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When Will Judgment Proof Injurers Take Too Much Precaution?

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Abstract

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ABSTRACT

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I. INTRODUCTION: TWO-POCKET PROBABILITY MODEL AND TWO-POCKET MAGNITUDE MODEL.

If the total assets of a potential injurer are less than the harm he may cause, he is said to be judgment proof.[1] The injurer's total assets can be regarded as a maximum upper threshold on his liability.

Summers (1983) and Shavell (1986) showed that judgment proof injurers tend to take less than optimal precaution. The reason is that, as not all accident losses are internalized, injurers bear the full marginal cost of any additional precaution but receive less than the full marginal benefits thereof (a reduction in the expected harm). In order to prove this result, Shavell (1986) used a probability model in which injurers can reduce the probability of an accident, $p(x)$, by taking more precaution, x, but they cannot influence the magnitude of the harm, h, which is considered to be exogenous.

In addition, he made the simplifying assumption that precaution expenses do not reduce the assets available for compensation in the case of an accident. In most real-world cases, however, the more the injurer spends in precaution, the less he will be able to pay in the case of an accident. This simplifying assumption is realistic only in two cases: non-monetary precautionary measures (which do not reduce the injurer's assets) and legal thresholds (where the law creates an artificial cap on the damages to be paid, and such a cap is set at a lower level than the injurer's total assets). These situations result in what could be called a *two-pocket* model: the injurer behaves as if he had two separate pockets. The first, limited by t, consists of the assets available for victim compensation; the second, unlimited, consists of the resources to be used to take precaution. The money spent on precaution, x, does not reduce the assets available for compensation, t.

In a two-pocket probability model, the injurer seeks to minimize the following function:

(1) $$\begin{cases} J(x) = p(x)h + x & if \quad h \le t \\ J_t(x) = p(x)t + x & if \quad h > t \end{cases}$$ [*Two-pocket probability model*].

While probability models can be appropriate to analyze some accident types (such as aircraft accidents), magnitude models (where more precaution reduces the magnitude of the loss and not the probability of an accident) are more appropriate to analyze other externality problems (such as nuisance, many types of environmental pollution and safety measures). Magnitude models, however, can lead to different analytical results. In Dari Mattiacci and De Geest (2001) we showed that the judgment proof effect in a two-pocket magnitude model is different from the effect in a two-pocket probability model. While the latter leads to systematic underprecaution, a magnitude model yields either optimal precaution or no precaution at all.

(2) $$\begin{cases} J(x) = ph(x) + x & if \quad h(x) \le t \\ J_t(x) = pt + x & if \quad h(x) > t \end{cases}$$ [*Two-pocket magnitude model*].

The reason is that, as the magnitude of the harm depends directly on precaution, the injurer can actually decide whether or not to go bankrupt by selecting his level of precaution. If he does not go bankrupt, *h(x)£t* [solvent zone], he will be able to pay the full harm, hence he will choose the optimal level of precaution. If he goes bankrupt, *h(x)>t* [judgment proof zone], any precaution will be worthless, as he will pay anyway all his assets; therefore, he will choose no precaution at all. The judgment proof effect generates therefore a binary outcome: the injurer decides either to be insolvent (no precaution) or to be solvent (optimal precaution).[2] See figure 1.

In two-pocket models in general, precaution is optimal or lower than optimal: overprecaution never results. This paper analyzes the remaining two possibilities, the one-pocket probability model and the one-pocket magnitude model: the injurer has only one pocket to pay both precaution expenses and damages. The more he spends on precaution, the less will be available to pay damages.

The one-pocket probability model has already been studied by Beard (1990), who showed that under certain conditions the judgment proof problem might lead to overprecaution. The fundamental intuition behind Beard (1990) can be reformulated as follows. The judgment proof problem distorts the injurer's incentives in two ways. First, it provides the injurer with an implicit harm subsidy: the greater the expected accident loss, the greater the portion thereof that will remain uncompensated in the case of an accident (an incentive to take less precaution in order to increase the expected accident loss). Second, it provides the injurer with an implicit precaution subsidy: the more the injurer spends on precaution, the greater the portion of the harm that will remain uncompensated (an incentive to take more precaution). In some cases, the precaution subsidy may dominate the harm subsidy, which induces the injurer to take too much precaution. Beard (1990) used a stochastic model; we will show that his result also holds in a non-stochastic model.

We will also analyze the one-pocket magnitude model – not examined by Shavell (1986) and Beard (1990) – and will find that overprecaution never occurs, because the harm subsidy (an incentive towards less precaution) always offsets the precaution subsidy (an incentive towards more precaution). The levels of precaution taken in the four models are depicted in figure 2.

Section II will present the reader with the one-pocket probability model, and show that overprecaution is possible. In section III, we will consider the one-pocket magnitude model, and demonstrate that overprecaution never results. Section IV will explain the logic behind our results. In section V, we will analyze mixed one-pocket probability-magnitude models. In the concluding section (section VI), we will discuss the policy implications of our findings.

II. THE ONE-POCKET PROBABILITY MODEL

In sections II to V, we will consider accidents between a victim (the party which suffers a loss) and an injurer (the party which does not suffer any loss). They are strangers to each other. For the sake of simplicity, we assume unilateral accidents: only the injurer can take precaution in order to reduce the expected harm. The rule in force is strict liability.

All functions used in this and the next sections will be assumed to be continuous and continuously differentiable to any desired order. Let:

x = the injurer's precaution costs, $x=[0,t]$;

t = the injurer's assets (maximum upper threshold on injurer's liability);

$J(x)$ = the injurer's expected total expenditure;

$SC(x)$ = social cost.

The injurer seeks to minimize his total expenditure, which is the sum of his precaution costs, x, and his expected liability expenses in the case of an accident. The socially optimal level of precaution is the level that minimizes the sum of the precaution costs and the expected harm. The injurer has limited assets t, therefore his exposure to liability may be less than the harm. In addition, the injurer's precaution costs x reduce the assets that are available to pay compensation to t-x.

In a one-pocket *probability* model, the injurer can reduce the probability of an

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accident by spending more on precaution, but he cannot reduce the magnitude of the harm, which is exogenous.

Let:

$p(x)$ = probability of an accident, $p=(0,1)$, $p'<0$, $p''>0$;

h = accident loss, i.e. magnitude of the harm (exogenous).

The social cost function is:

(3) $$SC(x) = p(x)h + x\,.$$

Let x^* denote the level of precaution which minimizes Exp. (3). The assumptions made guarantee convexity.

In a probability model, the harm h remains constant (irrespective of x). In case an accident occurs, the injurer's total expenditure is $h + x$ if he is not judgment proof. If he is judgment proof (that is, unable to compensate for the full harm h) his expenditure in the case of an accident is $(t - x) + x = t$. In the case of an accident, the injurer pays t-x (all assets minus what he spent in precaution) to the victim. Therefore, the injurer's expenditure function is:

(4) $$\begin{cases} J(x) = p(x)h + x & \text{if} \quad h + x \le t \\ J_t(x) = p(x)[t - x] + x & \text{if} \quad h + x > t \end{cases}$$ [*One-pocket probability model*].

$J(x)$ is clearly minimized by x^*. Let x_t denote the level of precaution that minimizes $J_t(x)$. The assumptions made guarantee convexity. The injurer will choose to take optimal precaution if his total expenditure is lower at x^* than at x_t, i.e. if $J(x^*) \le J_t(x_t)$. He will choose x_t otherwise. This condition can be rewritten as:

(5) $$t \geq \{p(x^*)h + x^* - [1 - p(x_t)]x_t\} / p(x_t) \ .$$

Lemma 1: The solution obtained by applying Exp. (5) always satisfies the conditions imposed by Exp. (4).

Let t^* denote the minimum t that verifies condition (5). If the injurer's assets are greater than t^* he will take x^*. On the contrary, if his assets are lower than t^* he will take x_t.

Proposition 1: In a one-pocket probability model, when t ($t<t^*$) is particularly low $x_t<x^*$ (underprecaution); when t increases (but is still lower than t^*) – the injurer is wealthier – also x_t increases, becomes equal to x^* (optimal precaution) and finally greater than x^* (overprecaution). When t becomes equal to or greater than t^* the level of precaution falls down to x^*.

Even an injurer, who is not judgment proof if he takes optimal precaution ($h+x^* \leq t$), might decide to take overprecaution ($x_t>x^*$) in order to be insolvent ($h+x_t>t$) and reduce his total costs.

It is also worthwhile to notice that the level of precaution taken in a one-pocket probability model is higher than or equal to the level of precaution taken in a two-pocket probability model,[3] as depicted by figure 2.

Beard (1990) showed that overprecaution is possible in a (one-pocket) stochastic probability model. Beard (1990, p.634) attributed his findings to a number of features of his model, including the stochastic elements. In this section, we have shown that overprecaution can also occur in a (one-pocket) *non-stochastic* probability model.

III. THE ONE-POCKET MAGNITUDE MODEL

In this section, we will consider a one-pocket *magnitude* model. The injurer can reduce the magnitude of the harm, but not its probability, which is exogenous.

Let:

p = probability of an accident (exogenous), $0<p<1$;

$h(x)$ = accident loss, i.e. magnitude of the harm, $h'<0$, $h''>0$;

The social cost function is:

(6) $$SC(x) = ph(x) + x\ .$$

Let x^* denote the level of precaution which minimizes Exp. (6). The assumptions made guarantee convexity.

The injurer pays the full harm to the victim only if $h(x)+x£t$ (the assets are large enough to pay both the precaution costs and the harm). Otherwise, the injurer pays t-x (all assets minus what he spent in precaution).

Therefore, the injurer's expenditure function is:[4]

(7) $$\begin{cases} J(x) = ph(x) + x & if \quad h(x) + x \leq t \\ J_t(x) = pt + (1-p)x & if \quad h(x) + x > t \end{cases}$$ [*One-pocket magnitude model*].

$J(x)$ is clearly minimized by x^*, while $J_t(x)$ is minimized by $x=0$. The injurer will choose to take optimal precaution if his total expenditure is lower at x^* than at $x=0$, that is, if $J(x^*) \leq J_t(0)$. He will chose $x = 0$ otherwise. This condition can be rewritten as:

(8) $$t \geq h(x^*) + x^*/p\ .$$

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<u>Lemma 2</u>: The solution obtained by applying Exp. (8) always satisfies the conditions imposed by Exp. (7).

Let t^* denote the minimum t that verifies condition (8). If the injurer's assets are larger than t^*, the injurer will take optimal precaution x^*. Note that, as $p<1$, the condition requires the injurer's assets to be higher than (not simply equal to) the sum of optimal precaution costs and optimal harm, hence the injurer will not go bankrupt if he takes optimal precaution. However, the condition does not require the assets to be large enough to pay any possible harm. The assets might be quite limited and the injurer might be potentially insolvent at low precaution levels. If condition (8) is not satisfied, the injurer will opt for no precaution at all, $x=0$. See figure 1 and figure 2.

<u>Proposition 2</u>: In a one-pocket magnitude model, when t is particularly low ($t<t^*$) a judgment proof injurer takes no precaution at all. When t is sufficiently high ($t \geq t^*$), the injurer takes optimal precaution. Overprecaution never results.

In our other study[5] we show that also a two-pocket magnitude model is subject to the same condition in Eq. (8), therefore the level of precaution taken by injurers with the same assets will be the same in one-pocket and two-pocket magnitude models.[6]

IV. THE LOGIC BEHIND THE DIFFERENT FINDINGS FOR THE ONE-POCKET MAGNITUDE MODEL AND THE ONE-POCKET PROBABILITY MODEL

Overprecaution is only possible in one-pocket models, since only these models allow an implicit precaution subsidy to the injurer: the more the injurer spends on precaution, the greater the portion of the harm that will remain unpaid in case he causes an accident which renders him insolvent (an incentive toward more precaution). The precaution subsidy contrasts the harm subsidy also generated by judgment-proofness: a portion of the harm is externalized on the victim (an incentive toward less precaution).[7] The stronger subsidy (in marginal terms) will prevail. If the marginal harm subsidy prevails, underprecaution will result; if the marginal precaution subsidy prevails, the outcome will be overprecaution; if they perfectly set-off each other, optimal precaution will be taken.

In the one-pocket probability model any of those outcomes might result, while in the one-pocket magnitude model, the precaution subsidy can never prevail over the harm subsidy. We will provide the reader first with an intuitive reason for this result to hold and then with a formal explanation.

On the one hand, in both the magnitude and the probability model, the precaution subsidy renders precaution less expensive, but does not reduce the cost of precaution to zero. In fact, the subsidy is “paid” to the injurer only if an accident occurs, as only in this case a reduction in the injurer’s assets is perceived as a benefit, i.e. as a decrease in the damages that will actually be paid to the victim. On the contrary, the cost of precaution is borne by the injurer even if an accident does not occur.

On the other hand, the harm subsidy has different effects under the two models. In the one-pocket probability model, the harm subsidy only concerns a fraction of the expected damages to be paid to the victim. If, for instance, the harm is equal to €100 and the injurer's assents are equal to €85, only 15% of the expected harm is subsidized. The injurer maintains an incentive to spend on precaution and to reduce the probability to pay €85. Therefore, the effect of the harm subsidy competes against the effect of the precaution subsidy and the outcome will depend on their relative weight.

In the magnitude model an insolvent injurer pays his total assets, €85, with a given probability and has no incentive to reduce the magnitude of the harm from €100 to, say, €90. Since 100% of the harm above the injurer's assets is subsidized, the harm subsidy is so powerful to reduce the marginal benefit of precaution to zero, and always overcomes the precaution subsidy – we have noted that the precaution subsidy does not reduce the marginal cost of precaution to zero. A formal interpretation follows.

The second Exp. in (7) depicts the cost function of a bankrupt injurer and can be rewritten as follows:

(9) $$J_t(x) = ph(x) + x - p[h(x) - t] - px$$

[*Harm subsidy and precaution subsidy in the one-pocket magnitude model*].

The first two terms in (9) represent the social cost function of Exp. (6); the third term, $p[h(x)-t]$, describes the expected harm subsidy: the portion of the harm that will remain uncompensated in the case of an accident. The fourth term, px, describes the expected precaution subsidy: the portion of the precaution costs that are subsidized as they reduce the assets available for compensation in the case of an accident.

The first derivative of Exp (9) depicts the marginal values of the four components just described and is given by the following Exp.:

(10) $ph' + 1 - ph' - p$

[*Marginal harm subsidy and marginal precaution subsidy in the one-pocket magnitude model*].

The first term in (10) depicts the optimal incentive to reduce the social cost by means of increasing precaution (marginal reduction in the expected accident loss, i.e. the marginal benefit of precaution); the second term depicts the optimal incentive to reduce the social cost by means of reducing precaution (the marginal cost of precaution). If the injurer were solvent, these two contrasting incentives would yield the optimal level of precaution, which optimally balances costs and benefits of precaution.

The judgment proof subsidies alter such an optimal balance. The third term refers to the harm subsidy, which equals and completely neutralizes the optimal incentive to increase precaution (first term). Because of the harm subsidy, the injurer has no incentive to increase his level of precaution.

The fourth term, the marginal precaution subsidy, is equal to the marginal cost of precaution in terms of absolute values (and thus it is equal to 1): if the injurer spends one more dollar in precaution, he will be able to pay one dollar less in the case of an accident. In terms of expected values, however, the marginal precaution subsidy is p, because the expenditure on precaution reduces the damage payment only if an accident occurs. Contrary to what we have noticed in relation to the marginal harm subsidy, the marginal precaution subsidy is never powerful enough to counteract the optimal incentive to reduce precaution ($p<1$): the result is that the injurer maintains some incentives to reduce precaution.

Therefore, the combined actions of the judgment proof subsidies only provide the injurer with incentives to reduce precaution, as the harm subsidy has a stronger effect than the precaution subsidy. Thus, his optimal choice will always be no precaution.

In the one-pocket probability model the harm subsidy is much weaker than in the magnitude model, and it is no longer sufficient to completely remove the effect of the (marginal) harm on injurer's expenditure. The second Exp. in (4) can be rewritten as follows

(11) $$J_t(x) = p(x)h + x - p(x)[h - t] - p(x)x$$

[*Harm subsidy and precaution subsidy in the one-pocket probability model*].

The first two terms in (11) represent the social cost function of Exp. (3), the third term, *p(x)[h-t]*, describes the expected harm subsidy and the fourth term, *p(x)x*, the expected precaution subsidy. The first derivative of Exp. (11) is

(12) $$p'h + 1 - p'[h - t] - [p'x + p(x)]$$

[*Marginal harm subsidy and marginal precaution subsidy in the one-pocket probability model*].

In the probability model, the marginal harm subsidy (third term) is *p'[h-t]*, while the marginal harm is *p'h* (first term, the optimal incentive to increase precaution). The harm subsidy only reduces the incentive to increase precaution, because the marginal harm subsidy is always smaller than the marginal harm. Therefore, the injurer maintains some incentives to increase precaution.

The marginal precaution subsidy (fourth term) is *p'x+p(x)*. Since *p'* is negative, the first term *p'x* reduces the second, which means that the marginal precaution subsidy is lower than 1 (second term, the optimal incentive to reduce precaution). In the probability model, the precaution subsidy reduces but does not completely remove the optimal incentives to reduce precaution.

Neither the harm subsidy nor the precaution subsidy is powerful enough to completely

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neutralize the optimal incentives to increase and to reduce precaution respectively. Consequently, the injurer maintains some incentives to increase precaution and some incentives to reduce precaution.

The result of the combined actions of the judgment proof subsidies is indeterminate: either of the two might prevail. Therefore, the outcome might be an increase in the incentives to take precaution over the optimal level (overprecaution) as well as a decrease therein (underprecaution). It is also possible that the two subsidies balance each other perfectly and do not alter the optimal incentives (optimal precaution results).

V. MIXED ONE-POCKET PROBABILITY-MAGNITUDE MODELS

So far, we have analyzed two stereotypical situations: a pure probability model and a pure magnitude model. In reality, injurers can often control through precaution both the probability of the accident and the magnitude of the harm. It is, therefore, worth analyzing briefly two mixed cases:

(13) $$SC(x) = p(x)h(x) + x,$$

in which the injurer can reduce both the probability and the magnitude with the same precautionary measure (joint probability-magnitude model), and

(14) $$SC(s,z) = p(s)h(z) + s + z;$$

in which the injurer can reduce the probability by using a precautionary measure s and the magnitude by using a different precautionary measure z (separate probability-magnitude model). We further assume that the product $p(s)h(z)$ is a strictly convex function of s and z. Let x^* and (s^*,z^*) be the levels of precautions that minimize Eq. (13) and Eq. (14)

respectively.

In the first case, Exp. (13), the injurer's expenditure function is:[8]

$$\begin{cases} J(x) = p(x)h(x) + x & \text{if} \quad h(x) + x \le t \\ J_t(x) = p(x)t + [1 - p(x)]x & \text{if} \quad h(x) + x > t \end{cases} \tag{15}$$

[*One-pocket joint-probability-magnitude model*].

J(x) is clearly minimized by x^*. Let x_t denote the level of precaution that minimizes $J_t(x)$. The injurer will choose to take optimal precaution if his total expenditure is lower at x^* than at x_t, that is, if $J(x^*) \le J_t(x_t)$. He will chose $x=0$ otherwise. This condition can be rewritten as:

$$t \ge \{p(x^*)h(x^*) + x^* - [1 - p(x_t)]x_t\} / p(x_t) \,. \tag{16}$$

<u>Lemma 3</u>: The solution obtained by applying Exp. (16) always satisfies the conditions imposed by Exp. (15).

As in the one-pocket probability model, x_t can be lower than, equal to or greater than x^*.

<u>Proposition 3</u>: In a one-pocket joint-probability-magnitude model, a judgment proof injurer ($t<h(x_t)+x_t$) might take underprecaution ($x_t<x^*$), optimal precaution ($x_t=x^*$) or overprecaution ($x_t>x^*$).

In a mixed one-pocket probability-magnitude model, a precaution subsidy exists. Whether this leads to overprecaution will depend on whether the magnitude component of the model prevails over the probability one.

In the second case, Exp. (14), the injurer can take two separate precautionary measures. It is important to notice that the threshold t affects directly precaution z as in the pure magnitude model.

(17)
$$\begin{cases} J(s,z) = p(s)h(z) + s + z & \text{if} \quad h(z) + s + z \leq t \\ J_t(s,z) = p(s)t + (1 - p(s))s + (1 - p(s))z & \text{if} \quad h(z) + s + z > t \end{cases}$$

[*One-pocket separate-probability-magnitude model*].

J(s,z) is clearly minimized by *(s*,z*)*. Let $(s_t, z=0)$ denote the level of precaution that minimizes $J_t(s,z)$[9]. The injurer will choose to take optimal precaution if his total expenditure is lower at *(s*,z*)* than at $(s_t, z=0)$, that is, if $J(s^*,z^*) \leq J_t(s_t,0)$. He will choose $(s_t, z=0)$ otherwise. This condition can be rewritten as:

(18)
$$t \geq \frac{p(s^*)h(z^*) + s^* + z^* - [1 - p(s_t)]s_t}{p(s_t)} .$$

Lemma 4: The solution obtained by applying Exp. (18) always satisfies the conditions imposed by Exp. (17).

Proposition 4: In a one-pocket separate-probability-magnitude model, a judgment proof injurer takes either the optimal level of both the magnitude-reducing and the probability-reducing precaution or no precaution at all with respect to the magnitude measure and underprecaution, optimal precaution or overprecaution with respect to the probability measure.

As in the one-pocket magnitude model, the level of z will either be optimal, z^*, or equal to zero, hence no overprecaution takes place. On the contrary s_t might be lower, equal or

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higher than s^*, but this is not only due to the precaution subsidy (which reduces the cost of precaution by $1-p(s)$), but also to a sort of substitution effect between z and s. In fact, t might be higher than $h(z^*)$, and hence an insolvent injurer may face a higher expected harm than a solvent one, and be led towards more precaution s. As in the one-pocket probability model, overprecaution might result with respect to s. In some cases, s_t might even be higher than s^*+z^*:

Corollary 4.1: In a one-pocket separate-probability-magnitude model a judgment proof injurer might spend in total for both forms of precaution more than a solvent injurer.

Consequently, when probability and magnitude depend on two different precautionary measures, judgment-proofness might yield overprecaution only with respect to the probability-reducing precaution, s. However, the result might be more relevant than in the pure probability model, as two forces push precaution forward: the precaution subsidy and the substitution effect. On the contrary, a magnitude-reducing measure z never experiences overprecaution.

VI. CONCLUDING REMARKS: POLICY IMPLICATIONS

A complete policy analysis would require empirical data, in order to determine whether in real situations injurers can affect the probability of an accident, the magnitude of the harm or both. Therefore, this section can only highlight some general policy implications of our analysis.

Many categories of accidents are subject to regulation. In most of the cases, the

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justification for regulatory intervention is the concern that tort law alone would fail to enhance optimal precaution, as injurers are judgment proof. Our analysis shows that it is important to distinguish between different categories of accidents.

In one-pocket probability and joint-probability-magnitude models, regulators should be concerned not only with underprecaution, but also with overprecaution, which might result as a consequence of the precaution subsidy created by bankruptcy. A regulatory standard coupled with tort liability will solve the underprecaution problem but will not prevent injurers from taking too much precaution. The solution to overprecaution is a regulatory norm that sets a maximum limit on injurers' precaution.

In one-pocket magnitude models, overprecaution never results, but no precaution at all might be the outcome; therefore, the main concern of the regulator should be to compel injurers to take precaution. In this case, regulation of minimum required level of precaution might suffice.

In one-pocket separate-probability-magnitude models, if injurer's assets are not particularly low, $t>h(z^*)$, it might be sufficient to regulate the magnitude-reducing precaution alone. If injurers are forced to choose z^*, then efficient precaution results automatically also with respect to s, which does not need to be regulated.[10]

Many safety measures are likely to be pure magnitude measures, and hence should be analyzed under our approach. Fire escapes, lifeboats, helmets and safety belts, for instance, reduce the magnitude of the harm, and do not affect at all the probability of an accident occurring.

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FOOTNOTES

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1 This terminology has been employed by Shavell (1986) in relation to two situations that may dilute the incentives to take precaution: the injurer's assets might be insufficient to pay for damage compensation and the victim might not always sue the injurer. Summers (1983) referred to both problems as "disappearing defendant" problems. Somewhat arbitrarily, we refer to the first as judgment-proofness and to the second as disappearing defendant. In our other study (Dari Mattiacci and De Geest, 2001), we note that the two are inherently different, especially in the case of magnitude or mixed models. The focus of this study is on judgment-proofness.

2 Note that there are two decisions to be taken here. An inframarginal decision, concerning whether or not to be insolvent, and a marginal decision concerning the level of precaution. If the inframarginal decision is optimal (i.e. if the injurer decides to be solvent), also the marginal decision will be optimal (as the first Eq. in (2) is minimized by x^*). On the contrary, if the inframarginal decision is sub-optimal (i.e. if the injurer decides to be insolvent) the marginal decision will also be sub-optimal (the injurer will take no

precaution, which minimizes the second Eq. in (2)). Note also that no inframarginal decision is available in the two-pocket probability model, as the magnitude of the harm and the level of the threshold are independent from x. In one-pocket models, an inframarginal decision is always available to the injurer, as it will be clear in the analysis of the next several sections.

[3] This outcome is due to the precaution subsidy generated by a one-pocket model: precaution produces an extra marginal benefit consisting of the reduction in the injurer's assets exposed to liability. The point can be easily proven by comparing the first derivatives in the two cases. It is noteworthy that both the level of precaution taken in the one-pocket model and the level of precaution taken in the two-pocket model may be lower than the optimal level of precaution, even though the level of precaution taken in the one-pocket model will always be equal to or higher than the level of precaution taken in the two-pocket model. Nevertheless, the level of precaution taken in the one-pocket model might also be equal to or higher than the optimal level.

[4] Note that $p(t-x)+x$ can be rewritten as $pt+(1-p)x$.

[5] See Dari Mattiacci and the Geest (2001).

[6] The reason for this result to hold is that in a two-pocket magnitude model the second equation in (2) is also minimized by $x=0$. Thus in both cases an insolvent injurer bears pt, while a solvent injurer bears $ph(x^*)+x^*$.

[7] Note that in two-pocket models judgment-proofness generates only the harm subsidy.

[8] Note that $p(x)(t-x)+x$ can be rewritten as $p(x)t+[1-p(x)]x$.

[9] The second equation is increasing in z, hence $z=0$ minimizes it. See also the proof of Proposition 2.

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[10] As we suggest in De Geest and Dari Mattiacci (2002).

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APPENDIX

Proof of Lemma 1

The solution obtained by applying Exp. (5) always satisfies the conditions imposed by Exp. (4). In fact, the opposite is impossible. Let us assume that the solution is x^*; if $h+x^*>t$, then $p(x^*)h+x^* > p(x^*)[t-x^*]+x^* > p(x_t)[t- x_t]+x_t$ (by definition of x_t), hence $J(x^*)>J_t(x_t)$ and the solution would be x_t, which contradicts the premise.

Therefore, if x^* is the solution for Exp. (5), then $h+x^*$£t in Exp. (4) must be satisfied. Let us now assume that the solution is x_t; if $h+x_t$£t, then $p(x_t)[t-x_t]+x_t \geq p(x_t)h+x_t >$ $p(x^*)h+x^*$ (by definition of x^*), hence $J(x^*)<J_t(x_t)$ and the solution would be x^*, which contradicts the premise. Therefore, if x_t is the solution for Exp. (5), then $h+x_t>t$ in Exp. (4) must be satisfied.

In addition to that, for the same reason at least either $h+x^*$£t or $h+x_t>t$ must be satisfied.

Proof of Proposition 1

The simplest case in which $x_t>x^*$ results is when $t=h+x^*$. In this case, the injurer is actually solvent at the optimal level of precaution, but will decide to take a level of precaution higher than optimal, which renders him insolvent.

In fact since by hypothesis $p'(x^*)h+1=0$, it is easy to verify that the first derivative of $J_t(x)$ in x^* is $p'(x^*)[t-x^*]-p(x^*)+1<0$, i.e. the injurer can take a level of precaution $x_t>x^*$ that renders him insolvent and decrease this way his total cost from $J(x^*)$ to $J_t(x_t)<J(x^*)$.

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Moreover, note that since the latter inequality is strict and $J_t(x)$ increases in t, the injurer will still take $x_t>x*$ if $t>h+x*$ up to $t*$ at which he will prefer $x*$. The former also proves that overprecaution might result irrespective of the smallness of $x*$ if compared to h, that is, also in situations in which the accident is particularly unlikely to occur and the expenditure on care might seem to be negligible in relation to the size of the harm.

To complete the analysis, let us now consider that $t<h+x*$; as both $J_t(x_t)$ and x_t decrease if t decreases, x_t is still the solution (see the proof of Lemma 1 for a more formal demonstration) and, as t decreases, x_t will be greater than $x*$ equal to $x*$ and lower than $x*$. Formally, $x*$ satisfies $p'(x*)h=-1$, x_t satisfies $p'(x_t)[t-x_t]-p(x_t)=-1$. The second derivative is positive in both cases. If $x_t>x*$, then $p'(x*)[t- x*]-p(x*)<-1$. By substituting $p'(x*)=-1/h$ in the former we obtain $t>h+x*-p(x*)h$. Hence if $h+x*-p(x*)h<t<h+x*$, the solution is $x_t>x*$; if $h+x*-p(x*)h=t<h+x*$ the solution is $x_t=x*$, if $t<h+x*-p(x*)h$ the solution is $x_t<x*$.

<u>Proof of Lemma 2</u>

The solution obtained by applying Exp. (8) always satisfies the conditions imposed by Exp. (7): $h(x*)+x*\leq t$ if $x*$ is the solution, and $h(0)>t$ if $x=0$ is the solution, in fact the opposite is impossible. The proof is similar to the proof of Lemma 1.

Let us assume that the solution is $x*$; if $h(x*)+x*>t$, then $ph(x*)+x* > p[t-x*]+x* > pt$, hence $J(x*)>J_t(0)$ and the solution would be $x=0$, which contradicts the premise. Hence if $x*$ is the solution for Exp. (8), then $h(x*)+x*\leq t$ in Exp. (7) must be satisfied. Let us now assume that the solution is $x=0$; if $h(0)\leq t$, then $pt+0 \geq ph(0)+0 > ph(x*)+x*$ (by definition of $x*$), hence $J(x*)<J_t(0)$ and the solution would be $x*$, which contradicts the

premise. Hence if $x=0$ is the solution for Exp. (8), then $h(0)>t$ in Exp. (7) must be satisfied. In addition to that, for the same reason, at least either $h(x^*)+x^*£t$ or $h(0)>t$ must be satisfied.

Proof of Proposition 2

The proof that the solution is unique and is either x^* or $x=0$ is straightforward. However, it is worthwhile noticing that, if $h(0)<t$, $x=0$ cannot be a solution as $J_t(x)$ is minimized by $x^\wedge>0$ such that $h(x^\wedge)+x^\wedge=t$. Nevertheless, $J_t(x^\wedge)=ph(x^\wedge)+x^\wedge$ is always greater than $J(x^*)=ph(x^*)+x^*$ (by definition of x^*) and therefore the injurer will always choose x^*, unless he is bankrupt at x^*, i.e. if $t<h(x^*)+x^*$. Hence, $x^\wedge$ could be the outcome only if both $h(0)<t$ and $t<h(x^*)+x^*$ were simultaneously true. However, this is impossible, as it can be proven by the following simple algebra. If $h(0)<t$, then $ph(0)+0<pt$. By definition of x, $ph(x^*)+x^*<ph(0)+0$. Therefore, we can write $ph(x^*)+x^*<pt$, which yields $t>h(x^*)+x^*/p$. As $p<1$, then we can write $t>h(x^*)+x^*$. The latter proves that if $h(0)<t$, then $t<h(x^*)+x^*$ can never result. Therefore, $x^\wedge$ can never be a solution of the injurer's minimization problem: the only two mutually exclusive possibilities are x^* and $x=0$. Figure 1 clarifies this issue.

Proof of Lemma 3

The solution obtained by applying Exp. (16) always satisfies the conditions imposed by Exp. (15): $h(x^*)+x^*£t$ if x^* is the solution, and $h(x_t)+x_t>t$ if x_t is the solution, in fact the opposite is impossible. The proof is similar to the proof of Lemma 1.

Let us assume that the solution is x^*; if $h(x^*)+x^*>t$, then $p(x^*)h(x^*)+x^* > p(x^*)[t-$

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$x^*]+x^* > p(x_t)[t-x_t]+x_t$, (by definition of x_t) hence $J(x^*)>J_t(x_t)$ and the solution would be x_t, which contradicts the premise. Hence if x^* is the solution for Exp. (16), then $h(x^*)+x^*£t$ in Exp. (15) must be satisfied. Let us now assume that the solution is x_t; if $h(x_t)+x_t£t$, then $p(x_t\ )[t-x_t]+x_t \geq p(x_t)h(x_t)+x_t > p(x^*)h(x^*)+x^*$ (by definition of x^*), hence $J(x^*)\leq J_t(x_t)$ and the solution would be x^*, which contradicts the premise. Hence if x_t is the solution for Exp. (16), then $h(x_t)+x_t>t$ in Exp. (15) must be satisfied. In addition to that, for the same reason at least either $h(x^*)+x^*£t$ or $h(x_t)+x_t>t$ must be satisfied.

Proof of Proposition 3

The proof is analogous to the one already given for the one-pocket probability model in Proposition 1.

The simplest case in which $x_t>x^*$ results is when $t=h(x^*)+x^*$. In fact since $p'(x^*)h(x^*)+p(x^*)h'(x^*)+1=0$, it is easy to verify that the first derivative of $J_t(x)$ in x^* is $p'(x^*)[t-x^*]-p(x^*)+1<0$, i.e. the injurer can always take a level of precaution $x_t>x^*$ that renders him insolvent and decrease this way his total cost from $J(x^*)$ to $J_t(x_t)<J(x^*)$. Moreover, note that since the latter inequality is strict and $J_t(x)$ increases in t, the injurer will still take $x_t>x^*$ if $t>h(x^*)+x^*$ up to t^* at which he will prefer x^*. When t decreases both $J_t(x_t)$ and x_t decrease, thus x_t will still be the solution and, as t decreases, will be higher than, then equal to and finally lower than x^*.

Proof of Lemma 4

The solution obtained by applying Exp. (18) always satisfies the conditions imposed by Exp. (17): $h(z^*)+s^*+z^*£t$ if (s^*,z^*) is the solution, and $h(0)+s_t>t$ if $(s_t,z=0)$ is the solution, in

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fact the opposite is impossible. The proof is similar to the one given for Lemma 1.

Let us assume that the solution is (s^*,z^*); if $h(z^*)+s^*+z^*>t$, then $p(s^*)h(z^*)+s^*+z^* > p(s^*)[t-s^*-z^*]+s^*+z^* > p(s_t)[t-s_t]+s_t$, (by definition of s_t) hence $J(s^*,z^*)>J_t(s_t,0)$ and the solution would be (s_t0), which contradicts the premise. Hence if (s^*,z^*) is the solution for Exp. (18), then $h(z^*)+s^*+z^*£t$ in Exp. (17) must be satisfied. Let us now assume that the solution is $(s_t,0)$; if $h(0)+s_t£t$, then $p(s_t\ )[t-s_t]+s_t \geq p(s_t)h(0)+s_t > p(s^*)h(z^*)+s^*+z^*$ (by definition of s^* and z^*), hence $J(s^*,z^*)\leq J_t(s_t,0)$ and the solution would be (s^*,z^*), which contradicts the premise. Hence if $(s_t,0)$ is the solution for Exp. (18), then $h(0)+s_t>t$ in Exp. (17) must be satisfied. In addition to that, for the same reason at least either $h(z^*)+s^*+z^*£t$ or $h(0)+s_t>t$ must be satisfied.

Proof of Proposition 4

The proof is analogous to the proof of Proposition 1.

Proof of Corollary 4.1

Let us assume that $h(z^*)+s^*+z^*=t$, then similarly to the proof of Proposition 1, it is easy to show that the injurer can always take levels of precaution $s_t>s^*+z^*$ and $z_t=0$, which render him bankrupt while decreasing his total cost. In fact the first partial derivative of $J(s,z)$ with respect to s ought to be $p'(s^*)h(z^*)+1=0$; thus, it is easy to verify that the first partial derivative of $J_t(s,z)$ with respect to s in (s^*, z^*) is $p'(s^*)[t-s^*-z^*]-p(s^*)+1<0$ and, since the left-hand side decreases if z decreases and f s_t increases, the injurer can take levels of precaution $s_t>s^*$ and $z_t=0$ such that $J_t(s_t,0)<J(s^*,z^*)$. Moreover, the proof given for Lemma 4 assures that if $(s_t,0)$ is the solution, then the injurer must be bankrupt at $(s_t,0)$,

which occurs only if $s_t > s^* + z^*$ and proves our claim.

FIGURES