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Abstract

This paper analyzes the market diffusion of a new product whose quality is uncertain. Consumers learn the product quality by observing the history of market outcomes. Firms cannot observe how consumers evaluate the product quality; instead, they learn by observing consumer behavior. New entry occurs gradually because of informational externalities. This dual uncertainty contributes to an S-shaped diffusion of the new product with declining prices.

JEL Classifications Code: D11, L11, L14.
Keywords: Experience Goods; Quality Uncertainty; Bilateral Learning; S-shaped Diffusion.

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1 Introduction

When a new product or service market opens, consumers are usually uncertain about its characteristics. An important source of product information is actual experience of the product. Another important source is learning from others. Consumers usually learn about product characteristics through their observations of past market outcomes that reflect the behavior of consumers who have experienced the product. The degree of their uncertainty depends on the history of market outcomes and it decreases as the number of past observations increases.

At the same time, firms face demand uncertainty regarding how consumers evaluate the quality of their products. This causes them to be uncertain about the profitability of the market. They learn the product quality by observing the behavior of consumers in the market. Therefore, both demand and supply are endogenously determined by consumer learning.

The aim of this paper is to explore theoretically the time pattern of market diffusion where both the market demand and the level of entry are endogenously determined by consumer learning. The significant result reported here is that consumer-based bilateral learning explains the S-shaped diffusion with decreasing prices.

Previous studies that estimate the time pattern of market diffusion find that it is initially too slow and tends to be S-shaped.[1] In terms of the entry of firms, the S-shaped time pattern of market diffusion implies that the level of new entries initially rises but eventually falls. In addition to the time pattern of diffusion, it is commonly observed that the market price of the product falls over time (Klepper and Graddy (1990)). In the theoretical literature, the phenomenon of S-shaped diffusion with a strictly declining market price has been viewed as the result of a reduction in production cost (Jovanovic and Lach (1989)). In this paper, we provide an alternative explanation to technological progress, instead examining the S-shaped diffusion in terms of informational externalities.

In this paper, we define a dynamic model of market diffusion with consumer-based bilateral learning. The product involved is assumed to be what Shapiro (1983) calls an "experience

[1] See, for example, Gort and Klepper (1982), who investigate the time pattern of the number of firms in 46 product markets. Note that S-shaped diffusion is not an isolated phenomenon. Empirical evidence shows that the time pattern of innovation, whether in usage or ownership, of new product technologies by households tends to be S-shaped. See the seminal work by Griliches (1957) and Mansfield (1968), together with the survey by Stoneman (2002).

good”, which means that consumers immediately realize the product’s quality once they try it. For all periods, there exist two types of consumer: experienced consumers who have already tried the product and inexperienced consumers who have not. Experienced consumers know the product quality but inexperienced consumers do not. Inexperienced consumers learn the product quality by observing the behavior of experienced consumers and update their beliefs in a Bayesian fashion. As they continually update their beliefs about the product’s quality, their expectations become more accurate in each period. At the same time, firms face uncertainty regarding how consumers evaluate the quality of their product. They learn the product quality in the same way that inexperienced consumers do. For each period, entry is determined by the zero expected profit condition. In the case of a high-quality product, bilateral learning positively affects market diffusion and market capacity grows.

We also define a market diffusion process with unilateral learning in which firms know the true quality of the product in the benchmark case. Then, we compare both market capacity expansions in terms of entry level, equilibrium price, and time pattern. The analysis in the case of a high-quality product provides several interesting results. The main results are as follows: (i) diffusion with bilateral learning leads to a lower entry level; (ii) although unilateral learning leads to constant equilibrium prices, bilateral learning leads to declining equilibrium prices over time; and (iii) bilateral learning is more likely to generate S-shaped diffusion.

The intuitive logic for these results is as follows. The only difference between these two market diffusions is whether or not firms know the quality of their product. Under bilateral learning, because of the presence of firm uncertainty about product quality, firms may earn negative profits following overentry when they overestimate the product quality. In this case, the expected profitability of entering the market under bilateral learning is lower than that under unilateral learning. Therefore, the level of entry is lower and the equilibrium price is higher than for the unilateral learning case.

However, as the market capacity increases, the degree of uncertainty decreases. From the firm’s viewpoint, this effect leads to two benefits. The first benefit is that this effect raises the expected profitability associated with the demand change through consumer learning. Note that this effect can be observed under both unilateral and bilateral learning. The second benefit is that it reduces the expected loss from overestimation through firm learning. Because unilateral learning does not have the second benefit, bilateral learning generates an additional increase in the expected profitability. This leads to more new entries than in the unilateral case. Therefore, bilateral learning speeds up market diffusion and reduces equilibrium prices. S-shaped diffusion is more likely to be observed in the case of bilateral learning because bilateral learning leads to fewer initial entries but more subsequent entries.

This paper relates to the literature on market diffusion. In the theoretical literature, market diffusion is regarded as the result of firm learning (Rob (1991)) or intertemporal consumption externalities (Vettas (2000) and Kitamura (2010)). One of the main differences between this paper and those studies is that the others do not simultaneously explain two empirical facts: S-shaped diffusion and strictly declining pricing.[2]

By extending the model in Rob (1991), Vettas (1998) analyzes the model of S-shaped diffusion with bilateral learning.[3] In the model in Vettas (1998), consumers are uncertain about product quality and firms are uncertain about the number of consumers. The model in Vettas (1998) and that in this paper differ in the demand structure; that is, the demand function in Vettas (1998) is horizontal but the demand function in this paper is downward sloping. This seemingly small difference leads to a crucial difference in the explanation of why market expansion is gradual. In the model in Vettas (1998), the reason is firm learning. In contrast, in the model in this paper, the reason is the downward-sloping demand function. When the demand function is horizontal, there is no cost of expansion. Therefore, Vettas (1998) assumes sequential resolution of uncertainty for firms regarding the number of consumers to derive gradual adjustment. In contrast, when the demand function is downward sloping, a rise in current output lowers current prices. This makes faster expansion more costly. Therefore, the downward-sloping demand function plays the role of adjustment costs, leading to gradual market expansion.

In addition, the difference in demand structure leads to the difference in market prices. In

[2]In a study of strictly declining prices without technological progress, Bagwell and Riordan (1991) show that the prices of high-quality products are initially high and strictly decline as a result of strategic behavior. In contrast, this paper shows strictly declining prices without strategic behavior but with firm learning.

[3]Bergemann and Välimäki (1997) also analyze market diffusion with bilateral learning. In their model, however, the number of firms is fixed exogenously.

the model in Vettas (1998), market prices increase over time. When the demand function is horizontal, updates on product quality directly increase the market price. In contrast, when the demand function is downward sloping, an update on product quality leads to a demand shift but does not necessarily lead to an increase in the market price.

The remainder of this paper is organized as follows. Section 2 defines the model. Section 3 introduces market diffusion with unilateral learning and market diffusion with bilateral learning. Section 4 compares properties of the bilateral learning diffusion with that of the unilateral learning diffusion. Section 5 contains concluding remarks. The proofs of all results are provided in Appendix.

2 Model

This section defines the model. We assume that time is discrete and the time horizon is infinite, and that a period consists of two stages: the first half and the second half. The product here is perishable.

2.1 Consumers

There are a number of mass unit consumers in each period. Each consumer has a different preference for a product and purchases a product in each half of every period if and only if his/her reservation price is higher than the market price. Let θ be the type of consumer, which is stationary for all periods and is uniformly distributed on the interval $[0, 1]$. The reservation price of type θ consumer is assumed to depend on his/her type and the product quality, q:

$$V(\theta, q) = \rho\theta + q, \tag{1}$$

where $\rho > 0$ is a preference parameter.[4] We assume that the product is an experience good: consumers do not know the product quality before they try it but once they try it, they perfectly realize its true quality. The following assumption provides the property of q.

Assumption 1. *q is uniformly distributed on the interval* $[q_L, q_H] \in \mathbb{R}_+$.

[4]Assumption 1 implies that every consumer's reservation price grows in the same way. When an increase in reservation price differs among consumers, for example, $V(\theta, q) = \rho + \theta q$, we have almost the same results, which leads to a more S-shaped time pattern of market diffusion.

At the beginning of each period, the set of consumers is composed of experienced consumers who have tried the product and inexperienced consumers who have not. Experienced consumers realize the actual product quality, $\tilde{q}$, and purchase the product if and only if the price is less than or equal to $V(\theta, \tilde{q})$. On the other hand, inexperienced consumers who do not know the true product quality form a belief regarding its quality by observing the behavior of experienced consumers in a Bayesian fashion.

Assumption 2. *At the beginning of (the first half of) each period, inexperienced consumers update their beliefs on the product quality by observing the history of market outcomes, which summarize the behavior of experienced consumers.*

The exact learning mechanism is characterized below. This assumption implies that the transmission of information indirectly occurs from experienced consumers to inexperienced consumers through the observation of past market outcomes.[5] Although the direct information from experienced consumers to inexperienced consumers is important for learning about product quality, we do not focus on this effect in this paper. Let q_t^h be the highest quality that inexperienced consumers expect in period t and q_t^l be the lowest quality in period t. It is easy to see that $q_1^h = q_H$ and $q_1^l = q_L$. The expected quality in period t is denoted by m_t. Then, we have the following lemma.

Lemma 1. *For all* $t = 1, 2, ...,$

$$m_t = \frac{q_t^h + q_t^l}{2}. \tag{2}$$

2.2 Firms

Firms are small and identical. At the beginning of each period, the set of firms is composed of two subsets: potential entrants and incumbents. There is no asymmetry of information between the two subsets. Under bilateral learning, firms do not know how consumers evaluate their product quality but they know its prior distribution and consumers' updating rules. Firms

[5] When the equilibrium price in the first half of period t is larger than that in period $t-1$, inexperienced consumers can update their beliefs at the beginning of the second half of period t. However, as mentioned below, we focus on the equilibrium where the price is nonincreasing, and consumers cannot update their beliefs at the beginning of the second half of the period along the equilibrium path. Therefore, allowing for the possibility of belief updating in the second half of the period does not alter our main results. See also footnote 13.

update their beliefs about the product quality in the same way as inexperienced consumers. On the other hand, under unilateral learning, firms know the consumers' evaluations of the quality of their product. The entry decision for potential entrants is assumed to be made at the beginning of the first half of each period.[6] Potential entrants enter the market with entry cost $c > 0$, which is the initial investment such as the purchase of machinery. We assume that the machinery is durable and continues to work over time. Machines can be operated at any level from zero to one marginal unit for each half of every period in an environment of constant returns to scale. To simplify the analysis, we assume that the scrap value of the machinery and the marginal operating cost are zero.

Let x_t and y_t be the numbers of incumbents and new entrants in period t, respectively. Because the entry cost is not recoverable and the marginal cost is zero, incumbents have no incentive to exit the market and, therefore, $y_t = x_t - x_{t-1} \geq 0$ for all $t = 1, 2, ...$ Assuming that $x_0 = 0$, we have $x_t = \sum_{\tau=1}^{t} y_\tau$ for all $t = 1, 2, ...$ Let $i > 0$ be a constant interest rate. Then, the discount factor is denoted by $\beta \equiv 1/(1 + i)$. Let $R_t(x_{t-1} + y_t)$ be the discounted sum of future expected profits in period t when there exist x_{t-1} incumbents and y_t new entrants in the market at the beginning of period t.[7] The discounted sum of future expected operating profits in period t is composed of the expected operating profits in both the first and the second halves of period t and the discounted continuation operating profits. The aim of firms is to maximize the discounted sum of future profits, which is denoted by $R_t(x_t) - c$. Potential entrants enter the market when the present value of expected profits is positive, i.e., $R_t(x_{t-1}) > c$.

2.3 Equilibrium

Each period's entry level and equilibrium price are endogenously determined in this model. Under both unilateral and bilateral learning, the equilibrium condition is determined by the market-clearing condition and the zero expected profit condition. Let d_t be the number of consumers who purchase the product in period t and f_t be the number of new consumers in

[6]If potential entrants could also enter the market in the second half of each period, they would have to choose between entering in the second half of the period without updating their beliefs about the quality and entering in the first half of the following period after updating their beliefs. This makes the analysis considerably complicated.

[7]Because entrants and incumbents are symmetric and the time horizon is infinite, they have the same present value of expected operating profits.

period t. If $d_0 = 0$, then $d_t = \sum_{\tau=1}^{t} f_\tau$ for all $t = 1, 2, ...$ Now, we define the competitive equilibrium as follows.

Definition. *Under both unilateral and bilateral learning, the competitive equilibrium consists of three sequences* $\{p_t, f_t, y_t\}$ *that simultaneously satisfy the following conditions.*

1. *The market clears for all* $t = 1, 2, ...$*:*

$$f_t = y_t. \tag{3}$$

2. *The market price is determined by the inverse demand of consumers for all* $t = 1, 2, ...$*:*

$$p_t = P_t(x_{t-1} + y_t). \tag{4}$$

3. *New entry satisfies the zero expected profit condition for all* $t = 1, 2, ...$*:*

$$R_t(x_{t-1} + y_t) \leq c, \text{ with equality if } y_t > 0. \tag{5}$$

3 Analysis

In this section, we explain the demand evolution with consumer learning and derive the competitive equilibrium under both unilateral and bilateral learning. After examining consumers' learning process, we analyze the diffusion with unilateral learning, where firms know the product quality, as a benchmark case. Then, we explore the diffusion with bilateral learning and show that there exists a unique equilibrium in which prices gradually decline.

3.1 Demand Evolution with Consumer Learning

In our model, inexperienced consumers learn the product quality by observing the history of market outcomes such as past market prices and the number of past sales. To make the learning process in this paper function well, we introduce two assumptions. First, we assume that the market price in each period satisfies the following properties.

Assumption 3. *The market price satisfies the following properties.*

1. *When inexperienced consumers and firms underestimate the product quality,* $m_t \leq \tilde{q}$*, the equilibrium price is constant during the period.*

2. *When inexperienced consumers and firms overestimate the product quality,* $m_t > \tilde{q}$*, the equilibrium price declines in the second half of the period.*

The intuition behind this assumption is as follows. Firms sell their product at a constant price during a period when they underestimate the product quality.[8] However, when they overestimate the product quality, they reduce the market price. If overestimation occurs, some experienced consumers do not repurchase the product in the second half of the period. Therefore, the market price must fall so that the second half sales equal the first half sales.

Second, we assume the following inequalities.

Assumption 4.

$$2\rho + \frac{7(1-\beta)q_H + (25-9\beta)q_L}{8(2-\beta)} > (1-\beta)c > 2q_H. \tag{6}$$

The first inequality implies that in the second half of each period, at least some experienced consumers who purchased the product in the first half of the period repurchase the product. The importance of this assumption is explained below. In addition, this inequality guarantees positive first period entry on the competitive equilibrium under both unilateral and bilateral learning. The second inequality implies that the diffusion rate is less than one for every period and it guarantees that there exist at least some inexperienced consumers and that learning exists for all periods.[9]

The demand evolution based on consumer learning is characterized as follows. At the beginning of the first period (first half of the first period), all consumers are inexperienced consumers whose expectation about the quality is $m_1 = (q_H + q_L)/2$. Then, the inverse demand function is:

$$P_1(x_1) = \begin{cases} \rho(1-x_1) + m_1 & 0 \leq x_1 \leq 1, \\ 0 & x_1 > 1, \end{cases} \tag{7}$$

[8] For example, if firms must pay the "menu costs" for price adjustment, they may be unwilling to change prices.

[9] The inequalities in (6) are derived by using the equilibrium capacity level under bilateral learning which we obtain in Proposition 2. For the details of the derivation of these inequalities, see footnotes 11 and 18.

and the equilibrium price in the first half of period 1 is $p_1^{fst} = \rho(1 - x_1^*) + m_1$, given the equilibrium market capacity in period 1, x_1^*. At the beginning of the second half of period 1, consumers who purchased the product become experienced consumers and realize its true quality, $\tilde{q}$. On the other hand, inexperienced consumers do not know the product quality. At the beginning of period 2, inexperienced consumers observe experienced consumers' repurchase behavior through the change in equilibrium price during period 1. There are two possible cases.

Case 1 $m_1 > \tilde{q}$

When inexperienced consumers overestimated the product quality at the beginning of period 1, some experienced consumers did not repurchase the product at the same price in the second half of the period. Therefore, the market price must decrease so that the second half sales equal the first half sales. Let p_1^{sec} be the equilibrium price in the second half of period 1; e_1^{sec} be the number of experienced consumers who repurchased the product; and u_1^{sec} be the number of inexperienced consumers. Then, the following three equations are simultaneously satisfied in the market equilibrium in the second half of period 1:

$$p_1^{sec} = \rho(1 - e_1^{sec}) + \tilde{q}, \tag{8}$$

$$p_1^{sec} = \rho(1 - x_1^* - u_1^{sec}) + m_1, \tag{9}$$

$$x_1^* = e_1^{sec} + u_1^{sec}, \tag{10}$$

where $e_1^{sec} > 0$ and $u_1^{sec} > 0$. The first equation represents the inverse demand of experienced consumers, the second equation is the inverse demand of inexperienced consumers, and the third equation indicates the feasibility condition. Since inexperienced consumers know p_1^{sec}, x_1^*, and m_1, then, from these simultaneous equations, they immediately know the true quality of the product, $\tilde{q}$, and learning stops. For the subsequent periods $t = 2, 3, ...,$ the inverse demand function becomes:

$$P_t(x_t) = \begin{cases} \rho(1 - x_t) + \tilde{q} & 0 \leq x_t \leq 1, \\ 0 & x_t > 1. \end{cases} \tag{11}$$

Note that, from equations (8)–(10), we have $e_1^{sec} = x_1^* - (m_1 - \tilde{q})/2\rho$. Therefore, when x_1^* is sufficiently small or $\tilde{q}$ is sufficiently low, no experienced consumers repurchase the product in the second half, i.e., $e_1^{sec} = 0$. In this case, inexperienced consumers cannot realize the true quality of the product, $\tilde{q}$, and learning does not stop.[10] To exclude this possibility, in our model, the first inequality of (6) in Assumption 4 guarantees that x_1^* is sufficiently large so that $x_1^* > (m_1 - q_L)/2\rho$ holds.[11] This means that, under Assumption 4, when overestimation occurs, at least some experienced consumers necessarily repurchase the product and that inexperienced consumers know the product quality regardless of the value of $\tilde{q}$.

Case 2 $m_1 \leq \tilde{q}$

When inexperienced consumers underestimated the product quality at the beginning of period 1, the equilibrium price was constant during the period, by Assumption 3. Since all experienced consumers repurchased the product at the same price, inexperienced consumers believe that the true quality of the product is at least as high as m_1 and update the lowest quality $q_2^l = m_1$. Then, the expected product quality in period 2 becomes $m_2 = (q_H + m_1)/2 = (3q_H + q_L)/4$. There are two cases. First, when inexperienced consumers overestimate the product quality in period 2, $m_2 > \tilde{q}$, the inverse demand function at the beginning of period 2 becomes:[12]

$$P_2(x_2) = \begin{cases} \rho(1 - x_2) + \tilde{q} & 0 \leq x_2 \leq x_1^* - (m_2 - \tilde{q})/\rho < 1, \\ \frac{\rho(2 - x_1^* - x_2) + m_2 + \tilde{q}}{2} & x_1^* - (m_2 - \tilde{q})/\rho < x_2 < x_1^* + (m_2 - \tilde{q})/\rho < 1, \\ \rho(1 - x_2) + m_2 & x_1^* + (m_2 - \tilde{q})/\rho \leq x_2 \leq 1, \\ 0 & x_2 > 1. \end{cases} \tag{12}$$

[10] This is because equation (8) vanishes when $e_1^{sec} = 0$.

[11] By substituting the equilibrium capacity level under bilateral learning which we obtain in Proposition 2 into $x_1^* > (m_1 - q_L)/2\rho$ and rearranging, we have the first inequality of (6) in Assumption 4.

[12] When inexperienced consumers overestimate the product quality in period 2, the highest reservation price among inexperienced consumers, $\rho(1 - x_1^*) + m_2$, can be higher than that among experienced consumers, $\rho + \tilde{q}$. Therefore, with overestimation, there can be two types of inverse demand function depending on whether $\rho + \tilde{q} > \rho(1 - x_1^*) + m_2$ or $\rho + \tilde{q} \leq \rho(1 - x_1^*) + m_2$. However, we can show that $\rho + \tilde{q} > \rho(1 - x_1^*) + m_2$ always holds under Assumption 4 and, therefore, we have equation (12) as the inverse demand function with overestimation in period 2. This argument also holds for all subsequent periods with overestimation. For details, see Section A in the separate technical appendix, available on the first author's website: http://www.geocities.jp/hiro4kitamura/index.html.

Second, when inexperienced consumers underestimate the product quality, $m_2 \leq \tilde{q}$, the inverse demand function becomes:

$$P_2(x_2) = \begin{cases} \rho(1 - x_2) + \tilde{q} & 0 \leq x_2 \leq x_1^* < 1, \\ \rho(1 - x_2) + m_2 & x_1^* < x_2 \leq 1, \\ 0 & x_2 > 1. \end{cases} \tag{13}$$

Given the capacity $x_2^* > x_1^*$, the equilibrium price in the first half of period 2 can be either $p_2^{fst} = [\rho(2 - x_1^* - x_2^*) + m_2 + \tilde{q}]/2$ or $p_2^{fst} = \rho(1 - x_2^*) + m_2$ with overestimation, while $p_2^{fst} = \rho(1 - x_2^*) + m_2$ with underestimation. However, since our goal is to show the existence of a competitive equilibrium with decreasing price under bilateral learning, we focus on the case where the equilibrium price is nonincreasing, i.e., $p_1^{fst} \geq p_1^{sec} \geq p_2^{fst} \geq p_2^{sec} \geq ....$ On this equilibrium path, since all experienced consumers purchase the product, the equilibrium price becomes $p_2^{fst} = \rho(1 - x_2^*) + m_2$ in the first half of period 2.[13]

When $m_2 > \tilde{q}$, since inexperienced consumers observe at the beginning of period 3 that the equilibrium price declined during period 2, they realize the true quality of the product and learning stops in the same way as in the case of $m_1 > \tilde{q}$.[14] On the other hand, when $m_2 \leq \tilde{q}$, inexperienced consumers observe at the beginning of period 3 that the equilibrium price was constant during period 2. They realize that they underestimated the product quality in period 2 and believe that the true quality of the product is at least as high as the expected quality in period 2. Then, they update their beliefs and their posterior distribution has the expected value $m_3 = (q_H + m_2)/2 = (7q_H + q_L)/8$.

For all $t = 3, 4, ...$, the same learning process continues as long as the true quality of the

[13] Of course, if the equilibrium price could be increasing, $p_t^{fst} = [\rho(2 - x_{t-1}^* - x_t^*) + m_t + \tilde{q}]/2$ could be realized in the first half of period $t = 2, 3, ...$ with overestimation. In Section B in the separate technical appendix, we describe the learning process of consumers when the equilibrium price is $p_t^{fst} = [\rho(2 - x_{t-1}^* - x_t^*) + m_t + \tilde{q}]/2$. In addition, if we allow for the possibility of belief updating in the second half of a period, when the equilibrium price in the first half of period t is $p_t^{fst} = \rho(1 - x_t^*) + m_t$ and it is larger than the price in period $t-1$, inexperienced consumers can update their beliefs at the beginning of the second half of period t. In this case, the learning process differs from what is explained here. This possibility is also discussed in Section B in the technical appendix.

[14] For $t = 2, 3, ...$, as in the case of $t = 1$, when overestimation occurs, experienced consumers repurchase the product in the second half and consumer learning necessarily stops if and only if $x_t^* > (m_t - \tilde{q})/2\rho$ is satisfied. Note that, since $m_t - \tilde{q}$ has the largest value for $m_t = m_1$ and $\tilde{q} = q_L$, and since x_t^* is nondecreasing, it is easy to see that $x_t^* \geq x_1^*$ and $(m_1 - q_L)/2\rho > (m_t - \tilde{q})/2\rho$ for all $t = 2, 3, ...$ Therefore, Assumption 4, $x_1^* > (m_1 - q_L)/2\rho$, implies that $x_t^* > (m_t - \tilde{q})/2\rho$ holds for any $\tilde{q} \in [q_L, q_H]$, and guarantees that consumer learning stops once overestimation occurs for all $t = 2, 3, ...$

product is higher than their expected quality: $m_t \leq \tilde{q}$. The expected quality in each period is:

$$m_t = q_H - 2^{-t}(q_H - q_L), \tag{14}$$

for all $t = 1, 2, ..., \dot{t}$, where $\dot{t}$ is the first $t \geq 1$ such that $m_t > \tilde{q}$. Let $\Delta m_t \equiv m_{t+1} - m_t$. Then, it is easy to see that $\Delta m_t > 0$ for all $t = 1, 2, ...,$ and that the value of Δm_t strictly decreases over time and converges to zero. The inverse demand function at the beginning of each period for all $t = 2, 3, ..., \dot{t} - 1$ becomes:

$$P_t(x_t) = \begin{cases} \rho(1 - x_t) + \tilde{q} & 0 \leq x_t \leq x^*_{t-1} < 1, \\ \rho(1 - x_t) + m_t & x^*_{t-1} < x_t \leq 1, \\ 0 & x_t > 1; \end{cases} \tag{15}$$

for $t = \dot{t}$:[15]

$$P_t(x_t) = \begin{cases} \rho(1 - x_t) + \tilde{q} & 0 \leq x_t \leq x^*_{t-1} - (m_t - \tilde{q})/\rho < 1, \\ \frac{\rho(2 - x^*_{t-1} - x_t) + m_t + \tilde{q}}{2} & x^*_{t-1} - (m_t - \tilde{q})/\rho < x_t < x^*_{t-1} + (m_t - \tilde{q})/\rho < 1, \\ \rho(1 - x_t) + m_t & x^*_{t-1} + (m_t - \tilde{q})/\rho \leq x_t \leq 1, \\ 0 & x_t > 1; \end{cases} \tag{16}$$

and, finally, for $t = \dot{t} + 1, \dot{t} + 2, ...$:

$$P_t(x_t) = \begin{cases} \rho(1 - x_t) + \tilde{q} & 0 \leq x_t \leq 1, \\ 0 & x_t > 1. \end{cases} \tag{17}$$

Inexperienced consumers observe the previous market outcome and update their beliefs at the beginning of every period. More precisely, they observe whether or not new experienced consumers in the first half of a previous period repurchase the product in the second half at the same price.

The process of consumer learning in our model is different from that in Vettas (1998). In Vettas (1998), since the demand function is horizontal, the market price is determined only by demand of inexperienced consumers in each period. This leads to $p_t = m_t$ for $t = 2, 3, ..., \dot{t} - 1$ and p_t increases as consumers update their beliefs upward. Therefore, by observing that the consumers who bought at p_{t-2} buy again at p_{t-1}, consumers can update their beliefs. In contrast, in our model, the demand function is downward sloping and we focus on the

[15] See footnote 12.

competitive equilibrium with nonincreasing price. Therefore, consumers cannot update their beliefs about product quality merely by observing that the consumers who bought at p_{t-2} buy again at p_{t-1}, since p_{t-1} is lower than or equal to p_{t-2}. Instead, by observing whether consumers who bought at p_{t-1} in the first half of the period buy again at the same price p_{t-1} in the second half, where only incumbent firms are active, consumers revise their beliefs.

3.2 Benchmark: Diffusion with Unilateral Learning

In this subsection, we define diffusion with unilateral learning in which firms know the product quality and examine its properties for the benchmark case. Because our goal is to determine the properties of successful diffusion, we explore the case of a high-quality product and assume that $\tilde{q} = q_H$.

Suppose that firms know the product quality but consumers do not. Note that the only difference from bilateral learning is that firms do not learn the product quality under unilateral learning. Firms know that the actual quality of the product is q_H, and that consumers always underestimate the product quality for all $t = 1, 2, \ldots$. Let $R^u_t(x_{t-1} + y_t)$ be the discounted sum of future operating profits in period t under unilateral learning. Since firms earn the same operating profits $\rho(1 - x_t) + m_t$ for all periods, $R^u_t(x_t)$ is composed of current profits, $2[\rho(1 - x_t) + m_t]$, and ongoing operating profits, $\beta R^u_{t+1}(x_{t+1})$. Firms enter the market as long as $R^u_t(x_{t-1} + y_t) > c$ and the level of new entries is determined by the zero profit condition. The following proposition shows that there exists a unique competitive equilibrium under unilateral learning.

Proposition 1. *Let x^u_t, y^u_t, and p^u_t be capacity level, new entry level, and market price, respectively, at the competitive equilibrium with nonincreasing price under unilateral learning. Suppose that $x^u_0 = 0$. Then, there exists a unique competitive equilibrium under unilateral learning that satisfies the following conditions.*

1. *For all $t = 1, 2, \ldots$, there exists unique $y^u_t > 0$ that satisfies the following difference equation:*
$$c = 2[\rho(1 - [x^u_{t-1} + y^u_t]) + m_t] + \beta c, \text{ for all } t = 1, 2, \ldots. \tag{18}$$

2. *$\{x_t^u, y_t^u, p_t^u\}$ satisfies the following equations:*

$$x_t^u = 1 - \frac{(1-\beta)c - 2m_t}{2\rho}, \text{ for all } t = 1, 2, ...; \tag{19}$$

$$y_t^u = \frac{\Delta m_{t-1}}{\rho}, \text{ for all } t = 2, 3, ...; \text{ and} \tag{20}$$

$$p_t^u = \frac{(1-\beta)c}{2}, \text{ for all } t = 1, 2, ... \tag{21}$$

3. *The competitive equilibrium under unilateral learning always exists under Assumption 4. In addition, it exists even if Assumption 4 does not hold; more precisely, it exists if and only if*

$$2\rho + q_H + q_L > (1-\beta)c > 2q_H. \tag{22}$$

It is easy to see that the market capacity with unilateral learning, x_t^u, is increasing in the expected value of product quality for inexperienced consumers, m_t. There exists a unique strictly increasing sequence $\{x_t^u\}$ such that $x_t^u \in [0, x^u]$ for all $t = 1, 2, ...,$ and $x_0^u = 0$ and $x_t^u \to x^u$ as $t \to \infty$, where $x^u = 1 - [(1-\beta)c - 2q_H]/2\rho$. One of the important features of unilateral learning is a constant equilibrium price. This implies that the benefit from each period's demand change because of consumer learning does not affect the equilibrium price. Finally, inequality (22) is obtained from $x_1^u > 0$ and $x^u < 1$. Compared with inequalities in (6) in Assumption 4, it is easy to see that $2\rho + q_H + q_L > 2\rho + [7(1-\beta)q_H + (25-9\beta)q_L]/8(2-\beta)$. Therefore, under unilateral learning, a competitive equilibrium exists even when Assumption 4 does not hold, i.e., $2\rho + q_H + q_L > (1-\beta)c > 2\rho + [7(1-\beta)q_H + (25-9\beta)q_L]/8(2-\beta)$.

In our model, the reason for gradual market expansion differs from that in Vettas (1998). In the model in Vettas (1998), market expansion is gradual because of firm learning. In contrast, in our model, it is gradual because of the downward-sloping demand function. When the demand function is horizontal, there is no cost of expansion and the market capacity expansion is instantaneous if there is no uncertainty. Therefore, Vettas (1998) assumes sequential resolution of uncertainty about the number of consumers to derive gradual adjustment. In contrast, when the demand function is downward sloping, a rise in the current market capacity lowers current prices. This makes faster expansion more costly and leads to gradual market expansion.

3.3 Diffusion with Bilateral Learning

In this subsection, we focus on the competitive equilibrium with decreasing price under bilateral learning and examine its existence. Let $R_t^b(x_{t-1} + y_t)$ be the discounted sum of expected future operating profits in period t under bilateral learning. Then, from the definition, when the market is in the transition process $y_t > 0$, $c = R_t^b(x_{t-1} + y_t)$ for all $0 < t \leq \dot{t} - 1$. $R_t^b(x_{t-1} + y_t)$ is composed of expected current period operating profits and the discounted continuation operating profits.

At the beginning of period t, firms know that they underestimate the product quality with probability $\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\}$, and overestimate the product quality with probability $\Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\}$. When they underestimate the product quality in period t, the equilibrium price does not change during the period and they earn the operating profits $\rho(1 - x_t) + m_t$ in both the first and the second halves of the period. In addition, since firms face the same problem in the following period, i.e., underestimation with $\Pr\{m_{t+1} < \tilde{q} \mid m_t < \tilde{q}\}$ and overestimation with $\Pr\{m_{t+1} > \tilde{q} \mid m_t < \tilde{q}\}$, the discounted continuation operating profit becomes $\beta R_{t+1}^b(x_{t+1})$.

On the other hand, when firms overestimate the product quality, the equilibrium price declines during the period and learning stops. Under Assumption 4, they earn expected operating profit $\rho(1 - x_t) + m_t$ in the first half and $\rho(1 - x_t) + (3m_t + m_{t-1})/4$ in the second half of the period.[16] In the subsequent periods, since the expected quality for the overestimations at the beginning of period t is $(m_t + m_{t-1})/2$, firms earn the same operating profits $\rho(1 - x_t) + (m_t + m_{t-1})/2$. Thus, the discounted continuation operating profit becomes $2\beta[\rho(1 - x_t) + (m_t + m_{t-1})/2]/(1 - \beta)$.

Therefore, under Assumption 4, the discounted sum of expected future operating profits in period t, $R_t^b(x_t)$, is composed of expected current profits:

$$\begin{aligned} &2[\rho(1 - x_t) + m_t] \Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} \\ &+ \left[\rho(1 - x_t) + m_t + \rho(1 - x_t) + \frac{3m_t + m_{t-1}}{4}\right] \Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\}, \end{aligned} \tag{23}$$

[16] The equilibrium price in the second half with overestimation is derived in the proof of Proposition 2 in Appendix.

and continuation operating profits:

$$
\begin{aligned}
&\beta R^b_{t+1}(x_{t+1})\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} \\
&+ \frac{2\beta}{1-\beta}\left[\rho(1-x_{t+1}) + \frac{m_t + m_{t-1}}{2}\right]\Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\},
\end{aligned} \tag{24}
$$

for all $t = 1, 2, ..., \dot{t}$.[17]

The following lemma shows that the probabilities of underestimation and of overestimation are constant and are $1/2$, respectively, for all periods.

Lemma 2. *For all periods $t = 1, 2, ..., \dot{t}$,*

$$
\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} = \Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\} = \frac{1}{2}. \tag{25}
$$

This result follows from the uniform distribution of q. From Lemma 2 and the zero expected profit condition, $R^b_t(x_t) = c$ for all $t = 1, 2, ..., \dot{t}$, we have the following proposition.

Proposition 2. *Let x^b_t, y^b_t, and p^b_t be capacity level, new entry level, and market price, respectively, at the competitive equilibrium under bilateral learning. Suppose that $x^b_0 = 0$ and Assumption 4 hold. Then, there exists a unique competitive equilibrium with declining prices under bilateral learning that satisfies the following conditions.*

1. *For all $t = 1, 2, ..., \dot{t}$, there exists unique $y^b_t > 0$ that satisfies the following difference equation:*

$$
\begin{aligned}
c = &\{2[\rho(1-[x^b_{t-1} + y^b_t]) + m_t] + \beta c\}\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} \\
&+ \{\rho(1-[x^b_{t-1} + y^b_t]) + m_t + \rho(1-[x^b_{t-1} + y^b_t]) + \frac{3m_t + m_{t-1}}{4} \\
&+ \frac{2\beta}{1-\beta}[\rho(1-[x^b_{t-1} + y^b_t]) + \frac{m_t + m_{t-1}}{2}]\}\Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\},
\end{aligned} \tag{26}
$$

where $m_0 = q_L$.

[17] For $t = 2, 3, ...$, if the equilibrium price in the first half of period t can be higher than that of period $t-1$, the equilibrium price in the first half of period t with overestimation can be $p^{fst}_t = [\rho(2 - x^*_{t-1} - x^*_t) + m_t + \tilde{q}]/2$, not $p^{fst}_t = \rho(1 - x^*_t) + m_t$. We assume that firms form the expectation that equilibrium price will be nonincreasing and that they compute the present value of expected profits based on this expectation. As shown in Proposition 2, since equilibrium price is actually decreasing, this expectation is fulfilled in equilibrium.

2. $\{x_t^b, y_t^b, p_t^b\}$ *satisfies the following equations:*

$$x_t^b = 1 - \frac{4c(1-\beta)(2-\beta) - (15-11\beta)m_t - (1+3\beta)m_{t-1}}{8\rho(2-\beta)}, \text{ for all } t = 1, 2, ..., \dot{t}; \quad (27)$$

$$y_t^b = \frac{(15-11\beta)m_t - 14(1-\beta)m_{t-1} - (1+3\beta)m_{t-2}}{8\rho(2-\beta)}, \text{ for all } t = 2, ..., \dot{t}; \text{ and} \quad (28)$$

$$p_t^b = \frac{4(1-\beta)(2-\beta)c + (1+3\beta)\Delta m_{t-1}}{8(2-\beta)}, \text{ for all } t = 1, 2, ..., \dot{t}-1, \quad (29)$$

where $\Delta m_t \equiv m_{t+1} - m_t$.

It is easy to see that there exists a unique strictly increasing sequence of $\{x_t^b\}$ such that $x_t^b \in [0, x^b]$ for all $t = 1, 2, ...$, and $x_0^b = 0$ and $x_t^b \to x^b$ as $t \to \infty$, where $x^b = 1-[(1-\beta)c-2q_H]/2\rho$. Furthermore, note that bilateral learning leads to the same entry level and equilibrium price as unilateral learning in the steady state.[18]

Furthermore, it is easy to see that market prices decrease over time. This phenomenon differs from that of Vettas (1998), where market prices increase as long as consumers underestimate the product quality. In Vettas (1998), since the demand function is assumed to be horizontal, prices are demand driven. Therefore, as consumers revise their beliefs upward, prices also increase. On the other hand, in our model, the demand function is downward sloping. Given a downward-sloping demand function, market prices are determined not only by demand but also by supply, because increases in outputs lead to lower prices. Therefore, in each period, although upward revision of consumers' beliefs shifts the demand upward, entry of new firms and increases in the output result in decreases in the price.

4 Comparison

In this section, we examine the properties of diffusion with bilateral learning by comparison with the diffusion with unilateral learning. In particular, we focus on the size of market capacity, the equilibrium price, and the shape of the diffusion. As in Subsection 3.2, we assume that $\tilde{q} = q_H$.

[18]From $x^b = 1 - [(1-\beta)c - 2q_H]/2\rho < 1$, we have the second inequality of (6) in Assumption 4.

4.1 Market Capacity and Equilibrium Price

We first compare both diffusions in terms of the size of market capacity and the equilibrium price. From the above discussion, we know that both learning styles lead to the same market capacity and equilibrium price in the long run. On the other hand, in the transition process, each diffusion has different market capacity and equilibrium price.

Proposition 3. *Suppose that Assumption 4 holds. Compared with the diffusion with unilateral learning, the diffusion with bilateral learning has the following properties:*

1. *Bilateral learning leads to a smaller market capacity and a higher equilibrium price than unilateral learning in every period:* $x_t^b < x_t^u$ *and* $p_t^b > p_t^u$ *for all* $t = 1, 2, ...,$ *and*

2. *Bilateral learning accelerates the diffusion by reducing equilibrium prices:* $y_t^b > y_t^u$ *and* $\Delta p_{t-1}^b < \Delta p_{t-1}^u = 0$ *for all* $t = 2, 3, ...$

The intuitive logic for Proposition 3 is as follows. In the environment of bilateral learning, firms always face the risk of overestimation with probability $1/2$. If overestimation occurs, the equilibrium price falls during the period and firms earn low operating profits in the subsequent periods. Because of the risk of overestimating product quality, the firms' discounted sum of expected future profits in the bilateral learning case is smaller than that of future profits in the unilateral learning case for all periods. Therefore, bilateral learning leads to a smaller market capacity than unilateral learning for all periods.

In addition, it is easy to see that the level of market capacity in the previous period does not influence the change in inexperienced consumers' demand in the current period, and that the demand of inexperienced consumers is the same across both learning styles for all periods. Because equilibrium prices in both learning styles are determined in the area of demand of inexperienced consumers, bilateral learning generates a higher equilibrium price than unilateral learning for all periods.

The second property follows from a difference in the effect of a decrease in the degree of uncertainty between unilateral learning and bilateral learning. In the environment of unilateral learning, a decrease in the degree of uncertainty in period t leads to a demand change through consumer learning.

On the other hand, the decrease in the degree of uncertainty in period t in the environment of bilateral learning leads to two benefits associated with a demand change and with a decrease in the expected loss from overestimation. First, as for unilateral learning, a decrease in the degree of uncertainty in period t leads to a demand change through consumer learning. Second, the decrease in the degree of uncertainty reduces the profit loss from overestimation through firm learning. This benefit increases the incentives for potential entrants to enter the market. Therefore, it speeds up diffusion and reduces the equilibrium price. Because of these two benefits, the speed of market capacity expansion with bilateral learning is faster than that with unilateral learning after period 2.

4.2 S-shaped Diffusion

We now explore the existence of S-shaped diffusion. We first examine the case with unilateral learning and then explore the case with bilateral learning. Let $\Delta y_t = y_{t+1} - y_t$. S-shaped diffusion implies that Δy_t is initially positive but is eventually negative.

The analysis of the shape of diffusion with unilateral learning starts from eventual concavity. We show that $\Delta y_t^u < 0$ for all $t = 2, 3, ...$ Because we have $\Delta y_t^u = (\Delta m_t - \Delta m_{t-1})/\rho < 0$, the number of new entries decreases for all $t = 2, 3, ...$[19] Therefore, the market diffusion path with unilateral learning eventually becomes concave. More importantly, it becomes S-shaped if and only if $\Delta y_1^u > 0$. More precisely, we have the following proposition.

Proposition 4. *Suppose that $x_0^u = 0$. Under Assumption 4, the diffusion with unilateral learning does not become S-shaped. However, under inequality (24), there exists an S-shaped diffusion path if and only if:*

$$(1 - \beta)c > 2\rho + \frac{q_H + 3q_L}{2}. \tag{30}$$

Inequality (30) implies that S-shaped diffusion with unilateral learning arises when the profitability of entering the market is small enough in the first period. The profitability of entry in the first period is $y_1^u = 1 - [(1 - \beta)c - (q_H + q_L)]/2\rho$. On the other hand, the second period profitability is $y_2^u = \Delta m_1/\rho$. By comparing y_1^u and y_2^u, it is easy to check that the first period entry depends on the value of β and c but the second period entry does not. Therefore,

[19] Note that the magnitude of the demand change is decreasing, i.e., $\Delta m_t - \Delta m_{t-1} < 0$ for all $t = 2, 3, ...$

higher discount and a higher value of the entry cost make the first period entry level lower but they do not make the second period entry level lower. This difference leads to the initial convexity of the market diffusion path with unilateral learning.

Next, we examine the case with bilateral learning. Note that:

$$\Delta y_t^b = -\frac{(q_H - q_L)(17 - 5\beta)}{2^{t+4}\rho(2 - \beta)} < 0 \text{ for all } t = 2, 3, ... \tag{31}$$

Therefore, the diffusion path under bilateral learning eventually becomes concave and it becomes S-shaped if and only if $\Delta y_1^b > 0$. More precisely, we have the following proposition.

Proposition 5. *Suppose that Assumption 4 holds. Then, the market diffusion path with bilateral learning becomes S-shaped if and only if:*

$$(1 - \beta)c > 2\rho + \frac{(13 - 17\beta)q_H + 3(17 - 5\beta)q_L}{16(2 - \beta)}. \tag{32}$$

Finally, we compare two diffusion paths. From Propositions 4 and 5, diffusion with unilateral learning does not become S-shaped under Assumption 4, but diffusion with bilateral learning does so even under Assumption 4. Therefore, bilateral learning is more likely to generate S-shaped diffusion than is unilateral learning. From Proposition 3, bilateral learning leads to a lower first period entry than unilateral learning. However, the number of new entries becomes larger after period 2. Therefore, we have $y_1^b < y_1^u$ and $y_t^b > y_t^u$ for all $t = 2, 3, ...$ This contributes to more S-shaped time patterns of diffusion with bilateral learning.

More importantly, S-shaped diffusion with bilateral learning is observed with declining equilibrium prices, whereas unilateral learning leads to constant equilibrium prices. In the previous studies, S-shaped diffusion with declining prices is viewed as a result of a reduction in production cost. This result implies that S-shaped diffusion with declining prices can be explained by the dual uncertainty even in the absence of technological progress.

5 Conclusion

This paper presents a dynamic model of market diffusion in which consumers are uncertain about the product quality and firms do not know how consumers evaluate it. We showed that consumer-based bilateral learning leads to several interesting results by comparing it with unilateral learning. The main results are that bilateral learning (i) leads to a lower entry level; (ii) leads to a declining equilibrium price; and (iii) is more likely to generate S-shaped time patterns.

Although a number of related studies explain S-shaped diffusion, they do not explain a declining equilibrium price without technological improvement. The advantage of this paper is that it shows that S-shaped diffusion with a declining price is possible without technological progress. Rather, the diffusion arises because of informational externalities that increase the market demand through consumer learning and increase the expected profitability of entry of firms through firm learning. Therefore, this paper establishes the theoretical link between informational externalities and market diffusion.

There are several issues requiring future work. First, there is the empirical importance of consumer-based bilateral learning. Second, there is concern about market diffusion with other means of consumer learning. For example, communication between consumers may exist and be a major source of learning.

Finally, there is concern over the generality of our results. In particular, there is concern about the robustness of our results with respect to the demand specification. Our model is restricted to a linear demand structure, which implies that the type of consumer is uniformly distributed. We predict that if we assume that the type of consumer follows a normal distribution, diffusion becomes more likely to be S-shaped because the number of high-type consumers is small but that of intermediate consumer is large. We trust that this study will assist researchers in addressing these issues in the future.

Appendix

Proof of Lemma 1

Let $f(q)$ be the probability density function of q and $F(q)$ be the distribution function. When inexperienced consumers believe that $\tilde{q} \in [q_t^l, q_t^h]$, then the posterior density function becomes:

$$f(q \mid q_t^l < q < q_t^h) = \frac{f(q)}{F(q_t^h) - F(q_t^l)} = \frac{1}{q_t^h - q_t^l} \text{ for all } t = 1, 2, ... \tag{33}$$

Therefore, the posterior density is uniform for all $t = 1, 2, ...$, and we have:

$$m_t = E[q \mid q_t^l < q < q_t^h] = \int_{q_t^l}^{q_t^h} \frac{q}{q_t^h - q_t^l} dq = \frac{q_t^h + q_t^l}{2} \text{ for all } t = 1, 2, ... \tag{34}$$

□

Proof of Proposition 1

In this proof, we derive condition (22) for the existence of the equilibrium. For the proof of the existence and uniqueness of the equilibrium, see Subsection C.1 in the separate technical appendix. From equation (19), for the equilibrium to exist under unilateral learning, the following two conditions must be satisfied:

$$\begin{aligned} x_1^u &= 1 - \frac{(1-\beta)c - 2m_1}{2\rho} > 0, \\ x^u &= 1 - \frac{(1-\beta)c - 2q_H}{2\rho} < 1, \end{aligned} \tag{35}$$

where $x^u = \lim_{t\to\infty} x_t^u$. From these inequalities, we obtain condition (22). □

Proof of Lemma 2

Note that:

$$\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} = \frac{1 - F(m_t)}{1 - F(m_{t-1})} = \frac{q_H - m_t}{q_H - m_{t-1}}. \tag{36}$$

Because $m_t = q_H - 2^{-t}(q_H - q_L)$, then we have:

$$\Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} = \frac{2^{-t}}{2^{1-t}} = \frac{1}{2} \text{ for all } t = 1, 2, ..., \tag{37}$$

and

$$\Pr\{m_t > \tilde{q} \mid m_{t-1} < \tilde{q}\} = 1 - \Pr\{m_t < \tilde{q} \mid m_{t-1} < \tilde{q}\} = \frac{1}{2} \text{ for all } t = 1, 2, ... \tag{38}$$

□

Proof of Proposition 2

In this proof, we derive the second half price in the overestimation case under bilateral learning. For the proof of the existence and uniqueness of the equilibrium, see Subsection C.2 in the separate technical appendix. Note that the firm's expected quality with overestimation is $(m_t + m_{t-1})/2$ for each period. In this case, some experienced consumers do not repurchase the product in the second half. Let x_t^b be the number of sales and consumers in the first half, e_t be the number of experienced consumers who repurchase the product in the second half, and u_t the number of inexperienced consumers who purchase the product in the second half. Then, the firm's expected equilibrium price in the second half $p_t^{b(sec)}$ must satisfy the following three equations:

$$p_t^{b(sec)} = \rho(1 - e_t^{sec}) + \frac{m_t + m_{t-1}}{2}, \tag{39}$$

$$p_t^{b(sec)} = \rho(1 - x_t^b - u_t^{sec}) + m_t, \tag{40}$$

$$x_t^b = e_t^{sec} + u_t^{sec}. \tag{41}$$

Note that Assumption 4 guarantees $e_t^{sec} > 0$.[20] Therefore, by solving the above equations, we have $p_t^{b(sec)} = \rho(1 - x_t^b) + (3m_t + m_{t-1})/4$ for all $t = 1, 2, ...$

Proof of Proposition 3

We first prove the first property in Proposition 3. Note that:

$$x_t^b - x_t^u = -\frac{\Delta m_{t-1}(1 + 3\beta)}{8\rho(2 - \beta)} < 0. \tag{42}$$

Therefore, $x_t^u > x_t^b$ for all $t = 1, 2, ...$ Because both learning styles lead to the same demand change for all $t = 1, 2, ...$, we have $p_t^u < p_t^b$ for all $t = 1, 2, ...$

We next prove the second property in Proposition 3. Note that:

$$y_t^b - y_t^u = \frac{(\Delta m_{t-2} - \Delta m_{t-1})(1 + 3\beta)}{8\rho(2 - \beta)} > 0 \tag{43}$$

and

$$\Delta p_t^b = \frac{(\Delta m_t - \Delta m_{t-1})(1 + 3\beta)}{8(2 - \beta)} < 0. \tag{44}$$

From equation (43), it is easy to see that $y_t^b > y_t^u$ for all $t = 2, 3, ...$ In addition, because $\Delta p_t^u = 0$, it is easy to see that $\Delta p_t^u > \Delta p_t^b$ from equation (44). □

[20] From the three equations (39)–(41), we have $e_t^{sec} = x_t^b - [m_t - (m_t + m_{t-1})/2]/2\rho$. Because of $(m_t + m_{t-1})/2 \in [q_L, q_H]$, Assumption 4 guarantees $e_t^{sec} = x_t^b - [m_t - (m_t + m_{t-1})/2]/2\rho > 0$ for all $t = 1, 2, ...$ See also Subsection 3.1 and footnote 14.

Proof of Proposition 4

Because the diffusion path eventually becomes concave, S-shaped diffusion arises if and only if $\Delta y_1^u > 0$. By equations (19) and (20), Δy_1^u can be rewritten as:

$$\begin{aligned}\Delta y_1^u &= y_2^u - y_1^u = y_2^u - x_1^u \\ &= \frac{q_H - q_L}{4\rho} - \left[1 - \frac{(1-\beta)c - (q_H + q_L)}{2\rho}\right].\end{aligned} \tag{45}$$

Therefore, we have $\Delta y_1^u > 0$ if and only if inequality (30) holds. Since we have:

$$\frac{q_H + 3q_L}{2} - \frac{7(1-\beta)q_H + (25-9\beta)q_L}{8(2-\beta)} = \frac{(3\beta+1)(q_H - q_L)}{8(2-\beta)} > 0, \tag{46}$$

inequality (30) and Assumption 4 do not hold simultaneously. On the other hand, because we have:

$$q_H + q_L > \frac{q_H + 3q_L}{2}, \tag{47}$$

there exist parameter values which satisfy inequalities (22) and (30) simultaneously. Therefore, unilateral learning does not lead to S-shaped diffusion under Assumption 4 but it can under inequality (22). □

Proof of Proposition 5

Because the diffusion path eventually becomes concave, the necessary and sufficient condition for S-shaped diffusion is $\Delta y_1^b > 0$. By equations (27) and (28), Δy_1^b can be rewritten as:

$$\begin{aligned}\Delta y_1^b &= y_2^b - y_1^b = y_2^b - x_1^b \\ &= \frac{(17-5\beta)(q_H - q_L)}{32\rho(2-\beta)} - \left[1 - \frac{8c(1-\beta)(2-\beta) - (15-11\beta)q_H - (17-5\beta)q_L}{16\rho(2-\beta)}\right].\end{aligned} \tag{48}$$

Therefore, we have $\Delta y_1^b > 0$ if and only if inequality (32) holds. Because we have:

$$\frac{7(1-\beta)q_H + (25-9\beta)q_L}{8(2-\beta)} - \frac{(13-17\beta)q_H + 3(17-5\beta)q_L}{16(2-\beta)} = \frac{(3\beta+1)(q_H - q_L)}{16(2-\beta)} > 0, \tag{49}$$

there exist parameter values which satisfy inequality (32) and Assumption 4 simultaneously.

□

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