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Tinbergen Rules the Taylor Rule

by

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Abstract

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This paper elaborates a simple model of growth with a Taylor-like monetary policy rule that includes inflation-targeting as a special case. When the inflation process originates in the product market, inflation-targeting locks in the unemployment rate prevailing at the time the policy matures. Although there is an apparent NAIRU and Phillips curve, this long run position depends on initial conditions; in the presence of stochastic shocks, it would be path dependent. Even with an employment target in the Taylor Rule, the monetary authority will generally achieve a steady state that misses both its targets since there are multiple equilibria. With only one policy instrument, Tinbergen's Rule dictates that policy can only achieve one goal, which can take the form of a linear combination of the two targets.

Glendower: I can call spirits from the vasty deep.

Hotspur: Why, so can I, or so can any man; But will they come when you do call for them? (Shakespeare, King Henry IV)

The title of this paper contains no solecism or missing conjunction. Tinbergen's (1952) Rule that the number of achievable policy goals can not exceed the number of policy instruments dictates that a mechanical monetary policy rule can fail to achieve its stated objectives of full employment and target inflation. The problem is particularly acute with inflation-targeting, defined by complete indifference to the employment level. This kind of policy can lock in an unemployment rate prevailing at the time the policy matures, as well as create the illusion that this rate represents a unique NAIRU (i.e., the zero of a Phillips curve in difference form) even when the long-run unemployment rate is path dependent. Moreover, the authorities cannot avoid this fate by including an employment target. The family of models that illustrate these statements below is designed to substantiate the misgivings that heterodox economists allegiant to the classical-Keynesian synthesis[1] have about inflation-targeting. Given the rising popularity of this approach to monetary policy, including the appointment of an avowed inflation-targeter, Ben Bernanke, as Chair of the Federal Reserve Board, it would seem appropriate to raise some questions about the wisdom of rule-based monetary policies.

There are four features of the models elaborated here that betray their classical-Keynesian origins. First, they assume that inflation originates in the product market and is propagated through the labor market by a wage-setting process that maintains a constant real wage. (In a more dynamic setting with technical change, this might be replaced with a constant wage share.) This assumption creates openings for path dependency in the employment rate (the

[1]Two recent works that limn out the contours of heterodox macroeconomic theory in an irenic spirit are Taylor (2004) and Foley and Michl (1999). The former puts somewhat more emphasis on the Keynes-Kalecki tradition, the latter on the classical tradition. For another attempt to substantiate the misgivings that heterodox economists have about monetary policy rules, see Freedman et al. (2004).

ratio of employed workers to the labor force)[2].

Second, they assume that all saving originates in profits; workers do no saving. This could easily be relaxed, with modest gains, as long as the class structure of accumulation retains a prominent place in the architecture.

Third, our first model below assumes that the labor force is endogenous, with labor supply elastic at some given reservation wage. Classical-Keynesian economists remain open-minded about whether (or to what extent) labor shortages constrain growth in modern economies, unlike their neoclassical counterparts who seem to take exogeneity of the (fully employed) labor force as part of their scientific dogma. Our second model pursues the theme of labor-constrained growth.

Finally, the production function is strictly fixed-coefficient, which prevents us from sweeping the crucial problem of capacity (or capital) utilization under the rug by means of incredible assumptions about technology.

These models have more Keynesian characteristics in the short run but classical characteristics in the long run, having descended with modification from an effort by Duménil and Lévy (1999) to distinguish between the fast adjustment process associated with effective demand and the slow adjustment process associated with accumulation.

1 Elements of the models

A central bank reaction function implies that the monetary authorities recognize that the inflation process depends on a Phillips curve-like relationship[3] and that

[2]In the most famous classical model of the labor market and accumulation, the Goodwin (1967) model, the unemployment rate and the wage share cycle around a 'center'. Formally, this means that wherever in the phase space the system begins, it will come right back around to the same point. Any displacement, in other words, would be permanent.

[3]We will refer to this as the Phillips curve, even though Phillips (1958) himself was concerned with the relationship between wage inflation and unemployment. It seems that general usage has expanded the term to include the relationship between price inflation and unemployment.

the rate of capacity utilization depends on the interest rate, through an IS-curve. We will model the inflation process (with p representing the inflation rate) as a function of the rate of capacity utilization, u,

$$p_{+1} = p + a(u - 1) \tag{1}$$

For convenience, we define the normal or desired rate of utilization as unity, and suppress time subscripts where they can easily be inferred. Note that normal utilization does not represent full utilization in an engineering sense. Firms are assumed to build capacity slightly ahead of demand, for example so as to accommodate fluctuations in orders without losing customers. The assumption is that they will respond to high demand partly by stepping up production and partly by raising prices faster in the next period. Inflation thus has an inertial element, perhaps because of expectations-formation or some other slow process.

Most textbook presentations of the Phillips curve (often unwittingly) make the implicit assumption that full employment in the labor market and normal capacity utilization (full employment of capital) correspond. In other words, they presume that sufficient capital has accumulated to make full employment possible; the well-behaved neoclassical production function is one device for achieving this legerdemain. By contrast, this paper is preoccupied with getting right the relationship between these two measures of slack.

Money wages are assumed to respond to prices one-for-one so that their ratio, the real wage, remains constant. Thus, the distribution of income is parametric. We will use π to represent the profit share. For simplicity, we assume, without loss of much generality, that workers live hand-to-mouth and consume their real wage, $w = (1 - \pi)x$, where x is labor productivity. We will assume that changes in utilization of capacity are Solow-neutral: they leave output-per-worker unchanged and they are fully reflected in the output-capital ratio. Empirical evidence suggests this is not too far from the truth, although labor hoarding and other effects may cause violations in practice.

The monetary authority operates according to a fixed Taylor-type rule, or a central bank reaction function[4]. Its policy instrument is the real interest rate, R. Because of decision and implementation lags, we assume that it responds to current conditions by setting the prevailing rate for the next period. One good justification for this lag is that central banks control only the short-term interest rate on interbank lending while the long-term rates that govern investment spending change much more slowly as expectations are digested by financial markets. In general, we write the central bank reaction function as:

$$R_{+1} = R_n + h_0(e - \bar{e}) + h_1(p - \bar{p}) \tag{2}$$

where R_n is the neutral (or natural) rate of interest, e is the employment rate, and bars identify target values.

To obtain an IS curve, we make use of an investment equation that is the donkey engine of neo-Kaleckian modeling. Investment is responsive to the degree of utilization, on the grounds that high utilization signals that demand is expanding faster than capacity. This equation can also be interpreted as an error-correction response function for investment, sensitive to deviations from the normal rate of utilization[5]. We will include the interest rate, on the grounds that investment that cannot be financed through internally generated funds (profits) will be sensitive to credit conditions. Normalizing by the capital stock, we have

$$\frac{I}{K} = d_0 - d_1 R + d_2 u$$

For simplicity, we will assume that a constant proportion, s of profits are saved. Thus, saving normalized by the capital stock is

$$\frac{S}{K} = s\pi\rho u$$

[4]This is not precisely the rule Taylor (1999) had in mind; he works with the nominal interest rate, for example, which places different restrictions on the inflation-coefficient needed for stability. And his rule targets the output gap (derived from the full employment level of output), rather than employment itself. We will take liberty on occasion and loosely refer to the reaction function as the "Taylor Rule."

[5]For a very clear presentation of recent debates about how to interpret this investment equation, consult Lavoie et al. (2004).

where ρ is the normal output-capital ratio, sometimes referred to as capital productivity.

The short run is assumed to be long enough to permit changes in utilization that eliminate any excess demand in the product market. Equating planned investment and saving, we obtain the IS curve

$$u = \frac{d_0 - d_1 R}{c} \tag{3}$$

where $c = s\pi\rho - d_2$ represents the marginal excess saving generated by an increase in utilization. Stability of the short-run adjustment mechanism (i.e., the multiplier) requires that $c > 0$, and we will assume that this condition prevails. We also assume that the monetary authority knows the structure of the IS curve, and can determine that the neutral rate of interest is

$$R_n = \frac{d_0 - c}{d_1}$$

Having determined the utilization rate in any short-run period through the IS equation, the rate of capital accumulation, g, can be obtained from either the investment or the saving equation above.

Note that this model operates along standard Keynes-Kalecki lines in the short run. The principle of effective demand reigns: investment determines saving through changes in utilization. An autonomous increase in investment (an upward shift in the intercept term of the investment equation) has a multiplier effect on utilization in the short run. An increase in the propensity to save has a deflating effect on utilization, sometimes called the paradox of thrift. A decrease in the real wage, or equivalently an increase in the profit share, also has a deflating effect on utilization, sometimes called the paradox of costs. (Firms experience a decline in their real labor costs, yet they wind up reducing output.) It is apparent that the paradox of costs is really a variant of the paradox of thrift; it occurs because a redistribution toward the high-saving category of income (profit) raises the social saving rate[6].

[6] Leaving profitability out of the investment equation, as we have done, makes the paradox

In the long run, as we will see, the model gravitates (or, to be more precise, can gravitate) toward the growth rate specified by the saving function, which of course is (a version of) Harrod's warranted rate of growth. With utilization at its normal level, the warranted rate is:

$$g^* = s\pi\rho$$

For ease of exposition, let us assume that the capital-employed labor ratio (usually denoted by k) equals unity; one unit of capital employs one unit of labor (we can always choose units so that this is true). We can define the employment rate, e, as the ratio of employed workers to the labor force, L. Finally, define the ratio of capital to the labor force (*not* to employment, which is why we use a Greek letter) as κ. Now we can see that the employment ratio depends on the utilization rate and the capital-labor force ratio, or

$$e = u\frac{K}{L} = u\kappa$$

For future reference, note that the unemployment rate will be $1 - e$. We work mostly with the employment rate. We will also ignore the fact that there is in principle an upper bound on the employment rate. A value greater than unity might reflect overtime or moonlighting.

We know that $u(R)$ evolves with the interest rate. Thus, to gain some appreciation of the dynamics of employment, we can concentrate on the capital-labor force ratio. Assuming that the rate of growth of the labor force is n, this ratio will grow by a factor $(1 + g)/(1 + n)$ each period. Substituting from the IS equation and the investment function, we can obtain the difference equation

$$\kappa_{+1} = (A - BR)\kappa$$

where $A = (1 + d_0 + d_0 d_2/c)/(1 + n)$ and $B = (d_1 + d_1 d_2/c)/(1 + n)$. We will model the labor force growth rate first as a variable, in which case this equation

of costs a foregone conclusion. Including a term for the profit share produces a richer array of outcomes. For a lucid survey of what is known about the paradox of costs, see Blecker (2002).

will be unnecessary. But when we model the labor force in the traditional way (with a natural rate of growth), this equation will play a central role.

The three equations, (1)-(3), define a system of autonomous non-linear difference equations in the general form

$$\begin{aligned} p_{+1} &= p(p, R) \\ \kappa_{+1} &= \kappa(\kappa, R) \\ R_{+1} &= R(p, \kappa, R) \end{aligned} \tag{4}$$

This system will have different dynamic properties depending on the closure with respect to the labor market.

2 Endogenous growth constrained by capital

Both classical (Foley and Michl, 1999) and Keynesian (Thirlwall, 2002) economists are open to the view that capitalist economies operate in a labor-surplus environment. Under these conditions, at least as a first approximation it makes little sense to include an employment target. We pursue the behavior of equation system (5) under the hypothesis of pure inflation targeting ($h_0 = 0$).

2.1 Stability and dynamics

In this case, the capital-labor force ratio, κ, drops out of the system altogether since it has no role to play. We are left with a two-dimensional linear autonomous system of difference equations:

$$\begin{pmatrix} p_{+1} \\ R_{+1} \end{pmatrix} = \begin{pmatrix} 1 & -ad_1/c \\ h_1 & 0 \end{pmatrix} \begin{pmatrix} p \\ R \end{pmatrix} + \begin{pmatrix} a(d_0/c - 1) \\ R_n - a\bar{p} \end{pmatrix} \tag{5}$$

The eigenvalues ($i = 1, 2$) of this system are

$$\lambda_i = \frac{-1 \pm \sqrt{1 - 4ad_1h_1/c}}{2}$$

The stability of the equilibrium of the system requires that the eigenvalues lie within the unit circle, with modulus less than unity. This condition reduces to

$$h_1 < \frac{c}{ad_1}$$

This stability condition makes some sense. An increase in the slope of the IS curve (decrease in c), an increase in the sensitivity of the inflation process (a), or an increase in the interest sensitivity of investment (d_1) all demand that the monetary authorities become less aggressive about inflation-fighting, lest they cross the tipping point and become a destabilizing influence.

A more meaningful constraint on policy is probably the condition that separates a stable node from a stable focus, which is that the discriminant be positive. Ruling out repeated over- and undershooting the steady state path imposes more restraint on the monetary authorities:

$$h_1 < \frac{c}{4ad_1}$$

In light of the concern voiced by central bankers over achieving what Alan Greenspan has called a "glide path" to a "soft landing," this is probably the more relevant constraint.

It is easy to see that the fixed point or equilibrium (assuming that it exists) of this little three-equation model occurs where inflation achieves its target level, $p = \bar{p}$, utilization achieves its normal level, $u = 1$, and the interest rate is neutral $R = R_n$. The growth rate of capital will than be given by the justly-famous Cambridge equation (in inverted form),

$$g^* = s\pi\rho = sr_n$$

where r_n represents the normal rate of profit. It is also clear that the monetary authorities must know the neutral rate of interest, $R_n = (d_0 - c)/d_1$, in order to achieve their target rate of inflation, something that may not be that easy in practice.

[Figures can be found at the end of the paper.]

Now we can substantiate the claim that this model operates along classical lines in the long run. An increase in capitalist saving, for example, will raise the rate of accumulation in the long run even though its impact effect, in the short run, will be to reduce utilization and growth. A decrease in the profit share (i.e., a real wage increase) will lower the long-run growth rate even though it activates the paradox of costs in the short run. Figure 1 illustrates a wage increase. Both growth and utilization rise in the short run (a response sometimes called "wage-led growth"), but recover their long-term values; the growth rate is permanently lower.

Of course, it is formally possible that an equilibrium with normal utilization could fail to exist, if the IS curve is improperly positioned. To rule this out, the investment equation must satisfy the inequality $d_0 + d_1 > g^*$. If this inequality is violated, the system falls into a Krugman-style liquidity trap that requires the monetary authorities to target a negative real interest rate, something which has proved difficult to achieve in practice. Dumenil and Levy (1999) avoid this thorny issue by setting up their model in terms of the money supply, effectively taking the position that it is the availability of finance or liquid capital that constrains the investment process. One advantage of the present set-up is that it creates a bridge to the structuralist Keynesian models, which could be interpreted as applying to a world in which the investment inequality is persistently violated because of a deficit of entrepreneurial animal spirits.

Finally, consider another trail that leads back to Harrod: the reaction function and Dumenil and Levy's money supply process are both needed to contain the Harrodian Instability that results when the warranted rate and actual rate diverge. The basic problem can be seen by thinking about the investment function as a short-run or temporary expediency. If the system settles into a short-run equilibrium, say above normal utilization, we might expect some kind of low-frequency response (not modelled here) by managers that would shift the function upward so that capacity grows faster than output. But this will just lead to a higher level of utilization in the next period. If we want to maintain this interpretation of the investment function[7], some mechanism must be incorporated to prevent a knife-edge equilibrium.

2.2 The labor market

This model elides the labor market from its inner mechanisms. To add realism, we can add a labor supply component, but this is just embroidery, like the finishing touches on a Thomas Kinkade painting. Let us assume that workers observe the current equilibrium, and make a decision about whether to participate that will be realized in the next period. There may be migration or immigration effects, for example, which account for the lag in labor supply response. Workers supply labor elastically at a reservation wage, w^r, which obviously must be less than the existing wage. They participate whenever the *expected* wage equals or exceeds the reservation wage. The expected wage is just the employment rate, reflecting the probability of securing employment, times the wage, or ew. Workers are effectively myopic, projecting the current employment level into the future through a form of adaptive expectations[8]. Equating the expected wage and the reservation wage then gives us the labor supply function:

$$L_{+1} = \left(\frac{w}{w^r}\right) eL$$

Beginning from some initial condition, $L(0)$, this equation couples up with the system above, equations (5), to generate the path of the labor force and the employment rate. Since the system eventually achieves steady state growth at the rate g^*, we can see that the labor force will also achieve this growth rate.

[7] Again, Lavoie et al. (2004) provides a succinct overview. Their resolution, more in the Keynesian spirit, is to allow managers to form expectations adaptively about what level of utilization is normal, so that in a conflict situation, the normal level of utilization adjusts toward the actual level.

[8] There is an obvious coordination problem here that we are ignoring. My favorite dance club is Dullsville when no one shows up, and a virtual soccer riot when it is too crowded. In between, it's terrific. Somehow, the clientele manage to hit that golden mean without any central direction.

Dividing both sides by L, rearrange, and we obtain the steady state employment rate:

$$e^* = \left(\frac{w^r}{w}\right)(1 + g^*)$$

This makes some sense. A higher wage, relative to the reservation wage, makes it more attractive to take a chance on participation. There will be more "wait unemployment" as a result, or equivalently, a lower employment rate. Faster growth raises the employment rate, apparently because of the rather naive treatment of expectations.

The important point is that a supply-based theory of the equilibrium unemployment rate is quite possible, and the model looks and feels like a real economy. In particular, the unemployment rate will show no historical trend, just as it does in real capitalist economies for which the statistical record goes back far enough, such as the U.K. (Layard et al., 1991). At this point we might be tempted to imagine that the monetary authority would want to include an employment target in the reaction function, but let us resist that temptation and turn to the alternative closure with an exogenous or predetermined labor force.

3 Exogenous growth constrained by the labor force

The most common assumption in growth models of the last half century has been that the labor force grows at its "natural" rate, n. The great Keynesian economist Roy Harrod would recognize the question that confronts us: what is the relationship between the warranted rate, $s\pi\rho$, and the natural rate? Let us finesse the question by assuming that the warranted and natural rates are equal, perhaps because the Kaldor-Pasinetti mechanism involving changes in the distribution of income has achieved this result[9]. Now the question is, will

[9] Alternatively, we might hypothesize that the natural rate adjusts to the warranted rate, putting us back in an endogenous growth setting.

the level of employment match the labor force?

3.1 Inflation-targeting

When the monetary authority pursues pure inflation targeting, the employment rate will be dependent on initial conditions. If there were any stochastic shocks, say to the labor force, the employment rate would be path dependent.

To see this, first let us recognize that the dynamics of the model continue to be controlled by the equation system (5) for the endogenous model above. Without any way for the employment rate to affect monetary policy or inflation, it will just tag along behind capital accumulation and capacity utilization. Assume for the sake of realism that inflation-targeting is adopted out of the steady state (e.g., during a period of putatively excessive inflation). Once the policy matures, and the system enters its steady state path, the rate of utilization will be at its normal level ($u = 1$) and the employment rate will equal the capital-labor force ratio, or $e^* = \kappa^*$. But the capital-labor force ratio obeys the difference equation

$$\kappa_{+1} = \frac{1+g}{1+n}\kappa$$

When the system enters the steady state, of course, $g = n$, and this equation reduces to

$$\kappa_{+1} = \kappa$$

or in other words, the capital-labor force ratio is a unit root process.

This means that the capital-labor force, and ultimately the employment ratio upon which it depends, would be path dependent in the presence of stochastic shocks. In the present, deterministic setting, they will settle down to whatever value they happen to have when the inflation-targeting regime matures and the system enters its steady state. If the monetary authority imposes inflation-targeting by choosing the current inflation rate as its target precisely when the system is in a steady state, then whatever the employment rate happens to be at that moment will remain the employment rate in the future.

Moreover, the inflation rate and unemployment rate $(1-e)$ will obey a Phillips-like relationship, complete with an apparent "natural rate of unemployment" or NAIRU. Substituting into the true Phillips curve, and solving for the change in inflation gives

$$\Delta p = a\left(\frac{1-\kappa}{\kappa}\right) - \frac{a}{\kappa}(1-e)$$

This could be called the pseudo-Phillips curve (in difference form) for this system. An unsuspecting econometrician might be tempted to estimate this relationship, thinking that it was generated by a stochastic process, when in fact, it is only reflecting the co-movements in u, κ, and p. Figure 2 illustrates through a numerical example, jazzed up by setting the response parameter above the threshold for a negative discriminant. In this case, unlike the endogenous model above, the zero of the Phillips curve (in its difference form), which represents the NAIRU or inflation-neutral unemployment rate, is path dependent and could be changed if the political will were available, thus compounding the damage done by rote econometrics.

This model illustrates why heterodox economists are suspicious of the claims (Bernanke, 1999) sometimes made on behalf of inflation targeting. In this system, a monetary authority that chooses to impose inflation targeting when the employment rate happens to be low will have effectively locked in mass unemployment. Any element of path dependency or hysteresis in the inflation-neutral unemployment rate validates this concern. The accumulated evidence may be frustratingly ambiguous on the behavior of the inflation-neutral unemployment rate, but a disturbing amount of empirical work reveals that it has path dependent (or at least time variant) qualities[10]. The common-sense view that high unemployment in Europe may have something to do with low growth rests comfortably with this model.

[10]The classic Layard et al. (1991) is a good source. More recently, the essays in Howell (2005) should undermine any misplaced enthusiasm for the empirical foundations of the orthodox, or any, theory of the natural rate of unemployment.

3.2 Taylor-like rule

Adding an employment target into the central bank reaction function seems like an easy fix, but it turns out that this guarantees little. Like Owen Glendower in the piquant prefatory quotation, the monetary authorities are destined for disappointment.

We are now in a position to examine the full model written out in equation system (5). First, aside from the trivial solution, it is clear that the steady state or equilibrium values will be

$$u^* = 1$$

$$R^* = R_n$$

$$g^* = n$$

In order to secure stable inflation, utilization must clearly be at its normal level (the inflation function is "accelerationist" with respect to utilization). This in turn requires that the interest rate achieve the neutral rate, and these two results together imply the third equality.

Inflation and (un)employment are a different story. They are constrained by the reaction function, equation (2), which settles down, as we just saw, on the neutral rate of interest. This implies that the equilibrium employment rate and inflation rate are going to be found on the line segment:

$$p^* = \bar{p} + (h_0/h_1)\bar{e} - (h_0/h_1)e^* \tag{6}$$

where $e^* \geq 0$ and, (let's say) $p^* \geq 0$. It is true that this segment contains the target point, $(\bar{p}, \bar{e})$. But it also contains an uncountable infinity of other points. It represents the *terminal surface* of the system. Where the system winds up is a contingency of initial conditions, implying that shocks render both the steady state inflation rate and the capital-labor force ratio (which determines the steady state employment rate) path dependent.

It is not unlikely that the central bank, following a Taylor-like rule, will achieve a stable position with excessive inflation (by its standards) and excessive unemployment (or inadequate employment) if the policy is initiated during a period of high inflation and low employment. Once again, a central bank with an overweening attachment to policy rules would have realized the worst fears of heterodox economists by locking in mass unemployment. If employment is satisfactory, but the authorities choose to disinflate by setting the inflation target below current inflation, the system will never recover that satisfactory employment level; workers will suffer unemployment of iatrogenic origin.

Figure 3 brings this point to life. This time the reaction function's parameters have been set within their thresholds (see below) for a nonnegative discriminant. There is some alternation in the real variables, but a monotonic path of disinflation prevails along the model's transient. Note that the previous simulation, with pure inflation-targeting, would look very much the same, except the terminal surface would be the horizontal line at $\bar{p}$. Since the transient moves in a southwesterly direction, it seems intuitive that a pure inflation-target would create more iatrogenic unemployment.

If the target rate of inflation were set above the current rate, the system could gravitate toward a point on the terminal surface to the right (southeast) of the goal post, $(\bar{e}, \bar{p})$, in Figure 3. In other words, monetary policy could actually raise the level of employment. Heterodox concerns that inflation targets tend to get set too low are not without foundation.

As above, it is possible (with some difficulty) to analyze the stability conditions imposed on the parameters of the central bank reaction function. Since the system is non-linear, the first step is to obtain the Jacobian of equation (5), evaluated at the equilibrium point (i.e., some point lying on the terminal surface defined above). The stable space consists of all the (h_0, h_1) pairs that satisfy the condition that the roots (eigenvalues) of the Jacobian lie within the unit disc. This space identifies local asymptotic stability only; global stability of non-linear systems can get tricky (Elaydi, 2005, Ch. 5). Some details are provided in the Appendix. The important point is that the stable space is not empty.

It is also possible to determine what values of the parameters of the reaction function iron out any oscillations, as we did above in the simpler case of a pure inflation target. In this case, the characteristic equation is cubic, and the relevant question is whether the polynomial discriminant is nonnegative (all roots are real) or negative (one real root, two complex conjugates). Thanks to the work of some brilliant Italian mathematicians in the 16th century (Cardano, Tartaglia), all the heavy lifting has been done and made available on the website of some brilliant mathematicians in the 21st century (Wolfram, Weisstein). Its hard to say with any degree of generality, but it appears that taming the oscillations calls for low values of h_0. On the other hand, h_1 needs to be chosen with the Goldilocks principle in mind: it can be too large or too small. Again, some details are provided in the Appendix.

3.3 Employment targeting

We might ask whether a populist[11] central bank could do better by adopting pure employment-targeting, letting the inflation rate seek its own level. The answer is no. Just setting $h_1 = 0$ in the reaction function results in an ill-conditioned model. By cutting equation (1) out of the system, we also cut utilization out of the system's feedback loops. The system itself becomes uncontrollable as a result.

And if the central bank replaced the inflation target by a utilization target in its reaction function, controllability would be restored but at a price. In this case, the terminal surface is a linear combination of the equilibrium unemployment rate and the equilibrium utilization rate. By analogy with the behavior of the system under a Taylor rule above, it is clear that only by a fluke will both targets be achieved. As a result, the inflation rate will either rise steadily (if the utilization rate settles into a value above unity) or drop continuously. The fact that the inflation rate refuses to seek its own level makes pure employment-targeting unworthy of further consideration in the context of this model (though

[11] The premise here is that workers and the poor are generally not hurt much by inflation, but bear the brunt of unemployment, so a central bank that identified with those groups might adopt a pure employment target, or at least deemphasize inflation.

it may be worthy in some other context).

4 Conclusion

In effect, the Taylor Rule suffers from a deficit of instruments in this model. It only allows the authorities one tool (the interest rate) for regulating growth directly. Thus, it is possible to hit a pure inflation target, but adding an employment target forces the system toward a stable point that combines the two targets. Indeed, the target we do achieve is a linear combination, expressed in equation (6), of the two nominal goals set by the monetary authority, exactly what you might expect from Tinbergen's Rule.

The authorities cannot expect to achieve their goals by implementing a Taylor Rule mechanically, but this model does permit the capital stock to be used as a "state variable" in the sense of control theory. Setting up the monetary authority's task as an optimal control problem might be one way to overcome the inadequacy of rule-based policy, and one lesson might be that the authorities should apply the response coefficients flexibly. In the language of optimal control theory, the monetary authority needs to take account of the terminal (transversality) condition. Otherwise, it will be destined to "efficiently" guide the economy toward an unwanted destination, as we have seen it do under a mechanical policy rule. But this raises the question, what objective function should a monetary authority optimize in a society with such divergent economic interests (workers, managers, rentiers, financial capital, industrial capital, retirees, to name a few)? Answering will require taking on some refractory issues surrounding the costs and benefits (and their distribution) of unemployment and inflation[12]

[12] It has always struck me that the Federal Reserve Board, perhaps the largest employer of Ph.D. economists in the USA, has produced no prominent corpus of research on the real costs of unemployment (including, for example, their health effects), and very little balanced work on the costs of inflation. Would it be unreasonable to suggest that resources directed toward these questions could have a salubrious effect, especially if they incorporated the full spectrum of viewpoints?

One final point concerns the original formulation of the inflation process in terms of the rate of capacity utilization. The main results would change materially if we replaced that formulation with the traditional Phillips curve, using the (un)employment rate as the measure of slack. In that case, the unemployment rate would have to get down to its inflation-neutral level in the steady state. And capacity utilization would be taken care of by the reaction function, assuming that the authorities have a good idea what the neutral rate of interest that achieves full utilization actually is. So even pure inflation targeting could work as its advocates have claimed, achieving "full employment" and controlling inflation.

The traditional Phillips curve attributes the source of inflation to the labor market. In the version of it that makes sense to heterodox economists, inspired by Rowthorn (1977), class conflict over the real wage erupts in rising inflation when workers are in a strong bargaining position as the result of low unemployment. Firms operate as Kalecki originally specified they would, setting prices as a mark-up over labor costs and thereby acting as a transmission mechanism for wage-inflation to propagate price-inflation. But there is nothing in this theory to prevent hysteresis effects from creating path dependencies in the inflation-neutral rate of unemployment.

The point is that inflation could well be either a product market phenomenon as it is here or a labor market phenomenon. Both theories have the numen of truth about them. Does it make sense to bet the job security of millions of workers on which one is closer to the truth, or that the inflation-neutral equilibrium (if there is only one) is completely path-independent? The models presented here suggest that it would be wise for central bankers to remain flexible, to retain employment targets as well as inflation targets, and to remain careful students of the evolving art and science of macroeconomics in all its pluralist manifestations.

APPENDIX

The Jacobian of equations (5), evaluated at the equilibrium employment rate, interest rate, and rate of utilization, is:

$$J = \begin{pmatrix} 1 & 0 & \frac{-ad_1}{c} \\ 0 & 1 & \frac{-nd_1e^*}{c(1+n)} \\ h_1 & \frac{d_0(h_0-h_1)-ch_1}{c} & \frac{-h_0d_1e^*}{c} \end{pmatrix}$$

The characteristic equation for the Jacobian is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda^3 + a_3 = 0$$

where the coefficients take the somewhat barbarous forms:

$$a_1 = \frac{d_1e^*h_0}{c} - 1$$

$$a_2 = \frac{d_1(ach_1(1+n) - e^*(c(h_0(1+n) + h_1n) + d_0n(h_1 - h_0)))}{c^2(1+n)}$$

$$a_3 = \frac{d_1e^*n(ch_1 + d_0(h_1 - h_0))}{c^2(1+n)}$$

Gandolfo (1997, pp. 90–91) points out that recent work on cubic equations has whittled the number of necessary and sufficient conditions for the roots of the characteristic equation to lie within the unit circle down to three, basically working out the implications of the Schur-Cohn criterion. These are:

(i). $1 + a_1 + a_2 + a_3 > 0$. This condition reduces to:

$$\frac{ad_1h_1}{c} > 0$$

which will always be satisfied (see text ruling out pure employment-targeting, or $h_1 = 0$).

(ii). $1 - a_1 + a_2 - a_3 > 0$. This condition reduces to a linear inequality of the form $h_0 < b_0 - b_1h_1$, with b_0 and b_1 functions of the parameters.

(iii). $1 - a_2 + a_1a_3 - a_3^2 > 0$. This condition reduces to a polynomial inequality in the form $h(h_0, h_1) < 0$.

The intersection of the sets defined by conditions (ii) and (iii) form the stable set of reaction function parameters. The stable set is illustrated in Figure 4 for the parameter values used in the simulations reported in the paper.

The condition for real roots to the characteristic equation makes use of Cardano's Formula and the polynomial discriminant defined by

$$D = Q^3 + P^2$$

where $Q = (a_1^2 - 3a_2)/9$ and $P = (2a_1^3 + 27a_3 - 9a_1a_2)/54$. These formulas, complete with historical background, were obtained from the World of Mathematics website (http://mathworld.wolfram.com) maintained by Wolfram Research and authored by Eric Weisstein. I have taken the liberty of reversing the signs given there, in order to maintain the convention associated with the quadratic equation that a negative discriminant gives complex roots.

The necessary and sufficient condition for all three roots to be real is that $D \geq 0$. This reduces to a polynomial inequality in the form $D(h_0, h_1) \geq 0$. Examination of its properties for the parameter values used in numerical simulations supports the statements about the threshold values of h_0 and h_1 reported in the text.

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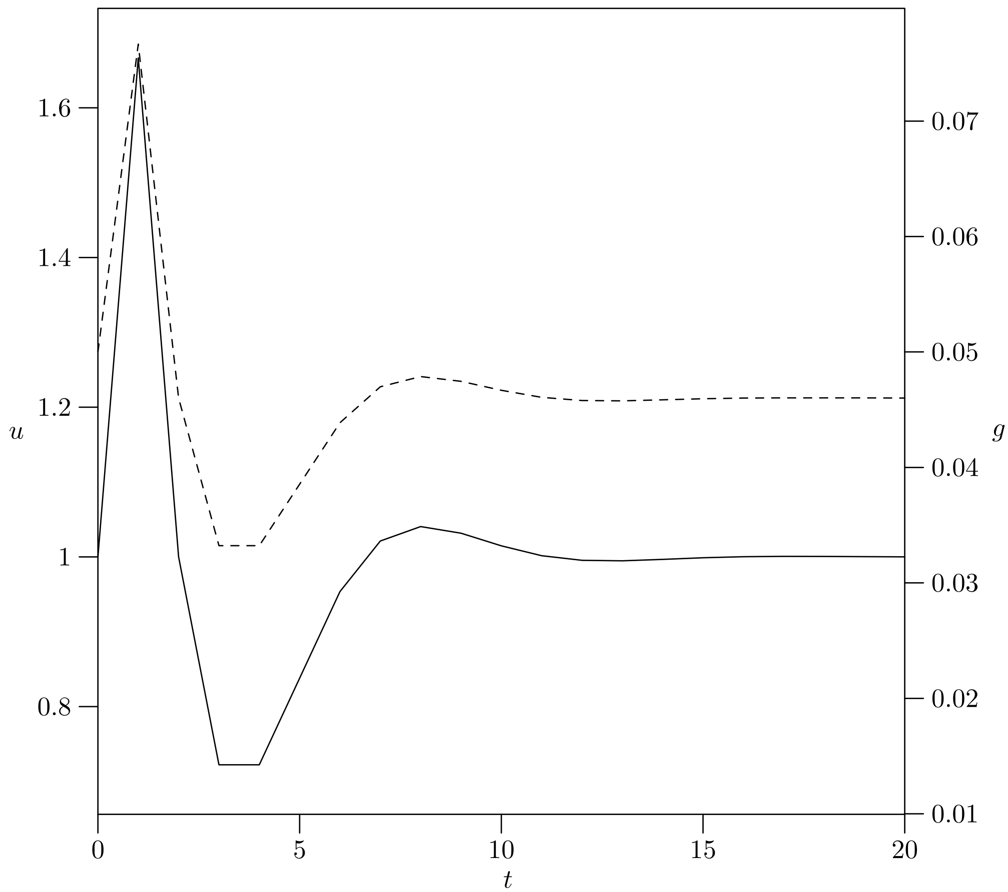

Figure 1: In the short run, a real wage increase has Keynes-Kaleckian effects on the utilization rate (left scale, solid line) and rate of accumulation (right scale, dashed line) but classical effects in the long run. The system began in a steady state, with $u(0) = 1$ and $g(0) = .05$. The profit share was reduced in period 1 forward from .0625 to .0575, and the neutral rate of interest was recalibrated in period 2 forward. The parameters are $h_0 = 0$, $h_1 = .05$, $\bar{p} = .05$, $s = .8$, $\rho = 1$, $d_0 = .1$, $d_1 = .1$, $d_2 = .04$, and $a = .5$. (Data have been joined by lines for visual effect only; the models are in discrete time.)

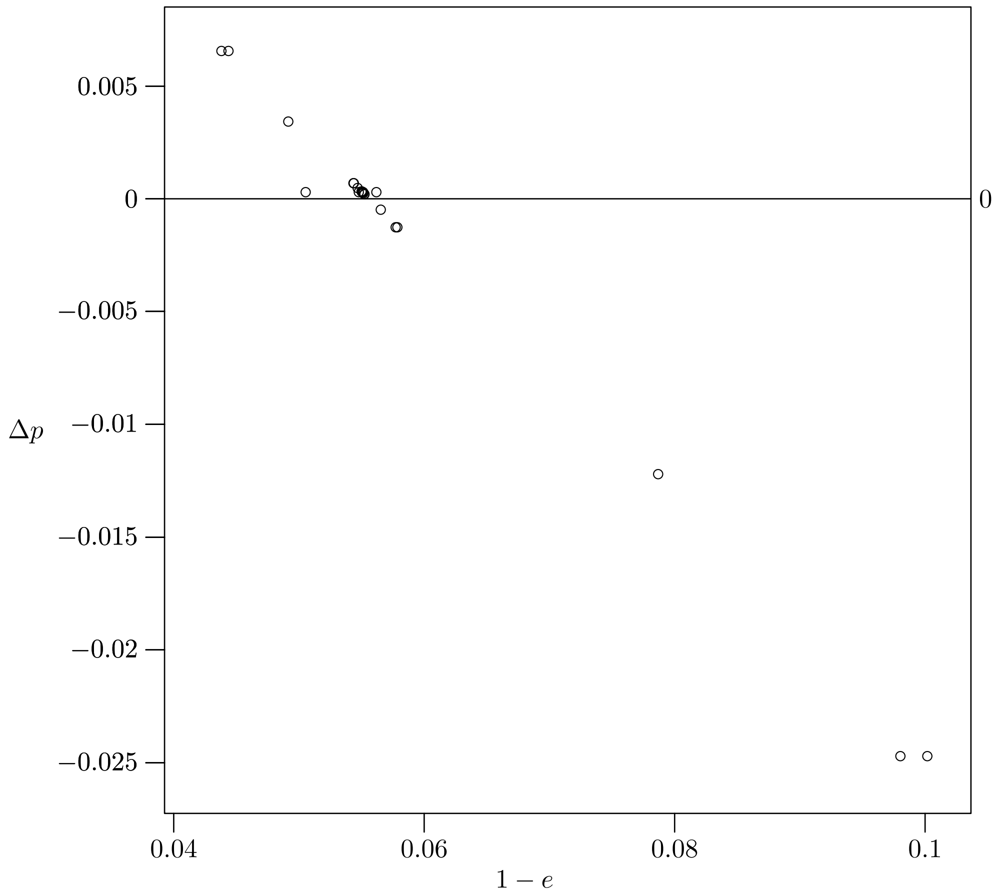

Figure 2: A pseudo-Phillips curve is generated by an episode of disinflation. In this simulation, the inflation target is 0.05, and the employment target is 0.95, making the unemployment target 0.05. The apparent inflation-neutral rate of unemployment is around 0.055. The system began with $p(0) = .1$ and $e(0) = .95 = \bar{e}$. The parameters are $h_0 = 0$, $h_1 = .1$, $n = .05$, $s = .8$, $\rho = 1$, $d_0 = .1$, $d_1 = .1$, $d_2 = .04$, and $a = .5$.

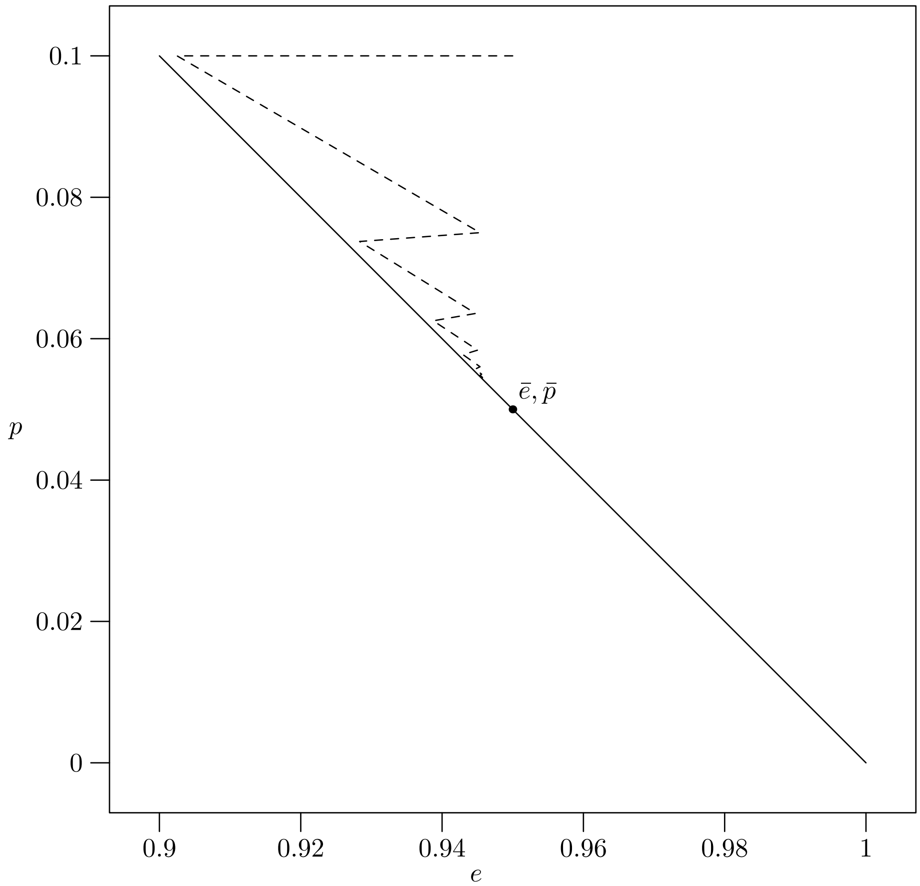

Figure 3: A Taylor-like rule, imposed when the system is operating at the target employment rate ($\bar{e}$) but above its target inflation rate ($\bar{p}$), generates a trajectory (dashed line) toward the terminal surface (solid line) created by the central bank reaction function, stopping short of the target point. All parameters are the same as in the previous figure, except $h_0 = .1$. (Data have been joined by lines for visual effect only; the models are in discrete time.)

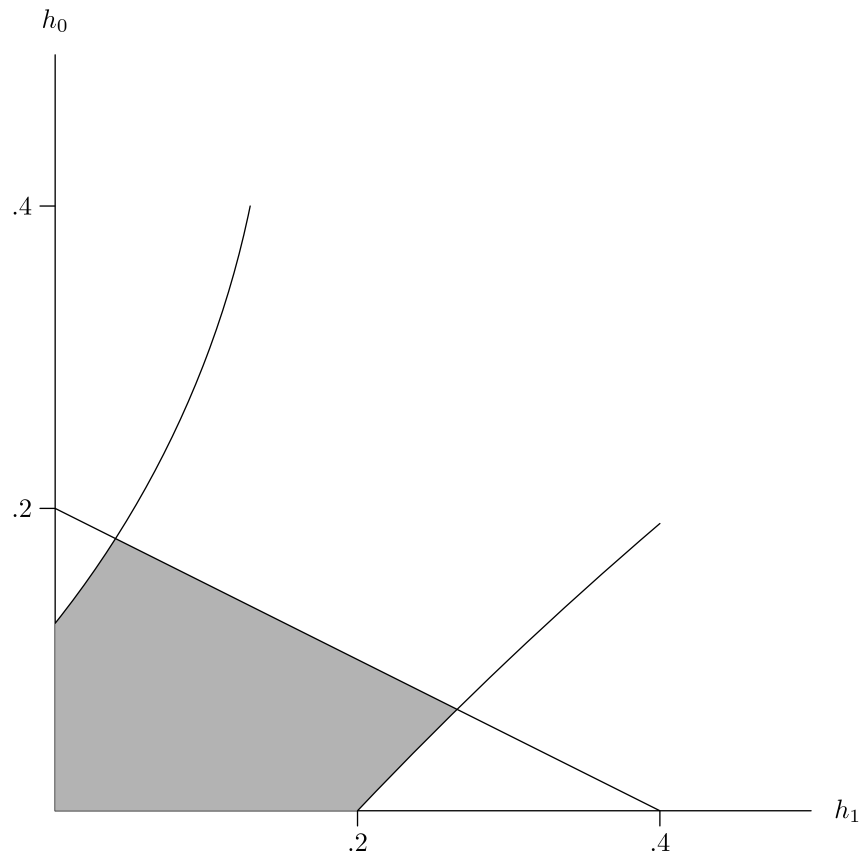

Figure 4: The stable space (shaded area) formed by conditions (ii) and (iii) of the Schur-Cohn criterion is illustrated for the parameter values: $n = .05$, $s = .8$, $\rho = 1$, $d_0 = .1$, $d_1 = .1$, $d_2 = .04$, and $a = .5$, $e^* = .945$.