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1 Introduction

A budget-balanced sharing contract induces inefficient effort provision in risk-neutral teams even if the team output is deterministic.[1] "Anti-Sharing" is an attempt to solve this sharing problem. It requires the team members to promise a fixed payment to an Anti-Sharer. They choose their effort and produce the team output which is collected by the Anti-Sharer. He then pays out the value of the actual output (net of the initially promised fixed payments) to each of the team members. Under this contract, the team members reap the full marginal product of their effort and, therefore, may have an incentive to spend higher effort than under a sharing contract.

When a team member becomes "internal" Anti-Sharer, this leads to a theory of the firm in the spirit of Alchian/Demsetz (1972). In their paper, one team member takes over a special role to solve the sharing problem: he perfectly monitors the other members and becomes residual claimant. Anti-Sharing does not

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[1]See Alchian/Demsetz (1972) and the general proof in Holmstrom (1982). The inefficiency of relative performance evaluation in the team context has been demonstrated by Choi (1993). For risk-averse teams, Rasmusen (1987) has shown that sharing contracts may implement first-best effort. Strausz (1999) has proposed an efficient sharing rule for sequential teams.

require the ability to monitor the other team members.[2] But we will show that in equilibrium the internal Anti-Sharer is unproductive. Thus, internal Anti-Sharing is - in general - unable to implement the first-best solution. However, the team may perform better than under a sharing contract.

If an outside player becomes "external" Anti-Sharer, and if he is working on a non-profit basis, even first-best efforts may be implemented. This is in line with the result of Cooter/Porat (2002), where an external "Anti-Insurer" induces the parties of a dangerous activity to spend efficient care. The model presented in this paper is also applicable to problems with stochastic output, and it mainly addresses internal Anti-Sharing.

Anti-Sharing is different from the outside enforcer of a non-balanced sharing rule mentioned in Holmstrom (1982, 327), as the payments to the Anti-Sharer are not contingent on whether the actual output reaches the efficient level. External Anti-Sharing is closely related to "bonding" mentioned by Holmstrom (1982, 328) in a footnote. Our model also differs from Varian (1994), where each agent pays the other agents for their efforts and demands a compensation for his own.

2 The model

2.1 Notation and assumptions

Consider n risk-neutral agents who spend effort $e_i \geq 0, i = 1..n$ to produce an output $Y(e_1..e_n)$. Individual efforts are assumed to be unobservable.[3] We denote the effort cost of agent i as $c_i(e_i)$ and assume $dc_i/de_i > 0 > d^2c_i/de_i^2$ as well as $c(0) = 0$. Players' utility functions are separable in wealth and effort cost. $E = (e_1..e_n)$ is the effort vector of all n players. $E_{(-i)}$ denotes the effort vector of all n players except player i: $E_{(-i)} = (e_1..e_{i-1}, e_{i+1}..e_n)$. Consequently, $E_{(-i,-j)}$ is the effort vector without the contributions of players i and j. For convenience, we write $E = (E_{(-i)}, e_i) = (E_{(-i,-j)}, e_i, e_j)$. The production function $Y(E)$ is twice differentiable, continuous, with positive but diminishing marginal returns, and with positive cross-partials.

2.2 Inefficiency of the Sharing Contract

The socially optimal effort E^* maximizes $Y(E) - \sum_{i=1}^{n} c_i(e_i)$, satisfying the first order conditions[4]

$$\frac{\partial Y(E^*_{(-i)}, e_i)}{\partial e_i} \stackrel{!}{=} \frac{dc_i(e_i)}{de_i} \;\forall\; i = 1..n \qquad (1)$$

As Holmstrom (1982) has demonstrated, a budget-balanced sharing contract does not induce the players to choose e_i^*. Let s_i denote the share player i receives

[2]Moreover, the concept of hierarchy embodied in Anti-Sharing is not based on formal authority.

[3]It is impossible to infer individual efforts from observed output if $n > 2$, or if the output is stochastic.

[4]Due to the assumptions we have made concerning the second derivatives, we the second-order conditions for a maximum are satisfied (and subsequently neglected).

from the output Y. A sharing contract is "budget-balanced" if $\sum s_i = 1$. Under such a contract, at least one player receives a share smaller than one. Therefore, at least for some players the incentives are insufficient even if all other players choose efficiently.

The individual payoff in equilibrium amounts to $\pi_i' = s_i Y' - c_i(e_i')$, where e_i' denotes the equilibrium effort. With positive cross-partials, this is smaller than the individual payoff for all agents if they choose efficiently, i.e., $\pi_i^* = s_i Y^* - c_i(e_i^*)$. Even an agent who is entitled to a share $s_i = 1$ would receive less than Y^*, since $\forall\ j \neq i:\ Y_{ij} > 0$ and $e_j' < e_j^*$. Therefore, the equilibrium is Pareto inefficient.

2.3 Internal Anti-Sharing

Now we introduce internal Anti-Sharing as an alternative to the sharing contract. Without loss of generality, we assign the role of the internal Anti-Sharer to the n^{th} team member. Each of the other $(n-1)$ agents promises to pay an amount $P_i \geq 0$ to the Anti-Sharer. Note that P_i is independent of the actual effort of player i. All players choose their effort, denoted as $\hat{e}_i$. This generates the actual output $Y(\hat{E})$, which is transferred to the Anti-Sharer. He then pays out its value to each of the other team members, net of P_i.

A team faces a two stage game: during the first stage, the players decide whether to switch from a sharing contract to an Anti-Sharing contract, endowing player n with this role, and in the second stage, they choose their effort. Let us first derive the Nash equilibria in the second stage, provided that the n players agree upon an internal Anti Sharing contract, i.e., the internal Anti-Sharing subgame. We analyze player n and the other $n-1$ players separately and prove the following

Proposition 1: *If one team member offers an Anti-Sharing contract to each team member, then it is individually rational for the internal Anti-Sharer to choose zero effort.*

Proof: Each one of the players $i = 1..(n-1)$ chooses his effort $\hat{e}_i$ so as to maximize $Y(\hat{E}_{(-i)}, e_i) - c_i(e_i) - P_i$. Thus, $\hat{e}_i, i = 1..(n-1)$ satisfy the following first-order conditions for an internal solution: $\partial Y(\hat{E}_{(-i)}, e_i)/\partial e_i = dc_i/de_i$. This would be equivalent to equation (1) above, the condition for a first-best outcome, provided that all players (including n) were doing the same. However, player n still needs to be analyzed. He receives the lump sum payments from the other players and the actual outcome, has to pay out $n-1$ times the actual outcome, and bears his own effort costs. Thus, he chooses $\hat{e}_n$ to maximize $\sum_{i=1}^{n-1} P_i + Y(\hat{E}_{(-n)}, e_n) - (n-1)Y(\hat{E}_{(-n)}, e_n) - c_n(e_n)$. The first derivative is

$$(2-n)\frac{\partial Y(\hat{E}_{(-n)}, e_n)}{\partial e_n} - \frac{dc_n}{de_n}. \qquad (2)$$

$n \geq 2$ implies that this expression is negative. Thus, the unique solution is $\hat{e}_n = 0$. $\square$

The reason for this result is the distortion of the Anti-Sharer's incentives to spend effort. On the one hand, he receives the actual output, but on the other hand, he has to pay its value out to the other $n-1$ team members. Therefore, his marginal net return from spending effort is negative. The application of the Anti-Sharing contract induces the internal Anti-Sharer to be unproductive in equilibrium. Given $\hat{e}_n = 0$, the team can only achieve second-best efficiency:

Proposition 2: *It is one Nash equilibrium in the internal Anti-Sharing subgame that the internal Anti-Sharer chooses $\hat{e}_n = 0$ while the other $(n-1)$ players choose second-best efficient efforts.*

Proof: The second-best efficient effort maximizes $Y(E_{-n}, 0) - \sum_{i=1}^{n-1} c_i(e_i)$. Now, the first-order conditions are

$$\forall i = 1..(n-1) : \frac{\partial Y(E_{(-n)}, 0)}{\partial e_i} \stackrel{!}{=} \frac{dc_i}{de_i} \tag{3}$$

Expecting $\hat{e}_n = 0$, each of the productive team members chooses his individual effort so as to maximize $Y(\hat{E}_{(-i,-n)}, e_i, 0) - c_i(e_i) - P_i$. The first-order condition is identical to expression (3). $\square$

Now we include the first stage of the game into the analysis. Two conditions have to be met for an internal Anti-Sharing contract to become effective: the Anti-Sharer's expected payoff equals the expected payoff of the productive team members, and each of the n players has to be made better off than under a sharing contract.

In the second-best efficient equilibrium with homogeneous players,[5] the Anti-Sharer's payoff is $(n-1)P_i - (n-2)Y(\hat{E}_{(-n)}, 0)$. Each of the other players collects $Y(\hat{E}_{(-n),0}) - c_i(\hat{e}_i) - P_i$. Setting the expected payoffs equal yields

$$P_i = \frac{(n-1)Y(\hat{E}_{(-n)}, 0) - c_i(\hat{e}_i)}{n}.$$

This is the fixed payment the $n-1$ productive players have to promise to the Anti-Sharer. A different P_i would implement Bertrand competition among the team members for the position of the Anti-Sharer. The resulting expected payoff for all team members is $[Y(\hat{E}_{(-n)}, 0) - (n-1)c_i(\hat{e}_i)]/n$. Under a budget-balanced sharing contract with equal shares, each team member expects $Y(e')/n - c_i(e'_i)$. All team members are, thus, better off if

$$Y(\hat{E}_{(-n)}, 0) - (n-1)c_i(\hat{e}_i) > Y(e') - n \cdot c_i(e'_i) \tag{4}$$

Rearrangement of inequality (4) highlights that Anti-Sharing can make all players better off if it induces an increase in output which exceeds n times the difference in individual effort costs, yet net of $c_i(\hat{e}_i)$, as the Anti-Sharer abstains from productive activity under the internal Anti-Sharing contract. This induces internal Anti-Sharing as an equilibrium which is subgame perfect, yet not unique.

3 Discussion and conclusion

The above analysis has demonstrated that, for n homogeneous team members, internal Anti-Sharing may provide a solution for the sharing problem of teams.

[5]The case of heterogenous team members is discussed in the final section.

However, this solution is neither first-best (as the internal Anti-Sharer will become unproductive in equilibrium) nor unique.

If members are heterogeneous, then the role of the internal Anti-Sharer should be assigned to the least productive player. However, the productivity of a player i consists of two components: his effort directly increases the output via his marginal productivity (($\partial Y/\partial e_i$), and it has an indirect impact via the positive cross-partials ($\partial^2 Y/\partial e_i e_j \forall j \neq i$). Our model adds a third aspect of productivity: even if the Anti-Sharer does not directly contribute effort, the strategic impact of his presence may foster the team output.

External Anti-Sharing, e.g. the Anti-Insurer in Cooter/Porat (2002), can be seen as a special case of internal Anti-Sharing with n heterogeneous team members: a team of $n-1$ productive members closes a contract with an n^{th} player who was unproductive in the first place. If $\partial Y/\partial e_n = 0$ and $\partial^2 Y/\partial e_i e_n = 0 \forall i = 1..n-1$, then the appearance of the external Anti-Sharer would even implement a first-best outcome, as there is no productivity loss. A first-best outcome can be implemented if the external Anti-Sharer collects zero payoff even though the mechanism is budget-balanced in the efficient equilibrium.

References

Alchian, A.A./Demsetz, H. 1972: Production, Information Costs, and Economic Organization; in: The American Economic Review 62(5), 777-795.

Choi, Y.K. 1993: Managerial Incentive Contracts with a Production Externality; in: Economics Letters 42(1), 37-42.

Cooter, R. /Porat, A. 2002: Anti-Insurance; in: The Journal of Legal Studies 31(2), 203-232.

Holmstrom, B. 1982: Moral Hazard in Teams; in: Bell Journal of Economics 13(2), 324-340.

Rasmusen E. 1987: Moral Hazard in Risk-Averse Teams; in: Rand Journal of Economics 18, 428-435.

Strausz, R. 1999 Efficiency in Sequential Partnerships; in: Journal of Economic Theory 85, 140-156.

Varian, H.R. 1994: A Solution to the Problem of Externalities when Agents are Well-Informed; in: The American Economic Review 84 (5), 1278-1293.