Merger Mechanisms*

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Abstract

A firm can merge with one of n potential partners. The owner of each firm has private information about both his firm's stand-alone value and a component of the synergies that would be realized by the merger involving his firm. We characterize incentive-efficient mechanisms in two cases. First, we assume that the value of any newly formed partnership is verifiable, hence transfers can be made contingent on the new information accruing after the merger. Second, we study the case of uncontingent rules. In the first case, we show that it is not optimal, in general, to redistribute shares of non-merging firms, and identify necessary and sufficient conditions for the implementability of efficient merger rules. In the second case, we show that the first-best can be obtained *i*) always, if the synergy values are privately known but the firms' stand-alone values are observable; *ii*) only with sufficiently large synergies, if the firms' stand-alone are privately known; and *iii*) never, if the set of feasible mechanisms is restricted to "auctions in shares".

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1 Introduction

We consider a model in which a firm can merge with one of many potential partners. If a merger takes place, the total value of the newly formed partnership can be higher or lower than the sum of the stand alone values. Efficiency dictates that the merger generating the highest synergy should take place, if and only if the added value is positive. We study the problem of designing merger mechanisms which implement efficient outcomes under the assumption that each firm's owner has private information about both its stand-alone value and the value of the synergies that would be realized if it merged.

We analyze two polar cases, depending on whether the value of the newly formed partnership is verifiable or not. In the first case, transfers of shares and payments can be made contingent on the realized value of the merger. In the second case only linear sharing rules, i.e. equity payments can be used.

In the verifiable case, we begin by showing that the surplus distribution among all agents depends only on how the shares of all non-merging firms are allocated, and not on the ownership structure of any new firm. Intuitively this is because, unlike the value of any new firm, the value of each non-merging firm is not verifiable. Therefore, while shares of new firms are equivalent to money, redistributing shares of any non-merging firm creates information rent for its owner.

This helps understand our next result, that is, efficient merger rules can be implemented if and only if they can be implemented with mechanisms which do not involve redistributions of non-merging firms. The problem of implementing efficient merger rules then boils down to a classic social choice problem for which it is sufficient to focus on Groves mechanisms. This allows us to identify necessary and sufficient conditions for the implementability of efficient merger rules.

For the non-verifiable case, we first analyze the special case in which firm 0 has no private information and zero stand-alone value (bankruptcy auctions). We show that if monetary transfer are ruled out, and the efficient merger rule p_* assigns different partners for different realizations of the firms types, the first-best cannot be implemented by any incentive compatible mechanism, unless the choice of the merger partner is trivial.

We then consider the problem of designing efficient merger mechanisms with a single potential partner in two cases: i) when each firm's stand-alone value is privately known, but it is common knowledge that the merger is always welfare enhancing, and ii) when the firms' stand alone values are known, but each firm observes a private signal about the merger's value. We show that a wedge exists between the first best and the incentive-efficient merger rule only in the second case.

Our paper is related to work by Hansen (1985), Cremer (1987), Samuelson (1987) and Rhodes-Kropf and Viswanathan (2000, 2003). Hansen (1985) has analyzed a model in which multiple firms compete to acquire a target firm, and shows that an (English) auction in which bidders offer fractions of the merged firm to the target generates a higher expected profit for the target than any auction in which only cash is used. Cremer (1987) has pointed out that with a combination of cash and shares the target can extract all gains from trade. Rhodes-Kropf and Viswanathan (2000) consider different securities and show that higher revenue is obtained from the security whose valuation is more sensitive to changes in the bidder's types.

All these results apply only to the case in which there is uncertainty only on the value of the joint asset, and not on the stand alone value of each firm. Samuelson (1987) argued that two-dimensional uncertainty would lead to different results. Rhodes-Kropf and Viswanathan (2003) consider a second price auction in a model of mergers where bidders have information about both the joint value of the asset and the stand alone value of their firm. None of the papers discussed above considers the design of merger mechanisms in environments where players have information about the joint value of the asset and their stand alone values.

Cramton, Gibbons and Klemperer (1987) have studied the problem of *dissolving* partnerships efficiently.[1] In their framework, efficiency requires that a single agent buys all his partners out of the company. Their main result is that the efficient outcome can be implemented even if each partner is privately informed about his willingness to pay for the entire firm, as long as the initial distribution of property rights is sufficiently balanced. In our framework, potential partners can pay with both money and stocks. Thus the assumption that firms are privately informed about the value of their own stocks adds a layer of adverse selection to the mechanism design problem.

[1] See also Jehiel and Pauzner (2002).

A central theme of our analysis will be identifying cases in which stocks are perfect substitutes for money and cases in which they are not.

The structure of the paper is as follows. Section 2 describes the model. In Section 3 we focus on the case in which the value of any new firm is *ex post* verifiable, so that any transfer of shares and money can be made contingent on the actual realization of the total value of the newly formed partnerships. In Section 4 we characterize efficient mechanisms under the assumption that no new information can be gathered once any merger takes place. Section 5 concludes. The appendix contains all proofs.

2 The Model

The owner of a firm, call it firm 0, faces a set $N := \{1, ..., n\}$ of potential merger partners. We will use the same index in the set $N_0 := \{0, 1, ..., n\}$ to denote both a firm and the agent who owns it. Each firm $j \in N_0$ has stand-alone value x_j. If firms 0 and $i \in N$ merge, the total net worth of the newly formed partnership is

$$r_i = x_0 + x_i + \gamma_i (w_i, v_i),$$

where w_i and v_i are random variables whose realizations are observed only by agent 0 and i respectively, and γ_i is the synergy function specifying the amount by which the total value r_i exceeds the sum of the two stand-alone values $x_0 + x_i$. The term $\gamma_i (w_i, v_i)$ can be interpreted as the expected synergy conditional on the pair (w_i, v_i). We assume that γ_i takes the separable form $\gamma_i (w_i, v_i) = g (w_i) + h (v_i)$, and after redefining the signals as $\widehat{w}_i := g (w_i)$ and $\widehat{v}_i := h (v_i)$ if necessary, we write without additional loss of generality

$$\gamma_i (w_i, v_i) = w_i + v_i.$$

To keep the analysis simple, we assume that any merger not involving firm 0, or involving more than two firms, generates negative synergies.

The random variables w_i, v_i, $i \in N$, and x_j, $j \in N_0$, are distributed independently, with respective supports $[\underline{w}_i, \overline{w}_i]$, $[\underline{v}_i, \overline{v}_i]$ and $[\underline{x}_j, \overline{x}_j]$, all closed intervals of real numbers. Agent 0's *type*

$\theta_0 := (x_0, w_1, ..., w_n)$ is drawn from the c.d.f. F_0 with support

$$\Theta_0 := [\underline{x}_0, \overline{x}_0] \times [\underline{w}_1, \overline{w}_1] \times ... \times [\underline{w}_n, \overline{w}_n].$$

For each $i \in N$, agent i's type $\theta_i := (x_i, v_i)$ is drawn from the c.d.f. F_i with support

$$\Theta_i := [\underline{x}_i, \overline{x}_i] \times [\underline{v}_i, \overline{v}_i].$$

As usual, we let symbols without subscripts denote variables pertaining to all agents, e.g. $\theta := (\theta_0, \theta_1, ..., \theta_n)$, $\Theta := \Theta_0 \times \Theta_1 \times ... \times \Theta_n$, and $F(\theta) \equiv \prod_{i \in N_0} F_i(\theta_i)$; and symbols with the subscript $-i$ denote variables pertaining to all agents in $N_0 \setminus \{i\}$, e.g. $\theta_{-i} := (\theta_0, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$, $\Theta_{-i} := \Theta_0 \times ... \times \Theta_{i-1} \times \Theta_{i+1} \times ... \times \Theta_n$, and $F_{-i}(\theta_{-i}) \equiv \prod_{j \in N_0 \setminus \{i\}} F_j(\theta_j)$.

A feasible outcome consists of the following three objects:

1. A (stochastic) *merger rule* $p = (p_0, p_1, \ldots, p_n) \in \Delta$, where Δ denotes the n-dimensional simplex, p_0 is the probability of no merger taking place, and p_i, $i \in N$, the probability that firms 0 and i merge.

2. A collection of *sharing rules* $s = \left(s^0, s^1, \ldots, s^n\right)$, where each $s^i = \left[s^i_{jk}\right]$ is a matrix of dimension $(n+1) \times (n+1)$ with s^i_{jk} denoting the fraction of firm j's total value going to agent k, when firm 0 and i merge ($i = 0$ denoting the case of no merger). For each $i, j \in N_0$, we have $\sum_{k \in N_0} s^i_{jk} = 1$, i.e. $s^i_{j\cdot} := \left(s^i_{j0}, s^i_{j1}, \ldots, s^i_{jn}\right) \in \Delta$. We adopt the convention that, when firm 0 and i merge, the new firm takes the name of 'firm 0', while firm i disappears. Thus s^i_{0k} denotes the fraction of the total value r_i going to agent k, and we set $s^i_{ik} = 0$, $\forall i \in N$ and $k \in N_0$. The set of feasible sharing rules can be defined as

$$S = \left\{s^i_{j\cdot} \in \Delta,\ i, j \in N_0 \mid s^1_{1\cdot} = ... = s^n_{n\cdot} = 0, \right\}$$

3. A collection of *monetary transfers* $t = \left(t^0, t^1, \ldots, t^n\right)$, where each $t^i = \left(t^i_0, t^i_1, \ldots, t^i_n\right) \in \mathbb{R}^{n+1}$, with t^i_j denoting the monetary transfer[2] to agent j when firm 0 and firm i merge, with $i = 0$

[2] Since all agents are risk neutral, restricting attention to deterministic monetary transfers is without loss of generality.

again denoting the case of no merger. We will impose budget balance, allowing for the strict inequality.[3] The set of feasible monetary transfers can be defined as

$$T := \left\{ t_j^i,\ i,j \in N_0 \mid \sum_{j \in N_0} t_j^i \leq 0\ \forall i \in N_0 \right\}$$

A feasible *mechanism* is a mapping $\mu : \Theta \to \Delta \times S \times T$. With slight abuse of notation and terminology, the mapping μ will be written as $\mu = (p, s, t)$, and the functions $p : \Theta \to \Delta$, $s : \Theta \to S$ and $t : \Theta \to T$ will be called the *merger rule*, the *sharing rule* and the *transfer rule*, respectively.

The focus will be on *implementing* a merger rule p. That is, given p, we will check for the existence of a sharing rule s and a transfer rule t such that the mechanism $\mu = (p, s, t)$ is *incentive-feasible*; that is, in the revelation game induced by the mechanism μ, truth-telling is individually rationality for each agent and forms a Bayesian-Nash equilibrium. Note that the budget balance constraint is embedded in the definition of feasible monetary transfers.

In the game induced by the mechanism (p, s, t), the expected utility of agent $j \in N_0$, when the true type profile is $\theta \in \Theta$ and the agents report $\theta' \in \Theta$, is

$$\begin{aligned} \widetilde{u}_j\left(\theta', \theta\right) &= p_0\left(\theta'\right)\left(\sum_{k \in N_0} s_{kj}^0\left(\theta'\right) x_k + t_j^0\left(\theta'\right)\right) \\ &\quad + \sum_{i \in N} p_i\left(\theta'\right)\left(s_{0j}^i\left(\theta'\right) r_i + \sum_{k \in N \setminus \{i\}} s_{kj}^i\left(\theta'\right) x_k + t_j^i\left(\theta'\right)\right); \end{aligned} \tag{1}$$

and the *interim* expected surplus of type $\theta_j \in \Theta_j$ of agent j, when he reports $\theta_j' \in \Theta_j$ and all other agents report their true types, is

$$\widetilde{U}_j\left(\theta_j', \theta_j\right) = \int_{\Theta_{-j}} \widetilde{u}_j\left(\theta_j', y, \theta_j, y\right)\ dF_{-j}(y).$$

A mechanism satisfies *individual rationality* if $\widetilde{U}_j\left(\theta_j, \theta_j\right) \geq x_j,\ \forall \theta_j \in \Theta_j,\ j \in N_0$. Redefining agent j's utility function as

$$\widehat{u}_j\left(\theta_j', \theta_j\right) = \widetilde{u}_j\left(\theta_j', \theta_j\right) - x_j, \tag{2}$$

[3] If the sum of the transfers is strictly negative the firms would be better off renegotiating the contract. However, it is easy to make the mechanism renegotiation proof prescribing that the money is given to an external party.

we can rewrite the IR constraints as

$$\widehat{U}_j\left(\theta_j, \theta_j\right) \geq 0, \;\; \forall \theta_j \in \Theta_j, \;\; j \in N_0, \tag{IR}$$

where

$$\widehat{U}_j\left(\theta_j', \theta_j\right) \equiv \int_{\Theta_{-j}} \widehat{u}_j\left(\theta_j', y, \theta_j, y\right) \; dF_{-j}\left(y\right). \tag{3}$$

Since we have just subtracted a constant to agent j's utility function, a mechanism induces participation and truthtelling as an equilibrium when the payoff functions are $\widehat{u}_j$ if and only if it does so when the functions are $\widetilde{u}_j$. From now on we will work with the 'normalized' utility functions $\widehat{u}_j$ given in (2).

Whether a certain mechanism satisfies incentive compatibility depends on the information available to the agents and to the mechanism designer. In particular, the set of incentive-feasible mechanisms differs depending on the information that becomes available after a merger. We will analyze the following two polar cases:

1. **No after-merger information.** In this case the mechanism design problem is the classic, static one. Invoking the revelation principle, we will restrict attention without loss of generality to incentive compatible and individually rational direct revelation mechanisms.

2. **Verifiable merger values.** In this case the total value of any newly formed firm is verifiable. The designer can use this information, together with agents' reports about their types, to determine the outcome. We will assume that no information about the non-merging firms becomes available after the agents' reports.

We will begin with the verifiable case.

3 Verifiable Merger Values

In this section we assume that, if firms 0 and i merge, the total value of the new firm, r_i becomes verifiable *ex post*. Given a mechanism $\mu = (p, s, t)$, let $P_j^i\left(\theta_j\right)$ denote the probability that firm 0 and firm i merge, conditional on agent j reporting $\theta_j \in \Theta_j$, and all other agents reporting their true types; that is

$$P_j^i(\theta_j) \equiv \int_{\Theta_{-i}} p_j(\theta_j, y)\, dF_{-i}(y), \qquad \theta_j \in \Theta_j, \quad i, j \in N_0. \tag{4}$$

with $i = 0$ denoting no merger. Also, for each $i \in N$, define

$$\Theta_i^{No} := \left\{ \theta_i \in \Theta_i \middle| P_i^i(\theta_i) = 0 \right\}, \qquad \Theta_i^+ := \Theta_i \backslash \Theta_i^{No};$$

and

$$\Theta_0^{No}(i) := \left\{ \theta_0 \in \Theta_0 \middle| P_0^i(\theta_0) = 0 \right\}, \qquad \Theta_0^+(i) := \Theta_0 \backslash \Theta_0^{No}(i).$$

In words, Θ_i^{No} is the set of reports that agent i can make for which there is no chance of his firm merging. Similarly, $\Theta_0^{No}(i)$ is the set of reports that agent 0 can make for which the merger with firm i has no chance. These sets can be non-empty. For example Θ_i^{No} is non-empty if the merger rule p is efficient, and $v_i + \overline{w}_i < 0$ for some v_i.

Since the value of any new firm r_i is verifiable, the mechanism can dictate that agents 0 and i pay large fines whenever r_i turns out to be different from the value implied by the agents' reports, i.e. whenever their reports θ_0' and θ_i' are such that $x_0' + x_i' + w_i' + v_i' \neq r_i$. Since in equilibrium agent i expects agent 0 to report his true type, it cannot be optimal for him to report (x_i', v_i') such that $x_i' + v_i' \neq x_i + v_i$, unless $(x_i', v_i') \in \Theta_i^{No}$ (in which case the lie would go undetected.) Thus the incentive compatibility constraints for agent $i \in N$ can be written as

$$\widehat{U}_i(\theta_i, \theta_i) \geq \widehat{U}_i\left(\theta_i', \theta_i\right), \quad \forall \theta_i' \in \mathcal{L}_i(\tau_i) \cup \Theta_i^{No}, \ \ \forall \theta_i \in \Theta_i, \tag{5}$$

where $\tau_i := x_i + v_i$ and $\mathcal{L}_i(\tau_i) := \{(x_i', v_i') \in \Theta_i | x_i' + v_i' = \tau_i\}$.

Similarly, since agent 0 expects all other agents to report their true types, it cannot be optimal for him to report θ_0' such that $x_0' + w_i' \neq x_0 + w_i$, unless $\theta_0' \in \Theta_0^{No}(i)$. Thus his incentive compatibility constraints can be written as

$$\widehat{U}_0(\theta_0, \theta_0) \geq \widehat{U}_0\left(\theta_0', \theta_0\right), \quad \forall \theta_0' \in \cap_{i \in N}\left(\mathcal{L}_0(\sigma_i) \cup \Theta_0^{No}(i)\right), \ \ \forall \theta_0 \in \Theta_0, \tag{6}$$

where $\sigma_i := x_0 + w_i$ and $\mathcal{L}_0^i(\sigma_i) := \left\{\theta_0' \in \Theta_0 | x_0' + w_i' = \sigma_i\right\}$, $i \in N$.[4]

In trying to implement a given merger rule p, the designer can use both shares and money to satisfy the incentive compatibility and the individual rationality constraints. It is natural to ask whether it is possible to restrict the class of mechanisms that we need to consider, given the multiplicity of ways in which payments can be made. Our first lemma establishes that neither the implementability of a given merger rule, nor the surplus distribution among the $n+1$ agents depend on how the shares of any new firm are allocated. Intuitively, this is because the value of the new firm is verifiable, and thus its shares are equivalent to money.

Lemma 1 *Suppose that the mechanism (p, s, t) is incentive-feasible, i.e. it satisfies budget balance, individual rationality and incentive compatibility. Then, for any sharing rule $\widetilde{s}$ in which the shares of all all non-merging firms are allocated as in s, i.e. $\widetilde{s}^0_{j\cdot} = s^0_{j\cdot}$ $\forall j \in N_0$, and $\widetilde{s}^i_{j\cdot} = s^i_{j\cdot}$ $\forall i, j \in N$, and the ownership structure of any new firm $\widetilde{s}^i_{0\cdot}$, $i \in N$, is arbitrary in Δ, there exists a feasible transfer rule $\widetilde{t}$ such that the mechanism $\left(p, \widetilde{s}, \widetilde{t}\right)$ is also incentive-feasible, and generates the same utility function $\widehat{u}_j$, defined in (1), for each $j \in N_0$.*

Lemma 1 allows us to focus, without loss of generality, to mechanisms in which agent 0 receives all shares of the merged firm, i.e. $s^i_{00}(\theta) = 1$, $\forall \theta \in \Theta$, $i \in N$. In this case the agents' utility functions can be written as

$$\begin{aligned} \widehat{u}_0(\theta', \theta) &= p_0(\theta')\left(t_0^0(\theta') + \sum_{k \in N} s^0_{k0}(\theta') x_k\right) + \sum_{i \in N} p_i(\theta')\left(t_0^i(\theta') + r_i + \sum_{k \in N \setminus \{i\}} s^i_{k0}(\theta') x_k\right) \\ &\quad - \left[1 - p_0(\theta') s^0_{00}(\theta')\right] x_0, \end{aligned} \tag{7}$$

[4] If r_i were observable to the merging owners, but not verifiable, it would still be possible to elicit truthfully the total value r_i, without having to pay any information rent. Formally, we would be using a two-stage direct revelation mechanism as in Mezzetti (2003).

and

$$\widehat{u}_i\left(\theta',\theta\right)=p_0\left(\theta'\right)\left(t_i^0\left(\theta'\right)+\sum_{k\in N_0\backslash\{i\}}s_{ki}^0\left(\theta'\right)x_k\right)+\sum_{j\in N\backslash\{i\}}p_j\left(\theta'\right)\left(t_i^j\left(\theta'\right)+\sum_{k\in N\backslash\{j,i\}}s_{ki}^j\left(\theta'\right)x_k\right)$$

$$+p_i\left(\theta'\right)\left(t_i^i\left(\theta'\right)+\sum_{k\in N\backslash\{i\}}s_{ki}^i\left(\theta'\right)x_k\right)-\left[1-\sum_{j\in N_0\backslash\{i\}}p_j\left(\theta'\right)s_{ii}^j\left(\theta'\right)\right]x_i,\quad i\in N. \tag{8}$$

Do the equivalence properties established in Lemma 1 also apply to the shares of each non-merging firm? In particular, is there any loss of generality in restricting attention to mechanisms in which the owner of each non-merging firm retains all its shares? If we could show that any implementable merger rule p can be implemented with a mechanism (p,s,t) where the sharing rule satisfies the "*Losers Untouched*" (LU) property, i.e.

$$s_{kk}^0\left(\theta\right)=s_{ii}^j\left(\theta\right)=1,\ \forall i,j,k\in N_0,\ i\neq j,\ \forall\theta\in\Theta, \tag{9}$$

then the extra flexibility given by the possibility of manipulating shares through s would be of no value; and we could narrow down the design problem, without loss of generality, to the specification of monetary transfers.

It turns out that neither equivalence property extends to the shares of the non-merging firms. That is, both the set of implementable merger rules and the surplus distribution among the agents depend on how the shares of the non-merging firms are allocated. Intuitively, this is because the value of each non-merging firm is not verifiable ex post, thus its shares are not equivalent to money.

However, we will show that a merger rule is *LU-implementable*, i.e. can be implemented with a mechanism whose sharing rule has the LU property, if and only if it satisfies a set of monotonicity conditions, spelled out in (14) below; and it will be straightforward to check that these monotonicity conditions are satisfied by any ex-post efficient merger rule.

With slight abuse of notation, we redefine the agents' types as $\theta_i:=(x_i,\tau_i)$ where $\tau_i:=x_i+v_i$, and $\theta_0:=(x_0,\sigma_1,...,\sigma_n)$ where $\sigma_i:=x_0+w_i$, $\forall i\in N$. For each $i\in N$, and each $\tau_i\in\left[\underline{x}_i+\underline{v}_i,\overline{x}_i+\overline{v}_i\right]$, we let $\left[\underline{x}^{\tau_i},\overline{x}^{\tau_i}\right]$ denote the interval of values of x_i which are consistent with τ_i, i.e.

$$\left[\underline{x}^{\tau_i}, \overline{x}^{\tau_i}\right] := \left\{x_i \in \left[\underline{x}_i, \overline{x}_i\right] \mid \exists v_i \in \left[\underline{v}_i, \overline{v}_i\right] \text{ such that } x_i + v_i = \tau_i\right\},$$

and define

$$U_i^{\tau_i}(\zeta) \equiv \widehat{U}_i\left(\left(\zeta, \tau_i\right), \left(\zeta, \tau_i\right)\right), \quad \zeta \in \left[\underline{x}^{\tau_i}, \overline{x}^{\tau_i}\right],$$

and

$$B_i^{\tau_i}(\zeta) \equiv 1 - E_{\theta_{-i}}\left[\sum_{j \in N_0 \setminus \{i\}} p_j\left(\left(\zeta, \tau_i\right), \theta_{-i}\right) \ s_{ii}^j\left(\left(\zeta, \tau_i\right), \theta_{-i}\right)\right], \quad \zeta \in \left[\underline{x}^{\tau_i}, \overline{x}^{\tau_i}\right]. \tag{10}$$

Similarly, for each $\sigma = (\sigma_1, ..., \sigma_n) \in \times_{i \in N}\left[\underline{x}_0 + \underline{w}_i, \overline{x}_0 + \overline{w}_i\right]$, we let $\left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right]$ denote the interval of values of x_0 which are consistent with σ, i.e.

$$\left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right] := \left\{x_0 \in \left[\underline{x}_0, \overline{x}_0\right] \mid \exists\left(w_1, ..., w_n\right) \in \times_{i \in N}\left[\underline{w}_i, \overline{w}_i\right] \text{ such that } x_0 + w_i = \sigma_i, \ i \in N\right\},$$

and define

$$U_0^\sigma(\zeta) \equiv \widehat{U}_0\left(\left(\zeta, \sigma\right), \left(\zeta, \sigma\right)\right), \quad \zeta \in \left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right],$$

and

$$B_0^\sigma(\zeta) \equiv 1 - E_{\theta_{-0}}\left[p_0\left(\left(\zeta, \sigma\right), \theta_{-0}\right) \ s_{00}^0\left(\left(\zeta, \sigma\right), \theta_{-0}\right)\right], \quad \zeta \in \left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right]. \tag{11}$$

The next lemma provides the standard characterization of the set of incentive compatibility constraints in (5) and (6) corresponding to each interval $\left[\underline{x}^{\tau_i}, \overline{x}^{\tau_i}\right]$, $\tau_i \in \left[\underline{x}_i + \underline{v}_i, \overline{x}_i + \overline{v}_i\right]$, $i \in N$, and $\left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right]$, $\sigma \in \times_{i \in N}\left[\underline{x}_0 + \underline{w}_i, \overline{x}_0 + \overline{w}_i\right]$. The proof is provided in the appendix, for the sake of completeness.

Lemma 2 *If the inequalities in (6) are satisfied, then B_0^σ is non-increasing and*

$$U_0^\sigma(x_0) = U_0^\sigma\left(\overline{x}_0^\sigma\right) + \int_{x_0}^{\overline{x}_0^\sigma} B_0^\sigma(\zeta)\, d\zeta, \quad \forall x_0 \in \left[\underline{x}_0^\sigma, \overline{x}_0^\sigma\right]. \tag{12}$$

For each $i \in N$, if the inequalities in (5) hold, then $B_i^{\tau_i}$ is non-increasing, and

$$U_i^{\tau_i}(x_i) = U_i^{\tau_i}\left(\overline{x}_i^{\tau_i}\right) + \int_{x_i}^{\overline{x}_i^{\tau_i}} B_i^{\tau_i}(\zeta)\, d\zeta, \quad \forall x_i \in \left[\underline{x}^{\tau_i}, \overline{x}^{\tau_i}\right]. \tag{13}$$

Lemma 2 has the following two immediate implications. First, the "envelope condition" in (13) shows that the information rent of each type $(x_i, \tau_i) \in \Theta_i^+$ of agent i increases with the fraction of firm i that is taken away from all types $(x_i', \tau_i) \in \Theta_i$, with $x_i' > x_i$, when firm i does not merge. Similarly, (12) shows that agent 0's ex-ante expected information rent decreases with the fraction of his firm that he retains when no merger takes place.

Second, if we impose the LU property in (11) and (10) we obtain

$$B_0^{\sigma}(\cdot) = 1 - P_0^0(\cdot, \sigma) \quad \text{and} \quad B_i^{\tau_i}(\cdot) = P_i^i(\cdot, \tau_i).$$

Since both B_0^{σ} and $B_i^{\tau_i}$ must be non-increasing, we have that the following monotonicity conditions are *necessary* for any (implementable) merger rule p to be also LU-implementable:

$$\begin{aligned} &\forall \sigma \in \times_{i \in N} [\underline{x}_0 + \underline{w}_i, \overline{x}_0 + \overline{w}_i], \quad P_0^0(\cdot, \sigma) \text{ is non-increasing} \\ &\quad \text{and} \\ &\forall i \in N,\ \tau_i \in [\underline{x}_i + \underline{v}_i, \overline{x}_i + \overline{v}_i], \quad P_i^i(\cdot, \tau_i) \text{ is non-decreasing.} \end{aligned} \tag{14}$$

The next proposition establishes that the conditions in (14) are also *sufficient*. Intuitively, this is because, as (12) and (13) show, applying the LU property can only reduce the information rent of any type of any agent, and thus can only makes it easier to satisfy both the budget balance and the individual rationality constraints.

Proposition 1 *Any implementable merger rule p which satisfies the monotonicity conditions in (14) is also LU-implementable.*

Consider now the *efficient* merger rule $p^* := (p_0^*, ..., p_n^*)$, defined[5] as

$$p_i^*(\theta) = \begin{cases} 1, & \text{if } v_i + w_i > \max\{0, v_j + w_j;\ j \in N \setminus \{i\}\}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

It is straightforward to check that p^* satisfies the monotonicity conditions in (14). Thus we have the following corollary to Proposition 1.

[5] Ties, being zero probability events, are ignored.

Corollary 1 *If the efficient rule p^* is implementable, it is also LU-implementable.*

The next subsection is devoted to identifying necessary and sufficient conditions under which the efficient rule p^* is implementable. In light of Corollary 1 we will be able to restrict attention, without loss of generality, to LU-implementable merger rules. Thus the design of the sharing rule s will be limited to determining the ownership structure of any new firm. As we will see, this can be done so that the residual problem of finding a transfer function for which p^* is implementable becomes a special case of a classic social choice problem studied in greatest generality in Williams [17], and Krishna and Perry [6].

3.1 The Feasibility of Efficient Merger Rules

Let the variable $a \in N_0$ denote any merger outcome. The social surplus generated by decision a, when the type profile is θ, can be written as

$$S(a,\theta) \equiv \sum_{j \in N_0} \widetilde{v}_j(a,\theta_i),$$

where

$$\widetilde{v}_0(a,\theta_0) = \begin{cases} 0, & \text{if} \quad a = 0, \\ \\ w_i, & \text{if} \quad a = i; \end{cases} \qquad \widetilde{v}_i(a,\theta_i) = \begin{cases} 0, & \text{if} \quad a \neq i, \\ \\ v_i, & \text{if} \quad a = i, \end{cases} \qquad i \in N.$$

Since $S(0,\theta) = 0$ and $S(i,\theta) = v_i + w_i$, we have that the efficient merger rule p^* defined in (15) satisfies

$$p^*(\theta) \in \arg \max_{(\pi_0,\dots,\pi_n) \in \Delta} \sum_{j \in N_0} S(i,\theta) \ \pi_i.$$

Since the merger values are verifiable, we can now find a sharing rule for which the surplus obtained by firm j from decision a, net of any cash transfer, is exactly $v_j(a,\theta)$. This is obtained by applying the LU property and setting $s^i_{00}(\theta) = \frac{\sigma_i}{\sigma_i+\tau_i}$ and $s^i_{0i}(\theta) = \frac{\tau_i}{\sigma_i+\tau_i}$ for all $\theta \in \Theta$. With this sharing rule in place, we have

$$\widehat{u}_j(a,\theta) = \widetilde{v}_j(a,\theta) + t_j, \ \ j \in N_0 \tag{16}$$

and the problem of finding a transfer rule for which p^* is implementable becomes essentially a special case of the general model studied in Williams [17]. To apply Williams' main result let $a^*(\theta) = i$ iff $p_i^*(\theta) = 1$, let $Z_j(\theta_j)$ denote the expected social surplus conditional on agent j having type θ_j, i.e.

$$Z_j(\theta_j) \equiv E_{\theta_{-i}}\left[\sum_{k \in N_0} \widetilde{v}_k\left(a^*(\theta_j, \theta_{-j}), \theta_i\right)\right],$$

and define

$$\underline{Z}_j := \inf_{\theta_j \in \Theta_j} Z_j(\theta_j).$$

Adapting Theorem 3 in Williams [17] to our setting, we are able to establish the following result.

Proposition 2 *The efficient merger rule p^* is implementable if and only if*

$$nE_\theta\left[\sum_{i \in N_0} \widetilde{v}_i\left(a^*(\theta), \theta_i\right)\right] \leq \sum_{i \in N_0} \underline{Z}_i. \tag{17}$$

Proposition (2) implies that, under a mild symmetry assumption, the ex-post efficient rule is implementable if and only if it is common knowledge that a merger should always take place, i.e. $p_0^*(\theta) = 0$, $\forall \theta \in \Theta$.

Corollary 2 *Suppose that all variables v_i, $i \in N$, have c.d.f. Φ_v, and support $[\underline{v}, \overline{v}]$, and all variables w_i, $i \in N$ have c.d.f. Φ_w, and support $[\underline{w}, \overline{w}]$. Then p^* is not implementable if $\overline{v} + \underline{w} < 0$.*

In the case with only one potential partner, i.e. $n = 1$, we have the following corollary to Proposition 2.

Corollary 3 *If $n = 1$, the efficient merger rule is implementable if and only if*

$$E\left[\max\{v + w, 0\}\right] \leq E\left[\max\{\underline{v} + w, 0\}\right] + E\left[\max\{v + \underline{w}, 0\}\right] \tag{18}$$

As implied by Corollary 2, if $\underline{v} + \overline{w} < 0$, efficiency is impossible. If instead $\underline{v} + \underline{w} \geq 0$, then condition (18) is satisfied for any distribution, since

$$E[v + w] \leq E[\underline{v} + w] + E[v + \underline{w}] \qquad \Longleftrightarrow \qquad \underline{v} + \underline{w} \geq 0.$$

Thus in this case the efficient merger rule $p^* (\theta) \equiv 1$ can always be implemented. The next proposition shows that this can be done without monetary transfers.

Proposition 3 *Suppose that $n = 1$ and $\underline{v} + \underline{w} \geq 0$. Then p^* is implementable with a mechanism that does not use cash.*

Intuitively, since the *ex post* value of the merger is verifiable, and the merger occurs with probability 1, it is possible to satisfy the individual rationality constraints of both agents using only the shares of the new firm. Also, the sharing rule depends only on the values of σ and τ_1, which are effectively observable ex-post; thus incentive compatibility is trivially satisfied.

4 Unverifiable Merger Values

In this section we assume that the total value of any new merged firm is not verifiable, hence redistributions of equity and monetary transfers can only be conditioned on the agents' reports about their types. We begin by looking at a special case which has been studied extensively in the literature.

4.1 The Inefficiency of Auctions in Shares

Suppose that firm 0 has no private information, i.e. θ_0 is known, and without additional loss of generality[6] set $x_0 = w_1 = ... = w_n = 0$, so that the total value of the new firm created by the merger of firms 0 and i is $x_i + v_i$. Consider the class of all mechanisms which can be described as "auctions in shares"; that is, monetary transfers are ruled out and all shares of each non-merging firm are retained by its owner (i.e. the LU property applies). As in Rhodes-Kropf and Viswanathan [12], we can interpret these mechanisms as a bankruptcy auctions, in which firm 0 is worth nothing, unless taken over by another firm.

This case was first studied by Hansen (1985), who also assumed that $x_i = 0$ for each $i \in N$. Hansen showed that an auction in shares can implement the efficient merger rule and generate a

[6] Since the variables $x_0, w_1, \ldots, w_n$ are known, we can redefine the synergy generated by the merger with of firm $i \in N$ as $v_i' = x_0 + v_i + w_i$.

higher expected revenue than any auction with cash. Related papers include Rhodes-Kropf and Viswanathan [13] which consider bankruptcy auctions in an environment where there is information about the joint value, and Rhodes-Kropf and Viswanathan [14] which analyzes a second price auction in which bidder i obtains $x_i + v_i$ if it merges and x_i otherwise.

We now show that Hansen's result hinges crucially on the assumption that $x_i = 0$ for each $i \in N$. The main result in this section is that, if each agent $i \in N$ has private information about both the stand-alone value of his firm x_i and the synergy term v_i, the efficient merger rule can be implemented with an auction in shares only if the choice of the merging partner is trivial.

To see this, consider any mechanism $(p, s, 0)$, where s satisfies the LU property. The interim expected surplus of type θ_i of agent i, when he reports θ_i' (and the others report their true types) is

$$\widehat{U}_i\left(\theta_i',\, \theta_i\right) = \left(x_i + v_i\right) B_i\left(\theta_i'\right) - x_i P_i\left(\theta_i'\right),$$

where

$$B_i\left(\theta_i'\right) \equiv E_{\theta_{-i}}\left[p_i\left(\theta_i', \theta_{-i}\right) s_{0i}^i\left(\theta_i', \theta_{-i}\right)\right], \tag{19}$$

and

$$P_i\left(\theta_i'\right) \equiv E_{\theta_{-i}}\left[p_i\left(\theta_i', \theta_{-i}\right)\right]. \tag{20}$$

First note that, since monetary transfers are ruled out, any two types $\theta_i = (x_i, v_i)$ and $\widetilde{\theta}_i = (\widetilde{x}_i, \widetilde{v}_i)$ such that $(\widetilde{x}_i, \widetilde{v}_i) = (kx_i, kv_i)$ for some $k > 0$ have identical preferences over the set of all feasible outcomes. Indeed, their utility functions are linear transformations of each other, i.e.

$$U_i\left(\theta_i',\, k\theta_i\right) = kU_i\left(\theta_i',\, \theta_i\right), \quad \forall \theta_i' \in \Theta_i.$$

As the next lemma establishes, this implies that, in any incentive compatible auction in shares, "proportional" types cannot be distinguished. That is, for each $i \in N$, both B_i and P_i must be constant along any line

$$R_i\left(\rho_i\right) := \left\{\left(x_i', v_i'\right) \in \Theta_i \,\middle|\, \frac{v_i'}{x_i'} = \rho_i\right\}, \quad \rho_i \in \left[\overline{x}_i + \underline{v}_i, \underline{x}_i + \overline{v}_i\right].$$

Lemma 3 *An 'auction in shares' satisfies incentive compatibility for agent i only if $B_i\left(\theta_i\right) = B_i\left(\theta_i'\right)$ and $P_i\left(\theta_i\right) = P_i\left(\theta_i'\right)$ $\forall \theta_i, \theta_i' \in \Theta_i$ such that $\theta_i' = k\theta_i$ for some $k > 0$.*

We are now ready to establish the impossibility result. Let P_i^* denote the interim probability function, defined as in (20), corresponding to the efficient merger rule p^*, i.e.

$$P_i^*(v_i) \equiv \int_{\Theta_{-i}} p_i^*(\theta_i, y)\, dF_{-i}(y), \quad i \in N.$$

Proposition 4 *If $P_i^*(\underline{v}_i) < P_i^*(\overline{v}_i)$ for some $i \in N$, then any merger rule p which is implementable with an auction in shares differs from p^* on a set of positive measure.*

The proof of Proposition is based on the idea that, while each function P_i, $i \in N$, corresponding to any incentive compatible auction in shares must be constant along any lines of proportional types, P_i^* depends only on v_i. Therefore we can have $P_i = P_i^*$ only if P_i^* is constant, i.e. when it is efficient for the same two firms to merge for any realization of the agent's type. Whenever the choice of the merging partner is non-trivial, efficiency cannot be attained.

4.2 Commonly Known Stand-Alone Values

In this subsection we show that, if all stand-alone values $x_0, x_1, \ldots, x_n$ are known, the efficient merger rule can always be implemented, without monetary transfers.[7] In other words, there exists a mechanism $(p^*, s, 0)$ which is incentive-feasible. This indicates that the real problem for efficiency comes from private information on stand-alone values, rather than on synergies.

The mechanism $(p^*, s, 0)$ can be described as follows. If, according to the agents' reports, no merger generates positive synergies, then nothing happens, i.e. each owner retains full ownership of his firm. Otherwise, the merger generating the highest synergy takes place, and agent $j \in N_0$ receives fraction $\frac{x_j}{\sum_{k \in N_0} x_k}$ of the new firm, as well as of each non-merging firm. Formally,

$$s_{jj}^0(v,w) = 1, \text{ and } s_{kj}^i(v,w) = \frac{x_j}{X}, \quad \forall k \in N_0 \backslash \{i\}, \ j \in N_0, \ (v,w) \in V \times W,$$

where $X := \sum_{k \in N_0} x_k$, $V := \times_{i \in N} [\underline{v}_i, \overline{v}_i]$ and $W := \times_{i \in N} [\underline{w}_i, \overline{w}_i]$.

[7] The results generalize immediately to the case in which x_i is i's private information, but it is sufficiently costly for i to report $\widehat{x}_i > x_i$, while reporting $\widehat{x}_i \leq x_i$ can be costless. This may be the case when x_i is given by tangible assets and it is costly to engage in fraudulent overvalution of the assets.

In the game induced by this mechanism, agent j's utility function, when the true type profile is (v, w) and the agent report (v', w'), is

$$\begin{aligned} \widehat{u}_j\left(v', w', v, w\right) &= p_0\left(w', v'\right) x_j + \sum_{i \in N} p_i^*\left(w', v'\right)\left(v_i + w_i + X\right) \frac{x_j}{X} - x_j \\ &= \frac{x_j}{X} \sum_{i \in N} p_i^*\left(w', v'\right)\left(v_i + w_i\right). \end{aligned}$$

It is immediate to see that for each agent truth-telling is a best response when the other agents tell the truth. Furthermore, in the truthtelling equilibrium individual rationality is always satisfied. We have thus proved the following result.

Proposition 5 *If the stand-alone values $x_0, x_1, \ldots, x_n$ are commonly known, the efficient merger rule p^* is implementable with a mechanism that does not use monetary transfers.*

This result provides a justification for the widespread use of non-cash transactions in mergers. The mechanism is particularly simple when there is only one potential partner. In this case, if the reported synergies are negative, the merger does not occur and each agent retains full ownership of his firm. If instead the reported synergies are positive, the merger occurs and each agent's share of the new firm is proportional to the stand-alone value of his firm, i.e. agent i obtains a share $\frac{x_i}{x_0+x_1}$. It is thus optimal to ignore the (claimed) synergies when deciding how to divide the new firm, using instead only the verifiable stand-alone values.

4.3 Known Synergies with One Partner

Suppose now that the synergies are known, while each agent has private information about the stand-alone value of his firm. For simplicity, we focus on the case with a single potential partner, and to make the problem interesting, we assume that the merger yields positive synergies, i.e. $\varpi := w + v > 0$. Letting $1 - p_0 = p_1 = p$, and $r_1 = r$, the *ex post* utility of agent $i = 0, 1$, when the true types are $x = (x_0, x_1)$ and the reported ones are $x' = (x'_0, x'_1)$, can be written as

$$\begin{aligned} u_i\left(x', x\right) &= p\left(x'\right)\left[r\ s^1_{0i}\left(x'\right) + t^1_i\left(x'\right)\right] + \left[1 - p\left(x'\right)\right]\left[x_i s^0_{ii}\left(x'\right) + x_{-i} s^0_{-ii}\left(x'\right) + t^0_i\left(x'\right)\right] - x_i \\ &= p\left(x'\right)\left[\left(\varpi + x_{-i}\right)\ s^1_{0i}\left(x'\right) + t^1_i\left(x'\right)\right] + \left[1 - p\left(x'\right)\right]\left[x_{-i} s^0_{-ii}\left(x'\right) + t^0_i\left(x'\right)\right] \\ &\quad - \left[1 - p\left(x'\right) s^1_{0i}\left(x'\right) - \left[1 - p\left(x'\right)\right] s^0_{ii}\left(x'\right)\right] x_i, \end{aligned}$$

and the *interim* expected utility from reporting x'_i, when the true value is x_i and the other agent reports his true type, as

$$\widehat{U}_i\left(x'_i; x_i\right) = R_i\left(x'_i\right) - B_i\left(x'_i\right) x_i,$$

where

$$B_i\left(x_i\right) \equiv E_{x_{-i}}\left[1 - p\left(x\right) s^1_{0i}\left(x\right) - \left[1 - p\left(x\right)\right] s^0_{ii}\left(x\right)\right],$$

and

$$\begin{aligned} R_i\left(x_i\right) &\equiv E_{x_{-i}}\left\{p\left(x_i, x_{-i}\right)\left[\left(\varpi + x_{-i}\right)\ s^1_{0i}\left(x_i, x_{-i}\right) + t^1_i\left(x_i, x_{-i}\right)\right]\right\} \\ &\quad + E_{x_{-i}}\left\{\left[1 - p\left(x_i, x_{-i}\right)\right]\left[x_{-i} s^0_{-ii}\left(x_i, x_{-i}\right) + t^0_i\left(x_i, x_{-i}\right)\right]\right\}. \end{aligned}$$

Standard arguments in mechanism design, as the ones used in Lemma 2, imply that, for any incentive compatible mechanism, B_i is non-increasing, and the function $U_i\left(x_i\right) \equiv \widehat{U}_i\left(x_i, x_i\right)$ satisfies the following envelope condition

$$U_i\left(x_i\right) = U_i\left(\overline{x}_i\right) + \int_{x_i}^{\overline{x}_i} B_i\left(y\right) dy, \quad \forall x_i \in \left[\underline{x}_i, \overline{x}_i\right]. \tag{21}$$

Since $B_i\left(\cdot\right) \geq 0$, individual rationality is satisfied for each $x_i \in \left[\underline{x}_i, \overline{x}_i\right]$, if $U_i\left(\overline{x}_i\right) \geq 0$.

Since the synergies are positive, we have $p^*\left(x\right) = 1$, for all $x \in \times_{i=1,2}\left[\underline{x}_i, \overline{x}_i\right]$, hence

$$B_i\left(x_i\right) = 1 - S_i\left(x_i\right),$$

where $S_i\left(x_i\right) \equiv E_{x_{-i}}\left[s^1_{0i}\left(x_i, x_{-i}\right)\right]$ is the expected share of the new firm for agent i of type x_i. Incentive compatibility requires $S_i\left(x_i\right)$ to be non-decreasing.

We say that a mechanism (p^*, s, t) is an *acquisition mechanism* if

$$s_i^A(x_i, x_{-i}) = \begin{cases} 1, & \text{for } x_i > x_{-i}, \\ 0, & \text{for } x_i < x_{-i}. \end{cases}$$

(Again, ties have probability zero and can be broken in any way). Acquisition mechanism are appealing because they are simple: the merger always occurs, and the agent whose firm has the larger stand-alone value becomes the sole owner of the new firm. Of course, in order to make the mechanism incentive compatible, the agent agent needs to be compensated with a cash transfer.

We now show that, if the hazard rate of the distribution of each stand-alone value is monotone, the sum of the utilities of the highest types $U_0(\overline{x}_0) + U_1(\overline{x}_1)$ is maximized by an acquisition mechanism among all mechanisms which implement the efficient merger rule. This result allows us to identify a necessary and sufficient condition for the the implementability of the efficient rule in the symmetric case.

Lemma 4 *Suppose that* $\frac{F_0(x_0)}{f_0(x_0)} > \frac{F_1(x_1)}{f_1(x_1)}$ *if and only if* $x_0 > x_1$. *Then an acquisition mechanism maximizes the sum* $U_0(\overline{x}_0) + U_1(\overline{x}_1)$ *among all mechanisms implementing the efficient rule. When the two firms are ex ante symmetric, the efficient rule can be implemented only if*

$$U_i^*(\overline{x}) = E(x) + \frac{\varpi}{2} - \left(\int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{y} f(y) F(x) \, dx dy + \int_{\underline{x}}^{\overline{x}} F(y) (1 - F(y)) \, dy \right) \geq 0.$$

In the symmetric case, a feasible transfer rule t for which the mechanism $\left(p_*, s^A, t\right)$ is an acquisition mechanism is

$$t_i^A(x_i, x_{-i}) = \begin{cases} -m(x_{-i}) - \frac{\varpi}{2}, & \text{for } x_i > x_{-i}, \\ m(x_i) + \frac{\varpi}{2}, & \text{for } x_i < x_{-i}. \end{cases}$$

That is, the "winner" i, i.e. the agent who becomes the owner of the new firm, pays $m(x_j) + \frac{\varpi}{2}$ to the loser j. The function m has to be chosen so that incentive compatibility holds. The expected utility of a firm of type x announcing $\widehat{x}$ under such mechanism is

$$\widehat{U}_i(\widehat{x}_i, x_i) = \left(\frac{\varpi}{2} + \int_{\underline{x}_i}^{\widehat{x}_i} [x_i + y - m(y)] \, dF(y) + m(\widehat{x}_i)(1 - F(\widehat{x}_i)) \right) - x_i.$$

For $\widehat{x}_i = x_i$ to be a maximizer at each x_i, it must be the case that

$$2(x_i - m(x_i)) f(x_i) + m'(x_i) [1 - F(x_i)] = 0, \quad \forall x_i \in (\underline{x}, \overline{x}).$$

Thus m must solve the following differential equation on the interval $[\underline{x}, \overline{x}]$

$$m'(t) - 2\mu(t)\, m(t) = -2\mu(t)\, t,$$

where $\mu(x) \equiv \frac{f(x)}{1-F(x)}$. The solution is

$$m(t) = \frac{m(\underline{x}) - 2\int_{\underline{x}}^{t} s\,(1-F(s))\, f(s)\, ds}{[1-F(t)]^2}.$$

To determine $m(\underline{x})$, observe that the total expected payment of type $\overline{x}$ is

$$M(\overline{x}) = \frac{\varpi}{2} + \int_{\underline{x}}^{\overline{x}} m(t)\, dF(t)\,. \tag{22}$$

From Lemma 4 we have

$$U_i(\overline{x}) = E(x) + \frac{\varpi}{2} - \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{y} f(y)\, F(x)\, dx dy - \int_{\underline{x}}^{\overline{x}} F(y)\,(1-F(y))\, dy. \tag{23}$$

Since type $\overline{x}$ always becomes the owner of the new firm, by direct calculation we have

$$U_i(\overline{x}) = E(x) + \varpi - M(\overline{x})\,. \tag{24}$$

Equating (23) and (24), and using (22) we obtain

$$\begin{aligned} & \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{y} f(y)\, F(x)\, dx dy + \int_{\underline{x}}^{\overline{x}} F(y)\,(1-F(y))\, dy \\ = \; & m(\underline{x}) \int_{\underline{x}}^{\overline{x}} \frac{dF(t)}{(1-F(t))^2} - 2\int_{\underline{x}}^{\overline{x}} \frac{\int_{\underline{x}}^{t} s\,(1-F(s))\, f(s)\, ds}{[1-F(t)]^2} dF(t)\,, \end{aligned}$$

which determines $m(\underline{x})$.

5 Conclusions

In standard trading environments where monetary payments are bounded by liquidity constraints, having ‘losers’ pay may facilitate the implementation of efficient outcomes, by minimizing the probability of large payments. We have shown however that, when each agent is privately informed about the stand-alone values of his firm, it is costly, in terms of information rents, to redistribute shares of firms which are not involved in mergers. In some cases the first best can be obtained without any monetary transfer, i.e. using only redistributions of shares of merging firms.

Appendix

Proof of Lemma 1. Suppose that the mechanism (p, s, t) is incentive-feasible, and consider a new sharing rule $\widetilde{s}$ which differs from s only in the ownership structure of any new firm $\widetilde{s}^i_{0\cdot} = \left(\widetilde{s}^i_{00}, \widetilde{s}^i_{01}, ..., \widetilde{s}^i_{0n}\right)$, $i \in N$. Define $\widetilde{t}$ as

$$\widetilde{t}^i_j(\theta) \equiv t^i_j(\theta) + r_i\left[s^i_{0j}(\theta) - \widetilde{s}^i_{0j}(\theta)\right].$$

Since s and $\widetilde{s}$ are both feasible, we have $\sum_{j \in N_0} s^i_{0j}(\theta) = \sum_{j \in N_0} \widetilde{s}^i_{0j}(\theta) = 1$, hence

$$\sum_{j \in N_0} \widetilde{t}^i_j(\theta) = \sum_{j \in N_0} t^i_j(\theta) \leq 0, \quad \forall \theta \in \Theta,$$

Thus $\widetilde{t}$ is feasible.

Now let $\widehat{u}_j\left(\theta', \theta\right)$ and $\widehat{u}^*_j\left(\theta', \theta\right)$ denote agent j's utility functions in the revelation game induced by (p, s, t) and $\left(p, \widetilde{s}, \widetilde{t}\right)$ respectively. Whenever $p_i\left(\theta'\right) > 0$, since r_i is verifiable, we have

$$\begin{aligned}
\widehat{u}^*_j\left(\theta', \theta\right) &= r_i \widetilde{s}^i_{0j}\left(\theta'\right) + \sum_{k \in N \setminus \{i\}} s^i_{kj}\left(\theta'\right) x_k + \widetilde{t}^i_j\left(\theta'\right) - x_j \\
&= r_i \widetilde{s}^i_{0j}\left(\theta'\right) + \sum_{k \in N \setminus \{i\}} s^j_{kj}\left(\theta'\right) x_k + t^i_k\left(\theta', r_i\right) + r_i\left[s^i_{0k}\left(\theta'\right) - \widetilde{s}^i_{0k}\left(\theta'\right)\right] - x_i \\
&= \widehat{u}_j\left(\theta', \theta\right).
\end{aligned}$$

Thus under the new mechanism the ex-post utility function of each agent is the same. Since the original mechanism was incentive-feasible, incentive compatibility and individual rationality are also satisfied by the new mechanism. ■

Proof of Lemma 2. For any $\sigma \in \times_{i \in N}\left[\underline{x}_0 + \underline{w}_i, \overline{x}_0 + \overline{w}_i\right]$, define $C^\sigma_0(\zeta) \equiv U^\sigma_0(\zeta) + \zeta\ B^\sigma_0(\zeta)$, $\forall \zeta \in \left[\underline{x}^\sigma, \overline{x}^\sigma\right]$. The inequalities in (6) imply

$$\begin{aligned}
U^\sigma_0(\zeta) &\geq C^\sigma_0\left(\zeta'\right) - \zeta\ B^\sigma_0\left(\zeta'\right) \\
&= U^\sigma_0\left(\zeta'\right) + \left(\zeta' - \zeta\right)\ B^\sigma_0\left(\zeta'\right), \quad \forall \zeta, \zeta' \in \left[\underline{x}^\sigma, \overline{x}^\sigma\right],
\end{aligned}$$

which in turn implies

$$\left(\zeta'-\zeta\right)\ B_0^{\sigma}\left(\zeta\right)\geq U_0^{\sigma}\left(\zeta\right)-U_0^{\sigma}\left(\zeta'\right)\geq\left(\zeta'-\zeta\right)\ B_0^{\sigma}\left(\zeta'\right),,\quad\forall\zeta,\zeta'\in\left[\underline{x}^{\sigma},\overline{x}^{\sigma}\right].\tag{25}$$

The two inequalities in (25) immediately imply that B_0^{σ} is non-increasing, hence continuous almost everywhere. Taking $\zeta' > \zeta$, dividing by $\zeta'-\zeta$, and letting $\zeta' \to \zeta$ yields

$$\frac{dU_0^{\sigma}\left(\zeta\right)}{d\zeta}=-B_0^{\sigma}\left(\zeta\right)\ \text{a.e.}\tag{26}$$

Since B_0^{σ} is bounded, (25) also implies that U_0^{σ} is Lipschitz, hence absolutely continuos. Therefore we can integrate both sides in (26) and obtain (12). The proof of the second half of the lemma is analogous. ■

Proof of Proposition 1. Suppose that (p,s,t) is incentive feasible, and without loss of generality (by Lemma **1**) suppose that $s_{00}^{i}\left(\theta\right)=1$ for all $\theta\in\Theta$, $i\in N$. Consider a new mechanism $\left(p,\widetilde{s},\widetilde{t}\right)$ where:

1. the sharing rule $\widetilde{s}$ is determined by the LU property, defined in (**9**), and by the fact that agent 0 becomes the sole owner of any new firm, $\widetilde{s}_{00}^{i}\left(\theta\right)=1$ for all $\theta\in\Theta$, $i\in N$;

2. the transfer rule $\widetilde{t}$ is designed so that the change in the total value received (both in shares and cash) by each type of each agent, for each realization of his opponents' types, leaves the lowest information rent which is consistent with the the incentive compatibility and individual rationality constraints; that is:

   for each agent $i\in N$

$$\begin{aligned}\widetilde{t}_i^{0}\left(\theta\right) &= t_i^{0}\left(\theta\right)+\sum_{k\in N_0\backslash\{i\}}s_{ki}^{0}\left(\theta\right)x_k-\left(1-s_{ii}^{0}\left(\theta\right)\right)x_i-\int_{x_i}^{\overline{x}_i^{\tau_i}}A_i^{\tau_i}\left(\zeta\right)d\zeta-u_i\left(\left(\overline{x}_i^{\tau_i},\tau_i\right),\theta_{-i}\right),\\ \widetilde{t}_i^{j}\left(\theta\right) &= t_i^{j}\left(\theta\right)+\sum_{k\in N\backslash\{i,j\}}s_{ki}^{j}\left(\theta\right)x_k-\left(1-s_{ii}^{j}\left(\theta\right)\right)x_i-\int_{x_i}^{\overline{x}_i^{\tau_i}}A_i^{\tau_i}\left(\zeta\right)d\zeta-u_i\left(\left(\overline{x}_i^{\tau_i},\tau_i\right),\theta_{-i}\right),\quad j\in N\backslash\left\{i\right\},\\ \widetilde{t}_i^{i}\left(\theta\right) &= t_i^{i}\left(\theta\right)+\sum_{k\in N\backslash\{i\}}s_{ki}^{i}\left(\theta\right)x_k-\int_{x_i}^{\overline{x}_i^{\tau_i}}A_i^{\tau_i}\left(\zeta\right)d\zeta-u_i\left(\left(\overline{x}_i^{\tau_i},\tau_i\right),\theta_{-i}\right),\end{aligned}$$

where $A_i^{\tau_i}(\zeta) \equiv B_i^{\tau_i}(\zeta) - P_i^i(\zeta, \tau_i)$, $\forall \zeta \in [\underline{x}_i^{\tau_i}, \overline{x}_i^{\tau_i}]$; and for agent 0

$$\begin{aligned}
\widetilde{t}_0^0(\theta) &= t_0^0(\theta) + \sum_{k \in N} s_{k0}^0(\theta) x_k - \left(1 - s_{00}^0(\theta)\right) x_0 - \int_{x_0}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta - u_0\left(\left(\overline{x}_0^\sigma, \sigma\right), \theta_{-i}\right), \\
\widetilde{t}_0^i(\theta) &= t_0^i(\theta) + \sum_{k \in N \setminus \{i\}} s_{k0}^i(\theta) x_k - \int_{x_0}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta - u_0\left(\left(\overline{x}_0^\sigma, \sigma\right), \theta_{-i}\right),
\end{aligned}$$

where $A_0^\sigma(\zeta) \equiv B_0^\sigma(\zeta) - \left[1 - P_0^0(\zeta, \sigma)\right]$, $\forall \zeta \in [\underline{x}_0^\sigma, \overline{x}_0^\sigma]$.

It can be checked that the transfer rule $\widetilde{t}$ satisfies budget balance, i.e.

$$\sum_{i \in N_0} \widetilde{t}_i^j(\theta) \leq 0, \quad \forall \theta \in \Theta, \quad j \in N_0$$

because the original transfer rule t does, all shares of all firms sum up to one, i.e. $s_{j\cdot}^i \in \Delta \; \forall i, j \in N_0$, $j \neq i$, and all functions $A_0^\sigma(\cdot)$ and $A_i^{\tau_i}(\cdot)$ take non-negative values for all σ, τ_i, and $i \in N$.

Denoting the payoff functions in the new mechanism with the superscript *, we have

$$\widetilde{u}_0^*\left(\theta', \theta\right) = \widehat{u}_0\left(\theta', \theta\right) - \int_{x_0'}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta - u_0\left(\left(\overline{x}_0^\sigma, \sigma\right), \theta_{-i}\right),$$

hence

$$\widehat{U}_0^*\left(\theta_0', \theta_0\right) = \widehat{U}_0\left(\theta_0', \theta_0\right) - \int_{x_0'}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta - U_0\left(\left(\overline{x}_0^\sigma, \sigma\right)\right), \tag{27}$$

and

$$\begin{aligned}
U_0^*(\theta_0) &= U_0(\theta_0) - \int_{x_0'}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta - U_0\left(\left(\overline{x}_0^\sigma, \sigma\right)\right) \\
&= \int_{x_0'}^{\overline{x}_0^\sigma} B_0^\sigma(\zeta) \, d\zeta - \int_{x_0'}^{\overline{x}_0^\sigma} A_0^\sigma(\zeta) \, d\zeta \\
&= \int_{x_0'}^{\overline{x}_0^\sigma} P_0^\sigma(\zeta) \, d\zeta \\
&\geq 0.
\end{aligned}$$

By the last inequality IR holds; and by the equality in (27) IC holds, because the original mechanism is incentive compatible. The proof that IC and IR hold for each agent $i \in N$ is analogous. ■

Proof of Proposition 2. The proof simply adapts the results of Williams [17] to our setting. Any efficient mechanism can be replicated by a Groves-Clarke mechanism in which the payments are given by

$$m_i(\theta_i, \theta_{-i}) = -\sum_{j \neq i} v_j(a(\theta_i, \theta_{-i}), \theta_j) + k_i$$

where k_i is a constant. In a Groves-Clarke mechanism truth-telling is a dominant strategy. Therefore, the only issue is whether we can find constants k_i such that each agent is willing to participate and guarantee budget balance. The expected transfers to the agents are

$$E\left(-\sum_{j \in N_0} m_j(\theta)\right) = E\left(\sum_{j \in N_0}\left(\sum_{i \neq j} v_i(a(\theta), \theta_i) - k_j\right)\right) = (n-1)\,E\left(\sum_{i \in N_0} v_i(a(\theta), \theta_i)\right) - \sum_{i \in N_0} k_i.$$

Therefore, budget balance requires

$$nE\left(\sum_{i \in N_0} v_i(a(\theta), \theta_i)\right) \leq \sum_{i \in N_0} k_i.$$

Individual rationality requires $\underline{Z}_j - k_j \geq 0$ for each j. This in turn implies

$$\sum_{i \in N_0} k_i \leq \sum_{i \in N_0} \underline{Z}_j$$

Therefore, a necessary condition for individual rationality and budget balance is

$$nE\left(\sum_{i \in N_0} v_i(a(\theta), \theta_i)\right) \leq \sum_{i \in N_0} \underline{Z}_j.$$

Finally, if the condition is satisfied then the Groves-Clarke mechanism where $k_i = \underline{Z}_i$ implements the efficient mechanism. ■

Proof of Corollary 2. Define $\varpi_i = v_i + w_i$, and let G denote the c.d.f. of ϖ_i and g its density. By symmetry, the variable $z := \max\{\varpi_1, \ldots, \varpi_n\}$ has c.d.f. G^n, therefore

$$E_\theta\left[\sum_{i \in N_0} \widetilde{v}_i(a(\theta), \theta_i)\right] = \int_0^{\overline{y}} n\; z\; g(z)\, G^{n-1}(z)\, dz$$

where $\overline{y} = \overline{v} + \overline{w}$. If $\overline{v} + \underline{w} < 0$, then the worst type of agent 0 is the one having $\theta_0 = (\underline{w}, \ldots, \underline{w})$, that is the lowest possible synergy for each merger, and in this case $Z_0 = 0$. For firm $i \in N$, the worst type is $\underline{v}$. Conditional on a merger with i not taking place, the expected social surplus is the expected value of the highest possible merger for firms other than i. Thus we have

$$\underline{Z}_i = \int_0^{\overline{y}} (n-1)\, yg(y)\, G^{n-2}(y)\, dy$$

Condition (17) then becomes

$$n \int_0^{\overline{y}} yg(y)\, G^{n-1}(y)\, dy \leq (n-1) \int_0^{\overline{y}} yg(y)\, G^{n-2}(y)\, dy$$

which can never be satisfied, since the distribution on left-hand side stochastically dominates the one on the right. Therefore in this case efficiency is impossible. ■

Proof of Proposition 3. Consider the following mechanism. For each announcement (θ_0, θ_1) the merger occurs with probability 1. If the observed value of the merger is equal to the one implied by the announcement, the new firm is divided as follows,

$$s_{00}^1(\theta) = \frac{\overline{x}^\sigma}{\sigma+\tau} + \alpha_0, \qquad s_{01}^1(\theta) = \frac{\overline{x}^\tau}{\sigma+\tau} + \alpha_1,$$

with $\alpha_0 \geq 0$, $\alpha_2 \geq 0$ and $\alpha_0 + \alpha_1 = 1 - \frac{\overline{x}^\sigma}{\sigma+\tau} - \frac{\overline{x}^\tau}{\sigma+\tau}$, and no monetary transfers are made. If the value of the merger is different from the one implied by the announcements, then $s_{00}^1 = s_{01}^1 = 0$, and the firm is given to a third party.

Individual rationality is satisfied, since

$$s_{0i}^1(\theta)(\sigma+\tau) - x_i \geq 0$$

for $x_0 \leq \overline{x}^\sigma$ and $x_1 \leq \overline{x}^\tau$. The sharing rule is feasible, since $\sigma + \tau \geq \overline{x}^\sigma + \overline{x}^\tau + \underline{v} + \underline{w}$. Incentive compatibility is also satisfied, since agents cannot lie about τ or σ and the shares are constant for any given pair (σ, τ). ■

Proof of Lemma 3. For any $\theta_i = (x_i, v_i) \in \Theta_i$ and k such that $k\theta_i = (kx, kv) \in \Theta_i$, incentive compatibility requires both $\widehat{U}_i(\theta_i, \theta_i) \geq \widehat{U}_i(k\theta_i, \theta_i)$ and $\widehat{U}_i(k\theta_i, k\theta_i) \geq \widehat{U}_i(\theta_i, k\theta_i)$. Also, since $\widehat{U}_i(\cdot, k\theta_i) = k\,\widehat{U}_i(\cdot, \theta_i)$, the second inequality is equivalent to $\widehat{U}_i(k\theta_i, \theta_i) \geq \widehat{U}_i(\theta_i, \theta_i)$. Combining this with the first inequality yields $\widehat{U}_i(\theta_i, \theta_i) = \widehat{U}_i(k\theta_i, \theta_i)$. Thus, omitting the subscript i to save notation, we have

$$(x+v)\, B(kx, kv) - x\, L(kx, kv) = (x+v)\, B(x, v) - x\, L(x, v). \tag{28}$$

In light of (28), it is sufficient to show that $B(kx, kv) = B(x, v)$, that is that B is constant along any line of proportional types. To this end, define $U(x, v) = (x+v)\, B(x, v) - x\, L(x, v)$. By standard mechanism design arguments, U must be continuous and convex, hence differentiable almost everywhere. At every point at which it is differentiable we have

$$\frac{\partial U}{\partial x} = B(x, v) - L(x, v), \tag{29}$$

$$\frac{\partial U}{\partial v} = B(x, v). \tag{30}$$

Suppose first that $\widehat{U}$ is twice continuously differentiable. Denoting derivatives with subscripts in the usual way, we have

$$\left.\frac{\partial\, \widehat{U}(x', v'; x, v)}{\partial\, v'}\right|_{((x',v')=(x,v))} = (x+v)\, B_v - x\, L_v = 0.$$

which implies

$$\frac{x+v}{x}\, B_v = L_v. \tag{31}$$

Differentiating (29) w.r.t. v, and (30) w.r.t. x, by the equality of the second mixed partial derivatives of U, we have

$$B_v - L_v = B_x. \tag{32}$$

Now the directional derivative of B, at any point (x, v), along the line of types which are proportional to (x, v), i.e. along the vector $\left(s, \frac{v}{x}s\right)$, is

$$\begin{aligned} \frac{dB\left(x+s, v+\frac{v}{x}s\right)}{ds} &= B_x + \frac{v}{x}B_v = B_v - L_v + \frac{v}{x}B_v \\ &= B_v\frac{x+v}{x} - L_v = 0 \end{aligned}$$

the second equality following from (32), and the last one from (31). This proves the Lemma when U is twice continuously differentiable.

We are left with the task of proving the Lemma in the case in which U is not twice continuously differentiable. Since U is in the feasible set, it is convex, and satisfies $\frac{\partial U(x,v)}{\partial v} = B(x,v)$, for any (x,v) in a set $K \subset \Theta$ of full measure. We now invoke Theorem H.4.5 in Mas Colell [7], which ensures that, for any positive integer m, there exists a smooth convex function U_m such that $\left|\frac{\partial U_m(x,v)}{\partial v} - \frac{\partial U(x,v)}{\partial v}\right| < \frac{1}{m}\ \forall (x,v) \in K$.

Since U_m is smooth, we know that $\frac{\partial U_m(x,v)}{\partial v} = B_m(x,v)$ is constant along any line of proportional types, i.e. $B_m(x,\alpha x) = b_m(\alpha)$ each (x,α) such that $(x,\alpha x) \in \Theta$. Thus, for each m we have $|B(x,\alpha x) - b_m(\alpha)| < \frac{1}{m}$, $\forall (x,\alpha)$ such that $(x,\alpha x) \in K$. Since, for each α, the sequence $(b_m(\alpha))$ is contained in the compact interval $[0,1]$, there exists a strictly increasing sequence (m_k) of positive integers such that $(b_{m_k}(\alpha))$ converges to some $b(\alpha) \in [0,1]$. Combining this with the previous inequality, and letting $k \to \infty$, we obtain

$$B(x,\alpha x) = b(\alpha)\,,\ \ \forall (x,\alpha) \text{ such that } (x,\alpha x) \in K.$$

This completes the proof. ■

Proof of Proposition 4. By Lemma 3, P_i must be constant along any line of proportional types. Therefore, we can find $x_i, x_i' \in [\underline{x}_i, \overline{x}_i]$ and $v_i', v_i \in [\underline{v}_i, \overline{v}_i]$ such that $P_i^*(v_i') > P_i^*(x_i, v_i)$ and $\frac{v_i'}{x_i'} = \frac{v_i}{x_i}$. By Lemma 3, in any auction in shares it must be the case that $P_i(x_i, v_i) = P_i(x_i', v_i')$, but efficiency requires $P_i^*(v_i') > P_i^*(v_i)$. ■

Proof of Lemma 4. By standard mechanism design arguments, we have $U_i'(x_i) = -B_i(x_i) = S_i(x) - 1$ for almost all $x_i \in [\underline{x}_i, \overline{x}_i]$. The *ex ante* expected surplus for agent i can be written as

$$\int_{\underline{x}_i}^{\overline{x}_i} U_i(x)\, dF_i(x) = \int_{\underline{x}_i}^{\overline{x}_i} \left[U_i(\overline{x}_i) + \int_x^{\overline{x}_i} B_i(y)\, dy\right] dF_i(x) = U_i(\overline{x}_i) + \int_{\underline{x}_i}^{\overline{x}_i} \left(\int_x^{\overline{x}_i} B(y)\, dy\right) dF_i(x)$$

Since

$$\int_{\underline{x}_i}^{\overline{x}_i}\left(\int_{x}^{\overline{x}_i} B_i(y)\,dy\right) dF_i(x) = \int_{\underline{x}_i}^{\overline{x}_i} B_i(x)\,F_i(x)\,dx,$$

we have

$$\int_{\underline{x}_i}^{\overline{x}_i} U_i(x)\,dF_i(x) = U_i(\overline{x}_i) + \int_{\underline{x}_i}^{\overline{x}_i} B_i(x)\,F(x)\,dx. \tag{33}$$

But we also know that

$$\int_{\underline{x}_i}^{\overline{x}_i} U_i(x)\,dF_i(x) = \int_{\underline{x}_i}^{\overline{x}_i}\int_{\underline{x}_{-i}}^{\overline{x}_{-i}} (x+y+v)\,s_i(x,y)\,dF_{-i}(y)\,dF_i(x) - \int_{\underline{x}_i}^{\overline{x}_i} M_i(x)\,dF_i(x), \tag{34}$$

where $M_i(x)$ is the expected payment of type x. Combining (33) and (34) and solving for $U_i(\overline{x}_i)$, we have

$$U_i(\overline{x}_i) + \int_{\underline{x}_i}^{\overline{x}_i} B_i(x)\,F(x)\,dx = \int_{\underline{x}_i}^{\overline{x}_i}\int_{\underline{x}_{-i}}^{\overline{x}_{-i}} (x+y+v)\,s_i(x,y)\,dF_{-i}(y)\,dF_i(x) - \int_{\underline{x}_i}^{\overline{x}_i} M_i(x)\,dF_i(x),$$

hence

$$\begin{aligned}
U_i(\overline{x}_i) &= \int_{\underline{x}_{-i}}^{\overline{x}_{-i}}\int_{\underline{x}_i}^{\overline{x}_i} (x+y+v)\,s_i(x,y)\,dF_i(x)\,dF_{-i}(y) - \int_{\underline{x}_{-i}}^{\overline{x}_{-i}}\left(\int_{\underline{x}_i}^{\overline{x}_i} B_i(x)\,F_i(x)\,dx\right) dF_{-i}(y) \\
&\quad - \int_{\underline{x}_i}^{\overline{x}_i} M_i(x)\,dF_i(x) \\
&= \int_{\underline{x}_{-i}}^{\overline{x}_{-i}}\int_{\underline{x}_i}^{\overline{x}_i}\left(x+y+v+\frac{F_i(x)}{f_i(x)}\right) s_{0i}^1(x,y)\,dF_i(x)\,dF_{-i}(y) \\
&\quad - \int_{a_i}^{b_i} M_i(x)\,dF_i(x) - \int_{\underline{x}_i}^{\overline{x}_i} F_i(x)\,dx.
\end{aligned}$$

Let $K_i = \int_{\underline{x}_i}^{\overline{x}_i} F_i(x)\,dx$. Since budget balance implies $\int_{a_1}^{b_1} M_1(x)\,dF_1(x) + \int_{a_2}^{b_2} M_2(x)\,dF_2(x) = 0$, and $s_{00}^1(x_0,x_1) = 1 - s_{01}^1(x_0,x_1)$ we have

$$U_0(\overline{x}_0) + U_i(\overline{x}_0) = \int_{\underline{x}_0}^{\overline{x}_0}\int_{\underline{x}_1}^{\overline{x}_1}\left(\frac{F_0(x_0)}{f_0(x_0)} - \frac{F_1(x_1)}{f_1(x_1)}\right) s_{00}^1(x_0,x_1)\,dF_1(x_1)\,dF_0(x_0) + \widehat{K}. \tag{35}$$

where

$$\widehat{K} = \int_{\underline{x}_0}^{\overline{x}_0}\int_{\underline{x}_1}^{\overline{x}_1} (x_0+x_1+\varpi)\,dF_1(x_1)\,dF_0(x_0) - \int_{\underline{x}_0}^{\overline{x}_0} F_0(y)\,dy.$$

Clearly, if $x_0 > x_1 \Leftrightarrow \frac{F_0(x_0)}{f_0(x_0)} > \frac{F_1(x_1)}{f_1(x_1)}$, an acquisition mechanism maximizes $U_0(\overline{x}_0) + U_i(\overline{x}_0)$.

Under symmetry, the expression in (35) becomes

$$2U_i(\overline{x}) = \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{y} \left(F(y) f(x) dxdy - F(x) f(y)\right) dxdy + 2E(x) + \varpi - \int_{\underline{x}}^{\overline{x}} F(y)\, dy,$$

hence the condition $U_i(\overline{x}) \geq 0$ can be written as

$$2E(x) + \varpi \geq \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{y} f(y) F(x)\, dxdy + \int_{\underline{x}}^{\overline{x}} F(y)(1 - F(y))\, dy.$$

Finally, we observe that individual rationality is satisfied because $U_i(\overline{x}_i) \geq 0$ implies $U_i(x_i) \geq 0$ for each $x_i \in [\underline{x}_i, \overline{x}_i]$. ■

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