

A Reply to “Option Value of Harvesting” Revisited (The Comment)

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Abstract *Li's results (1998) are correct and robust. The Comment offers an interesting perspective from real-options theory, albeit misguided—from a resource-economics point of view. In particular, the lack of sensitivity in the optimal harvest trigger to biomass uncertainty is questionable. This reply begins with a general overview of resource-economic principles and the role of real options in them. A specific reply to each point raised in The Comment will then be addressed.*

General

Let us review the standard case of the economically efficient harvester. He maximizes his harvest profit:

$$\Pi(E) = \text{revenue} - \text{cost} = p H - c E. \quad (1)$$

E is the fishing effort (the control variable), p the fish price, and c the unit cost of effort. The total harvest, H , is given by the Gordon-Schaefer average surplus yield (G-S ASY):

$$H = qEX = qEk \left(1 - \frac{qE}{r} \right), \quad (2)$$

where k is the ceiling stock size, r the natural stock growth rate, and q the fixed catchability coefficient. In equation (2) we used:

$$X = k \left(1 - \frac{qE}{r} \right), \quad (3)$$

where X (sustainable stock size) is a state variable resulting from G-S ASY. Substituting equation (2) into equation (1) and maximizing over the control variable, E , will give us the profit-maximizing effort:

$$E' = \frac{r}{2q} \left(1 - \frac{c}{pqk} \right). \quad (4)$$

We show E' in figure 1. It also shows the MSY (maximum sustainable/surplus yield) effort, E^* . In general, E' is more conservationist, while MSY E^* is not economically efficient. The parallel lines indicate how the efficient E' is obtained. The upper (lower) line represents the slope of the revenue (cost) curve. When the two slopes are equal (parallel lines), it implies the derivative of (1) with respect to E is set to zero. This is for profit maximization (and economic efficiency).

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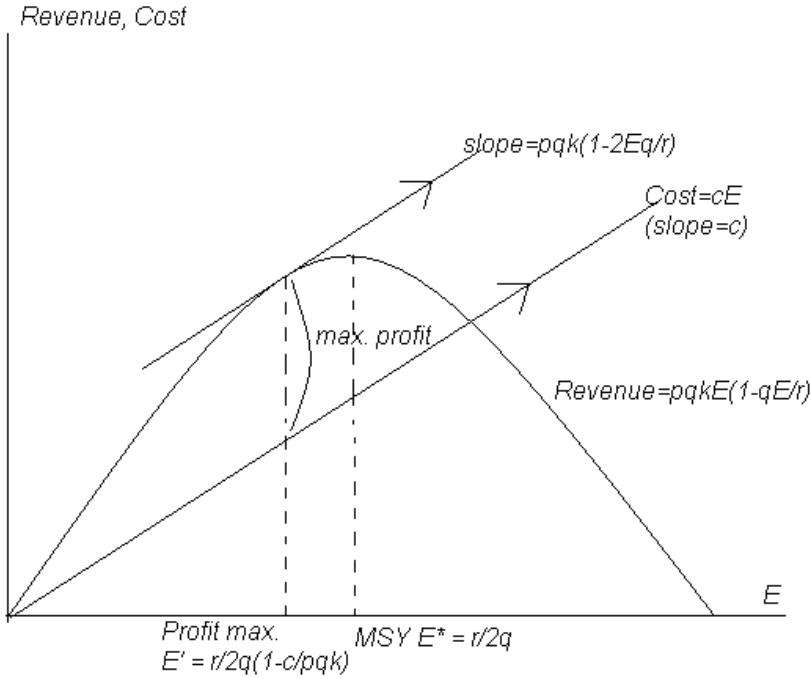


Figure 1. Economic Efficiency vs. Maximum Sustainable Yield

Next, we introduce the option value of harvesting into the profit equation (1):

$$\Pi(E) = \text{revenue} - \text{cost} - \text{value of option forgone.} \tag{5}$$

In other words, harvesting is justified only when revenue exceeds the cost of fishing and the value of the harvest option used. If we follow Li (1998), the harvest option value is (equation 8, p. 137):

$$\frac{pqEX}{\beta_1} = \frac{pqkE}{\beta_1} \left(1 - \frac{qE}{r} \right), \tag{6}$$

X is the stock size that justifies harvesting, and β_1 is a bigger-than-1 constant. In equation (6) we expressed X in terms of E using the G-S ASY.

At this stage, we are free to set E at any value that will maximize total profit, $\Pi(E)$. At the same time, stock sustainability is guaranteed through G-S ASY. Carrying out the maximization will give us the optimal effort E_{op} (Li 1998, equation 11, p. 138). The optimal harvest trigger, X_{op} , and CPUE follow immediately from equations (3) and (2), respectively.

The crucial point is that E is not constant. It can be set at any value to achieve economic efficiency. Before the harvest trigger is reached, $E = 0$. When the harvest trigger is reached, E is set at the value that maximizes total fishing profit (revenue – cost – option value). X cannot be controlled (directly) by the harvester. Substituting E out by X (equation 4 in The Comment) will remove the harvester’s ability to maximize profit for economic efficiency. Furthermore, the derivative with respect to X in the smooth-pasting condition (equation 10 in The Comment or Li 1998, equation 7, p. 137) should not in-

volve E . E is the variable of optimization in the profit-maximization stage after the option value is derived.

That E and X are unrelated in the option value stage is not an internal inconsistency with G-S ASY. When determining the option value, no harvesting is actually taking place (see Li 1998, line 26 (directly above equation 8), p. 137 – "...the level of profit (*i.e.*, option value) required before harvest is initiated..."). Once harvesting starts, the option to harvest is killed. The harvester proceeds to set an E that maximizes his profit for economic efficiency. At this stage, G-S ASY must be invoked for stock sustainability.

Specific (reference to The Comment)

1. E was related to X by G-S ASY (lines 2-3, abstract).

Reply: E and X are not related in the option value stage (approaching boundary) since harvesting is not taking place. The option to harvest is retained.

2. E and X were treated as unrelated to each other (lines 4-5, abstract).

Reply: E is the control variable for profit maximization. Hence, it is not affected by the stock-size derivative in the smooth-pasting condition (equation 10, The Comment). E should not be substituted out by stock size, X . X should be substituted out by E during profit maximization.

3. E and X are related to one another (lines 7-8, p. 77).

Reply: X is a state variable, which is the result of E (control variable). Therefore, X is a decreasing function of E at time of harvest.

4. When deriving the option value, E is treated as if it is unrelated to X (lines 9-10, p. 77).

Reply: E and X are unrelated in the option value stage since no harvesting is taking place. The harvest production function, $H = qEX$, is only notional *ex ante*. Once harvesting begins, the option is killed. The harvester then proceeds to set E that maximizes his harvest profit. Now G-S ASY must be enforced for stock sustainability.

5. Incorporate the G-S ASY relationship between E and X when deriving option value and optimal harvest trigger (lines 11-12, p. 77).

Reply: The G-S ASY relationship between E and X need not be invoked when deriving the option value. There is no harvesting at this point. As a result, E should not be expressed as a function of X in the value-matching (VM) and smooth-pasting (SP) conditions. These conditions are used to derive the option value, not the optimal harvest trigger. Consider the following sequence adopted in Li (1998): leave E as free variable in VM and SP → solve for the option value ($pqEX/\beta_1$) → insert option value in profit equation $\Pi(E)$ → substitute X out by E using G-S ASY → maximize $\Pi(E)$ over E → obtain optimal E_{op} → calculate optimal harvest trigger X_{op} .

6. The G-S ASF relationship was not enforced (lines 15-16, p. 77).

Reply: G-S ASF need not be enforced in the option value stage since no harvesting is actually occurring. Furthermore, it is not E but H (harvest yield) that is constrained

by the biological sustainability requirement. The harvest yield function $H = qEX$ (equation 2, this reply) where $X = k(1-qE/r)$ is the Gordon-Schaefer average surplus yield (G-S ASY). E is a free variable during profit optimization as long as G-S ASY is satisfied.

7. Without any constraints on E or H , option value would be higher, but H might get so large that the fish stock may be driven to extinction (lines 18-20, p. 77).

Reply: When the option value is higher, the optimal harvest trigger will be more conservative (larger). The harvester will less likely harvest. As a result, stock will less likely be driven to extinction.

8. E is related to X by the sustainable yield model of Gordon-Schaefer (G-S ASY). (equation 4, p. 78).

Reply: E should stay a free variable at the VM and SP stage. Equation (4) (in The Comment) should express X as a function of E . That is, X is a result of exerting E . Furthermore, this step should occur at the profit-maximization stage, after VM and SP.

9. The option value satisfies the ordinary differential equation (ODE) using standard techniques (equation 5, p. 78).

Reply: The ODE is derived from the following economic reasoning. A small random change in stock size, dX , over dt results in a small expected option value change of $E[F(X + dX)]$. This can be discounted back to t using the discount rate ρ ; *i.e.*, $F = e^{-\rho dt} E[F(X + dX)]$. $e^{-\rho dt}$ is approximately $1 - \rho dt$, and $E[F(X + dX)]$ can be expanded by Ito's lemma (Dixit and Pindyck 1994). That is, $E[F(X + dX)] = E[F(X) + dX F'(X) + 0.5 (dX)^2 F''(X)] = F(X) + \alpha X F'(X) dt + 0.5 \sigma^2 X^2 F''(X) dt$. On rearranging, we have the ODE.

10. E should not be unrelated to the fish stock at harvest X^* (lines 6-7, p. 79).

Reply: E is unrelated to X^* at the option value stage since fish stock is *not* at harvest. In fact, X^* (the optimal harvest trigger) is unknown at this stage. X^* (a function of the optimal effort E_{op}) is obtained only after E_{op} is determined during the profit-maximization stage. See Reply (12) below.

11. The optimal harvest trigger can be used to compute the optimal effort, optimal harvest size, and resulting CPUE (lines 1-2, p. 80).

Reply: The optimal effort should be obtained first, then use it to obtain optimal harvest trigger, harvest size, and CPUE. See Reply (5) above.

12. This is the optimal harvest trigger, X_{op} , from Li (1998) (equation 16, p. 80).

Reply: This is not X_{op} . X_{op} is not given in Li (1998). Li's X_{op} can be obtained as follows:

$$X_{op} = k \left(1 - \frac{qE_{op}}{r} \right) \quad (\text{using G-S ASY})$$

$$= \frac{1}{2}k \left(1 + \frac{c \left(\frac{\beta_1}{\beta_1 - 1} \right)}{pqk} \right)$$

(using the optimal effort, E_{op} , from Li 1998, equation 11, p. 138).

13. X^* is not very sensitive to uncertainty (lines 23-24, p. 80).

Reply: In figure 1 (p. 81, The Comment), σ (sigma) represents stock size volatility in percentage terms. This is a result of the geometric Brownian motion. As the commenter's harvest trigger, X^* , suggests, it more or less fluctuates around $5E+05$. At the same time, σ moves from 0 (no volatility) to 1 (a 100% increase or decrease in size). This lack of sensitivity to uncertainty is questionable. When stock could be totally wiped out in the next period, one would expect a much more conservative (higher) trigger than $5E+05$. Ditto when stock could be doubled next period--harvesting the stock now is tantamount to killing the goose that lays the golden egg.

14. Harvesting effort is not constrained in Li (1998) (Conclusion, line 1; p. 80).

Reply: E need not be constrained at the option value stage since fish stock is not at harvest. Harvest size/yield, H, should be constrained by G-S ASY for biological sustainability in the profit-maximization stage. This is adopted in Li (1998) (equation 10, p. 138).

15. The option aspect becomes less important when there are constraints on effort and harvest size (lines 3-4, p. 81).

Reply: The sustainability constraint (G-S ASY) is enforced on H (Li 1998, equation 10, p. 138) in the profit-maximization stage. The option aspect is important and becomes more so as X becomes more uncertain. This is because the larger-than-1 option-value multiple $\beta_1/(\beta_1 - 1)$ decreases fishing effort E_{op} (Li 1998, equation 11, p. 138) and increases the optimal harvest trigger X_{op} (Reply 12). Furthermore, the larger the stock uncertainty, σ , the higher the option-value multiple, the lower the effort, E_{op} , and the higher the optimal trigger, X_{op} .

Conclusions

Li's results (1998) are correct and robust. The Comment ("Option Value of Harvesting" Revisited) offers an interesting perspective from real-options theory, albeit misguided --from a resource-economic point of view. In particular, the lack of sensitivity in the optimal harvest trigger to biomass uncertainty is questionable.

References

- Dixit, A., and R. Pindyck. 1994. *Investment Under Uncertainty*. Princeton, NJ: Princeton University Press.
- Li, E. 1998. Option Value of Harvesting – Theory and Evidence. *Marine Resource Economics* 13:135-42.

