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# Single-Valuedness of the Demand Correspondence and Strict Convexity of Preferences: An Equivalence Result[☆]

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**Abstract**

If preferences are rational and continuous, then strict convexity implies that the demand correspondence is single-valued (e.g. Barten and Böhm, 1982, lemma 7.3). We show that if, in addition, preferences are strictly monotone then the converse is also true, namely single-valuedness of the demand correspondence implies strict convexity of preferences.

*Key words:* Strict convexity, Strict monotonicity, Demand correspondence, Single-valuedness, Demand function
*JEL:* D11

## 1. Introduction

A basic result in consumer theory is that, if preferences are rational (i.e. complete and transitive, e.g. Richter, 1971) and continuous, then strict convexity of preferences implies single-valuedness of the demand correspondence (see, for instance, Mas-Colell et al., 1995, proposition 3.D.2, and Barten and Böhm, 1982, lemma 7.3).[1] The result is useful since it provides a sufficient condition on preferences to obtain a property on demand which is very convenient when working with microeconomic models of consumption.

In this paper we show that, if preferences are rational, continuous and strictly monotone, then single-valuedness of the demand correspondence implies strict convexity of preferences.

To illustrate the relevance of our contribution consider the following issues.

Q1 Strict convexity of preferences may be considered as a rather strong assumption. Is there any weaker assumption yielding single-valuedness of the demand correspondence?

Q2 Some models take a demand function as a primitive. What are they assuming on the uderlying preferences?

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[1]A similar result can be found in Uzawa (1971, thereom 5), where monotonicity is also assumed to prove the existence of a demand function that is Lipschitz continuous and satistfies the SARP.

These questions cannot be answered on the sole basis that strict convexity implies single-valuedness of the demand correspondence. Our result enables us to solve the issues under the auxiliary assumptions of rational, continuous and strictly monotone preferences. In particular, we answer "no" to Q1 and "strict convexity" to Q2.

In section 2 we introduce definitions and preliminary results. In section 3 we present our main result and we then discuss it in section 4.

## 2. Preliminaries

Let $\succsim$ be a preference relation defined on $\mathbb{R}^n_+$, with $\succ$ and $\sim$ its asymmetric and symmetric parts, respectively. See Mas-Colell et al. Mas-Colell et al. (1995) for definitions of properties on preferences and related concepts.[2] Let $B(\mathbf{p}, w) = \{\mathbf{x} \in \mathbb{R}^n_+ : \langle \mathbf{p}, \mathbf{x} \rangle \leq w\}$ be the individual budget set where $\mathbf{p} \in \mathbb{R}^n_{++}$ is a vector of prices and $w \in \mathbb{R}_+$ denotes individual wealth. The demand correspondence is denoted by $\mathbf{x}^*(\mathbf{p}, w) = \{\hat{\mathbf{x}} \in \mathbb{R}^n_+ : \hat{\mathbf{x}} \succsim \mathbf{x}, \forall \mathbf{x} \in B(\mathbf{p}, w)\}$. Single-valuedness of the demand correspondence means that for all $(\mathbf{p}, w) \in \mathbb{R}^n_{++} \times \mathbb{R}_+$, we have that $\mathbf{x}^*(\mathbf{p}, w)$ is single-valued.

In order to prove our main result, we first show that, when preferences are not strictly convex, there exists a hyperplane supporting an upper contour set at multiple points. Lemma 1 makes the final step, that is, it proves the existence of the hyperplane under the assumption that the convex hull of the upper contour set is closed and not strictly convex.[3]

**Lemma 1.** *If $co(X)$[4] is closed and not strictly convex, then there exists a supporting hyperplane of $X$ containing at least two elements of $X$.*

*Proof.* $co(X)$ is convex but not strictly convex. Hence, there must exist $\tilde{\mathbf{y}}, \hat{\mathbf{y}} \in co(X)$, $\bar{\mathbf{y}} \in \partial co(X)$[5] such that $\tilde{\mathbf{y}} \neq \hat{\mathbf{y}}$ and $\bar{\mathbf{y}} = \beta\tilde{\mathbf{y}} + (1-\beta)\hat{\mathbf{y}}$ for some $\beta \in (0, 1)$. By Caratheodory's theorem there must exist $\tilde{\mathbf{x}}_\mathbf{1}, \tilde{\mathbf{x}}_\mathbf{2}, \ldots, \tilde{\mathbf{x}}_\mathbf{n+1} \in X$, $\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_{n+1} \in [0, 1]$, $\sum_i^{n+1} \tilde{\alpha}_i = 1$ such that $\tilde{\mathbf{y}} = \sum_1^{n+1} \tilde{\alpha}_i \tilde{\mathbf{x}}_\mathbf{i}$, and $\hat{\mathbf{x}}_\mathbf{1}, \hat{\mathbf{x}}_\mathbf{2}, \ldots, \hat{\mathbf{x}}_\mathbf{n+1} \in X$, $\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_{n+1} \in [0, 1]$, $\sum_i^{n+1} \hat{\alpha}_i = 1$, such that $\hat{\mathbf{y}} = \sum_1^{n+1} \hat{\alpha}_i \hat{\mathbf{x}}_\mathbf{i}$.

By the supporting hyperplane theorem, there exists $\mathbf{h} \in \mathbb{R}^n$ and $a \in \mathbb{R}$ such that (i) $\langle \mathbf{h}, \bar{\mathbf{y}} \rangle = a$ and (ii) $\langle \mathbf{h}, \mathbf{y} \rangle \geq a, \forall \mathbf{y} \in co(X)$. Since $\langle \mathbf{h}, \bar{\mathbf{y}} \rangle = \beta \sum_1^{n+1} \tilde{\alpha}_i \langle \mathbf{h}, \tilde{\mathbf{y}}_\mathbf{i} \rangle + (1-\beta) \sum_1^{n+1} \hat{\alpha}_i \langle \mathbf{h}, \hat{\mathbf{y}}_\mathbf{i} \rangle$, then (i) and (ii) imply $\langle \mathbf{h}, \tilde{\mathbf{x}}_\mathbf{i} \rangle = \langle \mathbf{h}, \hat{\mathbf{x}}_\mathbf{j} \rangle = a$, for $i, j = 1, \ldots, n+1$. Since $\tilde{\mathbf{y}} \neq \hat{\mathbf{y}}$, at least two out of $\tilde{\mathbf{x}}_\mathbf{1}, \tilde{\mathbf{x}}_\mathbf{2}, \ldots, \tilde{\mathbf{x}}_\mathbf{n+1}, \hat{\mathbf{x}}_\mathbf{1}, \hat{\mathbf{x}}_\mathbf{2}, \ldots, \hat{\mathbf{x}}_\mathbf{n+1}$ are distinct elements. Finally note that $\langle \mathbf{h}, \mathbf{x} \rangle \geq a, \forall \mathbf{x} \in X$ by (ii) because $X \subseteq co(X)$. □

We have now to prove that the convex hull of the upper contour set is indeed closed (lemma 2) and not strictly convex (lemma 3). We show that it is true under particular assumptions.

**Lemma 2.** *If $X$ is closed under $\geq$, has a lower bound and is closed, then $co(X)$ is closed.*

*Proof.* Suppose $\bar{\mathbf{x}} \in \partial co(X)$. We want to show that $\bar{\mathbf{x}} \in co(X)$. Since $\bar{\mathbf{x}} \in \partial co(X)$, there exists a sequence $(\mathbf{x^m})$ such that $\mathbf{x^m} \to \bar{\mathbf{x}}$, $\mathbf{x^m} \in co(X), \forall m \in \mathbb{N}$. By Caratheodory's theorem, $\forall m \in \mathbb{N}$, $\exists \lambda_1^m, \lambda_2^m, \ldots, \lambda_{n+1}^m \in [0, 1]$, $\sum_{i=1}^{n+1} \lambda_i^m = 1$, $\mathbf{x^m_1}, \mathbf{x^m_2}, \ldots, \mathbf{x^m_{n+1}} \in X$ such that $\mathbf{x^m} = \sum_{i=1}^{n+1} \lambda_i^m \mathbf{x^m_i}$. Since $[0, 1]$ is bounded, by the repeated

[2] We conform to standard mathematical notation and we make use of simple tools that can be found in any textbook of mathematical analysis; see Ok (2007) for a reference book.

[3] Debreu (1959) is the classical reference for separating (and supporting) theorems in economics, while Rockafellar (1997) is a notable reference book for convex analysis.

[4] $co(X)$ denotes the convex hull of $X$.

[5] $\partial co(X)$ denotes the frontier of the convex hull of $X$.

application of the Bolzano-Weierstrass theorem we can find a strictly increasing function $m : \mathbb{N} \to \mathbb{N}$ such that $\lambda_i^{m(k)} \to \lambda_i$, for $i = 1, \ldots, n+1$.

(i) $\lambda_i \in [0,1]$, for $i = 1, \ldots, n+1$, and $\sum_{i=1}^{n+1} \lambda_i = 1$, because $[0,1]$ is closed and $\sum_{i=1}^{n+1} \lambda_i^{m(k)} = 1$ for all $k \in \mathbb{N}$.

(ii) If $\lambda_i > 0$, then we can find a strictly increasing function $\hat{m} : \mathbb{N} \to m(\mathbb{N})$ such that $\mathbf{x_i^{\hat{m}(k)}} \to \mathbf{x_i} \in X$. This follows from $X$ being closed and the Bolzano-Weierstrass theorem, because $\left(\mathbf{x_i^{\hat{m}(k)}}\right)$ is bounded. In fact, $\left(\mathbf{x_i^{\hat{m}(k)}}\right)$ is clearly bounded from below by the lower bound. Furthermore, $\left(\mathbf{x_i^{\hat{m}(k)}}\right)$ must also be bounded from above since, otherwise, $\lambda_i > 0$ and $\mathbf{x^{\hat{m}(k)}} = \sum_{j=1}^{n+1} \lambda_j^{\hat{m}(k)} \mathbf{x_j^{\hat{m}(k)}}$ would imply that $\left(\mathbf{x_j^{\hat{m}(k)}}\right)$ is unbounded from below for some $j$, against the existence of a lower bound.

(iii) $\sum_{i:\lambda_i=0} \lambda_i^{\hat{m}(k)} \mathbf{x_i^{\hat{m}(k)}} \to \hat{\mathbf{x}} \geq \mathbf{0}$. First, $\sum_{i:\lambda_i=0} \lambda_i^{\hat{m}(k)} \mathbf{x_i^{\hat{m}(k)}}$ converges because it is the difference between two converging sequences, $\sum_{i:\lambda_i=0} \lambda_i^{\hat{m}(k)} \mathbf{x_i^{\hat{m}(k)}} = \mathbf{x^{\hat{m}(k)}} - \sum_{i:\lambda_i>0} \lambda_i^{\hat{m}(k)} \mathbf{x_i^{\hat{m}(k)}}$. Second, $\hat{\mathbf{x}} \geq \mathbf{0}$ because $\lambda_i = 0$ and $\left(\mathbf{x_i^{\hat{m}(k)}}\right)$ is bounded from below by the lower bound, for $i = 1, \ldots, n+1$.

Therefore, $\bar{\mathbf{x}} = \hat{\mathbf{x}} + \sum_{i:\lambda_i>0} \lambda_i \mathbf{x_i} = \sum_{i:\lambda_i>0} \lambda_i (\mathbf{x_i} + \hat{\mathbf{x}})$. We note that for $i = 1, \ldots, n+1$, $\mathbf{x_i} + \hat{\mathbf{x}} \in X$ by (ii) and $X$ being closed under $\geq$. We can conclude that $\bar{\mathbf{x}} \in co(X)$, since $\bar{\mathbf{x}}$ is expressed as convex combination, by (i), of elements of $X$. □

**Lemma 3.** *If $X$ is closed under $\geq$, has a lower bound, is closed and not strictly convex, then $co(X)$ is not strictly convex.*

*Proof.* Since $X$ is not strictly convex, there must exist $\mathbf{x_1}, \mathbf{x_2} \in X$, $\bar{\mathbf{x}} \notin in(X)$[6] such that $\mathbf{x_1} \neq \mathbf{x_2}$ and $\bar{\mathbf{x}} = \alpha \mathbf{x_1} + (1-\alpha)\mathbf{x_2}$ for some $\alpha \in (0,1)$. Clearly, $\mathbf{x_1}, \mathbf{x_2}, \bar{\mathbf{x}} \in co(X)$. If $\bar{\mathbf{x}} \in \partial co(X)$, we have proven that $co(X)$ is not strictly convex. Hence, suppose $\bar{\mathbf{x}} \in in(co(X))$. Define $Y = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} < \bar{\mathbf{x}}\}$.

(i) $Y \cap X = \varnothing$, since otherwise $\bar{\mathbf{x}} \in in(X)$ since $X$ is closed under $\geq$.

(ii) $Y \cap in(co(X)) \neq \varnothing$, by the hypothesis that $\bar{\mathbf{x}} \in in(co(X))$ since $in(co(X))$ is an open set.

(iii) $Y \cap ex(co(X)) \neq \varnothing$,[7] by the existence of a lower bound and the definition of $Y$.

(iv) $Y \cap \partial co(X) \neq \varnothing$, since $in(co(X))$ and $ex(co(X))$ are open sets.

Take whaterver $\hat{\mathbf{x}} \in Y \cap \partial co(X)$. By lemma 2, $\hat{\mathbf{x}} \in co(X)$. Hence by Caratheodory's theorem $\exists \lambda_1, \lambda_2, \ldots, \lambda_{n+1} \in [0,1]$, $\sum_{i=1}^{n+1} \lambda_i = 1$, $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_{n+1}} \in X$ such that $\hat{\mathbf{x}} = \sum_1^{n+1} \lambda_i \mathbf{x_i}$. This completes the proof, since for $i = 1, \ldots, n+1$, $\mathbf{x_i} \in co(X)$ and $\mathbf{x_i} \neq \hat{\mathbf{x}}$ because of (i). □

## 3. Main result

We are now ready to state our main result. The strategy of the proof is by contradiction and it consists of the following steps. First, we show that, if preferences are rational, strictly monotone and continuous, then non-strict convexity implies the existence of an upper contour set that is not strictly convex and that satisfies the assumptions of lemma 2 and lemma 3. At this point we apply lemma 1 and we obtain a hyperplane supporting the upper contour set at multiple points. Then, we show that such a hyperplane can be a budget hyperplane, since its normal vector has all positive components which can hence be used as prices. Finally, we check that the demand correspondence is indeed set-valued for such a budget set.

**Proposition.** *If preferences are rational, strictly monotone and continuous, then single-valuedness of the demand correspondence implies strict convexity of preferences.*

[6] $in(X)$ denotes the interior of $X$.

[7] $ex(X)$ denotes the exterior of $X$.

*Proof.* By contradiction, suppose preferences are not strictly convex.

(i) There must exist an upper contour set $\widehat{X}$ which is not strictly convex. Otherwise, for all $\mathbf{x} \in \mathbb{R}_{\geq}^{n}$, for all $\mathbf{x_1}, \mathbf{x_2} \in X$ upper contour set of $\mathbf{x}$, we have that $\beta\mathbf{x_1} + (1-\beta)\mathbf{x_2} \in in(X)$ for $\beta \in (0,1)$. Since $in(X)$ is an open set, we can find $\hat{\mathbf{x}} < \beta\mathbf{x_1} + (\mathbf{1}-\beta)\mathbf{x_2}$ sufficiently close to $\beta\mathbf{x_1} + (\mathbf{1}-\beta)\mathbf{x_2}$ to belong to $X$. Then, $\hat{\mathbf{x}} \succsim \mathbf{x}$. By strict monotonicity $\beta\mathbf{x_1} + (1-\beta)\mathbf{x_2} \succ \hat{\mathbf{x}}$, and hence $\beta\mathbf{x_1} + (1-\beta)\mathbf{x_2} \succ \mathbf{x}$ obtaining strict convexity of preferences.

(ii) $\widehat{X}$ is closed because preferences are continuous.

(iii) $\widehat{X}$ is closed under $\geq$, by strict monotonicity of preferences.

(iv) $\widehat{X}$ has a lower bound. For instance, $\mathbf{0} \leqq \mathbf{x}$, $\forall \mathbf{x} \in \widehat{X}$, since $\widehat{X} \subseteq \mathbb{R}_{+}^{n}$.

In the light of (i), (ii), (iii) and (iv) we can apply lemma 2 and lemma 3, and then lemma 1 to obtain that:

(v) $\exists \mathbf{x_1}, \mathbf{x_2} \in \widehat{X}$, $\mathbf{h} \in \mathbb{R}^n$, $a \in \mathbb{R}$ such that $\langle \mathbf{h}, \mathbf{x_1} \rangle = a$, $\langle \mathbf{h}, \mathbf{x_2} \rangle = a$ and $\langle \mathbf{h}, \mathbf{x} \rangle \geq a$, $\forall \mathbf{x} \in \widehat{X}$.

We now show that $\mathbf{h}$ has all positive components. Suppose not, and consider $\hat{\mathbf{x}} \in \mathbb{R}^n$ having 0 in the components where $\mathbf{h}$ is positive, and being postive in the components where $\mathbf{h}$ is non-positive. Note that $\langle \mathbf{h}, \hat{\mathbf{x}} \rangle \leq 0$. By strict monotonicity and continuity of preferences, $\mathbf{x_1} + \hat{\mathbf{x}} \in int(X)$. Take whatever $\tilde{\mathbf{x}}$ such that $\langle \mathbf{h}, \tilde{\mathbf{x}} \rangle < 0$, which surely exists since $\mathbf{h} \neq \mathbf{0}$. Since $\mathbf{x_1} + \mathbf{h} \in int(X)$, we have that $\mathbf{x_1} + \hat{\mathbf{x}} + \beta\tilde{\mathbf{x}} \in \widehat{X}$ for $\beta > 0$ sufficiently close to zero. However, $\langle \mathbf{h}, \mathbf{x_1} + \hat{\mathbf{x}} + \beta\tilde{\mathbf{x}} \rangle < a$ yielding a contradiction.

Consider prices $\mathbf{p} = \mathbf{h}$ and wealth $w = \langle \mathbf{p}, \mathbf{x_1} \rangle$. We have that $\mathbf{x_1}, \mathbf{x_2} \in B(\mathbf{p}, w)$ by (v). Finally, the lower contour sets of $\mathbf{x_1}$ and $\mathbf{x_2}$ must include $B(\mathbf{p}, w)$ by (v) and continuity of preferences. Therefore, we have that $\{\mathbf{x_1}, \mathbf{x_2}\} \subseteq \mathbf{x}^*(\mathbf{p}, w)$ in contradiction with the assumption of single-valuedness of $\mathbf{x}^*$. □

By combining our proposition with the result in Mas-Colell et al. (1995, proposition 3.D.2), or Barten and Böhm (1982, lemma 7.3),[8] we obtain the following equivalence result, which we state for the sake of reference.

**Equivalence result.** *If preferences are rational, strictly monotone and continuous, then single-valuedness of the demand correspondence is equivalent to strict convexity of preferences.*

## 4. Final remarks

We conclude this note with a couple of remarks. First, we note that both lemma 2 and lemma 3 are actually stronger than required for the proof of our main result. More precisely, the property for a set of being closed under $\geq$ amounts to monotonicity of the underlying preferences for an upper contour set, when preferences are also rational and continuous. However, we assume strict monotonicity in the proposition where our main result is stated. The reason is that with strict monotonicity any supporting hyperplane can be interpreted as a budget hyperplane; this is not the case when preferences are monotone but not strictly monotone (see figure 4 for a discussion). We could have restricted lemma 2 and lemma 3 to deal with the case of strictly monotone preferences, but we think that the current statements might be of some interest on their own.

Second and more importantly, we cannot exclude that the equivalence between single-valuedness of the demand correspondence and strict convexity of preferences might be obtained under weaker assumptions than rationality, continuity and strict monotonicity of preferences. However, figures 1-4 sketch examples where abandoning each of the three assumptions in turn, while maintaining the other two, causes the result to fail. Incidentally, these examples (especially those in figures 3 and 4) show that our proposition is less trivial than it might appear at first sight.

[8] Minor adjustments are necessary to adapt their proofs to our setting, but we neglect them since trivial.

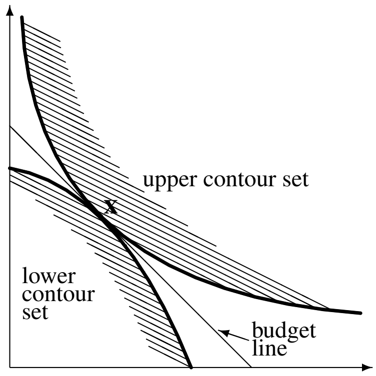


**Figure 1.**[9] *Non-complete (hence non-rational), continuous, strictly monotone preferences.* Preferences are strictly convex but the demand correspondence is empty for the budget line drawn in the figure,[10] since **x** is not comparable – hence neither superior, nor inferior, nor indifferent – with any other bundle in the budget line.

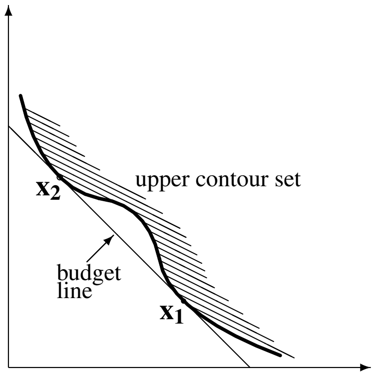


**Figure 2.** *Rational, non-continuous, strictly monotone preferences.* The upper contour set of $\mathbf{x_1}$ is closed at every point of the frontier except at bundle $\mathbf{x_2}$. Even if preferences are not strictly convex, the demand correspondence is not set-valued, because the failure of the continuity assumption causes tangency not to happen at multiple points.

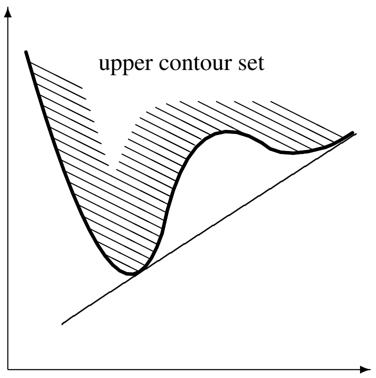


**Figure 3.** *Rational, continuous, non-monotone preferences.* The upper contour set has a slant asymptote, hence the straight line supports the upper contour set at a single point, despite non-strict convexity of preferences. In this example the hypotheses of lemma 2 and lemma 3 are not met (in particular the assumption of being closed under $\geq$), and this explains why we have a single tangency point.

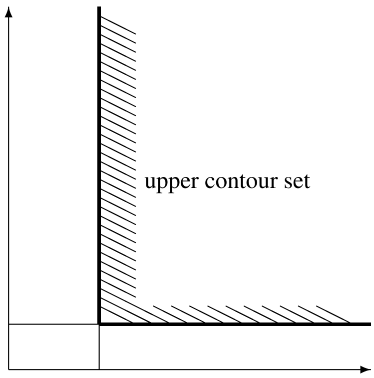


**Figure 4.** *Rational, continuous, monotone but not strictly monotone preferences.* Lemma 2 and lemma 3 can be applied. Lemma 1 then implies that there must exist lines supporting the upper contour set at multiple points. They are drawn in the figure as one vertical line and one horizontal line. However, such lines cannot be budget lines since their normal vector has a zero component, while prices must all be strictly positive.

[9] It is well-known that with non-rational preferences we can have an empty demand correspondence for some budget set. This example suggests that non-rationality may lead to an empty demand correspondence even if we assume strictly monotone and continuous preferences.

[10] Note that, when preferences are non-rational, the definition of $\mathbf{x}^*(\mathbf{p}, w)$ that we use, that is $\{\hat{\mathbf{x}} \in \mathbb{R}^n_+ : \ \hat{\mathbf{x}} \succsim \mathbf{x}, \forall \mathbf{x} \in B(\mathbf{p}, w)\}$, is no longer equivalent to $\{\hat{\mathbf{x}} \in \mathbb{R}^n_+ : \ \nexists \mathbf{x} \in B(\mathbf{p}, w), \mathbf{x} \succ \hat{\mathbf{x}}\}$.

## References


Barten, A. and P. Böhm (1982). Consumer theory. In K. Arrow and M. Intriligator (Eds.), *Handbook of Mathematical Economics*, pp. 381–429. Netherlands: North Holland.

Debreu, G. (1959). Separation theorems for convex sets. In T. Koopmans and A. Bausch (Eds.), *Selected Topics in Economics Involving Mathematical Reasoning*, pp. 48–79. SIAM Review.

Mas-Colell, A., M. Winston, and J. Green (1995). *Microeconomic Theory*. New York: Oxford University Press.

Ok, E. (2007). *Real Analysis with Economic Applications*. Princeton: Princeton University Press.

Richter, M. (1971). Rational choice. In J. Chipman, L. Hurwicz, M. Richter, and H. Sonnenschein (Eds.), *Preference, Utility and Demand*, pp. 29–58. New York: Harcourt Brace and Jovanovich.

Rockafellar, R. (1997). *Convex Analysis*. Princeton: Princeton University Press.

Uzawa, H. (1971). Preference and rational choice in the theory of consumption. In J. Chipman, L. Hurwicz, M. Richter, and H. Sonnenschein (Eds.), *Preference, Utility and Demand*, pp. 7–28. New York: Harcourt Brace and Jovanovich.

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