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The Hybrid Multidimensional Index of Inequality

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Abstract:
In this paper, we propose a new multidimensional inequality index that satisfies a fundamental set of desired properties. We discuss the case where the social evaluation function of welfare depends simultaneously on unidimensional and multidimensional forms of inequality. We show how this mixed social norm interferes with the most popular axioms conceived specifically for multidimensional indices of inequality. Illustrations of the proposed developments are made using the Cameroonian household surveys, conducted in 2001. It is shown that multidimensional inequality is more pronounced in the Cameroonian semi-urban and rural areas whereas the monetary inequality is more pronounced in urban area.

Keywords: Multidimensional inequality, social welfare, human development

JEL Classification: D63, D31, O15

1 Introduction

Why is the multidimensional inequality (MDI) important to be quantified and analyzed? Why should not we focus on income inequality only? When is MDI relevant? What does the concept *multidimensional inequality* refer to? In developing countries, there is a growing interest to focus on the distributive analysis of well-being with its multidimensional form. Indeed, it is impossible to monetize all types of goods and services. One can recall here that many non-monetized goods can be found in developing countries and where the public sector provides the main part of collective goods. In addition, for other goods, it is difficult to monetize them even in developed countries, like the expectancy life rate.

The developments of Sen (1982) about the welfare economics have contributed radically in redefining this concept. With the Sen's capability approach, we are based on the assessment of a person's advantage, individual differences in the ability to transform resources into valuable activities. Starting from this, the minimum required set of functioning will be sufficient for the person to have the freedom to choose and to lead freely the desired life style. Even if one can agree on the importance of assessing and monitoring the deprivation in these different dimensions of well-being, one can also agree for the need of assessing the inequality in distribution of these non monetary dimensions. For instance, even if the provision of a given public service, like education, has significantly expanded, the bad repartition of this expansion may prevent most of the population to reach these services and will increase the inequality in well-being in general.

The extension of the *Pigou-Dalton Principle* from unidimensional to the multidimensional form of well-being was pioneered by Kolm (1977). Even if the proposed set of social norms that determine the dominance criteria are well founded, they fail to establish a complete order of preferences. One can recall here that this is, sometimes, the case with the unidimensional inequality.[1] As noted by Lugo (2005) for the multidimensional inequality, the use of indices became more attractive to establish the complete order of preferences.

Among the fundamental axioms that concern to multidimensional inequality is *Uniform Majorization Principle* (UPM) (Kolm (1977)). The latter stipulates that the change in distribution is socially desirable if the latter is implied by a decrease in inequality within attributes without increasing the unidimensional inequality (for simplicity, we call it: *income inequality*). The interdependent form of equalization of attributes insures the unidirectional change in the two forms of

[1] A classical example is when the two Lorenz curves cross.

inequality. However, this reduces the set of mappings for which one can take a clear judgment about the nature of change in MDI.

The second fundamental axiom is the *Correlation Increasing Majorization* (**CIM**). This axiom was introduced by Tsui (1999) (See again Atkinson and Bourguignon (1982), Boland and Proschan (1988) , Tsui (1995), (1999) and (2002)). This axiom assumes that it is socially undesirable to have a higher correlation between components. In this paper, we propose to use the unidimensional inequality to catch indirectly the impact of this correlation. Indeed, for the same level of inequality within components, an increase in this correlation implies an increase in income inequality. This argument justifies in part why we propose linking the MDI form of inequality to the unidimensional inequality. Moreover, imposing this axiom enlarges the set of mappings for which one can take a clear judgment about the change in multidimensional inequality.

The rest of the paper is as follow. Section 2 presents the theoretical framework and the related developments for the proposed hybrid multidimensional index of inequality. Section 3 then studies the multidimensional inequality in Cameroon and this by used a national representative household survey, conducted in 2001. Section 4 summarizes the main results of the paper.

2 The theoretical framework

To what refer the concept *inequality*? In general, it refers to the disparity between individual incomes. Any reduction in disparity between two unequal incomes will reduce the level of inequality. Let us recalling the main set of principles that all inequality indices must obey.

1. *Anonymity principle:* Inequality indices are independent of any characteristic of individuals other than their standards of living (or the value of their welfare indicator).
2. *Population principle:* The population principle requires that inequality indices be invariant to replications of the population: merging two identical distributions should not alter inequality.
3. *The Pigou-Dalton transfer principle:* It stipulates that an income transfer from a poorer person to a richer person should register as a rise (or at least not as a fall) in inequality.

In the literature of inequality, there are two main distinct concepts of inequality, which are:

1. *Absolute inequality:* It refers to the disparity between incomes. Then, adding the same amount to all individuals will not interfere with the level of disparity between incomes.
2. *Relative inequality:* It refers to the disparity between income shares. Then, multiplying the incomes by a scalar will not interfere with the level of disparity between income shares.

If we assume that the social welfare will depends positively on average standards of living -income- and negatively on inequality, we can recall the two main compact social evaluation functions, denoted by $\mathcal{S}(.)$.

- *Absolute approach*: $\mathcal{S}(I^A, \mu) = \mu - I^A \in [0, \mu]$;
- *Relative relative*: $\mathcal{S}(I^R, \mu) = 1 - \frac{I^R}{\mu} \in [0, 1]$,

where I^A and I^R denote respectively the absolute and relative inequality indices, and μ the average income. Among the popular inequality indices that leads to these compact social welfare evaluation functions, one can cite that of Gini and that of Atkinson. The presented previous basic notions will help us introducing the main desirable social norms for the quantification of multidimensional inequality.

Let the matrix of achievements (or income components) of N individuals for the K dimensions be denoted by $X \in \mathbb{R}^{NK}$, that take the following form:

$$X = \begin{vmatrix} x_{1,1} & \cdots & x_{1,k} & \cdots & x_{1,K} \\ & & & & \\ x_{n,1} & & x_{n,k} & & \vdots \\ & & & & \\ x_{N,1} & x_{N,2} & \cdots & & x_{N,K} \end{vmatrix} \tag{1}$$

One can recall here that MDI indices are mappings that take the form $I : \mathbb{R}^{NK} \rightarrow \mathbb{R}$, and used to quantify the multidimensional inequality. These indices must obey a set of social norms, that we assume also represent the main ingredients to conceive the social evaluation function and assess the social welfare. Let the matrix $Y \in \mathbb{R}^{NK}$ be an alternative distribution of achievements obtained with a given mapping: $\gamma \in \Gamma$, such that, $\gamma : X \rightarrow Y \in \mathbb{R}^{NK}$. At this stage, let us focus on some subsets of mappings, which may be of interest to establish the ordinal social preferences, presented in Table (1):

Table 1: Multidimensional inequality and social norms

Subsets	***Normative constraints***	Y ***is weakly preferable than*** X ***by different norms***
$\Gamma^{A_{inc}}$	$X, Y \in \mathbb{R}^{NK}$	$Y \succeq_{A_{inc}} X$ if: $\mathcal{S}(I_y^A, \mu_y) \geq \mathcal{S}(I_x^A, \mu_x)$
$\Gamma^{A_{dim}}$	$X, Y \in \mathbb{R}^{NK}$	$Y \succeq_{A_{dim}} X$ if: $\mathcal{S}(\mathbf{I_Y^A}, \mu_y) \geq \mathcal{S}(\mathbf{I_X^A}, \mu_x)$
$\Gamma^{A_{inc,dim}}$	$X, Y \in \mathbb{R}^{NK}$	$Y \succeq_{A_{inc,dim}} X$ if: $\mathcal{S}(I_y^A, \mu_y, \mathbf{I_Y^A}, \mu_\mathbf{Y}) \geq \mathcal{S}(I_x^A, \mu_x, \mathbf{I_X^A}, \mu_\mathbf{X})$
$\Gamma^{R_{inc}}$	$X, Y \in \mathbb{R}^{NK}_{+*}$ and $\mu_x = \mu_y$	$Y \succeq_{R_{inc}} X$ if: $S(I_y^R, \mu_y) \geq S(I_x^R, \mu_x)$
$\Gamma^{R_{dim}}$	$X, Y \in \mathbb{R}^{NK}_{+**}$ and $\mu_{x_k} = \mu_{y_k} \forall k$	$Y \succeq_{R_{dim}} X$ if: $\mathcal{S}(\mathbf{I_Y^R}, \mu_\mathbf{Y}) \geq \mathcal{S}(\mathbf{I_X^R}, \mu_\mathbf{X})$
$\Gamma^{R_{inc,dim}}$	$X, Y \in \mathbb{R}^{NK}_{+**}$ and $\mu_{x_k} = \mu_{y_k} \forall k$	$Y \succeq_{R_{inc,dim}} X$ if: $\mathcal{S}(I_y^R, \mu_y, \mathbf{I_Y^R}, \mu_\mathbf{Y}) \geq \mathcal{S}(I_x^R, \mu_x, \mathbf{I_X^R}, \mu_\mathbf{X})$
Γ^{B}	$X, Y \in \mathbb{R}^{NK}_{+**}$	$Y \succeq_B X$ $Y = BX$, B is a bistochastic matrix.
Γ^{T}	$X, Y \in \mathbb{R}^{NK}_{+**}$	$Y \succeq_T X$ $Y = TX$, T is a Pigou-Dalton transfer matrix.

$\mathcal{S}()$: Social evaluation function.
I_x^A : Absolute income inequality in X.
μ_x : Average income in X.
$\mathbf{I_X^A} = \{I_{x_1}^A, I_{x_2}^A, \ldots, I_{x_K}^A\}$ and $I_{x_k}^A$ is the absolute inequality on component k in X.
$\mu_\mathbf{X} = \{\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_K}\}$ and μ_{x_k} is the average of component k in X.
$Y \succeq_{A_{inc}} X$: Y is weakly preferred than X with the absolute income inequality norm.
$Y \succeq_{R_{inc}} X$: Y is weakly preferred than X with the relative income inequality norm.

Γ^{A_*}: This set of mappings contains less restrictive conditions considering the domain of definition of X and Y and where values of attributes can be negative. Further, there are no conditions of equality of average income between X and Y or between those of their components. These less restrictive conditions are more appropriate with the well-known *absolute inequality* norm.

$\Gamma^{A_{inc}}$: Social preferences are based on *absolute income inequality*, without focusing on absolute inequality within components. This requires implicitly the *income-invariance constant* axiom (**IIC**), which implies that our order of preferences remains unchanged by adding a constant amount to income vector (or to any one of the components).

$\Gamma^{A_{dim}}$: Social preferences are based on the absolute inequality within components, and this, without focusing on absolute inequality of income. This requires implicitly the *component invariance constant* axiom (**CIC**) and implies that our order of preferences remains unchanged by adding a constant amount to each component: if $X \succeq Y \Rightarrow X + C \succeq Y + C$ where $C \in \mathbb{R}^{NK}$, $c_{i,k} = C_k \; \forall i$ and $C_k \in \mathbb{R}$.

$\Gamma^{A_{inc,dim}}$: Social preferences are based on the two forms of inequality (income and components). Here, axioms (**CIC**) and (**IIC**) are required.

Γ^{R_*}: With this subset of applications, we focus in establishing rules of social preferences considering the MDI with the *relative inequality* norm.

$\Gamma^{R_{inc}}$: Social preferences are based on the relative inequality of income, without focusing on the relative inequality within components. The notation $\mathbb{R}^{NK}_{+*}$ implies simply that at least, we have a positive value in one column. This insures that average incomes are not nil. The natural axiom, that is required for this case, is the *income-invariance scale* (**IIS**). In other words, multiplying all components (or income vector) by the same scalar does not interfere with the social order of preferences.

$\Gamma^{R_{dim}}$: Social preferences are based on the relative inequality within components, without focusing on relative inequality of income. The notation $\mathbb{R}^{NK}_{+**}$ indicates simply that, at least, we have a positive value in each column, and consequently, $\mu_k > 0 \; \forall k$. In addition, we assume that: $\gamma^{R_{dim}} : Y = VX$, when the sum of each row-stochastic matrix $V \in \mathbb{R}^{NN}_{+**}$ equals to 1. The natural axiom, that is required in this case, is the *component-invariance scale* (**CIS**). This axiom is equivalent to the well-known *Strong Homotheticity* axiom, which requires: $Y \equiv X$ if $Y = \Lambda X$ and Λ is a diagonal matrix with $\lambda_{i,i} \in \mathbb{R}^+_*$.

$\Gamma^{R_{inc,dim}}$: Social preferences are based on the two forms of inequality. The required axioms here are the $\mathbf{IIS}$ and $\mathbf{CIS}$. Also, this is equivalent to the well-known *Weak Homotheticity* axiom: $Y \equiv X$ if $Y = \lambda X$ and the scalar $\lambda \in \mathbb{R}^+_*$.

The two following subsets are special cases of $\Gamma^{R_{inc,dim}}$. Note that in general, we have: $\Gamma^T \subset \Gamma^B \subset \Gamma^{R_{i,k}} \subset \Gamma^{R_k} \subset \Gamma^{R_i}$.

Γ^T We assume that Y is obtained by multiplying X by a *Pigou-Dalton Transfer* matrix T. The matrix $T = \delta E + (1 - \delta)\Pi$, where E and Π are the identity and row-permutation matrices respectively and the scalar $\delta \in [0, 1]$. For instance, if the mapping γ^T implies simply a change in components of two individuals i_1 and i_2 ($\pi_{i,i} = 1 \; \forall i \neq i_1, i_2$), this mapping will reduce inequality within components and within income. One can recall here that with this norm, our indices obey the well-known *Uniform Pigou-Dalton Majorization Principle* ($\mathbf{UPM}$). However, clearly with this norm, one cannot establish a complete order of preferences. For instance, how can one say about the change in inequality if the transformation do not take the T scheme?

Γ^B We assume that Y is obtained by multiplying X by a bistochastic matrix $B \in \mathbb{R}^{NN}_{++}$. A matrix is bistochastic if the values of each row or column are non negative adding up to one. Except for the case where B is an identity matrix, the mapping γ^B reduces inequality in minimum within one attribute and that of income. One can recall here that this mapping rule was used to develop the well-known *Uniform Majorization Principle* ($\mathbf{UM}$). Kolm (1977) gives different intuitive interpretations on the nature of equalization operating by using intermediately the bistochastic B matrix. The interdependent equalization of components with this mapping form, makes the distribution of attributes more equal (non decreasing for weak preferences). The interdependence constraint implies in its turn, a decrease in income inequality. Evidently, the γ^B mappings may be used to establish the order of preferences when the decrease in component inequalities does not increase that of income. However one cannot take a clear judgment when the change in the two forms of inequality have opposite directions.

The society may be averse to inequality in achievements -components- with the consensus about the necessity of ensuring minimum levels to each person. For instance, the society may be highly averse to the inequality in education attainments and less to the access to public network transportation. The difference in social aversion toward the inequality across dimensions may justify focusing on the multidimensional form of inequality. In general, we can start our reflection with the following two main social preoccupations for the MDI purpose:

A1: *Multidimensional Sensitivity* (MDS): The society is sensitive to the inequality within each dimension -component-. This sensitivity may differ across dimensions.

A2: *Unidimensional Sensitivity* (UDS): The society is sensitive to the inequality in standards of living of persons (to simplify, we call it: *income inequality*).

However, [**A1**] and [**A2**] may have opposite effects. Assume that the poor has a level of a given component higher than that of the rich and the change is by performing more equalization within this component. How do the proposed axioms interfere with this example? Clearly, the decrease in inequality within component will contribute in decreasing MDI (**A1** acts for this first aspect). However, by considering the well-being at individual level, this change will contribute in increasing the MDI (**A2** acts for this second aspect).

For a given level of income inequality (individual income remains constant), the individual (social decision maker (SDM) for the provision of public goods) can improve the well-being by selecting the optimal bundle of attributes given the level of the individual income. A social decision maker will prefer the distribution that generates more social welfare. Then, for a given level of income inequality, there is an optimal level of inequality between attributes at individual level. A special case is when preferences are homothetic and homogeneous (the same for the whole population). In this case, individualistic preferences make income inequality equals to that of each attribute. When attributes are normalized by their averages, inequality within individual attributes must be nil.

With the welfarist approach, one has to specify explicitly the individualistic utility, like what was proposed by Atkinson and Bourguignon (1982), to assess accurately the impact of interaction between components on individual well-being. However, this approach may be criticized for the following reasons. First, using the individualistic measurement of well-being reduces the multidimensional concept to its unidimensional space, and we are interested simply to the inequality in standards of living or explicitly the quantified individual utility. Second, in general, when the horizontal aggregation is made at first stage (evaluated individual utility), it is difficult to disentangle the contribution of inequalities within components into MDI index. In other words, this empties the MDI concept from its meaning. Third, the proposed individualistic utility functions are not derived from empirical estimations, but are imposed to respect some ad-hoc axioms.[2] Obviously, for the ordinal preferences over income inequality, one can propose any concave social evaluation function to assess the level of inequality. However, this proposal cannot be transposed at the level of attributes for the individualistic utility. indeed, the latter must be well specified to continue to have its cardinal nature as a measurement of the individual well-being. Forth, this proposed index by Atkinson and Bourguignon (1982) was derived with the relative approach and the author assume the non existence of negative values of attributes. However, some methods, like those based on the factorial analysis, quantify components in $\mathbb{R}$.

One can note that with the (**CIM**) axiom, decreasing the correlation between individual attributes by keeping the levels of inequality within components unchanged (for instance, by interchanging individual attributes to decrease the correlation: $\begin{vmatrix} 1 & 6 \\ 2 & 9 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 6 \\ 1 & 9 \end{vmatrix}$), decreases income inequality. Bourguignon and Chakravarty (2003) raised some reservations about this axiom and where the latter seems to impose the substitution criteria between attributes. In our view, this axiom may be related to the impact on income inequality. It is clear that the increase in this correlation increases income inequality even with the additive utilitarian approach. In addition, the impact of change in the distribution of components on the individual well-being may depend on the nature of attributes (complements/substitutes/etc.). Further, when MDI indices do not obey the **CIM** axiom, they are not sensitive to the row inequality. Among the MDI indices that not obey the **CIM** axiom, is the unidimensional indices of inequality. For instance, one can use the Atkinson index to assess inequality in the distribution ($\{1, 2, 6, 9\}$ for our example above. The other case is when the MDI index is the average of inequalities within components.

At this stage, let us review the social evaluation function ($\mathcal{S}$), when the society is only sensitive to one form of inequality.

Perfect sensitivity to income inequality

Assume that the society is perfectly sensitive to the inequality of income and in-

[2]Precisely, the scale independence axiom requires an utility function with constant elasticity of substitution (CES) form.

sensitive to the inequality within components. This implies also that our set of multidimensional indices obey the well-known *Income Pigou-Dalton Transfer* axiom. Social welfare increases with the reduction in income inequality even if the change generates more inequality within components (the richer may have lower level in some components). Assume that the social evaluation function $\mathcal{S}^{inc}$ is used to assess the level of social welfare with the relative norm: $\mathcal{S}^{inc} = f(\mu, I)$. In this case, the usual unidimensional inequality indices can be used to assess the level of inequality and one can write:

$$I^{inc} = 1 - \frac{\mathcal{S}^{inc}}{\mu} \tag{2}$$

where $\mathcal{S}^{inc}$ is the level of social welfare. When the Gini social welfare is used, we have then:

$$I^{inc} = \sum_{k=1}^{K} \varphi_k C_k \tag{3}$$

where φ_k and C_k are the total income share and the coefficient of concentration of component k respectively.[3] One can note that in the case where we cannot establish a uniform scale of measurement across attributes, we can set $\varphi_k = 1/K\ \forall k$ or $\mu_k = \mu_l \forall k \neq l$ and the I^{inc} index continues to be sensitive to the correlation across components.

In what case this approach will be sufficient to assess the multidimensional inequality? Evidently this approach may be used if the distinction between components does not make any sense or, where they are perfect substitutes. In an extreme case, if all these components are monetized, one cannot distinguish between the dollar spent on a given component and that spent on another component. Among the well desired axioms that MDI indices must obey with this social norm, we cite:

A3: *Weak Anonymity* (**WAN**): MDI indices do not depend on the individual characteristics except its standards of living or its income for simplicity.

A4: *Weak Normalization (***WNM***)*: If each person has the average income, then the MDI indices equals to zero.

Perfect sensitivity to inequality within components

[3] For the decomposition of the Gini index by income components, see Pyatt (1976), Lambert and Aronson (1993) and Araar (2006).

Assume that the society is sensitive to the inequality within each component, but insensitive to income inequality. In such case, it is natural to assume that the social evaluation function is simply the sum of the K unidimensional social welfare functions. Let $\mathcal{S}_k$ denotes the social evaluation function of the k_{ith} dimension and $\mathcal{S}_k$ its level. The function $\mathcal{S}_k$ depends on the average of component k, denoted by μ_k, and inequality within the component.[4] We denote the social evaluation function with this pure multidimensional norm by $\mathcal{S}^{dim} = \sum_{k=1}^{K} \mathcal{S}_k$.[5] When $S_k = \mu_k(1 - I_k)$, the MDI index can be written as follows:

$$I^{dim} = 1 - \frac{\mathcal{S}^{dim}}{\mu} = \sum_{k=1}^{K} \varphi_k I_k \tag{4}$$

where I_k is the index of inequality of component k.[6]
When is this approach sufficient to assess the multidimensional inequality? Evidently, this approach may be used where components are perfect complements. Among the well desired axioms that MDI indices must obey with this social norm are:

A5: *Strong Anonymity* (**SAN**): MDI indices do not depend on the individual characteristics except their levels of achievements or income components for simplicity.

A6: *Strong Normalization* (**SNM**): If each person has the average component -achievement-, then MDI indices equals to zero.

These two situations, for which the society is sensitive to only one form of inequality, represent the two extreme cases of social preferences. It is rational to assume that the society may desire to reduce inequality within components without hurting, as much as possible, the situation of the poor. When the change in the distribution has two opposite effects, like the case where the reduction in inequality within a given component k increases the income inequality, the net effect on

[4]This simplified representation does not impose beforehand the component-separability criteria for the individualistic utility function.

[5]Note that, with the Gini social welfare function, $\mathcal{S}^{dim} = \mathcal{S}^{inc}$ if each of component ranks persons similarly to the income rank.

[6]Note that in this case, the index I^{dim} may be obtained with the counterfactual distribution of income where the poorer have the sum of all minimum levels of components, the second poorer have the sum of second minimum levels and so on.

$\mathcal{S}$ will depend on social preferences trade-off between these two effects (How the society is more sensitive to one type of inequality rather than the other). Assume that the vector of K parameters, denoted by λ, represents the social preferences for this issue. The hybrid social evaluation function can take the following linear form:

$$\mathcal{S} = \sum_{k=1}^{K} \lambda_k \nu_k^{dim} + (1 - \lambda_k)\, \nu_k^{inc} \tag{5}$$

where ν_k^{ω} is simply the contribution of the k_{th} component to the $\mathcal{S}$ social welfare function with norms $\omega \in [dim, inc]$.

To assess income inequality under uncertainty, Ben-Porath, Gilboa, and Schmeidler (1997) have proposed a multiple-priors functional form J. The context of the problematic that they treat is practically similar to that of MDI and we have to perform two successive stages of aggregations ($J_1 * J_2, J_2 * J_1$). For the inequality under uncertainty, one can estimate the expected income and thus, estimate income inequality or estimate the expected index of inequality of the different potential distributions of income. The notation $J_1 * J_2$, denotes the case where the operator J_2 is applied to each row matrix at the first stage (row aggregation at the first stage). The compact and convex function J takes the form: $J = \alpha(J_1 * J_2) + (1 - \alpha)(J_2 * J_1)$ where $\alpha \in [0, 1]$. As noted by Gajdos and Weymark (2005), when J_1 is the relative Gini operator and J_2 is the expected operator with uniform probability of distribution over K components, J respects the **CIM** axiom when $\alpha \neq 0$.[7] Thus, the MDI index that we propose may be viewed as a generalization of this result by focusing on the trade-off between the two forms of inequality considering contributions of each component. In the case where $\lambda_i > \lambda_j$, the society is more sensitive to the inequality within component i comparatively to that within j. When we use the Gini social welfare function, the relative ***MDI*** index takes the following form:

$$I_R = \sum_{i=1}^{K} \varphi_k \left[\lambda_k I_k + (1 - \lambda_k)\, C_k \right] \tag{6}$$

In general, performing more equalization in a given component (k) may have mitigated impact on MDI, and this, depending on I_k, C_k and the parameter λ_k.

[7] Even if the problematic of quantifying MDI that we discuss in this paper and that of inequality under uncertainty, treated by Ben-Porath, Gilboa, and Schmeidler (1997)) are different, the juxtaposition of these two problematics may be done by assuming that each of the terms ($\frac{\mu}{\mu_k} *$ component$_k$) represents the income vector in state k with probability ($\frac{1}{K}$).

Graphically, one can plot the difference between Lorenz curve of income and that of concentration of component k to expect the impact of the marginal component equalization on the MDI index, and this by assuming, for instance, that the parameter λ_k equals to 0.5. MDI indices, which are simultaneously sensitive to the two forms of inequality must obey the following axiom:

A7: *Joint Sensitivity (***JOS***)*: MDI indices will be sensitive to the uni and multidimensional inequalities simultaneously.

Under the *absolute inequality* norm, the proposed developments are practically similar and the absolute MDI index takes the following form:

$$I_A = \sum_{i=1}^{K} \left[\lambda_k AI_k + (1 - \lambda_k) AC_k\right] \tag{7}$$

where , AI_k and AC_k denotes the absolute Gini and absolute coefficient of concentration respectively. Moreover, the proposed index satisfies a set of desirable properties for MDI indices. Among the implications of the proposed compact hybrid index, one can cite the following:

- *Compact functional form*: For all possible values $\lambda_k \in [0, 1]$ one can prove easily that our index is bounded between 0 and 1. This result is because of $I_k \geq |C_k|$. Hence, its is sufficient to agree with this trivial weak inequality to prove that: $\sum_{i=1}^{K} \varphi_k \left[\lambda_k |C_k| + (1 - \lambda_k)(C_k)\right] \geq 0$ and $\sum_{i=1}^{K} \varphi_k \left[\lambda_k I_k + (1 - \lambda_k)(I_k)\right] \leq 1$.

- *UPM principle*: The proposed index obeys to the **UPM** principle for all $\lambda_k \in [0, 1]$. Recall that with the T transformation, inequality within components, as well as, within income decreases. Hence the decrease in the proposed index is trivial.

- *UM principle*: The proposed index obeys to the **UM** principle for all $\lambda_k \in [0, 1]$, since the B transformation makes inequality within components and that of income lower.

- *Correlation Increasing Majorization (***CIM***)*: This axiom was introduced by Tsui (1999). Recall that with this axiom, it is socially undesirable to have a higher correlation between components. When $\lambda_k \neq 1$ our index is sensitive to this form of correlation and respects this axiom.

- *Decomposability by component (***DEC***)*: The proposed index is decomposable by attributes or components and according to the two forms of inequality. This axiom must be strongly required for MDI indices to design easily the anti-inequality policies based on the contribution of each component.

What is the link between λ and the nature of components? As reported above, when the component k is a perfect substitute of the other set of components, it is appropriate to set λ_k to zero. In contrast, if the component is a perfect complement, λ_k will converge to one.

As reported before, when the comparison concerns the case where inequalities in all components and that of income decrease, it is easy to take a clear judgment about the nature of change in multidimensional inequality. The complication arises when the comparison concerns the case of increase in inequality of some components and a decrease for the others. This implies to consider the total impact at the individual level. Under the welfarist approach, one must define explicitly the functional form of the individualistic utility to assess the trade-off in such case. Evidently, if we have a complete information about the individualistic functional form, we can propose more refined MDI indices. However, one must take a trade-off between this simplified and operational proposed approach and that when an individualistic function is specified without any empirical validation.

3 Application

To illustrate how these proposed developments may be used to quantify and analyze the multidimensional inequality, we use the Cameroonian Household Survey (ECAM II: Enquête Camerounaise Auprès des Ménages) conducted by the National Institute of Statistics in 2001. This is a national survey with a sample of about 11,000 households selected randomly using two stages in the urban areas and three for the rural areas. Besides the detailed information on household expenditures, this sample contains rich information about the non monetary dimensions of well-being, such as the access to public goods. For our application, we focus on three dimensions of well-being, which are the housing, education and health basic infrastructures.

As is well-known, there is no unique indicator as a measure for a given dimension of well-being at the individual level. In general, one has to quantify this level starting from a set of primary indicators, which will be strongly related to the dimension that they represent (see the Appendix 1 for more information about the retained basic indicators). Starting from the fact that all used indicators are categorical, we propose to use the Multiple Correspondence Analysis (MCA) technique to estimate the individual normalized scores, and this, for each of the three retained dimensions of well-being. One can recall here that the MCA is the application of the simple correspondence analysis to multivariate categorical data, coded as an indicator matrix or a Burt matrix.[8] The level of well-being of the individual i for a given dimension is quantified as follows:

$$W_i = \frac{\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} w_{j_k} I_{i,j_k}}{K} \tag{8}$$

where K is the number of categorical variables, J_k the number of categories for indicator k, I_{i,j_k} the binary indicator taking 1 if the individual i has the category j_k and w_{j_k} is the normalized first axis score of the category j_k.[9]

To sum up, after reducing the information from the $\sum_k J_k$ dimensions to the fewest one, the reduced space preserves the main disparities in well-being. Precisely, the main part of this disparity is projected on the first axis -factor- of the reduced space. This is the reason for which one can use the categorical scores of the first axis as categorical weights. In Appendix 2, we show the housing categorical indicators in bi-dimensional reduced space. As one can observe, after inverting the sign of scores,[10] all categories related to the best quality of housing have higher scores.

In Figure 1, we plot the density curves of each of the three dimensions of well-being according to the household living area.[11] As expected, urban households have better scores in housing, education and sanitary infrastructures compared to those that live in semi-urban and rural areas. However, how is the level of multidimensional inequality within each Cameroonian region? To explore this and to quantify the MDI with the absolute approach, we use the proposed hybrid MDI index. For simplicity, we assume that the parameter λ_k is the same for all k. For the relative approach and since relative inequality indices are not defined with negative indicators of well-being, we perform a linear transformation by translat-

[8]See, for instance, Benazécri (1979), Greenacre (1984) and Greenacre (1993).

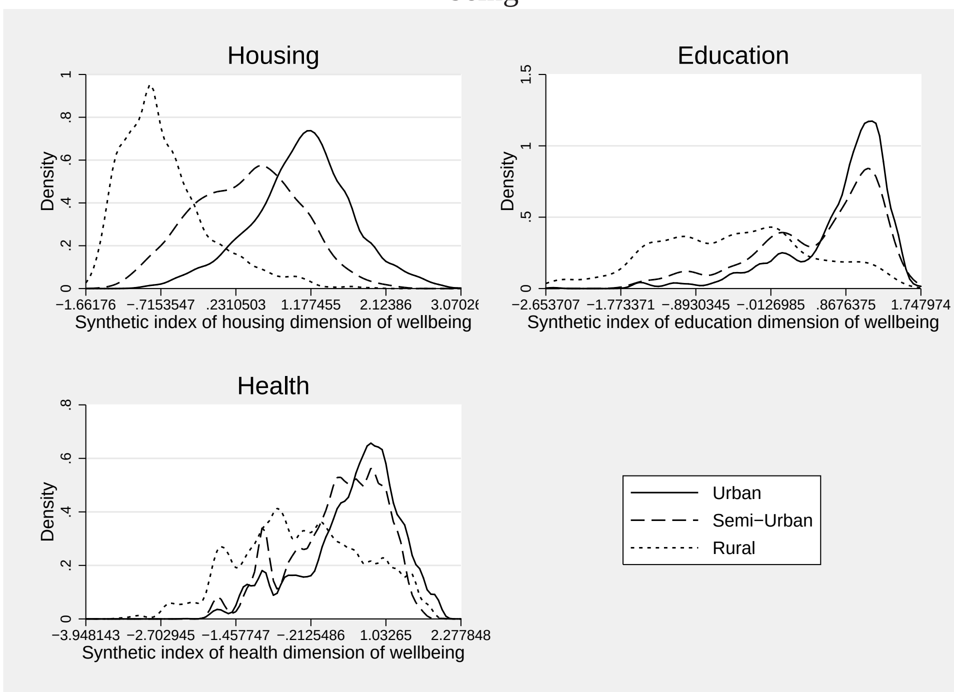
[9]Precisely, the normalized weight refers to the score on the first axis, normalized by the square root of its correspondent eigenvalue.

[10]We recall here that this inversion does not interfere with the score results obtained by using the MCA method.

[11]The DASP Stata package was used to carry the main part of results of the application. For more information, see also Araar and Duclos (2007).

ing individual scores by the distance between the minimum of scores and zero. Further, we normalize each dimension by its mean. In Tables 2 and 3 we present

Figure 1: Density curves of the synthetic indices for the three dimensions of well-being



results of MDI indices by the Cameroonian areas and provinces. Starting from these results, one can remark the higher level in multidimensional inequality in rural area, and this with both approaches. As shown again in Appendix 2, while the monetary inequality is what characterize the urban area, the non monetary inequality is more pronounced in rural area. Thus, the first conclusion that one can draw concerns the disparities in housing, education and health facilities in the Cameroonian semi-urban and rural areas. This result may induce policymakers to review the planned expansions in the provision of public services and reduce these disparities in priority within the more unequal provinces in Cameroon.

Concerning the sensitivity of the results to the choice of the parameter λ_k, we find that the rank based on MDI may change with the selected level of this parameter (for instance, see the ordinal rank between *Adamaoua* and *Extreme-*

Table 2: Estimated MDI indices by areas and provinces (*absolute approach*)

	$\lambda = 0.0$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.9$	$\lambda = 1.0$
Areas					
Urban	0.291	0.266	0.312	0.358	0.369
Semi-Urban	0.314	0.286	0.338	0.390	0.402
Rural	0.391	0.357	0.406	0.456	0.468
Provinces and main cities					
Douala	0.263	0.240	0.289	0.337	0.350
Yaounde	0.227	0.207	0.252	0.297	0.308
Adamaoua	0.512	0.466	0.495	0.525	0.532
Center	0.419	0.382	0.419	0.457	0.466
EST	0.403	0.367	0.409	0.450	0.460
Extreme-North	0.399	0.363	0.405	0.448	0.458
Litoral	0.328	0.299	0.346	0.392	0.404
North	0.459	0.418	0.461	0.504	0.515
North-west	0.401	0.365	0.398	0.432	0.440
West	0.368	0.335	0.382	0.429	0.441
South	0.369	0.336	0.372	0.408	0.417
South-west	0.432	0.394	0.441	0.488	0.499
Cameroon	0.519	0.473	0.515	0.557	0.568

North provinces for $\lambda_k = 0$ and $\lambda_k = 1$ in Table 3). This result confirms the sensitivity of MDI indices to the adopted properties that MDI indices must obey.

Among the desired properties of the proposed MDI index is its decomposability by components or dimensions of well-being. In Table 4, we present the results of decomposition of the relative MDI when the parameter λ_k is set to 0.5. The first remark is the higher contribution of housing dimensions to total MDI. This result is expected being given the high correlation between this dimension and the unequal monetary indicator. The second remark concerns the contribution of education which is higher compared to that of health. This may incite the policy makers to reexamine the planed expansions in public goods and to reduce disparities in access to this service.

Table 3: Estimated MDI indices by areas and provinces (*relative approach*)

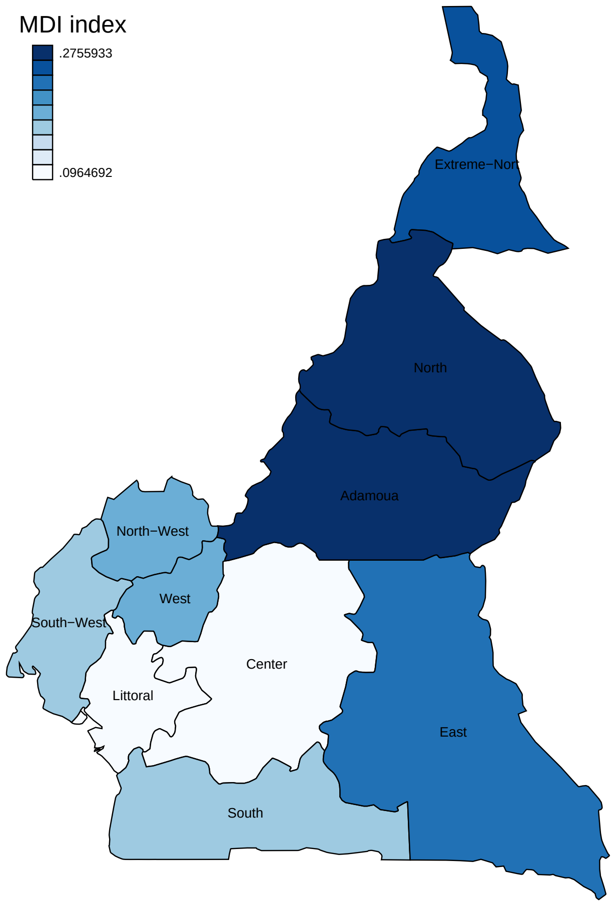
	$\lambda = 0.0$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.9$	$\lambda = 1.0$
Areas					
Urban	0.085	0.078	0.091	0.105	0.108
Semi-Urban	0.110	0.100	0.117	0.134	0.138
Rural	0.200	0.182	0.205	0.228	0.233
Provinces and main cities					
Douala	0.072	0.065	0.080	0.094	0.098
Yaounde	0.061	0.056	0.069	0.081	0.084
Adamaoua	0.238	0.217	0.229	0.241	0.244
Center	0.185	0.169	0.185	0.201	0.205
East	0.191	0.174	0.190	0.206	0.210
Extreme-North	0.229	0.209	0.232	0.255	0.261
Litoral	0.121	0.110	0.124	0.139	0.142
North	0.280	0.255	0.276	0.296	0.301
North-west	0.179	0.163	0.175	0.187	0.190
West	0.150	0.137	0.153	0.169	0.173
South	0.148	0.135	0.148	0.160	0.163
South-west	0.152	0.138	0.153	0.168	0.171
Cameroon	0.219	0.199	0.214	0.228	0.232

Table 4: Decomposition of relative MDI by well-being dimensions ($\lambda_k = 0.5$)

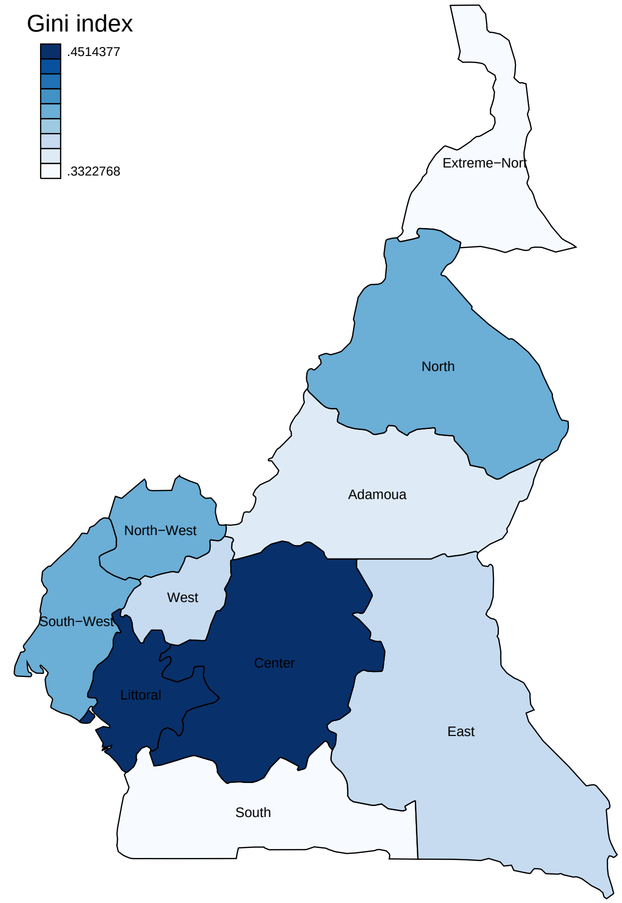
	Housing	Education	Health	Total (MDI)
The absolute contribution				
Urban	0.042	0.025	0.024	0.091
Semi-Urban	0.055	0.037	0.026	0.117
Rural	0.091	0.070	0.044	0.205
Cameroon	0.110	0.064	0.039	0.214
The relative contribution (in %)				
Urban	45.7%	27.5%	26.8%	100%
Semi-Urban	46.9%	31.3%	21.8%	100%
Rural	44.5%	34.2%	21.3%	100%
Cameroon	51.7%	30.0%	18.3%	100%

Figure 2: The map of monetary and non monetary inequalities (Cameroon 2001)

MDI index ($\lambda_k = 0.5$)



Gini index of total expenditures per equivalent adult



4 Conclusion

There is a growing interest in studying the negative aspects in distribution of wealth in developing countries during the last few decades. At the same time, the perception of individual well-being was revised to include the real abilities of the person to transform resources into valuable activities. In this line, there is consensus about the multidimensional nature of well-being. Besides the importance of assessing the deprivation in well-being, one can agree with the need of assessing inequality within the non monetary dimensions, like education, health and basic infrastructures and where the latter represents the main ingredients to develop the human capital and to give a chance to escape from poverty. Ethically, it is desirable that population will have equivalent opportunities to access to public goods. This requires in its turn to reduce disparities in the provision of these goods.

In this paper, we propose a new index that may help in assess and analyze the multidimensional inequality. The proposed hybrid MDI index has a more flexible functional form to reflect the antagonism in the multi aspects of social preferences. It satisfies the main desirable properties and allows to establish a complete order for the social welfare, and this by considering the multidimensional aspect of well-being. In addition, this index is easily interpretable considering its functional form and their easily understandable components. Moreover, this index is multi-level decomposable by components or dimensions, and by the uni- and multi-dimensional forms of inequality. This property allows shaping anti-inequality policies to fight simultaneously against the different forms of inequality.

Results of the application conducted with the 2001 Cameroonian data confirm the rural nature of multidimensional inequality. The decomposition of MDI index shows that housing dimension, that is more correlated with monetary indicators of well-being, is that which contribute the more to MDI. In addition, the contribution of education is higher than that of health. These results may guide policymakers in the revision of the planed expansion in provision of public goods, and this, by improving the repartition of access to public services. Anti-inequality policies will target more the higher unequal Cameroonian provinces and especially those less urbanized.

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Appendix 1 Basic indicators of the non monetary well-being dimensions

Dimension 1:	*Housing infrastructures and environmental facilities*
	Occupation status
	Type of housing
	Number of bedrooms
	Source of drinking water
	Source of lighting energy
	Source cooking energy
	Type of garbage vacation
	Sanitary facilities
	Wall materials
	Roof materials
	Ground materials
Dimension 2:	*Education and basic infrastructures*
	Knowing read and write
	Already attended schools
	First reason of the dissatisfaction of the nearly public primary school
	First reason of the dissatisfaction of the nearly private primary school
	Distance to go to the nearly public primary school (0,1,2,3,4,5 or 6km and more.)
	Distance to go to the nearly private primary school (0,1,2,3,4,5 or 6km and more.)
	Required Time to go the nearly primary public school (0-5min/6-15min/16-25min/26-35min/36-45min/ 46min or more)
	Required Time to go the nearly private public school (0-5min/6-15min/16-25min/26-35min/36-45min/ 46min or more)
Dimension 3:	*Health and basic infrastructures*
	Sector of consultation
	Type of sanitary center
	Appreciation of own health status
	First reason of the dissatisfaction of the nearly sanitary center
	Distance to go to the nearly sanitary center (0,1,2,3,4,5 or 6km and more.)
	Required Time to go the nearly sanitary center (0-5min/6-15min/16-25min/26-35min/36-45min/ 46min or more)

Appendix 2 Categorical coordinates in the bi-dimensional reduced space (Housing basic infrastructures (Cameroon 2001))