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Competition Between Payment Systems: Results

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Abstract

This paper is the second of two companion pieces. In the first we developed a model of competition between payment systems which extends that of Chakravorti and Roson (2006). Here we turn to the results which can be obtained from the Chakravorti and Roson model, from our extension of it, and from a third family of models which we develop in this paper. We obtain two main sets of findings.

First, we shed further light on how competing platforms will set their price level and pricing structure when endogenous multi-homing is allowed on both sides of the market. Our results challenge the general finding in the literature that the greater the propensity of one side of the market to single-home, the more attractive will be the pricing offered to its members by competing platforms. Our results confirm that while this finding generally holds when platforms charge both consumers and merchants on a purely per-transaction basis, it need not hold in the more realistic situation where platforms instead levy flat fees on consumers. Second, we extend findings of Hermalin and Katz (2006) showing that, in certain circumstances, platforms may offer less attractive pricing to the side of the market which holds the choice of payment instrument at the moment of sale.

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George Gardner and Andrew Stone

1. Introduction

A common finding in the literature on competition between payment systems has been that an increase in the relative propensity of either consumers or merchants to single-home – that is, to hold or accept only a single payment instrument (besides cash) – will lead payment networks to price more attractively to that side of the market.[1] The results we present in this paper challenge the universal applicability of this finding.

These results are obtained from numerical simulations of a model of payment system competition developed in Gardner and Stone (2009a). The contrary finding we obtain from this model stems from the fact that it avoids two key simplifying assumptions commonly made in the literature.

The first such assumption is that one side of the market may subscribe to at most one payment network – that is, *must* single-home. We impose no such restriction, so that consumer and merchant decisions are fully endogenous. Second, it is also commonly assumed that networks charge both consumers and merchants on a purely per-transaction basis. By contrast, in our framework consumers face a flat subscription fee for joining a network rather than per-transaction fees – which better matches the reality of (say) card payment markets, where cardholders are typically charged an annual fee.

Several existing models incorporate one or other of these modifications. However, relaxing both assumptions simultaneously creates a fundamental change, by allowing the effective price consumers pay for each transaction to vary across

1 For example, Rochet and Tirole (2003) list as one of six key insights that 'an increase in multihoming on the buyer [consumer] side facilitates steering on the seller [merchant] side and results in a price structure more favorable to sellers' (p 1013). Similarly, Guthrie and Wright (2003) note that 'when consumers hold only one card, the effect of competition between card schemes is to make it more attractive for each card scheme to lower card fees to attract exclusive cardholders to their network' (p 16).

individuals depending on how intensively they use each payment instrument. We demonstrate that having different consumers face different effective per-transaction prices for the same instrument has important consequences for both consumer and payment network behaviour. In particular, it breaks the positive nexus, previously found in the literature, between consumers' propensity to single-home and the attractiveness of the pricing they will be offered by competing platforms.[2]

We obtain our findings by comparing results from three main models: the framework developed in Chakravorti and Roson (2006), referred to here as the 'CR' model; our extension of this, set out in Gardner and Stone (2009a), which we refer to as our 'ECR' or 'Extended Chakravorti and Roson' model; and a third model developed in Section 4 of this paper. This latter model, which we refer to as our 'Per-transaction Pricing' or 'PTP' model, is identical to our ECR model in all respects except with per-transaction pricing rather than flat fees to consumers. A brief recap of the CR and ECR models is provided in Section 2. Section 3 presents simulation results for these two models, from which our main finding emerges. Section 4 then develops the PTP model and uses it to concretely confirm the key role of flat rather than per-transaction pricing in generating our contrary findings.

This paper also contains a second main set of findings. These concern how the assumption regarding which side of the market holds the choice of payment instrument at the moment of sale (typically taken to be consumers) affects competing platforms' relative pricing to merchants and consumers. For simplicity, we investigate this issue in the context of purely per-transaction pricing to both sides of the market, using our PTP model.

Clearly, once platforms have set their fees and all card holding and acceptance decisions have been made, holding the choice of payment instrument confers a benefit on consumers relative to merchants at the moment of sale. However, in terms of the pricing they will be offered by competing platforms *ex ante*, it is less clear whether having this choice would be expected to prove a blessing or a curse to consumers as a group.

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2 Our results also show that, even when platforms do use purely per-transaction pricing, an increasing propensity to single-home on one side leads to that side receiving more attractive relative pricing only if it also holds the final choice of instrument at the moment of sale.

Hermalin and Katz (2006) first noted that consumer choice at the moment of sale might lead to a phenomenon whereby competing platforms bias their pricing in favour of merchants rather than consumers. In a simple model, platforms would behave this way to minimise the impact on their profits from merchants seeking to alter consumers' payment choices via 'steering'.[3]

Our own results confirm this phenomenon, and also Hermalin and Katz's observation that, in a modelling framework with purely per-transaction pricing, the extent of the bias against the side with the choice of instrument declines as the per-transaction cost to platforms of processing payments increases. Further, we show that the strength of this bias also decreases: first, as platforms' per-subscriber costs of signing up new cardholders rise; and second, as consumers exhibit a greater innate propensity to single-home.[4] All of these results are presented in Section 5, while Section 6 concludes.

2. A Brief Recap of the CR and ECR Models

Before turning to the results from our ECR model of payment system competition, and its precursor the CR model, we briefly recap the key features of these models – a more detailed discussion is provided in Gardner and Stone (2009a). Note also that, throughout the remainder of this paper, we adopt the notation set out in Section 2 of that paper to denote key model quantities such as platform fees and consumer and merchant market fractions. For ease of reference, a table summarising this notation is provided in Appendix A of this paper.

2.1 Key Features of the CR and ECR Models

Both the CR and ECR models contain three types of agents: a set of C consumers, denoted Ω^c, a set of M merchants, denoted Ω^m, and the operators of two payment

3 We use the term 'steering' here in the formal sense of the refusal by a merchant to accept a platform's cards, so as to force consumers who multi-home to use the card of a rival platform preferred by the merchant.

4 In the latter case this is consistent with the intuition that while platforms face competitive pressure to forestall 'steering' by whichever side does not hold the choice of instrument (by pricing attractively to that side), this pressure should diminish as consumers' tendency to carry the cards of multiple platforms falls.

platforms, i and j. These platforms offer card payment services to consumers and merchants, in competition with the baseline payment option of cash.[5]

Each consumer is assumed to make precisely one transaction with each merchant, using either cash or one of the platforms' cards.[6] For a transaction to be made using a particular payment type two conditions must be satisfied.

First, both the consumer and merchant must have access to that instrument – so that, for a transaction to occur on (say) platform i, the consumer must hold a card from platform i and the merchant must accept platform i's cards. Second, the decision must be made to select that payment method in preference to other feasible options. Consistent with most treatments of payment sytems, this choice at the moment of sale is assumed to fall to the consumer. All consumers and merchants are assumed to hold/accept cash, so that cash is always a payment option.

The two platforms are assumed to face per-transaction costs of c_i for platform i and c_j for platform j, and to incur fixed costs g_i and g_j respectively for each consumer that they sign up. In terms of pricing, platforms charge flat fees, f_i^c and f_j^c, to each consumer, but levy no per-transaction fees on consumers (and offer no per-transaction rewards). Conversely, platforms impose no flat, up-front fees on merchants, but charge per-transaction fees f_i^m and f_j^m to merchants for the use of their cards. Platforms are assumed to be profit-maximising.

Consumers are assumed to receive a per-transaction benefit for paying by non-cash means, equal to h_i^c for payments made over network i and h_j^c for payments made over network j.[7] Consumers are heterogeneous in their benefits, which throughout

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5 For simplicity of exposition we take these payment systems to be card networks, but our analysis would apply just as well to non-card payment systems.

6 By fixing the number of transactions, independent of the pricing decisions of the platforms, this assumption is consistent with the 'derived demand' aspect of payments. However, it also explicitly rules out 'business stealing' considerations from both models (see Section 1 of Gardner and Stone 2009a).

7 Both merchants and consumers are, for simplicity, assumed to receive zero benefit if cash is used to make a payment. Also, for each consumer, the quantities $\{h_i^c, h_j^c\}$ do not vary from transaction to transaction.

this paper are assumed to be randomly and independently drawn from uniform distributions over the intervals $[0, \tau_i]$ for platform i and $[0, \tau_j]$ for platform j.[8]

In making their decisions about which cards to hold and use, consumers are assumed to maximise utility. Each consumer's total utility is taken to be the sum across all transactions of their benefit accrued on each, less any flat fees paid. In assessing this utility, each consumer is assumed to have a good understanding not only of the flat subscription fees charged by each platform, but also of the fraction of merchants who will choose to accept each platform's cards for given platform fees $\{f_i^c, f_i^m\}$ and $\{f_j^c, f_j^m\}$.

In our ECR model we assume that each consumer can choose to hold no cards, one card or cards from both networks; and, if they hold both, they can choose to use card i in preference to card j, or *vice versa*, where a merchant accepts both. By contrast, in the CR model consumers are assumed to be prohibited by *fiat* from holding more than one platform's card.

Like consumers, merchants are assumed to receive a per-transaction benefit for accepting non-cash payments, equal to h_i^m for payments received on network i and h_j^m for payments received on network j. Merchants are also heterogeneous in these benefits, which are again assumed to be randomly and independently drawn from uniform distributions, this time over the intervals $[0, \mu_i]$ and $[0, \mu_j]$ for platforms i and j.

In both the ECR and CR models it is assumed that each merchant can choose to sign up to both networks, one network, or neither network. Merchants make this choice based on maximising the total net benefit they will receive from doing so, taken to be the sum across all transactions of whatever per-transaction benefit they receive less any per-transaction fee charged. In assessing this, each merchant is once again assumed to have a good knowledge of both: the fraction of consumers who will sign up to each platform, for given platform fees $\{f_i^c, f_i^m\}$ and $\{f_j^c, f_j^m\}$;

8 This assumption of uniform and independent distributions was also used in Chakravorti and Roson (2006). It represents an interesting case and one which significantly simplifies analysis of the model. Note also that, since consumers' per-transaction benefits from using either platform are always non-negative, and they face no per-transaction fees, once consumers have chosen which cards to hold (if any) they will always prefer to pay by card than by cash, whenever possible.

and the fractions of those choosing to hold both cards who will then prefer to *use* card i over card j, or *vice versa*, at the moment of sale.

2.2 Implications of the 'No Consumer Multi-homing' Assumption in the CR Model

It may seem from this description that the differences between our ECR model and the CR model are not great, since they are identical in all respects except for their handling of potential multi-homing by consumers. The discussion in Section 3.3 of Gardner and Stone (2009a), however, already highlighted how far-reaching the consequences are of prohibiting multi-homing by consumers, as imposed in the CR model. In Appendix B we present additional results for the CR model which both augment those in Chakravorti and Roson (2006) and further draw out this point.

We obtain these additional results by deriving full *analytical* solutions of the CR model – under both monopoly and duopoly, and in each case with and without a constraint on platforms' total fees to consumers and merchants (in effective per-transaction terms). These analytical results complement the simulation-based analysis provided in Chakravorti and Roson. In addition to any intrinsic interest they might have, they illustrate how pervasive the effects of a prohibition on multi-homing on one side of the market can be. In particular, we use them to demonstrate that such a prohibition can vitiate not only the distinction between flat and per-transaction pricing by platforms, but also the distinction between which side of the market (consumers or merchants) holds the choice of payment instrument at the moment of sale. Given the frequent use in the literature of models in which, to simplify the analysis, one or both sides are prohibited from multi-homing, these observations may be of broader relevance than just the CR model.[9]

3. The Effects of Competition on Platform Pricing

We turn now to the results obtainable from our ECR model and from its restricted version, the CR model. These results relate both to the overall level of fees charged by competing platforms and, more strikingly, platforms' allocation of these fees across the two sides of the market.

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[9] Examples of papers which focus, partly or wholly, on models where multi-homing is prohibited on at least one side include Caillaud and Jullien (2001) and Armstrong and Wright (2007).

In the case of the CR model, the scenario results we report are computed using the analytical solutions to this model which we have been able to derive – see Gardner and Stone (2009b) for details. For our ECR model they are instead obtained via numerical simulation, since the greater complexity of this model renders it impractical to solve analytically. We report results to three decimal places but, for those obtained numerically, the solution grid used in solving each platform's fees means that these may not be precisely accurate in the third decimal place. Hence, in what follows we regard solutions in which all model variables differ only by ± 0.001 as equivalent.

For the scenarios considered below, the two platforms in the CR or ECR models are assumed to be identical in: the maximum benefits they provide to consumers and merchants ($\tau_i = \tau_j = \tau$, $\mu_i = \mu_j = \mu$); and their costs ($c_i = c_j = c$, $g_i = g_j = g$). We also focus throughout on symmetric model solutions, in which both platforms' fees to merchants are the same, as are their fees to consumers. Chakravorti and Roson (2006) investigated five such scenarios, and we concentrate on the three of these where $\tau = \mu = 1$. These scenarios correspond to situations where platforms: face no costs (Scenario 1); face no costs in signing up consumers, but do face per-transaction costs (Scenario 2); and face per-subscriber costs but no per-transaction costs (Scenario 3). Table 1 presents results for these three scenarios, for both the CR and ECR models, in the duopoly case.[10] Note that Table 1 also contains results for a third model, denoted the PTP (or Per-transaction Pricing) model, which is developed later; these results are discussed in Section 4.3.

From Table 1 we see that the ECR model outcome for Scenario 3 is the same as that for the CR model. This reflects the fact that, even though multi-homing by consumers is permitted in our ECR model, for this scenario the two platforms' Nash equilibrium price settings turn out to make it unattractive for any consumers to hold both platforms' cards.

For Scenarios 1 and 2, however, the ECR model outcomes are different from those for the CR model. In these cases, the fees which symmetric competing platforms will adopt in Nash equilibrium, when consumers are allowed to multi-home, turn out to be consistent with some consumers opting to do so – placing us firmly in

[10] The corresponding monopoly results are reported in Table 2 in Section 5. We omit them here because they turn out to be identical, for each scenario, across both the CR and ECR models.

Table 1: Profit-maximising Prices and Consumer and Merchant Fractions
The case of two symmetric platforms in duopoly competition, with $\tau = \mu$

Variable	Scenario 1 $g = c = 0$			Scenario 2 $g = 0, c = 0.5$			Scenario 3 $g = 0.2, c = 0$		
	CR	ECR	PTP	CR	ECR	PTP	CR	ECR	PTP
Platform fees									
f_i^c	0.180	0.086	–	0.247	0.148	–	0.360	0.360	–
$f_i^{c,*}$	0.236	0.155	0.314	0.449	0.336	0.494	0.449	0.450	0.536
$f_i^{c,**}$	–	0.242	–	–	0.540	–	–	2.250	–
f_i^m	0.236	0.305	0.262	0.449	0.532	0.478	0.200	0.200	0.172
Consumer market fractions									
D_0^c	0.056	0.024	0.099	0.202	0.113	0.244	0.202	0.203	0.287
$D_{i,\sim j}^c$	0.472	0.201	0.215	0.399	0.338	0.250	0.399	0.399	0.249
$D_{i,j;i}^c$	0.000	0.287	0.235	0.000	0.106	0.128	0.000	0.000	0.108
Merchant market fractions									
D_0^m	0.056	0.093	0.069	0.202	0.283	0.228	0.040	0.040	0.030
$D_{i,\sim j}^m$	0.180	0.354	0.336	0.247	0.275	0.296	0.160	0.160	0.246
$D_{i,j}^m$	0.584	0.199	0.260	0.303	0.167	0.180	0.641	0.640	0.478
Other									
Π_i	0.170	0.180	0.200	0.088	0.089	0.103	0.127	0.128	0.109
Vol_i	0.361	0.372	0.347	0.220	0.225	0.218	0.319	0.319	0.285
$f_i^{c,*} + f_i^m$	0.472	0.460	0.576	0.899	0.868	0.972	0.649	0.650	0.708
$AvgPPT_i$	0.472	0.484	0.576	0.899	0.894	0.972	0.649	0.650	0.708
$f_i^m/AvgPPT_i$	0.500	0.630	0.455	0.500	0.595	0.492	0.307	0.307	0.243

Notes: CR denotes the Chakravorti and Roson model; ECR denotes the Extended Chakravorti and Roson model; and PTP denotes the Per-transaction Pricing model (introduced in Section 4 below). For simplicity, results are shown in units such that C and M both equal 1, and the common value of τ and μ is also 1. Vol_i denotes the volume of transactions that take place on platform i, while $AvgPPT_i$ denotes the average price per transaction on platform i. Notation for all other variables is as in Gardner and Stone (2009a) – see also Appendix A.

the non-CR world. We now analyse the results in Table 1 for these two scenarios in greater detail.

3.1 Duopoly Results for the ECR Model

For Scenarios 1 and 2 in the ECR model, some consumers do find it optimal to hold the cards of both platforms ($D_{i,j;i}^c = D_{i,j;j}^c > 0$). This, in turn, induces some merchants to opt to steer cardholders, which accounts for the higher fraction of single-homing merchants in the ECR model in each of these scenarios (compare $D^m_{i,\sim j}$ values).

It is interesting next to compare platforms' total price levels in the two models, for Scenarios 1 and 2. Before doing so, however, it should be noted that the price level is not as well-defined a concept in our ECR model as it is in the CR model (or, more generally, in a world of purely per-transaction pricing). When platforms levy only per-transaction fees their price level may be defined simply as $f^{c,*} + f^m$, which represents the total price paid by consumers and merchants for any given transaction. By contrast, when consumers face flat rather than per-transaction charges and may multi-home, as in our ECR model, different consumers may face different effective per-transaction charges for using (say) platform i (depending on their preferences for using card i or card j).

Specifically, consumers who will use card i whenever possible face an effective per-transaction fee for doing so of $f_i^{c,*}$. However, those who subscribe to both platforms but prefer to use card j face a higher effective per-transaction fee for card i transactions of $f_i^{c,**}$ – since they pay the same flat subscription fee but will undertake fewer card i transactions. Hence, in our ECR model, the quantity $f^{c,*} + f^m$ no longer reflects the effective total price paid *universally* by consumers and merchants for any given transaction.

In Table 1 we adopt two approaches to handling this complication. The first is to continue to report $f_i^{c,*} + f_i^m$ for our ECR model, since this remains the effective total price paid for the bulk of transactions. This reflects that, for symmetric platform fee settings, consumers who prefer to use card i wherever possible (those in sets $\Omega^c_{i,\sim j}$ and $\Omega^c_{i,j;i}$) outnumber, sometimes by a significant margin, those who hold both cards but prefer to use card j (those in set $\Omega^c_{i,j;j}$). The second is to report also the *average* price per transaction, $AvgPPT_i$, obtained by dividing platforms' total revenue from consumers and merchants by their total number of transactions.

On either measure, the most notable feature of the price level results for Scenarios 1 and 2 is how little difference there is between the CR and ECR models in each case. While $AvgPPT_i$ is marginally higher for the ECR than for the CR model in Scenario 1 (0.484 versus 0.472), it is marginally lower in Scenario 2 (0.894 versus 0.899); and in neither case is the difference noteworthy. Hence, for these scenarios at least, permitting consumer multi-homing does not seem to significantly affect competing platforms' equilibrium price level settings.[11]

By contrast, the effect of allowing endogenous consumer multi-homing on platforms' *allocation* of their fees between the two sides of the market is striking. For Scenarios 1 and 2, these allocations are evenly balanced for the CR model, with $f^{c,*}$ equal to f^{m}. Once consumer multi-homing is permitted in the ECR model, however, they become strongly tilted in favour of consumers over merchants – by a factor of around 2 for Scenario 1, and around 1.6 for Scenario 2.[12]

While the strength of this shift is worthy of comment in its own right, more noteworthy still is its direction! The only difference between the CR and ECR models is that consumers in the CR model are prohibited from multi-homing – which is equivalent to their being imbued with an overwhelming propensity to single-home. A common finding in the literature to date on competition between payment systems has been that an increase in the tendency to single-home on one side of the market will lead platforms to price more attractively to that side. One might therefore have expected competing platforms, in Scenarios 1 and 2, to tilt their prices more heavily in favour of consumers in the CR model than in the ECR model. This, however, is the exact opposite of what we find. The results in Table 1 therefore immediately raise the question: why do platforms in our ECR model framework behave in a way contrary to that which might have been expected, based on the literature to date?

3.2 The Incentives Driving Platforms' Price Allocation Decisions

To address this question, it is useful to begin by reviewing the features of those models which have been used thus far to find a nexus between increases in a side's

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11 It also leaves intact Chakravorti and Roson's (2006) observation that this price level will be lower under duopoly than monopoly – compare the ECR model results in Table 1 with those in Table 2 in Section 5.

12 Of course, since some consumers pay a higher effective per-transaction price of $f^{c,**}$ for certain card transactions in the ECR model, these figures somewhat overstate the extent of the shift in favour of consumers. Nevertheless, in Scenario 2 merchants are required to pay almost 60 per cent of the *average* price per transaction, versus only 40 per cent for consumers; while in Scenario 1 even $f^{c,**}$ is still significantly below f^{m}.

propensity to single-home and more attractive pricing by platforms to that side. Identifying features common to these models, but not to our ECR framework, should help to isolate the factors driving our contrary finding.

When we conduct such a review – albeit only a partial one – one thing which stands out is that those models which have yielded such a nexus all effectively involve purely per-transaction pricing by platforms to both sides of the market (see, for example, Armstrong 2006 and Rochet and Tirole 2003). Hence, a natural candidate for a driver of our contrary finding is the presence of flat rather than per-transaction pricing to consumers in our ECR model.

One way to test this hypothesis explicitly is to: construct a third model of payments system competition, equivalent to our ECR model except with per-transaction rather than flat pricing to consumers; and then compare numerical simulations of this model with those for the CR and ECR models, for the same three scenarios considered in Table 1. We take up such an approach in Section 4.

First, however, it is instructive to ask how, intuitively, we would expect flat rather than per-transaction pricing to consumers to affect platforms' price allocation incentives, in the face of a change in consumers' propensity to single-home. Clearly, in a world of purely per-transaction pricing, the goal of profit-maximising platforms is to maximise their *volume* of transactions, for any given total price level. As one side becomes more inclined to single-home, research to date suggests that this entails courting that side more aggressively, since 'platforms have monopoly power over providing access to their single-homing customers for the multi-homing side ... [which] naturally leads to high prices being charged to the multi-homing side' (Armstrong 2006, p 669).

In a world of flat fees to consumers, however, it may be profitable for platforms to pursue a class of consumers who will actually bring relatively few transactions to the platform. For platform i (say) these are the consumers who would prefer to use card j over card i if possible, but who might still judge it worthwhile to subscribe to platform i in addition to platform j, provided the subscription fee is not too high. Such consumers will now pay the same revenue to platform i as its other subscribers (those who subscribe to platform i only and those who subscribe to both platforms but prefer to use card i over card j), despite bringing fewer transactions – and clearly the prospects of attracting such subscribers will rise the less inclined consumers are to single-home.[13]

Another way to put the same point is to note that, when platforms charge only per-transaction fees, all consumers will pay the same amount per transaction to (say) platform i, namely $f_i^{c,*}$. This would make consumers that hold both cards but prefer to use card j of limited value to platform i, given the relatively small number of transactions these consumers will make on the platform. By contrast, when platforms charge flat fees to consumers, those same subscribers pay a higher effective per-transaction fee, $f_i^{c,**}$, than subscribers who use card i whenever possible, giving platform i an incentive not to focus solely on maximising transaction volume, for a given level of $f_i^{c,*} + f_i^m$.

These observations suggest a way in which the use of flat rather than per-transaction fees to consumers might create a countervailing incentive for platforms to court consumers more aggressively, not as their tendency to single-home rises, but rather as it falls.

4. Another Model

In this section we pursue further the relationship between consumers' propensity to single-home and competing platforms' pricing to this side of the market. In Section 3.1 we observed that the usual relationship found in the literature – that competing platforms will price more favourably to consumers the greater their propensity to single-home – is overturned in our ECR model.

We now develop a third model of competing payment systems, equivalent to our ECR model except with per-transaction rather than flat pricing to consumers. We henceforth refer to this as our PTP (Per-transaction Pricing) model, to distinguish it from our ECR model. We use this model to explicitly test our hypothesis that it may be the presence of flat rather than per-transaction pricing which accounts for the usual relationship described above being overturned in our ECR model.

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[13] In the event that a platform's per-transaction costs exceeded its fees to merchants, these consumers would in fact be even more valuable to the platform than its 'regular' subscribers – since the platform, having accumulated its subscription revenue, would then be losing money for every transaction actually undertaken on its network.

4.1 A Model with Purely Per-transaction Pricing

Our new PTP model continues to allow for endogenous multi-homing on both sides of the market, but reverts to the more common assumption in the literature of purely per-transaction pricing by platforms to both consumers and merchants.[14] Fortunately, deriving the geometric frameworks to describe the aggregate card holding and acceptance decisions of consumers and merchants in this setting is much more straightforward than was the case for our ECR model.

We adopt the same notation used for our ECR model – with the exception that, as there are now no flat fees f_i^c and f_j^c to consumers, we take $f_i^{c,*}$ and $f_j^{c,*}$ to denote the per-transaction fees to consumers now directly set by each platform. Then, since platforms' pricing to merchants in our ECR model was also on a purely per-transaction basis, the geometric framework describing the aggregate behaviour of merchants here will actually be exactly the same as there (see Appendix A of Gardner and Stone 2009a for the detailed derivation). By this we mean that, for given $\{f_i^m, f_j^m\}$, this framework will look exactly like that shown in Figure 1 of Gardner and Stone (2009a) – repeated in Panel 1 of Figure 1 of this paper – with the slopes of Lines 1 and 2 given by the same equations in terms of the consumer market fractions $D_{i,\sim j}^c$, $D_{j,\sim i}^c$, $D_{i,j;i}^c$ and $D_{i,j;j}^c$.[15]

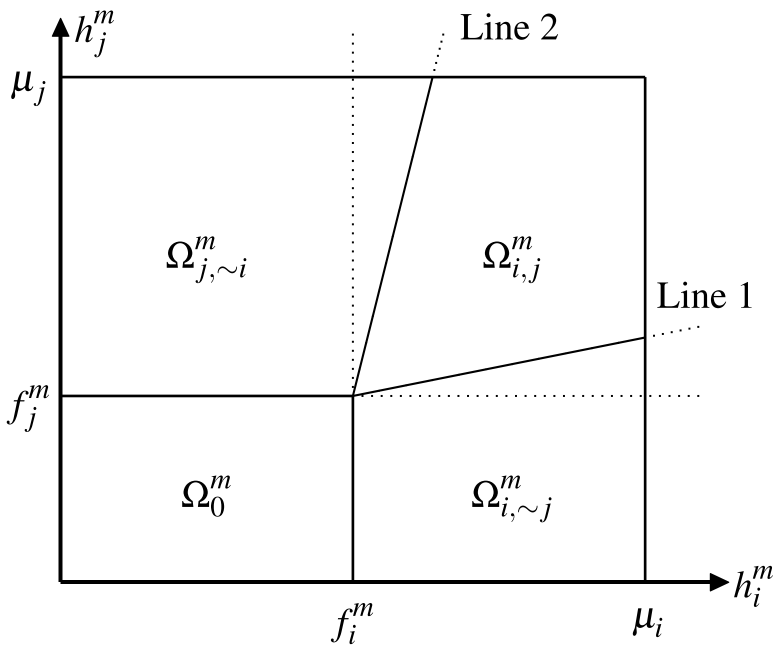
As for the geometric framework describing the aggregate behaviour of consumers, in our PTP model consumers' costs and benefits from the use of either platform's cards are now both determined on a purely per-transaction basis. Hence, since it is they rather than merchants who choose the method of payment at the moment of sale – so that no considerations of 'steering' arise – consumers will clearly choose to subscribe to a platform if and only if their per-transaction benefit from transacting on that platform exceeds the per-transaction consumer fee set by it. Moreover, those opting to subscribe to both platforms will clearly prefer to use card i over card j, when given the choice, if and only if the net benefit from doing

[14] Note that this does not make the model an entirely 'per-transaction model', since on the cost side it still allows for platforms to incur costs g_i and g_j which are per-subscriber rather than per-transaction. Only in the event $g_i = g_j = 0$ will the behaviour of all agents in our new model be determined completely by per-transaction considerations.

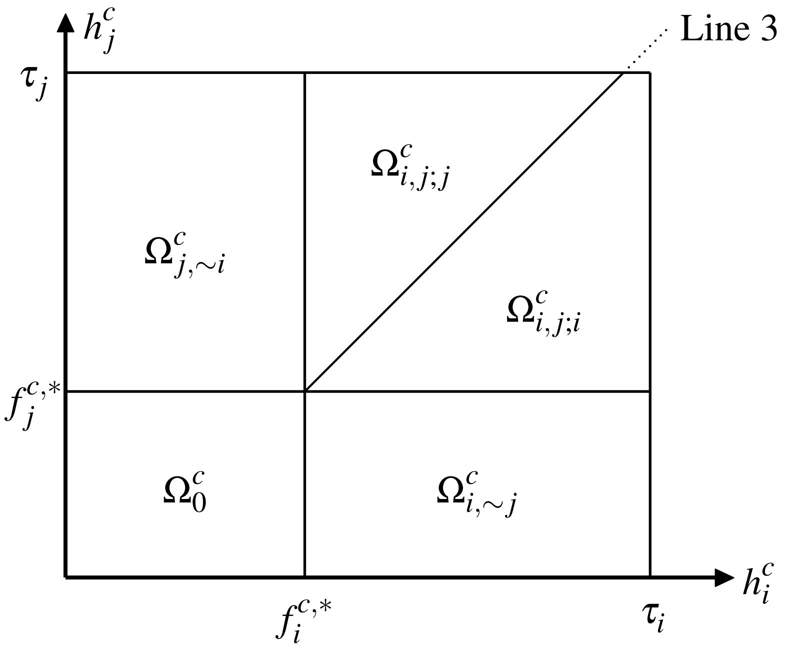
[15] Of course, the actual values of these consumer market fractions – and hence the slopes of these two lines – will typically differ between our PTP and ECR models, even in the event that the fees $\{f_i^m, f_j^m, f_i^{c,*}, f_j^{c,*}\}$ happen to coincide in the two.

Figure 1: Merchant/Consumer Population Breakdowns for the PTP Model

Panel 1: merchants



Panel 2: consumers



Notes: Panel 1 shows a representation of the population of all merchants in h_i^m, h_j^m-space, subdivided into the four subsets Ω_0^m, $\Omega_{i,\sim j}^m$, $\Omega_{j,\sim i}^m$ and $\Omega_{i,j}^m$. Lines 1 and 2 pass through the point (f_i^m, f_j^m) and have slopes $D_{i,j;j}^c/(D_{j,\sim i}^c + D_{i,j;j}^c)$ and $(D_{i,\sim j}^c + D_{i,j;i}^c)/D_{i,j;i}^c$ respectively. Panel 2 shows a representation of the population of all consumers in h_i^c, h_j^c-space, subdivided into the five subsets Ω_0^c, $\Omega_{i,\sim j}^c$, $\Omega_{j,\sim i}^c$, $\Omega_{i,j;i}^c$ and $\Omega_{i,j;j}^c$. Line 3 passes through the point $(f_i^{c,*}, f_j^{c,*})$ and has slope 1.

so, $(h_i^c - f_i^{c,*})$, exceeds that from using card j, $(h_j^c - f_j^{c,*})$. Hence, the population of all consumers will simply subdivide in their card choices as shown in Panel 2 of Figure 1 – with the line dividing $\Omega_{i,j}^c$ into $\Omega_{i,j;i}^c$ and $\Omega_{i,j;j}^c$ having slope 1 and passing through the point $(f_i^{c,*}, f_j^{c,*})$.

This new model – like the CR model – can be solved analytically in the symmetric case. The theoretical details are available in Gardner and Stone (2009b). Rather than go over these here, however, we next record a number of additional observations about our PTP framework, before turning to the results which flow from it about platforms' pricing to consumers and merchants.

4.2 Three Further Observations

In this section we make three further brief observations about the PTP model just described. The first is that, for this model, no issue of potential non-uniqueness arises in relation to the consumer and merchant market outcomes for given platform fees. As was discussed in detail in Section 4 of Gardner and Stone (2009a), for some platform fee choices such non-uniqueness can arise in our ECR model. Here, however, it cannot owing to the fact that consumers face only purely per-transaction pricing and benefits. In combination with their holding the choice of payment instrument at the moment of sale, this removes any feedback from merchant behaviour into consumers' card holding choices. Hence, the breakdown of the consumer side of the market is uniquely determined for any given platform fee choices $\{f_i^{c,*}, f_j^{c,*}\}$; and this, in turn, uniquely fixes the breakdown of the merchant side, for any given $\{f_i^{c,*}, f_i^m, f_j^{c,*}, f_j^m\}$.

A second feature of our PTP framework concerns its similarity to the CR model. While it obviously differs (by construction) from our ECR model in only one way – namely the use of per-transaction rather than flat pricing by platforms to consumers – it turns out that it *also differs from the CR model in only one respect*. For although the CR model is formally set up in terms of platforms charging flat fees to consumers, it is actually equivalent (as discussed in Appendix B) to a model in which both sides of the market are charged purely per transaction. Hence, the CR model is actually identical to our PTP model, save for the restriction that consumers in the CR model may at most subscribe to the cards of a single platform.

Finally, a third point about our PTP model is that it may actually be viewed as a special case of a more general model, incorporating an additional parameter κ.

This parameter represents the disutility to a consumer from holding more than one card – say, due to the cluttering of their wallet, or the hassle of having to check multiple periodic transaction statements. The option of including such a parameter was raised in relation to our ECR model, but not pursued there (see Section 3.3 of Gardner and Stone 2009a). In Section 5, however, it will be useful to take up this option in the context of our PTP model – as well as the option of then varying which side of the market, consumers or merchants, is assumed to hold the choice of payment instrument at the moment of sale.

4.3 Revisiting the Relationship between Platform Pricing and Consumers' Propensity to Single-home

We now turn to the simulation results which may be obtained from our PTP model. For the duopoly case, these were already shown in Table 1 for the three scenarios considered earlier.

In discussing these results we focus primarily on the price allocation aspects of platforms' fee choices. It is worth noting first, however, that the total price level charged by competing platforms in the PTP model, for both Scenarios 1 and 2, is higher than for either the CR or ECR model. Hence, when both sides of the market can multi-home, the use of purely per-transaction pricing allows platforms in these scenarios to extract markedly higher revenue per transaction – which in these cases also translates into higher profits relative to the CR and ECR models.

Greater average per-transaction revenue need not always lead to higher platform profits, however, as the results for Scenario 3 illustrate. Although *AvgPPT* is again higher there than for the CR or ECR models (0.708 versus 0.649), this higher revenue comes at the cost of substantially lower transaction volume (0.285 versus 0.319). With platforms here facing per-subscriber and not just per-transaction costs, this lower volume then turns out to be sufficient to make overall platform profit in the PTP model lower, for this scenario, than in the CR and ECR models (0.109 versus 0.127).

Turning now to the price allocation aspects of the results in Table 1, for all three scenarios we see that competing platforms' fees are more skewed against consumers in the PTP model than in the CR model. In the case of Scenarios 1 and 2, for example, fees to both consumers and merchants are higher in the PTP model, but the difference is greater for consumers – so that they now bear more than half of the total per-transaction price levied by platforms.[16]

In contrast to the findings in Section 3.1 from comparing the CR and ECR models, these results are consistent with the relationship previously found in the literature between competing platforms' price structures and the propensity of consumers to single-home. They suggest that, in a framework of purely per-transaction pricing, an overwhelming propensity for consumers to single-home – the only difference between the CR model and our PTP model – does imply more favourable pricing by platforms towards consumers than if the latter are more amenable to holding multiple cards.

Overall, therefore, these results formally confirm our hypothesis, in Section 3.2, that per-transaction pricing is the key driver of a positive relationship between consumers' propensity to single-home and the attractiveness of the pricing they will be offered by competing platforms. Where purely per-transaction pricing is used by platforms, our results tally with the existence of such a relationship. However, as discussed in Section 3.1, this need not hold when platforms instead levy flat fees on consumers (which is typically the case in the real world).

4.4 Multi-homing and Payment Instrument Choice

A final remark is in order regarding the relationship between the propensity to single-home and platforms' pricing. We just claimed, rather loosely, that our results confirm that where purely per-transaction pricing is used by platforms, a positive link exists between consumers' propensity to single-home and the attractiveness of the pricing they will be offered by competing platforms. These results are, however, for a world where consumers always have the final choice of payment instrument. Hence, they strictly only confirm a positive link between propensity to single-home *on the side with the choice of instrument* and the attractiveness of platforms' pricing to that side. The same *caveat* turns out to apply to the other papers in the literature reporting such a link, such as those quoted in Footnote 1.

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[16] In the case of Scenario 3 this difference is even more marked, in that platforms' per-transaction fees to merchants are actually lower in the PTP model than in the CR model, whereas fees to consumers are much higher.

This raises the question whether or not the more general result, loosely stated earlier, holds without this *caveat*. It turns out that the short answer to this is no: a positive link does *not* necessarily hold between propensity to single-home on the side without the choice of instrument and the relative attractiveness of platforms' pricing to that side – even when platforms' pricing is on a purely per-transaction basis. We return to the demonstration of this claim below.[17]

5. Payment Instrument Choice and Platform Pricing

We now turn in greater detail to the issue of how competing platforms' relative pricing to the two sides of the market is affected by which side holds the choice of payment instrument at the moment of sale. We focus on this for the case where platforms charge both sides of the market on a per-transaction basis, as in our PTP model. An attractive feature of this model is that – at least whenever there are no per-cardholder costs to platforms ($g = 0$) – the *only* asymmetry between the merchant and consumer sides relates to who holds this final choice of instrument. Hence, any tilting of platforms' pricing for or against consumers in this setting can be safely attributed purely to this factor.

In our basic PTP model it is consumers who have the final choice of payment instrument at the moment of sale. In principle, this might push competing platforms' pricing to the two sides of the market either way. On the one hand,

[17] In fact, this claim can already be verified, for the case $g = c = 0$, from the results in Table 1. In this case: (i) the only asymmetry between the two sides in our PTP model relates to who holds the final choice of instrument, so that the results for this model with consumer choice replaced by merchant choice would actually be exactly the same as those shown for Scenario 1, except with the roles of consumers and merchants reversed; and (ii) as discussed in detail in Appendix B, the CR model is actually equivalent to a PTP model with merchant rather than consumer choice (together with an overwhelming predisposition on the part of consumers to single-home). Looking at the results for Scenario 1 in Table 1 then shows that: when consumers have no particular innate propensity to single-home and merchants choose (PTP model with consumers and merchants switched), consumers must bear only around 45.5 per cent of the total per-transaction price charged by platforms to both sides ($f^{c,*} = 0.262$, $f^m = 0.314$); whereas when they have an overwhelming innate propensity to single-home and merchants choose (CR model), consumers must bear half of platforms' total per-transaction price ($f^{c,*} = f^m = 0.236$). Hence, in this case a greater predisposition to single-home by the side without the choice of instrument does not translate into more attractive relative pricing by the platforms to that side (even though it does improve the absolute pricing by competing platforms to consumers).

it could drive down each platform's relative pricing to consumers, as platforms compete to persuade consumers to direct transactions to them rather than a rival. Equally, however, it could lead platforms to tilt their relative pricing in favour of merchants, to minimise the impact on their profits from merchants seeking to influence consumers' payment choices via steering.

The results for Scenario 1 in Table 1 show that, for our PTP model with no per-cardholder or per-transaction costs to platforms ($g = c = 0$), it is the latter effect which wins out – with consumers being charged more per transaction than merchants (0.314 versus 0.262). This phenomenon, whereby (symmetric) competing platforms price less attractively to the side which holds the ultimate choice of instrument, was previously observed (in the context of network routing rules) by Hermalin and Katz (2006).[18] We refer to it henceforth as the Hermalin and Katz phenomenon.

Hermalin and Katz also observed that the strength of their phenomenon declines as platforms' per-transaction costs (c) rise. An illustration of this can be seen in our results, by comparing the PTP model results for Scenarios 1 and 2 in Table 1. In Scenario 2, while a higher c (0.5 versus 0) results in platforms charging higher per-transaction prices to both sides, the difference in f^m values is greater than it is for $f^{c,*}$. The upshot is that merchants must bear 49.2 per cent of platforms' total per-transaction fees in Scenario 2, compared with only 45.5 per cent in Scenario 1. More generally, this further finding of Hermalin and Katz can be replicated using our PTP model by simulating the model for a range of c values. Figure 2 shows platforms' allocations of their total per-transaction fees between the two sides of the market as c varies, clearly displaying the expected monotonic downward trend in the consumer share of platforms' total fees.[19]

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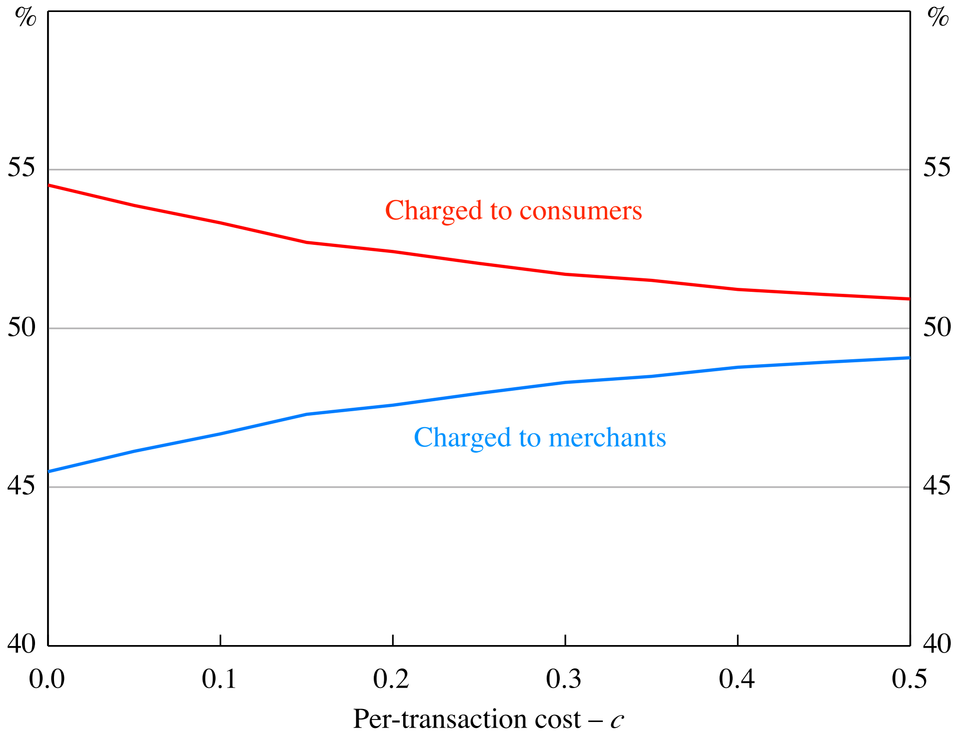
[18] Our PTP model is identical to the model used by Hermalin and Katz, except that it allows in principle for a cost (g) to platforms, for signing up subscribers, which is per-subscriber not per-transaction. In the case where $g = 0$ the two models are equivalent.

[19] Figure 2 shows (symmetric) platforms' proportional allocations of their total per-transaction fees across the two sides of the market, in our PTP model, as c increases. Part of the decline observable in Figure 2 is therefore due simply to the rise in platforms' total per-transaction fees which an increase in c causes. Part is also due, however, to the fact that, even in absolute terms, the gap between platforms' pricing to the two sides also narrows as c rises.

Figure 2: Platforms' Per-transaction Fee Allocations

PTP model with $g = 0$ and consumer choice – per cent of total fees



It is worth noting a simple but important point about the source of the tilting we observe in platforms' pricing, in favour of merchants, in our PTP model with $g = 0$. We noted earlier that platforms might be motivated to offer such favourable pricing to merchants to discourage them from steering consumers. It should be emphasised, however, that the key to merchants receiving preferential pricing is not their scope to steer *per se*, but their scope to do so *in the presence of competition between platforms*. With such competition, the threat of steering creates a risk for each platform, lest they lose transactions to their rival as a result of merchant steering. It is this possibility of lost profit which generates downward pressure on each platform's relative pricing to merchants as a bloc. By contrast, were both platforms operated by a monopoly provider of card payment services, any steering by merchants, to the extent that it simply shifted card payments between platforms, would be of no concern to the monopolist. Hence, while such steering might also see a small number of card payments lost to cash, the pressure on a monopolist to hold down prices to merchants, due to the threat of steering, would be much weaker than in the duopoly case. This can be formally confirmed by comparing the results for our PTP model for Scenarios 1 and 2 under monopoly (Table 2) with those for the duopoly case (Table 1).

Table 2: Profit-maximising Prices and Consumer and Merchant Fractions
The case of two symmetric platforms with a monopoly operator, with $\tau = \mu$

Variable	Scenario 1 $g=c=0$			Scenario 2 $g=0, c=0.5$			Scenario 3 $g=0.2, c=0$		
	CR	ECR	PTP	CR	ECR	PTP	CR	ECR	PTP
Platform fees									
f_i^c	0.375	0.375	–	0.324	0.324	–	0.473	0.471	–
$f_i^{c,*}$	0.500	0.500	0.370	0.593	0.593	0.522	0.593	0.593	0.620
$f_i^{c,**}$	–	2.000	–	–	1.310	–	–	2.892	–
f_i^m	0.250	0.250	0.432	0.453	0.453	0.538	0.203	0.205	0.294
Consumer market fractions									
D_0^c	0.250	0.250	0.137	0.352	0.352	0.272	0.352	0.352	0.384
$D_{i,\sim j}^c$	0.375	0.375	0.233	0.324	0.324	0.250	0.324	0.324	0.236
$D_{i,j;i}^c$	0.000	0.000	0.198	0.000	0.000	0.114	0.000	0.000	0.072
Merchant market fractions									
D_0^m	0.063	0.063	0.187	0.206	0.205	0.289	0.041	0.042	0.086
$D_{i,\sim j}^m$	0.188	0.188	0.320	0.248	0.248	0.282	0.162	0.163	0.266
$D_{i,j}^m$	0.563	0.563	0.174	0.299	0.300	0.146	0.635	0.632	0.384
Other									
Π_i	0.211	0.211	0.222	0.097	0.097	0.105	0.141	0.141	0.124
Vol_i	0.281	0.281	0.277	0.177	0.178	0.188	0.258	0.258	0.219
$f_i^{c,*} + f_i^m$	0.750	0.750	0.802	1.047	1.045	1.060	0.797	0.798	0.914
$AvgPPT_i$	0.750	0.750	0.802	1.047	1.045	1.060	0.797	0.798	0.914
$f_i^m/AvgPPT_i$	0.333	0.333	0.539	0.433	0.433	0.508	0.255	0.257	0.322

Notes: CR denote the Chakravorti and Roson model; ECR denotes the Extended Chakravorti and Roson model; and PTP denotes the Per-transaction Pricing model. For simplicity, results are shown in units such that C and M both equal 1, and the common value of τ and μ is also 1. Vol_i denotes the volume of transactions that take place on platform i, while $AvgPPT_i$ denotes the average price per transaction on platform i. Notation for all other variables is as in Gardner and Stone (2009a) – see also Appendix A.

In Scenario 1 we see that, when a monopoly owner operates both platforms, the relative pricing to merchants versus consumers is reversed, with consumers now receiving the (relatively) more favourable deal ($f^{c,*} = 0.370$ versus $f^m = 0.432$).[20] The same reversal is evident in the results for Scenario 2 – although, as in the duopoly case, the effect of c being higher is to mute the extent of the tilting of platforms' prices in favour of consumers. Overall, for our PTP model, possessing

[20] The total price level, $f^{c,*} + f^m$, is also higher under monopoly than under duopoly, as expected.

the final choice of payment instrument thus generates a benefit even *ex ante* for consumers in the monopoly case – unlike in the duopoly case – in terms of the pricing they receive from platforms compared with merchants.

Returning to the case of competing platforms, our results thus far have merely confirmed earlier findings of Hermalin and Katz (2006). We turn now to the task of trying to extend these findings, using our basic PTP model and some natural generalisations of it. One such possible extension is to ask: how does the strength (or even the direction) of the Hermalin and Katz phenomenon vary as platforms' per-subscriber costs of signing up new cardholders, $g_i \equiv g_j \equiv g$, rise?

This is not as straightforward as for the case of increasing c. Unlike there, a non-zero value for g in our PTP model creates an inherent asymmetry between the two sides of the market. Hence, we cannot simply simulate the model for a range of g values and then study how platforms' relative pricing to consumers and merchants changes over this range. Rather, to *isolate* the effect on platforms' relative pricing of which side has the choice of payment instrument it is necessary to construct a new model, identical to our basic PTP model in every respect except that the choice of instrument is instead taken to lie with merchants.[21] It is then possible to study how changes in g affect the Hermalin and Katz phenomenon by:

i. simulating both our basic PTP model and this new variant of it for a range of g values; and

ii. for each g value comparing how platforms' relative pricing to consumers and merchants differs between the two models.

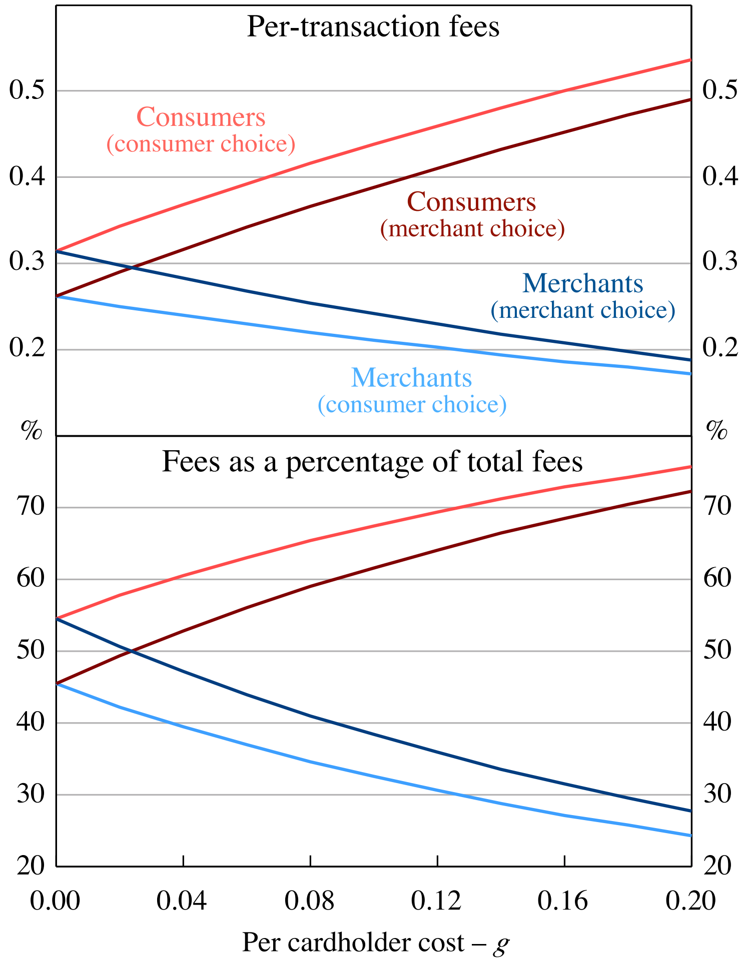
The results are shown in Figure 3. The top panel shows competing (symmetric) platforms' per-transaction prices to both consumers and merchants in the two models, as g varies. The bottom panel then shows these prices as a proportion of platforms' total per-transaction fees (so as to abstract from the increase in these total fees which accompanies an increase in g in both models).

Focusing first on the bottom panel of Figure 3, the upward slopes of the lines showing the consumer shares of platforms' total fees indicate that, in both models, the direct effect of a rise in g is to tilt competing platforms' pricing in favour of

[21] The key features of such a model are described briefly in Appendix C.

Figure 3: Platforms' Per-transaction Fee Allocations

PTP model with $c = 0$ and consumer or merchant choice



merchants and against consumers. However, of greater present interest is that the *gap* between the two consumer lines gradually decreases, as g increases. It is this gap which represents the effect on platforms' relative pricing due purely to which side of the market holds the final choice of payment instrument.

We see that as g rises the strength of Hermalin and Katz's phenomenon, whereby platforms price less favourably to whichever side holds the choice of instrument, gradually declines (just as happened with a rise in c). Indeed, our results show that this weakening of the phenomenon, as g rises, occurs not just in proportional terms (bottom panel) but even in absolute terms (top panel). For merchants it is clear in the top panel that this absolute difference between the prices charged to them by platforms, depending on who holds the choice of payment instrument, decreases in our PTP framework as g rises. Although it does not show through as strongly, this pricing gap also declines in absolute terms for consumers (falling from 0.052 when $g = 0$ to 0.046 when $g = 0.2$).

Finally, we would also like to investigate how, in a world of per-transaction pricing, the strength (or even direction) of Hermalin and Katz's phenomenon varies as a function of consumers' innate propensity to single-home. To address this issue we take up the option flagged in Section 4.2 and incorporate into our PTP model a new parameter κ – representing the disutility to consumers from holding more than one card. A higher value for κ then corresponds to a greater *innate* propensity on the part of consumers to single-home, independent of any price-related incentives.

We then consider two variants of this 'PTP model with κ' – one with consumer choice of the payment instrument and the other with merchant choice – and compare how competing platforms' pricing to the two sides of the market differs across the two models, for different values of κ.[22] As in the case of varying g, this approach allows us to isolate how the nature of the Hermalin and Katz phenomenon varies, as consumers' innate propensity to single-home increases.

Doing this in the case of $c = 0$ and $g = 0$ we find that, for small κ values up to a threshold of around 0.035, the situation is complicated by the apparent existence of multiple feasible Nash equilibria in the model with consumer choice. Understanding the factors generating this non-uniqueness of potential duopoly pricing outcomes, and how it might be resolved in practice by competing platforms, would be interesting. However, it would take us beyond our current focus on the Hermalin and Katz phenomenon, so we do not pursue it further in this paper. Rather, we focus on platforms' pricing behaviour in the two models for κ values above this threshold, for which uniqueness of the Nash equilibrium, under duopoly, holds in both.

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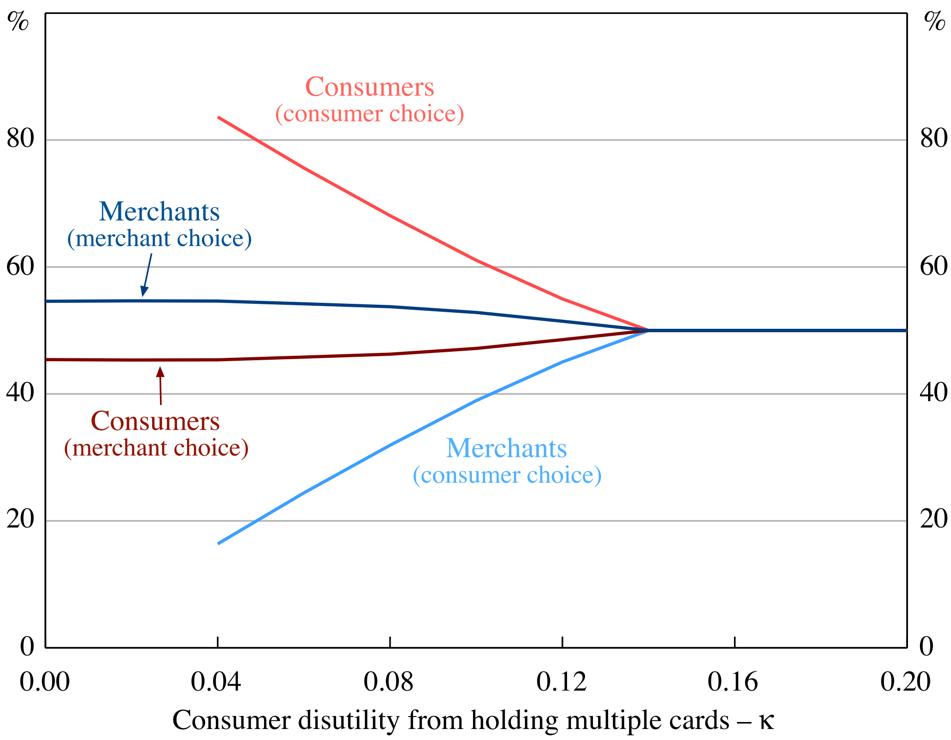
[22] Once again, the key features of these two further variants of our basic PTP model are discussed briefly in Appendix C. Detailed derivations of the geometric frameworks describing the decisions of consumers and merchants in each of these generalised models are provided in Gardner and Stone (2009b).

Figure 4: Platforms' Per-transaction Fee Allocations
PTP model with $g = c = 0$ and consumer or merchant choice – consumer and merchant fees as a percentage of total fees



For such κ the direction of the Hermalin and Katz phenomenon remains invariant – as usual, biasing platforms' pricing against the side with the final choice of payment instrument – but its strength once again declines as κ increases. This is evident in the narrowing gap between (say) the two consumer lines in Figure 4.[23] It accords with the intuition that, as consumers' propensity to single-home rises, the competitive pressure on platforms to forestall 'steering' by whichever side

[23] It should, however, be noted that for the model with consumer choice, the family of solutions for different κ shown in Figure 4 does *not* extend smoothly and monotonically back to a solution at $\kappa = 0$. For $\kappa = 0$ the only Nash equilibrium solution is the one to our basic PTP model, identified in Section 4, in which consumers' share of platforms' total fees is equal to 54.5 per cent and merchants' share to 45.5 per cent. To get from this unique solution at $\kappa = 0$ to the solutions shown in Figure 4 would seem to require some *strengthening*, rather than weakening, of the Hermalin and Katz phenomenon as κ rises, for some range of small κ values. However, as noted, understanding what might be driving this contrasting behaviour, or what would actually eventuate in our consumer choice PTP model for such κ, is beyond the scope of this paper.

does not hold the choice of instrument, by pricing attractively to that side, diminishes.[24]

This decline in the strength of the Hermalin and Katz phenomenon continues until competing platforms' preferred pricing, in both variants of the model with κ, makes multi-homing unpalatable to all consumers ($\kappa \geq 0.136$). Consumers and merchants are then charged equally in both models – at fee levels which are the same across the two models and invariant to further increases in κ.[25]

Finally, the results shown in Figure 4 also confirm the claim made at the end of Section 4.4: namely that, even when platforms' pricing is purely per-transaction, a positive link does *not* necessarily hold between the propensity to single-home on the side without the choice of instrument and the relative attractiveness of platforms' pricing to that side. This is evident in the results for the PTP model with merchant choice. As κ rises in this model, consumers become more inclined to single-home, yet the fraction of platforms' total fees which they must bear slowly increases rather than falls – indicating a negative, not positive, link in this case.

6. Conclusions

In this paper we have used our ECR model and several others to study competition between payment systems. Because of the complexity of such competition, all models necessarily involve some simplifications to make them tractable. For example, our ECR model does not incorporate 'business stealing' considerations, nor does it allow for separate issuers and acquirers (being a model of competing three-party rather than four-party payment networks).

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[24] In the case where consumers choose, this reflects the fact that as the number of consumers holding multiple cards falls, the scope for merchants to attempt to steer consumers' payment choices, and hence their incentive to do so, also clearly falls. Where *merchants* choose, it instead reflects the fact that – at least when consumers' per-transaction benefits are uniformly and independently distributed – the number of consumers inclined to 'steer' merchants, as a proportion of all consumers, declines as κ rises, in line with the decline in the number of consumers who would even contemplate holding both platforms' cards absent any steering considerations.

[25] For reasons which emerge from the detailed discussion in Appendix B, these model solutions for large κ in fact coincide not only with each other but also with that for the CR model, for $g = c = 0$, under duopoly (given in Table 1).

Our ECR model does, however, overcome several key limitations of earlier models used in the literature. Most notably it allows for flat rather than per-transaction pricing by platforms to consumers, and endogenises card holding and acceptance decisions by both consumers and merchants. These represent steps towards a more realistic treatment of payments system competition in a range of practical settings of interest. A number of key messages and subsidiary findings emerge from our analysis, which we would expect to be robust to the use of alternative modelling approaches.

The main message is that platforms' use of flat rather than per-transaction pricing to consumers has a potentially major impact on the behaviour of platforms, merchants and consumers. Caution is therefore called for in drawing any firm conclusions about market behaviour from models which assume purely per-transaction pricing. For example, our results show that introducing flat pricing by platforms to consumers calls into question what had been a consistent finding in the literature: that an increase in the propensity of consumers to single-home will necessarily lead competing platforms to bias their prices in favour of consumers relative to merchants.

One reason we would expect this general message to be robust to plausible changes to our specific ECR model is that, in a world where both sides of the market may choose to multi-home, the use of flat pricing leads to *different consumers facing different effective per-transaction prices* for the use of a given platform's card. This is not the case when platforms' pricing to both sides is on a purely per-transaction basis, or when (as in the CR model) consumers are prohibited by *fiat* from multi-homing. It thus represents a fundamental change (and one that better matches reality in, say, the case of competing credit card schemes).

A second, subsidiary finding of our analysis of flat versus per-transaction pricing is that, even in a world where platforms use purely per-transaction pricing, care needs to be taken in describing the relationship between a side's propensity to single-home and the relative attractiveness of the pricing it will be offered by competing networks. Various papers, in reporting there to be a positive link, have stated it in quite general terms. For example, Rysman (2007) asserts that 'the literature on two-sided markets establishes that, in a competitive market for payment networks, the side that multi-homes subsidizes the side that single-homes' (p 10).

Our results, however, not only show the need to qualify such statements as applying only to situations where platforms use solely per-transaction pricing, but also demonstrate the need for a further *caveat*. This relates to which side holds the choice of instrument at the moment of sale. Specifically, our results show that a positive link only holds between the propensity to single-home on the side *with the final choice of payment instrument*, and the relative attractiveness of platforms' pricing to that side. Where the propensity to single-home increases on the side without the final choice of payment instrument, this can lead to less attractive pricing to that side, in proportional terms, even where platforms' pricing to both sides is on a purely per-transaction basis.

Finally, as noted previously by Hermalin and Katz (2006), possessing the final choice of payment instrument may not be an unalloyed benefit to consumers because it may shift competing platforms' price structures in favour of merchants and against consumers. Of course, other factors – including some not incorporated in our various models, such as 'business stealing' considerations – will likely work to push these price structures in the opposite direction. However, even if these other factors dominate, it is still potentially helpful to be aware of this possible influence on platforms' pricing behaviour, and to understand how it would change with variations in key elements of platforms' cost structures or the preferences of market participants.

In this regard, the results from our basic PTP model, and various generalisations of it, extend the findings of Hermalin and Katz. These results show that the force of the Hermalin and Katz phenomenon falls off not only as competing platforms' per-transaction costs rise, but also as their per-subscriber costs increase, and as consumers' innate predisposition to single-home rises (at least above a certain low threshold level).

Appendix A: Model Notation

Table A1: List of Model Notation

Variable		Description
Consumer market segments (fractions)		
Ω^c		Set of all consumers
Ω^c_0	(D^c_0)	Subset (fraction) of consumers who choose not to hold any cards
Ω^c_i	(D^c_i)	Subset (fraction) of consumers who choose to hold card i
$\Omega^c_{i,\sim j}$	$(D^c_{i,\sim j})$	Subset (fraction) of consumers who choose to hold card i but not card j
$\Omega^c_{i,j}$	$(D^c_{i,j})$	Subset (fraction) of consumers who choose to hold both cards i and j
$\Omega^c_{i,j;i}$	$(D^c_{i,j;i})$	Subset (fraction) of consumers who choose to hold both cards and who prefer to use card i over card j whenever merchants accept both
Merchant market segments (fractions)		
Ω^m		Set of all merchants
Ω^m_0	(D^m_0)	Subset (fraction) of merchants that choose not to accept any cards
Ω^m_i	(D^m_i)	Subset (fraction) of merchants that choose to accept card i
$\Omega^m_{i,\sim j}$	$(D^m_{i,\sim j})$	Subset (fraction) of merchants that choose to accept card i but not card j
$\Omega^m_{i,j}$	$(D^m_{i,j})$	Subset (fraction) of merchants that choose to accept both cards i and j
Platform fees		
f^c_i		Flat fee charged to consumers to subscribe to card i
$f^{c,*}_i$		The flat fee f^c_i converted to per-transaction terms for a consumer in $\Omega^c_{i,\sim j}$ or $\Omega^c_{i,j;i}$ (that is, the quantity f^c_i/MD^m_i)
$f^{c,**}_i$		The flat fee f^c_i converted to per-transaction terms for a consumer in $\Omega^c_{i,j;j}$ (that is, the quantity $f^c_i/MD^m_{i,\sim j}$)
f^m_i		Per-transaction fee charged to merchants by platform i
Platform costs		
c_i		Cost incurred by platform i for each transaction processed over the platform
g_i		Flat cost to platform i of signing up each consumer
Other		
C	(M)	Total number of consumers (merchants)
τ_i		Maximum per-transaction benefit received by any consumer on platform i
μ_i		Maximum per-transaction benefit received by any merchant on platform i
h^c_i		Per-transaction benefit received by a given consumer on platform i
h^m_i		Per-transaction benefit received by a given merchant on platform i
Π_i		Total profit earned by platform i

Notes: For simplicity, where there is analogous notation for both platforms only that for platform i is shown. Consumer (merchant) market fractions represent the proportion of all consumers (merchants) that are members of the corresponding set.

Appendix B: Analytical Results for the Chakravorti and Roson (CR) Model

In this appendix we briefly re-visit the results obtained by Chakravorti and Roson (2006) for their original model. To better understand the impact of competition on platforms' pricing structures in this model, it is possible to derive full *analytical* solutions of the model under both monopoly and duopoly, in each case with and without a price level constraint – see Gardner and Stone (2009b) for the details. As discussed below, this allows us to identify a number of additional aspects of Chakravorti and Roson's results, which in turn provide insights into the implications of prohibitions on multi-homing by either side of the market (for the modelling of competition between payment systems).

For the case of identical platforms setting symmetric fees, under monopoly or duopoly, Chakravorti and Roson numerically investigate five scenarios. Three of these are situations where the maximum per-transaction benefits to consumers and merchants are equal ($\tau = \mu$), which we concentrated on in Section 3 and which we focus on again here. For ease of reference, Chakravorti and Roson's results for these three scenarios, reported previously across Tables 1 and 2, are gathered together and repeated in Table B1.[26]

Several of the main results observed by Chakravorti and Roson are evident in this table, including: that platforms' total effective per-transaction price level, $f^{c,*} + f^m$, will in every instance be lower under duopoly than under monopoly; and that the proportion of transactions occurring on either network, rather than by cash, will correspondingly always be higher with competition than under monopoly. However, a number of additional results, not explicitly noted by Chakravorti and Roson, are also apparent.

[26] The CR model results shown in Table B1 actually differ at times in the third decimal place from those, based on model simulations, reported in Chakravorti and Roson (2006). This just reflects that, by virtue of having derived analytic solutions to the CR model under both monopoly and duopoly, we can determine results for these scenarios to arbitrary precision.

Table B1: The Original Chakravorti and Roson Model
The case of two symmetric platforms with $\tau = \mu$

Variable	**Duopoly**			**Monopoly**		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
	$g = 0$ $c = 0$	$g = 0$ $c = 0.5$	$g = 0.2$ $c = 0$	$g = 0$ $c = 0$	$g = 0$ $c = 0.5$	$g = 0.2$ $c = 0$
Platform fees						
f_i^c	0.180	0.247	0.360	0.375	0.324	0.473
$f_i^{c,*}$	0.236	0.449	0.449	0.500	0.593	0.593
f_i^m	0.236	0.449	0.200	0.250	0.453	0.203
Consumer market fractions						
D_0^c	0.056	0.202	0.202	0.250	0.352	0.352
$D_{i,\sim j}^c$	0.472	0.399	0.399	0.375	0.324	0.324
Merchant market fractions						
D_0^m	0.056	0.202	0.040	0.063	0.206	0.041
$D_{i,\sim j}^m$	0.180	0.247	0.160	0.188	0.248	0.162
$D_{i,j}^m$	0.584	0.303	0.641	0.563	0.299	0.635
Other						
Π_i	0.170	0.088	0.127	0.211	0.097	0.141
Vol_i	0.361	0.220	0.319	0.281	0.177	0.258
$f_i^{c,*} + f_i^m$	0.472	0.899	0.649	0.750	1.047	0.797

Notes: For simplicity, results are shown in units such that C and M both equal one, and the common value of τ and μ is also 1. Vol_i denotes the volume of transactions that take place on platform i. Notation for all other variables is as in Gardner and Stone (2009a) – see also Appendix A.

B.1 Additional Results for the Duopoly Case

Focusing first on the duopoly case, one additional result which stands out is that, for Scenarios 1 and 2 where $g = 0$, symmetric competing platforms in the CR model will charge the same fee to both merchants and consumers in per-transaction terms ($f_i^{c,*} = f_i^m = f_j^{c,*} = f_j^m$).[27] In fact, this is a manifestation of the following more general result – which may, moreover, be proven rather than merely suggested numerically.

[27] One reason this may not have been remarked upon in Chakravorti and Roson (2006) is that, for their simulation results, they focus only on platforms' flat fees to consumers, f^c, and do not report these flat fees converted to per-transaction terms, $f^{c,*}$.

Result 1. *Suppose that two identical competing platforms in the CR model arrive at a symmetric, profit-maximising Nash equilibrium which is not a corner solution of the platforms' optimisation problem. Then the (common) fees charged by each platform to consumers and merchants, in per-transaction terms, will satisfy*

$$f^{c,*} - f^{m} = (\tau - \mu) + g^{*} \tag{B1}$$

where subscripts have been dropped in view of the symmetry assumption and where g^* denotes the per-subscriber cost to platforms expressed in per-transaction terms.

The proof of this result is provided in Gardner and Stone (2009b). Here we content ourselves with four observations about it, three brief and the fourth more substantial.

The first is that it is easily checked that Equation (B1) does indeed hold not just for Scenarios 1 and 2 in Table B1, but also for Scenario 3 with $g > 0$. The second is that Equation (B1) implies that in the CR model, under symmetric duopoly competition, the difference between platforms' consumer and merchant fees in per-transaction terms, $f^{c,*} - f^{m}$, will be independent of the common per-transaction cost c faced by each platform to process transactions. Note, however, that this does *not* mean that platforms' balancing of their fees between the two sides of the market, reflected in the ratios $f^{c,*}/(f^{c,*} + f^{m})$ and $f^{m}/(f^{c,*} + f^{m})$, will be invariant as c changes. These ratios will typically vary with changes in c, since such shifts will generally alter the total price level, $f^{c,*} + f^{m}$, which platforms will charge to the two sides in symmetric duopoly.[28]

A third observation about Equation (B1) is that it formally quantifies various observations in Chakravorti and Roson (2006) about how symmetric, competing platforms' allocations of their fees between consumers and merchants will depend on the relative values of $\tau_i \equiv \tau_j \equiv \tau$ and $\mu_i \equiv \mu_j \equiv \mu$.[29] Chakravorti and Roson's

[28] This latter point can be established rigorously using the complete analytical solution for platforms' profit-maximising fees under duopoly in the CR model – for the case of non-corner solutions – as derived in Theorem 2 of Gardner and Stone (2009b). It is illustrated informally by a comparison of Scenarios 1 and 2.

[29] Chakravorti and Roson refer to τ and μ as determining the degree of 'competitive pressure' on each side of the market, and note various findings regarding how the *relative* competitive pressure on the two sides will affect platforms' pricing allocations in their model.

findings do not indicate the precise way in which $f^{c,*}$ and f^m will vary relative to each other in the symmetric setting, as functions of τ and μ – something which Result 1 makes explicit, at least for the duopoly case.

Finally, a fourth and more substantial observation about Result 1 is that it also illustrates how pervasive the implications of a single-homing condition can be for the modelling of payments markets – an issue with potentially wider relevance than just the CR model. To see this, observe that in the event that $\tau = \mu$ and $g = 0$, Result 1 implies that platforms in symmetric duopoly in the CR model will always (as seen in Scenarios 1 and 2) set fees such that

$$f^{c,*} = f^m . \tag{B2}$$

This symmetric treatment of the two sides of the market is actually, at first glance, surprising. After all, in the CR framework there are three ways in which the model would appear to be *asymmetric* (even for $\tau = \mu$ and $g = 0$) in its treatment of consumers and merchants:

i. consumers face a flat fee for subscribing to either platform but no per-transaction fees, whereas merchants face per-transaction fees for using either network but no flat fees;

ii. consumers are assumed to have full control, and merchants no say whatsoever, over the choice of payment method at the moment of sale; and

iii. consumers, unlike merchants, are prohibited from multi-homing.

Any or all of these factors might have been expected to generate some bias in platforms' treatment of the two sides of the market, even where the platforms are themselves completely symmetric. To see why they do not, consider each in turn.

On the nature of each platform's fees to the two sides of the market, what Equation (B2) highlights is that in the CR framework the distinction between flat and per-transaction fees to consumers is actually an artificial one. For given merchant fees, $\{f_i^m, f_j^m\}$, the number of merchants who will accept each platform's cards is fully determined, and this information is assumed to be known to both consumers and platforms. Thus, for any given flat consumer fees, $\{f_i^c, f_j^c\}$, both platforms and consumers in the model will know, in view of the single-homing restriction on consumers, exactly what these fees correspond to in per-transaction terms, $\{f_i^{c,*}, f_j^{c,*}\}$, uniformly for all consumers; and conversely, were platforms instead setting purely per-transaction fees for consumers, both platforms and consumers would know what these fees would correspond to in terms of a uniform flat subscription fee for each platform. Hence, although the CR model is nominally set up in terms of flat fees to consumers, it can be viewed as equivalent to one in which both sides are charged on a purely per-transaction basis. This explains why the first of the three factors mentioned above does not, in fact, give rise to any asymmetry between platforms' treatment of consumers and merchants in the model.

The second factor, however, seems more problematic. Whether one would expect the fact that consumers hold the final choice of payment instrument at the moment of sale to lead platforms to favour them in their pricing, or instead to favour merchants, is an issue we discussed in Section 5. Either way, however, one might expect it to create an asymmetry in pricing between consumers and merchants.

It turns out, however, that the imposition of a 'no multi-homing' condition on the consumer side in the CR model actually renders null consumers' assumed control over the choice of payment instrument at the moment of sale. There is, therefore, *no* aspect of the CR model which would be altered if merchants instead held the choice of payment instrument at the moment of sale.

To see this, suppose that both consumers and merchants knew, even before making their card holding/acceptance decisions, that merchants held the choice of payment instrument at the moment of sale. Since merchants face only per-transaction fees, this would not alter the card acceptance decisions of any merchant relative to the usual CR model. These decisions would continue to be based purely on whether or not each platform's cards offered a net per-transaction benefit to the merchant – with each merchant continuing to have no incentive to steer consumers (now because they hold the power over the choice of payment instrument at the moment of sale, rather than because the prohibition on consumer multi-homing removes any scope for such steering).

Moreover, consumers' card holding decisions would then also be unaltered relative to the usual CR model. Each consumer would know: the number of merchants who will accept each platform's cards, for any given merchant fees $\{f_i^m, f_j^m\}$; and that, whatever card they choose to hold (if any), every merchant that accepts that card will prefer to choose it over cash at the moment of sale, exactly as they themselves would do. Hence, the incentives facing consumers in their card holding choices would also be exactly the same as in the usual CR model.

In the CR framework, therefore, granting the final choice of payment instrument to merchants, rather than consumers, would not alter either merchants' card acceptance or consumers' card holding decisions. Nor would it alter the model in any way in an *ex post* sense (since consumers hold at most one card, and merchants will always – like consumers – prefer a card payment to cash where possible). This explains why the second of the three factors mentioned above also fails to give rise to any asymmetry between platforms' treatment of consumers and merchants – as well as highlighting again how far-reaching the implications of a prohibition on multi-homing, even for just one side of the market, can be.

Finally, the third potential source of asymmetry in platforms' treatment of consumers and merchants was the prohibition on multi-homing by consumers but not merchants. Here the interesting point is that, unlike the first two factors, this *does* represent a genuine source of asymmetry. This is illustrated by the results for Scenarios 1 and 2 in Table B1 for the monopoly setting.

In that setting, even with $\tau = \mu$ and $g = 0$ the monopoly operator's preferred pricing treats merchants and consumers differently in both scenarios, being tilted in each case in favour of merchants and against consumers. Hence, it appears to be a particular artefact of duopoly competition in the CR model, rather than some more general consideration, which prevents this last factor from generating any actual asymmetry in platforms' pricing, in the symmetric duopoly setting.

B.2 Additional Results for the Monopoly Case

Turning briefly to the monopoly case, we note that we can also derive a counterpart to Result 1, as follows.

Result 2. *Suppose that the two platforms in the CR model are identical and that a monopoly operator selects symmetric fees for the two so as to maximise the monopolist's combined profit. Then, in the event these fees represent a non-corner solution of the monopolist's optimisation problem, the fees charged by each* *platform to consumers and merchants, in per-transaction terms, will satisfy*

$$f^{c,*} + 2f^{m} = \mu + c \tag{B3}$$

where subscripts have again been dropped in view of the symmetry assumption.

The proof of this result – which, as for the duopoly case, represents only one part of our full analytic solution of the CR model in the monopoly setting – is again provided in Gardner and Stone (2009b). Here, besides noting that it may be readily checked to hold for the three scenarios considered in Table B1, various points could again be observed about the implications of Equation (B3). For example, it implies that in the symmetric monopoly setting the quantity $f^{c,*} + 2f^{m}$ will be independent of the flat cost to platforms, g, of signing up consumers. Rather than explore such observations here, however, we turn to a different issue in relation to the CR model.

B.3 The Impact of Competition on Platforms' Price Structures

As already noted, Chakravorti and Roson (2006) demonstrated that competition between two identical platforms will drive down the total combined per-transaction prices charged by each to consumers and merchants. A separate question, however, is whether competition may also cause the allocation of this total price to be even more skewed against one side of the market than would be the case with a monopoly operator of the two platforms.

To answer this question within the CR model we do not want to simply compare the proportions of total price allocated by platforms to (say) merchants in the cases of symmetric duopoly and monopoly, for given values of the parameters τ, μ, g and c. This is because, given such values, the total price levels adopted by platforms in the two settings will typically be different.

To overcome this problem one natural approach is to consider corresponding scenarios (in terms of the parameters τ, μ, g and c) under both symmetric monopoly and duopoly – with an additional condition imposed that the total per-transaction price charged in each setting is exogenously fixed at some common level, k. With this restriction in place we can directly compare how platforms will allocate their fees between merchants and consumers under monopoly versus duopoly.

For such a comparison, it turns out that – at least in the case where platforms face no cost in signing up new cardholders – one can prove the following result for the CR model in the symmetric case.[30]

Result 3. *Suppose the two platforms in the CR model are identical and are restricted to charging a fixed total per-transaction price, k. Suppose also that the (common) cost to each platform of signing up new cardholders, g, is zero. Then competing platform operators in the case of symmetric duopoly will always skew their allocation of the total per-transaction price more strongly against merchants than will a monopoly operator setting profit-maximising symmetric fees for the two platforms. This will be the case for any choices of the remaining key model parameters* τ*,* μ *and c, provided these give rise to non-corner solutions of the platforms' optimisation problems in both the monopoly and duopoly settings.*

Hence, at least in the case where $g = 0$, platform competition will always shift symmetric platforms' price structures in favour of consumers and against merchants in the CR model, relative to the situation under monopoly.

[30] As usual, see Gardner and Stone (2009b) for a formal proof of this result. Note also that we expect this result may continue to hold for many, if not all, values of $g > 0$. However, we are not yet able to prove a general result along these lines.

Appendix C: Generalised Versions of Our PTP Model

In this appendix we briefly describe the main changes to our PTP model which would result from allowing for the two possible generalisations canvassed in Section 4.2, namely: having the merchant rather than consumer choose the payment instrument at the moment of sale; and introducing a parameter κ representing disutility to consumers from holding the cards of more than one platform. These generalisations give rise to three variants of the basic PTP model – corresponding to incorporating either one or both of the possible changes.

In each case we focus on the intuition underlying the changes which would result to the geometric frameworks determining the card holding and acceptance behaviour of consumers and merchants. Technical details of the derivations of these frameworks are provided in Gardner and Stone (2009b).

C.1 The PTP Model without κ but with Merchant Choice

Consider first the case of introducing merchant choice into the basic PTP model but with $\kappa = 0$. This affects the geometric frameworks described in Section 4.1 in a straightforward way, since the only asymmetry between merchants and consumers in both this and our main PTP model relates to who holds the choice of payment instrument at the moment of sale. Hence, granting this power to merchants rather than consumers simply switches the roles of these two groups: the card holding decisions of consumers are now described by a geometric framework exactly akin to that shown in Panel 1 of Figure 1 (with m-superscripts suitably replaced by c- or $c,*$-superscripts); and the corresponding card acceptance decisions of merchants are likewise described by a geometric framework exactly akin to that shown in Panel 2 of Figure 1 (with c- or $c,*$-superscripts replaced by m-superscripts).

C.2 The PTP Model with $\kappa > 0$ but Retaining Consumer Choice

In the case where we allow for $\kappa > 0$, but with consumer choice restored, the situation for merchants is simple. For them, the only difference resulting from the introduction of κ, relative to the basic PTP model, is indirect, through its effect on the consumer market fractions $\{D_0^c, \ldots, D_{i,j;j}^c\}$. Hence, the geometric framework describing merchants' card acceptance decisions is exactly as in

Figure 1 for the basic PTP model, with the slopes of Lines 1 and 2 again given by $D^c_{i,j;j}/(D^c_{j,\sim i}+D^c_{i,j;j})$ and $(D^c_{i,\sim j}+D^c_{i,j;i})/D^c_{i,j;i}$ respectively.[31]

As for the consumer side, here the introduction of $\kappa > 0$ does have a direct effect on which cards consumers choose to hold and use, as shown in Panel 2 of Figure C1 (for the case where $D^m_i < D^m_j$). Intuitively, the effect of κ is to diminish the incentive for consumers to hold both platforms' cards, pushing the boundaries of the region $\Omega^c_{i,j}$ up and to the right compared with the situation in the basic PTP model.

Specifically, the lower boundary of $\Omega^c_{i,j;i}$ is now given by the line

$$h^c_j = f^{c,\#}_j \equiv f^{c,*}_j + \frac{\kappa}{MD^m_{j,\sim i}}\,, \tag{C1}$$

reflecting the trade-off now facing those consumers who would choose to hold card i rather than card j if they could only hold one, and who would prefer to use card i over card j if they held both. These consumers will now opt to hold both platforms' cards rather than just card i if, and only if, their aggregate benefit from *also holding card* j, $MD^m_{j,\sim i}(h^c_j - f^{c,*}_j)$, exceeds the disutility, κ, now entailed by holding multiple cards.

Similarly, the left-hand boundary of $\Omega^c_{i,j;j}$ is now correspondingly given by the line

$$h^c_i = f^{c,\#}_i \equiv f^{c,*}_i + \frac{\kappa}{MD^m_{i,\sim j}}\,. \tag{C2}$$

Finally, the remaining boundary of $\Omega^c_{i,j;i}$ is now given by the line joining the points $(f^{c,\#\#}_i, f^{c,\#}_j)$ and $(f^{c,\#}_i, f^{c,\#\#\#}_j)$, where the quantities $f^{c,\#\#}_i$ and $f^{c,\#\#\#}_j$ are defined by

$$f^{c,\#\#}_i \equiv f^{c,*}_i + \frac{\kappa}{M}\left(\frac{D^m_j}{D^m_i D^m_{j,\sim i}}\right) \tag{C3}$$

and

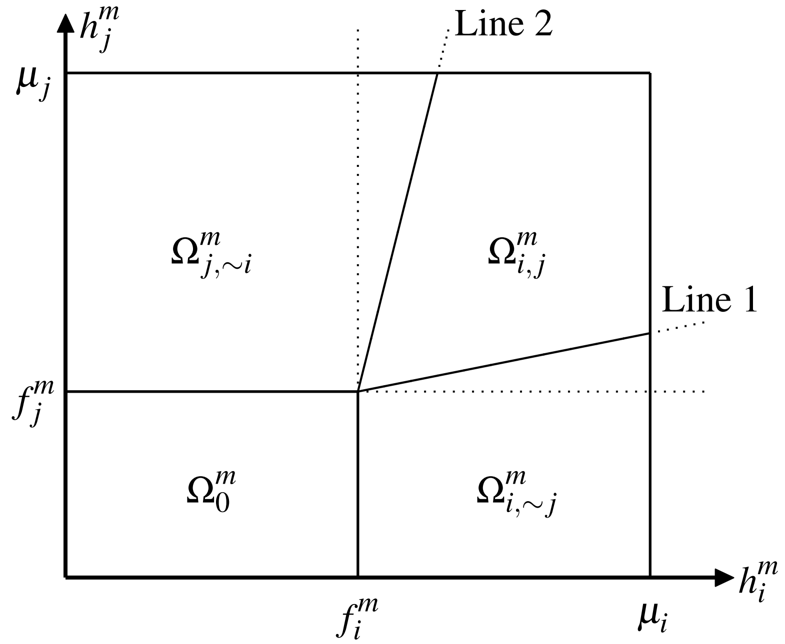
$$f^{c,\#\#\#}_j \equiv f^{c,*}_j + (f^{c,\#}_i - f^{c,*}_i)\,. \tag{C4}$$

[31] Of course, for any given $\{f^{c,*}_i, f^{c,*}_j, f^m_i, f^m_j\}$, the actual slopes of these lines will be different from the basic PTP model because of the different values taken in each case by the relevant consumer market fractions in the face of such fees.

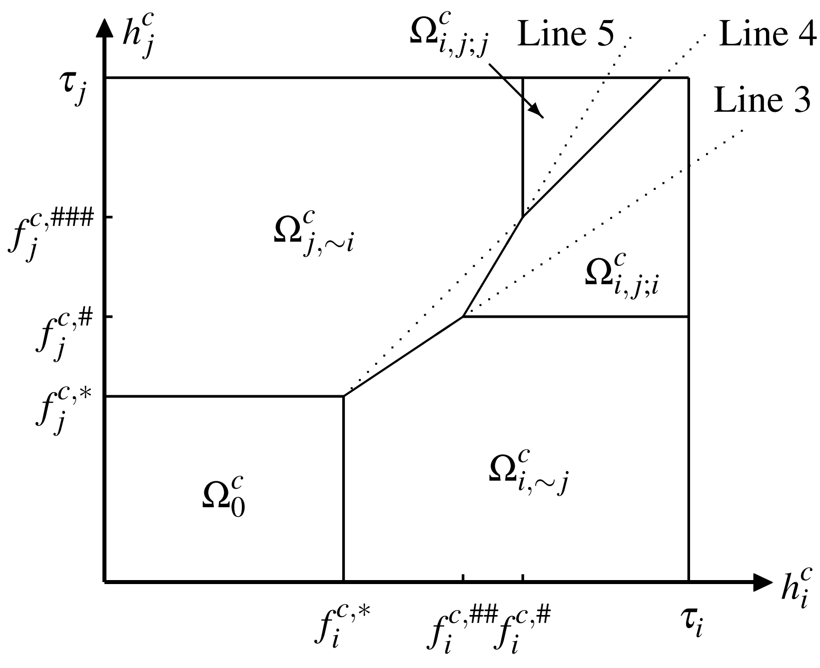
Figure C1: Representations of the Populations of Merchants and Consumers

The PTP model with $\kappa > 0$ and consumer choice, for the case $D_i^m < D_j^m$

Panel 1: merchants



Panel 2: consumers



Notes: Panels 1 and 2 show representations of the populations of merchants and consumers in h_i^m, h_j^m-space and h_i^c, h_j^c-space respectively, subdivided into the subsets Ω_0^m, $\Omega_{i,\sim j}^m$, $\Omega_{j,\sim i}^m$ and $\Omega_{i,j}^m$ for merchants, and Ω_0^c, $\Omega_{i,\sim j}^c$, $\Omega_{j,\sim i}^c$, $\Omega_{i,j;i}^c$ and $\Omega_{i,j;j}^c$ for consumers. Lines 1 and 2 have slopes $D_{i,j;j}^c/(D_{j,\sim i}^c + D_{i,j;j}^c)$ and $(D_{i,\sim j}^c + D_{i,j;i}^c)/D_{i,j;i}^c$ respectively, while Lines 3, 4 and 5 have slopes D_i^m/D_j^m, 1 and $D_i^m/D_{i,j}^m$ respectively.

This line (Line 5) has slope $D_i^m/D_{i,j}^m$. It represents the boundary along which consumers who would choose to hold card j rather than card i if they could only hold one, but who would prefer to use card i over card j if they held both, will be *indifferent between holding both platforms' cards or just that of platform* j (that is, will have $U_{i,j;i}^c = U_{j,\sim i}^c$ in utility terms).

C.3 The PTP Model with $\kappa > 0$ and Merchant Choice

Turning finally to the case where we allow for $\kappa > 0$, but switch to merchant choice of the payment instrument, here the situation for merchants is exactly akin to that for consumers in our basic PTP model. In particular, the introduction of κ now has no effect at all on merchants' card acceptance (and selection) decisions, even indirectly. Hence, as shown in Panel 1 of Figure C2, the geometric framework describing these decisions here is just the identical twin of that for consumers' card holding and use decisions in Figure 1.

As for the consumer side, here (Figure C2) the effect of the presence of κ is once again to push the boundaries of the region $\Omega_{i,j}^c$ up and to the right compared with the situation in the basic PTP model (Figure 1); while the inability to choose the payment instrument also now creates an incentive for 'steering' behaviour on the part of some consumers. The upshot is the geometric framework shown in Panel 2 of Figure C2, where Lines 3 and 4 are not horizontal or vertical (respectively) because of this 'steering' incentive, and where the quantities $f_i^{c,+}$ and $f_j^{c,+}$ are given by:[32]

$$f_i^{c,+} \equiv f_i^{c,*} + \frac{\kappa}{M}\left(\frac{D_j^m}{D_i^m D_j^m - D_i^m D_{i,j;i}^m - D_j^m D_{i,j;j}^m}\right) \tag{C5}$$

and

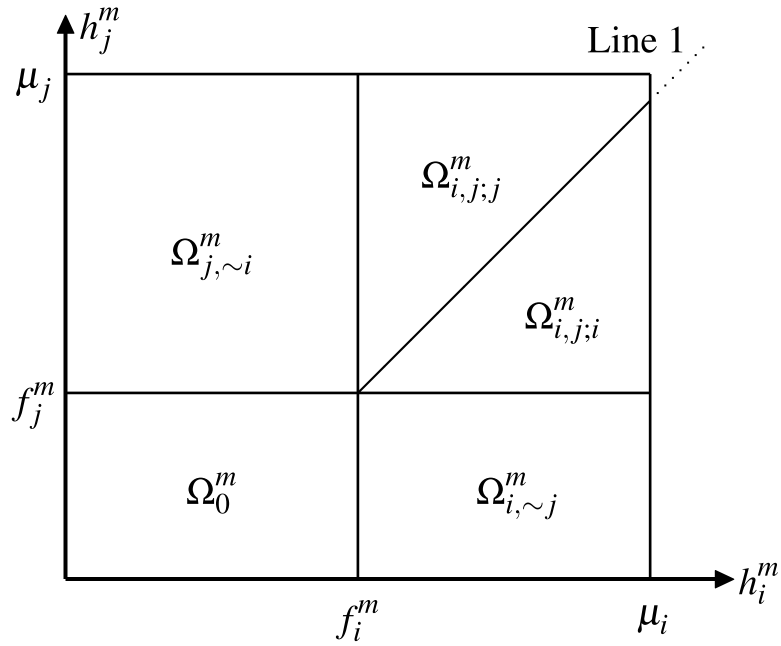
$$f_j^{c,+} \equiv f_j^{c,*} + \frac{\kappa}{M}\left(\frac{D_i^m}{D_i^m D_j^m - D_i^m D_{i,j;i}^m - D_j^m D_{i,j;j}^m}\right) . \tag{C6}$$

[32] See Gardner and Stone (2009b) for details.

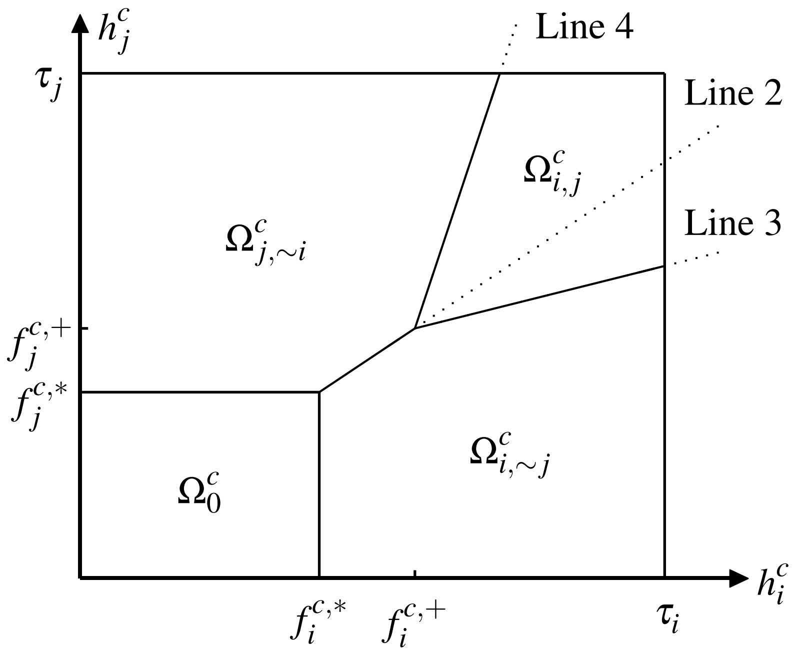
Figure C2: Representations of the Populations of Merchants and Consumers

The PTP model with $\kappa > 0$ and merchant choice

Panel 1: merchants



Panel 2: consumers



Notes: Panels 1 and 2 show representations of the populations of merchants and consumers in h^m_i, h^m_j-space and h^c_i, h^c_j-space respectively, subdivided into the subsets Ω^m_0, $\Omega^m_{i,\sim j}$, $\Omega^m_{j,\sim i}$, $\Omega^m_{i,j;i}$ and $\Omega^m_{i,j;j}$ for merchants, and Ω^c_0, $\Omega^c_{i,\sim j}$, $\Omega^c_{j,\sim i}$ and $\Omega^c_{i,j}$ for consumers. Line 1 has slope 1, while Lines 2, 3 and 4 have slopes D^m_i/D^m_j, $D^m_{i,j;j}/(D^m_{j,\sim i} + D^m_{i,j;j})$ and $(D^m_{i,\sim j} + D^m_{i,j;i})/D^m_{i,j;i}$ respectively.

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