Regulating a Monopoly with Universal Service Obligation: The Role of Flexible Tariff Schemes

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Abstract

This paper's purpose is to study the problem of a regulator of a utility monopoly, who has a universal service goal that is binding, in the sense that there is no two-part tariff that can induce efficient consumption, self-finance the firm, and guarantee universal access at the same time.

The optimal two-part tariffs that the regulator should set under the following three different regulatory rules are derived: no flexibility (the monopolist just offers the regulated plan), partial flexibility (the monopolist can offer alternative plans, but these -and the regulated one- must be available to all customers), and full flexibility (the regulated plan must be available to all customers, but not the alternative ones).

The solutions under the three schemes are characterized, and provide an unambiguous ranking of regulatory rules: total flexibility is weakly better than partial flexibility, with the latter being strictly better than no flexibility.

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1. Introduction

Regulators of network utilities, such as fixed telephony, electricity**,** and gas typically impose on monopolists (or the main operator) a universal service obligation (USO), which aims to guarantee the equal access of all consumers to the service at a reasonable cost. The universal service goal is typically based on equity and/or efficiency grounds (for those services that exhibit positive externalities) and it has been a central element of many industrial and regulatory policies.

These USOs are still quite common in developing and underdeveloped countries, where access to certain services --as in the case of fixed telephony-- is far from being universal.[1] In these countries, the universal access problem typically has two different dimensions: one of these is the rural dimension, which is mainly associated to underdeveloped networks and its larger costs in less populated areas;[2] and the other one is related to the (un)willingness of low income consumers to pay for services in areas where networks are available.[3]

In this paper, the focus is on this second dimension, and by imposing a universal access constraint on the regulator's problem, the optimal two-part tariffs he should set is derived under three different regulatory schemes:[4] one in which the monopolist is forced to offer a single regulated two-part tariff, and the other two in which he is able to offer alternative tariff schemes but must always offer the regulated tariff. The latter two schemes differ in the degree of flexibility permitted to the monopolist to price discriminate among consumers: in the first scheme, all tariffs -including the one set by the regulator- must be readily available for all customers, and in the second only the regulated tariff needs to be offered to all consumers, so the monopolist can tailor alternative tariff schemes for each consumer group.

Regulatory schemes where the main operator is granted flexibility to design alternative plans are seen more frequently in Latin America in the last few years. In 2004, the Chilean authority (SubTel) granted the regulated operator (Telefonica CTC) the flexibility to offer different tariff schemes in addition to the regulated one. A similar regulatory framework is using the Colombian authority (CRT) since 2005: the regulator defines the Basic Plan's maximum tariff, which must be offered with the firm's alternative plans. Several other Latin

[1] Although the results are general for most network utilities, the paper will refer mainly to the fixed telephony case.
[2] In Latin America, there have been two main approaches to foster telecom network development in rural areas. Some countries like Mexico, Brasil and Venezuela imposed network expansion obligations when they privatized their national monopolies, while other countries like Chile and Peru relied on reverse auctions for minimum subsidies to provide connectivity in remote areas (García-Murillo and Kuerbis, 2005).
[3] The number of main fixed lines per one hundred urban inhabitants in Latin America ranges from 3.7 in Paraguay and 4.4 in Bolivia to 21.7 in Argentina and 26.6 in Uruguay; while in developed nations with percentage levels of urban population similar to those of Uruguay and Argentina (above 85%) such as the UK or Germany the same figure is near 50 (source, ITU statistics at http://www.itu.int on Main Telephone Lines in 2007, and The United Nations World Population Prospects: The 2006 Revision at http://data.un.org on percentage of urban population).
[4] The two-part tariff refers to a monthly fixed charge and a per-unit price. In most Latin American countries, unlike for example the U.S., the per-minute price for local calls is not zero.

American countries (Brazil, Mexico, and Peru) allow their main fixed telephony operators to offer alternative plans beyond the regulated ones as well. The regulatory schemes, however, differ in several aspects: In Peru and Mexico a cap is set on a basket of plans; while in Brazil the regulator defines two plans that all operators must offer.

Two questions arising from the different schemes are: firstly, how much flexibility should the monopolist have to design tariff schemes different from the one designed by the regulator? And secondly, which is the regulator's optimal tariff plan for the different regulatory schemes?

This paper shows that the regulatory scheme which gives the most flexibility to the monopolist Pareto dominates the one in which all tariffs must be available to all customers for some parameter configurations (and is equally good for all other parameters), with this one strictly dominating the more rigid and traditional one in which the monopolist can only offer the regulated tariff. It is characterized when the regulatory schemes of partial and full flexibility are both efficient and in which cases this is only true for the latter one; and also how the optimally regulated tariff changes as the regulatory regime switches from the most rigid one to more flexible ones. The predicted changes are consistent with the changes observed in Chile and Colombia when more flexible regulatory regimes were introduced.

The results are derived in a simple setup in which the unique information asymmetry between the regulator and the monopolist is about the type of each consumer, which is assumed to be known to the firm but not to the regulator.[5] These results depend on the assumption that universal access is an issue. This is operationalized by assuming that there is no two-part tariff such that the monopolist breaks even and that the low valuation consumers are willing to subscribe to the network (if this were the case, a single two-part tariff would be first-best efficient).[6]

The case of partial flexibility, although weakly dominated, is relevant for reasons of political economy, because it could be hard for the regulator to explain why certain plans are available to some customers but not to others. It is precisely in the situation when a certain group would prefer the tariff scheme of a different one that the fully flexible regulatory scheme strictly dominates the one of partial flexibility.

The findings are in line with the literature on price delegation (Loeb and Magat, 1979; Armstrong and Vickers, 1991; Riordan, 1984; Sappington and Sibley, 1988; Sharkey and Sibley, 1993; Sibley, 1989; Bertoletti and Polleti, 1997; and Vogelsang, 1989 and 1990) in the sense that granting the informed party the possibility to set the prices constrained in some particular way is desirable.

[5] The extension to the case where neither the regulator nor the firm know each consumer type is straightforward: the case of full flexibility is identical to the case of partial flexibility, because the incentive compatibility constraints the monopolist should consider when designing the alternative plans are identical to the ones the regulator imposes in the case of partial flexibility.

[6] Throughout the paper it is implicitly assumed the regulator has no instruments other than tariffs and the regulatory scheme. This is a reasonable assumption for many countries, especially for the type of universal access problem addressed: the access of low income customers.

In particular, the efficiency result for the case of full flexibility corresponds to Vogelsang's (1989) result, once the initial set of prices is chosen to satisfy the universal access constraint and to leave no rent to the monopolist (after he optimally designs his alternative plans). The paper's contribution is three-fold: it analyzes alternative regulatory schemes in which the monopolist is granted different degrees of flexibility and it ranks them unambiguously; it identifies the optimally regulated tariffs and how they must change if the regulatory regime changes; and finally, it shows that in the particular setup in which consumers' preferences satisfy the single crossing property, there is no welfare loss if the regulator focuses on a single regulated tariff.[7]

The rest of the paper is organized as follows: the next section describes the model and solves the regulator's problem assuming that there is no universal service constraint (this being a useful benchmark to contrast our results). In Section 3, the regulator's optimal tariff problem under the three regulatory schemes is derived and the main results presented. In Section 4 final remarks are presented. All formal proofs are found in the Appendix.

2. The model

The Firm

The cost function for the firm is assumed

$$C(s,m) = A + h(s) + gm,$$

where A is a fixed cost, independent of the number of customers and the number of minutes, $h(s)$ is an avoidable fixed cost that depends on the number of subscribers s but not on total output m (from now on we will refer to this output as the number of minutes), and g is the constant marginal cost.[8]

The firm's only income comes from the two-part tariff received from consumers. It is assumed the government can not make direct transfers to the firm.

Consumers

There are 2 types of consumers that differ in their valuation of the service. Those of high valuation -*h*- (low valuation -*l*-) derive a utility u_h (u_l) from being connected, and $v_h(m)$ ($v_l(m)$) from consuming m minutes. Type h consumers are a fraction α of the total population which is normalized to one.

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[7] Given the particular asymmetric information assumption, in this setup the monopolist obtains no rent. If some asymmetry about costs (or total demand) were included, the low cost (high demand) monopolist would certainly obtain an informational rent, and the regulator should optimally define a menu of two-part tariffs rather than a single one.

[8] The simplicity of the cost function makes it easier to derive all results. In fact, nothing essential would change with a more general cost function as long as it had some fixed costs and the marginal cost is larger than consumers' marginal valuation for a sufficiently large m.

If a type $i = h$, l customer pays a fixed charge t, a per-minute price p, and consume m_i minutes, his total utility is $u_i + v_i(m_i) - t - pm_i$.

The assumptions on the functions v_i are standard: they are differentiable, increasing, strictly concave, and $v_h'(m) > v_l'(m)$ holds for any $m > 0$. Further, it is assumed that $\lim_{m\to\infty} v_l'(m_l) = \lim_{m\to\infty} v_h'(m_h) = 0$ and u_h is large enough.

For future reference the following values of m are defined

$$m_l^* \equiv v_l'^{-1}(g),$$
$$m_h^* \equiv v_h'^{-1}(g), \text{ and}$$
$$m_l^{SB}(\alpha) \equiv \{m_l : v_l'(m_l) - \alpha v_h'(m_l) = (1-\alpha)g\}.$$

m_l^* and m_h^* are first-best quantities (recall g is firm's marginal cost) and m_l^{SB} is the quantity a second-degree price discriminating monopolist would choose for low valuation consumers ($m_l^{SB} > 0$ is assumed).

The Regulator

The regulator maximizes the weighted sum of utilities of all consumers. In doing so, he will choose a unique regulated tariff (t, p).[9] Formally, his problem is given by

$$\max_{t,p} \alpha[u_h + v_h(m_h) - t - pm_h] + (1-\alpha)[u_l + v_l(m_l) - t - pm_l]$$

subject to the firm's self-financing constraint and consumer's optimal behavior, which are expressed as

$$t + (\alpha m_h + (1-\alpha)m_l)p - A - h(1) - g(\alpha m_h + (1-\alpha)m_l) \geq 0,$$
$$m_h = v_h'^{-1}(p),$$
$$m_l = v_l'^{-1}(p).$$

A participation constraint for consumers type l will be included later.

To help in the interpretation of the model and its results, it is useful to "translate" a two-part tariff –a point (t, p)– to a pair of points in the plane (m, T). Given (t, p), $(m_l, T_l; m_h, T_h)$ is such that $m_l = v_l'^{-1}(p)$, $T_l = t + m_l p$, $m_h = v_h'^{-1}(p)$, and $T_h = t + m_h p$; that is to say, the

[9] As will become clear, the regulator choosing a unique two-part tariff rather than a menu is without loss of generality.

quantities are the ones consumers would choose given p, and total payments T_l and T_h are equal to that which consumers would pay facing a two-part tariff (t, p).

Note that by defining a pair (m_l, T_l) the pair (m_l, T_l) is uniquely determined by the following equations:

$$m_h = \overline{m}(m_l) \equiv \{\overline{m} : v_l'(m_l) = v_h'(\overline{m})\}$$
$$T_h = \overline{T}(T_l, m_l) \equiv T_l + v_l'(m_l)(\overline{m}(m_l) - m_l).$$

The regulator's problem can therefore be written as

$$\underset{\{m_l, T_l\}}{Max}\ \alpha[u_h + v_h(\overline{m}(m_l)) - \overline{T}(T_l, m_l)] + (1-\alpha)[u_l + v_l(m_l) - T_l], \qquad (\text{OF}_\text{R})$$

subject to

$$\alpha\overline{T}(T_l, m_l) + (1-\alpha)T_l - A - h(1) - g(\alpha\overline{m}(m_l) + (1-\alpha)m_l) \geq 0 \qquad (\text{R1})$$

When participation of l-types is not an issue, the solution to the above problem is trivial: $m_l = m_l^*$, and $T_l = A + h(1) + m_l^* g$ (which imply $m_h = m_h^*$ and $T_h = A + h(1) + m_h^* g$); or, in terms of the two-part tariff, $(t, p) = (A + h(1), g)$.

3. Results

In this section, the options the regulator has when he faces a binding universal service obligation are analyzed.[10] This constraint is simply incorporated as a participation constraint for the low valuation customers, given by

$$u_l + v_l(m_l) - T_l \geq \underline{U}, \qquad (\text{R2})$$

where $\underline{U}$ is their reservation utility. To assume this constraint is binding amounts to assuming that the solution to the previous problem (OF_R subject to R1) does not satisfy this expression when participation is not an issue. That is,

$$u_l + v_l(m_l^*) - gm_l^* - (A + h(1)) < \underline{U} \qquad (\text{S1})$$

For the rest of the paper, (S1) is assumed to hold. This inequality by no means implies that, from a social point of view, it would be optimal to exclude low valuation customers. As long as their utility minus the marginal cost of connecting them exceeds their reservation utility it will be optimal to have them subscribing to the service. That is whenever

[10] The universal service is actually an obligation on the firm, but the regulator has to make sure the firm can satisfy it and, at the same time, make non-negative profits.

$$u_l + v_l(m_l^*) - gm_l^* - \frac{h(1) - h(\alpha)}{(1-\alpha)} \geq \underline{U}$$

holds.[11] For the results of the paper is irrelevant whether the above inequality holds, as it's assumed that the regulator seeks to maximize the consumers' surplus constrained by (R2). Therefore, in this setup, the cost per subscriber $h(s)$ plays a role identical to the fixed cost A.

A Single Two-part Tariff and no Flexibility

Under the no flexibility regime, the regulator's problem is to find a two-part tariff such that the monopolist breaks even and the universal service goal is fulfilled. By assumption (S1), the first-best solution discussed above will not suffice. The regulator must find a way to subsidize low valuation customers, and this can be done simply by raising the per-minute price and lowering the fixed charge. This will result in an indirect subsidy from type *h* to type *l* customers. Proposition 1 formalizes this result, which rationalizes what has been part of an implicit universal service policy in some countries (e.g., Chile prior to the introduction of a partial flexibility scheme in 2004, and Colombia since the implementation of the new tariff framework in 2005).

Proposition 1: *Let* (m_l, T_l) *be the solution to (OF$_R$) subject to (R1) and (R2) under (S1). Then* (m_l, T_l) *is such that (R1) and (R2) are satisfied as equalities and* m_l *is strictly lower than* m_l^*.[12]

The formal proof is given in the Appendix. Figure 1 illustrates the result. This depicts the case where $u_l + v_l(m_l^*) - gm_l^* - (A + h(1)) < \underline{U}$. The regulator must therefore lower t and raise p up to the point where the equations $\alpha Z_h + (1-\alpha) Z_l = A + h(1)$ and $u_l + v_l\left(v_l^{\prime -1}(p)\right) - t - p v_l^{\prime -1}(p) = u_l + v_l(m_l) - T_l = \underline{U}$ are satisfied.

FIGURE 1 ABOUT HERE

Flexible Tariffs

Two different flexibility schemes will be analyzed. Under the first one, called "partial flexibility", the monopolist is allowed to offer alternative plans to customers. However, all alternative plans, on top of the one designed by the regulator, must be available to all customers. In the second one, which named "full flexibility", this constraint is removed and the monopolist is allowed to design an alternative plan for each customer (but he must also

[11] This setup does not consider network externalitites that would add to the convenience of having low valuation customers connected.

[12] Note that a solution to this problem may not exist. Given the cost parameters, it could be the case that for any price above the marginal cost the quantity consumed by type *h* customers decreases so quickly that it is insufficient to compensate for the deficit the monopolist has with type *l* customers. By now it is assumed that this is not the case; later it will be characterized precisely when there is no solution.

offer the regulated plan to all consumers). For both schemes the timing of the game will be the same: the regulator defines a regulated tariff, the monopolist then designs the alternative tariffs, and finally consumers choose from the available tariffs.

Partial Flexibility

To solve the game by backward induction it is necessary to obtain first the firm's reaction function given a regulated plan. Since consumers are of only two types, it is possible with no loss of generality, to constrain the firm to choose just two alternative plans.

Proposition 2 below characterizes this reaction function,[13] but first it is necessary to define the following two functions:

Definition 1: $T_l'\left(m_l^R, T_l^R, m\right)$ *is a T such that low valuation individuals are indifferent between* $\left(T_l^R, m_l^R\right)$ *and* $\left(T_l'(\cdot), m\right)$*; therefore*

$$T_l'(m_l^R, T_l^R, m) \equiv T_l^R + v_l(m) - v_l(m_l^R).$$

Definition 2: $\widetilde{m}\left(m_l^R\right)$ *is an m such that type h consumers are indifferent between choosing the regulated plan -given by* $\left(\overline{m}\left(m_l^R\right), \overline{T}\left(T_l^R, m_l^R\right)\right)$*- and the plan* $(\widetilde{m}(m_l^R), T_l'(m_l^R, T_l^R, \widetilde{m}(m_l^R)))$. *Formally*

$$\widetilde{m}(m_l^R) \equiv \{\widetilde{m} : v_h(\overline{m}(m_l^R)) - v_l'(m_l^R)(\overline{m}(m_l^R) - m_l^R) - v_l(m_l^R) = v_h(\widetilde{m}) - v_l(\widetilde{m})\}.$$

Figure 2 illustrates these definitions.

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[13] The firm's reaction function can be stated in alternative ways:
1) As a mapping from $\Re_+^2$ -the pair $\left(p^R, t^R\right)$- to $\Re_+^4$ -the pair of two-part tariffs the firm will offer $\left((p_l, t_l), (p_h, t_h)\right)$, which in turn can be viewed as a pair $(m_l, T_l), (m_h, T_h)$ once the consumers' reaction is considered;
2) As a mapping from a subset of $\Re_+^4$ -those points $\left((m_l^R, T_l^R), (m_h^R, T_h^R)\right)$ that result from the pair $\left(p^R, t^R\right)$ when consumers' reaction is considered- to a subset of $\Re_+^4$ -alternative tariff plans $\left((m_l, T_l), (m_h, T_h)\right)$ that satisfy all incentive compatibility constraints; or
3) As a mapping from $\left(m_l^R, T_l^R\right)$ to the alternative tariff plans $\left((m_l, T_l), (m_h, T_h)\right)$. This interpretation is a valid one because $\left(m_h^R, T_h^R\right)$ is uniquely determined once $\left(m_l^R, T_l^R\right)$ is set by the functions $\overline{m}(m_l)$ and $\overline{T}(T_l, m_l)$. These definitions guarantee the alternative plans $\left((m_l, T_l), (m_h, T_h)\right)$ satisfy the incentive compatibility constraints.
The third option is followed as it can be easily represented in the plane (m, T).

FIGURE 2 ABOUT HERE

Additionally, two critical values of m are defined:

$$m_l^H : \widetilde{m}(m_l^H) = m_l^*$$
$$m_l^L : \widetilde{m}(m_l^L) = m_l^{SB}.$$

The problem the monopolist solves is

$$\underset{\{m_l, m_h, T_l, T_h\}}{Max} \alpha(T_h - gm_h) + (1-\alpha)(T_l - gm_l) \qquad (\text{OF}_\text{M})$$

subject to

$$v_l(m_l) - T_l \geq v_l(m_l^R) - T_l^R \qquad (\text{IC}_l^R)$$
$$v_h(m_h) - T_h \geq v_h(m_h^R) - T_h^R = v_h(\overline{m}(m_l^R)) - \overline{T}_h(T_l^R, m_l^R) \qquad (\text{IC}_h^R)$$
$$v_h(m_h) - T_h \geq v_h(m_l) - T_l \qquad (\text{IC}_h)$$

The first and second constraints guarantee that the alternative plans offered by the monopolist are actually preferred by all customers to the regulated plan. The third constraint is the standard incentive compatibility constraint, which ensures type *h* individuals actually prefer the alternative plan intended for them to the one designed for *l*-types. It is implicitly assumed that the regulated plan (m_l^R, T_l^R) is such that the *l*-types participation constraint is satisfied and can therefore be omitted.[14]

Proposition 2: *Assuming the regulated plan* (m_l^R, T_l^R) *satisfies* $u_l + v_l(m_l^R) - T_l^R \geq \underline{U}$,

a) If $m_l^R \geq m_l^H$, *then the menu of alternative plans the monopolist will propose is:*

$$\{(m_l^*, T_l^R + v_l(m_l^*) - v_l(m_l^R)), (m_h^*, \overline{T}(T_l^R, m_l^R) + v_h(m_h^*) - v_h(\overline{m}(m_l^R)))\}$$

b) If $m_l^R \in (m_l^L, m_l^H)$, *then the menu of alternative plans the monopolist will propose is:*

$$\{(\widetilde{m}(m_l^R), T_l^R + v_l(\widetilde{m}(m_l^R)) - v_l(m_l^R)),$$
$$(m_h^*, \overline{T}(T_l^R, m_l^R) + v_h(m_h^*) - v_h(\overline{m}(m_l^R)))\}.$$

c) If $m_l^R \leq m_l^L$, *then the menu of alternative plans the monopolist will propose is:*

$$\{(m_l^{SB}, T_l^R + v_l(m_l^{SB}) - v_l(m_l^R)),$$
$$(m_h^*, T_l^R + v_l(m_l^{SB}) - v_l(m_l^R) + v_h(m_h^*) - v_h(m_l^{SB}))\}.$$

[14] It is argued below (see footnote 19) that this is with no loss of generality, given that the ultimate goal is to solve the regulator's problem subject to this participation constraint.

Figure 3 illustrates the result of this proposition for three different regulated plans (one for each relevant segment of m_l^R). The regulated plans are depicted as black figures, while the corresponding white figures represent the firm's reaction to the regulated plans (i.e., the alternative plans it would offer).

FIGURE 3 ABOUT HERE

To minimize the notation, two functions are defined

$$\widehat{m}_l(m_l^R) \equiv \begin{cases} m_l^* & if\, m_l^R \geq m_l^H \\ \widetilde{m}(m_l^R) & if\, m_l^R \in (m_l^L, m_l^H) \\ m_l^{SB} & if\, m_l^R \leq m_l^L. \end{cases}$$

and

$$\widehat{T}_h(m_l^R, T_l^R) \equiv$$
$$\begin{cases} T_l^R + v_l'(m_l^R)(\overline{m}(m_l^R) - m_l^R) + v_h(m_h^*) - v_h(\overline{m}(m_l^R)) & if\, m_l^R \geq m_l^L \\ T_l^R + v_l(m_l^{SB}) - v_l(m_l^R) + v_h(m_h^*) - v_h(m_l^{SB}) & if\, m_l^R < m_l^L \end{cases}$$

so the firm's reaction function is $\left\{\left(\widehat{m}(m_l^R), T_l^R + v_l\left(\widehat{m}(m_l^R)\right) - v_l(m_l^R)\right); \left(m_h^*, \widehat{T}_h(m_l^R, T_l^R)\right)\right\}$.

Therefore, the problem for the regulator is to choose the plan $\left(m_l^R, T_l^R\right)$ that maximizes his objective function taking into account the self-financing and participation constraints and how the monopolist will react by offering alternative plans.

Formally, the problem he solves is

$$\underset{\{m_l^R, T_l^R\}}{Max} \quad \alpha\left[u_h + v_h(m_h^*) - \widehat{T}_h(m_l^R, T_l^R)\right] +$$
$$(1-\alpha)\left[u_l + v_l\left(\widehat{m}_l(m_l^R)\right) - T_l'\left(m_l^R, T_l^R, \widehat{m}_l(m_l^R)\right)\right] \qquad \text{(OF'}_R\text{)}$$

subject to

$$\alpha\widehat{T}_h(m_l^R, T_l^R) + (1-\alpha)T_l'\left(m_l^R, T_l^R, \widehat{m}_l(m_l^R)\right) -$$
$$A - h(1) - g(\alpha m_h^* + (1-\alpha)\widehat{m}_l(m_l^R)) \geq 0 \qquad \text{(R1')}$$

and

$$u_l + v_l(m_l^R) - T_l^R \geq \underline{U}. \quad \text{(R2')}$$

Note that the participation constraint for *l*-types (R2') is written in terms of the regulated plan rather than the alternative plan the monopolist will design. This is valid in this case because, as Proposition 2 shows, the utility that *l*-type consumers will get with the alternative plan is always the same they obtain with the regulated plan.

Proposition 3 below presents a formal solution to this problem, but first the results are illustrated by making use of Figure 3. The analysis starts at the regulated plan $\left(g, v_l\left(m_l^*\right) - \underline{U}\right)$, which translates to points *a*, *a'* for type *l* and *h* respectively. In this plan, the per-minute price equals the marginal cost and the fixed charge *t* is such that low valuation consumers get their reservation utility $\underline{U}$. Then, since the plan involves efficient quantities, the monopolist can not gain by designing alternative plans. The assumption (S1) implies, however, that this plan does not satisfy the self-financing constraint.

What are the options open to the regulator? As he changes the regulated plan to the left along the type *l* indifference curve $\underline{U}$ (e.g., to the plan represented by the black circle - $m_l^R \geq m_l^H$), the regulated plan becomes less and less attractive to consumers of type-*h* and, as a consequence, the alternative plan the monopolist designs for them will increase the money transfer T_h (and his benefits) while maintaining the efficient quantity m_h^*. This is done with no efficiency loss as long as $m_l^R \geq m_l^H$, because the monopolist will offer m_l^* for the *l*-types. So the first best will be attainable if and only if the monopolist profits induced by $m_l^R = m_l^H$ are non-negative.

If the monopolist still gets negative profits when $m_l^R = m_l^H$, the first best will be unattainable. To increase monopolist's profits further, the regulator keeps reducing m_l^R along the indifference curve $\underline{U}$ (e.g., the plan represented by the black square), but now there will be no menu of alternative plans that involves efficient quantities for both types and also satisfies the self-financing and incentive compatibility constraints at the same time. The profit maximizing monopolist will now choose to distort down the quantity m_l in order to satisfy $\left(IC_h^R\right)$ as an equality.

This will be the optimal response for the monopolist as long as $m_l^R \geq m_l^L$. However, if $m_l^R < m_l^L$ (e.g., the plan represented by the black triangle), then choosing $T_h = \overline{T}\left(T_l^R, m_l^R\right) + v_h\left(m_h^*\right) - v_h\left(\overline{m}\left(m_l^R\right)\right)$ and satisfying $\left(IC_h^R\right)$ would imply choosing $m_l < m_l^{SB}$, which is never optimal for the monopolist (note that the menu of alternative plans the monopolist chooses does not depend on m_l^R if $m_l^R < m_l^L$). Therefore, if the monopolist obtains negative profits when $m_l^R = m_l^L$, then the problem has no solution: the self-financing constraint and the participation constraint can not be satisfied simultaneously by any incentive compatible plan.[15]

Logically, the monopolist being able to be self-financing depends on the proportion of consumers of each type. Intuitively, if α is close to one, the extra profit the monopolist will need to make out of consumers of type h will be relatively small, and therefore, m_l^R could be close to m_l^*. As α decreases, this extra profit must increase, which amounts to a decrease in m_l^R, but never going beyond m_l^L. Two critical values of α can be defined. α^{FB} is such that if $\alpha \geq \alpha^{FB}$ the first best will be attainable:

$$\alpha^{FB} \equiv \{\alpha \in (0,1) \,:\, (1-\alpha)\left(u_l + v_l\left(m_l^*\right) - \underline{U} - gm_l^*\right) + \\ \alpha\left(u_l + v_l\left(m_l^*\right) - \underline{U} + v_h\left(m_h^*\right) - v_h\left(m_l^*\right) - gm_h^*\right) = A + h(1)\};$$

and α^{SB} is such that $\alpha < \alpha^{SB}$ implies that the self-financing constraint and the participation constraint for l-types can not be satisfied together:

$$\alpha^{SB} \equiv \{\alpha \in (0,1) \,:\, (1-\alpha)\left(u_l + v_l\left(m_l^{SB}(\alpha)\right) - \underline{U} - gm_l^{SB}(\alpha)\right) + \\ \alpha\left(u_l + v_l\left(m_l^{SB}(\alpha)\right) - \underline{U} + v_h\left(m_h^*\right) - v_h\left(m_l^{SB}(\alpha)\right) - gm_h^*\right) = A + h(1)\}.$$

Note that the definition of α^{SB} recognizes the fact that m_l^{SB} depends on α. It is now possible to formally state the solution to the regulator's problem.

Proposition 3: *Consider the maximization of OF' subject to (R1') and (R2'):*
a) If $\alpha \geq \alpha^{FB}$ *the first best is achievable. An optimal regulated plan is* $\left(m_l^R,\ u_l + v_l\left(m_l^R\right) - \underline{U}\right)$, *where* $m_l^R \geq m_l^H$ *satisfies*

$$(1-\alpha)(T_l'(m_l^R, T_l^R, m_l^*) - gm_l^*) + \alpha\left(\widehat{T}(m_l^R, T_l^R) - gm_h^*\right) = A + h(1).$$

b) If $\alpha \in \left[\alpha^{SB}, \alpha^{FB}\right)$, *the optimal regulated plan is* $\left(m_l^R, u_l + v_l\left(m_l^R\right) - \underline{U}\right)$, *where* $m_l^R \in \left[m_l^L, m_l^H\right)$ *and satisfies*

$$(1-\alpha)\left(T_l'\left(m_l^R, T_l^R, \widehat{m_l}(m_l^R)\right) - g\widehat{m_l}(m_l^R)\right) + \\ \alpha\left(\widehat{T}(m_l^R, T_l^R) - gm_h^*\right) = A + h(1).$$

c) If $\alpha < \alpha^{SB}$, *there is no regulated plan such that (R1') and (R2') are satisfied.*

[15] Of course, if this is the case in this model with partial flexibility it must also be the case in the no flexibility case.

The formal proof is relegated to the Appendix.

Note that when $\alpha > \alpha^{FB}$ there are many solutions, as the regulator can, at no social cost, give type-*l* individuals some rent. This is not possible for values of $\alpha \leq \alpha^{FB}$, since the extra rent would require further distortion of the quantity offered to *l*-types.[16]

Full Flexibility

The case where the monopolist is not obliged to offer all alternative plans to all consumers is now analyzed (but he must have the regulated plan readily available). As the incentive compatibility constraints between alternative plans become irrelevant (in particular IC_h), this case is analytically much simpler than the previous one. The monopolist will design - for each type- a plan with an efficient quantity and a payment such that the consumer is indifferent between this alternative plan and the regulated one. Formally, a regulated plan $\left(m_l^R, T_l^R\right)$ that satisfies the *l*-type participation constraint will induce a menu of alternative plans.

$$\left\{\left(m_l^*, T_l^R + v_l\left(m_l^*\right) - v_l\left(m_l^R\right)\right),\right.$$
$$\left.\left(m_h^*, \overline{T}\left(T_l^R, m_l^R\right) + v_h\left(m_h^*\right) - v_h\left(\overline{m}\left(m_l^R\right)\right)\right)\right\}.$$

The problem for the regulator is simply to find the point on the type-*l* indifference curve $\underline{U}$ such that the monopolist, after designing the menu of alternative plans, obtains zero profits. As in the partial flexibility case, the firm's profits increase as m_l^R decreases from m_l^*. In fact, the two profit functions are the same as long as $m_l^R \geq m_l^*$. But, unlike the partial flexibility case, $m_l^R < m_l^H$ does not imply any distortion of the quantities -$\left(IC_h\right)$ is irrelevant- and therefore, as m_l^R decreases profits will increase faster in the case of full flexibility. Moreover, profits still increase as m_l^R decreases, even for values of $m_l^R < m_l^L$. Proposition 4 formalizes this result.

Proposition 4: *Under complete flexibility and given a regulated plan* $\left(m_l^R, T_l^R\right)$ *that satisfies* $u_l + v_l\left(m_l^R\right) - T_l^R \geq \underline{U}$, *the monopolist will offer the menu of alternative plans*

[16] It was claimed above that restricting the regulated plans to satisfy the *l*-type participation constraint was with no loss of generality. The argument is the following: If the regulated plan does not satisfy it, the monopolist, maximizing its profits, will either choose a menu of alternative plans such that the component designed for *l*-types satisfies the participation constraint or it will simply focus on *h*-types. As the regulator would never choose a regulated plan such that the *l*-types are excluded, we can therefore focus on the first case. Now, if the monopolist designs a menu of alternative plans such that *l*-types participate, the menu will have the following characteristics: a) $m_h = m_h^*$, b) T_h will be such that IC_h is binding, c) $m_l \in [m_l^{SB}, m_l^*]$, and d) $T_l = \underline{u} + v_l(m_l) - \underline{U}$. But any such plan could be induced with a regulated plan (T_l^R, m_l^R), where $m_l^R \in [m_l^L, m_l^H]$ and T_l^R is such that the *l*-types participation constraint is satisfied as an equality.

$$\left\{\left(m_l^*, T_l^R + v_l\left(m_l^*\right) - v_l\left(m_l^R\right)\right),\right.$$
$$\left.\left(m_h^*, \overline{T}_h\left(T_l^R, m_l^R\right) + v_h\left(m_h^*\right) - v_h\left(\overline{m}\left(m_l^R\right)\right)\right)\right\}.$$

An optimal regulated plan is $\left(m_l^R, v_l\left(m_l^R\right) - \underline{U}\right)$, *where* m_l^R *satisfies*

$$\alpha\Big[v_l(m_l^R) - \underline{U} + v_l'(m_l^R)\Big(\widehat{m}_l(m_l^R) - m_l^R\Big) +$$
$$v_h(m_h^*) - v_h\Big(\widehat{m}_l(m_l^R)\Big) - gm_h^*\Big] + (1-\alpha)(v_l(m_l^*) - \underline{U} - gm_l^*) = A + h(1).$$

The formal proof is relegated to the Appendix.

This result is the same as that of Vogelsang (1989) for the particular case of a single initial two-part tariff. Interestingly, in this paper's setup, a single regulated two-part tariff is sufficient to induce an efficient outcome.[17]

From this proposition and the previous one it is possible to conclude that there will be some parameter configurations for which the monopolist is unable to provide a service to both types under the scheme of partial flexibility, but he will be able to do it under total flexibility.

Maintaining the assumption that (S1) holds, the results can be summarized in terms of the parameter α: when $\alpha \geq \alpha^{FB}$ the first best is attainable with both partial and full flexibility, but not under the no flexibility scheme; if $\alpha \in \left[\alpha^{SB}, \alpha^{FB}\right)$ the first-best can be attained only with full flexibility, with partial flexibility being strictly better than no flexibility; and if $\alpha < \alpha^{SB}$ the universal access and self-financing constraints can not be fulfilled together with partial or no flexibility, whith the full flexibility scheme still providing first best results.

4. Conclusions

The paper shows the convenience of allowing a regulated monopolist to price discriminate when the regulator has a binding universal access constraint. Two different flexibility schemes are analyzed, both sharing the features that the monopolist is obliged to offer a plan that the regulator has designed for all customers, but he is also allowed to offer alternative plans to his customers. Under the first scheme -partial flexibility- any alternative plan must be readily available to all customers, while in the second -full flexibility- this constraint disappears.

It is demonstrated that total flexibility is (weakly) superior to partial flexibility.[18] The intuition for this result has to do with the superior information the monopolist has and his

[17] Vogelsang (1989), and most of the literature cited in the Introduction, does not focus on the optimality of the initial prices but on the firm's reaction to those initial prices.

[18] They are equally good only when the universal access constraint is not too strong and the first best is attainable under the two schemes.

ability to use it. Under complete flexibility the monopolist can always exploit the superior information to induce efficient quantities, whereas under partial flexibility he is constrained in the same way as a second-degree price discriminating monopolist would be.[19]

Optimally regulated tariffs are derived for each regime and it is fully characterized when the first best is attainable. As the regulatory scheme changes from the most inflexible one – in which the regulator defines a unique tariff– to the more flexible ones –in which the firm can offer alternative plans– the fixed charge of the regulated two-part tariff must decrease while the per-unit price must increase. Such changes were observed, for example, in Colombia since 2005 with the implementation of the new regulatory scheme, and in Chile in 2004 when the main telecom firm was granted the right to offer alternative plans in addition to the regulated one.[20]

Although Pareto inferior, the scheme of partial flexibility could be relevant for reasons of political economy as it might be quite difficult for a regulator to explain how it is possible that the regulated firm can offer some plans to certain consumers but not to all of them. In fact, to the best of our knowledge, all the countries that allow their operators to offer alternative plans do it in the spirit of what we called partial flexibility rather than full flexibility.

[19] The constraint arises not because of any information asymmetry between consumers and the monopolist -as in the canonical case of screening-, but because the regulator imposes that all plans must be available to all customers.

[20] We do not claim that regulators have allowed firms to offer alternative plans because they have followed a similar line of reasoning in ours. Their decisions probably had to do more with incipient competition in the telephony market and/or the monopolist's need of offering bundled services (e.g., triple play).

A. Appendix

Proof of Proposition 1: Let λ and ξ be the multipliers for (R1) and (R2) respectively. Since we assumed the solution to (OF) restricted only by (R1) does not satisfy (R2), it follows that $\xi > 0$.

Differentiating with respect to T_l, we get

$$\lambda = 1 + \xi > 0.$$

Therefore, the two constraints are binding in the solution.

For the sake of contradiction assume $m_l \geq m_l^*$. Note that (R1) can be written as

$$T_l = A + h(1) + gm_l - \alpha\left(v_l'(m_l) - g\right)\left(\overline{m}(m_l) - m_l\right), \qquad \text{(R1'')}$$

and replacing in (R2) we get

$$u_l + v_l(m_l) - gm_l - A - h(1) + \alpha\left(v_l'(m_l) - g\right)\left(\overline{m}(m_l) - m_l\right) = \underline{U}. \qquad \text{(R2'')}$$

Since $v_l(m_l) - gm_l \leq v_l(m_l^*) - gm_l^*$ and $\alpha(v_l'(m_l) - g)(\overline{m}(m_l) - m_l) \leq 0$, the above equality contradicts (S1). Therefore, $m_l < m_l^*$.

Proof of Proposition 2: The monopolist's problem is

$$\underset{\{m_l, m_h, T_l, T_h\}}{Max} \alpha\left[T_h - gm_h\right] + (1-\alpha)\left[T_l - gm_l\right]$$

subject to

$$v_h(m_h) - T_h \geq v_h\left(\overline{m}(m_l^R)\right) - T_l^R - v_l'(m_l^R)\left[\overline{m}(m_l^R) - m_l^R\right] \qquad (\text{IC}_h^R \text{ - } \lambda)$$

$$v_l(m_l) - T_l \geq v_l(m_l^R) - T_l^R \qquad (\text{IC}_l^R \text{ - } \mu)$$

$$v_h(m_h) - T_h \geq v_h(m_l) - T_l \qquad (\text{IC}_h \text{ - } \rho)$$

$$v_l(m_l) - T_l \geq v_l(m_h) - T_h \qquad (\text{IC}_l \text{ - } \sigma)$$

As standard in this type of problem, the constraint (IC_l) does not bind and can therefore be ignored. The first order conditions for this problem are:

$$T_h: \quad \alpha - \lambda - \rho = 0 \qquad (1)$$

$$m_h: \quad -\alpha g + \lambda v_h'(m_h) + \rho v_h'(m_h) = 0 \qquad (2)$$

$$T_l: \quad (1-\alpha) - \mu + \rho = 0 \qquad (3)$$

$$m_l: \quad -(1-\alpha)g + \mu v_l'(m_l) - \rho v_h'(m_l) = 0 \qquad (4)$$

To these conditions we must add the non-negativity of the multipliers, the constraints themselves, and the fact that if a multiplier is strictly positive then its associated inequality constraint must be satisfied as an equality.

From (1) and (2) it is clear that $m_h = m_h^*$. Note also that (3) implies $\mu > 0$.

a) Consider the case $m_l^R \geq m_l^H$.
Assume $\rho > 0$. Then (3) and (4) imply $m_l < m_l^*$. Using (IC_l^R) and (IC_h) as equalities, (IC_h^R) can be rewritten as

$$
\begin{aligned}
v_h(m_l) - v_l(m_l) &\geq v_h(\overline{m}(m_l^R)) - v_l'(m_l^R)[\overline{m}(m_l^R) - m_l^R] - v_l(m_l^R) \\
&= v_h(\widetilde{m}(m_l^R)) - v_l(\widetilde{m}(m_l^R)) \\
&\geq v_h(m_l^*) - v_l(m_l^*).
\end{aligned}
$$

This contradicts that $m_l < m_l^*$ (the last weak inequality follows from the definition of m_l^H, the assumption that $m_l^R \geq m_l^H$ and that $v_h'(m) > v_l'(m)$ for any m). Therefore, $\rho = 0$.

Conditions (3) and (4) imply that $m_l = m_l^*$. Moreover, $\lambda = \alpha > 0$ and $\mu = (1-\alpha) > 0$. Therefore (IC_h^R) and (IC_l^R) can be used as equalities to obtain

$$
\begin{aligned}
T_h(m_l^R) &= T_l^R + v_l'(m_l^R)[\overline{m}(m_l^R) - m_l^R] + v_h(m_h^*) - v_h(\overline{m}(m_l^R)) \\
T_l(m_l^R) &= T_l^R + v_l(m_l^*) - v_l(m_l^R).
\end{aligned}
$$

b) Consider the case $m_l^R \in (m_l^L, m_l^H)$.
Following the steps in a), it is clear that if $\rho = 0$ then $m_l = m_l^*$, and given that $m_l^R < m_l^H$, constraint (IC_h) can not be satisfied. Therefore $\rho > 0$.

Assume $\lambda = 0$. Then $\rho = \alpha$ and $\mu = 1$. By (4) $m_l = m_l^{SB}$, and using (IC_h^R) and (IC_h), it obtains

$$
v_h(\overline{m}(m_l^R)) - T_l^R - v_l'(m_l^R)[\overline{m}(m_l^R) - m_l^R] \leq v_h(m_l^{SB}) - T_l.
$$

Substituting T_l from (IC_l^R) and rearranging

$$
\begin{aligned}
v_h(\overline{m}(m_l^R)) - v_l'(m_l^R)[\overline{m}(m_l^R) - m_l^R] - v_l(m_l^R) &\leq v_h(m_l^{SB}) - v_l(m_l^R) \\
v_h(\widetilde{m}(m_l^R)) - v_l(\widetilde{m}(m_l^R)) &\leq v_h(m_l^{SB}) - v_l(m_l^R)
\end{aligned}
$$

which can not hold since

$$
\begin{aligned}
v_h(\widetilde{m}(m_l^R)) - v_l(\widetilde{m}(m_l^R)) &\geq v_h(\widetilde{m}(m_l^L)) - v_l(\widetilde{m}(m_l^L)) \\
&= v_h(m_l^{SB}) - v_l(m_l^{SB}) \\
&> v_h(m_l^{SB}) - v_l(m_l^R).
\end{aligned}
$$

Therefore $\lambda > 0$. Using (IC_h^R), (IC_l^R), and (IC_h) as equalities the desired result is obtained.

c) Consider the case $m_l^R \leq m_l^L$.
Assume constraint (IC_h^R) is not binding. Then $\rho = \alpha$ and $\mu = 1$. (4) implies that $m_l = m_l^{SB}$, and from (IC_l^R) and (IC_h) as equalities it follows

$$
\begin{aligned}
T_l &= v_l(m_l^{SB}) - v_l(m_l^R) + T_l^R \\
T_h &= v_h(m_h^*) - v_h(m_l^{SB}) + v_l(m_l^{SB}) - v_l(m_l^R) + T_l^R.
\end{aligned}
$$

Note that the proposed solution satisfies (IC_h^R), which after simple algebra, can be written as

$$
\begin{aligned}
v_h(m_l^{SB}) - v_l(m_l^{SB}) &\geq v_h(\overline{m}(m_l^R)) - v_l'(m_l^R)[\overline{m}(m_l^R) - m_l^R] - v_l(m_l^R) \\
&= v_h(\widetilde{m}(m_l^R)) - v_l(\widetilde{m}(m_l^R)).
\end{aligned}
$$

The inequality holds since $m_l^R \leq m_l^L$ implies $\widetilde{m}(m_l^R) \leq m_l^{SB}$.

Proof of Proposition 3: The regulator's problem is

$$
\underset{\{m_l^R, T_l^R\}}{Max} \quad \alpha\left[u_h + v_h(m_h^*) - \widehat{T}_h(m_l^R, T_l^R)\right] + (1-\alpha)[u_l + v_l(m_l^R) - T_l^R]
$$

subject to

$$
\begin{aligned}
\alpha\widehat{T}_h(m_l^R, T_l^R) + (1-\alpha)\left[T_l^R + v_l\left(\widehat{m}_l(m_l^R)\right) - v_l(m_l^R)\right] - \\
A - h(1) - g(\alpha m_h^* + (1-\alpha)\widehat{m}_l(m_l^R)) \geq 0 \qquad &(\text{R1' - }\phi) \\
u_l + v_l(m_l^R) - T_l^R \geq \underline{U}. \qquad &(\text{R2' - }\chi)
\end{aligned}
$$

Note that from Proposition 2 it is clear that $\frac{\partial \widehat{T}_h(\cdot)}{\partial T_l^R} = 1$, so from the first order condition (differentiating with respect to T_l^R) it follows that $\phi = \chi + 1 > 0$. Therefore, the self-financing constraint must hold as an equality in the solution.

a) For $\alpha \geq \alpha^{FB}$, the proof proceeds in two steps. First it shows that the first best can be induced by $m_l^R = m_l^H$ for the extreme case that $\alpha = \alpha^{FB}$. Second, it shows that the firm's profit function is strictly decreasing in m_l^R (for $m_l^R \geq m_l^H$) as the regulated plan moves along the *l*-type indifference curve $\underline{U}$ (i.e., assuming $T_l^R = u_l + v_l(m_l^R) - \underline{U}$). Since the profit function is continuous, using the Intermediate Value Theorem it can then be concluded that for every $\alpha \geq \alpha^{FB}$, there is a unique $m_l^R \geq m_l^H$ such that the monopolist makes zero profits. This is so, because the monopolist would get strictly positive profits if $\alpha \in (\alpha^{FB}, 1)$ and $(m_l^R, T_l^R) = (m_l^H, u_l + v_l(m_l^R) - \underline{U})$ (note that, given the regulated plan, the profit function for the monopolist is strictly increasing in α and equal to zero if $\alpha = \alpha^{FB}$), with strictly negative profits if $(m_l^R, T_l^R) = (m_l^*, u_l + v_l(m_l^R) - \underline{U})$ (by assumption S1).

1) Consider $m_l^R = m_l^H$. By proposition 2, the monopolist will offer efficient quantities, so it suffices to show he gets no rent.

By definition, α^{FB} satisfies

$$\alpha^{FB}(v_l(m_l^*) + v_h(m_h^*) - v_h(m_l^*) - gm_h^*) + (1 - \alpha^{FB})(v_l(m_l^*) - gm_l^*) - A - h(1) - \underline{U} + u_l = 0.$$

Substituting $m_l^R = m_l^H$ and $\alpha = \alpha^{FB}$ in the firm's profit function, it follows

$$\alpha^{FB}[v_l(m_l^H) + v_l'(m_l^H)(\overline{m}(m_l^H) - m_l^H) - v_h(\overline{m}(m_l^H)) + v_h(m_h^*) - gm_h^*] + (1 - \alpha^{FB})[v_l(m_l^*) - gm_l^*] + u_l - \underline{U} - A - h(1),$$

so it is enough to show that

$$v_l(m_l^H) + v_l'(m_l^H)(\overline{m}(m_l^H) - m_l^H) - v_h(\overline{m}(m_l^H)) = v_l(m_l^*) - v_h(m_l^*),$$

which follows immediately from the definition of m_l^H and $\widetilde{m}(\cdot)$.

2) The firm's profit function for values of $m_l^R \geq m_l^H$, taking into account Proposition 2, and choosing $T_l^R = u_l + v_l(m_l^R) - \underline{U}$ is

$$\alpha[v_l(m_l^R) + v_l'(m_l^R)(\overline{m}(m_l^R) - m_l^R) + v_h(m_h^*) - v_h(\overline{m}(m_l^R)) - gm_h^*] + (1 - \alpha)[v_l(m_l^*) - gm_l^*] + u_l - \underline{U} - A - h(1)$$

Differentiating with respect to m_l^R and taking into account that $v_l'(m_l^R) = v_h'(\overline{m}(m_l^R))$, by definition of $\overline{m}(m_l^R)$ it follows that

$$\alpha[v_l''(m_l^R)(\overline{m}(m_l^R) - m_l^R)] < 0.$$

b) For $[\alpha^{SB}, \alpha^{FB}),$ the proof proceeds in five steps: first it is shown that $\chi > 0,$ so the two constraints are satisfied as equalities to determine the solution (i.e., it must be the case that $T_l^R = u_l + v_l(m_l^R) - \underline{U}$ and the monopolist gets no rent). Second, it is shown that if $\alpha = \alpha^{SB}$ and $m_l^R = m_l^L(\alpha^{SB})$ (recall that m_l^L is related to $m_l^{SB},$ which depends on α. Therefore, m_l^L also depends on α), the self-financing constraint is satisfied as an equality, so $m_l^L(\alpha)$ is the solution if $\alpha = \alpha^{SB}.$ Third, it is shown that profits are positive if $\alpha \in (\alpha^{SB}, \alpha^{FB})$ and $m_l^R = m_l^L(\alpha).$ Fourth, it is shown that for any $\alpha \in (\alpha^{SB}, \alpha^{FB}),$ if $m_l^R = m_l^H$ then profits are strictly negative. By the intermediate value theorem, for every $\alpha \in (\alpha^{SB}, \alpha^{FB})$ there will be an $m_l^R \in (m_l^L, m_l^H)$ such that profits are zero. To conclude, it is shown that the profit function is monotonous on $m_l^R,$ so for every α the particular m_l^R that makes profits equal to zero is unique.

1) From the first order condition (differentiating with respect to T_l^R) it follows $\phi = \chi + 1.$

We define γ

$$\gamma \equiv \widehat{m}'(m_l^R) = \widetilde{m}'(m_l^R) = -\frac{v_l''(m_l^R)(\overline{m}(m_l^R) - m_l^R)}{v_h'(\widetilde{m}) - v_l'(\widetilde{m})} > 0.$$

From the first order condition (when we differentiate with respect to m_l^R) it follows:

$$- \alpha\frac{\partial \widehat{T}_h(\bullet)}{\partial m_l^R} + (1-\alpha)v_l'(m_l^R) + \chi v_l'(m_l^R) +$$
$$\phi\left[\alpha\frac{\partial \widehat{T}_h(\bullet)}{\partial m_l^R} + (1-\alpha)\Big(v_l'\Big(\widehat{m}(m_l^R)\Big)\gamma - v_l'(m_l^R)\Big) - (1-\alpha)g\gamma\right] = 0$$

and substituting $\phi = \chi + 1$ it follows

$$\chi\alpha\frac{\partial \widehat{T}_h(\bullet)}{\partial m_l^R} + (\chi+1)(1-\alpha)\gamma\Big(v_l'\Big(\widehat{m}(m_l^R)\Big) - g\Big) = 0.$$

The second term of the left hand side is strictly positive: from part a) of the proof it is clear that, given $T_l^R = u_l + v_l(m_l^R) - \underline{U},$ then if $m_l^R \geq m_l^H$ and $\alpha < \alpha^{FB}$ the monopolist makes strictly negative profits, therefore $m_l^R < m_l^H$ is required, and by Proposition 2, $\widehat{m}(m_l^R) < m_l^*$. Therefore, χ must be positive (note that $\frac{\partial \widehat{T}_h(\cdot)}{\partial m_l^R} = -v_l'(m_l^R) + v_l''(m_l^R)(\overline{m}(m_l^R) - m_l^R) < 0$).

2) The firm's profit function for $m_l^R = m_l^L(\alpha^{SB})$, $\alpha = \alpha^{SB}$, taking into account Proposition 2 and choosing $T_l^R = u_l + v_l(m_l^R) - \underline{U}$ (to minimize the notation we use $m_l^L(\alpha^{SB}) = m_l^L$ and $m_l^{SB}(\alpha^{SB}) = m_l^{SB}$), is given by

$$\alpha^{SB}[v_l(m_l^L) + v_l'(m_l^L)(\overline{m}(m_l^L) - m_l^L) - v_h(\overline{m}(m_l^L)) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha^{SB})[v_l(\widetilde{m}(m_l^L)) - g\widetilde{m}(m_l^L)] + u_l - \underline{U} - A - h(1).$$

By definition, $\widetilde{m}(m_l^L) = m_l^{SB}$, so the above expression is

$$\alpha^{SB}[v_l(m_l^L) + v_l'(m_l^L)(\overline{m}(m_l^L) - m_l^L) - v_h(\overline{m}(m_l^L)) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha^{SB})[v_l(m_l^{SB}) - gm_l^{SB}] + u_l - \underline{U} - A - h(1).$$

By definition of $\widetilde{m}(\cdot)$

$$v_l(m_l^L) + v_l'(m_l^L)(\overline{m}(m_l^L) - m_l^L) - v_h(\overline{m}(m_l^L)) = v_l(\widetilde{m}(m_l^L)) - v_h(\widetilde{m}(m_l^L))$$

and, by the definition of m_l^L, $\widetilde{m}(m_l^L) = m_l^{SB}$. Therefore, the previous expression can be rewritten as

$$\alpha^{SB}[v_l(m_l^{SB}) - v_h(m_l^{SB}) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha^{SB})[v_l(m_l^{SB}) - gm_l^{SB}] + u_l - \underline{U} - A - h(1);$$

which, by the definition of α^{SB}, is equal to zero.

3) Consider the profit function for the monopolist (which is a function of α as m_l^R is set equal to $m_l^L(\alpha)$):

$$\alpha[v_l(m_l^{SB}(\alpha)) - v_h(m_l^{SB}(\alpha)) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha)[v_l(m_l^{SB}(\alpha)) - gm_l^{SB}(\alpha)] + u_l - \underline{U} - A - h(1).$$

The above expression is zero when $\alpha = \alpha^{SB}$. Differentiating this function with respect to

α, and considering that by the definition of m_l^{SB} it holds that $v_l'(m_l) - \alpha v_h'(m_l) = (1-\alpha)g$, this simplifies to

$$v_h(m_h^*) - v_h(m_l^{SB}(\alpha)) - g(m_h^* - m_l^{SB}(\alpha)) > 0,$$

so profits are strictly positive for any $\alpha > \alpha^{SB}$ if $m_l^R = m_l^L(\alpha)$.

4) Note first that $\widetilde{m}(m_l^H) = m_l^*$ does not depend on α. Then, for an $\alpha \in (\alpha^{SB}, \alpha^{FB})$ and considering Proposition 2, the profit function for the monopolist when $m_l^R = m_l^H$ and $T_l^R = u_l + v_l(m_l^H) - \underline{U}$ is

$$\alpha[v_l(m_l^H) + v_l'(m_l^H)(\overline{m}(m_l^H) - m_l^H) - v_h(\overline{m}(m_l^H)) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha)[v_l(\widetilde{m}(m_l^H)) - g\widetilde{m}(m_l^H)] + u_l - \underline{U} - A - h(1).$$

By definition of $\widetilde{m}(\cdot)$ and considering $\widetilde{m}(m_l^H) = m_l^*$ this reduces to

$$\alpha[v_l(m_l^*) - v_h(m_l^*) + v_h(m_h^*) - gm_h^*] + (1-\alpha)[v_l(m_l^*) - gm_l^*] + \\ u_l - \underline{U} - A - h(1).$$

Given the definition of α^{FB} and since we are considering $\alpha < \alpha^{FB}$ the above expression is strictly negative.

5) For a given $\alpha \in [\alpha^{SB}, \alpha^{FB})$ the profit function for the monopolist, considering Proposition 2 and $T_l^R = u_l + v_l(m_l^H) - \underline{U}$, is:

$$\alpha[v_l(m_l^R) + v_l'(m_l^R)(\overline{m}(m_l^R) - m_l^R) - v_h(\overline{m}(m_l^R)) + v_h(m_h^*) - gm_h^*] + \\ (1-\alpha)[v_l(\widetilde{m}(m_l^R)) - g\widetilde{m}(m_l^R)] + u_l - \underline{U} - A - h(1).$$

By the definition of $\widetilde{m}(\cdot)$ and rearranging terms this simplifies to

$$-\alpha v_h(\widetilde{m}(m_l^R)) + \alpha v_h(m_h^*) - \alpha g m_h^* + v_l(\widetilde{m}(m_l^R)) - (1-\alpha)g\widetilde{m}(m_l^R) + \\ u_l - \underline{U} - A - h(1)$$

If we differentiate with respect to m_l^R we obtain

$$[v_l'(\widetilde{m}(m_l^R)) - \alpha v_h'(\widetilde{m}(m_l^R)) - (1-\alpha)g]\widetilde{m}'(m_l^R).$$

The first factor is strictly negative for any $m_l^R > m_l^L,$ as $\widetilde{m}(m_l^R) > m_l^{SB}.$ The second factor, given by

$$\widetilde{m}'(m_l^R) = -\frac{v_l''(m_l^R)[\overline{m}(m_l^R) - m_l^R]}{v_h'(\widetilde{m}(m_l^R)) - v_l'(\widetilde{m}(m_l^R))},$$

is strictly positive, so the profit function is strictly monotone in m_l^R on the interval (m_l^L, m_l^H).

c) Note that, given Proposition 2, if $m_l^R < m_l^L,$ the alternative plans offered by the monopolist are the same as if $m_l^R = m_l^L.$ Therefore, following step b.2) above, profits for the firm are expressed by

$$\alpha[v_l(m_l^{SB}) - v_h(m_l^{SB}) + v_h(m_h^*) - gm_h^*] + (1-\alpha)[v_l(m_l^{SB}) - gm_l^{SB}] + u_l - \underline{U} - A - h(1).$$

It was shown that profits are zero if $\alpha = \alpha^{FB}$: the firm is making losses with type *l* customers that are just compensated by profits with *h*-types. Therefore, profits are strictly negative if $\alpha < \alpha^{FB}$.

Proof of Proposition 4: Since the monopolist can offer a different plan to each individual, his problem can be presented as two separate problems, one for each type of consumer.

$$\underset{\{m_i, T_i\}}{Max}\, T_i - gm_i$$

subject to

$$v_i(m_i) - T_i \geq v_i(m_i^R) - T_i^R; \quad i = h, l.$$

Note that the constraint must bind (otherwise we could raise T_i and increase the benefits). Substituting T_i in the objective function and differentiating with respect to m_i it is obtained that $v_i'(m_i) - g = 0,$ which implies $m_i = m_i^*.$ The constraint implies $T_i = v_i(m_i^*) - v_i(m_i^R) + T_i^R.$

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Figure 1

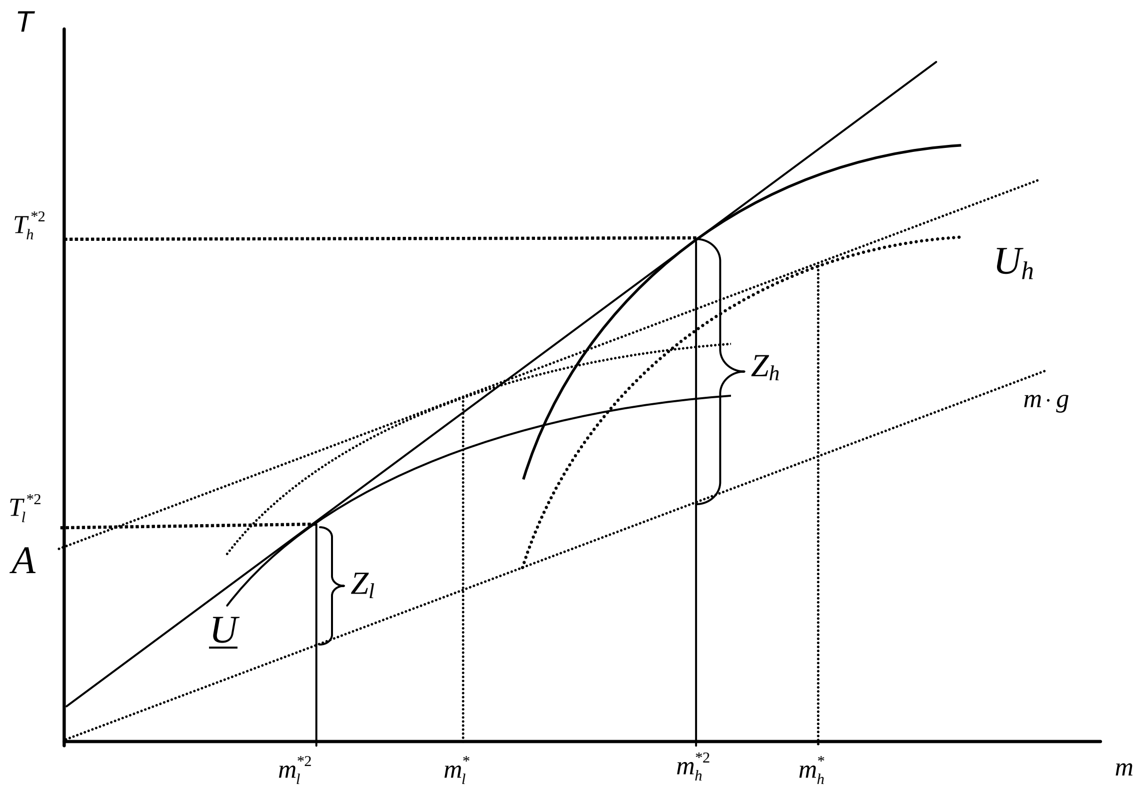

Figure 1: The optimal tariff scheme that satisfies universal access and self-financing constraints

Figure 2

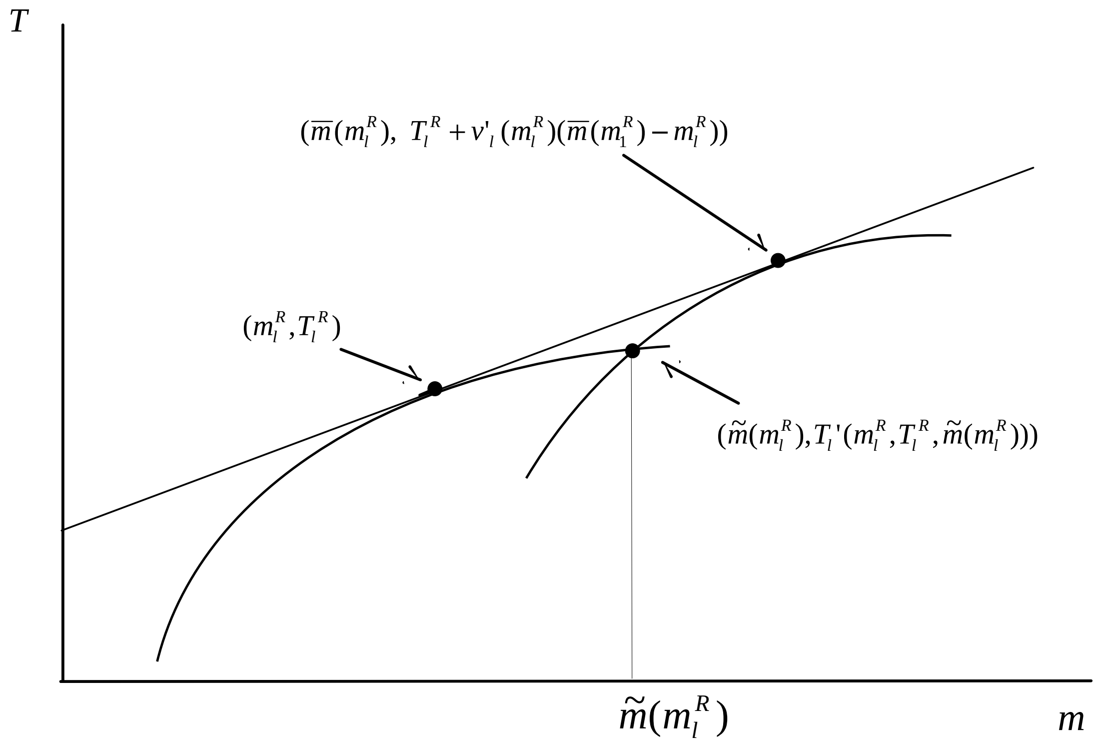

Figure 2: Definition of $\widetilde{m}\left(m_l^R\right)$ and $T_l'\left(m_l^R, T_l^R, m\right)$

Figure 3

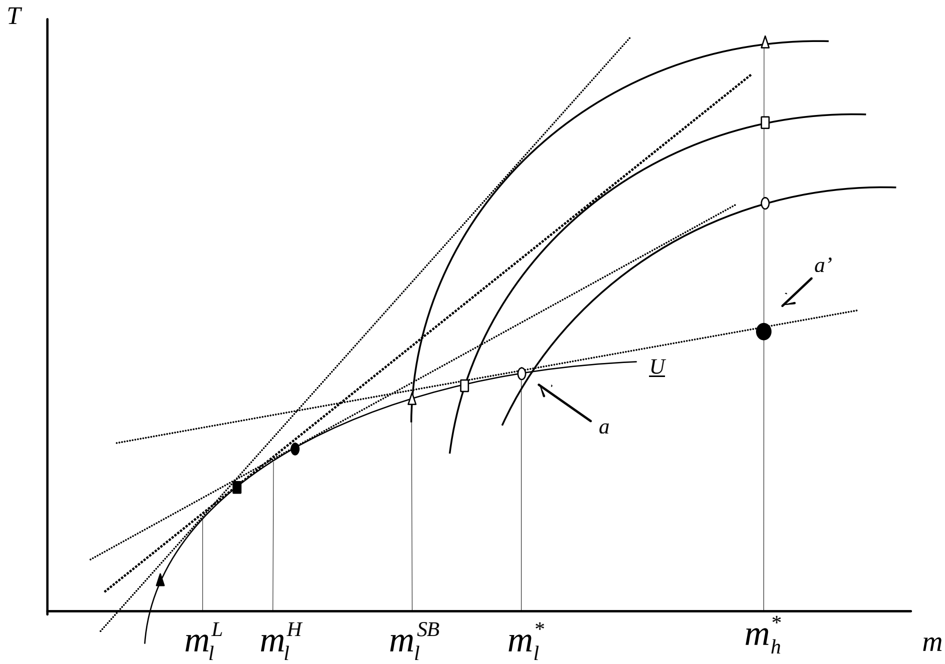

Figure 3: Firm's optimal alternative plans under partial flexibility