Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

A General Framework to Evaluate Economic Efficiency with an Application to British SME

Jan Průša

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

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A General Framework to Evaluate Economic Efficiency with an Application to British SME

Jan Průša*

*IES, Charles University Prague
E-mail: jan.prusa@gmail.com

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Abstract:
This article formalises the idea of money-metric production frontiers, which we propose as a general framework for nonparametric evaluation of economic efficiency. As we show in our methodological discussion, this improves the flexibility and economic interpretation of our model.
The empirical part is the first attempt to test the existence of a size-efficiency relationship among small businesses in the United Kingdom. It is based on a unique panel both with respect to size — ranging from agriculture to services — and to the ten year time span. We employ statistically robust methods to estimate and analyse sectoral efficiency. Our analysis yields three main insights: (1) Average sectors are expected to be two to four times less efficient than those on the efficient frontier. Great dispersion of efficiency scores highlights the importance of dynamic out-of-equilibrium modelling. (2) There is no evidence of a general economy-wide size-efficiency relationship. (3) Economic efficiency remained constant over the past ten years.

Keywords: Small and medium enterprises; economic efficiency; firm size; robust efficiency estimation.

JEL: D24, L25, L26

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1 Introduction

Research on small and medium enterprises (henceforth SME) has recently received much attention. The question that has been examined most intensively can be posed with Yang & Chen [47]: "Are small firms less efficient?" They list nine studies (table 1 *ibid*) which find a positive size-efficiency relationship, although their own results for Taiwan's electronics industry are heterogeneous. Taymaz [40] analyses Turkish manufacturing industries and asks a similar question, but he is more concerned with efficiency dynamics. He confirms that higher efficiency implies higher probability of survival.

Our case study of the United Kingdom contributes to research on efficiency of SME. We extend previous studies by two main features.

Firstly, numerous articles on the efficiency-size relationship were motivated by technical efficiency and returns to scale (e. g. Alvarez & Crespi [1]) and derived their models from static microeconomic framework. The shortcomings of these simplifications are well understood and have been extensively covered in parametric applications. Yet not much discussion has been devoted to overcoming them in nonparametric estimation. Therefore we focus on *economic* efficiency and propose a more general solution which we contend is more suitable to evaluate economic efficiency.

Secondly, we use a large dataset based on firm-level survey and compare most of the sectors in the economy, which allows us to test whether previous results were sector-specific or whether they extend to the whole economy. We analyse efficiency scores and test if they are size or time dependent.

The most important institution conducting research specifically aimed at SME in the United Kingdom is the Centre for Business Research at the University of Cambrigde.[1] Most of the recent papers are concerned with institutional and structural issues, such as financing (Hughes [21], Cosh *et al.* [10]), innovation (overview by Hoffman *et al.* [20], Cosh *et al.* [11]), or subcontracting (Wynarczyk & Watson [46]). To the best of our knowledge, no recent article examined SME efficiency. This offers room for our analysis.

2 Understanding Inefficiency

2.1 A Neoclassical Firm

The simplest neoclassical model of production, such as in chapter 5 of Mas-Colell *et al.* [30], approximates the long-run equilibrium and maintains that firms know their technology represented by a production function. By assumption of profit maximizing or cost minimizing behaviour, firms attain both technical and allocative efficiency. Hence, allocation at the firm level is efficient. If all firms are facing the same technology and the same prices, they will all lie on the same aggregate production frontier.

[1] [http://www.cbr.cam.ac.uk/].

This means that both types of inefficiency (technical and allocative) occurring either within firms or across firms are assumed away. However, both our daily experience and empirical evidence show that there are big differences in production abilities among firms. Better economic performance of some firms compared to others is the crucial issue which we investigate in this article.

2.2 Introducing Frictions in the Neoclassical Paradigm

Varying economic performance in the short run implies that a realistic economic theory has to model efficiency differentials. One approach is to attribute them to market interactions of firms and to focus on market structure. In this setup higher profits are associated with more monopolistic structure. Such analysis is certainly valid, but it cannot account for differences among similar firms, that is firms which operate in the same market with comparable products. In other words, the theory needs to depart from the symmetry assumption.

In 1937, Ronald Coase [8] posed himself the following question:

> Our task is to attempt to discover why a firm emerges at all in a specialized exchange economy.

After this groundbreaking article, more realistic models of firms have been developed:

- ⋆ A more dynamic view of firm's capital, which explicitly takes into account different 'vintages of capital'. This term, used e.g. by Johansen [22], was later generalized to 'technology', but the original literal description is quite instructive about the nature of firms in reality.[2]

- ⋆ The theory of transaction costs and institutional economics, as described in Moschandreas [32, chapter 3]. Profit-seeking versus Rent-seeking. This was further developed into the theory of incomplete contracts and imperfect monitoring and the principal-agent problem.

2.3 X-Efficiency

In 1966, Harvey Leibenstein [25] argued that a significant proportion of empirically documented inefficiencies stem from sources other than technical and allocative inefficiency. He introduced a new term: *X-efficiency*, and developed a theory based on this definition.

Frantz [17] points out that the difference between X-efficiency and neoclassical paradigm lies in the main assumption: While the latter assumes maximising behaviour in all circumstances, the former allows for situations where individuals are consciously not optimising. In Leibenstein's own words [25, p. 407]:

> The simple fact is that neither individuals nor firms work as hard, nor do they search for information as effectively, as they could.

[2] For a study implementing the original vintage model see Wickens [43].

Leibenstein's article was followed by an intensive discussion. Stigler [39], De Alessi [13] and others defended the neoclassical paradigm, arguing that it developed enough tools to handle inefficiencies (see section 2.2). Yet as noted by Frantz [17], X-efficiency lies outside neoclassical paradigm, and hence cannot be refuted by neoclassical arguments.

2.4 Austrian Theory of Production

The Austrian school regards entrepreneurs as those who pursue arbitrage, and concentrates on dynamics of the economy. Continuous dynamic adjustment is driven by entrepreneurs who exploit profitable opportunities as they emerge.

Accordingly the neoclassical paradigm suffers from the static equilibrium-always view. Sautet [34, p. 10] calls this the 'market theory problem', which is (emphasis in original):

> the *inconsistency* involved in trying to answer questions that would not exist in an equilibrium-always world.

Inefficiency can be regarded as one example of the market theory problem. With regard to efficiency, Sautet writes (p. 49 *ibid*):

> Understanding competition as a process helps explain empirical phenomena that cannot be explained by standard neoclassical theory, such as the persistent dispersion of returns that is wider among firms of the same industry than across industries (Rumelt 1984, 1987) and the different rates of growth among firms of the same industry (Penrose 1995 [1959]).

Therefore the Austrian school offers yet another theoretical explanation for 'inefficiencies'.

2.5 Modelling Production

To proceed we formalize the methodological framework introduced above.

2.5.1 Technology

The production set is defined as all feasible input-output vectors $(\boldsymbol{x},\boldsymbol{y})$, as in Tulkens & Eeckaut [42]:

$$\mathscr{Y} = \{(\boldsymbol{x},\boldsymbol{y}), \boldsymbol{x} \in \Re^r_{0,+}, \boldsymbol{y} \in \Re^s_{0,+} \mid (\boldsymbol{x},\boldsymbol{y}) \text{ is feasible}\}.^3$$

The points that are technically efficient are given by:

$$\mathrm{Eff}(\mathscr{Y}) = \{(\boldsymbol{x},\boldsymbol{y}) \in \mathscr{Y} \mid \forall[\boldsymbol{x}^1 \le \boldsymbol{x},\ \boldsymbol{y}^1 \ge \boldsymbol{y},\ (\boldsymbol{x}^1,\boldsymbol{y}^1) \ne (\boldsymbol{x},\boldsymbol{y})] :\ (\boldsymbol{x}^1,\boldsymbol{y}^1) \notin \mathscr{Y}\}.$$

$\mathrm{Eff}(\mathscr{Y})$ is known as the production frontier. If we can find a functional form, we have:

$$(\boldsymbol{x},\boldsymbol{y}) \in \mathrm{Eff}(\mathscr{Y}) \iff \mathscr{T}(\boldsymbol{x},\boldsymbol{y}) = 0,$$

where $\mathscr{T}(\cdot)$ is the transformation function. For scalar output, this simplifies to the production function $f(\boldsymbol{x}) = y$.

[3] We explicitly include 0 in the notation to indicate that vectors $\boldsymbol{x},\boldsymbol{y}$ are nonnegative.

2.5.2 Simple Cost and Profit Functions

Suppose that input prices $\boldsymbol{w}$ and output prices $\boldsymbol{p}$ are fixed. Refining the definition by Greene [19, p. 142], we can write a cost function for scalar output y as:

$$\mathscr{C}(\boldsymbol{w}, y) = \arg\min_{\{\boldsymbol{x}\}} \{\boldsymbol{w}'\boldsymbol{x} |\ f(\boldsymbol{x}) \geq y\}.$$

More generally, optimization over both inputs and outputs yields the profit function:

$$\Pi(\boldsymbol{p}, \boldsymbol{w}) = \arg\max_{\{\boldsymbol{x},\boldsymbol{y}\}} \{\boldsymbol{p}'\boldsymbol{y} - \boldsymbol{w}'\boldsymbol{x} |\ (\boldsymbol{x}, \boldsymbol{y}) \in \mathscr{Y}\},$$

which is by a contradiction argument equivalent to:

$$\Pi(\boldsymbol{p}, \boldsymbol{w}) = \arg\max_{\{\boldsymbol{x},\boldsymbol{y}\}} \{\boldsymbol{p}'\boldsymbol{y} - \boldsymbol{w}'\boldsymbol{x} |\ (\boldsymbol{x}, \boldsymbol{y}) \in \mathrm{Eff}(\mathscr{Y})\}.$$

2.5.3 Efficiency Decomposition

Along the lines of the definitions above, efficiency can be decomposed into technical and allocative component. Technical efficiency (operating on the frontier $\mathrm{Eff}(\mathscr{Y})$) is intuitively straightforward, although we must specify whether the feasibility of $\mathscr{Y}$ is with respect to a specific firm or whether we consider aggregate technology.

Being restricted to the most productive points of $\mathscr{Y}$, a firm achieves allocative efficiency iff it chooses the most profitable point of $\mathrm{Eff}(\mathscr{Y})$.

However, mathematical representation of this optimal choice by cost or profit functions requires strong structural assumptions. As indicated by the arguments of $\mathscr{C}(\cdot)$ or $\Pi(\cdot)$, these functions are derived for exogenous prices, but research suggests that this exogeneity is rare in practice.[4] To arrive at a valid framework for empirical application, we need to examine the structural assumptions in more depth.

3 Empirical Methodology of Efficiency Measurement

3.1 Identification I: Specification of Variables

Although it is possible in theory to separate technical and allocative efficiency, identification of the components requires detailed data on both quantities and prices, which is usually unavailable. Because technologies are simply too complex, economics came up with the concept of basic factors of production: capital, labour, materials, energy, and land (which is commonly omitted). This concept is widely accepted, not least because it simplifies aggregation. We take the most general form from Burnside [5, equation 2.1]:

$$y = f(\text{capital, labour, energy, materials, technology}). \tag{1}$$

[4] Fabiani *et al.* [16] present evidence on pricing behaviour of 11,000 firms. The result relevant for our discussion is that 54% of firms use markup pricing, while only 27% of firms use competitors' price as the main price setting factor (section 3.1 *ibid*).

Burnside's treatment provides a link between micro and macro level production; and a thorough discussion of assumptions which underlie specific choices of variables or pricing structures. Yasar *et al.* [48] use specification (1) for a firm-level study.[5]

Researchers usually take sales or value added (possibly in logs) for output, depending on whether materials and energy have been subtracted.[6] As regards inputs, researchers employ tangible and intangible assets for capital; and a combination of employees, hours worked and wages for labour. Dynamic models include investment in the form of acquisition of assets and depreciation. Technology is treated by separate models, and these go beyond the scope of our article.

3.2 Exogeneity of Prices

Suppose that a researcher has detailed data on both quantities and prices. Under standard regularity conditions, cost or profit functions are sufficient statistics for technology, but only for fixed prices, that is when firms treat prices as exogenous. The same conclusion applies to aggregation. Mas-Colell *et al.* [30, p. 149] note:

> If firms maximise profits taking prices as given, then the production side of the economy aggregates beautifully.

Estimation of $\mathscr{C}(\cdot)$ or $\Pi(\cdot)$ implicitly relies on this structural assumption about price setting. However, when analysing large cross-sections of firms, exogeneity assumptions are likely to be too restrictive: Not only because we are uncertain about how prices are actually formed, but especially because the structure of price setting will certainly differ across firms and markets.

Indeed, if the situation were symmetric, we could just work with a representative firm. Yet it is the *asymmetry* assumption that justifies efficiency analysis; that is asymmetry for which economists developed a variety of explanations, ranging from transaction costs to entrepreneurship.

3.3 Identification II: Which Units Can be Mixed?

Let us return to specification of variables. Due to their aggregate nature, general factors of production induce a measurement problem, which is especially apparent for capital. Capital is supposed to represent machinery, but since it is a term too broad, any measure of capital suitable for comparison must be monetary.

This however created a considerable amount of confusion. Studies on production often combine data in physical and monetary units without proper discussion. In his study on production functions, Johansen [22, chapter 9] analyzes output of Norwegian tankers. While he measures output as ton-miles per day, the inputs — fuel and labour per day — are measured

[5] They call technology as "total factor productivity".

[6] Other measures which are applied to assess performance of companies are surveyed by Murphy *et al.* [33, table 2].

in Norwegian *kroner*. This approach could be justified, say if output of all tankers was traded at the same price. Whether this explanation would be reasonable or not we leave aside. More surprising is that the author does not attempt at all to explain this specification.

More recently Biørn *et al.* [4] specify their micro-based *production* function as follows: Output in tonnes, and inputs as capital and materials in Norwegian *kroner*, labour in man-hours and energy in kWh (appendix B9.2 *ibid*).[7]

These examples reveal that empirical studies have not made a clear distinction between technical and allocative efficiency. In our view this results in dubious interpretation. We propose a solution to these inconsistencies in section 4.

3.4 Functional Specification

Cobb-Douglas is the most popular formulation due to mathematical simplicity. Other functions were considered and the debate on their adequacy reemerged in the context of macroeconomic growth models, see e.g. Duffy & Papageorgiou [15] for the constant elasticity of substitution function, and Kneller & Stevens [24] for the translog specification.

We do not pursue this discussion because we do not employ parametric specification. However let us note here that parametric research has developed detailed models of production addressing endogeneity of prices and other related issues. Systems of equations were proposed by Marschak & Andrews [29, see eqs. 1.29-1.31] and have grown to complex demand–supply models for differentiated products as in Berry *et al.* [3].[8] It is the more striking that efficiency studies did not integrate these results.

4 Money-metric Production Frontiers

4.1 Definition

In previous sections we noted that in empirical literature on efficiency it is the exception rather than the rule to discuss the step from $\mathrm{Eff}(\mathscr{Y})$ to $\Pi(\cdot)$ and the underlying assumptions. This holds especially for the price exogeneity assumption, which affects the whole model building procedure, and it also applies to combining data on quantities and prices in a single equation.

Our treatment rests on a more general approach that addresses some of the issues explained in the preceding exposition. Let us define the *money-metric production frontier* $\mathrm{Eff}(\mathscr{M})$, where:

$$
\begin{aligned}
\mathscr{M} &= \{(\boldsymbol{wx}, \boldsymbol{py}), \boldsymbol{wx} \in \Re^r_{0,+}, \boldsymbol{py} \in \Re^s_{0,+} \mid (\boldsymbol{wx}, \boldsymbol{py}) \text{ is feasible}\}, \qquad (2)\\
\boldsymbol{wx} &= (w_1 x_1, \ldots, w_r x_r)', \\
\boldsymbol{py} &= (p_1 y_1, \ldots, p_s y_s)',
\end{aligned}
$$

[7] Their justification is rather anecdotal. The authors prefer tonnes to *kroner* for output because of possible mismeasurement in sales. On the other hand, they do not mind using arbitrary constant depreciation rates for capital and transforming fuels to kWh using “estimated average energy content” (*ibid*).

[8] Other significant studies which use parametric models include Klette & Griliches [23], Melitz [31], Levinsohn & Petrin [26] and De Loecker [14].

and the definition of $\mathrm{Eff}(\mathscr{M})$ is analogous to that of $\mathrm{Eff}(\mathscr{Y})$. The notation indicates that firms participated in some form of bargaining, so that inputs and outputs are money-valued. Yet no explicit structure is placed on the bargaining process, so that $\mathscr{M}$ is a more general and flexible basis for efficiency measurement.

4.2 Knowing What We Do Not Know

4.2.1 Measuring Economic Efficiency

Several comments about definition (2) are in order.

Firstly, some studies in fact adopt the approach in (2), without explicitly stating it. Yang & Chen [47] use a production frontier kernel which is strictly money-valued, to which they add other regressors (see table 2 *ibid*).[9]

Our contribution is that by defining $\mathscr{M}$ we formalize the existing idea of money-valued production *and* give it a theoretical underpinning. As we saw in section 3.3, the explanation for interchanging physical and monetary units has so far been neglected. Unlike the mixture models from section 3.3, definition (2) provides a valid and consistent framework for efficiency analysis.

Secondly, $\mathscr{M}$ and its frontier provide the most general description of production. Because it is purely empirical, we contend that it has relevance to real economy. Research on efficiency attempted to uncover both technical and allocation processes in firms and their separation. As a consequence, questions concerning interpretation or validity of assumptions faded into the background.

The focus on technical efficiency is itself surprising, since technology per se is not the subject of economics, although with the words of Sautet [34, p. 4] "it has some influence on economic issues". One approach to overcome the problems arising from separation of technical and allocative efficiency is a more detailed structure building.[10]

On the other hand, our generalized approach abandons identification of efficiency components for the reasons that we discussed above: (1) data are not easily measurable and hence not available; (2) structural assumptions required for identification are questionable for large cross-sections or aggregated datasets; (3) identification in empirical studies has been handled with neither good precision nor with great success; and finally (4) we want to learn mainly about the economic process, not the technique relation, since what matters in the end is the money-valued outcome.

[9] Paradoxically, they consistently use "technical efficiency", which highlights the lack of discussion of the underlying methodology.

[10] We already mentioned the complete demand–supply model by Berry *et al.* [3] for automobile industry. Note carefully that this specification is only for one product, not for the automotive industry as a whole.

4.2.2 Aggregation

Thirdly, the crucial assumption for aggregation is to view economic environment as a technology pool *and* a market pool. Hence technology is aggregate omnipresent knowledge that is available for everybody, and similarly for market opportunities. It follows that our observations are all drawn from one $\mathscr{M}$, rather than each firm or sector having its own $\mathscr{M}_i$. If we only had one (cross-section) or a few (time series) observations for each $\mathscr{M}_i$, we could not estimate much. Note that aggregation in the case of $\mathscr{M}$ can be justified precisely on the grounds of no specific pricing structure, which would normally differ across companies.

Fourthly, we cannot expect $\mathscr{M}$ to be convex. The replication argument which justifies convexity of $\mathscr{Y}$ is likely to fail here: The feasibility of a given $(\boldsymbol{wx}, \boldsymbol{py})$ relies not only on general availability of aggregate technology (as for $\mathscr{Y}$), but also on the unique bargaining abilities of the entrepreneur. This point will be important for computational implementation.

4.2.3 Panel Data

Finally it is necessary to bear in mind one crucial property of efficiency estimation: In a cross-section, the point estimator of inefficiency for individual firms is necessarily based on a single observation of each firm. This complicates statistical inference for individual firms. Moreover, it is improbable that panel data could provide a remedy: Long time series for single firms are generally not available, and further questions arise with a dynamic specification of efficiency. One would then ask why the feasible set should remain constant over time. Greene [18, p. 277] notes:

> For panels which involve more than a very small number of periods, this [time invariant efficiency] is a significant and possibly unreasonable assumption.

Therefore, it seems that inefficiency should be treated as time varying instead. We return to this issue in section 5.2.3.

5 Application to SME in the UK

5.1 The Data

The data that we use to test our hypotheses are extracted from the Annual Business Inquiry organised by the Office for National Statistics.[11] Compared to the publicly available version, our data are sizebanded according to the number of employees to distinguish different classes of SME. Hence the dataset can be summarised:

- ⋆ Four-digit Standard industrial classification (SIC) including all sectors from agriculture to services.
- ⋆ Sizebands 1-10, 11-25, 26-50, 51-100, 101-250, and more than 250 employees.

[11] Detailed information about this product is found at [http://www.statistics.gov.uk/abi/].

- ⋆ Variables: Number of firms; number of employees; wage costs; total employment costs ($EMPCOST$); net capital expenditure (NCE); turnover; gross value added (GVA).
- ⋆ Years 1998–2007.

Because of data confidentiality, about a third of observations involved missing information, and these had to be omitted. We further deleted observations with negative GVA or NCE. Still, the resulting dataset contained $N_0 = 16,826$ observations, with more than $1,500$ datapoints for each year.

5.2 General Model Specification

5.2.1 Fitting a Model of Production

We specify our model of production as:

$$GVA = h(NCE; EMPCOST), \tag{3}$$

so that $\boldsymbol{wx} = (NCE, EMPCOST)$ and $py = GVA$. This directly follows equation (1) as a widely accepted formulation in the literature. Energy and material costs do not enter (3) because they were already subtracted from total sales, yielding GVA.[12]

To this formulation we apply a robust nonparametric efficiency estimator described in section 5.3. The method for analysis of efficiency scores is outlined in section 5.4. Prior to that, we discuss our model building.

5.2.2 Measurement of Capital

Measurement poses a challenge especially for capital. Studies in efficiency analysis commonly use data on fixed assets. However, it was pointed out to us[13] that from the viewpoint of economic calculation what we ideally want to measure is a flow variable; that is: How much does the use of given capital assets cost? Or equivalently: How much would it cost to hire these capital assets for the time required for production?

Researchers are aware of this problem, and some of them use depreciation to extract a flow proxy from the stock of capital. Nonetheless, if this is accomplished by a constant depreciation rate, as in Biørn *et al.* [4], it does not bring any additional information.

Although it seems that ‘net capital expenditure’ could be adequate as a flow variable, NCE has its own shortcomings. It captures one-off acquisitions and disposals of capital, and hence offers no guide as to how these values are distributed over time. Because it is in fact the sum of positive and negative investments, it is very volatile, and in addition it can result in spuriously low or even negative values.

[12] Subtracting costs of energy and materials is in effect a parametric operation. Nevertheless, there are at least two reasons why it should not influence information in the data significantly: (1) In this case the parametrization is intuitive; and (2) prices can be reasonably treated as given for energy and materials.

[13] The suggestion comes from a senior consultant at a leading management consultancy.

The best option would be to combine both a stock and a flow measure of capital. Since the former is not available to us, we have to continue with the latter, bearing in mind its flaws.

5.2.3 Pooling Observations

Pooling Across Sections

We attempt to provide an economy-wide analysis of efficiency of SME. To accomplish this, we utilize maximum available information and pool observations across sections. If a vector $(\boldsymbol{wx}, py)$ is observed, it has to be feasible by definition. Hence once an observation is made, it is only natural to add it to the money-metric production set $\mathscr{M}$ defined in (2). Contrary to models involving detailed structure, our version provides framework which is flexible enough that pooling across heterogeneous sectors is economically meaningful.

Pooling Over Time

Using observations from different time periods raises major difficulties in any econometric analysis, because they cannot be regarded as independent.

Methodology for nonparametric methods is provided in Tulkens & Eeckaut [42]. They consider two main approaches: Either we construct a frontier for each year separately, or we update the frontier every year with new observations. This offers a decomposition of frontier shift and firm specific efficiency change.

We decided to use 2007 as the reference year against which efficiency is measured, so that the number of reference observations to construct $\widehat{\mathscr{M}}_{2007}$ was $1,526$.[14] This involves a significant computational simplification (see section 5.3.3), but only a minor loss of information. The relative efficiency ranking across years and across sectors remains the same, we only forfeit the absolute efficiency ranking for each given year. This simplification further makes comparison of efficiency scores more intuitive than in Tulkens & Eeckaut [42].

It must be noted that once a subset of observations is used as reference set, some observations might get 'super-efficient', that is they might achieve scores higher than one.

5.2.4 Data Processing

Outliers

When we run a simple free disposal hull efficiency measure from Cooper *et al.* [9, equation 4.69] as a preliminary test with all $16,826$ observations, most of the efficiency scores lied within a reasonable interval of three standard deviations. However a small number of scores were wildly away, such that in a few cases the computer was effectively attaching them zero efficiency on a standardized interval $[0,1]$.

Therefore we decided to employ outlier detection suggested by Wilson [44] to trim $\simeq 0.5\% \leftrightarrow 80$ observations. The principle of Wilson's measure is to compare the volume spanned by the

[14] Outliers are excluded in this figure.

whole dataset to the volume spanned by a subset where one or more points are deleted. For technical details see Wilson [44]. The number of observations was cut to $N = 16,746$.

Heteroscedasticity

A convenient property of nonparametric estimators is that data do not need any standardization. Nonparametric estimators automatically deal with heteroscedasticity, and consequently we do not have to scale the data as, for example, cost per unit of value added. This in turn means that we do not have to adopt any prior parametric assumption, which would normally be required for scaling.

This property has another important implication: The dataset does not have to be deflated by an inflation index. Given that the same deflating measure would be applied to all observations in a given year, it would not change the relative position of an observation as compared to other observations from that year. Deflating the data would only affect the relative spread of efficiency scores over time, yet this effect would be spurious because an aggregate inflation index does not reflect relative inflation in each sector.

5.3 Evaluation of Efficiency

5.3.1 Order-m Estimator

We employ the nonparametric estimator of efficiency by Cazals *et al.* [6]. The estimator is based on Assumption 4.2.1 of Simar & Wilson [37], which can be modified to our framework: Money-valued inputs and outputs are a pair of *i.i.d.*[15] multidimensional random variables $(\mathcal{WX}, \mathcal{PY})$ with a probability density on the support $\mathscr{M}$, with the property $\Pr((\boldsymbol{wx}, \boldsymbol{py}) \in \mathscr{M}) = 1$ so that there is no statistical noise.

The robustness of this estimator comes from the fact that we are comparing an observation $(\boldsymbol{wx}_0, \boldsymbol{py}_0)$ not to the whole sample, but to a randomly drawn subset of the sample. Averaging over the subset-dependent efficiency scores gives expected efficiency.

5.3.2 Convexity considerations

To evaluate efficiency relative to the money-metric frontier $\mathrm{Eff}(\mathscr{M})$, we must decide whether the empirical counterpart to equation (2) is a convex or nonconvex hull of available observations.[16]

We claim that the non-convex approach (FDH) is preferable. Convexity of the reference frontier is based on the replication argument. But $\mathscr{M}$ incorporates a variety of factors: technology, market structure, negotiation, managerial abilities etc. Reasoning based on replication is likely to fail here, and FDH is more appropriate. Moreover, convexity is only important in

[15] Independent and identically distributed.

[16] The convex approach is called "data envelopment analysis" (DEA), the non-convex is called "free disposal hull" (FDH).

small samples: When the number of observations grows, approximation of the true frontier in $\widehat{\mathscr{M}}$ will approach strict convexity even for FDH.[17]

5.3.3 Monte-Carlo Simulation

Order-m expected efficiency can be estimated as integration 3.5 in Cazals *et al.* [6], which does not have an analytical solution. Cazals *et al. ibid* proposed a four step Monte-Carlo algorithm. We take the computation from Daraio & Simar [12, p. 72] and adjust it to our money-metric frontier:

[1] Draw a sample with replacement among $\boldsymbol{wx}_i$ such that $\boldsymbol{py}_i \geq \boldsymbol{py}_0$ and denote this sample $(\boldsymbol{wx}_{1,b}, \ldots, \boldsymbol{wx}_{m,b})$.

[2] Compute

$$\tilde{\tilde{\theta}}_{OM,b}(\boldsymbol{wx}_0, \boldsymbol{py}_0) = \min_{q=1,\ldots,m} \left\{ \max_{j=1,\ldots,r} \left(\frac{wx^j_{q,b}}{wx^j_0} \right) \right\}.^{18}$$

[3] Redo [1]-[2] for $b = 1, \ldots, B$, where B is large.

[4] $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_0, \boldsymbol{py}_0) = \frac{1}{B} \sum_{b=1}^{B} \tilde{\tilde{\theta}}_{OM,b}(\boldsymbol{wx}_0, \boldsymbol{py}_0)$.

The simulated efficiency estimator $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_0, \boldsymbol{py}_0)$ lies in $(0, 1)$ for inefficient observations.

After experimenting with the behaviour of $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_0, \boldsymbol{py}_0)$ in smaller subsamples, and taking into account computational aspects, we finally specified $q = 150 \simeq 10\%$ and $B = 100$. The computational burden is considerable: Our specification required 75 minutes to compute.[19] This is one of the factors why we use only one year to construct the reference set $\widehat{\mathscr{M}}_{OM,2007}$.

5.4 Analysis of Efficiency Scores

5.4.1 Two-Stage Regressions

In our analysis, we would like to go further and find regular patterns in efficiency scores. Regressing estimates on explanatory variables other than those included in the production process — we shall denote them $\boldsymbol{z}$ — is widespread.

This practice was heavily criticised by Simar & Wilson [37, ch. 4.6]. The problem with a second stage regression is that estimates of efficiency are biased and serially correlated, and

[17] It must be noted that the use of FDH was questioned by Thrall [41]. The criticism regards efficiency decomposition: Because FDH frontiers are not convex, some points on the 'efficient' frontier will necessarily be allocatively inefficient. But firstly this conclusion of Thrall was opposed by Cherchye *et al.* [7]; and secondly once our frontier is money-valued, this concern is irrelevant. Another complication of FDH is identification of returns to scale, but solutions are now available (Soleimani-damaneh & Reshadi [38]).

[18] Note that wx^j is the j-th element of vector $\boldsymbol{wx}$. This min-max algorithm is computationally equivalent to eq. 4.69 in Cooper *et al.* [9], see eqs. 2.26-2.27 in Daraio & Simar [12, p. 37].

[19] On a computer with 3 GHz processor and 2GB RAM.

by construction induce dependence between the error term and explanatory variables in the second stage regression.

A full statistical model which incorporates second stage analysis of efficiency scores, and which mitigates the above shortcomings, was developed by Simar & Wilson [36]. Their model is based on the assumption that a vector of additional variables $\boldsymbol{z}$ directly influences efficiency, so that for the joint distribution holds $G(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \neq G(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{z})$. This form of statistical dependence is crucial, since as Simar & Wilson [36, p. 39] argue:

> otherwise, there would be no motivation for the second-stage regression.

5.4.2 Reformulation for Money-Metric Frontiers

Our model of efficiency frontier $\mathrm{Eff}(\mathscr{M})$, as we formulated it in section 4, is concerned with overall economic efficiency. This raises the question which variables belong to $\boldsymbol{z}$ in the distribution $G(\boldsymbol{wx}, \boldsymbol{py}|\boldsymbol{z})$

The measure of performance in our model is strictly monetary, so that it attempts to approximate profitability. Hence conditioning (i.e. environmental) variables $\boldsymbol{z}$ must be economic concepts concerning both external and internal environment in which firms operate. The former (external) could be captured by information on market structure, e.g. concentration indices. The latter (internal) are related to organization, management and entrepreneurship. For example, Man *et al.* [28] developed a conceptual model of entrepreneurial success, which consists of (1) competitive scope, (2) Entrepreneurial competencies, and (3) Organizational capabilities (see figure 4 *ibid*).

Nevertheless analysis of these factors lies beyond the scope of this article, not least because no such information is present in our dataset.

5.4.3 Ex-post Analysis

The data described above does not include any environmental variables, but we still would like to understand if some sectors show better performance than others, or whether efficiency improved over time. Obviously, by no reason should time or sectoral classification influence efficiency in the economic sense; this information is only collected ex-post. Does the sceptical view of Simar & Wilson [36] mean that we cannot infer anything about efficiency patterns in this case?

We want to see if efficiency score can be significantly explained along a sectoral classification. Therefore what we attempt is a decomposition motivated by 'unobserved components' class of models. A regression based on separation efficiency effects across three dimensions — time, sector, and firm size — cannot be justified in the sense of Simar & Wilson [36]. However we contend that it can still be useful from the empirical viewpoint, as a complement to pure descriptive analysis of efficiency scores.

5.4.4 Regression Specification

The model we employ in the second stage reads:

$$\begin{aligned}\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_{i,t}, py_{i,t}) &= \alpha + \beta \cdot NEF_{i,t} + \sum_{t=1999}^{2007} \delta_t \cdot YEAR_{i,t} + \sum_{u \in \{SIC1\}} \delta_u \cdot SIC_{i,t} + \\ &\quad \sum_{v \in \{EG\}} \delta_v EG_{i,t} + \xi_i + \epsilon_{i,t}, \qquad (4)\end{aligned}$$

where NEF is average number of employees per firm in the given four-digit SIC sector i[20], $YEAR$ is year dummy, SIC is sector dummy based on one-digit aggregated SIC[21], and EG is dummy for number of employees, grouped as 1-10, 11-25, 26-50, 51-100, 101-250, and >250.

To estimate this model, we interpret ξ_i as random effects. Because the variables included in (4) represent ex-post clustering, it is reasonable to assume zero correlation between ξ_i and regressors.

Further, contrary to Simar & Wilson [36], we use a robust measure of efficiency where scores are distributed on both sides of the efficient frontier. Therefore there is no need to compute truncated normal regression, instead ϵ is viewed as Gaussian.

The most pressing problem in (4) is the correlatedness among $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_{i,t}, py_{i,t})$. To obtain a meaningful covariance matrix we applied bootstrapping. We did not use the algorithm proposed by Simar & Wilson [35]. The shortcoming of their procedure is that they add new information into the sample, because estimates are updated each time based on draws from truncated normal distribution. This yields consistency if the underlying model of truncated normal distribution is correctly specified. Yet it also decreases the robustness of such an approach when the underlying distribution is not close to truncated normal. Hence we used simple bootstrap available directly in **STATA**, where covariance matrix is computed for repeatedly drawn subsamples from the data.

6 Results

6.1 First Stage

Computations were implemented in the statistical package **R**, using library FEAR by Wilson [45]. See table 1 for a summary.

Efficiency scores $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i)$ have to be regarded as relative ratios against the efficient level equal to one. Hence in table 1 the mean of $\simeq 0.477$ means that average sectors are less than half efficient as compared to best performers. Recall that this result holds even after accounting for outliers, who are ranked as 'superefficient'.

Interpreting efficiencies is not clear cut. Our measure is monetary, so that the pnly driving factor is costs per unit of value added. We built our model so that we are not able to distinguish

[20] Computed as Total number of employees divided by Number of firms.

[21] One-digit SIC has more detailed classes A-O, but we could not regroup our observations this way. Instead, we created groups according the first digit of the four-digit SIC, which yielded ten clusters.

technical and allocative efficiency. However, the great advantage of our approach is that it directly accounts for quality as it is perceived by buyers, because all production is priced.

Table 1 conveys one fundamental message. The wide dispersion of efficiency scores implies the need for more dynamic models of short-run out-of-equilibrium adjustment. Static equilibrium analysis helps us define and understand concepts of efficiency. Nonetheless our results suggest that imposing equilibrium conditions in empirical work on sectors that are not narrowly defined could potentially be misleading.

Visualising data with number of observations this large would require sophisticated tools and more space, because standard scatterplot matrix proved to be disorderly. Due to limited space, we illustrate only the most important relationship between efficiency and size of companies. In figure 1, we use the method of hexagon binning[22] to approximate the two-dimensional distribution, where the colour of each hexagon represents the number of observations in its area. Displayed are 13,871 observations restricted to satisfy $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i) \in [0,1]$ and $NEF_i \in (0,250]$. From the clusters in the figure it is apparent that the majority of observations do not achieve full efficiency.

6.2 Second Stage

Results from the previous section still suffered from extreme points, with the farthest observation being 100 times more efficient than the unit reference frontier. Wilson's method to detect outliers ex-ante, as described in section 5.2.4, proved unsatisfactory, so we omitted approximately 1% of observations before conducting the second stage analysis, yielding $N_2 = 16567$.[23]

Regression (4) was implemented in the package **STATA** using maximum likelihood estimation. The first dummy in each group was automatically dropped due to perfect multicollinearity. The results are summarised in table 2.

6.2.1 Overall Significance

Although the regression is significant as a whole according to the Wald test, most of the individual dummies are not. It must be noted that dropping either one of the three dummy groups resulted in insignificant regressions. Hence it appears that the clusters capture a good portion of information on distribution of the efficiency scores.

Nonetheless most of the effects alone do not move efficiency in a definite direction. Specifically, only three dummies have their confidence interval with both limits of the same sign.[24]

[22] Library 'hexbin' for **R**, see Lewin-Koh [27].

[23] Precisely, we removed 82 observations with $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i) < 0.0485$, 80 observations with $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i) > 4.56$, and 17 observations where $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i)$ could not be computed.

[24] We obtained the same result with dummies coding two-digit SIC groups.

6.2.2 Size Effect

The coefficient on average number of employees per firm, β, is not significant, and this result is robust to dropping EG dummies. The EG dummies are in fact the most dominant effects, with δ_{GE3} and δ_{GE4} being close to significant and δ_{GE5} and δ_{GE6} being strongly negative. Moreover, the magnitude of the effect is increasing with the number of employees, which suggests that with more employees economic efficiency *worsens*.

We are aware that our finding with respect to size is not sufficiently significant, and that it appears counterintuitive. The conclusion that we draw is more cautious: The results hint that a positive relationship between size and efficiency proposed by earlier studies is limited to certain sectors and does not apply to the economy as a global principle.

6.2.3 Time Effect

Although two of the time effects are significant, the overall message is blurred as no clear direction of the effect over time can be seen. We investigated this further by including a simple time trend $\gamma \cdot t$ in (4), where we followed Battese & Coelli [2]. The result was insignificant both with or without year dummies, so we do not report it here.

The hypothesis that *economic* efficiency changed over time was therefore strongly rejected. This statement must however be read in its positive sense, not normative. For example, in one possible underlying scenario technical efficiency might have improved due to growth of labor productivity, but this might have been compensated by higher wages, so that overall the effect cancelled out. Because GVA less wages and capital costs can be viewed as a proxy proportional to profits, our results reveal that the share of revenues from entrepreneurial activities going to equity holders remained constant over time.

6.2.4 Mean Efficiency

Finally, significance of α statistically confirms the outcome of table 1: Average sectors are expected to be between quarter to half efficient relative to the best practice frontier (see confidence interval in table 2). This once again underlines not only the dynamic nature of competition, but also the magnitude of competitive pressures in the markets.

7 Conclusions

In the previous section we presented detailed efficiency analysis of British SME. We would like to stress the robustness of our work and its complementarity to previous research. Both these advantages are based on these features of the article: Firstly, we proposed a general methodological framework for nonparametric evaluation of economic efficiency which we call *money-metric efficiency frontier* $\mathrm{Eff}(\mathscr{M})$. This clarifies and extends the approach of previous papers. Secondly, our dataset ranges from agriculture to services, and this allowed us to test economy-wide hypotheses which were not yet examined. Thirdly, we employed state-of-the-art robust method for efficiency estimation. The nonparametric nature seems especially suitable for our large dataset.

Our results are related to economic efficiency, which we modelled as creation of value added relative to costs of inputs. The findings can be summarised in the following stylised propositions:

1. Efficiency scores across sectors are very dispersed, which implies great heterogeneity within the economy. Specifically, we contend that it calls for more focus on out-of-equilibrium competitive and adjustment processes in further research.

2. We do not find significant evidence of an economy-wide size-efficiency relationship. Small samples benefit from better defined structure, but our finding implies that previous studies' results documenting a positive size-efficiency relationship are specific to either technical efficiency or to narrow sectors. We find mild evidence that the largest firms creating less value added per unit of costs, which might be due to reporting bias.

3. Economic efficiency remains relatively stable over time. In our view this constitutes evidence that wealth gains from presumed technology advances are evenly distributed across stakeholders in firms (i.e. owners and providers of labour and capital).

4. Average sectors are expected to be two to four times less efficient than those on the efficient frontier. We interpret this as an indicator for the magnitude of competitive pressures in the markets.

Two extensions of our second stage analysis are straightforward: Firstly, we did not structurally address the dependence of efficiency scores between size groups (EG) within one SIC sector. This would require a more detailed three-level model, where we would consider possible combinations of interaction effects between the three levels time–sector–EG. Secondly, we could specify a dynamic regression with lagged efficiency score among explanatory variables $\boldsymbol{z}$, using GMM estimation. These extensions are left for further research.

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Table 1: Box plot statistics for efficiency scores $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i)$.

min	1Q	median	3Q	max	mean
0.03958	0.18374	0.31484	0.47763	0.91753	0.46722

min $= 1Q - 1.5(3Q - 1Q)$, max $= 3Q + 1.5(3Q - 1Q)$.
$\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i) = 1 \Rightarrow (\boldsymbol{wx}_i, py_i)$ is expected to be efficient according to the approximation of $\mathrm{Eff}(\widehat{\mathscr{M}}_{OM,2007})$.
$\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i) < 1 \Rightarrow (\boldsymbol{wx}_i, py_i)$ is inefficient.
$N = 16746$. Number of superefficient observations: $891 \simeq 5.3\%$.

Figure 1: Distribution of $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i)$ against NEF_i.

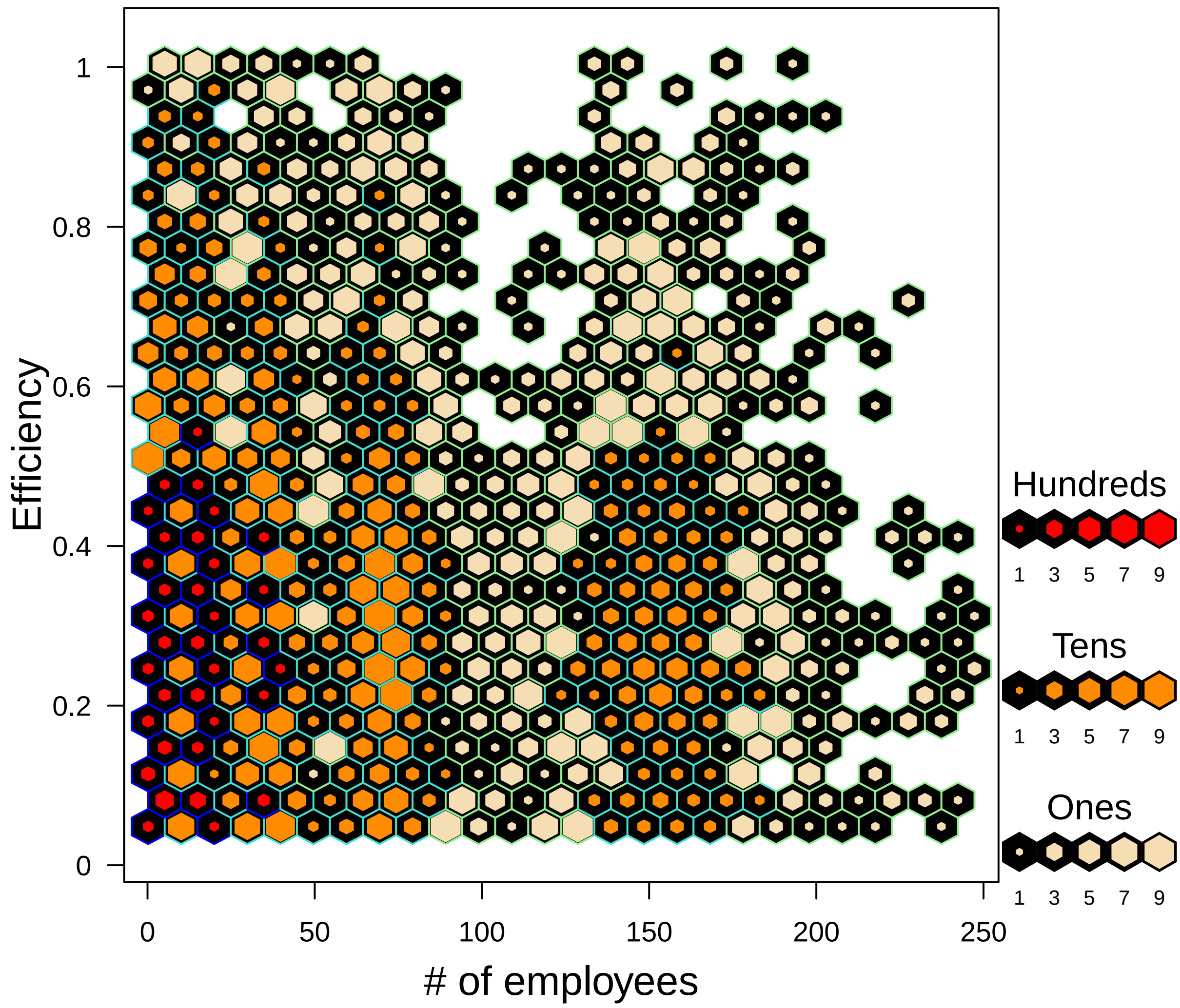

Table 2: Maximum likelihood estimation of the model (4).

	Coefficient	Bootstrap Std. Err.	p-value	95% Conf. Interval	
β	0.0000131	0.0000124	0.291	−0.0000112	0.0000375
δ_{1999}	−0.0154784	0.0115287	0.179	−0.0380743	0.0071176
δ_{2000}	−0.014184	0.0139489	0.309	−0.0415233	0.0131552
δ_{2001}	−0.0292996	0.0113953	0.010	−0.051634	−0.0069652
δ_{2002}	−0.0127928	0.0161132	0.427	−0.0443741	0.0187886
δ_{2003}	−0.0099504	0.0129875	0.444	−0.0354054	0.0155046
δ_{2004}	−0.01348	0.0124282	0.278	−0.0378388	0.0108789
δ_{2005}	−0.0190152	0.0128486	0.139	−0.044198	0.0061676
δ_{2006}	−0.0297586	0.0156952	0.058	−0.0605206	0.0010033
δ_{2007}	−0.0071706	0.0143761	0.618	−0.0353472	0.021006
δ_{SIC1}	0.049848	0.0677879	0.462	−0.0830139	0.1827099
δ_{SIC2}	0.0597635	0.0682027	0.381	−0.0739114	0.1934384
δ_{SIC3}	0.0535592	0.0675033	0.428	−0.0787448	0.1858631
δ_{SIC4}	0.0388944	0.0676146	0.565	−0.0936279	0.1714167
δ_{SIC5}	0.0633962	0.0679409	0.351	−0.0697656	0.196558
δ_{SIC6}	0.098012	0.071844	0.172	−0.0427996	0.2388236
δ_{SIC7}	0.0611964	0.0673043	0.363	−0.0707175	0.1931103
δ_{SIC8}	0.0302315	0.0723416	0.676	−0.1115555	0.1720184
δ_{SIC9}	0.0436897	0.0684226	0.523	−0.0904161	0.1777955
δ_{GE2}	−0.0042957	0.012248	0.726	−0.0283013	0.0197099
δ_{GE3}	−0.0226775	0.0137472	0.099	−0.0496215	0.0042664
δ_{GE4}	−0.0285102	0.0149814	0.057	−0.0578732	0.0008529
δ_{GE5}	−0.0347542	0.0135295	0.010	−0.0612716	−0.0082368
δ_{GE6}	−0.0445577	0.0214728	0.038	−0.0866435	−0.0024718
α	0.3721508	0.0677007	0.000	0.2394598	0.5048418
σ_ξ	0.1115971	0.0055925		0.1011572	0.1231145
σ_ϵ	0.3720866	0.0085735		0.3556566	0.3892756
Wald χ^2 (df)	68.00		0.000		
♯ of obs.	16567				

Dependent variable is $\hat{\hat{\theta}}_{OM}(\boldsymbol{wx}_i, py_i)$.

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Univerzita Karlova v Praze, Fakulta sociálních věd
Institut ekonomických studií [UK FSV – IES] Praha 1, Opletalova 26
E-mail : ies@fsv.cuni.cz http://ies.fsv.cuni.cz