Milan's Cycle as an Accurate Leading Indicator for the Italian Business Cycle

Matteo M. Pelagatti Department of Statistics Università degli Studi di Milano-Bicocca Email: matteo.pelagatti@unimib.it Valeria Negri Research Department Assolombarda Email: valeria negri@assolombarda.it

Abstract

A coincident business cycle indicator for the Milan area is built on the basis of a monthly industrial survey carried out by Assolombarda, the largest territorial entrepreneurial association in Italy. The indicator is extracted from three time series concerning the production level and the internal and foreign order book as declared by some 250 Assolombarda associates.

This indicator is potentially very valuable in itself, being Milan one of the most dynamic economic systems in Italy and Europe, but it becomes much more interesting when compared to the Italian business cycle as extracted form the Italian industrial production index. Indeed, notwithstanding the deep differences in the nature of the data, the indicator for Milan has an extremely high coherence with the Italian cycle and the former leads the latter by approximately 4-5 months. Furthermore there is a direct relation between the amplitude of the cycle and the leading time of the Milan indicator.

Key Words: Leading indicator, unobserved components model, structural time series model, local business survey.

JEL Classification: C22, C32, C53, E32, L60

I. INTRODUCTION

Assolombarda, the largest local entrepreneurial association in Italy, represents more than 5,945 associated firms in the Province of Milan and every month publishes on its web site the main results of the Assolombarda Business Survey of Manufacturing Sector in the Milan Area.

Although the Survey time series has now reached a more than adequate length (it started in 1992), to the best of our knowledge (apart from section 5 in Centro Studi Assolombarda 2002) there is no published study on the relationship between the content of this qualitative survey and the Italian business cycle. This article seeks to fill this gap. Indeed, it is well known that Milan has one of the most dynamic and reactive productive system in Europe and it is reasonable to expected that the survey data may reveal this leading role of Milan on the Italian industry.

For the extraction of the business cycle signals, both for Milan and for Italy we relied on Unobserved Components Models (UCM, but also Structural Time Series Models, see Harvey 1989 and Durbin and Koopman 2001). This choice presents several advantages: i) the methodology may be applied to one or more time series in a consistent way, ii) data do not have to be previously seasonally adjusted, iii) trends, cycles and seasonal components appears directly in the model and may be coherently estimated and predicted

The results presented in this article are a by-product of a research conducted by the authors for Assolombarda, whose main goal was the construction of a business cycle indicator for the Milan area. Giuseppe Panzeri, until his recent premature decease director of the Centro Studi Assolombarda, is the person who had the original idea for this indicator and who placed faith in our abilities of carrying out this project. This article is dedicated to his memory.

by Kalman filter techniques. Since we extracted the business cycle for Milan from a pool of time series, while the Italian cycle was obtained from the industrial production index alone, point i) is a natural requirement in this analysis. As far as point ii), it is well known that seasonal-adjustment techniques (specially if not model-based) may introduce serious distortions in the power spectrum of a time series and this may reflect in the cycle extraction. Point iii) is relevant as well, given that econometric business cycle models are usually built also for forecasting.

The main result presented in this paper may be summarised as follows. The industrial cycle indicator of Milan based on a choice of time series of the Assolombarda Business Survey is extremely coherent with the Italian business cycle extracted from the industrial production index. As far as the phase of the two business cycle signals, Milan's cycle tends to lead the Italian cycle by an average of approximately 4-5 months. The leading time is also related to the amplitude of the cycle: wide cycle turning points are anticipated by Milan's industry by as much as 7-11 months, while mild cycles tends to be lead by only 0-4 months.

The rest of the paper is organised in the following way. Section 2 reviews the models employed for extracting the business cycle signals. Section 3 describes the Assolombarda Business Survey and the time series we use. Section 4 contains the results of the analysis and Section 5 concludes.

II. REVIEW OF UNOBSERVED COMPONENT MODELS

There is a wide range of methods for extracting business cycles from time series, ranging from model-based methods like Markov-Switching (Hamilton 1989, Krolzig 1997, Pelagatti 2007) and Unobserved Component Models (from now on UCM, Harvey 1989, Harvey & Trimbur 2003) to band-pass filtering approaches (Baxter & King 1999, Christiano & Fitzgerald 2003, Iacobucci & Noullez 2005). Model-based approaches benefit from the advantage of being readily extensible to the multivariate context and the optimal predictor of the cycle may be easily derived. We chose unobserved components models because of their usually very good forecasting performance and because the optimal filter for extracting the business cycle may also be seen as a generalization of the Butterworth band-pass filter (Harvey & Trimbur 2003), common in the signal-processing literature (Pollock 1999). In this section we review the concepts relevant to our application.

A. Univariate Models

An univariate UCM expresses an observable variable y_t as sum of unobservable components, such as trend, cycle, seasonality and noise:

$$y_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t. \tag{1}$$

The trend component is generally specified as local linear trend:

$$\mu_{t} = \mu_{t-1} + \beta_{t-1} + \eta_{t} \beta_{t} = \beta_{t-1} + \zeta_{t}$$
(2)

where η_t and ζ_t are normally independently distributed (NID) random sequences with zero mean and variances σ_{η}^2 and σ_{ζ}^2 . This specification includes as special cases the linear trend (both variances set to zero), the random walk with drift ($\sigma_{\zeta}^2 = 0$), the random walk ($\sigma_{\zeta}^2 = 0, \beta = 0$) and the very smooth *integrated random walk* ($\sigma_{\eta}^2 = 0$).

The cycle is specified as the first component of the vector process defined by

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix},$$
(3)

where κ_t and κ_t^* are independent NID sequences with common variance σ_{κ}^2 . The frequency parameter λ controls the average period of the cycle $(2\pi/\lambda)$, while the damping factor $\rho \in [0, 1]$ determines the concentration of the power spectrum around the frequency λ . When ρ is strictly smaller than one, the cycle is stationary, while $\rho = 1$ implies a unit root at frequency λ . Harvey & Trimbur (2003) propose a generalisation of this stochastic cycle, that allows for a greater concentration of the spectral density around λg and, thus, smoother cycles. The *n*-order cycle they propose is given by $\psi_t := \psi_{n,t}$, where $\psi_{n,t}$ is defined by the recursion

$$\begin{bmatrix} \psi_{1,t} \\ \psi_{1,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} \psi_{1,t-1} \\ \psi_{1,t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}$$

$$\begin{bmatrix} \psi_{i,t} \\ \psi_{i,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} \psi_{i,t-1} \\ \psi_{i,t-1}^* \end{bmatrix} + \begin{bmatrix} \psi_{i-1,t-1} \\ \psi_{i-1,t-1}^* \end{bmatrix},$$
(4)

for i = 2, ..., n. For n = 1 the cycle (3) is obtained. Applications to real data have demonstrated that, while there is a substantial increase in smoothness when moving from order 1 to order 2, higher orders change the extracted cyclical components only marginally (cf. Section VIII. in Harvey & Trimbur 2003). In the application in Section 4, we adopted order 2 cycles.

Since a deterministic seasonal component, which is a zero-mean periodic function, can be represented as a linear combination of sines and cosines at Fourier frequencies, a possible stochastic seasonality specification can be obtained by summing the stochastic cycles of equation (3) at Fourier frequencies. Since seasonality is usually assumed nonstationary, the damping factor in equation (3) has to be set to one:

$$\gamma_{j} = \sum_{j=1}^{\lfloor s/2 \rfloor} \gamma_{j,t},$$

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^{*} \end{bmatrix} = \begin{bmatrix} \cos \lambda_{j} & \sin \lambda_{j} \\ -\sin \lambda_{j} & \cos \lambda_{j} \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^{*} \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^{*} \end{bmatrix},$$

$$\lambda_{j} = \frac{2\pi}{s}j,$$
(5)

where s is the number of "seasons" in a year (e.g. 12 form monthly data) and the $\omega_{j,t}$'s (starred and not starred) are independent zero-mean NID sequences with common variance σ_{ω}^2 .

The noise ε_t is a zero-mean NID sequence with variance σ_{ε}^2 .

The spectral properties of the optimal filters for extracting the cycle component in an univariate UCM were thoroughly examined by Harvey & Trimbur (2003).

B. Mutivariate Models

The UCM introduced above can be easily generalised to the mutivariate case. Let y_t be a $N \times 1$ vector of observable time series. The system of equations that relates the observable variates to the unobservable components is now

$$\boldsymbol{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\gamma}_t + \boldsymbol{\varepsilon}_t, \tag{6}$$

where each component is now a vector of the same dimensions as y_t . Each element of every single vector of unobservable components is defined exactly in the same way as seen above.

The vector of trend components is given by

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$$

$$\beta_t = \beta_{t-1} + \zeta_t$$
(7)

where η_t and ζ_t are independent NID sequences with covariance matrices, respectively, Σ_{η} and Σ_{ζ} . Notice that when the matrices Σ_{η} and Σ_{ζ} are not full rank some form of cointegration arises. For instance, if $\Sigma_{\zeta} = 0$ and $rank(\Sigma_{\eta}) = k < N$ we have an I(1) cointegrated system with cointegration rank r = N - k. Tests for common stochastic trends in the UCM framework were proposed by Nyblom and Harvey (2000).

As for the cycle, we adopt the *similar cycle* model introduced by Harvey and Koopman (1997):

$$\begin{bmatrix} \boldsymbol{\psi}_t \\ \boldsymbol{\psi}_t^* \end{bmatrix} = \begin{bmatrix} \rho \begin{pmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{pmatrix} \otimes \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}_{t-1} \\ \boldsymbol{\psi}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\kappa}_t \\ \boldsymbol{\kappa}_t^* \end{bmatrix}, \quad (8)$$

with covariance matrix of the shocks given by

$$E\left(\left[\begin{array}{cc}\boldsymbol{\kappa}_t\\\boldsymbol{\kappa}_t^*\end{array}\right]\left[\begin{array}{cc}\boldsymbol{\kappa}_t&\boldsymbol{\kappa}_t^*\end{array}\right]\right) = \boldsymbol{I}_2\otimes\boldsymbol{\Sigma}_{\boldsymbol{\kappa}}.$$
(9)

The similar cycle model is a quite appropriate choice for business cycle analysis, as it imposes the same average period and the same damping factor (i.e. the same shape for the spectral density) for all the cycles, but it does not imply that the all the cycles must be proportional to each other as in a common cycle model. The common cycle model is a special case of the similar cycle model that arises when the covariance matrix Σ_{κ} has rank one.

When only one common cycle seems to drive the whole vector of time series, then this may be imposed directly in the model by constraining Σ_{κ} to be of rank one, otherwise a common trend can be recovered by extracting the first principal component of the similar cycles based on the correlation matrix associated with Σ_{κ} . The generalization of the similar cycle model to higher order cycles of the type in equation (4) is straightforward and will be omitted.

The seasonal components are, again, obtained superimposing $\lfloor s/2 \rfloor$ stochastic sinusoids like (8) with unitary damping factor:

$$\gamma_{j} = \sum_{j=1}^{\lfloor s/2 \rfloor} \gamma_{j,t},$$

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^{*} \end{bmatrix} = \left(\begin{bmatrix} \cos \lambda_{j} & \sin \lambda_{j} \\ -\sin \lambda_{j} & \cos \lambda_{j} \end{bmatrix} \otimes I_{2} \right) \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^{*} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_{j,t} \\ \boldsymbol{\omega}_{j,t}^{*} \end{bmatrix},$$

$$\lambda_{j} = \frac{2\pi}{s}j,$$
(10)

where the covariance matrix of each vector of shock terms is

$$E\left(\left[\begin{array}{c}\boldsymbol{\omega}_{j,t}\\\boldsymbol{\omega}_{j,t}^{*}\end{array}\right]\left[\begin{array}{c}\boldsymbol{\omega}_{j,t}&\boldsymbol{\omega}_{j,t}^{*}\end{array}\right]\right)=\boldsymbol{I}_{2}\otimes\boldsymbol{\Sigma}_{\boldsymbol{\omega}},\ j=1,\ldots,\lfloor s/2\rfloor.$$
(11)

III. THE DATA

Since January 1992 the Research Department of Assolombarda has been conducting a qualitative business survey of the manufacturing sector in the Milan Area. The survey reflects the methodology used by the ISAE (Istituto di Studi a Analisi Economica), and therefore it implements the methodology harmonized by the European Commission.

Frequency, sample size and variables

The survey is carried out on a monthly basis and the sample size is about 250 manufacturing firms located in the Province of Milan and associated to Assolombarda. The sample is stratified by 7 sectors of activity (Food, Plastic and Chemical Industry, Fashion System, Mechanics, Metallurgy, Electronics and Other Industries) and 2 firm sizes (less than 100 employees, 100 or more employees).

The surveyed variables are: assessments on production, domestic and foreign order books, domestic and foreign turnover, stocks and level of employees, as well as expectations about production, domestic and foreign order books and Italian economic situation.

For every variable, respondents signal the current situation with reference to the previous month, choosing among three answering options: "increase" (+), "unchanged" (=) or "decrease" (-).

Aggregation and weighting of data

The technique adopted for data computation is based on balances between positive and negative answering options, measured by percentage points of total answers.

The balance for each stratum is computed as a simple count of the answers. Having obtained the results for each stratum, the balance for the whole manufacturing sector is calculated as a weighted average of the balances by strata. The weighting is used to improve the quality of the sample, by correcting any possible discrepancy of representation. Weighted coefficients reflect the relative importance in term of workers per stratum in the population of the Milan Area, as derived from official statistics (Istat Census).

IV. THE ANALYSIS

After a preliminary analysis of the time series available in the Assolombarda business survey, we decided to work on just three of them: assessments on production (Production), domestic order book (Internal Orders) and foreign order book (External Orders). This choice was due to different considerations. The very high (spectral) coherence among the three time series, the high level of the signal (the cycle) with respect to the noise (all the rest) and the economic importance of the information they carry. Moreover, the orders were found to lead the production by some 1-2 months.

A. The three Assolombarda survey series

There is a general consensus among economists in defining business cycles as being composed of those frequencies in the band corresponding to 1.5 to 8 years. From the spectral density estimates in Figure 1, it is clear that, apart from seasonal variations, most of the energy of the three Assolombarda series is concentrated at business cycle frequencies (shaded area).

Figure 2 report the spectral coherence of the two Order time series with the Production. Again, apart from seasonal variations, we have a peak very close to 1 around the



Fig. 1. Spectral densities of the Assolombarda survey time series

frequency corresponding to approximately 3 years surrounded by a plateau over most of the business cycle frequencies.



Fig. 2. Coherence between the Production and the two Order series

As for the phase between the Production and the two Order series, we proceeded as follows. We extracted the frequencies in the business cycle band using the Baxter-King filter (Baxter & King 1999), which is the fixed and symmetric *finite impulse response* (FIR) filter that best approximates the ideal filter. At this stage we did not implement the UCM/Kalman filter machinery for extracting the cycle, since the resulting filters are time varying and asymmetric, and therefore induce nonstationarity and phase shifts in the extracted cycles. Then, we computed the cross-correlations between the Production cycle and the cycles of the two Order series. As Table I shows, the maximum correlation is reached when both Order series lead the Production series by one month.

 TABLE I

 CROSS-CORRELATIONS BETWEEN THE PRODUCTION AND THE TWO ORDER SERIES

Production with	Lead 0	Lead 1	Lead 2	Lead 3	Lead 4	Lead 5
Internal Orders	0.970	0.977	0.950	0.890	0.800	0.684
External Orders	0.948	0.963	0.942	0.887	0.801	0.686

For the determination of the phase between two time series pairs, we tried a different approach as well. We built bivariate UCMs for the vectors $y_t = [\text{prod}_t, \text{orders}_{t+i}]'$ with different leading times *i* and then we looked at the estimated disturbance matrices $\hat{\Sigma}_{\kappa}$.

The correlation of the two cycle disturbances was virtually 1 for $i = \{1, 2\}$, suggesting leading times of one or two months.

B. Milan's cycle and the Italian business cycle

The pieces of information collected in the preliminary analysis of last subsection entail the estimation of an UCM like (6) for the vector

$$\boldsymbol{y}_t = [\text{prod}_t, \text{int.orders}_{t+1}, \text{ext.orders}_{t+1}]'$$

with trend component modelled as an integrated random walk (i.e. $\Sigma_{\eta} = 0$), similar cycles of order 2 and no further restriction. The integrated random walk (IRW) is a standard choice for a trend when a cycle is present in the model (cf. Harvey & Trimbur 2003 and Valle e Azevedo *et al.* 2006). Indeed, the IRW has a (pseudo) spectral density that is much more concentrated in a neighbourhood of the zero frequency than the local linear trend and this yields a better identification of the cycle.

TABLE II							
ESTIMATED COVARIANCES AND CORRELATIONS FOR THE MULTIVARIATE MOI	DEL						

Component	Covariance-correlation matrix			
<u>Slava</u>	$\begin{bmatrix} 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{bmatrix}$			
Slope	$\begin{bmatrix} 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{bmatrix}$			
Cycle	$\begin{bmatrix} 0.214 & 0.996 & 0.971 \end{bmatrix}$			
period = 41.7	0.232 0.254 0.958			
$\rho = 0.91$	0.239 0.256 0.282			
	$\begin{bmatrix} 1.122 & -0.951 & -0.884 \\ 0.596 & 0.220 & 0.986 \end{bmatrix}$			
Seasonal	-0.586 0.339 0.986			
	$\begin{bmatrix} -0.319 & 0.196 & 0.116 \end{bmatrix}$			
	12.01 0.591 0.192			
Noise	15.95 60.72 0.686			
	4.970 39.87 55.70			

Order of the variables: Production, Internal Orders, External Orders. Covariances in the diagonal and lower triangular part.

Correlations in the upper triangular part.

Table II reports the relevant ML estimates of the model's parameters. As expected by the visual inspection of Figure 2, the correlations of the cycle disturbances and, hence, of the cycles are extremely high (all above 0.95).



Fig. 3. Extracted cycles for the three Assolombarda survey series

The high correlation among the cycles is also evident in Figure 3, where the three extracted (smoothed) cycles are represented.

In order to extract a single cycle indicator for Milan we standardized the three series and extracted the first principal component (PC) based on the estimated correlation matrix of the cycle disturbance (which is identical to the correlation matrix of the cycles). The first PC accounts for more than 98% of the variability of the three series and the corresponding eigenvector [0.338, 0.336, 0.333] gives only a marginal preference to the Production series.

For comparing the cycle of Milan to the Italian one, we applied an univariate UCM to the Italian Industrial Production Index (IPI). We used, again, an IRW trend and imposed to the second-order cycle component the same period and damping parameter that we obtained for the model for Milan. This choice does not affect much the business cycle signal extracted for Italy, in fact the estimation of these two parameters on the IPI series is very close to the imposed one, but it guarantees that the same (spectral) definition of the cycle holds both for Milan and for Italy.



Fig. 4. Extracted business cycle signals for Milan and Italy

Figure 4 compares the cycle extracted for Milan with the Italian business cycle. The similarity of the two cycles is remarkable if we consider that one was extracted from a qualitative survey based on few hundreds Assolombarda associates, while the other was obtained from the official Italian IPI as published by Istat. It stands clear that Milan tends to lead Italy by a varying number of months, with the exception of the 2004 peak. Moreover, there seems to be a direct relation between the amplitude of the cycle and the leading power of Milan.

Table III reports the turning points for the two cycles. The average lead of Milan with respect to Italy is approximately 4 months for the troughs and 5 months for the peaks. We examined the relation between lead times and the cycle amplitudes. We computed cycle amplitudes by taking the vertical distance between two consecutive extremes. The amplitude of the first trough was computed by multiplying the absolute value of the cycle at its minimum by 2: this amounts in assuming that the (non available) previous peak has the same norm as this trough.

Figure 5 reports scatter plots and correlations for troughs and peaks with respect to the amplitudes of the cycles of Milan and Italy. The correlation tends to be higher with the Italian cycle amplitude, but these cannot be used for forecasting future lead times, while correlations with Milan's cycle may be exploited for this end. In particular, the last Milan trough's date and amplitude let expect a turning point for Italy around July 2008.

TABLE III	
BUSINESS CYCLE DATING FOR MILAN AND ITA	LY

Troughs				Peaks			
Milan	Italy	Lead	Milan	Italy	Lead		
Oct-92	Aug-93	10	Dec-94	Jul-95	7		
Jul-96	Nov-96	4	Sep-97	Dec-97	3		
Dec-98	Feb-99	2	Mar-00	Oct-00	7		
Oct-01	Jan-02	3	May-02	Aug-02	3		
May-03	May-03	0	May-04	Apr-04	-1		
Feb-05	Apr-05	2	Apr-06	Mar-07	11		
Mar-08							
	Mean	3.5		Mean	5		



(T) and (P) stand for Troughs and Peaks.

Fig. 5. Extracted business cycle signals for Milan and Italy

V. CONCLUSION

In this study we proposed a business cycle indicator for the city of Milan, based on three of the many time series collected by Assolombarda in its monthly business survey. In particular, we used production, internal orders and foreign orders. These series are built as differences between the number of entrepreneurs that claim an increase in the relevant quantity and those that declare a decrease in the same quantity.

The three series chosen for extracting the cycle indicator are extremely coherent at business cycle frequencies and the Orders tend to lead the Production by one month. The business cycle indicator for Milan is derived using an unobserved component model with similar higher-order cycles of type proposed by Harvey and Trimbur (2003). The indicator is obtained as the first principal component of the three estimated (smoothed) similar cycles, whose correlations are all over 0.95.

A reference cycle for Italy was extracted using a second order cycle in an unobserved

component model applied to the Italian industrial production index. The similarity of the Italian cycle and of the indicator for Milan is striking if we consider the completely different nature of the data we used. Moreover the cycle extracted for Milan tends to lead the Italian cycle by an average of 4-5 months, with leading times positively correlated with the amplitude of the cycles.

A further step for future research may be the construction of a (possibly non-linear) UCM model that makes explicit use of the relation between the amplitude and the phase between the survey series and the Italian cycle for forecasting relevant macroeconomic quantities such as production, output, employment, etc.

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