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WORKING PAPER SERIES

2009

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WP09/20

October 2009

UCD SCHOOL OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4

The Choice of Modeling Firm Heterogeneity and Trade Restrictions*

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WORKING PAPER: COMMENTS WELCOME

October 23, 2009

Abstract

There has been great focus in the recent trade theory literature on the introduction of firm heterogeneity into trade models. However, these models tend to rely heavily on symmetry assumptions and assume melting iceberg transport costs as the only form of trade restrictions. Moreover, a standard assumption is that firms differ across marginal cost, yet empirical evidence suggests this is not the only important source of heterogeneity. I provide a highly tractable model, in which firms differ across fixed costs, that qualitatively maintains the main results of these models, but allows for asymmetric changes in trade restrictions, a necessary step towards studying strategic trade policy. In addition, I highlight the differences in the effects on product variety associated with changes in an *ad valorem* tariff, iceberg transport costs, and additional beachhead costs to become an exporter. This is important as there are potential offsetting effects on firm entry.

JEL classification: F10; F13; F15

Keywords: Intra-industry Trade; Trade policy; Firm heterogeneity; Monopolistic competition

1   Introduction

There has been great focus in the recent trade theory literature on the introduction of firm heterogeneity into trade models. Beginning with Melitz (2003) and Jean (2002), one of this literature's key results is that increased trade restrictions lead to increases in average productivity for exporters and decreases in average productivity for domestic firms. This stems from increased trade barriers leading the least productive exporters to revert to only sales in their own markets, the reduced competition from which permits entry by the least productive domestic firms. These models, along with Helpman, Melitz, and Yeaple (2004), Yeaple (2005), and Baldwin and Forslid (2006), have provided a huge advancement in the literature on intra-industry trade since its conception with Krugman (1979, 1980). However, these gains come at the cost of restrictive symmetry assumptions. In this paper, I provide a more tractable model that maintains the key results of Melitz (2003) yet allows for more flexibility in the analysis.

In particular, whereas these papers commonly model trade restrictions by symmetric iceberg transport costs, my approach allows me to both analyze additional types of trade barriers and to allow such costs to differ.[1] These features are important for several reasons. First, although iceberg trade costs are equivalent to *ad valorem* tariffs in some settings, they are not equivalent in the case of monopolistic competition. Therefore one cannot take the lessons learned from the existing literature and blindly apply them to the changes in tariffs. Furthermore, tariffs generate income for the importer at the expense of the exporter whereas iceberg costs are modeled as pure losses, which has implications for aggregate welfare analysis. In addition, the evidence of Hummels and Skiba (2004) finds that a per-unit transport cost is more consistent with the data than iceberg costs (confirming the Alchian-Allen hypothesis).[2]

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[1] "Iceberg" transport costs are defined as a firm needing to ship more than one unit of good in order for one unit to arrive; the additional units "melt" away.

[2] Alchian and Allen (1964) hypothesize that transport costs lead firms to export only high-quality goods, leaving lower-quality goods for home consumption.

Second, it is true that symmetric trade costs simplify the analysis a great deal and are a reasonable assumption when dealing with transport costs since the distance between countries is the same regardless of the country of origin. Furthermore, one might expect that differences in fueling and other miscellaneous shipping costs would be minor. However, due to taxes on fuel and other regulatory differences, the cost of getting from A to B need not equal those of getting from B to A. Similarly, while symmetric changes in tariffs might be reasonable between members of the World Trade Organization by rules of reciprocity, this does not necessarily apply to trade policy changes between members and non-members. Third, in order to analyze strategic trade policy, it is necessary to derive best responses, a task which requires analysis of asymmetric tariffs. While the models of Melitz and others certainly have their uses, their complexity does not allow them to be used to study these issues in a tractable way.

In addition to the typical barriers to trade (tariffs and transport costs), I am able to consider the effect of "foreign beachhead costs", that is, those fixed costs necessary to engage in exporting.[3] This is often a minor consideration, but with the rapid technological growth and service industries being created to facilitate business operations, these beachhead costs are becoming increasingly important. Thomas Friedman explains in his book, *The World is Flat*, "...UPS also has a financing arm – UPS Capital – that will put up the money for the transformation of your supply chain, particularly if you are a small business and don't have the capital...UPS is creating enabling platforms for anyone to take his or her business global or vastly improve the efficiency of his or her global supply chain" (p. 173). This has direct implications for these beachhead costs and needs to be considered in conjunction with investigating changes in other trade restrictions, as they may have conflicting results.

To move in this direction, this paper makes several key modifications to the basic model that greatly ease the analysis of a situation with heterogeneous firms and endogenous entry. Among these are changes to the utility function of the representative consumer that reduce

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[3]The term 'beachhead' costs was coined by Baldwin (1988).

so-called income feedback effects and an assumption of fixed cost heterogeneity rather than variable cost heterogeneity. Although the latter is not necessary for the main results, it does aid greatly in simplifying the analysis. Moreover, recent data analysis suggests fixed cost heterogeneity is needed to explain a firm's decision of where to export. For instance, Eaton, Kortum, and Kramarz (2007), using a Melitz-type model calibrated to a French data set, illustrate the need for firm and market specific shocks to the fixed cost of entry to match the data. Furthermore, Lawless and Whelan (2008) reiterate the importance of variation in fixed and variable trade costs across firms in explaining trade flows for Irish owned firms. Additionally, as Jørgensen and Schröder (2008) point out, fixed cost heterogeneity is more appropriate with so-called "original brand name manufacturers" that differ in the power of their brand name – a result of marketing and other fixed cost activities.

With the changes I employ, it is relatively simple to undertake tasks such as determining the welfare impact of opening up for (even limited) trade, deriving the effect of changes in trade barriers on productivity, and contrasting the impact of the various trade barriers on the number of varieties.[4] It is worth noting that for some of these, qualitatively similar results have been found by others. Nevertheless, my model allows for both the exploration of new issues (such as the relative impact of various trade barriers) as well as a more simple derivation of the pre-existing results.

The paper proceeds as follows. Section 2 sets up the model and characterizes the equilibrium. Section 3 analyzes the results and compares the model to the existing literature. Section 4 concludes.

2 The Model

There are two countries labeled k and j. Each country is endowed with $\bar{L}$ units of labor which is the sole factor of production. Without loss of generality, let $\bar{L}_k \geq \bar{L}_j$. There are two

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[4]The number of varieties has an obvious theoretical impact on welfare. Additionally, Broda and Weinstein (2006) show that growth in product variety has been an important source of gains from trade in the U.S. from 1972-2001.

sectors. Sector 1 is the numeraire and consists of a homogeneous good (y) that is produced under constant returns to scale, freely traded, and sold in a perfectly competitive market. Sector 2 consists of a continuum of heterogeneous goods, each variety of which is indexed by i. As is standard in the Melitz-type model, this is produced under increasing returns to scale in a monopolistically competitive market with free entry. Unlike sector 1, this market may face both transportation costs and tariff barriers. With the exception of the differing labor endowments and tariff rates, countries are identical. Therefore, analyzing the situation for country k informs us of the analogous situation for country j.

2.1 Sector 1

The price of y is normalized to 1 in each market. Assuming that one unit of labor is needed for production, this will normalize the wage in each country to unity. Finally, I assume that in equilibrium a positive amount of y is produced in each country.

2.2 Consumers

The representative consumer in country k has quasi-linear preferences with an embedded Dixit-Stiglitz utility function which displays love for variety over the heterogeneous good;

$$U_k = \mu \ln(C_{xk}) + C_{yk}, \quad C_{xk} = \left(\int_0^{N_k} x_k(i)^\alpha di \right)^{\frac{1}{\alpha}}, \quad \mu > 0 \tag{1}$$

where $\varepsilon = 1/(1 - \alpha) > 1$ is the elasticity of substitution, N_k is the total mass of varieties in country k, and C_{yk} denotes aggregate consumption of the numeraire. Note that although it is tempting to interpret C_{xk} as aggregate consumption of the heterogeneous good, it is not. This is the first departure from the standard Melitz-type model, and is done for specific reasons. Quasi-linear utility will isolate the decision whether to become an exporter or not without any income feedback effects; providing a model that allows for asymmetric changes in trade restrictions (e.g. unilateral tariff policy) to be easily analyzed. Moreover, this

specification allows me to compare the differences between an *ad valorem* tariff and iceberg trade costs on productivity and variety without having to account for the income effects of the tariff or the "wasteful" costs of iceberg transport costs. Finally, I assume that income in each country is sufficiently large that both y and x goods are consumed.

Consumers maximize utility subject to their budget constraint:

$$\int_0^{N_k} p_k(i)x_k(i)di + C_{yk} \leq I_k \tag{2}$$

where $p_k(i)$ is the price of variety i and I_k is aggregate income in country k.[5] The solution to this problem yields a demand function for the heterogeneous good of variety i in country k:

$$x_k(i) = \frac{p_k(i)^{-\varepsilon}\mu}{\int_0^{N_k} p_k(i)^{1-\varepsilon}di}. \tag{3}$$

Note that $\int_0^{N_k} p_k(i)x_k(i)di = \mu$ by virtue of the quasi-linear preferences.

2.3 Heterogeneous Firms

There is a continuum of entrepreneurs. At time zero, every entrepreneur is given a unique draw that indexes its variety and productivity type. Once the entrepreneur is aware of her type, she decides two things; whether to create a firm and where to sell. If a firm is created, it must incur a fixed cost measured in units of labor. This cost is referred to as a 'beachhead' cost and can be interpreted as forming a distribution and servicing network. It is indexed by i, and will be dependent on the market(s) being served by the firm. Subsequent production exhibits constant returns to scale with labor as the only factor of production. The unit-labor requirement for a firm is normalized to one.

This is the second departure from the standard Melitz-type model which assumes firms are heterogeneous across marginal cost and draw their type from a probability distribution

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[5]Recall that under perfect competition, the price of y is equal to one.

with certain probability of "death".[6] Marginal cost heterogeneity complicates the analysis a great deal and will have similar qualitative results to that of fixed cost heterogeneity. In addition, as Eaton, Kortum, and Kramarz (2007) and Lawless and Whelan (2008) suggest, fixed cost heterogeneity is an important consideration for the real world. I do not claim to be the first to employ fixed cost heterogeneity; Davies and Eckel (2007) do so in studying tax competition with endogenous entry, and Jørgensen and Schröder (2006, 2008) use fixed *export* cost heterogeneity to show that a reciprocal reduction of small tariffs reduces welfare.

There are two available markets for a potential firm, each with a corresponding fixed cost. A firm can choose to serve only the domestic market and pay $f(i)$ or it can choose to additionally serve the foreign market through exports and pay an extra $\gamma f(i)$.[7] I assume that $\gamma > 1$; $f'(i) > 0$ and $f''(i) \geq 0$ denote the first and second derivatives respectively. Thus, entrepreneurs with higher ability correspond to a lower index i. These fixed cost differences are the source of firm heterogeneity. A firm, therefore, faces the following menu of fixed costs (measured in units of labor):[8]

Table 1: Fixed Cost Menu

Firm Type	Fixed Cost
domestic only	$f(i)$
domestic and exporter	$(1+\gamma)f(i)$

Goods that are exported from country k to country j are subject to melting-iceberg transport costs, $\sigma \geq 1$, where a firm must ship σ units in order for one unit to arrive at its destination. I assume that transport costs are symmetric and thus omit country

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[6]Typically, the Pareto distribution is used to find specific examples. Though entrepreneur types are not determined stochastically, the mapping from one's type to a firm's fixed cost is general enough to mimic certain distributions. For instance, a distribution that yielded a large mass of "high" types (i.e. very efficient firms) could be accounted for with a mapping that increases very slowly.

[7]Note that with wages equal to 1, these are equivalent to labor requirements for setting up firm activities.

[8]Constant expenditure on the heterogeneous good in each country (equal to μ), along with identical technologies and entrepreneurs implies that the condition $\gamma > 1$ is sufficient to insure a firm that serves the foreign market will also serve the domestic market.

subscripts.[9] I do not investigate the effect of a per-unit transport cost; since marginal costs are normalized to one, this would have the same effect as iceberg transport costs.[10] Additionally, an exporting firm from country k is subject to an *ad valorem* tariff τ_j, where I define $t_j \equiv 1 + \tau_j$. Furthermore, I assume that a government is unable to distinguish a particular firm's type, so any tariff is an across-the-board tariff applied to all exporters. Note that tariffs can differ across countries.

The decision to become a firm and which market to service depends on the associated profit for each type. Recall that the numeraire yields wages equal to one in both countries, thus the operating profits from serving the domestic market are

$$\pi_D^k(i) = p_k(i)q_k(i) - q_k(i) - f(i). \tag{4}$$

Given the nature of monopolistic competition, the price will be a constant mark-up over marginal cost and be equal to $\frac{1}{\alpha}$. From market clearing, set $q_k(i) = x_k(i)$, and the firm has the following profit function for supplying to the domestic market only:

$$\pi_D^k(i) = B_k - f(i) \tag{5}$$

where

$$B_k = \left(\frac{1}{\varepsilon \alpha^{1-\varepsilon}}\right) \frac{\mu}{\mathcal{P}_k^{1-\varepsilon}}$$

and $\mathcal{P}_k = P_k^{\frac{1}{1-\varepsilon}} = \left(\int_0^{N_k} p_k(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index of the heterogeneous good.

Since preferences are identical across both countries, it follows that the total expenditure on the heterogeneous good is equal to μ in both markets. Furthermore, recall that technologies and the mass of entrepreneurs are also identical across countries. This, along

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[9]This assumption is only done for notational ease. In order to investigate asymmetric changes in transport costs, on need only add a country subscript to σ.

[10]This is not the case when firms differ across marginal costs.

with $\gamma > 1$, is sufficient to ensure that a firm that exports will always serve the domestic market. Thus, the decision to become an exporter stems purely from the *additional* profits from serving the foreign market. The profit from exports for the firm in country k exporting to country j is:

$$\pi_X^k(i) = t_j^{-\varepsilon} \sigma^{1-\varepsilon} B_j - \gamma f(i). \tag{6}$$

Note that since the iceberg transport cost only affects marginal cost, this is passed through onto the consumer in the price. That is, the price of a good produced in country k, but sold in country j is $p_j^k = \frac{\sigma}{\alpha}$. Moreover, since firms take the aggregate price index as given, the tariff is also completely passed through onto the consumer, but done so differently than iceberg transport costs.[11] This difference may be a bit confusing. It has been a typical conception that iceberg transportation costs have identical results as an *ad valorem* tariff, and in some respects that still holds true for monopolistic competition; the price consumers pay is identical in both scenarios and the quantity demanded is the same. However, because in monopolistic competition, the price is equal to a constant markup over marginal cost, the amount paid in tariffs is also a constant markup over the "tax" paid in iceberg transportation costs. Therefore, the marginal cost associated with an *ad valorem* tariff is greater than that of iceberg transportation costs.[12]

2.4 Equilibrium

Firms will enter each market as long as there are positive profits, that is, until equations (5) and (6) are driven to zero. Thus, define the cut-off firms as the firms that draw the values

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[11]This claim is shown in Appendix A.

[12]Note that, in perfect competition, price equals marginal cost and the standard result of iceberg costs having the same effect as an ad valorem tariff still holds.

in the index (i) that solves the following equalities:

$$B_k = f(i_{kD}) \tag{7}$$

$$\frac{B_j}{\gamma t_j^\varepsilon \sigma^{\varepsilon-1}} = f(i_{kX}) \tag{8}$$

$$B_j = f(i_{jD}) \tag{9}$$

$$\frac{B_k}{\gamma t_k^\varepsilon \sigma^{\varepsilon-1}} = f(i_{jX}) \tag{10}$$

The indices i_{kD} and i_{jD} represent the firms that are indifferent between producing the heterogeneous good and not producing at all in country k and j respectively. The indices i_{kX} and i_{jX} represent the firms that are indifferent between serving both the domestic and foreign markets and serving only the domestic market. Furthermore, the terms on the left-hand side of the equalities represent the variable profit for a particular firm and are functions of the total mass of firms (domestic and foreign).

Figure 1 illustrates the profits of firms in country k including those who export and those who only sell domestically, assuming that the function $f(i)$ is linear. It can be seen that the greater the index i, the greater the fixed cost to enter a market, and thus the lower the profits. The intersection with the horizontal axis represents the index in which profits are zero for operating in that particular market. Note that the line representing export profits defines the profits from exporting in addition to serving the domestic market. In other words, firms with an index $i \in [0, i_{kX}]$ make profits from exporting *and* serving the domestic market, and firms with an index $i \in (i_{kX}, i_{kD}]$ make profits from only serving the domestic market. Firms with an index $i > i_{kD}$ do not produce.

After careful inspection of the equilibrium conditions, it can be seen that this is, in fact, two systems of two equations and two unknowns: equations (7) and (10) and equations (8) and (9).[13] Moreover, due to the symmetry it is sufficient to only focus on one country. I will focus on the output market in country k, and thus equations (7) and (10). For future use, it

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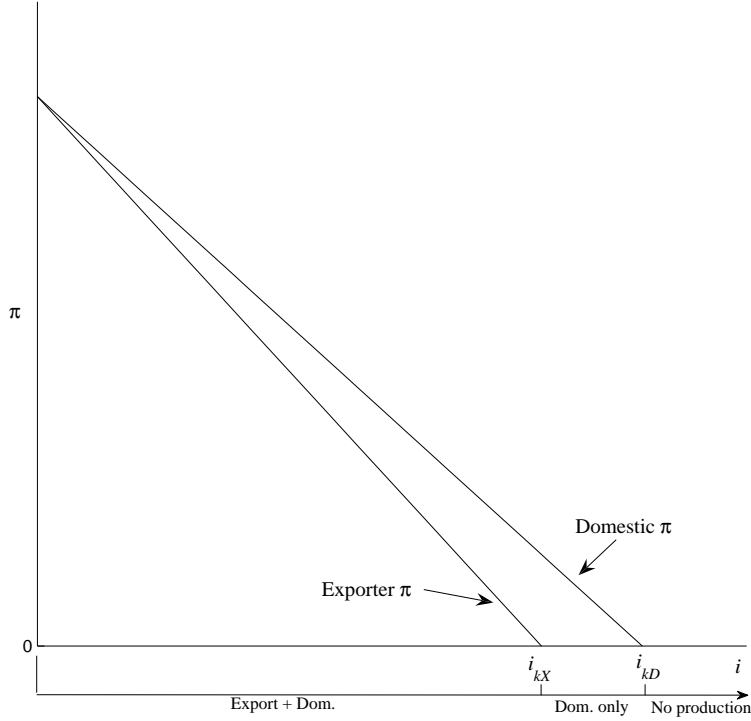[13]This nice simplification stems from the utility specification used.

Figure 1: Profits from production in country k with free trade

will be helpful to rewrite the equilibrium conditions, (7) and (10), in the following manner:

$$f(i_{kD}) = \frac{\mu}{\varepsilon(i_{kD} + (t_k\sigma)^{1-\varepsilon}i_{jX})} \tag{11}$$

$$f(i_{kD}) = \gamma t_k^{\varepsilon}\sigma^{\varepsilon-1}f(i_{jX}). \tag{12}$$

3   Results

Although, I cannot explicitly solve for the cutoff values without assuming a functional form of the fixed cost mapping $f(i)$, I am still able to characterize the comparative statics. Totally differentiating this system of equations (11) and (12) yields the following comparative statics:

$$\frac{\partial i_{jX}}{\partial \tau_k} = \frac{-f(i_{jX})}{t_k} \left[\frac{(t_k\sigma)^{1-\varepsilon}f'(i_{kD})i_{jX} + \varepsilon f(i_{kD})\big[\delta(i_{kD})+1\big]}{\psi} \right] < 0 \tag{13}$$

$$\frac{\partial i_{jX}}{\partial \sigma} = \left[\frac{-\alpha\varepsilon f(i_{jX})f(i_{kD})}{\sigma} \right] \left[\frac{\delta(i_{kD})+1}{\psi} \right] < 0 \tag{14}$$

$$\frac{\partial i_{jX}}{\partial \gamma} = \frac{-f(i_{jX})f(i_{kD})\delta(i_{kD})}{\gamma\psi} \left[1 + \frac{i_{jX}}{(t_k\sigma)^{\alpha\varepsilon}i_{kD}} \right] < 0 \tag{15}$$

$$\frac{\partial i_{kD}}{\partial \tau_k} = \frac{\gamma\varepsilon f(i_{jX})^2\big[\alpha\delta(i_{jX})+1\big]}{\psi} > 0 \tag{16}$$

$$\frac{\partial i_{kD}}{\partial \sigma} = \left[\frac{\alpha\gamma t_k\varepsilon f(i_{jX})^2}{\sigma} \right] \left[\frac{\delta(i_{jX})+1}{\psi} \right] > 0 \tag{17}$$

$$\frac{\partial i_{kD}}{\partial \gamma} = \frac{t_k f(i_{jX})^2}{\psi} > 0 \tag{18}$$

where

$$\psi \equiv f'(i_{jX})\big[\delta(i_{kD})+1\big] + (t_k\sigma)^{1-\varepsilon}f(i_{jX})f'(i_{kD})\big[\delta(i_{jX})+1\big], \text{ and}$$

$$\delta(z) = \frac{zf'(z)}{f(z)}.$$

The term $\delta(z)$ represents the elasticity of fixed costs with respect to the index i, evaluated at z. Equations (13) through (15) represent the effect of changes in trade restrictions (either through a tariff, transport cost, or foreign beachhead cost) on the cutoff firm serving the foreign market. It follows that increases in trade restrictions decreases this cutoff, or in other words the mass of exporting firms has decreased. By decreasing the mass of exporting firms, there is now less competition in the domestic market and the foreign firms still producing are now charging a higher price relative to domestic producers. This decreased competition makes being a domestic firm more profitable, thus increasing the mass of domestic firms – illustrated by equations (16) through (18). The fact that increased trade restrictions, in general, have these results is not surprising given the results of Melitz (2003) and Helpman, Melitz, and Yeaple (2004). What is important, and will be elaborated on further in proceeding sections, is that different trade restrictions correspond to different magnitudes in firm

cutoff changes. This is a new result stemming from my more flexible model.

3.1 Average Productivity

One key result from the standard model is that exposure to trade increases average domestic firm productivity. This is done by less efficient domestic firms exiting the market as competition increases from the existence of foreign firms. This result rings true in my model as well (as seen from equations (16) - (18)), though with a slightly different interpretation. Unlike the standard Melitz-type model, where average productivity is defined as the average amount of labor needed to actually produce one unit of output, average productivity here is defined at the total labor usage (including fixed cost) per unit of output.[14] As trade restrictions decrease, whether by lowering tariffs, iceberg transport costs, or foreign beachhead cost (represented by $\gamma f(i)$), less efficient (in terms of a higher fixed cost to enter) domestic firms exit. Thus, with the least efficient domestic firms exiting, the average productivity of remaining firms increases.

3.2 Moving from Autarky to Trade

In addition to the effect trade has on average firm productivity, the now standard Melitz (2003) model found additional important implications that trade has on firms and overall welfare. I address these results in the following two sections.

3.2.1 Exporter Profit and Market Share

Melitz (2003) found that "only a portion of the firms–the more efficient ones–reap benefits from trade in the form of gains in market share and profit" (p. 1719). My model yields a comparable result. Furthermore, since I am comparing this model to that of the Melitz-type, I will only consider the case with symmetric iceberg transport costs (σ) and set tariffs equal to zero ($t_k = 1$). Since I am considering symmetric changes in iceberg transport costs,

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[14]Recall that fixed costs are measured in units of labor.

the equilibrium cutoffs will be identical in each country. Thus, for notational ease, I define $i_{jX} = i_{kX} = i_X$, and $i_{jD} = i_{kD} = i_D$ for this section. Furthermore, let a superscript A denote autarky and T denote trade.

When countries open up to trade, there is increased competition and domestic profits decrease. However, some domestic firms are efficient enough to sell and make profits abroad. It is apparent that less efficient firms lose profit from opening to trade. What is not readily apparent is whether the new profits from exporting are greater than the loss in domestic profits for the more efficient firms. Thus, a firm who will choose to export in trade will gain if the following inequality holds:[15]

$$\pi^A(i) - \pi^T(i) \;=\; \underbrace{\left[f(i_D^A) - f(i_D^T)\right]}_{(+)} + \underbrace{\gamma\left[f(i) - f(i_X^T)\right]}_{(-)} < 0.$$

It is important to note that not *all* exporting firms will gain, since the least efficient exporting firm (the firm with the index i_X) makes zero profits abroad. However, if the most efficient firm (the firm with the index 0) is made better off from trade, then by continuity there must be a positive mass of firms better off from trade. This will depend on the functional form of $f(i)$, the entry(exiting) decision of exporting(pure domestic) firms, and the level of trade restrictions – in this case, iceberg transport cost. I provide a simple proof in Appendix B illustrating the ambiguous effect trade has on firm profits. This departure from Melitz is due mainly from firms differing across fixed cost and not marginal cost. When firms are heterogeneous across marginal cost, the most efficient firm realizes higher revenue relative to every other firm because it charges a lower price and consequently faces a higher level of demand. However, in my model firms charge the same price and thus receive the same level of revenue as every other firm in existence.

This highlights an interesting distinction from the Melitz (2003) model in the context of

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[15]This form follows from the equilibrium conditions (7) and (10).

the political economy. In the standard model where some firms gain profits in the presence of trade, there would exist firms lobbying for more open trade. Yet in my model, depending on the functional form of the fixed cost mapping, this gain to certain producers is absent, further driving a wedge between maximizing consumer and producer surplus. The magnitude of such a wedge is an important investigation for the political economy literature.

Melitz (2003) also finds that the most efficient firms would lose domestic market share from opening up to trade, but this loss would be outweighed by the gains in foreign market share. This holds true in my model as well.

Proposition 1. *The gain in foreign market share of a firm that becomes an exporter is greater than the loss in domestic market share when going from autarky to trade.*

Proof. The domestic market share of a firm that exports is the individual firm demand divided by total market demand is

$$
s_X^d = \frac{x_d(i)}{X_d} = \frac{\frac{p_d(i)^{-\varepsilon}\mu}{\int_0^{N_d} p_d(i)^{1-\varepsilon}di}}{\int_0^{N_d} \frac{p_d(i)^{-\varepsilon}\mu}{\int_0^{N_d} p_d(i)^{1-\varepsilon}di}} = \frac{p_d(i)^{-\varepsilon}}{\int_0^{N_d} p_d(i)^{-\varepsilon}di} = \frac{1}{i_D^T + \sigma^{-\varepsilon}i_X^T},
$$

and its foreign market share is

$$
s_X^f = \frac{x_f(i)}{X_f} = \frac{\frac{p_f(i)^{-\varepsilon}\mu}{\int_0^{N_f} p_f(i)^{1-\varepsilon}di}}{\int_0^{N_f} \frac{p_f(i)^{-\varepsilon}\mu}{\int_0^{N_f} p_f(i)^{1-\varepsilon}di}} = \frac{p_f(i)^{-\varepsilon}}{\int_0^{N_f} p_f(i)^{-\varepsilon}di} = \frac{\sigma^{-\varepsilon}}{i_D^T + \sigma^{-\varepsilon}i_X^T}.
$$

Therefore, the combined market share of a firm that exports is

$$
s_X^d + s_X^f = \frac{1 + \sigma^{-\varepsilon}}{i_D^T + \sigma^{-\varepsilon}i_X^T} = \frac{\sigma^\varepsilon + 1}{\sigma^\varepsilon i_D^T + i_X^T}.
$$

Comparing this with the market share of the same firm in autarky, it follows that the gain in foreign market share of a firm who becomes an exporter is greater than the loss in domestic market share

$$
s^A - s^T \frac{1}{i_D^A} - \frac{\sigma^\varepsilon + 1}{\sigma^\varepsilon i_D^T + i_X^T} = \frac{\sigma^\varepsilon(i_D^T - i_D^A) + i_X^T - i_D^A}{(\sigma^\varepsilon i_D^T + i_X^T)\, i_D^A} < 0 \tag{19}
$$

15

This inequality follows from the fact that $i_D^T < i_D^A$ and $i_X^T < i_D^A$. □

3.2.2 Welfare Effects

Increased welfare from trade is the cornerstone of international economics. Does my model maintain this critical result? Again, I will only consider the case with symmetric iceberg transport costs (σ) and set tariffs equal to zero ($t_k = 1$) to compare my model to Melitz (2003). The following proposition illustrates that this critical result is maintained in my model as well.

Proposition 2. *A country will experience an increase in welfare from trade.*

Proof. See Appendix B. □

3.3 Additional Results

In this section, I clarify results similar to that of the existing literature and provide further results not previously explored. In terms of the effects of trade restrictions on product variety, there is less consensus in the literature. In Melitz (2003), the effect on the total mass of varieties in a particular country is left ambiguous. Baldwin and Forslid (2006) address this issue and find that increased trade restrictions, in fact, have a counterintuitive pro-variety effect for the importing country. However, Melitz and Ottaviano (2008) find that increased trade restrictions have an anti-variety effect. In all three models (as in most models dealing with such issues), trade restrictions are modeled as the standard iceberg transportation cost.[16] The corresponding effects on the mass of varieties in country k (Note that $N_k = i_{kD} + i_{jX}$) are as follows:

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[16]Though the main focus of this literature is on the effects of changes in iceberg transportation costs, these papers do investigate changes of foreign beachhead costs as well ($\gamma f(i)$ in my model) – with the exception of Meltiz and Ottaviano (2008) where beachhead costs are omitted for increased tractability. To my knowledge Jørgensen and Schröder (2006, 2008), and Demidova and Rodríguez-Clare (2007) are the only papers to use an *ad valorem* tariff to represent trade restrictions in a Melitz-type model.

$$\frac{\partial N_k}{\partial \tau_k} = \frac{\varepsilon \gamma f(i_{jX})^2}{\psi} \left\{ 1 + \alpha \delta(i_{jX}) - (t_k \sigma)^{\alpha \varepsilon}[1 + \delta(i_{kD})] - \frac{i_{jX}}{\varepsilon i_{kD}} \delta(i_{kD}) \right\} \qquad (20)$$

$$\frac{\partial N_k}{\partial \sigma} = \frac{\alpha \varepsilon \gamma t_k f(i_{jX})^2}{\sigma \psi} \left\{ 1 + \delta(i_{jX}) - (t_k \sigma)^{\alpha \varepsilon}[1 + \delta(i_{kD})] \right\} \qquad (21)$$

$$\frac{\partial N_k}{\partial \gamma} = \frac{\gamma t_k f(i_{jX})^2}{\psi} \left\{ 1 - (t_k \sigma)^{\alpha \varepsilon}[1 + \delta(i_{kD})] - \frac{i_{jX}}{i_{kD}} \delta(i_{kD}) \right\} < 0 \qquad (22)$$

The model presented here is consistent with the existing literature just mentioned, in that the effect on variety is ambiguous (i.e. the signs of equations (20) and (21)). I provide conditions in which there is an associated anti-variety effect here.

The following proposition pins down the condition that ensures an anti-variety effect associated with increases in iceberg transport costs.

Proposition 3. *There is an anti-variety effect associated with increases in iceberg transport costs if and only if*

$$\frac{1}{(t_k \sigma)^{\alpha \varepsilon}} < \frac{1 + \delta(i_{kD})}{1 + \delta(i_{jX})}$$

Proof. Proof is by direct calculation. $\qquad \square$

This is a sufficient and necessary condition. A more restrictive condition for an anti-variety effect, although one that is perhaps more intuitive, is if the elasticity of $f(i)$ with respect to the index is nondecreasing in i. Examples would include both linear, exponential, and power functions of i. There is a similar result regarding tariffs and will be discussed further shortly.

The next proposition addresses the decision to use iceberg transport costs to additionally proxy for tariff policy. There is an important distinction between the forms of trade restrictions which stems from the two having different variable profit elasticities $(\epsilon_{v\pi_X^j})$, where

$$\epsilon_{v\pi_X^j}^{t_k} - \epsilon_{v\pi_X^j}^{\sigma} = \frac{-f'(i_{jX})\left[f'(i_{kD})\mu + \varepsilon f(i_{kD})^2\right]}{f'(i_{jX})\left[f'(i_{kD})\mu + \varepsilon f(i_{kD})^2\right] + \varepsilon \gamma t_k f(i_{jX})^2 f'(i_{kD})} < 0.$$

The variable profit is more elastic with respect to tariffs than iceberg transport costs. The different effects then feed into different changes in variety.

Proposition 4. *If $\alpha t_k \leq \sigma$ and $\frac{\partial N_k}{\partial \sigma} < 0$, then there is an anti-variety effect associated with an increase in the tariff and this effect is greater in magnitude than the anti-variety effect associated with an increase in iceberg transport costs.*

Proof. Define the following:

$$\frac{\partial N_k}{\partial \tau_k} - \frac{\partial N_k}{\partial \sigma} = \frac{\varepsilon \gamma f(i_{jX})^2}{\psi} \Psi$$

where $\frac{\varepsilon \gamma f(i_{jX})^2}{\psi} > 0$ and

$$\Psi \equiv (\sigma - \alpha t_k) \left[1 - (t_k \sigma)^{\alpha \varepsilon} [1 + \delta(i_{kD})] \right] + (\sigma - t_k) \alpha \delta(i_{jX}) - \frac{\sigma i_{jX}}{\varepsilon i_{kD}} \delta(i_{kD}).$$

From Proposition 3, it follows that

$$1 + \delta(i_{jX}) < (t_k \sigma)^{\alpha \varepsilon} [1 + \delta(i_{kD})].$$

Therefore,

$$\Psi < -\alpha \varepsilon \sigma \delta(i_{jX}) - \frac{\sigma i_{jX}}{\varepsilon i_{kD}} \delta(i_{kD}) < 0.$$

$\square$

This proposition, in conjunction with Proposition 3, implies that if there is an anti-variety effect associated with iceberg transport costs, for at least certain parameter values, there will be an anti-variety effect – of greater magnitude – associated with an increase in tariffs. In addition, I can find some interesting results in a second special case, where $\sigma = t_k$.

Proposition 5. *Evaluated at the same level, the effect of a change in tariff (t_k) on the mass of varieties is less (or more negative) than the effect of a change in ice-berg costs (σ) when $\sigma = t_k$.*

Proof. Let $\sigma = t_k = \rho \geq 1$

$$\left.\frac{\partial N_k}{\partial \tau_k}\right|_{\sigma=t_k=\rho} - \left.\frac{\partial N_k}{\partial \sigma}\right|_{\sigma=t_k=\rho} = \frac{\gamma f(i_{jX})^2}{\psi}\left\{1 - \rho^{\alpha\varepsilon} - \left[\rho^{\alpha\varepsilon} + \frac{i_{jX}}{i_{kD}}\right]\delta(i_{kD})\right\} < 0 \qquad (23)$$

Therefore,

$$\left.\frac{\partial N_k}{\partial \tau_k}\right|_{\sigma=t_k=\rho} < \left.\frac{\partial N_k}{\partial \sigma}\right|_{\sigma=t_k=\rho}. \qquad \square$$

This result leads to an interesting observation. The difference between the variety effects associated with changes in an *ad valorem* tariff and iceberg costs, exactly equals that of a change in γ divided by ρ. That is

$$\left.\frac{\partial N_k}{\partial \tau_k}\right|_{\sigma=t_k=\rho} = \left.\frac{\partial N_k}{\partial \sigma}\right|_{\sigma=t_k=\rho} + \left(\frac{1}{\rho}\right)\left.\frac{\partial N_k}{\partial \gamma}\right|_{\sigma=t_k=\rho}.$$

Though it is perhaps unrealistic to assume all three exogenous variables share the same value, this does highlight potential offsetting effects. The existing literature has investigated the effects of individual types of trade liberalization/restrictions. This model allows for easily comparison of these effects. These potential dampening/magnifying effects on variety is an important consideration for tariff policy given recent increases in transportation cost (primarily through higher fuel prices) and decreases in costs to become an exporter (primarily through increases in computing technologies).

Propositions 4 and 5 do not ensure that the iceberg transport costs have a qualitatively different result than that of an *ad valorem* tariff. However, in conjunction with Proposition 3, it is possible to have a pro-variety effect associated with an increase in transportation costs and an anti-variety effect associated with an increased tariff. In this scenario – assuming transport costs are increasing – if a country is solely concerned with the welfare properties of product variety, the government could raise its tariff and realize little changes in total product variety. Similarly, the government could lower tariff levels and experience a greater than expected *increase* in total product variety. In the case where both trade restrictions

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have an anti-variety effect, these results would be reversed.

4 Conclusion

I have provided a tractable model of intra-industry trade with heterogeneous firms that maintains the key results of the existing literature. The use of fixed cost heterogeneity is motivated by empirical and anecdotal evidence, and its algebraic ease. Additionally, one of this model's key contributions is it allows for asymmetric changes in trade restrictions. This paves the way for exploration of strategic trade policy and trade agreement in the context of the frontier of trade theory. In this vein of trade policy, I show that not all trade restrictions are created equally, that is the effects of changes in iceberg transport costs is not necessarily isomorphic to changes in an *ad valorem* tariff. These differences along with the effect of changes in the often overlooked beachhead costs have important policy implications. For instance, I find that for a government to maintain the number of varieties in the face of rising transport costs, it is not always the case a decrease in tariffs is called for. The method in which we model trade restrictions has important implications and needs to be implemented in models with greater care.

<div align="center">APPENDIX</div>

A   The Pricing Scheme of an Exporter

Proposition A1. *Given monopolistic competition and a continuum of firms, a tariff is completely passed through onto the foreign consumers.*

Proof. Solving the profit maximization problem for the firm in country k exporting to country j is:

$$\max_{p_k(i)} \ t_j p_k(i) x_j(i) - x_j(i) - \tau_j p_k(i) x_j(i) - \gamma f(i) \tag{A-1}$$

The first order condition is

$$\frac{\partial \pi_X^k(i)}{\partial p_k(i)} = \frac{(1-\varepsilon)t_j^{-\varepsilon}\mu p_k(i)^{-\varepsilon}}{P_j^{1-\varepsilon}} - \frac{t_j^{-\varepsilon}\mu p_k(i)^{1-\varepsilon}}{\left(P_j^{1-\varepsilon}\right)^2}\frac{\partial P_k^{1-\varepsilon}}{\partial p_k(i)} + \frac{\varepsilon t_j^{-\varepsilon}\mu}{p_k(i)^{1+\varepsilon}P_j^{1-\varepsilon}} + \frac{\left(t_j p_k(i)\right)^{-\varepsilon}\mu}{\left(P_j^{1-\varepsilon}\right)^2}\frac{\partial P_k^{1-\varepsilon}}{\partial p_k(i)} = 0$$

Note that

$$\frac{\partial P_k^{1-\varepsilon}}{\partial p_k(i)} = 0$$

Thus the first order condition becomes:

$$\frac{(1-\varepsilon)t_j^{-\varepsilon}\mu p_k(i)^{-\varepsilon}}{P_j^{1-\varepsilon}} + \frac{\varepsilon t_j^{-\varepsilon}\mu}{p_k(i)^{1+\varepsilon}P_j^{1-\varepsilon}} = 0$$

$$(1-\varepsilon) + \frac{\varepsilon}{p_k(i)} = 0$$

Therefore, the exporting firm's optimal price is

$$p_k(i) = -\frac{\varepsilon}{1-\varepsilon} = \frac{1}{\alpha}$$

which is the same constant markup as the domestic firm and means the tariff is completely passed through to the consumer. □

B   Moving from Autarky to Trade

B.1   Heterogeneous Firm Profits

Recall the following conditions:

$$\frac{\partial i_X}{\partial \sigma} = \left[\frac{-\alpha\varepsilon f(i_X)f(i_D)}{\sigma}\right]\left[\frac{\delta(i_D)+1}{\psi}\right] < 0 \tag{B-1}$$

$$\frac{\partial i_D}{\partial \sigma} = \left[\frac{\alpha\gamma\varepsilon f(i_X)^2}{\sigma}\right]\left[\frac{\delta(i_X)+1}{\psi}\right] > 0 \tag{B-2}$$

where

$$\psi \equiv f'(i_X) \left[\delta(i_D) + 1 \right] + \sigma^{1-\varepsilon} f(i_X) f'(i_D) \left[\delta(i_X) + 1 \right]$$

Proposition B1. *The gain in profit for the most efficient firm from opening to trade is ambiguous.*

Proof. A firm will gain from trade in the form of increased total profits if

$$\pi^A(i) - \pi^T(i) = \left[f(i_D^A) - f(i_D^T) \right] + \gamma \left[f(i) - f(i_X^T) \right] < 0 \tag{B-3}$$

If I evaluate (B-3) at the iceberg transport cost that is sufficient to insure Autarky ($\sigma^{\alpha\varepsilon} = \frac{f(i_D^A)}{\gamma f(0)}$), then this becomes:[17]

$$
\begin{aligned}
\pi^A(i) - \pi^T(i) &= -f'(i_D^A) \left[\left. \frac{\partial i_D}{\partial \sigma} \right|^A \right] - \gamma f'(0) \left[\left. \frac{\partial i_X}{\partial \sigma} \right|^A \right] \\
&= -f'(i_D^A) \left[\frac{\alpha\gamma\varepsilon f(0)^2}{\sigma} \right] \left[\frac{\delta(0)+1}{\psi} \right] + \gamma f'(0) \left[\frac{\alpha\varepsilon f(0) f(i_D^A)}{\sigma} \right] \left[\frac{\delta(i_D^A)+1}{\psi} \right] \\
&= \frac{\alpha\varepsilon\gamma f(0)}{\sigma\psi} \left\{ f'(0) f(i_D^A) \left[\delta(i_D^A) + 1 \right] - f'(i_D^A) f(0) \left[\delta(0) + 1 \right] \right\}.
\end{aligned}
$$

Since $f(i)$ is well-behaved, it follows that $\delta(0) = 0$. Therefore,

$$\pi^A(i) - \pi^T(i) = \frac{\alpha\varepsilon\gamma f(0)}{\sigma\psi} \left\{ f'(0) f(i_D^A) \left[\delta(i_D^A) + 1 \right] - f'(i_D^A) f(0) \right\} \tag{B-4}$$

will be negative and the most efficient firm will gain from opening to a "small" amount of trade if and only if

$$\delta(i_D^A) + 1 < \frac{f'(i_D^A) f(0)}{f'(0) f(i_D^A)}.$$

This condition will hold for any fixed cost mapping in which $f'(0) = 0$. For instance if $f(i)$ takes the form of a power function $f(i) = \eta i^n + \lambda$ where $\eta > 0$, $\lambda \geq 0$, and $n > 1$, then (B-4)

---

[17]Note, I need to multiply the derivatives by negative since I'm decreasing σ.

becomes

$$
\begin{aligned}
\pi^A(i) - \pi^T(i) &= \frac{-\alpha\varepsilon\gamma f(0)}{\sigma\psi}\left\{f'(i_D^A)f(0)\right\} \\
&= \frac{-\alpha\varepsilon\gamma f(0)}{\sigma\left[\sigma^{-\alpha\varepsilon}f(0)f'(i_D^A)\right]}\left\{f'(i_D^A)f(0)\right\} \\
&= \frac{-\alpha\varepsilon f(i_D^A)}{\sigma} \\
&= \frac{-\alpha\mu}{\sigma i_D^A} < 0
\end{aligned}
$$

However, this is not the case with a strictly linear case. Let $f(i) = \eta i + \lambda$, then (B-4) becomes:

$$
\begin{aligned}
\pi^A(i) - \pi^T(i) &= \frac{\alpha\varepsilon\gamma f(0)}{\sigma\psi}\left\{f'(0)f(i_D^A)\left[\delta(i_D^A) + 1\right] - f'(i_D^A)f(0)\right\} \\
&= \frac{\alpha\varepsilon\gamma\lambda}{\sigma\psi}\left\{2\eta^2 i_D^A\right\} \\
&= \frac{2\alpha\gamma\lambda\eta\mu}{2\eta i_D^A + \lambda + \gamma\lambda^2} > 0
\end{aligned}
$$

Thus the effect trade has on overall firm profits is ambiguous and depends on the functional form of $f(i)$. $\qquad\square$

B.2 Welfare Effects

Again, for notational ease, I will drop the country labels. This only holds because I am investigating symmetric changes in transport costs.

Proposition B2. *A country will experience an increase in welfare from trade.*

Proof. The indirect utility function for a country is

$$
V = \mu\ln\left(C_x\right) + I - \mu \tag{B-5}
$$

where

$$I = L + \int_0^{i_X} \pi_X(i)di + \int_0^{i_D} \pi_D(i)di$$

Differentiating (B-5) with respect to σ yields:

$$\frac{\partial V}{\partial \sigma} = \frac{\mu}{C_x}\frac{\partial C_x}{\partial \sigma} + \frac{C_X}{\alpha \varepsilon} + \frac{\sigma}{\alpha \varepsilon}\frac{\partial C_X}{\partial \sigma} - \frac{B}{\sigma \alpha \varepsilon}\frac{\partial i_X}{\partial \sigma} + \frac{1}{\alpha \varepsilon}\frac{\partial C_D}{\partial \sigma} - B\frac{\partial i_D}{\partial \sigma} \tag{B-6}$$

where

$$C_x = \left(\int_0^{N_k} x_k(i)^\alpha di\right)^{\frac{1}{\alpha}} \tag{B-7}$$

$$\frac{\partial C_X}{\partial \sigma} = -\left(\frac{\varepsilon C_X}{\sigma} + \frac{C_X}{P}\frac{\partial P}{\partial \sigma}\right) + \frac{\mu \alpha^\varepsilon}{\sigma^\varepsilon P}\frac{\partial i_X}{\partial \sigma} \tag{B-8}$$

$$\frac{\partial C_D}{\partial \sigma} = -\frac{C_D}{P}\frac{\partial P}{\partial \sigma} + \frac{\mu \alpha^\varepsilon}{P}\frac{\partial i_D}{\partial \sigma} \tag{B-9}$$

The use of quasilinear utility allows me to simplify this even more. Dixit and Stiglitz (1977) show that

$$C_x = \frac{Is(\mathcal{P})}{\mathcal{P}}$$

where $s(\mathcal{P}_k)$ is the propensity to consume the heterogeneous good. Quasilinear utility implies that $I_k s(\mathcal{P}_k) = \mu$. Thus

$$
\begin{aligned}
\mu &= P^{\frac{1}{1-\varepsilon}}C_x \\
\Rightarrow 0 &= \frac{1}{1-\varepsilon}P^{\frac{\varepsilon}{1-\varepsilon}}C_x\frac{\partial P}{\partial \sigma} + P^{\frac{1}{1-\varepsilon}}\frac{\partial C_x}{\partial \sigma} \\
\Rightarrow \frac{1}{\varepsilon \alpha P}\frac{\partial P}{\partial \sigma} &= \frac{1}{C_x}\frac{\partial C_x}{\partial \sigma}
\end{aligned}
$$

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Using this and equations (B-8) and (B-9), equation (B-5) can be reduced to

$$
\begin{aligned}
\frac{\partial V}{\partial \sigma} &= \frac{\mu}{\varepsilon \alpha P} \frac{\partial P}{\partial \sigma} - \frac{1}{\varepsilon \alpha} \left(\frac{C_D + \sigma C_X}{P} \frac{\partial P}{\partial \sigma} \right) - C_X \\
&= \frac{1}{\varepsilon \alpha} \left(\frac{\mu - C_D - \sigma C_X}{P} \frac{\partial P}{\partial \sigma} \right) - C_X \\
&= \frac{1}{\varepsilon^2 \alpha} \left(\frac{\mu}{P} \frac{\partial P}{\partial \sigma} \right) - C_X < 0
\end{aligned}
$$

Therefore, a country will gain welfare when transport costs are reduced. $\qquad \square$

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