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On Revenue and Welfare Dominance of Ad Valorem Taxes in Two-Sided Markets

BY

HANS JARLE KIND, MARKO KOETHENBUERGER AND GUTTORM SCHJELDERUP

**Norges
Handelshøyskole**

NORWEGIAN SCHOOL OF ECONOMICS AND BUSINESS ADMINISTRATION

On Revenue and Welfare Dominance of Ad Valorem Taxes in Two-Sided Markets

Hans Jarle Kind*, Marko Koethenbueger† and Guttorm Schjelderup‡

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Abstract

A benchmark result in public economics is that it is possible to increase both tax revenue and welfare by making a monopoly subject to ad valorem taxes rather than unit taxes. We show that such revenue and welfare dominance does not hold in two-sided markets.

Keywords: Ad Valorem Taxes, Unit Taxes, Two-Sided Markets, Revenue-Dominance, Welfare-Dominance, Monopoly.

JEL Codes: H2; D4; L1.

*Norwegian School of Economics and Business Administration, Bergen, Norway. E-mail: hans.kind@nhh.no.

†University of Copenhagen, Department of Economics, Studiestraede 6, DK 1455 Copenhagen, Phone: +45 35324417, Fax: +45 35323000, Email: marko.koethenbueger@econ.ku.dk.

‡Norwegian School of Economics and Business Administration, Bergen, Norway. E-mail: guttorm.schjelderup@nhh.no.

It is well known that ad valorem and unit taxes are equivalent in perfectly competitive markets, in the sense that appropriately chosen they yield the same output and tax revenue. In the case of industries characterized by imperfect competition, ad valorem and unit taxes have quite different effects. As shown by Suits and Musgrave (1953) ad valorem taxes revenue-dominate unit taxes under monopoly, i.e. for any unit tax it is possible to find an ad valorem tax which generates higher tax revenues while leaving quantity choices unaffected. Subsequent literature shows that ad valorem taxes welfare-dominate, and even Pareto-dominate, unit taxes under monopoly; e.g. Delipalla and Keen (1992) and Skeath and Trandel (1994). The findings are supportive for the widespread use of ad valorem taxes in practice. Almost all fiscally important commodity taxes are levied as ad valorem taxes.

In this paper we examine the effect of ad valorem taxes and unit taxes in two-sided monopoly markets. The defining characteristic of a two-sided market is that there exists a platform firm that caters to two distinct groups of customers, whose demand is connected through positive quantity spillovers from at least one of the groups to the other (positive externalities). Thus, the firm enables value-creating interactions between two different groups of end-users, and the pricing decision reflects the demand externalities between the groups; see e.g. Rochet and Tirole (2003, 2006). Two-sided platform firms operate in many economically significant industries, such as the media sector, the financial sector (payment card systems), real-estate brokerage, and the computing industry (computer operating systems, software, game consoles etc.).¹

Contrary to what is the case in one-sided markets, we show that a shift from ad valorem to unit taxes which holds monopoly output fixed in two-sided markets may lead to higher tax revenue. This is true if the quantity spillovers from one end-user group to the other are sufficiently strong. In the same vein we find that unit taxes may yield higher welfare than ad valorem taxes in two-sided markets.

We would like to emphasize that our analysis should not be confused with the standard theory of two goods being complements. Complements are used to describe a situation where an increase in the *price* of one good causes a consumer to reduce consumption of both goods, as measured by the change in his or her compensated demand (see e.g. Kreps 1990, p. 61). A two-sided market, in contrast, consists of two distinct groups of customers, and

¹See Evans 2003 for further examples.

the groups may respond differently to changes in *output* on the other side of the market (see Rochet and Tirole (2003) for a general discussion). An example from the media industry may be clarifying; a newspaper serves two distinct customer groups: readers and advertisers. The price of a newspaper is irrelevant for advertisers *per se*, as are advertising prices for the readers. However, to the extent that a higher newspaper price translates into reduced sales of newspapers, demand for ads will typically fall.² A lower advertising volume (e.g. due to higher advertising prices), on the other hand, may either increase or decrease demand for newspapers, depending on whether ads are perceived as a good or a bad.

1 Analysis

We consider a two-sided market with two different groups of customers, where group $i = A, B$ buys x^i units of good i at price p^i . Customer group i has an inverse demand function $p^i(x^i, x^j)$ where the own-price effect as usual is negative ($p_{x^i}^i < 0$). A characteristic of a two-sided market is that there are positive externalities from at least one side of the market to the other. For example, higher sales of good j may increase the willingness to pay for good i , that is, $p_{x^j}^i > 0$.

Let t^i and τ^i denote the ad valorem and the unit tax rate, respectively, on good i . The monopolist's profit under ad valorem taxation is

$$\pi = \frac{x^A p^A}{1 + t^A} + \frac{x^B p^B}{1 + t^B} - C(x^A, x^B),$$

whilst profit under unit taxes equals

$$\pi = x^A (p^A - \tau^A) + x^B (p^B - \tau^B) - C(x^A, x^B).$$

The cost function $C(x^i, x^j) \geq 0$ satisfies the standard conditions $C_{x^i} > 0$ and $C_{x^i x^i} \geq 0$.

The first-order condition for profit maximization under ad valorem taxation is

$$\pi_{x^i} = 0 \quad \Rightarrow \quad \frac{p^i + x^i p_{x^i}^i}{1 + t^i} + \frac{x^j p_{x^i}^j}{1 + t^j} - C_{x^i} = 0. \quad (1)$$

²Other things equal, the willingness to pay for an ad is increasing in the size of the audience, whether we consider newspapers, TV stations or other media products.

The first-order condition under unit taxes is likewise given by

$$\pi_{x^i} = 0 \quad \Rightarrow \quad (p^i - \tau^i + x^i p_{x^i}^i) + x^j p_{x^i}^j - C_{x^i} = 0. \quad (2)$$

From equations (1) and (2), it follows that quantity choices are identical under unit and ad valorem taxation iff

$$\tau^A = \frac{t^B - t^A}{1 + t^B} (x^B p_{x^A}^B) + t^A C_{x^A} \quad (3)$$

and

$$\tau^B = \frac{t^A - t^B}{1 + t^A} (x^A p_{x^B}^A) + t^B C_{x^B}. \quad (4)$$

In order to bring forward our main result in the simplest possible manner, we shall make an assumption that reduces the complexity of the model without affecting either standard results or the qualitative insight. We assume:

Assumption: *The ad valorem tax rate is zero on good A and positive on good B, that is, $t^A = 0$ and $t^B > 0$.*

This assumption reduces (3) and (4) to

$$\tau^A = \frac{t^B}{1 + t^B} (x^B p_{x^A}^B) \quad \text{and} \quad \tau^B = -t^B (x^A p_{x^B}^A) + t^B C_{x^B}. \quad (5)$$

We define tax revenue under respectively ad valorem and unit taxation as

$$R^{AVT} = \frac{t^B}{1 + t^B} x^B p^B \quad \text{and} \quad R^{UT} = \tau^A x^A + \tau^B x^B.$$

Using equation (5), we can express the change in tax revenue going from ad valorem to unit taxes as

$$R^{AVT} - R^{UT} = t^B \underbrace{\left[\frac{p^B}{1 + t^B} + x^A p_{x^B}^A - C_{x^B} \right]}_{+} x^B + \left[-\frac{t^B}{1 + t^B} p_{x^A}^B x^B x^A \right]. \quad (6)$$

The fact that the first bracketed term on the right-hand side of (6) is positive can readily be verified from first order condition (1) for good B.

Let us consider what would be the outcome if the markets in question were one-sided, meaning that $p_{x^A}^B = p_{x^B}^A = 0$. Then the second term in (6)

would be zero, and we clearly have $R^{AVT} - R^{UT} > 0$.³ Thus, the standard argument in the literature in favor of ad valorem taxation is reproduced: for any given unit tax imposed on a monopoly, there exists an ad valorem tax that revenue-dominates the unit tax.

In order to see that this result need not hold in a two-sided market, suppose that $p_{x^A}^B > 0$. Then the last term on the right-hand side of (6) is negative. Unit taxation may now generate higher tax revenue than does ad valorem taxation. Ceteris paribus, this is more likely to happen the larger the spillover effect $p_{x^A}^B$. In particular, inserting (1) (for good B) into (6), the spillover effect must satisfy

$$p_{x^A}^B > -p_{x^B}^B \frac{x^B}{x^A} \quad \Leftrightarrow \quad \eta_{p^B, x^A} > |\eta_{p^B, x^B}|, \quad (7)$$

where $\eta_{p^B, x^A} \equiv p_{x^A}^B x^A / p^B > 0$ and $\eta_{p^B, x^B} \equiv p_{x^B}^B x^B / p^B < 0$. Condition (7) holds if, in absolute values, the willingness to pay for good B is more elastic with respect to good A than with respect to good B . To see the intuition for this result, it is useful to rewrite first-order condition (1) for good A under ad valorem taxation as

$$p^A + x^A p_{x^A}^A = C_{x^A} - \frac{x^B p_{x^A}^B}{1 + t^B} \quad (8)$$

The left-hand side of (8) may be interpreted as the perceived marginal revenue on good A , and the right-hand side as the perceived marginal cost. The latter is decreasing in $p_{x^A}^B$, since larger quantity spillovers to good B reduce the opportunity cost of producing good A . Note that the value of this spillover is smaller the higher the ad-valorem tax on good B .

The first-order condition for unit taxation, equation (2), can likewise be written as

$$p^A + x^A p_{x^A}^A = C_{x^A} - \tau^A - x^B p_{x^A}^B. \quad (9)$$

Suppose that we start out in an equilibrium with $t^B > 0$, and that $p_{x^A}^B > 0$ is "large". By removing the ad valorem tax, we then see from equation (8) that the perceived marginal costs of good A decrease significantly, thus giving the monopoly incentives to produce more of that good. Thus, the government will have to impose a high unit tax on good A to keep output at the initial

³By appealing to the continuity argument it is clear that this result also holds for sufficiently small spillover effects, i.e. if the two-sidedness is not very pronounced.

level.⁴ This explains why unit taxes yield higher tax revenue than ad valorem taxes for sufficiently high values of $p_{x^A}^B$.

The following example demonstrates in a simple way that tax revenue may rise if we switch from ad valorem to unit taxes. Let the inverse demand functions be given by

$$p^A = 1 - x^A \text{ and } p^B = 1 - x^B + \gamma x^A, \quad (10)$$

which means that $p_{x^B}^A = 0$ and $p_{x^A}^B = \gamma > 0$. We define the cost function as $C(x^A, x^B) = (0.1)(x^A + x^B)$, and set $t^A = 0$ and $t^B = 0.2$. Solving $d\pi/dx^A = d\pi/dx^B = 0$, we obtain

$$x^A = \frac{2(27 + 11\gamma)}{5(24 - 5\gamma^2)} \quad \text{and} \quad x^B = \frac{3(88 + 45\gamma)}{25(24 - 5\gamma^2)}. \quad (11)$$

These first-order conditions yield an optimum with non-negative prices if $\gamma < 1.24$ (for higher values of γ the non-negativity constraint on p^A is binding). Using (10) and (11) in (6) show that unit taxes yield higher tax revenue than ad valorem taxes ($R^{AVT} - R^{UT} < 0$) for $\gamma > 1.05$, as illustrated by Figure 1. It should be stressed that the qualitative result does not hinge on the assumption $t^A = 0$; it is straightforward to construct examples where $R^{AVT} - R^{UT} < 0$ when all taxes are positive.

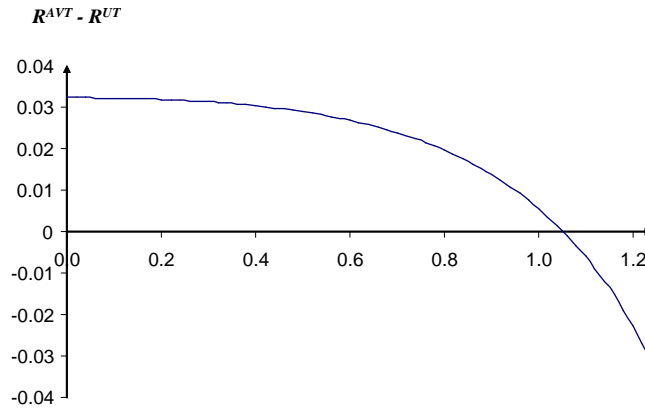


Figure 1: Tax Revenue Comparison.

Finally, can we draw any welfare implications from the analysis? Assume that $p_{x^A}^B$ is sufficiently large to make revenue from unit taxes higher than

⁴Note from equation (9) that τ^A has no direct effect on the value of the spillover $p_{x^A}^B$.

from ad valorem taxes, other things equal (i.e. (7) holds). Instead of keeping output constant when switching from ad valorem taxes to unit taxes, the government might decide to keep tax revenue constant. It could then impose lower unit taxes than those which are given by equations (3) and (4). This in turn implies that output will increase subsequent to the switch from ad valorem to unit taxes. Provided the monopoly undersupplies the two goods from a social point of view, welfare will consequently increase.⁵

It is thus clear that neither revenue dominance nor welfare dominance of ad-valorem taxes extends from one-sided to two-sided markets. In a wider sense, the lesson is that in-depth knowledge of industry structure is more important than what has previously been the conception.

References

- [1] Evans, D. S. (2003), Some empirical aspects of multi-sided platform industries. *Review of Network Economics*, 2 , 191 - 209.
- [2] Delipalla, S. and M. Keen (1992), The comparison between ad valorem taxes and specific taxation under imperfect competition. *Journal of Public Economics* 49, 351-367.
- [3] Kind, H.J., M. Koethenbueger, and G. Schjelderup (2008), Efficiency-enhancing taxation in two-sided markets. *Journal of Public Economics* 92, 1531–1539.
- [4] Kreps, D.M. (1990), *A Course in Microeconomic Theory*. Harvester Wheatsheaf.
- [5] Rochet, J.C., and J. Tirole (2003), Platform competition in two-sided markets. *Journal of the European Economic Association* 1, 990-1029.
- [6] Rochet, J.C. and J. Tirole (2006), Two-sided markets: A progress report. *Rand Journal of Economics* 35, 645-667.
- [7] Skeath, E. S. and G. A. Trandel. (1994), A Pareto comparison of ad valorem and unit taxes in non-competitive environments. *Journal of Public Economics* 53-71.

⁵It can be shown that the goods are underprovided in our example. See Kind et al. (2008) for a general discussion of when goods are under- or overprovided in a two-sided monopoly market.

- [8] Suits D. and R. Musgrave (1953), Ad valorem and unit taxes compared. Quarterly Journal of Economics 67, 598–604.