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On the existence of the true value of a probability.
Part I: Determinism versus aleatorism

Alberto H. Landro*

Abstract

Objetivist models are based on the deterministic hypothesis that postulates the existence of probability, which is cognoscible only in an asymptotic manner. On the other hand, subjectivist models consider the aleatoristic hypothesis according to which there is no truth about probability. However, both hypotheses may only be compared through stochastic models, which are not strictly falsifiable. Therefore, neither the hypothesis stating the existence of a true value regarding the probability of occurrence of an event nor de Finetti´s postulate which sustains that "probability does not exist" are strictly verifiable.

1.- An introduction to the theory of chance

The conceptualization of chance arose associated with the idea of lack of sufficient information about the causal structure that supposedly determines the behaviour of factual phenomena: the observer has a piece of information-whether it is objective in nature or consists of the knowledge of multiple characteristics data and origins that constitute their personal experience (subjective) on this phenomenon-that is incomplete (due to the sort of universal solidarity which links the processes and makes nature appear as infinitely complicated) encouraging, therefore, that the reasons of a part of the phenomenon's behaviour remain ignored for him.

This classical Thomistic notion of chance ignorance-for a long time the only notion accepted by moral theology-implies a deterministic conception of the outside world to the observer based on certain metaphysical assumptions: i) that the world to which the phenomena belong is real, ii) that there are objective laws that govern their behaviour and iii) that these laws are inherent to the phenomena and they are also rational and asymptotically cognoscible.

The inadequacy of the classical method to explain "…an unstable world we know through a finite window"[1], in which an infinitesimal change in the knowledge of the observer, despite having deterministic equations, leads to a realization of the phenomenon to any other of his infinite set of possible realizations in which the irreversibility is the rule and the reversibility is the exception, gave rise to a new formulation-aleatorist-which essentially differed with classical dynamics in the application of the concept of process state at a given instant as the result of an evolution oriented over time.

As a result, it can be concluded that accepting the classical hypothesis-deterministic-is equivalent to assuming that every phenomenon is explicable on the assumption that-in

* These viewpoints are personal and do not necessarily represent the position of Universidad del CEMA. This work with some modifications was presented at the 30th Conference of the AAEP, 2010.

[1] Prigogine, I., Nicolis, G. (1977).

the limit-it is the necessary consequence of an infinite set of factors that define its causal structure. On the contrary, accepting the notion of objective partial aleatorism or the thermodynamic interpretation-aleatorist-involves replacing the classical concept of chance-ignorance (epistemological) by the chance-absolute (ontological), replacing the statement "the observer can never know" by the statement “neither the observer nor nature can know".

In either case, the presence of such a thing called chance that inevitably appears in the vision that every observer has about the behaviour of all phenomena, generates a feeling of uncertainty which formal quantitative representation is given by the probability.

This appreciation of the concept of probability as inferential logic of uncertain knowledge gives rise to a fundamental question: how can a feeling of uncertainty be characterized by a numerically defined probability?

The different scenarios that led to this question gave rise to a notion of objective probability based on the concept of expectation and, then, on an essentially dual interpretation that assimilated the probability, either to a deductive expression based on the symmetry of the aleatorism inherent to some events-**classical definition**-either to the frequency with which certain phenomena are verified-**frequentist definition**. In the first case, the probability is determined by the possible ways to present the results of a phenomenon; in the second one, by the observed frequencies of such results.

Just a little later, and with the aim of approaching a neo-Bayesian conception of the notion of identifying model of the probability, a third logistic interpretation arose, which assimilates the probability to an indefinite logical relationship between a proposition and a knowledge body. The item added to the logistic conceptualization of the inevitable involvement in the process of induction of individual-assessor as a transforming mechanism of information, led to a more general **subjective (personalist) definition** of probability.

Given the failure in the attempt to find a universal definition of the notion of probability through a more or less complex formula, the possibility of a return to a somewhat objectivist interpretation was posed from less stringent definitions based on a variant of logicism, known as the **propensity theory**, which combines the concept of probability to that of the potential possibilities.

In addition to obtaining an explicit theoretical definition of probability (related to an axiomatic system consistinf of itself), each of these interpretations insisted on obtaining a consequent inferential structure, defined by explicit or implicit rules of interpretation, which characterized its role of identifying model of the true value of that elusive and purely theoretical measure of uncertainty called “probability”.

2.-The probability models and their rules of interpretation

2.1.-The classical model

The classic definition suffers from unavoidable failures that restrict Laplace’s dogmatic pretension of enshrining it as the only valid model of the true nature of probability, basically, its purely deductive nature (which prevents the definition of rules of interpretarion), the underniable circularity and the impossibility of its application outside the scope of the phenomena of ideal existence in which the physical mechanism that generates randomness includes, symmetrically, all possible outcomes.

It is well-known treatment that the literature has been devoted to the tautology that encloses the equiprobability assumption of the possible outcomes and especially its attempt at justification from the (subjective) principle of insufficient reason which, given its tendency to generate paradoxes that prevent the determination of unique probability values, is not obviously valid as an argument against circularity.

2.2.-The frequentist model

The frequentist definition is only suitable for calculating the probability of occurrence of those phenomena considered repeatable. Talking about the probability of occurrence of a single phenomenon or the probability for a proposition to be true or false has no sense in the frequentist context. The probabilities calculated from this interpretation are objective and, therefore, independent of the opinion of the individual-assessor.

The postulate of Quetelet, A., (1835) (1848) on the assimilation of the gravitational laws to the constant causes that govern society and the works of Fechner, G. Th. (1866) (1871) about the existence of a variant of partial indeterminism in the behaviour of factual phenomena led to the concept of "collective object" (Kollektivgegenstand") or "collective series" ("Kollektivereihe"), defined as a heterogeneous group of individuals that vary randomly with respect to a common attribute (in particular, a quantifiable attribute). In its simplest terms, the "Kollektiv" can be considered as a sequence of results obtained from a series of repeated observations on equal terms, each of which admits only two possible alternatives. This concept of "Kollektivgegenstand" prospered with the flourishing of empiricism developed by the Vienna Circle in the work of-among others-the philosopher and psychologist Lipps, G.F. (1898)(1901)(1905) and the astronomers Helm, G. (1902) and Bruns, H. (1897)(1898)(1905)(1906) and culminated in the reformulation of the concept of probability by von Mises, R.M.E. (1912)(1919a)(1919b)(1928) and Reichenbach, H. (1935) "…in order to replace or supplement the rigid causal structure of classical theory"[2].

von Mises, R. (1928) considered the need to distinguish between "empirical collectives" (which consist of a finite number of elements that are observable and that give rise to what some authors have called finite frequentism) and "mathematical collectives" (comprising a sequence of infinite elements that give rise to what is known as hypothetical frequentism) and assumed that the empirical collectives comply with two fundamental principles: the law of statistical frequency stability and the law of irregularity.

Based on these principles and on the assumption that infinite sequences are mathematical abstractions or idealizations of empirical reality which are necessary to obtain an acceptable mathematical representation of probability, von Mises established

[2] von Mises, R. (1921).

the (highly debatable) postulate whereby a finite empirical collective could represent an infinite mathematical collective in analytical terms. It should be noted that von Mises was an empiricist and that his analysis was always based on an operating philosophy by which the theoretical principles must be defined in terms of observable phenomena according to the characteristics of an empirical collective. According to this operational interpretation, the nature of repeatable phenomena is such that: i) it is possible, by abstraction, to obtain some mathematical concepts that allow to formulate the empirical laws that govern their behaviour, ii) using abstraction once again and from such empirical laws, it is possible to define the axioms of mathematical theory associated with that behaviour and iii) from this mathematical theory, it is possible to discover consequences that allow the explanation and prediction of other repeatable phenomena.

The positivist-operationalism of von Mises's ideas is primarily due to the influence of Ernstr Mach's work (in particular, "The science of mechanics: A critical and historical account of its development"). His development of the probability theory followed the same pattern as the development of mechanics that Mach achieved: he introduced the law of stability of statistical frequencies (assumed valid from observation) and he based his definition of probability on such law (the definition of a theoretical concept-probability-identifiable in terms of limit behaviour of an observable model-relative frequency), but he provided no link between observation and theory beyond the controversial use of the limits of a finite sequence of observations and justification from the application of the concept of limit in theoretical physics.

Among the many changes that von Mises's frequentist interpretation was subjected to, the most important was undoubtedly the one attributed to Reichenbach, H. (1935), who sought to obtain a definition of probability through an axiomatic way and to justify its intuitive meaning. Regarding the first question, Reichenbach tried a solution based exclusively on the set theory and on logic operations, obtaining a (purely formal) definition of probability expressed as a relationship between two kinds of propositions.

Regarding the second question, Reichenbach sought to broaden the scope of the frequentist interpretation to non-repeatable events, by defining what he called "reference classes" consisting of similar events to the analysed one and he considered the theory of probability as the discipline that assesses unknown probabilities of derived collectives from known probabilities of origin collectives. But this generalization encountered the insurmountable difficulty which means the impossibility of determining objective selection rules, universally accepted, of the events that must integrate these reference classes.

To avoid any kind of regularity in the sequences of events which make the basis, both von Mises's definition and Reichenbach's made an attempt to provide their probability models with a strictly mathematical content by enforcing complicated conditions that inevitably restricted the concept of total chance/aleatorism and led to the conclusion that it was impossible to give the notion of "absolute irregularity"[3] a mathematically precise nature.

Al these conditions led to transform the frequentist model into a purely mathematical theory that, instead of dealing with favourable results and possible results as in the

[3] See Landro, A.H. (2010b).

classical model, deals with limits which are abstract mathematical entities in which the proofs of the theorems are obtained, from the definition of probability, only by using logical-mathematical methods.

2.3.-The logistic model

As an extension to the definition of probability "a posteriori" of the frequentist interpretation, arise the proposal of the so-called logical interpretation, which led to a model in which the general notion of probability (which results into a degree of rational belief or similar idea about the occurrence of a phenomenon) is exclusively a function of a certain state of knowledge defined by a set of arguments, intrinsic or extrinsic to this phenomenon, that the observer has through the perception of a logical relationship among the propositions[4]. A probability, $p(A/B)$, conceived as an (indefinite) relationship between a proposition (A) and a body of knowledge (B), between a "*…statement and another statement (or set of statements) that represents the evidence*"[5], conditioned by the truth of such evidence. Where the event A can, therefore, be represented by a subset $A \subset \Omega$ such that $A = \{w / S(w) \text{ is true}\}$, so that every event has a single group A and vice versa, that is, a set A in the space of events corresponds to each proposition $S(w)$ of the propositional space and vice versa.

The logistic interpretation was based on the contributions made by Augustus de Morgan, John Venn, Harold Jeffrey, and, in particular, John Maynard Keynes, followed by the members of the Vienna Circle, Bernard Bolzano, Ludwig Wittgenstein, Friedrich Waismann, and in particular, Rudolf Carnap and Karl Popper.

The starting point of Keynes's approach was precisely to define a partial link theory as a generalization of the theory of total link of deductive logic and to consider probability as an assessment of that partial link, so that it is not possible to speak of the probability of a hypothesis but only of its probability conditioned by some evidence partly linked to it. Then, given a set h of propositions and a conclusion consisting of a set of propositions a, if h partly implies in a degree α, then, identifying the partial link degrees with the rational belief degrees, Keynes concluded that, given h there will be a degree α of rational belief in a, that is, a relationship of degree probability α between a and h. Note that Keynes assimilates his probabilistic model to a degree of rational belief but not simply to a degree of individual belief. That is, he considers probabilities as values objectively fixed by the observer which are comparable to intuitively known logical relations, but using a Platonic concept of the term "objective", that is, not referring to "things" of the material world, but "something" in a Platonic world made up of abstract ideas, similar to that postulated by the Cambridge philosophers, which included objective ideas, ethical qualities (with the idea of "virtue" occupying a prominent place) and mathematical entities.

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[4] Ramsey, F.P. (1931): "*According to this interpretation, the theory of probability is considered as a branch of logic, the logic of partial belief and non-conclusive argument.*"

[5] . Kyburg, H.E.; Smokler, H.E. (1980).

2.4. - The subjectivist model

If we add the inability to stop considering the intervention of the individual-evaluator as an information source or as an observations transformer mechanism to this logistic conceptualization in the induction process, the subjectivist-more general-model of Bernoullian probability emerges, according to which the existence of probability assessments that do not coincide with each other even for similar states of knowledge is understandable given that, in this aleatorist context, the maximum objectivity to which one can aspire is a kind of concordance of individual assessments, a certain intersubjectivity.

Beyond some curious previous background, it can be considered that the subjective theory of probability was introduced independently by Ramsey, F.P. (in Braithwaite, R.B. (1931)) and de Finetti, B. (1930a) (1930b) (1930c) (1931a) (1931b) (1937). Ramsey raised his proposal-strictly of an antilogicist nature-from a detailed critique of the Keynesian interpretation, while de Finetti's work-strictly of an antifrequentist nature-originates in the proposal of E. Czuber who, in his 1903 memoirs and in the second revised and expanded edition published in 1908-1910, provided one of the best expositions on the paradoxes of geometric probability (see Keynes, J.M. (1921)) and he concluded that there is no need to assume compliance with the condition of insufficient reason.

In subjectivist terms, the probability of occurrence of an event E could be interpreted as the price (bet) p that an individual considers fair to pay an opponent for the right to receive a unit amount, to be payable if E is verified. The fairness condition implies the indifference between being one player or the other one, between paying or charging p to collect or to pay 1 when E is verified. In that case, it is said that the assessment of the probability is "coherent" as it does not place any of the players in the position of winning for certain. That is, if p is a coherent evaluation in Ramsey-de Finetti's sense of the probability of occurrence of an event E to an individual, since the price is understood here as a linear scale, the evaluation of the probability of not occurrence of E (that is, the occurrence of $\overline{E}$) for such individual should be $p(\overline{E}) = q = p(1 - E) = 1 - p(E) = 1 - p$.

It is worth remembering that, according to the objectivist interpretation, it is said that an event E has a probability of occurrence $p(E)$, taking an event not as a well-defined individual case, but as all the events of a certain type (it should be noted that, in the objectivist context, the probability is considered a real property of a special type of physical situations called events). By contrast, the subjectivist interpretation, based on an aleatorist conception, considers probability always to belong to individual events and, whenever a probability is assigned, it is necessary to think of it as subordinate to each observer's interpretation of a set of particular information (considering an individual event a case that, for an individual that in certain circumstances cannot certainly ensure its occurrence is random).

2.5. - The propensionalist model

The interpretations discussed above consider that the notion of probability is represented by a more or less canonical version. While his failure to obtain a universal definition would seem to refute this hypothesis, this should not be considered as an acknowledgement of the impossibility of identifying the true value of the probability through a formula but, perhaps, as the need for less stringent definitions from a diffuse set of propositions[6]. This principle gave rise to a new objectivist model of probability based on the theory of propensities.

This propensionalist model was introduced by Popper, K.R. (1957b), developed in his works in 1959b, 1983 and 1990, and continued by a group of philosophers of science such as D.W. Miller and J.H. Fetzer[7].

The problem that gave rise to this theory entailed deciding about the possibility of identifying "unique" objective probabilities about the occurrence of single events. In principle, Popper (1934) considered a single event as von Mises's particular element of a collective and suggested that his "single probability" could be assimilated to his probability in the collective considered as a whole but, later, in his works in 1957b and 1959b, he deserted that frequentist interpretation.

Note that the word propensity suggests a type of orderly enumeration, which makes a difference with the frequentist viewpoint which, as it was presented, shows that the probabilities can only be introduced in physical situations (that is, in "occasions of whole display", according to Peirce, C.S.'s nomenclature (1910)) for which it is possible to define a collective. In Popper's propensionalist model, it is absolutely legitimate to postulate the existence of probabilities on sets of conditions, although they do not support a sufficiently large number of repetitions, which implies an indisputably significant expansion of the set of situations in which the theory of probability applies, with respect to the frequentist interpretation. The probabilities of individual events should be considered as primarily dependent on the set of conditions to which the event is referred more than to the event itself[8].

Popper's proposal was modified by Miller, D.W. (1994) (1996) who, in his intention to solve the foundational problem of unique probabilities' identification, dissociated propensities from repeatable conditions and proposed an association with the states of the universe, transforming propensionalism from a scientific theory into a metaphysical one.

In order to avoid the metaphysical character of the model, unlike Popper's original proposal and Miller's modification, Fetzer, J.H. (1982) (1993) abandoned the idea of

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[6] Resort to what Bachelard, G. (1953) called the *"conscience of the non-rigorous"*.

[7] Other significant contributions to the propensionalist interpretation were made by Hacking, I. (1965) and Mellor, D.H. (1971) (see Salmon, W.C. (1979)).

[8] This position could also be considered as a variant of logicism aimed at trying to return to an objectivist interpretation, associating the concept of probability to the so-called potential possibilities of V.A. Fock, according to which one can admit that the set of possible results exists only in the mind of the observer, not as his own creation, but as something that the observer must admit (see Omelyanoskij, M.E.; Fock, V.A. (1972)).

associating propensities to a complete state of the universe and suggested linking them to a complete set of relevant conditions so that, to falsify a "conjectured" value of a propensity, "conjectures" about a list of these relevant conditions should be presented. Now well, given the insurmountable difficulties that the formulation entails and, consequently, the falsifation of the required conjectures, it can be concluded that in Fetzer's model the propensities also show a more metaphysical than scientific aspect and, therefore, his concept of probability cannot be generally extended to the singular cases for which the assignment of a probability of occurrence remains subjective.

3. - The criteria for falsifation of probabilistic models

From the principles of non-existence of metaphysical assumptions about the true nature of probability, and in order to evaluate the explicit explanatory capacity of the frequentist, subjectivist and propensionalist models, the corresponding criteria of falsification were developed as a means to contrast the correlation between a probabilistic theory and the corresponding model obtained from the observable results of the phenomena[9].

With respect to the **frequentist model**, it should be noted that in von Mises's conception, the properties of the collectives are not expressed in relation to actual phenomena or to effective observation procedures, but are regarded as axioms, so it is assumed that their consequences are deductible in a rigorous way. Now then, those consequences will be true only if the axioms that originate them are true. In particular, considering the convergence of the sequence of frecuencies (that is, to the transfer from finite ferquentism to hypothetical frequentism) as an axiom and not as an event that could reasonably be true, requires recognition of certain principles of metaphysical determinism regarded as unacceptable when they contradict the conditions inherent in the definition of the falsifation criteria.

In this respect, von Mises proposed a more restrictive criterion of falsification consisting of an extension of the postulate known as the exclusion law of the game systems, which requires not only the stability of relative frequencies for certain specific results, but the invariance of these frequencies to a selection, according to a rule, of a sequence located in the original sequence. Critics to this proposal refer not only to their arguments based on the inaccurate or semi-mathematical notion of game system or localized selection, but to the not fully specified notion of admissible selection. To correct these shortcomings, Church, A. (1934) (1936a) (1936b), Turing, A. (1936) and Wald, A. (1937) proposed a more precise method of specification of the located selections based on the definition of computable function, obtaining precisions that can be considered fully valid in the ambit of classical mathematics, but not admissible in the ambit of constructivist mathematics. In this context, according to the algorithmic complexity theory, one can easily conclude that it is not possible to justify the aleatorism of a sequence, that is, that Wald-Church-Turing's proposal does not provide

[9] Note that the purely deductive nature of the classical model prevents the definition of the rules of interpretation and that the logical model considers that the probabilities are comparable to intuitively-known logical relations, which are nor always quantifiable and, in many cases, they are not even comparable among them (see Landro, A.H. (2010a)).

a strict characterization of the property of irregularity, of its relationship with the condition of stochastic independence, and therefore, of the axiom of statistical convergence of relative frequency. It follows that the condition of irregularity ("Regellosikeit") assumes, even in this case, a purely subjective character.

On the other hand, it proved to be undeniable that the probabilities interpreted as a relative frequency limit are always conditioned by a particular empirical collective and the fact that the notion of empirical series does not match that of the mathematical sequence (in which the law that univocally determines its elements is known) leads to the conclusion that the convergence of frequencies is not comparable to the analytic operation of passing the limit.

As for the **personalist probabilistic model**, in accordance with the considerations in Section 2.4., it is easy to conclude that any falsification criterion associated with this interpretation caters exclusively to the falsifation of the proposition derived from the sense of personal uncertainty of the observer, regardless the consideration of any reference to external phenomena.

Some authors try to overcome this restriction interpreting de Finetti's exchangeability condition as a link between personal and physical probabilities. In his representation theorem, de Finetti, B. (1937) shows that given a finite sequence of exchangeable events, $\{E_1, E_2, ..., E_n\}$, when $n \to \infty$, the distribution function of the variable $E^*_{(n)} = \frac{1}{n}(E_1 + E_2 + ... + E_n)$, $F_n(\pi) = p\left(E^*_{(n)} \le \pi\right)$, converges (except at points of discontinuity) to a limit function $F(\pi)$ and, as a corollary, establishes the link between the concepts of exchangeability and independence: *"Let $p_x(E)$ be the probability attributed to a generic event E when events $E_1, E_2, ..., E_n$ are considered independent and equally likely with probability π, assuming that events E_i are exchangeable with limited distribution $F(\xi)$, the probability $p(E)$ of the same generic event is given by*

$$p(E) = \int_0^1 p_x(E)\,dF(x)$$

. This property can be expressed as follows: the probability distributions p for the case of exchangeable events are linear combinations of distributions p_x for the case of equiprobable independent events, the weighs in the linear combinations are expressed as $F(x)$".

Now, with respect to the scope of use of this representation theorem, it should be noted that it is illusory, in this context, to suppose that a model can be built on something that has no empirical meaning, such as the events of infinite domain[10]. In the inference ambit, the representation theorem should be taken in accordance with its weakest formulation, whereby the necessary and sufficient condition for events E_n to be exchangeable is that, conditioned by a random element p, the joint probability distribution for any finite sequence is the same. Therefore, it can be concluded that, from a purely formal point of view, the exchangeable events are comparable to events

[10] See Landro, (2010a).

considered independent with constant but unknown probability (p), where p is distributed according to the mixed distribution that the representation theorem postulates.

Nevertheless, it should be noted that: **i)**the existence of (objective) probability p is a purely mathematical condition due to some particular extension of a consistent distributions family of a finite dimension to a law for the sequence $\{E_1, E_2,..., E_n,...\}$; **ii)** according to Regazzini, E.; Petris, G. (1992), there are sequences of exchangeable events whose relative frequencies of success do not converge stochastically in a precise sense[11] and **iii)** according to de Finetti, B. (1931), given a sequence $E^{(j)}(X)\left(j=1,2,...,[X];\ X=0,1,2,...,n\right)$ of exchangeable events and an event $E^{(n+1)}$: the result of the $(n+1)^{\text{ith}}$ trial is successful, it is shown that the probability of occurrence of $E^{(n+1)}$, conditioned by the alleged occurrence of $E^{(j)}$, is defined by a function of variables X and n, as $p\left(E^{(n+1)}/E^{(j)}(X)\right)=\frac{X+1}{n+1}\frac{p(X+1,n+1)}{p(X,n)}=f(X,n)$, such that $\lim_{n\to\infty} f(X,n)=\frac{X}{n}$. This leads to the conclusion that the apparent objectivity of probability $f(X,n)$ is nothing but a metaphysical illusion. That, in fact, different observers with different allocations of initial probabilities based only on the condition of coherence, by combining this condition with the property of exchangeability and assuming that $\lim_{n\to\infty}\frac{p(X+1,n+1)}{p(X,n)}=1$, will converge towards a final evaluation of the probability equal to $\frac{X}{n}$. That the observer changes his initial probabilities, $p\left(E^{(j)}\right)$ into final probabilities by a Bayesian conditioning. That is, although different observers can start from a model based on different initial probabilities, from an increase of the evidence, its final probabilities will usually tend to converge producing the illusion of the existence of an objective probability (which, in terms of subjective interpretation, is only a void-of-meaning metaphysical concept).

Because of the restrictions already mentioned in Section 2.5 with respect to the **propensionalist approximation** to the probabilities of individual events by defining classes of reference, one can immediately conclude that: **i)** some probabilities can be considered preferable to others and **ii)** the degree of preference with respect to probabilities varies directly with the magnitude of the evidence on which they are based, but this preference relation does not imply the existence of an objective singular probability.

Let a singular event E be that can be classified as an element of a sequence of conditions, $S_1, S_2, S_3 ....$ such that $S_1 \supset S_2 \supset S_3 \supset ....$ Suppose it is owned statistical information that allows for good estimates ($p_1, p_2, p_3,...$) of the objective probability of the occurrence of E with respect to the conditions $S_1, S_2, S_3,....$ Then, according to the

[11] See Landro, (2010a).

foregoing considerations, it is shown that probability p_2 is preferable to p_1, that p_3 is preferable to p_2 and so on. In particular, if the conditions S_i are replaced by the reference class of all elements, S, a probability estimate can be obtained associated with the most restricted reference class.

A first problem related to the identificatory possibilities of the propensionalist model derived from the application of this principle obviously occurs when there is not a single reference class of selected maximum restrictions. But it should be noted that, even if that class exists, the adoption of the criterion that consists of assimilating the probability on the occurrence of a single event to its relative frequency in the most restricted reference class to which the event belongs, could lead to a wrong decision. It could happen that circumstances that did not constitute statistical data in a reference class were known but which, however, provided substantial grounds to correct the assignment of probabilities. In this case, not considering such qualitative evidence can lead to assignments of probabilities with a less satisfactory basis than the one that could have been obtained from a complete analysis of the evidence. The general procedure for assigning probabilities to single events should then: i) assign the event to the most restricted reference class for which reliable statistical data exist (assuming there is a class with these features) and calculate the relative frequency (r) of the event occurrence in that class, and ii) consider any non-statistical information that is relevant for the occurrence of the event in such circumstance and, in light of this information, correct the relative frequency. In case that there is more than one reference of maximum restriction with relative frequencies $r_1, r_2, r_3, \ldots$, a relative frequency should be selected and corrected using non-statistical information. Conversely, if there were not any kind of acceptable reference, the assignment of probability should be based on non-statistical information.

While this method of assignment of probabilities seems reasonable, it is undeniable that it includes many subjective elements and that, therefore, it does not seem suitable as an identificatory model of a single objective probability, particularly in those cases in which there is no statistical information obtained from sufficiently log sequences of observations. In case of having a series of observations without circumstances beyond the statistics, the model to identify the (theoretical) probability on the occurrence of an event from the (observable) relative frequency will be, according to Popper, K.R.'s nomenclature (1934), "*impervious to strict falsifation*"[12]. A vain attempt at solution to this difficulty proposed by Popper was to appeal to the notion of "*methodological falsifation*" according to which, even though the propositions about probabilities are not strictly falsifiable, they can be used (and in fact they are in experimental sciences) as

[12] Let a coin be on which characteristics there is no information and such that it is supposed that the probability of obtaining the result "face" on a given throw is equal to p. Then, the probability of obtaining m times the result "face" in a sequence of n throws independent of such coin will be given by $p(m/n) = \binom{n}{m} p^m (1-p)^{n-m}$. So, no matter how many throws are made, how many times the result "face" is obtained, or which is the supposed value of $p \in (0,1)$, the probability $p(m/n)$ will be non-null. This implies that the hypothesis "*the probability of obtaining the result "face" in a throw is equal to* p" is "*immunized*" with respect to "*strict falsifation*" (see Gillies, D.A. (1990))

falsifiable arguments using statistical test[13]. It should be noted that, according to this procedure, any hypothesis can be methodologically refuted even if, from a strictly logical point of view, it is not falsifiable and, therefore, that this criterion of falsification does not solve the problem of identifying the probability.

4. - Borel's law of large numbers and Cournot's principle

With the aim of isolating the problem of identification of any metaphysical assumption about the true nature of probability, Borel, E. (1905) (1909a) (1909b) proposed the first attempts at linking the theory of set measure with the quantification of the probability (a probability conceived as a mathematical entity exclusively defined in formal terms) by the formulation of the well-known strong law of large numbers, according to which, given a binomial phenomenon whose possible results are E and $\overline{E}$ and denoted by:

$$X_n = X_{n(}(\omega)\begin{cases} 1; if \omega \in E \\ \\ 0; if \omega \in \overline{E} \end{cases}$$

the random variable representing the result of the n -ith observation with probability $p(X_n = 1) = p$ and by $Y^{(n)} = \sum_{j=1}^{n} X_j$ denoting the random variable representing the number of times that the result E occurs in a sequence of n observations, it is verified that $\left(\frac{Y^{(n)}}{n} - p\right) \xrightarrow{ccc} 0$.

This result and the later works by Faber, G. (1910), Hausdorff, F. (1914), Cantelli, F.P. (1916a) (1916b) (1917a), Kolmogorov, A.N. (1929) (1931) (1933a) (1933b) (1933c) and Prohorov, Y.V. (1956) achieved a rigorous formalization of the relationship between the measure theory, the geometric interpretation of probability and the concept of independence in repeatable sequences of events. The falsification criterion associated with this identification model was based on Cournot's principle, according to which it is possible to ensure that a quasi-impossible event will not happen. That is, a criterion of falsification that considers only extreme probabilities (0 or 1)[14] to be related with external references in a conceptually significant way.

Despite the insistence of many authors on considering that Borel's proposal does not need to assume the physical existence of probabilities, given he interprets probability as a purely theoretical term existing in the ambit of ideas and not directly related to the ambit of factual phenomena, one can immediately conclude that his acceptance of

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[13] Popper (1934): "*…a physicist is usually faced with the dilemma of deciding whether a particular probabilistic hypothesis should be accepted as "empirically confirmed" or whether it should be rejected as 'practically falsified'*".

[14] Note that this criterion does not allow distinguishing between two absolutely continuous probabilistic models $\hat{p} \approx \hat{p}^*$; in other words, it only allows distinguishing between two models that match with respect to those events to which a zero probability is assigned, but not necessarily for other events with non-extreme assignments.

Cournot's principle as a link between his interpretation of the notion of probability and the ambit of observations is indisputable proof of the propensionalist character of his proposal. Consequently, to this identificatory criterion, apparently abstract and completely independent of any interpretation of probability, based on the postulates of the strong law of large numbers, the same considerations made to the propensionalist model can be applied, with respect to its falsification.

5. - Conclusion

The presence of such a thing called hazard that inevitably appears in the vision that **all** observers have about the behaviour of **all** phenomena, generates a feeling of uncertainty, which quantification led to different interpretations of the notion of probability.

Some of these interpretations (classical, frequentist, logicist and propensionalist), based on a deterministic conception generated in the epistemological assumption of chance-ignorance, obtained objectivist definitions of probability associated with inferential structures defined by rules of interpretation, explicit or implied, which define their role of identificatory models of the true value of probability.

However, with regard to these rules of interpretation, it should be noted that: **i)** the classical model suffers from unavoidable failures basically related to its purely deductive nature which prevents the testing of its agreement with the observable results of the phenomena; **ii)** the frequentist model is based on the collectives' properties assumed as axioms that are not strictly demonstrable; **iii)** the logistic model considers probabilities as values comparable to logical relationships in an ambit consisting of abstract ideas and iv) the propensionalist model, despite Fetzer's changes to Popper and Miller's proposal, cannot avoid its metaphysical character. Consequently, from the principle of non-existence of assumptions about the true nature of probability, it is not possible to define strict falsification criteria for these models.

This inability of objectivist models to identify the alleged true value of the probability of an event occurrence seems to support the deterministic hypothesis which postulates the "proteiform"[15] character of the probability. Meanwhile, the acceptance of the subjectivist model that, based on an aleatorism concept generated in the ontological assumption of absolute chance, postulates the validity of probability assessments which do not coincide with each other, provided they meet the condition of consistency, leads to the conclusion that there is no truth about the probability, that there is no real probability but infinite versions of the same probability.

Yet, both the deterministic and the aleatorism interpretations constitute, in turn, stochastic models, so it can be concluded that neither the classical hypothesis which postulates the existence of a true value of the probability of occurrence of an event, only asymptotically knowable, nor de Finetti's hypothesis summarized in the postulate that "*probability does not exist*" are strictly verifiable.

[15] Costantini, D.; Geymonat, L. (1982).

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