“Option Value of Harvesting” Revisited

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Abstract This article corrects an internal inconsistency in Li’s (1998) model of the option value of fishery harvesting. In that model, the harvesting effort was related to the fish stock at harvest by the Gordon-Schaefer average sustainable yield model. However, when deriving the option value, the harvesting effort and fish stock at harvest were treated as unrelated to each other. I show that, when this inconsistency is rectified, the option value is smaller, and as a result the optimal harvest trigger is lower (or moves closer). Further, the optimal harvest trigger becomes less sensitive to the degree of uncertainty regarding the evolution of the fish biomass.

Key words Fishery harvesting, optimal trigger, real option.

JEL Classification Codes Q22, C61.

Introduction

Li’s (1998) “Option Value of Harvesting: Theory and Evidence” was one of the early papers to use option theory for fishery harvesting decisions. Under the assumptions of uncertainty, irreversibility, and the ability to time/delay the harvest, it derives the optimal harvesting trigger and related outputs (effort, harvest size, and cost per unit of effort [CPUE]) when the fish stock (biomass) follows a stochastic lognormal process. The harvesting effort in Li’s (1998) model is determined by considerations of long-term sustainability using the Gordon-Schaefer model, which implies that the harvesting effort and the fish stock at harvest are related to one another.

However, when deriving the option value, the harvesting effort is treated as if it is unrelated to the fish stock. In order to correct this inconsistency, I re-work Li’s (1998) model but incorporate the Gordon-Schaefer relationship between harvesting effort and fish stock when deriving the option value and optimal harvest trigger.

Therefore, in our model, when determining the option value and the optimal harvest trigger, the effort level is constrained by a biological sustainability requirement from the Gordon-Schaefer model, as opposed to Li (1998) where the relationship was not enforced. This constraint results in a smaller option value, as expected. Since option value is smaller, the option will be exercised earlier. Thus, in our model, the optimal exercise trigger will be lower. Note that without any constraints on effort or harvest size the option value would be higher, but the harvest size might get so large that the fish stock may be driven to zero (or extinction) if it makes economic sense (see Clark 1990).

The main result is that the optimal harvest trigger is lower than in Li’s (1998) model, consistent with the above discussion. Moreover, although this trigger is an increasing function of the uncertainty or volatility of fish stock evolution (similar to Li 1998), it is less sensitive than implied by Li’s model. While the directions of the comparative static results remain the same as in Li (1998), the magnitudes can be very different.

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The Model

The model is the same as in Li (1998). That is, the fish stock or biomass \( X_t \) evolves stochastically as a Geometric Brownian Motion Process:

\[
dX = \alpha X dt + \sigma X dz,
\]

where \( \alpha \) is the intracycle stock movement, \( \sigma \) is the volatility (uncertainty) of the process, and \( dz \) the increment of the standard Weiner Process.

The production or harvest function is given by:

\[
H(E,X) = qEX,
\]

where \( E \) is the harvesting effort level and \( q \) is the fixed catchability coefficient. Equation (2) shows the amount harvested. If the unit catch price is \( p \) and the cost per unit of effort is \( c \), the profit or payoff from the harvest is given by:

\[
\pi(E,X) = pqEX – cE.
\]

Of course, there is no reason for harvesting immediately; \( i.e. \) there is an option to harvest. In a real-option setting, harvest will occur when the fish stock, \( X \), rises to some critical level, say \( X^* \), called the harvest trigger. Following Li (1998), I assume that the effort level, \( E \), is related to the fish stock at harvest by the Gordon-Schaefer average sustainable yield model:

\[
E = \frac{r}{q} \left( 1 - \frac{X^*}{K} \right),
\]

where \( r \) is the intrinsic natural growth rate and \( K \) is the ceiling stock size.

Option to Harvest

As mentioned above, the ability to harvest can be viewed as an option to harvest. Let this option value be given by \( F(X) \). Assuming a discount rate of \( \rho \), it can be shown using standard techniques that the option value must satisfy the ordinary differential equation (ODE):

\[
0.5\sigma^2 X^2 F''(X) + \alpha X F'(X) - \rho F(X) = 0.
\]

The general solution to the ODE (5) is \( F(X) = A X^{\beta_1} + B X^{\beta_2} \), where \( A \) and \( B \) are constants to be determined by the boundary conditions, and \( \beta_1 \) and \( \beta_2 \) are the positive and negative solutions, respectively, to the quadratic equation:

\[
0.5\sigma^2 \beta(\beta-1) + \alpha \gamma - \rho = 0.
\]

As \( X \to 0 \), we must also have \( F(X) \to 0 \), which implies the constant \( B \) in \( F(X) \) is zero. Then we can write the option value as:

\[
F(X) = A X^{\beta_1}.
\]
The exponent $\beta_1$ is given by:

$$\beta_1 = 0.5 - \alpha / \sigma^2 + \sqrt{2 \rho / \sigma^2 + (0.5 - \alpha / \sigma^2)^2}. \quad (8)$$

In order to determine the constant $A$ and the optimal option exercise boundary (or the optimal harvest trigger) $X^*$, I use the following boundary conditions (as in Li 1998):

$$F(X^*) = \pi(E, X^*) \quad \text{(value matching)} \quad (9)$$

$$F'(X^*) = \frac{\partial \pi(E, X^*)}{\partial X^*} \quad \text{(smooth-pasting)} \quad (10)$$

Equations (9) and (10) can be solved for the optimal harvest trigger, $X^*$, and the constant, $A$, (and thus the option value). Up to this point, our model is identical to Li (1998). Note, however, in equations (9) and (10) the effort, $E$, should not be unrelated to the fish stock at harvest ($X^*$), since we know that they are related in a specific manner by means of the Gordon-Schaefer average sustainable yield model. If we substitute for $E$ using equation (4), then equations (9) and (10) can be written:

$$A(X^*)^{\beta_1} = p q E X^* - c E = r p X^* \left(1 - \frac{X^*}{K}\right) - \frac{c r}{q} \left(1 - \frac{X^*}{K}\right) \quad (9a)$$

$$A \beta_1 (X^*)^{\beta_1 - 1} = r p \left(1 - \frac{2 X^*}{K}\right) + \frac{c r}{q K} \quad (10a)$$

The solution to equations (9a) and (10a) is slightly more complicated than in Li’s (1998) model, because $X^*$ is now derived from a quadratic equation. Equations (9a) and (10a) give us a quadratic equation that can be solved for $X^*$:

$$(X^*)^2 + U X^* - V = 0, \quad (11)$$

where:

$$U = \left(\frac{\beta_1 - 1}{2 - \beta_1}\right) \left(\frac{c}{p q} + \frac{K}{K + \frac{c}{p q}}\right)$$

and

$$V = \frac{c K}{p q \left(2 / \beta_1 - 1\right)}.$$

Solving equation (11), we get the optimal harvest trigger:\[1\]

$$X^* = -U / 2 + \sqrt{(U / 2)^2 + V}. \quad (12)$$

\[1\] There are two solutions to the quadratic equation, but the other solution was discarded because it gave meaningless results.
The optimal harvest trigger, $X^*$, can then be used to compute the optimal effort, optimal harvest size, and the resulting CPUE:

$$E^* = r\left(1 - \frac{X^*}{K}\right)$$  \hspace{1cm} (13)

$$H^* = qE^*X^*$$  \hspace{1cm} (14)

$$\text{CPUE} = \frac{H^*}{E^*}. \hspace{1cm} (15)$$

It can be shown by direct differentiation that the directions of the comparative static results are the same as in Li’s (1998) model.

**Comparison with Li’s Optimal Harvest Trigger**

If $E$ is treated as unrelated to $X$ in equations (9) and (10), the optimal harvest trigger would be given by:

$$X_{\text{op}} = \frac{c}{pq(1-1/\beta_1)}. \hspace{1cm} (16)$$

This is the optimal harvest trigger from Li’s (1998) model (although it is not explicitly specified in his paper, it follows in a very straightforward manner from his equations 3, 6, and 7, as can be easily verified). However, this would be incorrect for the following reason. The decision-maker knows that the effort level will be related to the fish stock at harvest by equation (4), but is nevertheless disregarding the relationship when deriving the harvest trigger. This is not rational behavior, and is clearly sub-optimal.

Comparing $X_{\text{op}}$ (from equation 16) with our optimal harvest trigger, $X^*$, (from equation 12), we find:

(i) $X^*$ is smaller than $X_{\text{op}}$,\(^2\) and

(ii) $X^*$ is less sensitive to uncertainty ($\sigma$) than $X_{\text{op}}$.

The first result is consistent with the earlier discussion; the option value is smaller, since our model puts in an additional (biological sustainability) constraint on harvesting effort when determining $X^*$. Since a smaller option value now needs to be overcome for harvesting, the optimal harvesting trigger is lower in our model. The relationship between $X^*$ and $X_{\text{op}}$ is shown in figure 1 as a function of the uncertainty $\sigma$. I also note from figure 1 that $X^*$ rises with uncertainty, but at a slower rate than $X_{\text{op}}$; thus, $X^*$ is not very sensitive to uncertainty. This is also as expected; since the value of waiting is less important as a result of the sustainability constraint, uncertainty has a smaller role in the constrained model.

**Conclusion**

We show that when the harvesting effort is constrained in Li’s (1998) model, by specifying a relationship between harvesting effort and fish stock at harvest (the Gordon-
The Schaefer average sustainable yield model, the option value is smaller and the optimal harvest trigger is lower. This is not surprising, since this is what we would expect to result from such a constraint. Thus, the option aspect becomes less important when there are such constraints on the effort (and thus the harvest size). The option aspect would become more important if the constraint was removed, but then there might be other implications to worry about; e.g., biological sustainability.

**References**

