or twenty years when they were first organized by unions in the 1930s and 40s. Even a relatively new plant organized during WWII would have been at least 35 years old by 1980. The lack of union success in organizing new plants and firms since the

¹⁶ Hoerr, 1988, page 87.

1950s left unions concentrated in the oldest pockets of the economy, the ones most vulnerable to restructuring. In a sense, the cumulative failure of unions to organize new work places from 1955 to 1975 exacerbated the impact of restructuring on unionization rates from 1976 to 1985.

Conclusions

Decomposing the union decline into elections, structural change, and restructuring reveal trends during the 1980s that would otherwise not be apparent. Unions are currently winning a miniscule number of new members through certification elections but they are also losing fewer to structural change. This means that most of the fluctuations in unionization rates during the 1980s can be attributed to restructuring which had a particularly devastating impact in the early 1980s and then receded to much lower levels by the late 1980s. The net effect is that unions are now declining at a rate of .47 percentage points a year, the slowest rate of decrease since the late 1960s.

The real challenge in predicting where unions will be ten years from now isn't merely to extrapolate current trends, but to anticipate new developments that may disrupt the gradual dissolution of unions. There is, for example, a good possibility that the record number of plant closures which devastated unions in the late 1970s and early 1980s

will now subside since many of the oldest plants have been closed. Perhaps the slower union decline in 1989 is already evidence of this effect.

There is also the possibility that the very benefits that companies hope to achieve from a nonunion labor force will at some future date stimulate a resurgence in the union movement. Lower wages and benefits, fewer workrules, and more management discretion may be appealing to companies trying to rid themselves of a union workforce but these are the same conditions that motivated the original explosion in union organizing in the 1930s and 1940s. Companies are likely to find that they can take full advantage of their newly won nonunion status in the short run only at the risk of making unions more appealing in the long run.

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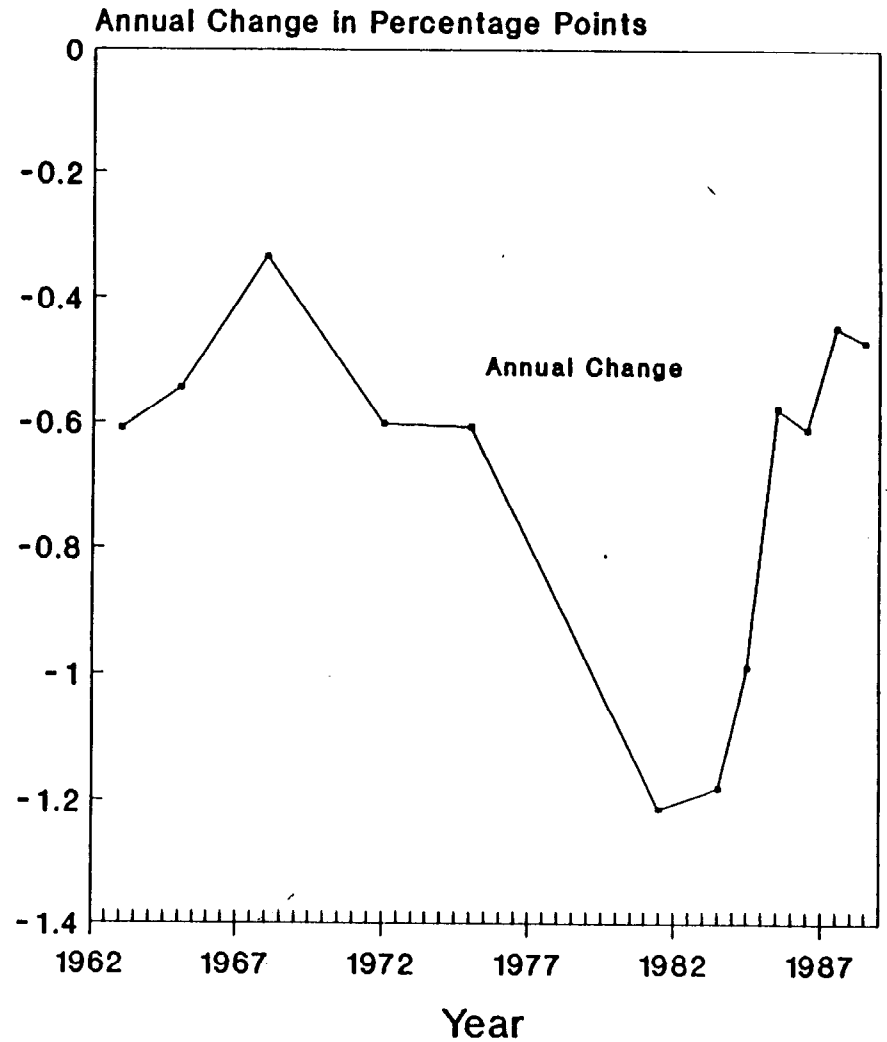
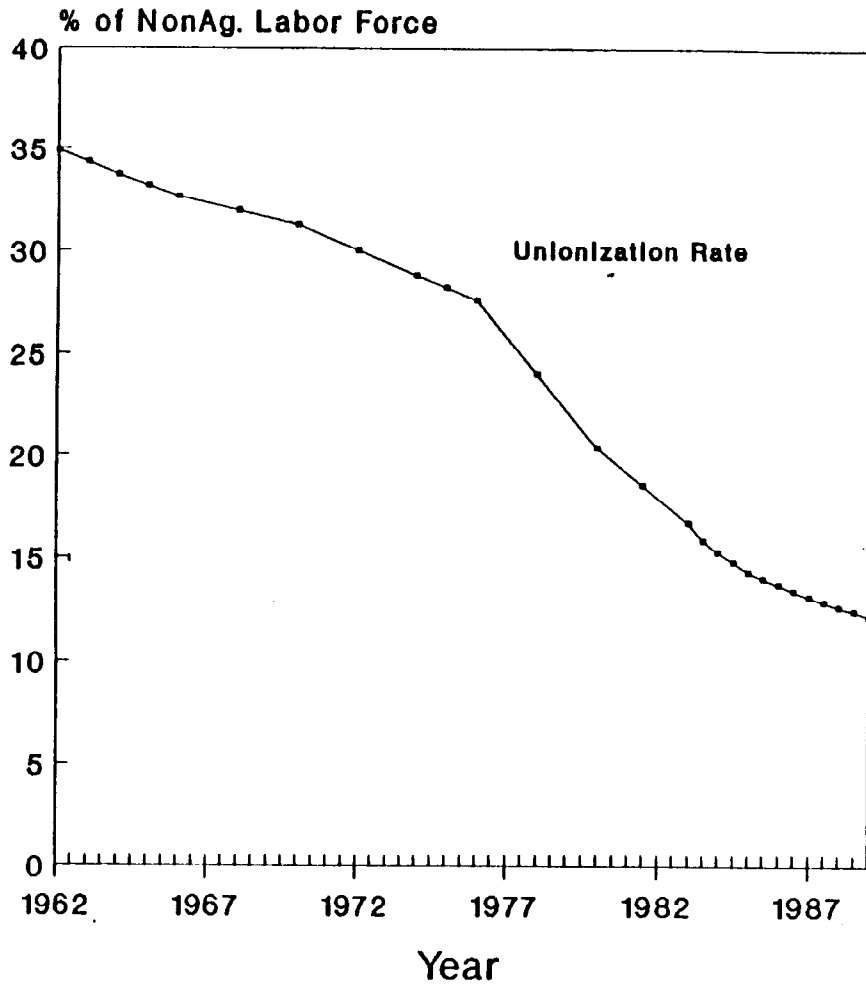
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Figure 1

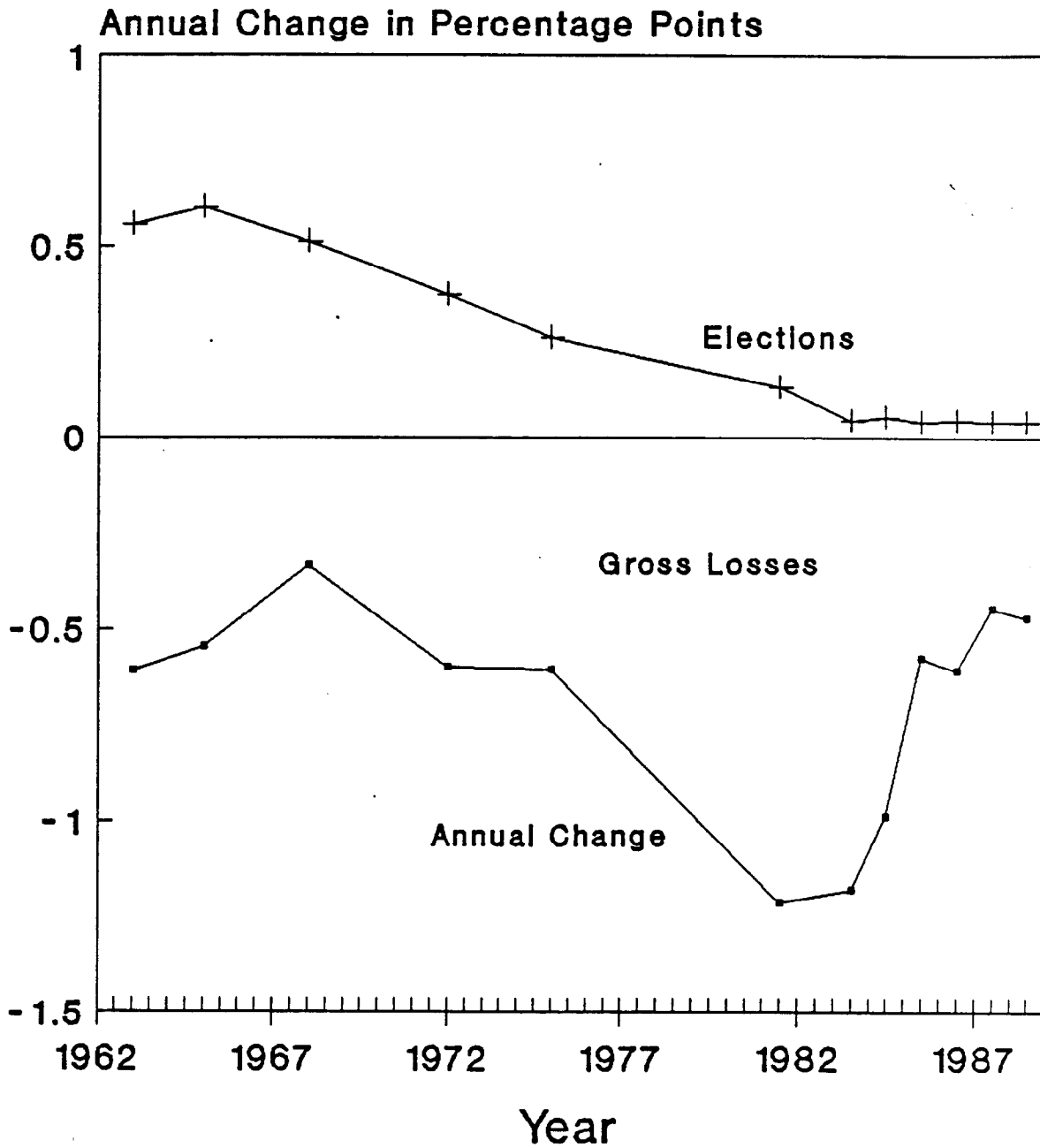
U.S. Private Sector Unionization Rate



Source: BLS, Directory of National Trade Unions and Employee Associations and Employment and Earnings.

Figure 2

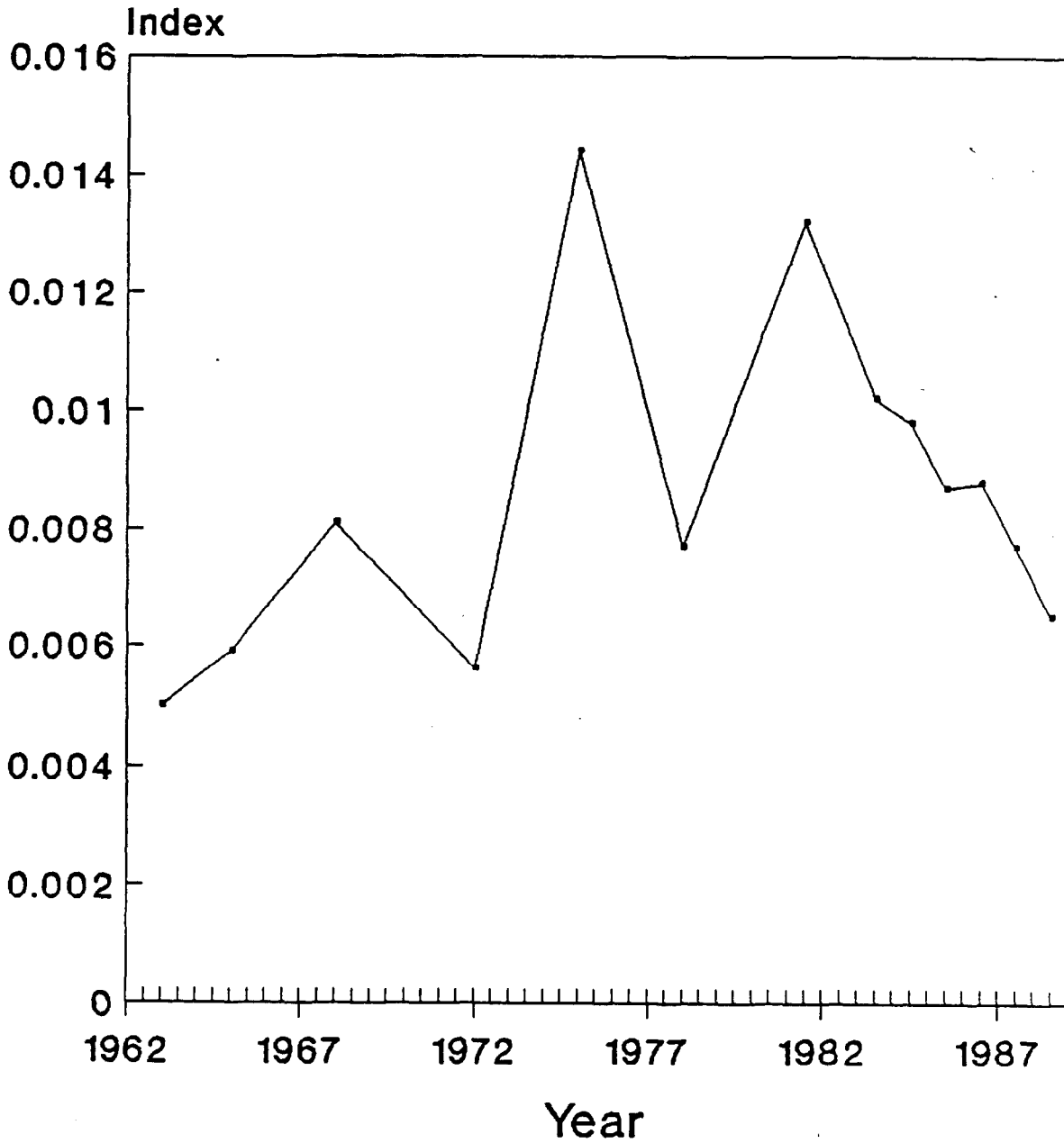
Annual Change in Unionization and the Contribution of NLRB Elections



Estimates provided by Bill Dickens, 1962 to 1980; NLRB Annual Reports, BLS data, and author's calculations, 1981 to 1989.

Figure 3

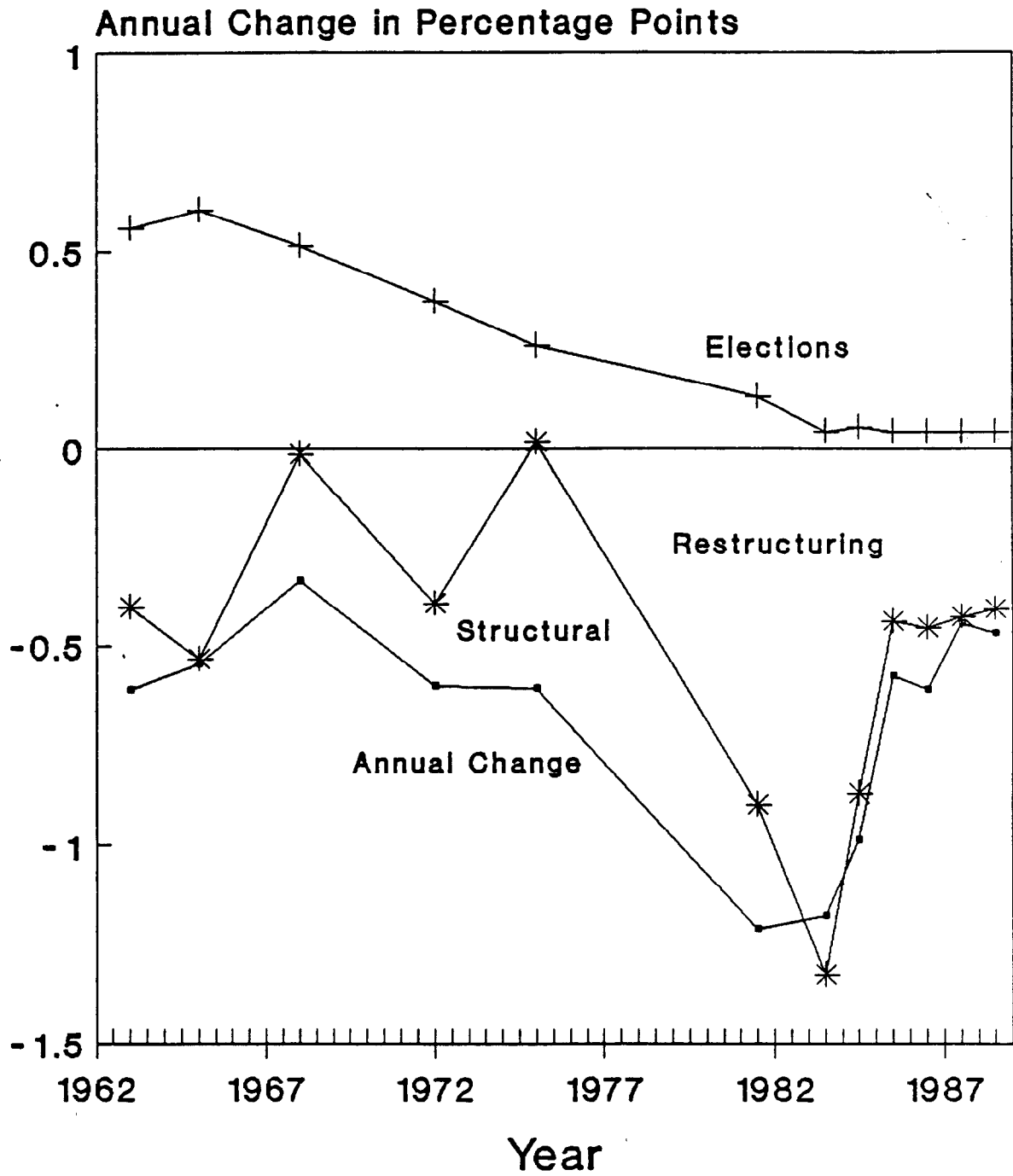
Index of Structural Change Annual Change in Relative Employment Between Sectors



The Index is equal to one-half the absolute value of changes in employment shares summed over each major industry.

Figure 4

Annual Change in Unionization Elections and Structural Change



TECHNICAL APPENDIX

Accounting for the Union Decline in the 1980s: Elections
Structural Change and Restructuring. Thomas Karier

In the first section changes in unionization are decomposed using calculus. While this is a useful method it involves a certain amount of error when real increments are used to approximate exact differentials in the formula. Therefore a second method using increments is presented in the second section which is somewhat more precise.

Union membership by sector	=	u^i
Nonunion workers by sector	=	n^i
Employment by sector	=	$L^i = u^i + n^i$
Total union membership	=	$U = \sum u^i$
Total employment	=	$E = \sum L^i$
Unionization Rate	=	U/E
Employment share	=	$s^i = L^i/E$

Total Differential Method

We begin with the definition of the unionization rate as a weighted average of unionization within each sector.

$$1) \quad U/E = \sum s^i \left(\frac{u^i}{L^i} \right) = \sum s^i \left(\frac{u^i}{u^i + n^i} \right)$$

At this point it is convenient to drop the summation sign and superscripts. The total derivative of equation 1 is defined as,

$$2) \quad d(U/E) = \left(\frac{u}{L} \right) ds + \left(\frac{sn}{L^2} \right) du - \left(\frac{su}{L^2} \right) dn$$

In other words, the change in unionization in equation 2 can be decomposed into changes in employment shares between sectors (structural change), and changes in the number of union members (u) and nonunion workers (n) within sectors.

We can now introduce the assumption that all changes in union membership or nonunion workers must arise either from elections (subscript e) or other changes primarily related to employment growth (subscript g).

$$3) \quad \begin{aligned} du &= du_e + du_g \\ dn &= dn_e + dn_g \end{aligned}$$

Substituting equation 3 into 2 we have,

$$4) \quad d(U/E) = \frac{u}{L} ds + \frac{sn}{L^2} du_e + \frac{sn}{L^2} du_g \\ - \frac{su}{L^2} dn_e - \frac{su}{L^2} dn_g$$

This can be simplified further by recognizing the fact that each union member gained from elections reduces the number of nonunion workers ($du_e = -dn_e$). Substituting this into equation 4 we have,

$$5) \quad d(U/E) = \frac{u}{L} ds + \frac{s}{L} du_e + \frac{sn}{L^2} du_g - \frac{su}{L^2} dn_g$$

This gives us the basic form for decomposing the total change in unionization into structural change (the ds term), elections (du_e), and restructuring (du_g and dn_g). The actual calculation involves an approximation since increments between two periods must be used to approximate the exact differentials (i.e. Δs for ds).

Total Increment Method

Each decomposition spans at most four years and usually one year during the 1980s. This reduces the amount of error involved when using real increments as an approximation. Another way to further reduce this error is to derive an equation similar to equation 5 but using only total increments. Although this requires considerably more algebra it results in a slightly more precise method of decomposition.

In this case the total increment in the unionization rate between period 2 and 1 is given by,

$$6) \quad \Delta(U/E) = (U_2/E_2) - (U_1/E_1)$$

where the subscripts identify the period. Although the decomposition can be accomplished directly by substituting equation 1 into 6 and solving, there are two simple rules¹ which are very helpful. These are described as follows¹

$$7) \quad \text{If } c=ab \text{ then } \Delta c = \bar{a}\Delta b + \bar{b}\Delta a \quad \text{where } \bar{a} = (a_2+a_1)/2 \\ \bar{b} = (b_2+b_1)/2$$

$$\text{If } c = 1/(a+b) \text{ then} \\ \Delta c = -(c_2c_1)\Delta a - (c_2c_1)\Delta b$$

The decomposition is accomplished by applying these general rules to equation 1. As in the case of total differentials,

¹ These rules are not proven here but they are straightforward to derive.

the change in union membership within an industry must be further decomposed into an election term (Δu_e) and an employment term (Δu_g), where,

$$\begin{aligned} 8) \quad \Delta u &= \Delta u_e + \Delta u_g \\ \Delta n &= \Delta n_e + \Delta n_g \\ \Delta u_e &= -\Delta n_e \end{aligned}$$

Solving all of this is a tedious algebraic problem but the result is,

$$9) \quad \Delta(U/E) = \left(\frac{\bar{u}}{\bar{L}}\right)\Delta s + \bar{s}\left(\frac{1}{L_2}\right)\Delta u_e + \bar{s}\left(\frac{n_1}{L_2 L_1}\right)\Delta u_g - \bar{s}\left(\frac{u_1}{L_2 L_1}\right)\Delta n_g$$

$$\text{Where } \left(\frac{\bar{u}}{\bar{L}}\right) = (1/2)[u_2/L_2 + u_1/L_1] \quad \text{and, } \bar{s} = (1/2)[s_2 + s_1]$$

Although this equation closely resembles equation 5 it has one important advantage. Since it was derived in terms of increments, no approximation is involved in using real data. In fact even if the observations are many years apart, the components calculated in equation 9 will always exactly equal the change in total unionization. It is this formula which is used to decompose unionization in the paper.

All of the variables in equation 9 are available from Bureau of Labor Statistic publications except for Δu_e , Δu_g , and Δn_g which represent union changes from elections and growth and the nonunion change from growth in each sector. Since total gains from elections are available (see page 4), these were allocated to each sector in proportion to each sector's share of total union membership.² This provided an estimate of Δu_e . Since Δu is known, Δu_g and Δn_g can be derived from the equations in 8 above.

² This assumption corresponds well with evidence from NLRB elections.