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On Various Ways of Measuring Unemployment, with Applications to Switzerland

by

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ABSTRACT

This paper begins with an examination of various ways of measuring unemployment and, borrowing ideas from the poverty measurement literature, proposes four new general unemployment indices. The first of these is parallel to the Sen poverty index; the second, to the Sen index's generalization by Shorrocks; the third, to the FGT poverty index; and the fourth, to the Watts poverty index. The authors then present an empirical illustration based on Swiss data compiled at the state, or canton, level, using the so-called Shapley decomposition to determine the contribution of three components—the traditional unemployment rate, the average unemployment duration, and the inequality in the unemployment durations—to the differences between the values of the four proposed indices, both within a given canton and within Switzerland as a whole. The paper concludes with a discussion of the assumptions made about the maximum unemployment duration for the purposes of the study, and their impact on the results obtained.

Keywords: FGT Poverty Index, Inequality, Sen Poverty Index, Shapley Decomposition, Switzerland, Unemployment Duration, Watts Poverty Index

JEL Classifications: I32–J64

I. INTRODUCTION

Starting with the pathbreaking work of Sen (1976), numerous studies during the past thirty years have attempted to measure the extent of poverty. Part of this work has been theoretical, taking either an ordinal or a cardinal approach to the measurement of poverty (see Zheng 1997 for a good survey of the work in this field), but there have also been many empirical studies of the extent of poverty and these works have generally taken a look at what is known as the "Three I's of poverty" (see Jenkins and Lambert 1997), that is, its incidence (the percentage of poor in the population), its intensity (how far are the poor from some agreed upon poverty line), and the inequality of poverty (how unequal are the incomes of the poor). Despite the numerous studies that have been published, one has to stress the fact that the most popular measure of poverty remains, for both politicians and the public at large, the headcount ratio, which gives the percentage of poor in the population.

This need for a simple index probably explains why in another field, unemployment, a simple measure such as the unemployment rate remains the most popular measure of unemployment. In recent years, however, there have been some attempts to derive more sophisticated measures of unemployment that would take into account not only the percentage of individuals who are unemployed but also the mean duration of unemployment and even the inequality of these durations (see, for example, the works of Sengupta 1990; Paul 1992; Shorrocks 1992 and 1993; Riese and Brunner 1998; and more recently, Basu and Nolen, forthcoming). Some of these works have also stressed the importance of the distinction between the total unemployment duration experienced by an individual and the various spells of unemployment that he experienced, but clearly the literature on unemployment measurement is much less abundant than that on income inequality or poverty measurement.

The purpose of this paper is to borrow some of the ideas that have appeared in the studies that have just been cited, propose some measures of unemployment that are more sophisticated than the unemployment rate, and apply them to data on unemployment in the various Swiss cantons during the period 1993–2005. The paper is organized as follows. Section II discusses various ways of measuring unemployment and, borrowing ideas from the poverty measurement literature, proposes four more general unemployment indices which are parallel to the Sen

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poverty index, to its generalization by Shorrocks, to the Foster, Greer, and Thorbecke (FGT), and to the Watts poverty indices. Section III then gives an empirical illustration based on Swiss data at the level of the "canton." Using the so-called Shapley decomposition, it computes the contribution to the difference between the value of each of these four unemployment indices in a given "canton" and in Switzerland as a whole, of three components measuring, respectively, the impact of differences in the traditional unemployment rate, in the average unemployment duration, and in the inequality in the unemployment durations. The paper ends by discussing the impact on the results obtained by assumptions made concerning the maximum unemployment duration.

II. THE METHODOLOGY

A. On Various Ways of Measuring Unemployment

Two indicators are commonly used to measure unemployment. The first one is the unemployment rate, which measures total unemployment as a proportion of the total labor force. This measure is obtained by asking individuals at some point of time (t) whether they are currently unemployed. The second indicator refers to the mean duration of unemployment. However, as stressed by Sengupta (1990) and Shorrocks (1993), there is much less agreement among economists about the way this mean duration should be measured. Several suggestions have in fact been made.

The first one is based on answers to a question like, "If you are currently unemployed, for how long have you been unemployed?" When this type of data is taken into account to compute durations of unemployment, one in fact looks at the distribution of "interrupted spells of employment" (Shorrocks 1993).

A second possibility was suggested by Akerlof and Main (1981). It looks at the distribution of the completed spells of unemployment of those who are currently (at some time, t) unemployed. In other words, whereas the first approach looks "backward," the second one looks "forward."

A third approach would also take a "backward look" and ask persons who are unemployed at some time (t) for how long they have been unemployed during a given period in the past (e.g., a year), no matter whether this unemployment duration included one or more spells of unemployment.

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Rather than looking at the mean duration of unemployment according to each of the three approaches previously mentioned, we could also look at the distribution of these durations and compute some index of inequality of these durations, such as the Gini index.

We can, however, think of a way of extending the first approach, which stresses the concept of "interrupted spell of unemployment." This approach often assumes that the unemployment rate, which serves as reference, is that observed in December. It is, however, possible to base computation of the unemployment rate on data which are available for each of the 12 months and compute the expected monthly unemployment rate over a period of 12 months.

Let v_{ij} be an indicator of unemployed defined as follows:

 $v_{ij} = 1$ if individual i was unemployed in month j and $v_{ij} = 0$ otherwise.

The expected monthly unemployment rate (U/N) during year t will then be defined as

$$(U/N) = (1/12) (1/N) (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} v_{ij})$$
(1)

This is, in fact, the way an annual unemployment rate is defined.

We may similarly define the mean duration of unemployment (among the unemployed) as the expected mean duration over all of the 12 months for which data are available. Let D_{ij} denote the cumulative number of days of unemployment of individual i in month j. The expected mean duration of unemployment in year t, on the basis of the first approach and assuming we have data for 12 months, may then be defined as

$$D_{A} = (1/12)(1/N) \sum_{i=1 \text{ to } N} \sum_{j=1 \text{ to } 12} (D_{ij} \times v_{ij})$$
(2)

The present section has thus shown that depending on the approach adopted, one may obtain very different values for the unemployment rate and the mean cumulative unemployment duration, as well as for the Gini index of these cumulative unemployment durations. However, there is an additional issue. We have hitherto analyzed separately three types of indicators of unemployment: the traditional unemployment rate, the mean duration of unemployment, and a measure of the inequality of these durations. The next section, using some previous work of Shorrocks (1993), will show that it is possible to construct a new measure of unemployment that will take into account all these three aspects of unemployment.

B. Deriving a More Complete Measure of Unemployment

As mentioned previously, the data that are available often give the cumulated number of days at month j during which individual i has been unemployed without interruption (see the hypothetical example presented in Table 1).

Let now D_{ij} denote this cumulated number of days of unemployment. We may therefore write that

$$D_{ij} = D_{i,j-1} + d_{ij}$$
 if $v_{ij} = 1$ and $D_{ij} = 0$ if $v_{ij} = 0$. (3)

Let now V_{ij} be the cumulated value of v_{ij} , that is,

$$V_{ij} = V_{i,j-1} + v_{ij} \text{ if } v_{ij} = 1 \text{ and } V_{ij} = 0 \text{ if } v_{ij} = 0$$
(4)

To illustrate the inequality in these cumulated days of unemployment, we can draw the following graph, which has been called unemployment duration profile curve by Shorrocks (1993).

In Figure 1 we plotted on the horizontal axis the cumulative values of the months of unemployment during year t of those who were unemployed in year t, that is, of $(1/12) (1/N) v_{ij}$. This means, in fact, that we plotted the cumulative values $(1/12) (1/N) V_{ij}$.

On the vertical axis, we plotted the cumulative values of the cumulative number of days of unemployment among the unemployed, that is, of $(1/12) (1/N) D_{ij}$. On both the horizontal and vertical axes, the individuals are ranked by decreasing values of these cumulated durations D_{ij} .

Such a plot gives us the curve OHAM. It is easy to see, using (1) and (4), that the horizontal coordinate of A (the length OB) is equal to the annual unemployment

rate (U/N), where U is the total number of individuals unemployed in year t, and N is the size of the labor force.

The vertical coordinate of A (the length AB) will be equal to

$$(1/12) (1/N) (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} D_{ij}) = D_{LF}$$

which is actually equal to the *average cumulative duration of unemployment (in days) per individual in the labor force.*

It is then easy to derive that the slope of OA will be equal to the ratio:

$$(1/12) (1/N) (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} D_{ij}) / (1/12) (1/N) (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} v_{ij})$$
$$= (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} D_{ij}) / (\Sigma_{i=1 \text{ to } N} \Sigma_{j=1 \text{ to } 12} v_{ij})$$
(5)

which, in fact, represents the average cumulative number of days of unemployment (D_A) among individuals who have been unemployed at some time in year t.

Let now $G(D_{ij})$ refer to the Gini index of the cumulative unemployment durations D_{ij} and let OPA denote the straight line OA. The area OHAB is therefore equal to the sum of the area OHAP and of the triangle OPAB. The area of this triangle OPAB is clearly equal to (1/2) (U/N) (U/N) (D_A), since the tangent of the slope AOB is equal to (AB/OB) = D_A.

The area that lies between the curve OHA and the line OPA, by construction, looks like the area lying between a Lorenz curve and a diagonal, and such an area is generally equal to half the Gini index of the variable whose cumulative values have been plotted. However, since this "diagonal" OPA does not end at a point whose coordinates are (1,1), but at point A whose coordinates are (U/N) and (U/N) D_A , it is easy to derive that the area between the curve OHA and the line OPA is equal to (1/2) $G(D_{ij})$ (U/N) (U/N) $D_A = (1/2) G(D_{ij})$ (U/N) D_{LF} .

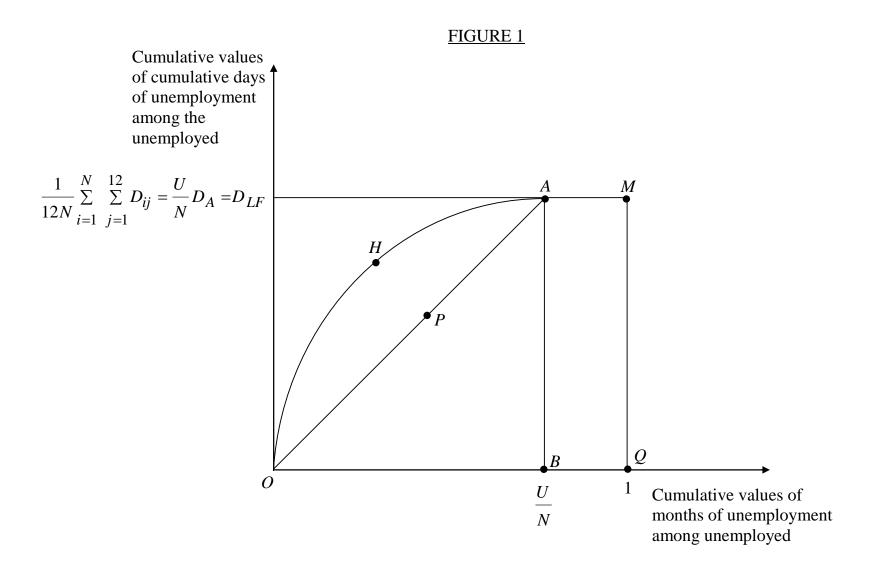
The sum (M) of the two areas OHA and OPBA will therefore be equal to

$$M = (1/2) (U/N) (U/N) D_A (1 + G(D_{ij})) = (1/2)(U/N) D_{LF} (1 + G(D_{ij}))$$
(6)

This indicator (M) may be considered a generalized measure of unemployment. As may be observed, M is an increasing function of the probability for an individual to be unemployed during a randomly selected month [which is really the way the annual unemployment rate (U/N) is defined]. This indicator (M) also increases with the average value D_{LF} in the whole labor force of the cumulative number of days of unemployment. Finally, M will be higher the more unequal these cumulative number of days of unemployment durations are since it increases with $G(D_{ij})$.

When these cumulative durations of unemployment are standardized, it turns out (see the proof in Appendix 1) that this indicator (M) is, in fact, equal to half the value of the product of the unemployment rate (U/N) times "Sen's Unemployment Index," an index which is obtained by applying to the measurement of unemployment Sen's (1976) index of unidimensional of poverty.¹

¹ This is, in fact, only the asymptotic value of Sen's (1976) index, that is, it assumes that the size of the labor force is big enough.



More precisely, let E_{ij} be equal to $(D_M - D_{ij})$, where $D_M = \{Max \ D_{ij}\}$ represents the maximal number of days of cumulative unemployment (it could correspond to one, two, or even more years). E_{ij} therefore represents the number of days during which individual i was employed during the period under consideration (one, two, or even more years, depending on how it is defined). Let now e_{ij} be equal to the ratio E_{ij}/E_M , where $E_M = \{Max \ E_{ij}\}$ is evidently equal to $D_M = \{Max \ D_{ij}\}$. Let also f_{ij} be equal to the ratio D_{ij}/D_M and f_A to the ratio D_A/D_M .

When applied to the analysis of unemployment, Sen's (1976) poverty index may therefore be written as

$$S_{U} = (U/N)[f_{A} + (1 - f_{A}) G(e_{ij})]$$
(7)

where $G(e_{ij})$ is the Gini index of the cumulative relative employment duration e_{ij} . Note that, as in the case of Sen's poverty index, the formulation for S_U holds only if U, the total number of unemployed individuals in year t, is big enough.

Appendix 1 shows also that another possible measure of unemployment is twice the area lying under the curve OHAM, that is the area OHAMQBO, in Figure 1. We then obtain (see Appendix 1) what Shorrocks (1995) called "The Revisited Sen Poverty Index" (S_{UR}), which, in the case of unemployment measurement, will be expressed as

$$S_{UR} = \{ [(U/N)^{2} ((1-f_{A}) G(e_{ij})] + [(U/N) f_{A} (2-(U/N))] \}$$
(8)

Another very popular poverty index is the so-called FGT index (Foster, Greer, and Thorbecke 1984). When applied to the measurement of unemployment, this index (FGT_U) will be expressed as

$$FGT_{U} = (1/12)(1/N) \sum_{i=1 \text{ to } U} \sum_{j=1 \text{ to } 12} (f_{ij})^{\alpha}$$
(9)

It is easy to observe that when $\alpha=0$, FGT_U = (U/N) and that when $\alpha=1$, FGT_U = (U/N) f_A

Let us now take the case of α =2.We may then write that:

$$FGT_{U} = (1/12)(1/N) \Sigma_{i=1 \text{ to } U} (f_{ij})^{2}$$

= (1/12)(U/N){(1/U) $\Sigma_{i=1 \text{ to } U} [(f_{ij} - f_{A}) + f_{A}]^{2}}$
$$FGT_{U} = (1/12)(U/N) \{ Var (f_{ij}) + (f_{A})^{2} = (1/12)(U/N) (f_{A})^{2} \{ 1 + \text{Coef. Var. } (f_{ij}) \}$$
(10)

where Var(.) refers to the variance and Coef. Var (.) to the coefficient of variation of a variable. So in the case where α =2, we observe, as in the case of the indices S_U and S_{UR}, that the index FGT_U is a function of the unemployment ratio (U/N), the average cumulative unemployment duration D_A, and of a measure of the dispersion of the relative cumulative unemployment durations, in this case, the coefficient of variation of these relative cumulative durations.

Finally we can also apply to the analysis of unemployment the poverty index defined by Watts (1969). When applied to the measurement of unemployment this index will be written as

$$W_{\rm U} = (1/N) \sum_{i=1 \text{ to } {\rm U}} \log(E_{\rm M}/E_{\rm ij})$$
(11)

Expression (11) may however be also written as

$$W_{U} = (U/N) \left[\sum_{i=1 \text{ to } U} (1/U) \log(E_{M}/E_{A}) + \sum_{i=1 \text{ to } U} (1/U) \log(E_{A}/E_{ij}) \right]$$
(12)

where E_A is equal to the average of the employment durations E_{ij} .

Note, however, that the first expression under square brackets on the R.H.S. of (12) may also be written as

$$W_{\rm R} = \log \left(E_{\rm M} / E_{\rm A} \right) \tag{13}$$

so that W_R somehow measures the percentage difference between the maximum cumulative duration of employment and the average cumulative duration of employment. In other words, W_R somehow indicates to which percentage of the

maximal employment duration the average cumulative unemployment duration corresponds.

The second expression under square brackets on the R.H.S. of (12) may be written as

$$L_U = \log(E_A) - \log(E_G) \tag{14}$$

where E_G refers to the geometric mean of the cumulative employment durations E_{ij} . It is then easy to observe that L_U measures the percentage difference between the arithmetic and the geometric means of the cumulative employment durations E_{ij} . Since the gap between the arithmetic and a geometric mean of a variable is usually considered as an indicator of the inequality of the distribution of this variable (see Champernowne 1953), the indicator L_U , in fact, measures the inequality of the cumulative employment durations among the individuals who were unemployed at least part of year t. This indicator is also known as the Bourguignon (1979)-Theil (1967) Inequality Index.

Combining expressions (11) to (14) we end up with

$$W_{\rm U} = (U/N) (W_{\rm R} + L_{\rm U})$$
 (15)

Like the indices S_U , S_{UR} , and FGT_U defined earlier the index, W_U is a function of three components measuring, respectively, the unemployment rate, the gap between maximal employment duration and the average cumulative unemployment duration, and finally the inequality in the employment durations among those who were unemployed at least part of the time in year t.

C. Comparing Unemployment Measures in Different Areas

The four indices (S_U , S_{UR} , FGT_U, and W_U) that have been defined previously may be computed for each area (j) in a given country, as well as for the whole country. The difference between the value that a given index (one of the four previously mentioned) takes for the whole country and for a given area may then be decomposed, using the so-called Shapley decomposition procedure, into three components (see Appendix 2) that measure, respectively, the extent of differences between the country as a whole and a given area in the unemployment rate, the gap between the maximal and average values of the cumulative number of days of unemployment, and finally in the inequality of the cumulative number of days of unemployment (employment) among those who were unemployed at least part of the year.

III. AN EMPIRICAL ILLUSTRATION

The concepts that have been previously presented have been applied to data on unemployment in the 26 Swiss areas, which are called "cantons," for the period 1993 to 2005. To illustrate these concepts, we have used the approach where unemployment is measured via the information on the expectancy of the interrupted spells of unemployment over the whole year. But we clearly could have used one of the three other approaches.²

As was just mentioned, we look at the values of the cumulative durations of unemployment as they are given each month of the year for the various unemployed individuals. More precisely, we work with the expectancy of these cumulative durations of unemployment on the basis of data on cumulative unemployment for each of the 12 months of the year. As maximal value for these cumulative durations we assumed again that it was equal to 365 days.

In Table 1 we give data on the unemployment rate (the expectancy of the monthly unemployment rates, which is also the value of the annual unemployment rate), on the average value observed during the year of the cumulative unemployment durations, and finally on the Gini index of these cumulative durations of unemployment—for Switzerland as a whole and for each canton in 2005. It appears that the highest rates of unemployment are observed in the cantons of Geneva, Vaud, and Tessin and the lowest in the cantons of Uri, Appenzell AI, and Obwalden. As far as average durations of unemployment are concerned, the highest averages are observed in the cantons Geneva and Vaud. The lowest average durations are observed in the cantons of Graubünden, Uri, and Obwalden. Finally, the highest levels of unemployment duration inequality are observed in the cantons of Valais, Graubünden, and Obwalden and the lowest in the cantons of Geneva, Neuchâtel, and Vaud.

In Table 2 we give the values in 2005 (for Switzerland as a whole, as well as for each canton) of the three indices of unemployment which we have defined

 $^{^2}$ In fact, computations similar to those presented in this section, but based on the other three approaches, are available from the authors upon request.

previously, the Sen index (S_U), Shorrocks' generalization of the Sen index (S_{UR}), and the Foster, Greer, and Thorbecke Index (FGT_U). It appears that the highest values of the unemployment indices are observed in the cantons of Geneva, Vaud, Tessin, and Neuchâtel, and the lowest in the cantons of Uri, Appenzell AI, Obwalden, and Graubünden.

In Tables 3 through 5 we give, for each of the three unemployment indices previously mentioned, the results of the Shapley decomposition of the gap between the national value of these indices and that observed in each canton. Such a breakdown allows one to identify the respective contributions to this gap of differences in the unemployment rate and in the average durations of unemployment, as well as in the inequality of unemployment (or employment, depending on the index selected) durations.

It appears that in the four cantons with the highest positive (Geneva and Vaud) or negative (Uri and Obwalden) gap, the contribution of differences between the unemployment rate in these cantons and that in Switzerland account for 60% to 78% of the overall gap, depending on the index of unemployment which is used. Note, however, that for these four cantons, the contribution of the two other factors (differences in the average unemployment durations and differences in the inequality of unemployment durations) cannot be ignored, this being especially true for the average unemployment duration.

Analyzing the Impact of the Maximum Unemployment Duration

In this section, we want to analyze the impact that the choice of a maximum duration of unemployment may have on the results of the "Shapley decomposition." For simplicity, we will only consider the case where we take a "backward looking" approach and measure unemployment via the information on the interrupted spells of unemployment as they are observed in the month of December. Here also we limit the analysis to the year 2005.

We will compare three cases:

- The maximum duration is assumed to be 365 days (as in section C).
- The maximum duration of unemployment is that actually observed in December 2005.
- The maximum duration of unemployment is 5000 days, which is slightly above the greatest unemployment duration observed in all years for which data are available (1994 to 2005).

We present the results only for the decomposition for the Sen index of unemployment (S_U) .³

Finally, in each table we first give the actual gap between the value of the index in Switzerland and its value in a given canton, and then the contributions (in percentage terms) of the three determinants of the indices of unemployment, namely the actual unemployment rate, the average duration of unemployment, and the inequality of unemployment (or employment) durations. All these results are presented in Tables 6 through 8.

 $^{^{3}}$ Results relative to the decomposition of the Shorrocks' generalization S_{UR} of Sen's unemployment index and of the Foster, Greer, and Thorbecke FGT index of unemployment are available upon request from the authors.

TABLE 1. "Expected" Interrupted Spells of Unemployment, 2005: Value of the Unemployment Rate (U/N), of the Average Value D_A of the Cumulative Unemployment Durations, and of the Gini Index of Unemployment Durations [G(D_{ij})] for Switzerland and the Various Cantons⁴

Canton	Unemployment rate (U/N)	Average value of the	Gini Index of unemployment
		unemployment	durations [G(D _{ij})]
		durations D _A	
ZH	0.0402	173.50	0.4122
BE	0.0283	153.19	0.4373
LU	0.0307	166.45	0.4211
UR	0.0131	123.26	0.4570
SZ	0.0231	154.88	0.4291
OW	0.0161	126.72	0.4679
NW	0.0196	130.60	0.4600
GL	0.0250	147.73	0.4487
ZG	0.0315	183.39	0.3986
FR	0.0309	158.71	0.4313
SO	0.0337	165.07	0.4151
BS	0.0406	179.56	0.3978
BL	0.0330	177.11	0.3926
SH	0.0328	174.84	0.4084
AR	0.0219	196.02	0.3537
AI	0.0147	155.57	0.4209
SG	0.0297	168.79	0.4080
GR	0.0216	119.32	0.4696
AG	0.0325	168.17	0.4180
TG	0.0307	164.34	0.4137
TI	0.0486	182.25	0.3869
VD	0.0533	209.16	0.3432
VS	0.0396	134.22	0.4725
NE	0.0433	203.05	0.3482
GE	0.0737	234.42	0.2963
JU	0.0422	192.79	0.3715
Switzerland	0.0376	179.68	0.3990
as a whole			

⁴ Similar tables for the other years are available upon request from the authors.

TABLE 2. "Expected" Interrupted Spells of Unemployment, 2005: Value of the Three Indices of Unemployment—the Sen Index (S_U), Shorrocks's Extension (S_{UR}) of the Sen Index, and the Foster, Greer, and Thorbecke (FGT_U)—for Switzerland and the Various Cantons

Canton	Sen Index (S _U) of Unemployment	Shorrocks' Extension (S _{UR}) of the Sen Index of Unemployment	The Foster, Greer, and Thorbecke Index (FGT _U) of Unemployment
ZH	26.95	37.72	13.89
BE	17.08	23.57	8.06
	19.88	27.74	9.95
UR	6.46	8.84	2.57
SZ	14.02	19.50	6.62
OW	8.21	11.14	3.40
NW	10.23	13.95	4.31
GL	14.63	20.06	6.75
ZG	22.14	31.36	11.88
FR	19.24	26.64	9.30
SO	21.55	30.16	10.64
BS	27.93	39.48	14.66
BL	22.32	31.73	11.50
SH	22.11	31.09	11.46
AR	15.89	23.30	8.73
AI	8.92	12.50	4.21
SG	19.31	27.18	9.65
GR	10.40	14.07	4.12
AG	21.24	29.68	10.69
TG	19.57	27.44	9.60
TI	33.66	47.82	17.75
VD	41.06	60.07	23.94
VS	21.45	28.82	9.39
NE	32.47	47.48	18.43
GE	61.33	92.18	38.94
JU	30.57	43.99	16.82
Switzerland as a whole	25.92	36.63	13.64

TABLE 3. "Expected" Interrupted Spells of Unemployment, 2005: Shapley Decomposition of the Difference between the Value of the Sen Index (S_U) for Switzerland as a Whole and for Each Canton

Canton	Gap between the national and cantonal values of the Sen Index (S _U)	Contribution of differences in the unemployment rate [Δ(U/N)]	Contribution of differences in the average unemployment duration per member of the labor force [Δ(D _{LF})]	Contribution of differences in the degree of inequality of the employment durations [∆G(e)]
ZH	1.036	1.714	-0.408	-0.269
BE	-8.840	-6.029	-1.548	-1.263
LU	-6.032	-4.647	-0.779	-0.606
UR	-19.456	-14.561	-2.659	-2.236
SZ	-11.892	-9.404	-1.333	-1.156
OW	-17.703	-12.962	-2.624	-2.117
NW	-15.682	-10.977	-2.584	-2.122
GL	-11.283	-8.095	-1.784	-1.404
ZG	-3.775	-4.262	0.213	0.273
FR	-6.680	-4.411	-1.259	-1.010
SO	-4.363	-2.628	-0.905	-0.829
BS	2.016	2.057	-0.007	-0.034
BL	-3.598	-3.140	-0.155	-0.303
SH	-3.810	-3.316	-0.288	-0.206
AR	-10.032	-11.174	0.802	0.340
AI	-16.999	-14.843	-1.115	-1.041
SG	-6.610	-5.349	-0.632	-0.629
GR	-15.519	-9.417	-3.359	-2.743
AG	-4.672	-3.434	-0.694	-0.544
TG	-6.344	-4.563	-0.914	-0.867
TI	7.743	7.578	0.187	-0.022
VD	15.146	11.474	2.110	1.562
VS	-4.471	1.219	-3.221	-2.469
NE	6.550	4.072	1.523	0.955
GE	35.417	27.542	4.447	3.428
JU	4.654	3.228	0.858	0.568

TABLE 4. "Expected" Interrupted Spells of Unemployment, 2005: ShapleyDecomposition of the Difference between the Value of Shorrocks's Generalization (Sur)of the Sen Index of Unemployment for Switzerland as a Whole and for Each Canton

Canton	Gap between the national and cantonal values of the Foster, Greer, and Thorbecke Index (S _{UR})	Contribution of differences in the unemployment rate [Δ(U/N)]	Contribution of differences in the average unemployment duration per member of the labor force $[\Delta(D_{LF})]$	Contribution of differences in the degree of inequality of the employment durations [ΔG(e)]
ZH	1.091	2.382	-1.281	-0.010
BE	-13.057	-8.338	-4.676	-0.042
LU	-8.896	-6.458	-2.417	-0.021
UR	-27.793	-20.041	-7.683	-0.069
SZ	-17.133	-13.058	-4.038	-0.037
OW	-25.487	-17.786	-7.636	-0.065
NW	-22.685	-15.086	-7.533	-0.066
GL	-16.567	-11.164	-5.358	-0.046
ZG	-5.269	-5.966	0.687	0.010
FR	-9.987	-6.108	-3.844	-0.035
SO	-6.469	-3.656	-2.783	-0.030
BS	2.848	2.872	-0.023	-0.001
BL	-4.900	-4.404	-0.485	-0.011
SH	-5.542	-4.626	-0.909	-0.007
AR	-13.329	-15.943	2.604	0.011
AI	-24.133	-20.715	-3.385	-0.032
SG	-9.448	-7.468	-1.959	-0.021
GR	-22.562	-12.881	-9.595	-0.086
AG	-6.952	-4.775	-2.157	-0.019
TG	-9.191	-6.356	-2.805	-0.030
TI	11.186	10.596	0.591	-0.001
VD	23.437	16.261	7.103	0.073
VS	-7.808	1.661	-9.373	-0.096
NE	10.853	5.782	5.033	0.039
GE	55.546	39.401	15.938	0.207
JU	7.358	4.547	2.788	0.023

TABLE 5. "Expected" Interrupted Spells of Unemployment, 2005: ShapleyDecomposition of the Difference between the Value of the Foster, Greer, and ThorbeckeIndex (FGT_U) of Unemployment for Switzerland as a Whole and for Each Canton

Canton	Gap between the national and cantonal values of the Foster, Greer and Thorbecke Index (FGT _U)	Contribution of differences in the unemployment rate [Δ(U/N)]	Contribution of differences in the standardized average unemployment duration [Δ(f _A)]	Contribution of differences in the coefficient of variation of the standardized unemployment durations (Δf _{ij})
ZH	0.243	0.893	-0.637	-0.013
BE	-5.589	-3.016	-2.182	-0.390
LU	-3.692	-2.387	-1.174	-0.131
UR	-11.071	-6.842	-3.256	-0.973
SZ	-7.028	-4.704	-1.892	-0.432
OW	-10.244	-6.170	-3.273	-0.801
NW	-9.331	-5.256	-3.271	-0.804
GL	-6.892	-4.012	-2.457	-0.422
ZG	-1.760	-2.266	0.350	0.155
FR	-4.344	-2.230	-1.825	-0.289
SO	-3.007	-1.342	-1.347	-0.318
BS	1.013	1.081	-0.012	-0.056
BL	-2.140	-1.636	-0.243	-0.261
SH	-2.187	-1.732	-0.453	-0.003
AR	-4.911	-6.015	1.371	-0.266
AI	-9.432	-7.429	-1.588	-0.415
SG	-3.997	-2.746	-0.958	-0.293
GR	-9.521	-4.422	-4.014	-1.084
AG	-2.955	-1.769	-1.053	-0.133
TG	-4.043	-2.323	-1.354	-0.366
TI	4.103	3.993	0.302	-0.192
VD	10.295	6.375	3.914	0.006
VS	-4.252	0.592	-4.136	-0.708
NE	4.788	2.230	2.717	-0.159
GE	25.293	16.057	9.470	-0.234
JU	3.174	1.738	1.464	-0.028

Let us first consider the first case where the maximum duration of unemployment is still 365 days (Table 6). It appears that although the greatest relative contribution to the unemployment indices is that of the unemployment rates, there are cases where the contribution of the average unemployment duration is quite high in comparison to that of the unemployment rate. When using the index S_U this is, for example, the case of the cantons of Zurich (ZH) and of Valais (VS).

If we now take the case where the maximum unemployment duration is that actually observed in December 2005 (Table 7), we observe a somehow different picture. The cantons of Zurich (ZH) and Valais (VS) are no longer the only ones (for the index S_U) for which the contribution of the average unemployment duration (in percentage term) is important. This is now also the case for the cantons of Basel Stadt (BS), Graubünden (GR), and even Jura (JU).

Finally, the results of the cases where we take 5000 days as maximum unemployment duration (Table 8) are very close to those where the maximum unemployment duration is that actually observed in December 2005 so that we will not analyze them separately.

In all the results what is quite striking is the growing role of the average unemployment duration when a longer maximal unemployment duration is selected and even, to some degree, the more important impact of the inequality of unemployment durations. Take, for example, the case of the canton of Geneva (GE), for which the value of the index of unemployment, whatever the index, is always the highest of all cantons. Here we observed that 60% to 74% (depending on the index) of the gap between the value of the index in Switzerland as a whole and in the canton of Geneva was the consequence of differences in unemployment rates when the maximum unemployment duration is 365 days. However, when a longer maximal unemployment duration is selected (either the maximal duration observed or 5000 days), this impact of the unemployment rate goes down to 60%, sometimes even to 45% (depending on the index). Moreover, even the impact of the inequality in unemployment (or employment) durations is now greater. For the canton of Geneva, depending on the index, it varied from 0% to 11% when the maximum unemployment duration is 365 days. With a greater maximal duration, the impact of this inequality in unemployment durations for the canton of Geneva varies now from 0% to 23%.

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IV. CONCLUDING COMMENTS

This paper attempted to borrow some ideas from the poverty measurement literature to propose some more sophisticated measures of unemployment which take into account not only the unemployment rate, but also the average duration of unemployment and the inequality in the distribution of these durations. It also applied the so-called Shapley decomposition to decompose the difference between the value of an unemployment index at the national and regional level into three contributions reflecting the three aspects of unemployment that have just been mentioned.

An empirical illustration based on Swiss data for the period 1993–2005 seems to confirm the usefulness of such an approach. It also showed the relative sensibility of the decomposition results to the maximum unemployment duration that has been selected.

TABLE 6. Interrupted Spells of Unemployment, 2005: Shapley Decomposition in Percentage Terms of the Difference between the Value of the Sen Index (S_U) for Switzerland as a Whole and for Each Canton, under the Assumption That the Maximum Duration of Unemployment Is 365 Days

Canton	Gap between the actual values of the Sen Index (S _U) at the national and cantonal levels	Contribution to this gap (in percentage) of differences in the unemployment rate [Δ(U/N)]	Contribution to this gap (in percentage) of differences in the average unemployment duration per member of the labor force $[\Delta(D_{LF})]$	Contribution to this gap (in percentage) of differences in the degree of inequality of the employment durations $[\Delta G(e)]$
ZH	1.344	43.68	33.26	23.07
BE	-7.692	63.40	20.23	16.37
LU	-5.114	81.85	9.54	8.60
UR	-18.040	79.92	11.14	8.95
SZ	-10.151	83.28	8.73	7.99
OW	-15.484	67.86	17.20	14.94
NW	-13.521	64.54	18.83	16.63
GL	-10.848	55.46	24.74	19.79
ZG	-4.315	116.73	-8.25	-8.48
FR	-6.787	47.56	28.25	24.19
SO	-5.481	53.56	22.93	23.51
BS	2.855	49.28	27.25	23.47
BL	-3.860	94.48	2.93	2.59
SH	-4.958	48.04	25.55	26.40
AR	-8.234	120.56	-12.30	-8.26
AI	-13.817	85.56	7.23	7.21
SG	-6.474	82.92	8.88	8.20
GR	-14.548	56.94	24.02	19.04
AG	-4.522	78.70	10.71	10.60
TG	-5.362	72.45	12.78	14.77
TI	11.159	101.10	-0.10	-1.00
VD	12.131	74.03	15.02	10.96
VS	-4.021	-148.10	146.13	101.96
NE	4.598	88.28	9.24	2.48
GE	32.034	73.88	14.87	11.24
JU	1.806	127.81	-8.70	-19.11

TABLE 7. Interrupted Spells of Unemployment, 2005: Shapley Decomposition inPercentage Terms of the Difference between the Value of the Sen Index (S_U) forSwitzerland as a Whole and for Each Canton, under the Assumption That theMaximum Duration of Unemployment Is That Observed in 2005

Canton	Gap between the actual values of the Sen Index (S _U) at the national and cantonal levels	Contribution to this gap (in percentage) of differences in the unemployment rate [Δ(U/N)]	Contribution to this gap (in percentage) of differences in the average unemployment duration per member of the labor force $[\Delta(D_{LF})]$	Contribution to this gap (in percentage) of differences in the degree of inequality of the employment durations
711	-0.127	40.61	02 10	$[\Delta G(e)]$
ZH		-49.61	83.46	66.14
BE LU	-1.049 -0.689	49.86	32.51 21.63	17.64
	-0.689	66.62 75.11	16.05	11.76 8.84
SZ	-1.334	67.07	20.26	12.68
OW	-1.884	58.55	26.27	12.08
NW	-1.601	59.34	20.27	13.55
GL	-1.256	53.18	30.89	15.92
ZG	-0.499	113.20	-8.60	-4.60
FR	-0.691	53.48	32.03	14.49
SO	-0.713	45.30	31.56	23.14
BS	-0.047	-314.89	217.02	197.87
BS	-0.594	66.78	15.85	17.37
SH	-0.455	60.35	23.35	16.30
AR	-1.012	107.72	-10.78	3.07
AI	-1.700	73.47	16.06	10.47
SG	-0.854	68.50	17.80	13.70
GR	-1.768	49.75	33.18	17.07
AG	-0.661	58.55	24.36	17.10
TG	-0.772	54.40	25.91	19.69
TI	1.123	111.22	-6.32	-4.90
VD	1.931	56.21	30.07	13.72
VS	-0.535	-122.85	159.18	63.67
NE	0.969	50.57	36.64	12.80
GE	4.924	60.22	27.78	12.00
JU	0.462	58.75	30.24	11.02

TABLE 8. Interrupted Spells of Unemployment, 2005: Shapley Decomposition in Percentage Terms of the Difference between the Value of the Sen Index (S_U) for Switzerland as a Whole and for Each Canton, under the Assumption That the Maximum Duration of Unemployment Is 5,000 Days

Canton	Gap between the actual values of the Sen Index (S _U) at the national and cantonal	Contribution to this gap (in percentage) of differences in the Unemployment Rate [Δ(U/N)]	Contribution to this gap (in percentage) of differences in the average unemployment duration per member of the	Contribution to this gap (in percentage) of differences in the degree of inequality
	levels		labor force [Δ(D _{LF})]	of the employment
				durations
				$[\Delta G(e)]$
ZH	-1.191	-49.41	83.31	66.11
BE	-9.837	49.83	32.57	17.60
LU	-6.458	66.63	21.68	11.69
UR	-19.518	75.08	16.09	8.82
SZ	-12.509	67.06	20.28	12.66
OW	-17.666	58.52	26.32	15.15
NW	-15.016	59.34	27.11	13.55
GL	-11.776	53.23	30.92	15.85
ZG	-4.678	113.38	-8.70	-4.68
FR	-6.476	53.51	32.07	14.42
SO	-6.689	45.29	31.64	23.06
BS	-0.438	-316.86	218.91	197.95
BL	-5.569	66.71	15.93	17.36
SH	-4.268	60.28	23.39	16.33
AR	-9.487	107.65	-10.76	3.11
AI	-15.944	73.44	16.09	10.47
SG	-8.007	68.57	17.77	13.65
GR	-16.581	49.76	33.21	17.03
AG	-6.202	58.50	24.41	17.09
TG	-7.243	54.40	25.90	19.70
	10.530	111.23	-6.31	-4.92
VD VS	18.113	56.21	30.14	13.65 63.42
VS NE	-5.013 9.084	-122.70 50.58	159.29 36.73	12.69
GE	46.181	60.21	27.84	12.09
JU	4.332	58.77	30.29	11.93
JU	4.332	38.//	30.29	10.94

APPENDIX 1 On the Link between the Unemployment Duration Profile Curve and Sen's Index of Poverty (When Applied to the Measurement of Unemployment)

Recall [see expression (6)] that the area OHAB may be expressed as

$$M = (1/2) (U/N) (U/N) D_A (1 + G(D_{ii}))$$
(A-1)

If we now normalize the durations D_A and D_{ij} by dividing them by their maximal value (D_M), we may write that

$$(M/D_M) = (1/2) (U/N) (U/N) (D_A/D_M) (1 + G(D_{ij}/D_M))$$
(A-2)

or as

$$(M/D_M) = (1/2) (U/N) (U/N) (f_A) (1 + G(f_{ij}))$$
(A-3)

where $f_{ij} = (D_{ij}/D_M)$ and $f_A = (D_A/D_M)$.

But $f_{ij} = (1 - e_{ij})$ where $e_{ij} = (E_{ij}/E_M) = (E_{ij}/D_M)$

We may therefore write that

$$G(f_{ij}) = G(1 - e_{ij})$$
 (A-4)

However, using the well-known formulas for expressing the Gini index of a sum of components (see, for example, Silber 1989), we may rewrite $G(1-e_{ij})$ as

$$G(1-e_{ij}) = (1/(1-e_A))(Ps.G(1) + ((-e_A)/(1-e_A))(Ps.G(e_{ij}))$$
(A-5)

where Ps. $G(e_{ij})$) refers to the Pseudo-Gini and e_A is equal to the average value of the standardized employment durations e_{ij} (e_A .= (E_A/E_M)).

Note, however, that the Pseudo-Gini of a vector of the constant 1 is 0 so that the first expression on the R.H.S. of (A-2) is zero. Since (see Silber 1989), in the second expression on the R.H.S. of (A-2), the Pseudo-Gini of e_{ij} implies that the

elements e_{ij} are ranked by decreasing values of the expressions $(1-e_{ij})$, we easily derive that

$$Ps. G(e_{ij})) = -G(e_{ij})$$
(A-6)

Combining (A-4) and (A-6) we obtain:

$$G(f_{ij})) = G(1-e_{ij}) = ((-e_A)/(1-e_A))(-G(e_{ij})) = (e_A/(1-e_A))G(e_{ij})$$
(A-7)

and since by definition $e_A = (1 - f_A)$, we conclude that

$$G(f_{ij}) = ((1-f_A)/f_A) G(e_{ij})$$
 (A-8)

Combining now (A-3) and (A-8) we end up with

$$\begin{split} (M/D_M) &= (1/2) \ (U/N) \ (U/N) \ f_A \ (1 + G(f_{ij})) \\ &= (1/2) \ (U/N) \ (U/N) \ f_A \ (1 + (((1 - f_A)/f_A) \ G(e_{ij}))) \\ &= (1/2) \ (U/N) \ (U/N) \ f_A \ (1/f_A)(f_A + ((1 - f_A) \ G(e_{ij}))) \\ &= (1/2) \ (U/N) \ (U/N) \ (f_A + ((1 - f_A) \ G(e_{ij}))) \\ &\leftrightarrow (M/D_M) = (1/2) \ (U/N) \ S_U \end{split}$$
(A-9)

where S_U is the application to unemployment measurement of the Sen index of poverty [see expression (7)].

Finally, since the area BAMQ is equal to $(1-(U/N))((U/N)D_A$, it is easy to conclude, using (A-9), that the area OHAMQBO is equal to

$$\begin{split} &D_{M} \left[(1/2) (U/N) (U/N) (f_{A} + ((1 - f_{A}) G(e_{ij}))) + (1 - (U/N))((U/N)f_{A}] \right] \\ &= D_{M} \left[(1/2) \left\{ \left[(U/N)^{2} ((1 - f_{A}) G(e_{ij})) \right] + \left[(U/N)f_{A} ((U/N) + 2(1 - (U/N))) \right] \right\} \right] \\ &= D_{M} \left[(1/2) \left\{ \left[(U/N)^{2} ((1 - f_{A}) G(e_{ij})) \right] + \left[(U/N)f_{A} (2 - (U/N)) \right] \right\} \right] \end{split}$$
(A-10)

Expression (A-10) is in fact equal to the application to unemployment measurement of what Shorrocks (1995) has called "The Revisited Sen Poverty Index," which has better properties than Sen's (1976) original index.

APPENDIX 2

On the Concept of the Shapley Decomposition

The concept of the Shapley (1953) decomposition is a technique borrowed from game theory but extended to applied economics by Shorrocks (1999), and Sastre and Trannoy (2002). Let us explain it briefly.

Assume an indicator (*I*) is a function of three determinants (a,b,c) and is written as I = I(a,b,c). *I* could be an index of inequality but more generally any function of variables, this function being linear or not.

There are obviously 3!=6 ways of ordering these three determinants *a*, *b*, and *c*:

$$(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a)$$
 (B-1)

Each of these three determinants may be eliminated first, second, or third. The respective (marginal) contributions of the determinants a,b,c will hence be a function of all the possible ways in which each of these determinants may be eliminated. Let, for example, C(a) be the marginal contribution of a to the indicator I(a,b,c). If a is eliminated first, its contribution to the overall value of the indicator I will be expressed as I(a,b,c) - I(b,c), where I(b,c) corresponds to the case where a is equal to zero. Since expression (1) indicates that there are two cases in which a appears first, and may thus be eliminated first, we will give a weight of (2/6) to this possibility.

If *a* is eliminated second, it implies that another determinant has been eliminated first (and been assumed to be equal to 0). Expression (A-1) indicates that there are two cases in which this possibility occurs, the one denoted in (1) as (b,a,c)and the one denoted (c,a,b). In the first case, the contribution of *a* will be written as I(a,c) - I(c), while in the second it is expressed as I(a,b) - I(b). To each of these two cases we evidently give a weight of (1/6).

Finally, if *a* is eliminated third, it implies that both *b* and *c* are assumed to be equal to 0. Expression (31) indicates that there are two such cases, the one denoted (b,c,a) and the one denoted (c,b,a). Since we may assume that when each of the three determinants is equal to 0, the indicator *I* is equal to 0, we may write that the contribution of *a* in this case will be equal to I(a) - 0 = I(a) and evidently we have to give a weight of (2/6) to such a possibility since there are two such cases.

We may therefore summarize what we have just explained by stating that the marginal contribution C(a) of the determinant *a* to the overall value of the indicator *I* may be written as:

$$C(a) = (2/6)[I(a,b,c) - I(b,c)] + (1/6)[I(a,c) - I(c)] + (1/6)[I(a,b) - I(b)] + (2/6)I(a)$$
(B-2)

One can similarly determine the marginal contribution C(b) of b and C(c) of c and then find out that

$$I(a,b,c) = C(a) + C(b) + C(c)$$
 (B-3)

This Shapley decomposition may also be applied in a similar way to the case where one wants to understand the respective contributions to the change over time in the value of the indicator *I*, this change being written as ΔI , of the variations over time in the values of the three determinants *a*, *b*, and *c*, these variations being expressed as Δa , Δb , and Δc .

In our case, ΔI would refer, for example, to the difference between the value of the Sen index in a given canton and its value in the whole of Switzerland, Δa to the difference between the unemployment rate (K) in the canton and in Switzerland, Δb to the difference between the average unemployment duration in the canton and in Switzerland, and finally Δc to the difference between the inequality in unemployment durations in the canton and in Switzerland.

APPENDIX 3 Names of the Cantons and Their Abbreviation

Abbreviation	Full name of canton
ZH	Zürich
BE	Bern
LU	Luzern
UR	Uri
SZ	Schwyz
OW	Obwalden
NW	Nidwalden
GL	Glarus
ZG	Zug
FR	Freiburg
SO	Solothurn
BS	Basel Stadt
BL	Basel Land
SH	Shaffausen
AR	Appenzell – Ausser Rhoden
AI	Appenzell – Inner Rhoden
SG	Sankt Gallen
GR	Graubünden
AG	Aargau
TG	Thurgau
TI	Tessin
VD	Waadt
VS	Vallis
NE	Neuenburg
GE	Genf
JU	Jura

REFERENCES

- Akerlof, G.A., and B.G.M. Main. 1980. "Unemployment Spells and Unemployment Experience." *American Economic Review* 71(5): 885–893.
- Basu, K., and P. Nolen. forthcoming. "Unemployment and Vulnerability: A New Class of Measures, its Axiomatic Properties, and Applications." in P. Pattanaik, K. Tadenuma, Y. Xu and N. Yoshihara (eds.), *Rational Choice and Social Welfare*. Heidelberg: Springer.
- Bourguignon, F. 1979. "Decomposable Income Inequality Measures." *Econometrica* 47(4): 901–920.
- Champernowne, D. G. 1953. "A Model of Income Distribution." *Economic Journal* 63(250): 318–351.
- Foster, J.E., J. Greer, and E. Thorbecke. 1984. "A Class of Decomposable Poverty Measures." *Econometrica* 52(3): 761–766.
- Jenkins, S. P., and P. Lambert. 1997. "Three 'I's of Poverty Curves with an Analysis of UK Poverty Trends." *Oxford Economic Papers* 49(2): 317–327.
- Paul, S. 1992. "An Illfare Approach to the Measurement of Unemployment." *Applied Economics* 24(7): 739–743.
- Riese, M., and J.K. Brunner. 1998. "Measuring the Severity of Unemployment." *Journal of Economics* 67(2): 167–180.
- Sastre, M., and A. Trannoy. 2002. "Shapley Inequality Decomposition by Factor Components: Some Methodological Issues." *Journal of Economics* Supplement(9): 51–89.
- Sen, A.K. 1976. "Poverty an Ordinal Approach to Measurement." *Econometrica* 44: 219–231.
- Sengupta, M. 1990. Unemployment Duration and the Measurement of Unemployment. Mimeo. Canterbury, New Zealand: University of Canterbury.
- Shapley, L.S. 1953. "A Value for n-Persons Games." in H. W. Kuhn and F. W Tucker (eds.) Contributions to the Theory of Games II (Annals of Mathematical Studies 28). Princeton, NJ: Princeton University Press.
- Shorrocks, A.F. 1992. Spell Incidence, Spell Duration, and the Measurement of Unemployment. Mimeo. University of Essex.

—. 1993. On the Measurement of Unemployment. Mimeo. University of Essex and Southern Methodist University.

. 1995. "Revisiting the Sen Poverty Index." *Econometrica* 63(5): 1225–1230.

—. 1999. *Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value*. Mimeo. University of Essex.

- Silber, J. 1989. "Factors Components, Population Subgroups, and the Computation of the Gini Index of Inequality." *The Review of Economics and Statistics* LXXI(1): 107–115.
- Theil, H. 1967. Economics and Information Theory. Amsterdam: North Holland.
- Watts, H. 1968. "An Economic Definition of Poverty." in D.P. Moynihan (ed.) On Understanding Poverty. New York: Basic Books.
- Zheng, B. 1997. "Aggregate Poverty Measures." *Journal of Economic Surveys* 11(2): 123–162.