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Abstract

Patents have long been regarded as the 'gold standard' of intellectual property protection. In "Little patents and big secrets: managing intellectual property", Anton and Yao (2004) call this traditional view into question by finding that firms keep their most important innovations secret. This model modifies key assumptions made by Anton and Yao by accounting for patenting costs, patentability standards, and the fact that patents provide protection in competitive situations where secrecy fails. The latter aspect counteracts the empirically substantiated fact that, in situations where both appropriation mechanisms are applicable, secrecy provides more protection. It is found that firms keep small inventions secret, use both mechanisms for medium inventions, and patent their most important innovations. This result reestablishes the traditional view that patents are crucial to provide R&D incentives and is yet consistent with main empirical findings on the issue.

Keywords: *Heterogeneous inventions · innovation size · intellectual property rights · patents · patent filing fees · patentability standards · renewal fees · secrecy · technology evolution*

JEL-Classifications: *K11 · L16 · L50 · O32 · O34*

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1   Introduction

How do firms protect new inventions? For decades patents have been in the limelight of the economic debate on innovation incentives.[1] The traditional rationale is that, unlike other appropriation mechanisms, patents provide a temporary exclusive right which inventors trade off against two disadvantages. For one, patentees must incur filing and renewal fees to obtain and maintain the exclusive right. For another, a patent requires disclosing technical details, providing rivals with useful information on the invention. By keeping it secret instead, firms can avoid compulsory disclosure and fees but lose the ability to legally prevent rivals from using the idea. Numerous studies suggest that secrecy typically provides more protection than patents.[2] Consistent with this finding, Arora et al. (2008) recently found that patenting an average invention in U.S. manufacturing industries yields a negative 'patent premium', i.e., a negative proportional increment of the invention's value generated by patenting it. However, according to their results, in each industry, at least some inventions are profitable to patent, suggesting that they differ regarding certain characteristics that impact the firms' decision between secrecy and patenting. This provides an explanation why firms rely on both appriopriation mechanisms to protect their intellectual property.[3] Yet, it raises the question which mechanism firms choose for what kind of inventions.

The goal of this paper is to explore how firms decide between secrecy and patenting when innovations are heterogeneous in quality, i.e., they differ regarding the size of the technological step forward induced by them. Assuming a similar heterogeneity concept, Anton and Yao (2004) (henceforth: AY) address this question in a seminal model, where knowledge disclosure has an ambivalent effect on R&D profits: albeit facilitating imitation by reducing the information asymmetry between an innovator and his rivals, partial disclosure can signal a strong competitive position and dissuade rivals from imitating. AY find that, along the lines of the view that patents typically offer weaker protection, firms protect high-quality inventions using secrecy and partial disclosure to signal a strong position. By contrast, firms find it optimal to patent and partially disclose medium-quality inventions, as a trade-off arises between profits from the technological lead and royalties from licensing. Only for low-quality inventions firms patent and fully disclose, since the rivals' profits from imitating small inventions are too low to justify the risk of infringement.

AY make the strong case that firms disclose to a degree that lets rivals infer a large technological lead and reduce production accordingly. Their answer to the question which appropriation mechanism is optimal for different kinds of inventions yet builts upon three

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[1]   See, e.g., Nordhaus (1969), Scherer (1972), Tandon (1982), Scotchmer (1991), and Denicolo (1996).
[2]   See Levin et al. (1987), Harabi (1995), McLennan (1995), Cohen et al. (2000) and Arundel (2001).
[3]   See Mansfield (1986) and Arundel/Kabla (1998). See, e.g., Hall/Ziedonis (2001) for another explanation.

questionable assumptions. Firstly, AY neglect filing and renewal fees which, in most patent regimes, are required to receive and maintain a patent. For instance, Rassenfosse and Pottelsberghe (2008) and Pottelsberghe and Francois (2009) find such costs to have impact on patenting behavior. Hence, especially for low-quality inventions, where fees relative to value are high, filing and renewal fees call the optimality of patents into question. Secondly, patentability standards require inventions to be 'non-obvious', 'novel', and 'useful' in order to qualify for patent.[4] AY model probabilistic patent rights to incorporate the risk of failing to meet these standards. However, they implicitly assume this risk to be uniformly distributed over qualities, while one expects low-quality inventions to face a higher risk, since a smaller technological impact makes them less likely to be 'non-obvious', 'novel' and 'useful'. This assumption thus overrates the viability of patents for low- and medium-quality inventions. Moreover, it calls into question that firms in the AY-model can freely determine the amount of knowledge disclosed by a patent. Partial disclosure reduces an invention's chance to be patentable and, since a patent only protects what is specified in it, impairs its ability to provide protection against subsequent inventions. Thirdly, AY take an invention as given and limit their analysis to competitive behavior-aspects of the decision between secrecy and patenting, neglecting its impact on how to achieve the invention. In a model with homogeneous inventions, Kultti et al. (2007) (henceforth: KTT) show that both mechanisms differ regarding their influence on R&D investments. KTT argue that patents can compensate for providing weaker protection by also providing it, if more than one firm simultaneously discovers an idea. Hence, especially for large inventions, AY underestimate the effectiveness of patents by assuming a given technological lead instead of analyzing how it emerges.

By contrast, this paper investigates the 'R&D side of the story', where firms have to determine the optimal amount of R&D investment before choosing between secrecy or patenting to appropriate returns. Moreover, it scrutinizes how relaxing the other two assumptions by AY influences the decision between secrecy and patenting for different invention qualities. To that extent, a simple innovation model is set up, which extends the KTT-framework by introducing heterogeneous inventions, filing and renewal fees, and patentability standards. Unlike AY, the model does not account for an alterable amount of disclosure. Patentability standards and the necessity for protection against subsequent inventions precludes firms from being able to determine how much to disclose when patenting, while in case of secrecy signalling a strong position via disclosure is unnecessary, since Bertrand competition ensures that a technological leader drives his rivals out of the market. However, instead of assuming a given technological lead, it extends the AY-analysis by also accounting for situations where

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[4]   In the U.S., an idea must differ from prior art in a 'non-obvious' way to a person with ordinary skill in the field, be 'novel' to others at filing date and yield a 'useful' benefit for society (35 U.S.C. §§ 101-103a).

more than one firm takes the lead.

As it turns out, modifying and relaxing the assumptions made by AY turns their result on its head. The model presented here predicts that firms prefer to keep low-quality inventions secret in order to save filing fees or because qualities are too low to meet patentability standards. For higher qualities, firms increase their propensity to patent, i.e., the willingness to seek patent protection, as the aspect that patents provide protection in more situations than secrecy overcompensates filing fees. Since a higher rivals' propensity to patent the same invention decreases each firm's probability of receiving the exclusive right, for medium-quality inventions the model predicts that firms are indifferent between both mechanisms and play a mixed strategy. Only for high qualities, the patent value exceeds the one from secrecy, so that firms prefer to patent and renew their inventions to the full statutory term.

These results are well in line with the finding that secrecy is regarded to typically provide more protection. Throughout the model, secrecy is assumed to yield a higher probability to protect a given technological lead than patenting. Yet, patents can compensate for this disadvantage by providing protection in situations where secrecy fails, i.e., if more than one firm simultaneously discovers the same invention. In practice, this is not unlikely. Standard-izsation within industries as well as the fact that all firms face the same market requirements narrow down the possible paths the future development of a technology takes.[5] Kultti et al. (2006) discuss implications of resulting 'simultaneous innovations' for patent policy. Our model applies this rationale to the firms' decision on how to protect different kinds of inventions, underlining the crucial importance of patents for intellectual property protection.

The model also contributes to the empirical plausibility of current theories on patenting behavior by providing a framework that is consistent with major empirical findings on the issue. For further reference, table 1 summarizes these findings under five 'stylized facts' of patenting behavior. The legal and economic literature on patents provides explanations for each of those facts separately, but no theoretical approach yet managed to explain their occurrence simultaneously in a unifying framework. In a first attempt to theoretically explain why not all inventions are patented (fact 1), Horstmann et al. (1985) see patents as a signal to rivals whether it is worthwhile to imitate. In accordance with fact 1 and the finding that patents increase the value of at least some inventions (fact 5), they find that the optimal propensity to patent is neither zero, since patents increase profits from innovation, nor is it one, since then the rivals would certainly imitate and reduce the innovator's profits. However, Horstmann et al. (1985) cannot explain that secrecy typically provides more protection than patenting (fact 4) simultaneously with fact 1 and 5, as this would drive the propensity to patent in their model to zero. In a complementary framework, Harter (1994) finds that,

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[5]   See Kultti et al. (2006) and (2007). See also Rahnasto (2003) and Varian et al. (2004).

Stylized fact	Description and references
1. *Not all inventions are patented.*	According to Mansfield (1986), U.S. manufacturing firms patent between 49 and 97% of patentable inventions. Arundel and Kabla (1998) use survey data with 604 of the largest European industrial firms to show that the fraction of patented inventions on average is 35.9% for product innovations (24.8% for process innov.). See table 2 for more details. See also Brouwer/Kleinknecht (1999).
2. *Not all patent applications are granted.*	Quillen/Webster (2009) report that between 1995 and 2007 the average percentage of successful applications was 68% in the U.S., 45% at the European Patent Office, and 52% in Japan. Similar numbers have been reported by, e.g., Lemley (2001).
3. *Not all granted patents are renewed to the full statutory term.*	Lemley (2001) reports U.S. patent renewal data indicating that fees are paid for only 82% of patents at the 3.5 year-level, 57% at the 7.5 year-level, and 37% at the 11.5 year-level. Pakes (1986) and Pakes/Simpson (1989) find that only 7% of French and 11% of German patents are renewed to the full statutory term. More recently, Baudry/Dumont (2009) confirmed the 7%-estimate for French patents; see fig. 7 in the Appendix. See also Deng (2003).
4. *Secrecy typically provides more protection than patenting.*	Cohen et al. (2000) find that U.S. manufacturing firms rate secrecy to provide qualitatively more protection than patents. This is consistent with evidence from other countries: Harabi (1995), McLennan (1995), Arundel (2001). See also Levin et al. (1987).
5. *Patenting increases the value of at least some inventions.*	Arora et al. (2008) use the same data as Cohen et al. (2000) but differentiate inventions regarding size of their patent premium. Based on a simultaneous equation model and the assumption that the premiums are normally distributed, Arora et al. use observations of each firm's share of inventions that are patented to estimate mean and variance of the patent premium distribution. Consistent with fact 4, they find that patenting reduces the value of an average invention of U.S. manufacturing firms by 40%. However, in all industries the distribution's variance is high enough that at least some inventions exhibit a positive premium. The authors estimate the expected value of those inventions to be on average 50% higher than without patenting. This is well in line with Schankerman (1998) and Lanjouw (1998) who use European renewal data to estimate that patenting is equivalent to a subsidy to R&D investment of 15 to 25% and about 10% respectively.

Table 1: 'Stylized facts' of patenting behavior.

consistently with fact 5, an innovator normally chooses patents over secrecy, even though compulsory disclosure impairs the efficiency of patents. Only if a patent discloses enough information, so that the rival's decision is to imitate when without a patent he would have chosen not to imitate, an innovator, in line with fact 4, prefers secrecy. Gallini (1992) analyzes optimal patent strength assuming costly imitation and secrecy. She theoretically shows that an innovator will choose secrecy over patents, if patent length is too short to outpace secrecy (fact 4). Yet, if the statutory term is sufficiently long for patents to, consistent with fact 5, yield a positive premium, the innovator will choose to patent all inventions. Since the vast majority of countries obtain a uniform patent length, Gallini (1992) offers no specification where firms rely on both appropriation mechanisms.

In their seminal model, KTT took an important step towards bringing theories on patenting behavior in line with the stylized facts. Assuming simultaneous innovations, they were the first to show that specifications of patent protection exist where some inventions are patented (fact 1), even though secrecy provides more protection (fact 4). They also account for cases where patents still provide less protection than secrecy but enough protection to yield a positive patent premium (fact 5). However, due to the assumption of homogeneous innovations, their model cannot explain facts 1, 4, and 5 simultaneously. Moreover, it does not explain why not all applications are granted and not all granted patents are renewed. To the best of my knowledge, no theoretical model to date can simultaneously explain the occurrence of all five stylized facts in a unifying framework.

This paper intends to make up for this shortcoming. By assuming heterogeneous innovations, it explains why not all inventions are patented *and* how a patent premium occurs, even though secrecy yields more protection. Furthermore, the heterogeneity assumption enables us to also account for the fact why not all patents are renewed to the full statutory term. This is achieved without assuming a specific probability distribution of innovation size in order not to take side in the ongoing debate on which skewed distribution fits the data best.[6] By additionally assuming uncertainty in the patent examination process, the model can account for *all* five stylized facts. The resulting framework constitutes a new generation of patenting behavior theory that is well in line with the stylized facts and may provide a basis for empirical and legal studies on the issue.

The paper is organized as follows. Section 2 outlines the basic framework. Section 3 deals with firms' equilibrium R&D decision when all inventions are kept secret. In Section 4 the equilibrium R&D decision is scrutinized for the case where firms can choose between secrecy and patenting to appropriate returns. Section 5 introduces the decision whether or not to renew a patent, and section 6 deals with patentability standards. Section 7 shows that main results are robust to relaxing the assumption of non-drastic innovations – an aspect of particular interest in light of heterogeneous innovations. Section 8 concludes the paper.

2 The model

Outline. Consider an infinite horizon, discrete-time economy with a continuum of consumer-good industries indexed by $j \in [0, 1]$. In each industry, two risk-neutral firms $i \in \{A, B\}$ compete in prices à la Bertrand and engage in R&D to improve production technology.[7] Unlike standard quality-ladder R&D models assuming a patent race, the model presented

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[6] See, e.g., Scherer and Harhoff (2000), Harhoff et al. (2003), and Silverberg and Verspagen (2007).
[7] Assuming a duopoly streamlines the analysis. The model can easily be extended to include a free-entry-condition determining the number of firms but at the cost of a more intricate IO-analysis.

here particularly accounts for 'simultaneous innovation', i.e., the discovery of an invention by more than one firm at the same time.[8] KTT argue that this phenomenon *"especially characterizes [...] industries [...] where standardization limits the possible paths for future technologies and, accordingly, firms concentrate their R&D activities on the same fields"*.[9] In the following, I account for the notion of technology paths by assuming that, at the beginning of each period, one idea on how to improve technology occurs in each industry.[10] Along the lines of O'Donoghue et al. (1998), ideas differ from innovations. While ideas arise free of charge during production and outline a rough sketch of how technology improvements should look like, innovations constitute the knowledge on how to implement the improvements. Ideas and the corresponding innovations are heterogeneous in quality q, which is assumed to be drawn from an ex ante known probability density $\psi(q)$ with cumulative distribution $\Psi(q)$. The quality of an idea measures the extent to which the resulting invention meets a patent office's patentability requirements, novelty, non-obviousness, and usefulness, so that a higher q represents a larger technological step forward. Quality is non-observable prior to implementation. A new idea immediately becomes common knowledge of all firms active in an industry, but in order to make use of it and to discover the inherent q, firms must invest in R&D to convert the idea into a marketable innovation.

R&D investments (in units of labor) consist of a compulsory fixed component f and an optional variable component $\mathcal{G}$. By investing fixed costs f, firms discover at the end of a period how to implement an idea that occurred at the beginning. For simplicity let us assume that firms must pay f in order to stay in business.[11] A regular fixed costs payment thus ensures a continuous implementation of ideas with a one-period lag and, thereby, constitutes the standard technology path taken in each industry. However, due to Bertrand competition, firms cannot cover fixed costs by simply following the standard path. Instead, they additionally have to invest a variable amount $\mathcal{G}$ to increase the chance of discovering how to implement an idea immediately after its occurrence. R&D outcome is uncertain and follows a memoryless Poisson process with arrival rate φ. For simplicity I assume $\mathcal{G} = \frac{\varphi}{\beta}$, where β captures R&D productivity, so that φ is a measure for the R&D intensity chosen by each firm and thus the variable of interest in the model.[12] It pins down the probability that a firm successfully converts an idea into an innovation within one discrete

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[8] See, e.g., Schumpeterian growth models, such as Aghion/Howitt (1992) and (1998).

[9] See Kultti et al. (2007), p. 23. See also Rahnasto (2003), Varian et al. (2004), and Kultti et al. (2006).

[10] Each industry is assumed to solely rely on its own technology, which in reality is clearly not the case. However, in many industries a firm generally focuses on improving technology in its field of expertise while relying on suppliers and business partners to bring forward the extant technologies used.

[11] This ensures that firms follow a stable technology path, which streamlines the analysis. In practice, firms must catch up with an industry's technology to stay in business in the long run but may skip some steps in the short run. See Aghion et al. (2001) for a model without a maximum lead of one step.

[12] Linear R&D costs simplify the analysis, but main results hold for any weakly convex and monotonic $\mathcal{G}$.

time period to $1 - e^{-\varphi}$.[13] A higher R&D investment increases a firm's success probability and, therewith, the ability to temporarily outpace its rivals. Albeit not occurring with certainty, the resulting technological lead compensates for both types of R&D costs.

The optimal R&D decision of firms can be summarized by the Bellman equation

$$\rho \mathcal{V} = \max_{\varphi} \left\{ V \left(1 - e^{-\varphi} \right) - \frac{\varphi}{\beta} - f \right\} , \qquad (1)$$

where $\rho > 0$ is a time discount rate. Expression (1) is the key equation of the model. The investment value $\mathcal{V}$ (written as an annuity) consists of the two kinds of R&D costs and the success probability times the appropriation value of a one-period technological lead, V. V can be seen as a placeholder for either secrecy or patent value specified below.

The model modifies the traditional understanding of what drives technological progress in this kind of models. What matters for R&D incentives is not *whether* a certain technology level is realized but *when*. In today's knowledge driven society, firms can achieve a new technology level by simply managing to stay in business. In time, even the most venturous invention will turn into common knowledge that can be absorbed at relatively small cost (here: f). Technological progress is instead driven by the prospect of a temporary technological lead. This notion contrasts standard R&D models where innovation incentives result from each technological step forward. The advantage of this specification is simple: Modelling a technology path enables us to account for simultaneous innovations and their implications for appropriation mechanisms neglected by standard R&D models with vertical innovations. Moreover, it ensures tractability of the model, since firms in each industry simultaneously face the same invention qualities and, therewith, the same decisions. Yet, despite this simultaneity, the model incorporates the uncertainty associated with technological progress via the assumption of heterogeneous innovations. In that regard, it only differs from familiar theories in that it models a standardized time interval with varying invention sizes instead of a standardized invention size occurring at varying points in time.

Consumption and profits. Consider a continuum of risk-neutral agents, each endowed with one unit of labor and holding a balanced portfolio of shares in all firms. Agents spend the same amount ξ in each industry as consumer preferences are logarithmic: $U_0 \equiv \sum\limits_{t=0}^{\infty} \frac{ln\,X_t}{(1+\rho)^t}$. For now, I assume aggregate output X_t to follow $ln\,X_t = \int_0^1 ln\,x_{jt}\,dj$, where x_{jt} denotes the output of industry j.[14] Given this we can focus on an exemplary industry where demand takes the simple unit-elastic form $x_{jt} = \frac{\xi}{p_{jt}}$ for all $j \in [0,1]$.

Output is produced by both firms using labor L_{ijt} and industry-specific productivity $\mathcal{A}_{jt}$:

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[13]   This specification follows KTT and ensures a non-negative complementary success probability $e^{-\varphi}$.

[14]   In section 7, I replace this Cobb-Douglas specification by the more general CES form.

$x_{ijt} = \mathcal{A}_{jt} L_{ijt}$. $\mathcal{A}_{jt}$ captures all steps forward taken by industry j's standard technology path until time t. Marginal costs of a firm following this path are $m_{ijt} = \frac{1}{\mathcal{A}_{jt}}$, where wage is exogenous and normalized to unity. Firms that successfully outpace the standard path can improve upon marginal costs, yielding $m_{ijt} = \frac{1}{\mathcal{A}_{jt} \gamma_{jt}}$. $\gamma_{jt} > 1$ is the size of the productivity leap resulting from an idea implemented at the beginning of the tth period and immediately follows from the drawn quality q. In fact, q is modelled to be a linear transformation of γ, whose exact value follows from the firms' profit maximization.

With unit-elastic demand, the technological leader maximizes profits by charging limit price $p_{it} = \frac{1}{\mathcal{A}_{jt}}$.[15] As a result, operating profits take the form

$$\pi_{jt} = \left(1 - \frac{1}{\gamma_{jt}} \right) \xi \ . \tag{2}$$

Profits strictly increase in innovation size and consumer spending. For notational simplicity, I choose $q(\gamma) = \left(1 - \frac{1}{\gamma} \right)$, so that each $\gamma \in (1, \infty]$ relates to one $q \in (0, 1]$. An invention with a quality close to zero leads to virtually no productivity increase, while $q = 1$ represents an infinite sized productivity leap reducing marginal production costs to zero. Profits thus take the convenient form $\pi_{jt} = q_{jt} \xi$, where quality indicates the percentage of consumer spending that the technology leader can turn into profits.

Appropriation mechanisms. Firms have two options to appropriate returns: secrecy or patenting. In practice, neither of them provides perfect protection. Parallel research on similar projects, reverse engineering and informal knowledge spillover impair the effectiveness of secrecy, while patents are frequently challenged in lawsuits, require additional resources to prove infringement, and have fairly limited scope and duration.[16] Let us assume that, if a firm relies on secrecy, there is a probability $1 - \eta_s$ that the invention becomes publicly available within one period. In case of public availability rivals can realize the same productivity leap as the innovator free of charge. Hence, $\eta_s \in [0, 1]$ captures the extent of protection provided by secrecy. $\eta_s = 1$ implies perfect protection until the rivals catch up due to spending f, while for $\eta_s = 0$ the innovation becomes public immediately after its occurrence.

Similarly, patent protection is measured by probability η_0 that a patentee can exclude others from using his invention, where $\eta_0 \in [0, 1]$.[17] This specification of patent protection ensures comparability between secrecy and patenting. Along the lines of KTT, a single randomization determines whether a patented invention becomes publicly available. It simultaneously captures patent life, the scope of protection, probability and success of costly

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[15]　The fact that limit pricing always maximizes profits is a result of the unit-elastic demand assumption, where profits strictly increase in prices. See section 7 for a more detailed discussion.

[16]　See Bessen/Meurer (2005) regarding the significance of patent litigation. See also Hall/Ziedonis (2007).

[17]　Note that Bertrand competition rules out licensing, as no profits accrue if more than one firm produces.

litigation, and other uncertainties.[18] If this random event does not occur, the innovator receives a perfect property right until the invention becomes common knowledge with the discovery the subsequent idea.[19] While firms can rely on secrecy free of charge, patenting requires the payment of filing costs c_0. Moreover, firms can maintain a patent to its maximum statutory term and increase protection to η_1, where $\eta_1 > \eta_0$, but in return they must incur renewal costs c_1. Most patent regimes require the payment of more than one renewal fee to obtain the maximum term, but for simplicity I restrict the analysis on a representative renewal decision with an adequately adjusted fee.[20]

Both appropriation mechansims are assumed to be mutually exclusive. In practice, firms forfeit their right to patent after a one year 'grace period' of commercial usage of the invention, and patenting comprises compulsory disclosure of technical details, precluding secrecy as an option.[21] In the model, firms are thus unable to reverse an appropriation decision.

Timing of decisions. The timing of the model within one period can be summarized by four consecutive stages shown in figure 1. After the occurrence of a new idea, at the first stage, firms form expectations about the inherent q and decide whether to remain in the market via the payment of compulsory fixed costs f. They will choose to do so as long as the investment value implied by (1) is positive. At stage two, firms determine the optimal R&D intensity to maximize (1) and learn about the idea's quality in case of successful implementation. Knowing q and their own research success but without information about the rival's success and the exact patentability standard the patent office sets for the invention, at stage three, innovators choose whether to rely on secrecy or patenting to protect their invention. If firms choose patenting and successfully take the patentability hurdle set by the patent office, at stage four, they have to decide whether or not to renew the patent to its full statutory term.

To solve the model let us focus on the steady-state. The solution concept is subgame-perfect equilibiria, so I solve the model proceeding backwards. Note that the required spending of fixed costs f ensures that at the beginning of each period, firms in each industry are symmetric with regard to production, R&D, and probability density $\psi(q)$. The implication of this assumption is twofold. For one, firms choose identical steady-state R&D intensities φ^*, so in the following we can focus on a representative firm. Where useful, I denote the rival's intensity, which is exogenous to the firm under scrutiny, by $\overline{\varphi}$. For another, since the

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[18] See Gilbert/Shapiro (1990), Klemperer (1990), and Denicolo (1996) for different dimensions of patents.
[19] I assume patents to be designed such that by the time the innovation becomes common knowledge no subsequent innovation is prohibited and the rival can invent around the patent to achieve the same productivity level with a non-infringing technical solution.
[20] In most countries renewal fees must be paid anually, starting 2 to 4 years after *filing*, while in the U.S. fees are due 3.5, 7.5, and 11.5 years after patent *grant*. In Canada, renewals start with the 2nd anniversary of filing, in Germany with the 3rd, and in the U.K. with the 4th. In Japan renewal fees are due within the first year from *grant* – lump-sum for the first 3 years and anually starting with the 4th.
[21] For the U.S., see 35 U.S.C. § 102(b) and 112. See also Denicolo/Franzoni (2004) for a discussion.
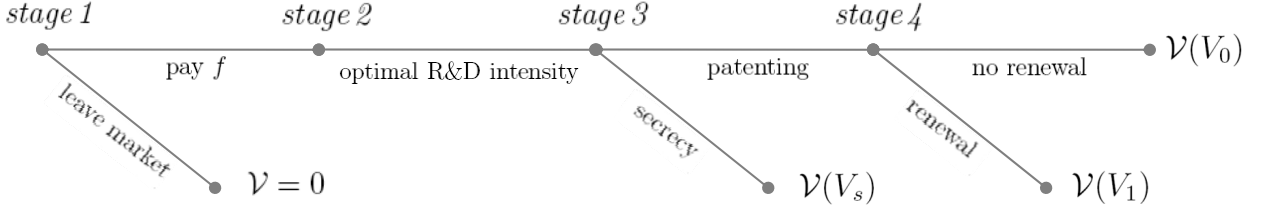
Figure 1: Timing of the model.

productivity level $\mathcal{A}_{jt}$ does not enter profits given by (2), at the beginning of each period an identical patenting game is played in each industry. The variety in outcomes implied by the stylized facts follows from the heterogeneity of innovations.

3   Equilibrium R&D investment without patents

A closed-form solution of (1) requires a detailed specification of V. To begin with, let us focus on the case without patenting, where $V = V_s$ is the value of a technological lead protected by secrecy. The likelihood of such a lead does not solely depend on a firm's own success probability $1 - e^{-\varphi}$, since due to symmetry the rival invests the same amount $\mathcal{G}$ to also obtain the lead. Market structure can thus take three different forms:

i) With probability $(1 - e^{-\varphi})^2$ both firms outpace the technology path and compete neck-to-neck at an improved productivity level $\mathcal{A}_{jt}\gamma_{jt}$ (simultaneous innovation case).

ii) With probability $2(1 - e^{-\varphi})e^{-\varphi}$ the industry is in a leader-follower situation, where one firm succeeds and produces at $\mathcal{A}_{jt}\gamma_{jt}$ while the other fails and remains at $\mathcal{A}_{jt}$.

iii) With probability $e^{-2\varphi}$ both firms fail and compete neck-to-neck at $\mathcal{A}_{jt}$.

Secrecy is useless in the simultaneous innovation case, since a rival has already obtained knowledge on how to implement the idea. Instead, secrecy only yields profits, if the rival fails to innovate, so that given (2) we have

$$V_s = q\,\xi\,\eta_s\,e^{-\overline{\varphi}}\,. \tag{3}$$

The secrecy value is linear in q and positive for all $q \in (0, 1]$, as no filing costs accrue. A higher R&D intensity φ decreases V_s, as the probability that the rival succeeds rises.

Recall from the previous section that, by the time they choose the optimal R&D intensity φ^*, firms have no information about the idea's quality. Therefore, they form expectations about quality depending on the underlying distribution $\psi(q)$. Let us denote by $\tilde{q}$ the expected quality of the innovation under scrutiny, where $\tilde{q} = \int_0^1 q\,\psi(q)\,dq = 1 - \int_0^1 \Psi(q)\,dq$ is the

average mean quality weighted by probability density $\psi(q)$. φ^* thus follows from maximizing

$$\rho \mathcal{V} = \max_{\varphi} \left\{ \tilde{q} \, \xi \, \eta_s \, e^{-\overline{\varphi}} \left(1 - e^{-\varphi} \right) - \frac{\varphi}{\beta} - f \right\} , \tag{4}$$

which yields the closed-form solution for the optimal R&D intensity[22]

$$\varphi^* = \frac{1}{2} \, ln(\beta \, \tilde{q} \, \xi \, \eta_s) . \tag{5}$$

Intuitively, the optimal R&D intensity increases in profits, R&D productivity and protection provided by secrecy. Existence and uniqueness of a positive optimal intensity is ensured by

Lemma 1. *A unique and positive optimal equilibrium R&D intensity exists for all R&D productivity levels β equal to or higher than minimum R&D productivity $\tilde{\beta} = \frac{1}{\tilde{q} \, \xi \, \eta_s}$.*

Proof. See Appendix A.1. □

The existence of a positive φ^* does not necessarily imply that firms invest in research, since the optimizing behavior at stage two neglects the payment of compulsory fixed costs at stage one. To ensure that firms in equilibrium invest in R&D, i.e., $\mathcal{V}(\varphi^*) > 0$, the model requires an additional parameter requirement, the 'β-f-relation', summarized in

Lemma 2. *Firms invest in R&D, if $f \leq \beta^{-1} \left(\sqrt{\beta \, \tilde{q} \, \xi \, \eta_s} - 1 - \frac{1}{2} \, ln(\beta \, \tilde{q} \, \xi \, \eta_s) \right)$.*

Proof. See Appendix A.1. □

For $f = 0$ the β-f-relation equals the minimum R&D productivity level implied by Lemma 1. Yet, higher fixed costs require either a higher R&D productivity, higher profits or more protection from secrecy to ensure a positive investment value $\mathcal{V}$.

The β-f-relation in Lemma 2 and the minimum R&D productivity in Lemma 1 outline the necessary parameter combinations to solve the model. Moreover, since firms only choose patenting over secrecy, if the former yields a higher value, no stricter condition than the β-f-relation for all $\beta \geq \tilde{\beta}$ is needed for the analysis in the remainder of the paper.

4 The case with patents

In the previous section secrecy was the firm's only option to protect its intellectual property. Let us now turn to the case with patents, where firms have to trade off the secrecy value from equation (3) against the patent value (without renewals) given by

$$V_0 = q \, \xi \, \eta_0 \left(e^{-\overline{\varphi}} + \frac{1 - e^{-\overline{\varphi}}}{1 + \overline{\sigma}} \right) - c_0 . \tag{6}$$

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[22] Note that for the firm under scrutiny $\overline{\varphi}$ is exogenous. After maximizing, I use $\overline{\varphi} = \varphi^*$ to derive (5).

$\overline{\sigma} \in [0,1]$ is the rival's propensity to patent, i.e., the willingness to seek patent protection for an invention based on the same idea as the firm under scrutiny. $\frac{1}{1+\overline{\sigma}}$ thus measures the probability of receiving the patent right on the invention.

The patent value in (6) differs from (3) in three regards.[23] Firstly, unlike secrecy, patenting includes filing cost c_0. Secondly, while secrecy is only a viable means of protection, if the rival failed to innovate, a patent also protects the invention in case of simultaneous innovations, i.e., if both firms succeed in R&D.[24] Thirdly, without patentability standards, the probability of being granted a patent in case of a simultaneous innovation depends on the rival's propensity $\overline{\sigma}$ of seeking a patent as well. If the rival also files a patent application ($\overline{\sigma} = 1$), the probability of receiving the patent is $\frac{1}{2}$, since both inventions are based on the same idea. If, however, the rival chooses not to patent ($\overline{\sigma} = 0$), the firm under scrutiny will be granted the patent for sure. Due to symmetry, both firms choose identical propensities to patent σ, whose exact value follows from trading off secrecy against patenting.

The trade-off between secrecy and patenting is governed by two opposing effects. Secrecy outperforms patenting by providing qualitatively more protection at no cost. In line with stylized fact 4, let us henceforth focus on the case where $\eta_s > \eta_0$. However, despite this disadvantage, firms might prefer patents, as they are applicable in more situations than secrecy. In other words, patenting provides less protection for a technological lead but obtaining this lead based on a patent is more likely than based on secrecy. In (6) this is captured by the term $\frac{1-e^{-\overline{\varphi}}}{1+\overline{\sigma}}$. For patents to outperform secrecy at least for some q, this expression must overcompensate the disadvantage $\eta_s > \eta_0$. Otherwise patents cannot make up for the head start of secrecy resulting from filing costs, and no patenting occurs.

The investment value for the case with patenting can be written as

$$\rho \mathcal{V} = \max_{\varphi} \left\{ \max_{\sigma} \left\{ (1-\sigma) V_s + \sigma V_0 \right\} (1 - e^{-\varphi}) - \frac{\varphi}{\beta} - f \right\} . \tag{7}$$

Firms will choose secrecy or patenting (and thereby σ) depending on which yields the higher value. Figure 2 illustrates the underlying process. For q close to zero, it becomes apparent from (3) and (6) that secrecy is optimal, as $\lim_{q \to 0} V_s > \lim_{q \to 0} V_0$ due to filing costs c_0. For low qualities both firms thus choose $\sigma = 0$ until q is sufficiently high to yield $V_0|_{\overline{\sigma}=0} = V_s$. The cut-off quality associated with this value equality is denoted by q_0'. Marginally higher qualities than this cut-off, however, do not yield $V_0 > V_s$, as both firms increase the propensity to patent, which in turn lowers the patent value just enough for $V_s = V_0(\overline{\sigma})$ to hold. This

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[23] Recall that firms know the invention's inherent q when deciding to patent at stage three. Hence, the probability density function $\psi(q)$ in $\tilde{q}$ is only relevant for the optimal R&D decision at stage two, while the patent value relevant for the decision at stage three only depends on q.

[24] This would be the only case where firms choose patents, if they knew the rival's success probability, since with information about the rival's failure, they could save c_0 and rely on secrecy without risk.
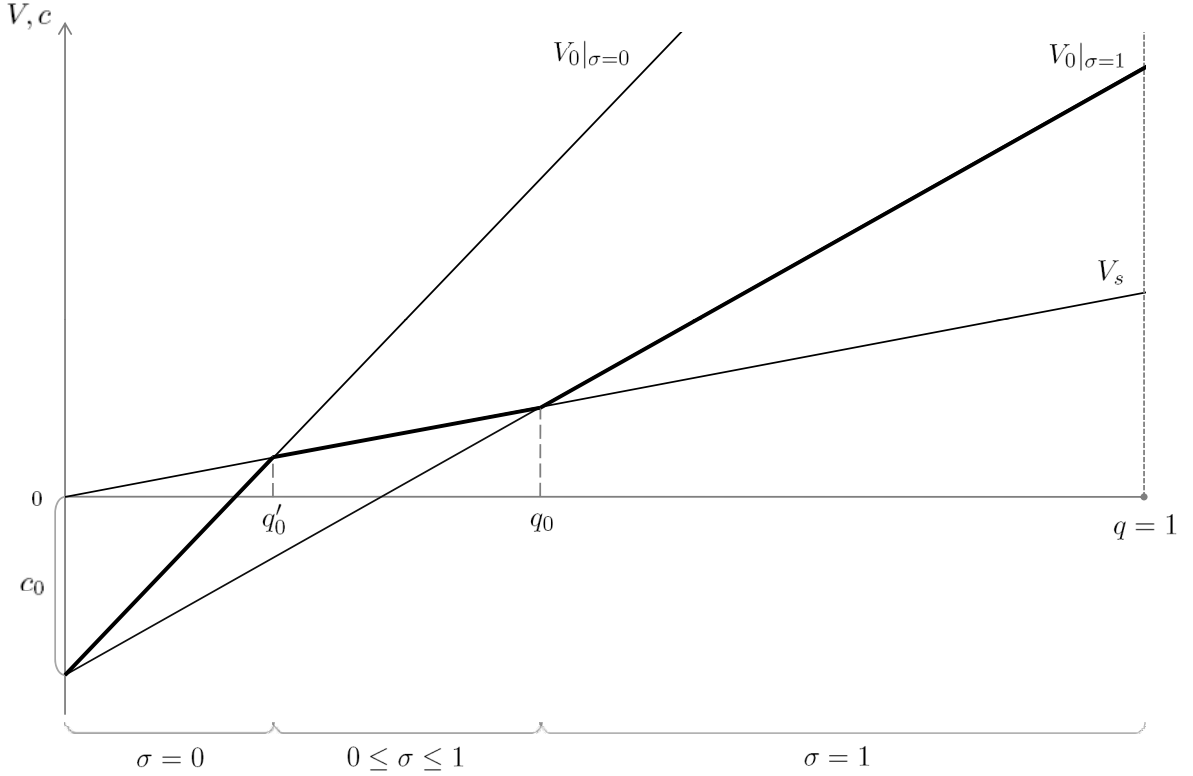
Figure 2: Patent and innovation value for the case where q_0 exists.

expression can be seen as a simple reaction function following from the maximization w.r.t. σ in (7) and ensures a stable equilibrium behavior of a firm given the response of its rival. As long as the resulting σ lies within its bounds zero and unity, both firms are indifferent between secrecy and patenting and play a mixed strategy as σ rises with higher qualities.[25] Since σ cannot exceed one, we can derive a second cut-off q_0, above which higher q unambiguously yield $V_0|_{\sigma=1} > V_s$, and both firms choose to solely rely on patents.

Before I characterize the optimal decision between secrecy and patenting in the following Proposition, we have to take a look at the parameter restrictions necessary to ensure that cut-offs q_0' and q_0 are within $q \in (0,1]$. Moreover, what justifies focusing on the case shown in figure 2, in which patents can compensate for their disadvantages associated with filing costs and $\eta_s > \eta_0$? The answer lies in the stylized facts of patenting behavior. If even for $q = 1$ we have that $V_0|_{\bar\sigma=0} \leq V_s$, no patenting occurs. This, however, conflicts with stylized fact 1 stating that the propensity to patent is larger than zero. Hence, at least for the highest qualities possible, $q_0' \in (0,1)$ must hold. This empirical plausibility is ensured by

Lemma 3. *The percentage of innovations that are patented is between zero and one, if* $\eta_0 > \eta_s\, e^{-\varphi^*} + c_0\,\xi^{-1}$ *holds. Then, cut-off* $q_0' = \frac{c_0\,\xi^{-1}}{\eta_0 - \eta_s\, e^{-\varphi^*}}$ *exists.*

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[25] σ can thus be seen as a buffer between secrecy and patent value which ensures that for certain qualities both appropriation mechanisms yield the same profits. The corresponding σ follows from $\max_{\sigma}\{(1-\sigma)\,V_s + \sigma\,V_0(\bar\sigma)\} \Leftrightarrow V_s = V_0(\bar\sigma)$ and $\sigma = \bar\sigma$. It is given by $\sigma(q) = \frac{q\,\xi\,\eta_0\,(1-e^{-\varphi})}{q\,\xi\,(\eta_s-\eta_0)\,e^{-\varphi}+c_0} - 1$.

Proof. See Appendix A.2. □

Cut-off q_0' captures the main intuition behind the coexistence of secrecy and patenting in the model. Firms use both means of protection, if $q_0' \in (0, 1)$. Albeit secrecy provides more efficient protection, the cut-off can be positive, since secrecy is only applicable if the rival fails to innovate. This occurs with probability $e^{-\varphi^*}$, potentially yielding a positive denominator of q_0', as $\eta_0 > \eta_s e^{-\varphi^*}$. If this difference additionally compensates for filing costs relative to spending, $c_0 \xi^{-1}$, q_0' is between zero and unity and both means of protection are used.

While in line with fact 1, Lemma 3 is insufficient to additionally account for stylized fact 5 stating that patenting increases an invention's value yielding a patent premium. For this to be the case $V_0|_{\sigma=1} > V_s$ and $q_0 \in (0, 1)$ must hold, which corresponds to

Lemma 4. *A 'patent premium' and the corresponding cut-off* $q_0 = \frac{2\, c_0\, \xi^{-1}}{\eta_0\, (1+e^{-\varphi^*}) - 2\, \eta_s\, e^{-\varphi^*}}$ *exist, if equilibrium R&D investment satisfies* $\eta_0 > \eta_s \frac{2\, e^{-\varphi^*}}{1+e^{-\varphi^*}} + \frac{2\, c_0\, \xi^{-1}}{1+e^{-\varphi^*}}$.

Proof. See Appendix A.3. □

The corresponding minimum R&D intensity, below which patenting does not yield a positive premium, is $\varphi^*_{min} = ln\left(\frac{2\, \eta_s - \eta_0}{\eta_0 - 2\, c_0\, \xi^{-1}}\right)$. It is the minimum parameter requirement to bring the model in line with stylized facts 1, 4 and 5 (see Proposition 2 below).

We can now summarize the optimal decision between secrecy and patenting in

Proposition 1. *If Lemmas 2, 3 and 4 hold, so that cut-offs q_0' and q_0 exist in equilibrium and $0 < q_0' < q_0 < 1$, then, as optimal appropriation mechanism firms will choose*

   i) secrecy, if $0 < q \leq q_0'$,

   ii) a mixed strategy, if $q_0' < q \leq q_0$, and

   iii) patenting, if $q_0 < q \leq 1$,

for a given innovation quality q.

Proof. Proposition 1 follows immediately from the discussion above. □

Proposition 1 provides the basis for the derivation of the optimal R&D intensity at stage two. For all $0 < q < q_0'$, secrecy yields a higher value than patenting. Since in the mixed strategy-area σ endogenously adjusts to ensure that both values are equal, secrecy value (3) can be used for all $0 < q < q_0$. Only for higher qualities, the patent value (with $\bar{\sigma} = 1$) becomes relevant. In Appendix A.4 it is shown that this gives rise to investment value

$$\rho\, \mathcal{V} = \left[V_s|_{q=\tilde{q}} \left(\frac{\tilde{q}_0}{\tilde{q}}\right) + V_0|_{q=\tilde{q}} \left(1 - \frac{\tilde{q}_0}{\tilde{q}}\right) - c_0 \left(\frac{\tilde{q}_0}{\tilde{q}} - \Psi(q_0)\right) \right] (1 - e^{-\varphi}) - \frac{\varphi}{\beta} - f \,, \quad (8)$$

where $\tilde{q}_0 = q_0 \, \Psi(q_0) - \int_0^{q_0} \Psi(q) \, dq$ is the expected quality for all $q \in (0, q_0)$, and q_0 is given by Lemma 4. The optimal R&D intensity in the case with patenting maximizes this expression.

As in the case without patenting, the investment value consists of the expected value of the innovation minus costs. In contrast to equation (4), however, the expected value in (8) depends on a combination of secrecy and patent value weighted by an expression of the probability that the drawn quality is higher or lower than the cut-off quality q_0. This expression, $\frac{\tilde{q}_0}{\tilde{q}}$, is the weighted average mean of all qualities within interval $(0, q_0]$ in terms of the expected quality over all qualities possible. It is a measure of the probability that a drawn quality lies in the interval $(0, q_0]$. If $q_0 = 1$, which corresponds to the case where no positive patent premium exists, both intervals coincide implying $\tilde{q}_0 = \tilde{q}$, and the fraction becomes one.[26] As a result, the weight for the patent value becomes zero, so that the secrecy value is relevant for all qualities. If, however, the cut-off quality equals zero, $\tilde{q}_0$ also drops to zero, and the patent value is the value accruing for all qualities. Note that filing costs are weighted by $\frac{\tilde{q}_0}{\tilde{q}}$ solely due to notational reasons.[27] Yet, the term $c_0 \, \Psi(q_0)$ is added, since only for qualities higher than q_0 filing costs accrue, so that a formulation of V_0 over the whole interval requires a corresponding compensation.

The cumulative distribution of q assumes a crucial role in the solution of the model, as it influences the weight of both values in (8). Without a more detailed specification of $\Psi(q)$, it is not possible to derive a closed-form solution for the equilibrium R&D intensity φ^* for the case with patenting. However, it becomes apparent from (8) that R&D incentives at least sustain the optimal intensity derived for the case without patenting, since firms only patent if it yields more profits than secrecy.[28] The lower bound of possible equilibrium R&D intensities resulting from (8) is thus given by equation (5) in the previous section. Based on this closed-form solution, it can be shown that there exist parameter combinations for which even a 'worst case' R&D intensity meets the minimum requirement implied by Lemma 4. This suffices to bring the model in line with the stylized facts, giving rise to

Proposition 2. *The model is line with facts 1, 4, and 5 for any parameter combination satisfying* $c_0 < \left(\frac{\eta_0}{2}(1 + e^{\varphi^*}) - \eta_s \right) \xi \, e^{-\varphi^*}$, *where* $\varphi^* \geq \frac{1}{2} ln(\beta \, \tilde{q} \, \xi \, \eta_s)$.

Proof. See Appendix A.5. □

According to Proposition 2, firms can cover filing costs, if they invest enough in R&D to overcompensate the fact that patents offer less efficient protection. The Proposition shows that in this model parameter specifications exist which concur with stylized facts 1, 4, *and*

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[26]   Note that $\tilde{q}_0$ is *not* the expected value of q_0, so that having $q_0 = 1$ with certainty corresponds to $\tilde{q}_0 = \tilde{q}$.

[27] c_0 is independent of q, so $\frac{\tilde{q}_0 \, c_0}{\tilde{q}}$ compensates for what is added by the weight of V_0 times the c_0 in V_0.

[28]   In (8) this 'worst case' occurs, if the probability of drawing a quality smaller than the cut-off q_0 is 1.

5. This results from the assumption of heterogeneous inventions inducing that the expected quality, which firms base their R&D investment decision on, and the actual quality, which firms learn in time for the patenting decision, diverge. The degree of divergence depends on probability density $\psi(q)$ which I assumed to be of general form. A closed-form solution of the model beyond our results requires a specific definition of $\psi(q)$. Two aspects suggest that the density ought to be highly skewed towards low qualities with a fairly low probability for qualities closer to 1. Firstly, a quality of degree one suggests that marginal production costs of consumer goods drop to zero by only taking one technological step forward. Since this would supersede any further technological progress, qualities of such a high degree are implausible. Secondly, in the empirical literature the skewness has been well established, and the debate shifted towards the question which highly skewed distribution of the size of innovations fits the data best, lognormal or Pareto.[29] Since the Pareto distribution exhibits the convenient feature that, after a technology leap occurred, the remaining higher qualities are distributed independently of the surpassed state-of-the-art and still follow Pareto, it might be suitable to assume this distribution in a theoretical model of this kind.[30] Nevertheless, the results in this model hold for any probability density.

5   Introducing the renewal decision

Most patent regimes require the payment of renewal fees to maintain a valid intellectual property right. This aspect changes the investment value to

$$\rho\,\mathcal{V} = \underset{\varphi}{max}\left\{\underset{\sigma}{max}\left\{(1-\sigma)\,V_s\,,\ max\left\{\sigma\,V_0\,,\sigma\,V_1\right\}\right\}(1-e^{-\varphi}) - \frac{\varphi}{\beta} - f\right\}\,. \tag{9}$$

Firms choose whether or not to maintain a patent right to its full statutory term by paying renewal fees c_1.[31] Let us again focus on the case in line with stylized fact 4, so that $\eta_s > \eta_1 > \eta_0$. The renewal decision is based on the trade-off between patent value without renewals (equation (6)) and the patent value with renewals given by

$$V_1 = q\,\xi\,\eta_1\left(e^{-\overline{\varphi}} + \frac{1-e^{-\overline{\varphi}}}{1+\overline{\sigma}}\right) - c_0 - c_1\,, \tag{10}$$

which differs from the former only with regard to the amount of patent strength and the additional fees. Firms will choose to pay c_1, if $V_1 > V_0$. Since the renewal fees accrue independently of q, at least for very small qualities, firms will prefer not to renew the patent

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[29]   See Scherer (1998), Scherer/Harhoff (2000), Harhoff et al. (2003), Silverberg/Verspagen (2007).
[30]   See Eaton/Kortum (1999), p. 545.
[31]   Recall from the introduction that instead of modelling many small renewal steps, I facilitate the analysis by all steps in one renewal decision with an accordingly adjusted fee c_1.

right. According to stylized fact 3, in practice at least some patents are renewed to the maximum statutory term. Hence, there must exist a 'renewal cut-off', $q_1(\sigma)$, which separates renewed and non-renewed qualities. This is ensured by

Lemma 5. *The percentage of patents that are renewed is between zero and one, if the optimal R&D intensity in equilibrium satisfies i)* $\eta_1 > \eta_0 + \frac{(1+\sigma)\,c_1\,\xi^{-1}}{(1+\sigma\,e^{-\varphi^*})}$, *and ii)* $\eta_1 < \eta_0 + \frac{c_1}{c_0}\left(\eta_0 - \eta_s\,e^{-\varphi^*}\right)$. *Then, renewal cut-off* $q_1(\sigma) = \frac{(1+\sigma)\,c_1\,\xi^{-1}}{(\eta_1 - \eta_0)\left(1 + \sigma\,e^{-\varphi^*}\right)}$ *exists.*

The proof for Lemma 5 will be delivered together with one for Proposition 3. The renewal cut-off depends on σ, since the rival's propensity to patent is determined in decision stage three, which due to backward induction is solved after the renewal stage four. Aspect *i)* in Lemma 5 is the minimum requirement for renewals of at least some patents. Intuitively, it requires that the additional protection received is high enough to compensate the associated renewal fees (relative to spending), given the equilibrium R&D success probabilities. Aspect *ii)* ensures that, along the lines of stylized fact 3, not all patents are renewed (maximum requirement). In this empirically plausible case, the renewal decision is as follows:

Proposition 3. *If Lemma 5 holds, so that* $q_1(\sigma) \in (0,1)$, *in equilibrium firms will choose*

    i) not to pay renewal fees c_1, *if* $0 < q \leq q_1(\sigma)$

    ii) to pay renewal fees c_1, *if* $q_1(\sigma) < q \leq 1$.

for a given innovation quality q.

Proof. See Appendix A.6.   □

The renewal decision potentially affects the decision between secrecy and patenting at stage three, since, if renewals are worthwhile enough, firms weigh secrecy against patents with renewal instead of patents without. To that extent, we have to distinguish between two cases. Firstly, if renewals yield relatively little additional protection, the renewal cut-off affects only qualities located in the pure strategy 'patenting'-area, and $q_0|_{\sigma=1}$ remains to be the relevant cut-off separating mixed and pure patenting strategy in Proposition 1. Figure 3 illustrates an example for this case (henceforth: benchmark case), where $q_1|_{\sigma=1} > q_0|_{\sigma=1}$. The investment value in (9) becomes

$$\rho\,\mathcal{V} = \left[\int_0^{q_0} V_s\,\psi(q)\,dq + \int_{q_0}^{q_1} V_0|_{\sigma=1}\,\psi(q)\,dq + \int_{q_1}^1 V_1|_{\sigma=1}\,\psi(q)\,dq\right](1 + e^{-\varphi}) - \frac{\varphi}{\beta} - f, \quad (11)$$

where $q_1 = q_1|_{\sigma=1}$. In stage two, the equilibrium R&D intensity maximizes this expression, while Lemma 2 ensures that firms continue R&D in stage one.
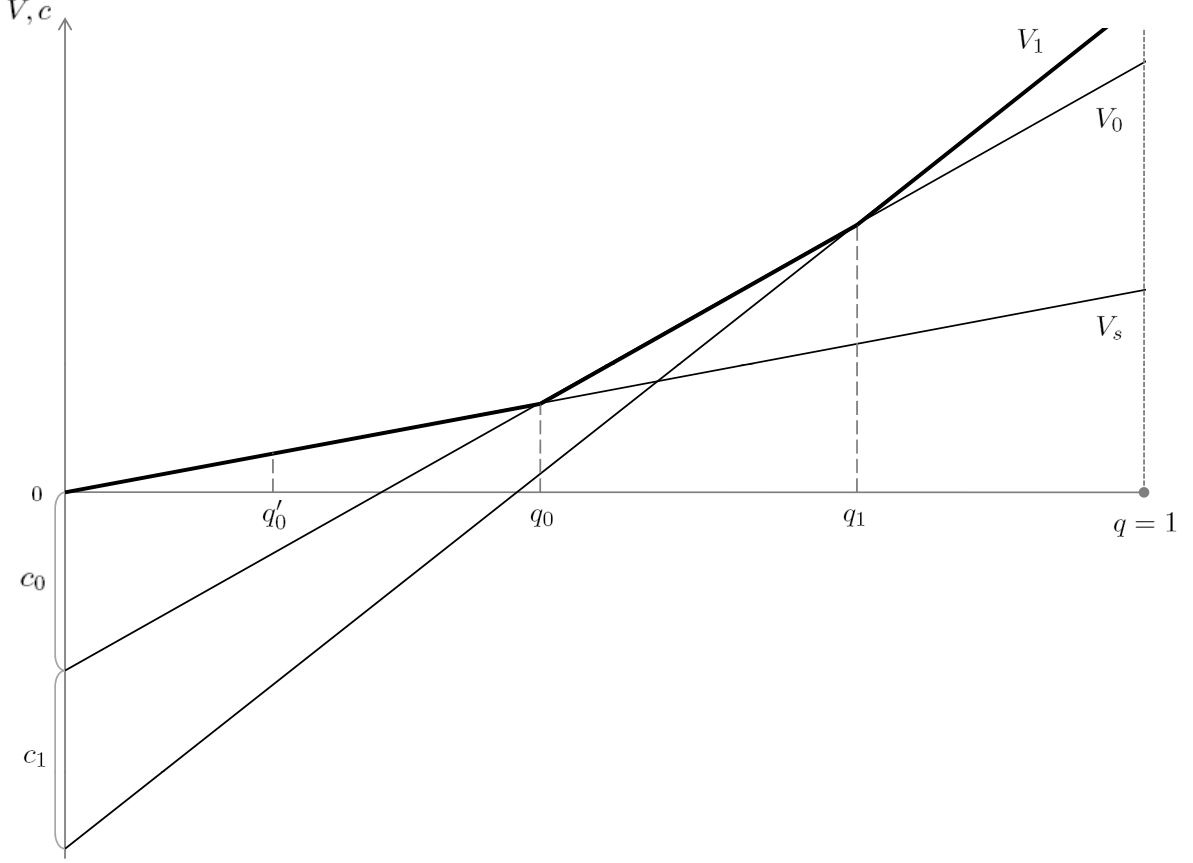
Figure 3: Patent and innovation value with and without renewals.

Secondly, if renewals are profitable enough to also be considered for patents in the mixed strategy area starting at $q_0' = q_0|_{\sigma=0}$, so that $q_1|_{\sigma=1} < q_0|_{\sigma=1}$, $q_1|_{\sigma=1}$ replaces $q_0|_{\sigma=1}$ as relevant cut-off between mixed and pure strategy. As a result, the rival's propensity to patent, σ, is determined by the trade-off between V_s and V_0 until it is high enough for renewed patents to outperform non-renewed ones ($V_1 > V_0$). I henceforth denote this level by σ'. It represents the highest propensity to patent for which firms choose a mixture between secrecy and non-renewed patents. It is the only σ for which patents with and without renewals yield identical profits and thus follows from $q_0(\sigma) = q_1(\sigma)$, yielding $\sigma' = \frac{c_0\,(\eta_1-\eta_0)\,e^{-\varphi}-c_1\,(\eta_0\,e^{-\varphi}-\eta_s)}{c_1\,(\eta_0-\eta_s)-c_0\,(\eta_1-\eta_0)}$. For all $\sigma > \sigma'$, firms trade off secrecy against patent value with renewals (V_s and V_1). Therefore, if $\sigma' < 1$, $q_1|_{\sigma=1}$ becomes the cut-off relevant for the decision in Proposition 1. Along the lines of equation (8), the investment value then becomes

$$\rho\,\mathcal{V} = \left[V_s|_{q=\tilde{q}}\left(\frac{\tilde{q}_1}{\tilde{q}}\right) + V_0|_{q=\tilde{q}}\left(1 - \frac{\tilde{q}_1}{\tilde{q}}\right) - c_0\left(\frac{\tilde{q}_1}{\tilde{q}} - \Psi(q_0)\right)\right](1 - e^{-\varphi}) - \frac{\varphi}{\beta} - f\,, \qquad (12)$$

where $\tilde{q}_1 = q_1\,\Psi(q_1) - \int_0^{q_1} \Psi(q)\,dq$ and q_1 is given by Lemma 5. In stage two, the equilibrium investment maximizes (12), and Lemma 2 ensures positive investment in stage one.

6 Introducing patentability standards

So far the analysis has neglected the fact that not all patent applications lead to patent grants (stylized fact 2). Patent offices set patentability standards, which constitute minimum requirements with regard to the quality of inventions. The decision on whether the requirements in each individual case are met falls to patent examiners. Different interpretations about what the patentability standards imply, a lack of technical expertise on the part of the examiners, or simply the patent office's work overload result in uncertainty in the application decision for each invention. If the patent office made no mistake and could perfectly enforce impartial standards, firms could save filing cost and only file patents that are granted for sure. Since stylized fact 2 suggests that in practice this is not the case, let us assume that each application faces a different patentability cut-off $q_\mathcal{S}$ drawn from a probability distribution $\upsilon(q_\mathcal{S})$ over all $q_\mathcal{S} \in (0,1]$ with cumulative distribution $\Upsilon(q_\mathcal{S})$.[32] Firms form expectations about $q_\mathcal{S}$ to be able to decide whether or not to patent at stage three. Expectations are based on experience and correspond to the target patentability standards, $\overline{q}_\mathcal{S}$, which the patent office intends to enforce. As expected and eventually realized patentability standards diverge, even high-quality inventions may fail to be granted, which, along the lines of fact 2, results in a grant rate between zero and one.

Note that the introduction of patentability standards itself would not further complicate the model. The uncertainty introduced via $q \sim \psi(q)$ can easily be used to explain fact 2, if we accept that this precludes us from simultaneously explaining why firms rely on both appropriation mechanisms.[33] Since one goal of the model is to account for *all* five stylized facts, it becomes necessary to assume an additional probability distribution to introduce uncertainty in the patent examination process. This fairly complicates the model. For tractability, I limit the following analysis to a simple probability distribution, which has yet enough structure to get an idea of the intuition behind the results.

Let us assume that $\upsilon(q_\mathcal{S})$ takes the form of a logistic density with cumulative distribution $\Upsilon(q_\mathcal{S}) = \frac{1}{1+e^{(\overline{q}_\mathcal{S} - q_\mathcal{S})\mathcal{I}}}$.[34] $\overline{q}_\mathcal{S}$ is the patent office's target requirement, and $\mathcal{I}$ is a measure for its ability to implement it.[35] If $\mathcal{I} \to \infty$, the patent office makes no mistakes and perfectly

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[32]   Alternatively, we can assume that the patent office perfectly enforces $q_\mathcal{S}$ but, due to information asymmetry between firms and patent office, cannot correctly evaluate quality. While leading to the same results, analysis would be more intricate than the error margin in enforcing standards modelled here.

[33]   In that case firms would not learn the exact q until stage four, so that, based on expectations on q at stage three, they would choose either secrecy or patenting for all inventions.

[34]   This specification ensures that $q_\mathcal{S}$ cannot fall below zero, but it can exceed one. Yet, we can interpret standards larger than one similarly to $q_\mathcal{S} = 1$, i.e., the patent office does not grant the envisaged patent for sure. Since the target cut-off $\overline{q}_\mathcal{S}$ is well below unity, the associated error margin is acceptable for the gained notational simplicity. Alternatively, one can modify $\upsilon(q_\mathcal{S})$ to be a 'beta distribution' yielding a logistic shaped cumulative distribution over the interval $q_s \in (0,1]$. This eliminates the imprecision, but at the cost of more intricate solutions.

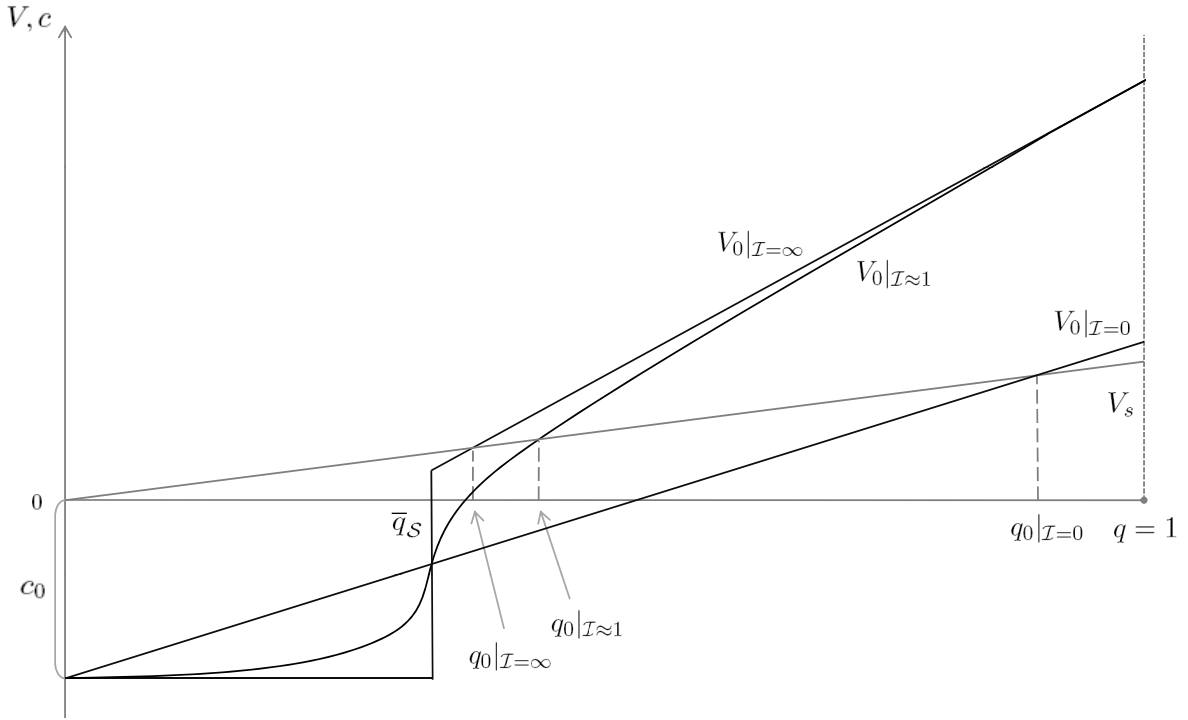[35]   To that extent, $\mathcal{I}$ is an inverse measure of the variance of distribution $\upsilon(q_\mathcal{S})$.

Figure 4: Patent and innovation value with patentability standards for different $\mathcal{I}$-levels.

enforces $\overline{q}_{\mathcal{S}}$ objectively for every application, so that qualities smaller than $\overline{q}_{\mathcal{S}}$ fail, while those larger than $\overline{q}_{\mathcal{S}}$ are granted for sure. If, however, $\mathcal{I}$ is zero, the patent office implements arbitrary standards, regardless of its target $\overline{q}_{\mathcal{S}}$. A lower $\mathcal{I}$ hence increases the risk of vain patent applications and thus influences patent values. Firms are assumed to use their knowledge on the distribution of standards to assess the probability of a successful application. The resulting *expected* patent value (without renewals) equals

$$E(V_0) = \frac{q\,\xi\,\eta_0}{1 + e^{(\overline{q}_S - q)\,\mathcal{I}}} \left(e^{-\overline{\varphi}} + \frac{1 - e^{-\overline{\varphi}}}{1 + \sigma} \right) - c_0 \,. \tag{13}$$

The compared to (6) additional term measures the probability of being granted a patent.[36]

Patentability standards leave the intuition behind our main results unchanged. Yet, for two reasons they affect the relevant cut-offs. Firstly, except in extreme cases, where $\mathcal{I} \to \infty$ and $\mathcal{I} = 0$, the patent value is no longer linear in q. An example for an $\mathcal{I}$ that introduces nonlinearity is shown in figure 4. Secondly, the risk of a failed application reduces the expected profit from patenting relative to secrecy, changing the relevant cut-offs for Proposition 1. The patenting cut-offs for $\sigma = 1$ and $\sigma = 0$ are implicitly given by

$$q_0 = \frac{c_0\,\xi^{-1}}{\left(\frac{\eta_0\,(1 + e^{-\varphi^*})}{2\,(1 + e^{(\overline{q}_S - q_0)\,\mathcal{I}})} - \eta_s\,e^{-\varphi^*} \right)} \quad \text{and} \quad q_0' = \frac{c_0\,\xi^{-1}}{\left(\eta_0\,\left(1 + e^{(\overline{q}_S - q_0')\,\mathcal{I}}\right)^{-1} - \eta_s\,e^{-\varphi^*} \right)} \,. \tag{14}$$

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[36] More specifically, $\Upsilon(q_{\mathcal{S}}) = \frac{1}{1 + e^{(\overline{q}_S - q)\,\mathcal{I}}}$ is the probability that for an application with quality q a $q_{\mathcal{S}} \leq q$ is drawn, in which case the application is successful. That is why q replaces $q_{\mathcal{S}}$ in $\Upsilon(q_{\mathcal{S}})$.

It follows that with patentability standards the optimal decision between secrecy and patenting is given by Proposition 1, where q_0 and q_0' follow from (14).

Similarly, in Appendix A.7 the expected value with renewals, $E(V_1)$, and the renewal cut-off are derived for the benchmark case. Using the latter and the cut-offs from (14), the optimal R&D investment follows from maximizing the investment value given by (11). The β-f-relation ensures that stage one of the R&D process has a positive solution.

7 Drastic vs. non-drastic innovations

The linearity of secrecy and patent value in q increases the model's tractability, but it is conditional on limit-pricing resulting form the assumption of unit-elastic demand. In a duopoly with Bertrand competition, a technological leader sets a price which, *i)* ensures that his rival does not produce and, *ii)*, if possible, maximizes profits. If the profit-maximizing price is too high to drive the rival out of the market, the leader must resort to limit pricing to ensure being the only producer. This is the 'non-drastic innovation'-case analyzed above, where inventions are too small to allow a lower price setting.[37] If, however, inventions are large and, therewith, 'drastic' enough for the leader's profit maximizing price to drive his rival out of the market, firms do not need to resort to limit pricing. The fact that in the prior analysis any innovation quality leads to limit-pricing is a result of unit-elastic demand. In this special case, profits strictly increase in prices, so that a profit-maximizing price is always too high to drive the rival out of the market and all inventions are 'non-drastic'. In order to show that the above results are robust to the assumption of unit-elastic demand, this section analyzes the more general CES case and, therewith, drastic *and* non-drastic innovations.

Let us modify preferences such that aggregate output follows $ln\,X_t = ln\,\left(\int\limits_0^1 x_{jt}^{\alpha}\,dj\right)^{\frac{1}{\alpha}}$, where $0 < \alpha \leq 1$ is a measure for the love-of-variety.[38] $\alpha = 1$ implies that sector-wise produced consumer goods are perfect substitutes, while α approaching zero corresponds to the Cobb-Douglas case discussed above. For any α between zero and unity, consumer goods of one industry are imperfect substitutes for those produced in another industry.[39] The resulting demand in each industry j takes the form $x_{jt} = \frac{p_{jt}^{\frac{1}{\alpha-1}}\xi}{\mathcal{P}_t^{\frac{\alpha}{\alpha-1}}}$, where $\mathcal{P}_t = \left[\int\limits_0^1 p_{jt}^{\frac{\alpha}{\alpha-1}}\,dj\right]^{\frac{\alpha-1}{\alpha}}$ is the aggregate price index in the economy.

Given this modified demand and marginal costs from section 2, the profit maximizing price is $p_{jt}^{drastic} = \frac{1-q_{jt}}{\alpha\,\mathcal{A}_{jt}}$, where I use the fact that $q_{jt} = \left(1 - \frac{1}{\gamma_{jt}}\right)$. However, the leader is only able to charge this price without losing market share, if it is low enough to drive the laggard

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[37] See Aghion/Howitt (1998) and Acemoglu (2009) regarding 'drastic' and 'non-drastic' innovations.
[38] See Li (2001) for a similar preference specification.
[39] Note that in any case, consumer goods produced within each industry remain perfect substitutes.

out of the market. This will be the case, if the technological step forward measured by q_{jt} is 'drastic' enough to compensate the markup $\frac{1}{\alpha}$ and reduce $p_{jt}^{drastic}$ under the marginal cost level of the laggard. Yet, for smaller 'non-drastic' innovations the profit maximizing price is too high to drive the rival out of the market. In this case the leading firm must charge limit price $p_{jt}^{limit} = \frac{1}{\mathcal{A}_{jt}}$ to maintain its market share.

It becomes apparent that given $0 < \alpha \leq 1$ there is a cut-off quality q_p separating drastic and non-drastic innovations. More specifically, equating both prices yields $q_p = 1 - \alpha$. All qualities $q_{jt} > q_p$ imply drastic innovations, enabling the leader to charge its profit maximizing price. By contrast, $q_{jt} \leq q_p$ lead to non-drastic innovations, which require the leader to charge the limit price. It follows that with the CES specification profits become

$$\pi_{jt} = \begin{cases} q_{jt} \, \xi \, (\mathcal{A}_{jt} \, \mathcal{P}_t)^{\frac{\alpha}{1-\alpha}} & \text{if } q_{jt} \leq q_p \\ (1-\alpha) \, \xi \left(\frac{\alpha \, \mathcal{A}_{jt} \, \mathcal{P}_t}{1 - q_{jt}} \right)^{\frac{\alpha}{1-\alpha}} & \text{if } q_{jt} > q_p \end{cases} \tag{15}$$

Albeit more intricate than profits resulting from unit-elastic demand, CES profits under limit pricing are still linear in q. Yet, in case of drastic innovations, profits grow exponentially in quality (since $0 < \alpha \leq 1$). Note that, if $q_{jt} = q_p = 1 - \alpha$, profit maximization and limit pricing lead to the same results. Hence, profits in (15) are continuous and monotonically increasing for all $0 \leq q \leq 1$. Moreover, firms will only choose profit maximization over limit pricing, if it makes them better off. The mere existence of drastic innovations thus does not change the intuition behind previous results. However, the introduction of the love of variety parameter, the price index, and the fact that the technology level enters profits may change the exact location of patenting and renewal cut-offs within the interval $q \in (0, 1]$. See Appendix B for the results regarding patenting and renewal decisions as well as optimal R&D intensity. An example for the solution of the model with drastic innovations is illustrated in figure 6. We can conclude that the intuition behind the main Propositions in the model carry over to the more general case.

8   Conclusion

The contribution of this paper is twofold. For one, it extends upon the seminal contribution of Kultti et al. (2006) and (2007) and takes a further step towards bringing economic theory on patenting behavior in line with *all* stylized facts on the issue outlined in table 1. For another, it addresses the question how firms decide between secrecy and patenting when the amount of R&D investment is endogenous and R&D outcome is heterogeneous. To that extent, it accounts for the possibility of simultaneous innovation, so that patents

can compensate for providing qualitatively less protection than secrecy by providing it also in situations where secrecy fails. The model predicts that low-quality inventions are kept secret, firms play a mixed strategy for medium-quality inventions, and firms patent and renew high-quality inventions. These results contradict previous findings by Anton and Yao (2004) who neglect patenting costs, patentability standards and the question how to obtain a technological lead. Instead they assume exogenous R&D investment and focus on competitive behavior-aspects of the 'secrecy vs. patenting'-decision resulting from licensing and disclosure. In practice, both results can prevail depending on the significance of royalties from licensing and the degree of technological standardization which increases the probability of simultaneous innovation in each industry. The question for which industries and under which circumstances the 'R&D side of the story' or the 'competitive behavior'-aspects determine firms' patenting decision is an empirical one and clearly deserves further study.

Other possible extensions of the model include a distinction between process and product innovations. Unlike KTT, this paper models technological progress as process innovations reducing marginal production cost. Li (2001) shows how such a set-up can easily be modified to include product innovations via a quality index in consumption. However, product innovations are easier to reverse engineer, since they are freely available on markets, implying a higher risk of being imitated. This calls into question that secrecy provides identical protection for both types of innovations.[40]

The model presented here accounts for knowledge spillovers in R&D. However, these spillovers occur exclusively at the end of each period by the time an invention becomes common knowledge. More protection via secrecy has no impact on spillovers and thus has a strictly positive impact on R&D. Recent empirical evidence, however, suggests that stronger secrecy laws impair knowledge spillovers between firms and, consequently, reduce R&D.[41] Hence, extending the model to account for spillovers within each standard time interval might be an interesting topic for future research.

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[40] E.g., Levin et al. (1987) find a significantly less effectiveness of secrecy for product innovations. See also Arundel (2001), who explains secrecy's effectiveness for product innovations with the fact that most technology products have a considerable pre-market development phase, during which no difference to process innovations exist and lead-time advantages can be established.

[41] See, e.g., Png (2011).

Appendices

Appendix A

A.1

Objective. Show that firms invest in R&D, if the β-f-relation given by Lemma 2 holds, and that a unique and positive optimal intensity exists for all β larger than the minimum R&D productivity.

Proof. The concavity of V_s in φ due to the term $1 - e^{-\varphi}$ along with weak convexity of $\mathcal{G}(\varphi)$ and the fact that both expressions are monotonic ensures uniqueness. Since $ln(1) = 0$, φ^* given by (5) is positive, if $\beta\,\tilde{q}\,\xi\,\eta_s > 1$. This is the case for all $\beta \geq \frac{1}{\tilde{q}\,\xi\,\eta_s}$, implying minimum R&D productivity $\tilde{\beta}$.

To derive the β-f-relation I use (4) and (5) rewrite $\mathcal{V}(\varphi^*) \geq 0$ to

$$f \leq \tilde{q}\,\xi\,\eta_s\,(1 - e^{-\varphi^*})\,e^{-\varphi^*} - \frac{\varphi^*}{\beta} = \tilde{q}\,\xi\,\eta_s\left(1 - e^{-\frac{ln(\beta\,\tilde{q}\,\xi\,\eta_s)}{2}}\right)e^{-\frac{ln(\beta\,\tilde{q}\,\xi\,\eta_s)}{2}} - \frac{ln(\beta\,\tilde{q}\,\xi\,\eta_s)}{2\,\beta}\ ,$$

so that rearranging yields the expression in Lemma 2:

$$f \leq \left(\tilde{q}\,\xi\,\eta_s - \frac{\tilde{q}\,\xi\,\eta_s}{\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s}}\right)\frac{1}{\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s}} - \frac{ln(\beta\,\tilde{q}\,\xi\,\eta_s)}{2} \quad \Leftrightarrow \quad f \leq \frac{1}{\beta}\left(\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s} - 1 - \frac{1}{2}\,ln(\beta\,\tilde{q}\,\xi\,\eta_s)\right)\ .$$
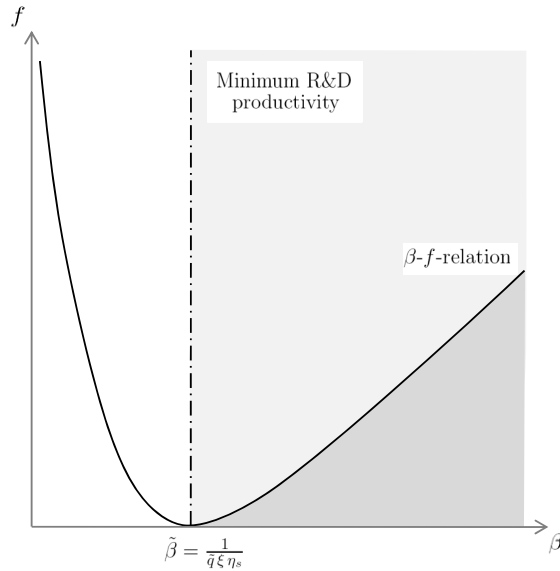

Figure 5: Minimum R&D productivity $\tilde{\beta}$ and β-f-relation for exemplatory $\tilde{q} \in (0,1]$ and $\eta_s \in (0,1]$.

Based on this expression it can be shown that the β-f-relation and the minimum R&D productivity intersect at $[\beta = \frac{1}{\tilde{q}\,\xi\,\eta_s}, f = 0]$. For $f = 0$, the β-f-relation yields $\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s} = 1 + \frac{1}{2}\,ln(\beta\,\tilde{q}\,\xi\,\eta_s)$, which is exactly satisfied for $\tilde{\beta} = \frac{1}{\tilde{q}\,\xi\,\eta_s}$. Figure 5 illustrates both parameter conditions. Smaller β are ruled out by the minimum requirement in Lemma 1, which allows only $\beta \geq \tilde{\beta}$ (light gray area).

For higher β than $\tilde{\tilde{\beta}}$, the β-f-relation is the relevant parameter condition, so that the dark gray area in figure 5 marks the relevant parameter combinations. $\square$

A.2

Objective. Show that the percentage of innovations that are patented is between zero and one, if φ^* satisfies $\eta_0 > \eta_s e^{-\varphi^*} + \frac{c_0}{\xi}$.

Proof. Due to filing costs, at least for infinitesimal small qualities firms choose secrecy. Hence, firms rely on patents besides secrecy, if at least for the highest qualities possible $V_0|_{\sigma \neq 0} = V_s$ holds, where firms choose a mixed strategy between secrecy and patenting. Since this corresponds to qualities where also $V_0|_{\sigma=0} > V_s$, even though for all $\sigma \neq 1$ the patent value cannot exceed the one granted by secrecy, I can use the simpler expression with $\sigma = 0$ to proof Lemma 3.

Based on $V_0|_{\sigma=0} > V_s$ I use (3) and (6) and $\varphi = \overline{\varphi} = \varphi^*$ to write

$$q \, \xi \, \eta_0 \left[e^{-\varphi^*} + (1 - e^{-\varphi^*}) \right] - c_0 > q \, \xi \, \eta_s \, e^{-\varphi^*} \; . \tag{16}$$

Since the highest possible q is one, we can use this fact to simplify and solve the inequality for η_0, yielding $\eta_0 > \eta_s e^{-\varphi^*} + \frac{c_0}{\xi}$. Note that assuming equality in (16) and without setting $q = 1$, solving for q yields cut-off quality $q_0' = \frac{c_0 \, \xi^{-1}}{(\eta_0 - \eta_s \, e^{-\varphi*})}$. $\square$

A.3

Objective. Show that a patent premium exists, if φ^* satisfies $\eta_0 > \eta_s \frac{2 e^{-\varphi^*}}{1+e^{-\varphi^*}} + \frac{2 c_0 \xi^{-1}}{1+e^{-\varphi^*}}$.

Proof. In order to yield a patent premium, at least the highest q must satisfy $V_0|_{\sigma=1} > V_s$, which yields

$$q \, \xi \, \eta_0 \left[e^{-\varphi^*} + \frac{1}{2} (1 - e^{-\varphi^*}) \right] - c_0 > q \, \xi \, \eta_s \, e^{-\varphi^*} \; . \tag{17}$$

While solving this for q gives us $q_0 = \frac{c_0 \, \xi^{-1}}{(1+e^{-\varphi^*}) \frac{\eta_0}{2} - \eta_s \, e^{-\varphi^*}}$, as above, I set $q = 1$ (since the inequality must hold at least for marginally smaller q than the highest quality), from which immediately follows $\eta_0 > \eta_s \frac{2 e^{-\varphi^*}}{1+e^{-\varphi^*}} + \frac{2 c_0 \xi^{-1}}{1+e^{-\varphi^*}}$. The corresponding minimum R&D intensity is derived by solving this expression for φ^*: $\varphi^*_{min} = ln \left(\frac{2 \eta_s - \eta_0}{\eta_0 - 2 c_0 \xi^{-1}} \right)$. $\square$

A.4

Objective. Show that the investment value given by (8) follows from Proposition 1.

Proof. Based on Proposition 1 and the fact that for the mixed strategy interval secrecy and patent value are identical, we can write the investment value

$$\mathcal{V} = \left(\int_0^{q_0} q \, \xi \, \eta_s \, e^{-\overline{\varphi}} \, \psi(q) \, dq + \int_{q_0}^1 \left(q \, \xi \, \eta_0 \left(\frac{1}{2} + \frac{1}{2} e^{-\overline{\varphi}} \right) - c_0 \right) \psi(q) \, dq \right) (1 - e^{-\varphi}) - \frac{\varphi}{\beta} - f \; . \tag{18}$$

Now suppose that Lemma 4 holds, such that $q_0 = \frac{c_0\,\xi^{-1}}{(1+e^{-\overline{\varphi}})\frac{\eta_0}{2} - \eta_s\,e^{-\overline{\varphi}}}$. The equilibrium investment is given by the first derivative of (18) with respect to φ, which can only be implicitly solved as long as no specific probability density $\psi(q)$ is assumed. The expression $1 - e^{-\varphi}$ ensures that second and third order conditions are non-zero.

The target expression (8) differs from (18) only with regard to how profits are expressed, while the terms for R&D and adaptation costs remain unchanged. I thus focus on rearranging the integrals

$$E(V) = \int_0^{q_0} q\,\xi\,\eta_s\,e^{-\overline{\varphi}}\,\psi(q)\,dq + \int_{q_0}^1 \left(q\,\xi\,\eta_0 \left(\frac{1}{2} + \frac{1}{2}\,e^{-\overline{\varphi}} \right) - c_0 \right) \psi(q)\,dq\,. \tag{19}$$

Note that the integrals capture V_s given by (3) and V_0 given by (6) respectively. (19) thus consists of two areas, the one under the innovation value V_s over the interval of 0 to q_0 and the one under V_0 over the interval q_0 to 1. Expression (8) follows from the notational trick to reformulate the latter area over a quality interval 0 to 1. The resulting value is an area over $q = 0$ and $q = 1$ which is defined by a curve $E(V)$ that lies between between V_s and V_0. As it turns out, the exact distance of $E(V)$ to secrecy and patent value crucially depends on where the cut-off q_0 is located. We can thus derive $E(V)$ as an expression of V_s, weighted by the probability that a drawn q is smaller than q_0, and V_0, weighted by the probability that q is larger than q_0, over the interval of all qualities. Mathematically this follows from

$$E(V) = \int_0^{q_0} V_s\,\psi(q)\,dq + \int_{q_0}^1 V_0\psi(q)\,dq = \int_0^1 V_s\,\psi(q)\,dq + \int_{q_0}^1 (V_0 - V_s)\,\psi(q)\,dq\,, \tag{20}$$

where $V_0 = V_0|_{\sigma=1}$, since the cut-off q_0 determines qualities above which patenting yields a premium.

The next step is to find the integral over $q = 0 \to 1$, which equals the term on the rhs of (20). I thus have to derive the slope $\mathcal{Z}$ of a new value function that yields the same profits over $q = 0 \to 1$ as the last integral in (20) over $q = q_0 \to 1$, given probability density $\psi(q)$. As the second interval is smaller, naturally $\mathcal{Z}$ must be smaller than the slope of $(V_0 - V_s)$, which for notational simplicity is $\mathcal{N}$, i.e., $\mathcal{N} = \xi\,\eta_0 \left(\frac{1}{2} + \frac{1}{2}\,e^{-\overline{\varphi}} \right) - \xi\,\eta_s\,e^{-\overline{\varphi}} > \mathcal{Z}$. Note that c_0 is independent of q but dependent on ψ, as only in the interval $q = q_0 \to 1$ filing costs accrue. I thus rewrite (20) to

$$E(V) = \int_0^1 V_s\,\psi(q)\,dq + \int_{q_0}^1 q\,\mathcal{N}\,\psi(q)\,dq - c_0(1 - \Psi(q_0))\,. \tag{21}$$

The last term on the rhs scales down c_0 by the probability that q is higher than q_0. This captures the fact that not for all qualities filing costs accrue. Since this term is irrelevant for the reformulation of the slope, I can write

$$\int_{q_0}^1 q\,\mathcal{N}\,\psi(q)\,dq = \int_0^1 q\,\mathcal{Z}\,\psi(q)\,dq$$

$$\left(q\,\Psi(q)|_{q_0}^1 - \int_{q_0}^1 \Psi(q)\,dq \right) \mathcal{N} \;=\; \left(q\,\Psi(q)|_0^1 - \int_0^1 \Psi(q)\,dq \right) \mathcal{Z}$$

$$\left(1 - q_0\,\Psi(q_0) - \int_{q_0}^1 \Psi(q)\,dq \right) \mathcal{N} \;=\; \left(1 - \int_0^1 \Psi(q)\,dq \right) \mathcal{Z}\,.$$

The term in brackets on the rhs is the aforementioned expected quality $\tilde{q} = \int_0^1 q\,\psi(q)\,dq$. The corresponding term on the lhs is the expected quality over an interval $q = q_0 \to 1$, which captures the probability that a drawn q is higher than the cut-off. I further simplify to

$$\mathcal{Z} = \left(\frac{1 - q_0\,\Psi(q_0) - \int_0^1 \Psi(q)\,dq + \int_0^{q_0} \Psi(q)\,dq}{1 - \int_0^1 \Psi(q)\,dq} \right) \mathcal{N} \quad \Leftrightarrow \quad \mathcal{Z} = \left(1 - \frac{\tilde{q}_0}{\tilde{q}} \right) \mathcal{N}\,,$$

where $\tilde{q}_0 = q_0\,\Psi(q_0) - \int_0^{q_0} \Psi(q)\,dq$ is the expected quality over an interval $q = 0 \to q_0$. Using this expression to replace the last integral on the rhs of (20) yields

$$\begin{aligned}
E(V) &= \int_0^1 q\,\xi\,\eta_s\,e^{-\overline{\varphi}}\,\psi(q)\,dq + \int_0^1 q\left(1 - \frac{\tilde{q}_0}{\tilde{q}}\right)\mathcal{N}\,\psi(q)\,dq - c_0(1 - \Psi(q_0)) \\
&= \int_0^1 V_s\left(\frac{\tilde{q}_0}{\tilde{q}}\right)\psi(q)\,dq + \int_0^1 \left[q\,\xi\,\eta_0\left(\frac{1}{2} + \frac{1}{2}\,e^{-\overline{\varphi}}\right)\left(1 - \frac{\tilde{q}_0}{\tilde{q}}\right) - c_0 + c_0\,\Psi(q_0)\right]\psi(q)\,dq \\
&= V_s|_{q=\tilde{q}}\left(\frac{\tilde{q}_0}{\tilde{q}}\right) + V_0|_{q=\tilde{q}}\left(1 - \frac{\tilde{q}_0}{\tilde{q}}\right) + c_0\left(\Psi(q_0) - \frac{\tilde{q}_0}{\tilde{q}}\right)\,,
\end{aligned}$$

where again I use (3) and (6). The investment value in (8) immediately follows from this expression. $\qquad\square$

A.5

Objective. Show that the minimum parameter requirement to bring the model in line with stylized facts 1, 4, and 5 is $c_0 < \left(\frac{\eta_0}{2}(1 + e^{\varphi^*}) - \eta_s\right)\xi\,e^{-\varphi^*}$, where $\varphi^* \geq \frac{1}{2}\ln(\beta\,\tilde{q}\,\xi\,\eta_s)$.

Proof. To bring the model in line with the stylized facts, φ^*_{min} implied by Lemma 4 must be at least as high as the optimal intensity under secrecy given by (5):

$$\begin{aligned}
\frac{1}{2}\ln(\beta\,\tilde{q}\,\xi\,\eta_s) &\geq \ln\left(\frac{2\,\eta_s - \eta_0}{\eta_0 - 2\,c_0\,\xi^{-1}}\right) \\
\eta_0 - 2\,c_0\,\xi^{-1} &\geq \frac{2\,\eta_s - \eta_0}{\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s}} \\
c_0 &\leq \frac{\eta_0\,\xi}{2}\left(1 + \frac{1}{\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s}} - \frac{\eta_s\,\xi}{\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s}}\right) \\
c_0 &\leq \xi\,e^{-\varphi^*}\left(\frac{\eta_0}{2}\left(1 + e^{\varphi^*}\right) - \eta_s\right)
\end{aligned}$$

where we use (5) to write $e^{\varphi^*} = \sqrt{\beta \tilde{q} \xi \eta_s}$. This yields the expression in Proposition 2. Note that Lemma 1 ensures that $\beta \tilde{q} \xi \eta_s > 1$, so that due to $1 + e^{\varphi^*} > 2$ at least very small $\eta_s > \eta_0$ can be evened out. Additional R&D investment subsequently ensures that firms can afford payment of positive c_0. $\qquad\square$

A.6

Objective. Show that Lemma 5 ensures that at least some patents are renewed and that the renewal cut-off given by $q_1(\sigma) = \frac{(1+\sigma) c_1 \xi^{-1}}{(\eta_1 - \eta_0)(1 + \sigma e^{-\varphi^*})}$ exists. Also show that $\sigma' = \frac{c_0 (\eta_1 - \eta_0) e^{\varphi} + c_1 (\eta_s - \eta_0 e^{\varphi})}{c_1 (\eta_0 - \eta_s) + c_0 (\eta_0 - \eta_1)}$.

Proof. Patent values with and without renewals are given by (6) and (10). Equating both expressions ($V_0 = V_1$) yields the renewal cut-off given in Lemma 5. Since firms decide to maintain the patent, if $V_1 > V_0$, the parameter requirement for renewals is

$$q \xi \eta_1 \left(e^{-\varphi^*} + \frac{1 - e^{-\varphi^*}}{1 + \sigma} \right) - c_1 - c_0 > q \xi \eta_0 \left(e^{-\varphi^*} + \frac{1 - e^{-\varphi^*}}{1 + \sigma} \right) - c_0 .$$

For at least one renewal to exist, it suffices that $q = 1$, so that the minimum requirement is

$$\eta_1 > \eta_0 + \frac{(1 + \sigma) c_1 \xi^{-1}}{(1 + \sigma e^{-\varphi^*})} .$$

In order to ensure that not all patents are renewed we must have that $q'_0 = q_0(\sigma = 0) < q_1(\sigma = 0)$, as at least for some q in the mixed strategy area the patent value without renewals must exceed both V_s and V_1. The minimum requirement for this to be the case is met, if the rival's probability to patent is zero. Using q'_0 from Lemma 3 and $q_1(\sigma = 0)$ given above, this yields

$$\frac{c_0}{(\eta_0 - \eta_s e^{-\varphi^*})} < \frac{c_1}{(\eta_1 - \eta_0)(e^{-\varphi^*} + (1 - e^{-\varphi^*})}$$

$$\eta_1 - \eta_0 < \frac{c_1}{c_0} \left(\eta_0 - \eta_s e^{-\varphi^*} \right) ,$$

which is the maximum additional protection provided by renewals given filing and renewal fees c_0 and c_1 to be still in line with the data.

σ' follows from equating $V_0(\sigma)$ and $V_1(\sigma)$ given by (6) and (10), yielding

$$\frac{(1 + \sigma) c_1 \xi^{-1}}{(\eta_1 - \eta_0)(1 + \sigma e^{-\varphi})} = \frac{(1 + \sigma) c_0 \xi^{-1}}{\eta_0 (1 + \sigma e^{-\varphi}) - (1 + \sigma) \eta_s e^{-\varphi}}$$

$$c_1 (\eta_0 + \sigma \eta_0 e^{-\varphi} - \eta_s e^{-\varphi} - \sigma \eta_0 e^{-\varphi}) = c_0 (\sigma \eta_1 e^{-\varphi} - \sigma \eta_0 e^{-\varphi} + \eta_1 - \eta_0) .$$

Solving this for σ and rearranging yields $\sigma' = \frac{c_0 (\eta_1 - \eta_0) e^{\varphi} + c_1 (\eta_s - \eta_0 e^{\varphi})}{c_1 (\eta_0 - \eta_s) + c_0 (\eta_0 - \eta_1)}$. $\qquad\square$

A.7

Objective. In the case where patentability standards are drawn from a logistic distribution $v(q_{\mathcal{S}})$, derive the expected patent value with renewals and the renewal cut-off with patentability standards.

Proof. Similar to (13), I use the cumulative distribution $\Upsilon(q_\mathcal{S})$, where $q_\mathcal{S} \leq q$ to write the expected patent value with renewals

$$E(V_1) = \frac{q\,\xi\,\eta_1}{1+e^{(\bar{q}_S-q)\,\mathcal{I}}} \left(e^{-\overline{\varphi}} + \frac{1-e^{-\overline{\varphi}}}{1+\sigma}\right) - c_0 - c_1 ,\tag{22}$$

Equating (22) with (13) implicitly yields the renewal cut-off $q_1(\sigma) = \frac{(1+\sigma)\,c_1\,(1+e^{(\bar{q}_S-q_1)\,\mathcal{I}})}{(\eta_1-\eta_0)\,(1+\sigma\,e^{-\varphi^*})\,\xi}$. $\qquad \square$

Appendix B

Since all industries face the same density $\psi(q)$, over the course of a long time period there should be no industry difference with regard to productivity $\mathcal{A}_{jt}$, as long as all industries have the same starting point $\mathcal{A}_{j0}$. In that case I can once again take a look at a representative industry in steady-state and drop all dependencies on j apart from quality. For simplicity I also drop subscript t.

In order to show how the CES profits given by (15) influence the cut-offs resulting from patenting and renewal decisions while leaving the intuition behind their results unchanged, we first have to modify the β-f-relation in Lemma 2. Since firms only choose to charge the profit maximizing price if it outperforms the limit price, it is sufficient for the *minimum* requirement to use the profits for non-drastic innovations, $\pi = \tilde{q}\,\xi\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}}$, and apply them over all $q \in (0, 1]$. The resulting optimal R&D intensity is $\varphi^* = \frac{1}{2}\,ln(\beta\,\tilde{q}\,\xi\,\eta_s\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}})$. Hence, the β-f-relation becomes

$$f \leq \beta^{-1}\left(\sqrt{\beta\,\tilde{q}\,\xi\,\eta_s\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}}} - 1 - \frac{ln(\beta\,\tilde{q}\,\xi\,\eta_s\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}})}{2}\right) .$$

The next step is to find the patenting cut-off q_0. It follows from equating the value provided by secrecy with the patent value. Given (15), the former is

$$V_s = \begin{cases} q\,\xi\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}}\,\eta_s\,e^{-\overline{\varphi}} & \text{if } q \leq q_p \\ (1-\alpha)\,\xi\,\left(\frac{\alpha\,\mathcal{A}\mathcal{P}}{1-q}\right)^{\frac{\alpha}{1-\alpha}}\,\eta_s\,e^{-\overline{\varphi}} & \text{if } q > q_p \end{cases}\tag{23}$$

while the latter can be written as

$$V_0 = \begin{cases} q\,\xi\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{1-\alpha}}\,\eta_0\,\frac{1}{2}\,(1+e^{-\overline{\varphi}}) - c_0 & \text{if } q \leq q_p \\ (1-\alpha)\,\xi\,\left(\frac{\alpha\,\mathcal{A}\mathcal{P}}{1-q}\right)^{\frac{\alpha}{1-\alpha}}\,\eta_0\,\frac{1}{2}\,(1+e^{-\overline{\varphi}}) - c_0 & \text{if } q > q_p \end{cases}.\tag{24}$$

Equating both expressions yields the patenting cut-off for the CES specification

$$q_0 = \begin{cases} \frac{2\,c_0\,\xi^{-1}\,(\mathcal{A}\mathcal{P})^{\frac{\alpha}{\alpha-1}}}{\eta_0\,(1+e^{-\varphi^*})-2\,\eta_s\,e^{-\varphi^*}} & \text{if } q_0 \leq q_p \\ 1 - \alpha\,\mathcal{A}\mathcal{P}\left(\frac{2\,c_0\,\xi^{-1}}{(1-\alpha)\,(\eta_0\,(1+e^{-\varphi^*})-2\,\eta_s\,e^{-\varphi^*})}\right)^{\frac{\alpha-1}{\alpha}} & \text{if } q_0 > q_p \end{cases}\tag{25}$$

Note that I focused on the payoff relevant case where $\sigma = 1$.

The renewal cut-off q_1 will be derived by equating (24) with the CES patent value with renewals

$$
V_1 = \begin{cases} q\,\xi\,(\mathcal{A}\,\mathcal{P})^{\frac{\alpha}{1-\alpha}}\,\eta_1\,\tfrac{1}{2}\,(1+e^{-\overline{\varphi}}) - c_0 - c_1 & \text{if } q \le q_p \\ (1-\alpha)\,\xi\,\left(\frac{\alpha\,\mathcal{A}\,\mathcal{P}}{1-q}\right)^{\frac{\alpha}{1-\alpha}}\,\eta_1\,\tfrac{1}{2}\,(1+e^{-\overline{\varphi}}) - c_0 - c_1 & \text{if } q > q_p\ , \end{cases} \tag{26}
$$

which, using the expressions in section A.6, yields

$$
q_1(\sigma) = \begin{cases} \frac{(1+\sigma)\,c_1\,\xi^{-1}\,(\mathcal{A}\,\mathcal{P})^{\frac{\alpha}{\alpha-1}}}{(\eta_1-\eta_0)\,(1+\sigma\,e^{-\varphi^*})} & \text{if } q_1 \le q_p \\ 1 - \alpha\,\mathcal{A}\,\mathcal{P}\,\left(\frac{(1+\sigma)\,c_1\,\xi^{-1}}{(1-\alpha)\,(\eta_1-\eta_0)\,(1+\sigma\,e^{-\varphi^*})}\right)^{\frac{\alpha-1}{\alpha}} & \text{if } q_1 > q_p\ . \end{cases} \tag{27}
$$

As previously discussed, the renewal cut-off depends on σ, since it is determined at stage three of the R&D process, while the renewal decision constitutes stage four.
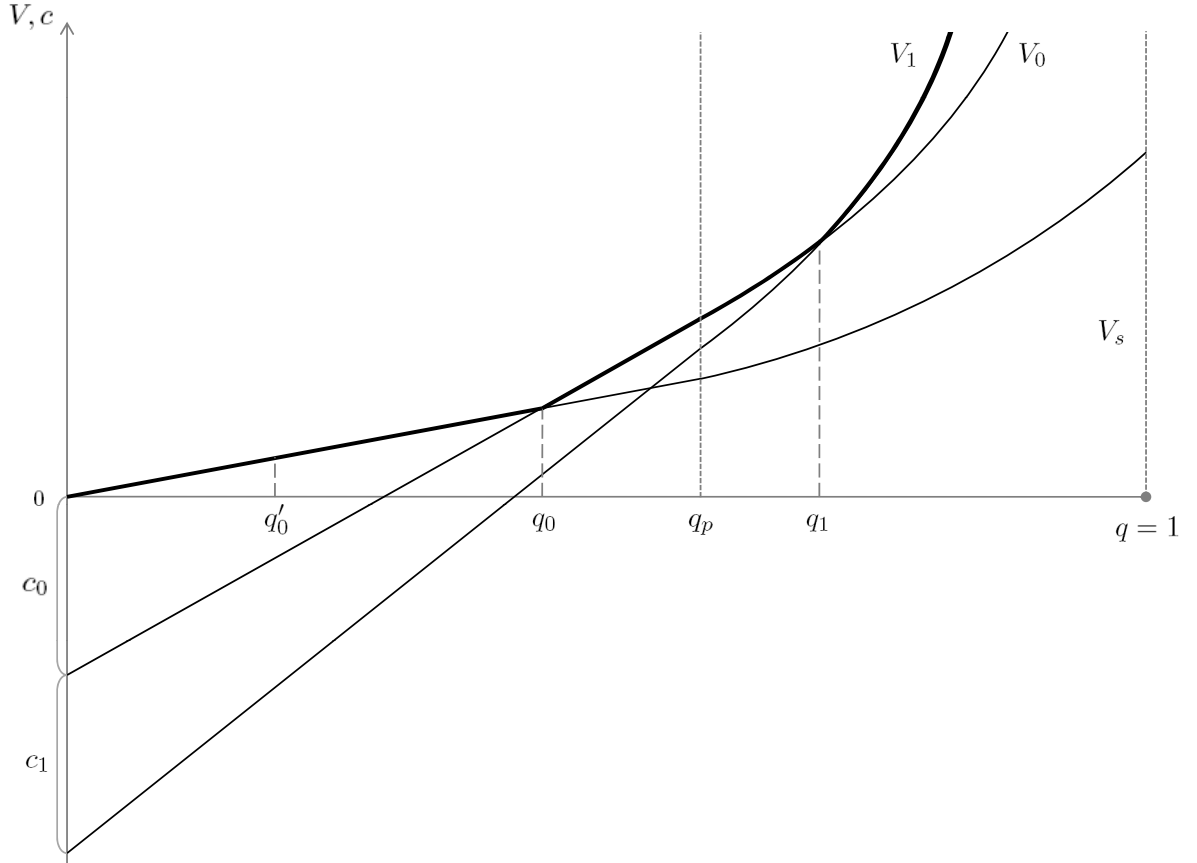

Figure 6: Patent and innovation value with drastic and non-drastic innovations.

Given these cut-offs and the modified β-f-relation, the model can be solved similarly to above. Figure 6 shows an example for the behavior of value functions and cut-offs under the CES specification. The cut-off $q_p = 1 - \alpha$ represents the innovation quality at which firms can switch from limit pricing to the profit-maxizing price under drastic innovations. As a result, to the right of q_p the value functions grow exponentially in q.

Appendix C

Sector	Sales weighted patent propensity (in %)		
	N	Product innovations	Process innovations
Mining	11	27.7	32.5
Food, beverages & Tobacco	42	26.1	24.7
Textiles, clothing	9	8.1	8.1
Petroleum refining	17	22.6	29.0
Chemicals	88	57.3	39.0
Pharmaceuticals	32	79.2	45.6
Rubber & Plastic products	20	33.7	27.6
Glass, clay, ceramics	35	29.3	20.2
Basic metals	13	14.6	15.1
Fabricated metal products	42	38.8	39.4
Machinery	69	52.4	16.3
Office & computing equip.	8	56.8	20.9
Electrical equip.	26	43.6	21.5
Communication equip.	37	46.6	22.7
Precision instruments	24	56.4	46.8
Automobiles	46	30.0	17.0
Other transport equip.	30	31.2	10.9
Power utilities	14	29.5	26.5
Transport & telecom services	23	20.5	12.4
Other	18	-	-
All firms	604	35.9	24.8

Table 2: Patent propensity by sector for European firms, 1990 to 1992. (Source: Arundel/Kabla (1998), p. 133.)
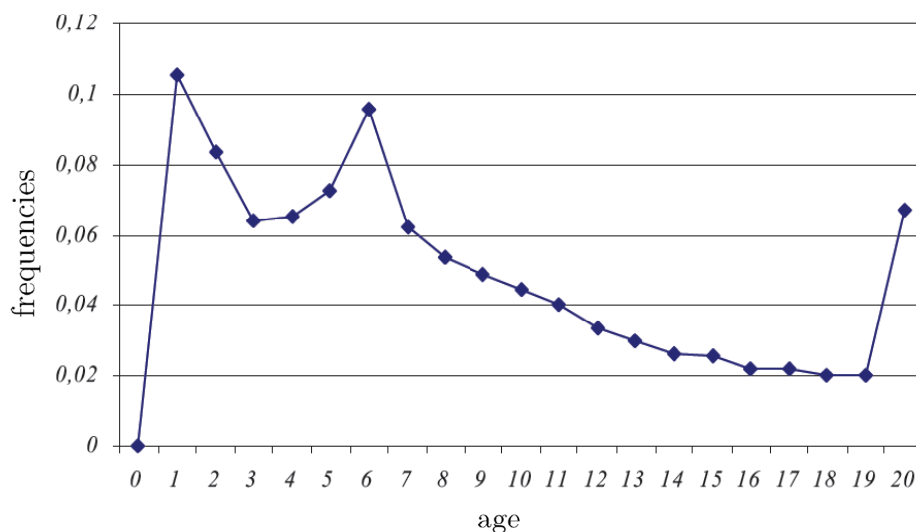

Figure 7: Average drop out frequencies of granted and rejected patent applications. (Source: Baudry/Dumont (2009), figure 3.)

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