DOCUMENTO DE TRABAJO. E2011/ 03

Output taxation by a revenue-raising government under signaling

MANEL ANTELO

Centro de Estudios Andaluces
CONSEJERÍA DE LA PRESIDENCIA

E2011/03

Output taxation by a revenue-raising government under signaling*

Manel Antelo**
Universidad Santiago de compostela

ABSTRACT

In this paper the behavior of a tax-collecting government (a tax office) when imposing a quantity-tax to firms is analyzed along a two-period signaling model. Each taxpayer privately knows its technological attributes, while third parties—the tax office among them—have only a prior belief about this fact, so firms can be tempted to behave opportunistically. In monopoly, signaling is always costly in terms of output deviation and the tax office reacts by setting, a smaller tax in (asymmetric information) period 1 than it would under symmetric information. In oligopoly, signaling can be either costly or costless. In the former case, the tax imposed by the tax office to each firm is below that imposed under symmetric information, while it is equal in the latter case. Besides, fiscal revenue under signaling is unambiguously lower than under symmetric information, even when tax size is the same in both contexts. Finally, it is shown that signaling (and the resulting tax office behavior upon tax setting) may lead to greater social welfare than under symmetric information either in monopoly or oligopoly markets.

Keywords: Output-tax, tax office, asymmetric information, signaling
JEL classification codes: H21, D82

---

*The author acknowledges comments and suggestions by Antonio R. Sampayo. I would also like to thank the financial aid received from the Galician Autonomous Government (Xunta de Galicia) through Research Project INCITE09201042PR.

** Departamento de Fundamentos da Análisis Económica, Universidad de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela (Spain). E-mail: manel.antelo@usc.es

Impuestos en un contexto de información asimétrica y señalización

Manel Antelo

Una de las tareas que conlleva más dificultad para un gobierno que carece de información sobre la estructura de costes de las empresas a regular, la demanda de mercado que sirven y otras variables económicas relevantes es conseguir que dichas empresas revelen de forma honesta su información privada. Este problema es importante en muchos contextos y, como tal, ha recibido mucha atención por parte de la literatura. Uno de los aspectos en el que este problema es relevante es, sin duda, el que se refiere a la relación entre el gobierno y los negocios o entre el gobierno y las personas físicas a la hora de establecer los impuestos. Por ejemplo, mientras que en el caso de los asalariados públicos podemos pensar que no existe asimetría de información entre estos agentes y la agencia tributaria, la situación puede ser radicalmente distinta cuando nos referimos a profesionales autónomos, empresas, etc. En este caso, es factible pensar que la información del gobierno a la hora de establecer los impuestos adecuados es inferior a la que tienen los agentes gravados. En un contexto de asimetría informativa podemos plantear dos preguntas relevantes. En primer lugar, ¿qué tipo de estrategia de señalización puede inducir a los agentes gravados a comportarse de forma honesta? En segundo lugar, ¿debería el gobierno alterar de forma significativa su política impositiva (con respecto a la que practicaría en condiciones de información simétrica) para desalentar el comportamiento deshonesto? La respuesta a estas y otras cuestiones similares tiene, sin duda, importantes implicaciones para el análisis normativo.

En este artículo investigamos el comportamiento de una agencia tributaria a la hora de establecer un impuesto por unidad producida a empresas que tienen información privada sobre sus atributos tecnológicos y, por ende, sobre sus beneficios. Para ello utilizamos, como punto de comparación, una situación en la que ambos agentes (empresas y gobierno) comparten la misma información. Construimos un modelo en el que la agencia tributaria y la o las empresas interaccionan a través de un juego de dos periodos, en el cual las empresas señalizan su tipo o nivel de eficiencia en el primer periodo y, por tanto, el segundo periodo se convierte en un periodo de información simétrica. Cada empresa conoce de forma privada sus características tecnológicas, mientras que cualquier otro agente participante en el juego impositivo (la agencia tributaria entre ellos) sólo tiene una probabilidad sobre este aspecto. Por lo tanto, el hecho de que las empresas tengan información privada las induce a comportarse de forma oportunista y obtener ventaja de ello.

*The author acknowledges comments and suggestions by Antonio R. Sampayo. I would also like to thank the financial aid received from the Galician Autonomous Government (Xunta de Galicia) through Research Project INCITE09201042PR.

** Departamento de Fundamentos da Análisis Económica, Universidad de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela (Spain). E-mail: manel.antelo@usc.es

En estas condiciones, si en el mercado existe una sola empresa (monopolio) objeto de imposición, dicha empresa tiene interés en ser percibida por la agencia tributaria como ineficiente con el fin de pagar un impuesto menor en el segundo periodo. Esto significa que el tipo ineficiente necesita enviar una señal a la agencia tributaria que no sea imitable por el tipo eficiente. Si la señal es la cantidad producida en el primer periodo, la señalización es siempre costosa en el sentido de que el monopolista ineficiente está obligado a reducir la cantidad que produce con relación a la que produciría en condiciones de información simétrica. La agencia tributaria, anticipándose a este comportamiento, reacciona a esta contracción del output estableciendo un impuesto inferior al que fijaría en información simétrica.

Por el contrario, si las empresas gravadas por el gobierno son varias (oligopolio), cada una se enfrenta a dos incentivos de signo opuesto. Por una parte, un incentivo vertical, por el cual quiere indicar a la agencia tributaria que es ineficiente. Por otra parte, un incentivo horizontal, por el cual pretende ser considerada por la empresa rival como eficiente. En términos netos, no obstante, el incentivo vertical domina al horizontal. Ahora bien, el proceso de señalización de cada empresa (ineficiente) ya no es costoso inequívocamente, como sucede en el caso de monopolio. En determinadas circunstancias continúa siendo costoso, pero en otras deja de serlo. En particular, si la diferencia tecnológica entre el tipo eficiente y el ineficiente de cada empresa es suficientemente amplia, a cada empresa ineficiente, para señalizarse como tal ante el resto de participantes en el juego, le basta con producir en el periodo 1 la cantidad que maximiza su beneficio en dicho periodo.

En consecuencia, cada empresa eficiente reacciona produciendo en dicho periodo la cantidad que también maximiza su beneficio en el citado periodo. En estas condiciones el impuesto que fija la agencia tributaria a cada empresa es inferior al que fijaría en ausencia de señalización (en el primer supuesto) e igual (en el segundo). En cualquier caso, y al igual que sucede cuando existe un monopolio, el ingreso que obtiene la agencia tributaria en condiciones de información asimétrica es inferior al que obtendría en condiciones de información simétrica.

En definitiva, la percepción de que agentes económicos como los profesionales pagan relativamente menos impuestos que otros como los asalariados se ve refrendada por los hallazgos de nuestro modelo. Es decir, se puede interpretar como una política impositiva derivada de un contexto informativo diferente: mientras que los profesionales tienen información privada, los asalariados carecen de esa ventaja informativa respecto a la agencia tributaria. Pues bien, es justamente este contexto informativo diferente es el que induce un comportamiento distinto por parte de la agencia tributaria.

*The author acknowledges comments and suggestions by Antonio R. Sampayo. I would also like to thank the financial aid received from the Galician Autonomous Government (Xunta de Galicia) through Research Project INCITE09201042PR.

** Departamento de Fundamentos da Análisis Económica, Universidad de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela (Spain). E-mail: manel.antelo@usc.es

Por último, en el artículo se analiza el impacto de la información asimétrica, la señalización (y el subsiguiente comportamiento del gobierno en materia fiscal) sobre el bienestar agregado, definido como la suma del excedente del consumidor, los beneficios de las empresas y el ingreso fiscal del gobierno.

El resultado que se deriva es que la información asimétrica puede conducir a un mayor nivel de bienestar social que la información simétrica.

*The author acknowledges comments and suggestions by Antonio R. Sampayo. I would also like to thank the financial aid received from the Galician Autonomous Government (Xunta de Galicia) through Research Project INCITE09201042PR.

** Departamento de Fundamentos da Análisis Económica, Universidad de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela (Spain). E-mail: manel.antelo@usc.es

1. Introduction

One of the most difficult tasks for a government which lacks information about firms' cost structure and market demand, as well as other relevant economic variables, is inducing voluntary information disclosure in regulated firms. This problem has drawn much attention (Loeb and Magat, 1979; Cech, 1991; Kim and Chang, 1993; and so on) due to both its theoretical relevance and practical consequences. For example, in the government-to-business (G2B) context, tax fraud is estimated 2-2.5% of GDP; that is, about 200-250 billion Euro per year at EU level (EU Commission, 2006). Such tax fraud can be unambiguously understood as the outcome of an asymmetric information problem (Liu and Tan, 2008).

As Liu and Tan (2008) indicate, the value added tax (VAT) collection is a fitting example in the G2B context. The players involved here include firms, obliged to declare and pay VAT, and the corresponding auditing government agency (tax office) that controls VAT collection. If symmetric information prevailed, the tax office would be fully aware of the exact operations performed by taxpayers, who would declare and pay their corresponding VAT honestly. However, under asymmetric information conditions taxpayers dispose of private information about their own operating details and actual transaction values that are unknown or unknowable to other agents—including the tax office—at least without considerable effort and monitoring costs. In addition, optimizing agents have obvious incentives to exploit private information to their own interest. In fact, these agents will try to do everything they can to reduce tax burden by hiding or even falsifying certain private information to get tax advantage. Whether such incentive is easily achieved with no tax-office awareness or the penalty of defaulting is not severe enough, taxpayers choose to cheat and commit tax fraud. Needless to say that in such circumstances the tax office must adopt the necessary measures in terms of participation and incentive compatibility constraints to encourage honest tax payment. What kind of signaling strategy can effectively encourage taxpayers to behave honestly? Should the tax office significantly alter its tax policy (relative to that in symmetric information) to discourage dishonest behavior? These are open questions with profound implications for normative analysis.

This paper aims at constructing a period-by-period tax scheme that induces a signaling game in both monopolistic and oligopolistic markets. The expected taxes under asymmetric information in both cases are shown to be only affected by the either costly or costless nature of the signaling activity. Regardless of the market structure formed by taxpayers, expected taxes under asymmetric information are smaller than under symmetric information when signaling involves a cost in the form of deviations in firms' outputs beyond distortions due to the mere existence of asymmetric information. Contrariwise, when signaling is costless and the distortions on firms' outputs are simply those resulting from asymmetric information, the tax office behaves as when under symmetric information when taxing firms. Overall, the model predicts a non-decreasing time pattern on taxes as information evolves from asymmetric to complete.

Regarding the different pattern observed on tax revenue between asymmetric and symmetric information conditions, two facts are relevant. First, the above-mentioned signaling effect has a negative impact on tax revenue through reduced per-unit tax in period 1, when signaling is costly. Second, the asymmetric information effect, either with or without signaling effect, reduces the aggregate expected output of taxpayers under asymmetric information relative to symmetric information conditions. Hence, the tax office is worse under asymmetric information, regardless of whether signaling is costless or costly. The nuance is that in the latter case both asymmetric information and signaling effect reinforce each other and leave the tax office in worse position than in absence of signaling effect.

From a social viewpoint, this paper shows that both signaling-related distortions and the resulting tax office behavior may lead social welfare to be above the level of symmetric information. In addition, this result is more likely to emerge in oligopoly than in monopoly, in spite of the fact that under-taxation asymmetric information leads to is less pronounced in oligopoly. In particular, we find that, in case of monopoly and parameters μ (the probability of the firm becoming efficient) and c (the cost level in case of an inefficient firm)[1] being sufficiently high, expected welfare under asymmetric information excels that under symmetric information. Furthermore, the conditions on the values of parameters μ and c that lead the society as a whole to be better under asymmetric than under symmetric information are slightly less strict in oligopoly than in monopoly. In oligopoly, yet not in monopoly, signaling improves welfare relative to a symmetric information context whenever it is either costless or, being costly, its cost is small.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 considers, as a benchmark, the behavior of the tax office if firms' information concerning their technological attributes is publicly observed both in monopoly and oligopoly. Section 4 examines the impact of asymmetric information on both tax size and fiscal revenue. Section 5 analyses the negative impact of asymmetric information and the resulting tax office behavior on social welfare. Finally, Section 6 sketches tax office behavior upon facing highly heterogeneous taxpayers. Section 7 outlines our conclusions.

[1] Parameter c also measures the difference between the marginal cost of the two types, since the low-cost type is assumed to have zero marginal cost.

2. The model

Consider a single industry comprising (one or several) firms that produce a good along two production periods (t=1,2). To get the explicit solution of the model, firms are assumed to produce for a market with a periodical linear inverse demand given by

$$p_t(q_t) = 1 - q_t, \tag{1}$$

being p_t the unit price in period t when q_t units of output are sold. If there is only one firm in the industry, then q_t is the output produced by such monopolistic firm. If there are two firms, A and B, then $q_t = q_t^A + q_t^B$, where q_t^A and q_t^B denote the period-t output of firms A and B, respectively. In both cases, firms choose output levels, letting the market demand given in (1) determine the unit price of the good.

We also consider a tax office that imposes taxes in each production period so as to maximize the period fiscal revenue. Taxes (not necessarily the same for all firms) are publicly-known fixed amounts per output unit to be revised at the beginning of the second period. There is no tax office-taxpayers negotiation, yet the former has all the bargaining power to set taxes.

In this stylized framework, taxpayers may have or not the same efficiency level. This is translated into the marginal production cost of each firm: a random variable whose realization is constant across periods but initially known only to that firm. Knowledge of the remaining tax-game participants is restricted to the distribution function drawn for each firm's cost. In particular, the marginal cost of each firm i, $\tilde{c}^i$, can adopt either a low (zero, for simplicity's sake) or a high value (the strictly positive value c), each randomly assigned. Specifically, it is common knowledge that

$$\tilde{c}^i = \begin{cases} 0, \text{ with Probability } \mu \\ c, \text{ with Probability } 1-\mu \end{cases} \tag{2}$$

where $\mu \in (0,1)$. In case of monopoly, (vertical) asymmetric information relies on the fact that the tax office only knows the distribution in (2). In turn, in case of two taxpayers, i=A,B, vertical and horizontal asymmetric information exists, as both the tax office and the rival firm j (j=A,B; $j \neq i$) only have knowledge of the prior beliefs assumed in (2).

Corner solutions shall not be discussed. Given the demand stated in (1), this implies considering only the range of parameter c indicated in the following assumption:

Assumption A1. The bad realization c of marginal cost (which also measures the difference in marginal costs) is such that $0 < c < 1/3$.

This assumption ensures that production is always positive for any taxpayer, regardless of its cost and other players' beliefs about such cost, as well as the type of its rival (when applicable).[2]

The timing of the tax-game is as follows. At the beginning of period 1, asymmetric information holds and the tax office, acting as a Stackelberg leader in setting taxes for this period, announces and imposes a per-unit tax T_1^s,[3] before observing the firms' output choice with the aim of maximizing the expected tax revenue of that period. The only common knowledge in this period is the distribution of firm cost in (2). Then, firms act as Stackelberg followers and choose their first-period output level, q_1^s. At the beginning of period 2, the firms' outputs are publicly observed, from which uninformed players update their prior beliefs. Next, given this posterior belief formed after observing the output of each firm in period 1, the tax office publicly announces and imposes period 2 per-unit taxes T_2^*,[4] to each type of firm. Finally, firms choose their profit-maximizing output for period 2, q_2^*.

The Perfect Bayesian equilibrium can be used to solve the proposed tax-game, as the firms' period-1 outputs constitute a Bayesian-monopoly equilibrium (when one only firm exists) or a Bayesian-Cournot equilibrium (in case of several firms). Period-2 outputs are chosen optimally, after updating the set of a-priori probabilities and the a-posteriori beliefs that satisfy Bayes' rule (when applicable). To examine the role played by information transmission on tax size, we focus on the outcome of the separating Perfect Bayesian equilibrium of the proposed signaling tax-game as compared to that of the tax-game under symmetric information. For the sake of simplicity, no between-period discount factor is applied and all tax-game participants are assumed to be risk-neutral.

3. Taxes under symmetric information

As a benchmark, we begin considering tax office behavior in a symmetric information context where the firms' information on marginal production cost is shared by the tax office (and also by other firms when several firms produce in the market). In this context the timing of the proposed tax-game is as follows: at the beginning of the first period nature chooses the type of each firm (low- or high-cost), and such type is publicly observed. Then, the tax office sets the first-period tax as a function of the type of firms and these produce accordingly. Both tax size and the firms' output level for the first period remain valid for the second period. In this case the following result may be established.

[2] This assumption is removed in Section 6.
[3] Superscript *s* denotes signaling, whereas the subscript numeral stands for period 1.
[4] Superscript ∗ indicates a symmetric information context.

Lemma 1. *Regardless of the existence of either a monopoly or a duopoly, under symmetric information the tax office sets, in each production period t, the tax* $T_{tL}^{*}=1/2$ *for a low-cost firm, and the tax* $T_{tH}^{*}=(1-c)/2$ *for a high-cost one.*

Proof. See Appendix.

From Lemma 1 the per-period firm's output, $q_t^*(\tilde{c})$, as well as the corresponding values of the firm's profit, $\pi_t^*(\tilde{c})$, and the tax office's revenue, $R_t^*(\tilde{c})$, in each period t are listed in Table 1.

Table 1. Equilibrium values in the monopolistic tax-game with symmetric information (t=1,2)

$\tilde{c}$	$q_t^*(\tilde{c})$	$\pi_t^*(\tilde{c})$	$R_t^*(\tilde{c})$
0	$1/4$	$1/16$	$1/8$
c	$(1-c)/4$	$(1-c)^2/16$	$(1-c)^2/8$

Likewise, Table 2 shows the equilibrium values of the corresponding tax-game in case of duopoly.

Table 2. Equilibrium values in the Cournot-duopolistic tax-game with symmetric information (i=A,B; t=1,2)

$(\tilde{c}^i,\tilde{c}^j)$	$q_t^{i*}(\tilde{c}^i,\tilde{c}^j)$	$\pi_t^{i*}(\tilde{c}^i,\tilde{c}^j)$	$R_t^*(\tilde{c}^i,\tilde{c}^j)$
$(0,0)$	$1/6$	$1/36$	$1/6$
$(0,c)$	$(1+c)/6$	$(1+c)^2/36$	$(1-c+c^2)/6$
$(c,0)$	$(1-2c)/6$	$(1-2c)^2/36$	$(1-c+c^2)/6$
(c,c)	$(1-c)/6$	$(1-c)^2/36$	$(1-c)^2/6$

From Table 1, the per-period output of the (monopolistic) firm calculated at tax-game onset, the value of the firm's expected profit and the tax office's expected revenue are respectively given by $\overline{q}_t^* = (1/4)[1-(1-\mu)c]$, $\overline{\pi}_t^* = (1/16)[1-2(1-\mu)c+(1-\mu)c^2)]$ and $\overline{R}_t^* = (1/8)[1-2(1-\mu)c+(1-\mu)c^2)]$, t=1,2. Similarly, the equilibrium values in Table 2 render the following for the duopolistic case: $\overline{q}_t^{i*} = (1/6)[1-(1-\mu)c]$, $\overline{\pi}_t^{i*} = (1/36)[1-(1-\mu)c]^2$, i=A,B, and $\overline{R}_t^* = (1/6)[1-2(1-\mu)c+(1-\mu^2)c^2)]$, t=1,2.

Finally, Lemma 1 allows us to conclude that, under symmetric information conditions, firms are expected to pay the tax $\overline{T}_t^* = (1/2)[1-(1-\mu)c] = \overline{T}_t^{i*}$ in each period t (t=1,2), regardless of the presence of one only or several firms in the market. In other words, the size of tax expected to be paid by firms does not depend on the firm's expected output level. In addition, the fact that the tax chosen for a high-cost firm is below that for a low-cost one reduces the expected tax, $\overline{T}_t^*$ or $\overline{T}_t^{i*}$, according to the firms' probabilities of being inefficient and/or their degree of inefficiency, if they become inefficient.

4. Taxes under asymmetric information

Things may change drastically in case of asymmetric information and the tax office does not commit to maintaining the same tax across periods. In this case, at the beginning of the first period, nature chooses the type (low-cost or high-cost) of the firms, and such type is privately observed, while third parties—the tax office and rival firms—ignore it. The tax office sets an output tax valid for the first period and production takes place. Then, once production has been publicly observed, the tax office (if a monopoly prevails) or both the tax office and the rival firm (if a duopoly exists) infer, in a separating equilibrium, the correct firm type. At the beginning of period 2, symmetric information is then restored and the game replicates that of any period in Section 3. We look for a separating Perfect Bayesian equilibrium of this game by examining monopoly and oligopoly cases separately.

4.1. A single taxpayer

To determine the separating Perfect Bayesian equilibria of the two-period signaling game, we work by backward induction from period 2 to period 1. Given the monopolist's and tax office's behavior in period 2, when the firm is honest and when misleading, the following result can be stated.

Lemma 2. *The monopolist wishes to be understood by the tax office as inefficient.*

Proof. See the Appendix.

The tax paid by the monopolist in period 2 if understood as inefficient is smaller than when perceived as efficient. Hence, its production level in period 2 will be greater in the first case, as well as its profit amount.

We now concentrate in period 1. In this period the tax office is unable to distinguish one type of firm from other, so the imposed tax must be the same for both types. In addition, a separating Perfect Bayesian equilibrium implies that each type of taxpayer will produce a different quantity in this period, so the tax office, after observing such outputs, will update its prior assessment and will infer actual firm cost in period 2. Thus, the monopolist, if efficient, cannot do better in period 1 than choosing its profit-maximizing output, namely:

$$q_{1L}^{s} = \frac{1 - T_1^s}{2} = q_{1L}^{m}, \tag{3}$$

where superscripts *s* and *m* stand for signaling and monopoly regime, respectively, and numeral and *L* subscripts denote that the output produced in period 1 corresponds to a low-cost firm that maximizes its profits without considering period 2.

On the other hand, these are the incentive compatibility conditions defining a separating equilibrium:

$$\Pi_{1H}(T_1^s, q_{1H}^s) + \Pi_{2H}^m(T_{2H}^*) \geq \Pi_{1H}(T_1^s, q_{1H}^m) + \Pi_{2H}(T_{2L}^*), \tag{4}$$

and

$$\Pi_{1L}(T_1^s, q_{1H}^s) + \Pi_{2L}(T_{2H}^*) \leq \Pi_{1L}^m(T_1^s) + \Pi_{2L}^m(T_{2L}^*). \tag{5}$$

Condition (4) refers to the incentive compatibility constraint faced by the inefficient monopolist. It states that the profit collected in period 1 from producing the signaling output q_{1H}^s and thus moving away from the efficient firm, together with period-2 profit, once revealed as inefficient, exceeds the sum of period-1 profit obtained from producing the amount corresponding to an inefficient (myopic) monopolist, q_{1H}^m, plus the period-2 profit once it is misrepresented and pays the corresponding tax T_{2L}^*. Likewise, condition (5) denotes the incentive compatibility for an efficient monopolist. It shows that overall profit for such a firm when misrepresented as inefficient (by producing the inefficient firm's amount, q_{1H}^s, in period 1) does not exceed period-1 profit by producing the output stated in (3) plus period-2 profit once the tax office infers its efficiency and it is then imposed the corresponding (greater) tax, T_{2L}^{*}.

Particularizing (3)-(5) for the demand and cost structures considered in the model, the emerging result may be recorded as follows.

Lemma 3. *In the separating Perfect Bayesian equilibrium of minimum cost, the following holds:*

(i) *The tax office charges, in period 1, the tax* $T_1^s = 1/2 - (1/4)(1-\mu)\sqrt{2c+c^2}$ *to both types of monopolist*

(ii) *The taxpayer produces, in period 1, the output* $q_{1H}^s = 1/4 - (1/8)(1+\mu)\sqrt{2c+c^2}$ *, if it is inefficient, and the output* $q_{1L}^s = 1/4 + (1/8)(1-\mu)\sqrt{2c+c^2}$ *otherwise.*

(iii) *In period 2, both the tax office and the taxpayer behave as under symmetric information.*

Proof. See the Appendix.

To be perceived by the tax office as inefficient in period 2 and thus pay a small tax, the inefficient firm is obliged to produce in period 1 a strictly smaller output than it would in a symmetric information context; namely, $q_{1H}^s < q_{1H}^m$. Indeed, if the output produced in period 1 were one as $q_{1H}^s = q_{1H}^m$, the efficient firm would find profitable to misrepresent by producing q_{1H}^m and the separating equilibrium would thus break.

Comparison of taxes in both informational scenarios renders the following proposition.

Proposition 1. *Relative to the symmetric information context, signaling leads the tax office to reduce the tax in monopoly; namely,* $T_1^s < \overline{T}_1^*$. *Such under-taxation amounts to* $T_1^s - \overline{T}_1^* = -(1/4)(1-\mu)[\sqrt{2c+c^2} - 2c]$.

Under-taxation in period 1 is exclusively due to the cost of signaling. As stated in Lemma 3, to prevent the efficient producer's imitation, the output of the inefficient firm in period 1 must remain below that produced under symmetric information. In addition, the tax charged in period 1 exacerbates the lessening of the inefficient firm's output and also decreases the efficient firm's output. Thus, to minimize production cutback (and the resulting diminution on tax revenue), the tax size is smaller than it would in expected terms in case of symmetric information. Such difference only vanishes when $\mu = 1$ or c=0. Otherwise, under-taxation in period 1 is more pronounced as the firm is more likely to be a high-cost one. Regarding parameter c, under-taxation is not, however, monotonic, as it increases with c for small values of c, but decreases with c within the $([2\sqrt{3}-3]/3, 1/3)$ interval.

Taking part (i) of Lemma 3 and Table 1 into account, asymmetric information leads the efficient firm to pay a smaller tax than under symmetric information, $T_1^s < T_{1L}^*$, because it benefits from the inefficient firm's incentive to be perceived as such (that is, from the fact that the inefficient firm reduces its period-1 output and thus forcing the tax office to counteract this reduction by decreasing the tax imposed to both firm types below that expected under symmetric information). The inefficient firm, on the other hand, also pays a smaller tax under asymmetric information, $T_1^s < T_{1H}^*$, but only when parameters μ and c satisfy $\mu < 1 - \frac{2c}{\sqrt{2c+c^2}}$ (that is, when either the firm is very likely a high-cost firm or μ is large enough, when c is small). This is due to the large cost of signaling, which leads to highly significant tax size reductions. Otherwise, the tax paid by the inefficient firm under asymmetric information is greater than under symmetric information, $T_1^s > T_{1H}^*$. In sum, the presence of asymmetric information benefits the efficient firm more than the inefficient one.

The under-taxation faced by the efficient taxpayer under asymmetric information leads it to over-produce as $q_{1L}^s - q_{1L}^* = (1/8)(1-\mu)\sqrt{2c+c^2}$. An inefficient taxpayer, however, under-produces relative to a symmetric information context, $q_{1H}^s - q_{1H}^* = -(1/8)[(1+\mu)\sqrt{2c+c^2} - 2c]$ due to the interaction of two effects: the signaling effect and the over- or under-taxation relative to symmetric information. In the former, both effects are reinforced, while they are counteracted but the signaling effect dominates under-taxation in the latter. Overall, the output of the monopolist, as calculated at the beginning of the tax-game, decreases in

$$q_1^s - \overline{q}_1^* \equiv \mu(q_{1L}^s - q_{1L}^*) + (1-\mu)(q_{1H}^s - q_{1H}^*) = -\frac{1}{8}(1-\mu)[\sqrt{2c+c^2} - 2c], \qquad (6)$$

with respect to the expected output under symmetric information. Such reduction hinges on the fact that signaling-related reduction in inefficient firm's output is a first-order effect, whereas under-taxation-related output increase is a second-order effect. Thus, the former is dominant.[5] In other words, though the efficient firm produces more under asymmetric than under symmetric information (taxes depend on output), the signaling-related reduction in the inefficient firm's output reduces production expected in period 1 below that under symmetric information.

Finally, the first-period revenue for the tax office amounts to

$$R_1^s = \frac{1}{8}\left[1-(1-\mu)\frac{\sqrt{2c+c^2}}{2}\right]^2, \tag{7}$$

which is always lower than $\overline{R}_t^* = (1/8)[1-(1-\mu)(2c-c^2)]$, the expected fiscal revenue in period 1 under symmetric information. Namely, when taxing a monopoly, the revenue loss due to (costly) signaling amounts to

$$R_1^s - \overline{R}_1^* = -\frac{1}{32}(1-\mu)[4\sqrt{2c+c^2} - 2(5-\mu)c + (3+\mu)c^2]\,. \tag{8}$$

because both the value of the expected tax and the firm's expected output are reduced.

4.2. Several taxpayers

As in subsection 4.1, we work by backward induction to look for the separating Perfect Bayesian-Cournot equilibrium of the tax-game when several firms take part. In period 2, the profits that firms obtain in equilibrium may be compared to those obtained off-the-equilibrium. This yields the following result.

Lemma 4. *Each duopolistic firm wishes to be perceived as an inefficient taxpayer.*

Proof. See the Appendix.

Now, apart from the (vertical) incentive of each firm *i* (*i*=A,B) to convince the tax office it is inefficient so as to pay a lower tax in period 2, there is also a (horizontal) incentive: being perceived as an efficient firm by rival firm *j*. This is due to the fact that outputs are strategic substitutes: irrespective of the firm *i*'s true cost, whenever firm *j* (*j*=A,B; *j*≠*i*) believes that firm *i* is efficient and, consequently, that it will produce a high output level, the best firm *j* can do is

[5] Output reduction vanishes when c=0 or when μ=1.

decreasing its output level in period 2; firm i then increases its production and profits in such period. Thus, two opposite effects arise in oligopoly. However, the vertical incentive to face lower taxes prevails, as the taxes paid by the firms in period 2 have first-order impact on their profits, whereas a decrease in the rival's output in period 2 has a second-order impact on their profits. The former is then of greater magnitude than the latter.

In view of Lemma 4, each efficient firm is tempted to misrepresent itself when its gain in period 2 (resulting from its deception) outweighs losses in period 1 (from producing output level q^i_{1H} rather than $q^i_{1L,\text{II}}$).[6] On the contrary, inefficient firms, for whom such behavior of efficient firm would be detrimental in period 2, may find it profitable to set period-1 output q^i_{1H} so low that deception is not advantageous for efficient firms. This coercion requires fulfillment of the incentive compatibility condition:

$$E\pi^i_{1L}(q^i_{1L}|0;\tilde{c}^j) + E\pi^i_{2L,\text{CI}} \geq E\pi^i_1(q^{is}_{1H}|0;\tilde{c}^j) + E\pi^i_{L2,\text{M}}, \tag{9}$$

where $E\pi^i_{1L}(q^i_{1L}|0;\tilde{c}^j)$ denotes period-1 expected profit when producing output q^i_{1L} and the rival firm maintains its equilibrium output, and subscript M denotes monopolistic behavior. Period-1 output for efficient firms in the equilibrium of the two-period game, q^i_{1L}, is given by:

$$q^i_{1L} = \arg\max_q E\pi^i_1(q^i|0;\tilde{c}^j), \tag{10}$$

whereas the optimal output in period 1 for inefficient firms consists on producing the amount that maximizes single period profit in period 1, namely:

$$q^{i*}_{1H} = \arg\max_q E\pi^i_1(q^i|c;\tilde{c}^j). \tag{11}$$

Finally, the incentive compatibility condition for inefficient taxpayers refers that two-period expected gains in the equilibrium exceed the sum of period-1 expected profit from producing q^{i*}_{1H} and period-2 expected profit. Formally:

$$E\pi^i_{1H}(q^{is}_{1H}|c;\tilde{c}^j) + E\pi^i_{H2,CI} \geq E\pi^i_1(q^{i*}_{1H}|c;\tilde{c}^j) + E\pi^i_{H2,\text{M}}. \tag{12}$$

Analysis of conditions (9)-(12) leads to the following lemma.

[6] Subscript numeral indicates period 1, and subscripts H, L and II denote, respectively, a high-cost firm, a low-cost firm, and incomplete information.

Lemma 5. *Tax* T_1^{is} *for each firm i (i=A,B) leads to separating Perfect Bayesian-Cournot equilibria in the proposed signaling tax-game. Period-1 outputs* q_{1H}^{is} *and* q_{1L}^{is} *for inefficient and efficient firms, respectively, that shape the separating equilibrium of minimum cost depends on the difference in production costs and prior beliefs as follows:*

(i) *If* $c < 1/(3+\mu)$, *then* $q_{1H}^{is} = \frac{1-T_1^{is}}{3} - \frac{(2+\mu)\sqrt{2c+(3-2\mu)c^2}}{18}$ *and* $q_{1L}^{is} = \frac{1-T_1^{is}}{3} + \frac{(1-\mu)\sqrt{2c+(3-2\mu)c^2}}{18}$.

(ii) *If* $c \geq 1/(3+\mu)$, *then* $q_{1H}^{is} = \frac{1-T_1^{is}}{3} - \frac{(2+\mu)c}{6}$ *and* $q_{1L}^{is} = \frac{1-T_1^{is}}{3} + \frac{(1-\mu)c}{6}$.

Proof. See the Appendix.

Part (i) of the lemma states that, in the separating Perfect Bayesian equilibrium of minimum cost, a downward distortion on total period-1 expected production exists relative to the expected output under symmetric information. Whenever the difference in technological attributes between firms is sufficiently narrow as $c < 1/(3+\mu)$, two effects emerge under asymmetric information relative to symmetric information. First, the mere existence of asymmetric information leads the inefficient (efficient) firms to produce in period 1 less (more) than under symmetric information. Second, the cost of signaling leads inefficient taxpayers to reduce their production, which in turn leads efficient taxpayers to increase their period-1 output above the level of symmetric information. Part (ii) states that the second effect no longer holds if the difference in production costs between firms is high enough as $c \geq 1/(3+\mu)$, and signaling becomes costless. The content of this part is important because it shows that competition plays a key role in signaling processes, up to the point that when compared to monopoly (where signaling is always costly), the presence of several firms in the market may lead to costless signaling.

Given the firms' period-1 output predicted by Lemma 5, the tax office sets the period-1 tax T_1^i (i=A,B) to maximize its revenue in such period. The resulting tax is recorded in Lemma 6.

Lemma 6. *The tax charged by the tax office to each firm in period 1,* T_1^{is}, *is as follows.*

(i) *If* $c < 1/(3+\mu)$, *then* $T_1^{is} = \frac{1}{2} - \frac{(1-\mu)\sqrt{2c+(3-2\mu)c^2}}{6}$.

(ii) *If* $c \geq 1/(3+\mu)$, *then* $T_1^{is} = \frac{1}{2} - \frac{(1-\mu)c}{2}$.

Proof. See the Appendix.

If the difference in production costs is as low as listed in part (i), both asymmetric information and the cost of signaling lead the tax office to charge firms a smaller period-1 tax than that expected under symmetric information, $T_1^{is} < \overline{T}_1^{i*}$. Such under-taxation allows inefficient firms to reduce their period-1 output up to becoming unattractive so as to mislead efficient firms. Likewise, thanks to tax reduction, efficient firms can raise their period-1 output to optimize their response to rivals. In addition, under-taxation leads to as-low-as-possible distortions on firms' period-1 output to minimize tax revenue reduction as much as possible relative to the symmetric information context. However, if the difference in production costs is relatively as large as established in part (ii), the tax office needs not to reduce the period-1 tax, which is the same as expected under symmetric information, $T_1^{is} = \overline{T}_1^{i*}$. Overall, $T_1^{is} \leq \overline{T}_1^{i*} \equiv \overline{T}_2^{i*}$, that is, (expected) taxes are never time-decreasing as information evolves from asymmetric to complete.

The result of part (ii) emerges in oligopoly but not in monopoly, due to the emerging horizontal incentive that countervails the vertical incentive of (each) efficient taxpayer to misrepresent as inefficient. Therefore, for sufficiently high values of parameter c (which represents the difference between cost realizations), inefficient taxpayers need no deviate from their profit-maximizing output under incomplete information conditions to separate themselves from efficient taxpayers.

Taking Lemmas 1 and 6 into account, the following result is obtained.

Proposition 2. *Relative to symmetric information, signaling leads the tax office not to increase taxes to oligopolistic taxpayers; namely,* $T_1^{is} \leq \overline{T}_1^{i*}$. *The difference on taxes between both informational contexts amounts to*

$$T_1^{is} - \overline{T}_1^{i*} = \begin{cases} -\frac{1}{6}(1-\mu)[\sqrt{2c+(3-2\mu)c^2} - 3c], \text{ if } c < \frac{1}{3+\mu} \\ 0, \qquad \text{if } c \geq \frac{1}{3+\mu}. \end{cases}$$

That is, whenever signaling is costless, efficient firms pay a lower tax than under symmetric information, $T_1^{is} < T_{1L}^{i*}$, whereas inefficient firms pay a higher one, $T_1^{is} > T_{1H}^{i*}$. On the contrary, if a (costly) signaling effect emerges, efficient taxpayers pay a lower tax than under symmetric information, but inefficient taxpayers pay a lower tax only when $\mu < 1 - \frac{3c}{\sqrt{2c+(3-2\mu)c^2}}$. Otherwise, inefficient taxpayers pay a higher tax than under symmetric information, $T_1^{is} > T_{1H}^{i*}$. Overall, and as occurs in monopoly, the presence of asymmetric information in oligopoly benefits efficient taxpayers more than inefficient taxpayers, since the tax decrease to the former is unambiguous but not the decrease to the latter. In addition, comparison of Propositions 1 and 2 allows concluding that under-taxation in oligopoly is less pronounced than in monopoly. In a sense, when signaling in oligopoly is costly, the presence of competition allows the tax office to screen firms' efficiency by under-taxing them less noticeably than in monopoly. This may be understood as a trade-off between signaling (and the resulting under-taxation) and competition: (more) competition leads the tax office to reduce per-unit taxes in a smaller amount.

Finally, comparing the taxes paid by firms in both informational contexts and in both market structures, it can be easily checked that—when asymmetric information leads a firm to pay a lower per-unit tax than under complete information—such advantage is smaller in oligopoly than in monopoly. On the contrary, if asymmetric information makes an (inefficient) firm pay a higher tax than under complete information, the presence of competition then aggravates this situation relative to a monopoly. To sum up, while under symmetric information per-unit taxes do not vary with market structure, this no longer holds under asymmetric information. In this case, the presence of several firms tends to increase per-unit taxes relative to the context which includes one only taxpayer.

With taxes set as prescribed by Lemma 6, inefficient firms' period-1 output is

$$q_{1H}^{is} = \begin{cases} \frac{1}{6} - \frac{(1+2\mu)\sqrt{2c+(3-2\mu)c^2}}{18}, \text{ if } c < \frac{1}{3+\mu} \\ \frac{1}{6} - \frac{(1+2\mu)c}{6}, \text{ if } c \geq \frac{1}{3+\mu} \end{cases} \tag{13}$$

while efficient firms's is

$$q_{1L}^{is}=\begin{cases}\dfrac{1}{6}+\dfrac{(1-\mu)\sqrt{2c+(3-2\mu)c^2}}{9}, \text{ if } c<\dfrac{1}{3+\mu}\\ \dfrac{1}{6}+\dfrac{(1-\mu)c}{3}, \text{ if } c\geq\dfrac{1}{3+\mu}\end{cases} \tag{14}$$

Both the asymmetric information and the signaling effects have an impact on firms' production to the point that, when compared with outputs produced under symmetric information, yield

$$q_{1H}^{is}-q_{1H}^{i*}=\begin{cases}-\dfrac{1}{18}[(1+2\mu)\sqrt{2c+(3-2\mu)c^2}-3(1+\mu)c], \text{ if } c<\dfrac{1}{3+\mu}\\ -\dfrac{1}{6}\mu c, \qquad \text{if } c\geq\dfrac{1}{3+\mu}\end{cases} \tag{15}$$

and

$$q_{1L}^{is}-q_{1L}^{i*}=\begin{cases}\dfrac{1}{18}[2(1-\mu)\sqrt{2c+(3-2\mu)c^2}-3(1-\mu)c], \text{ if } c<\dfrac{1}{3+\mu}\\ -\dfrac{1}{6}(1-\mu)c, \qquad \text{if } c\geq\dfrac{1}{3+\mu}\end{cases} \tag{16}$$

That is, if the only source of productive distortions is asymmetric information, high-cost (low-cost) firms produce less (more) than under symmetric information. On the other hand, costly signaling leads to a second distortion source: the cost of signaling through which high-cost (low-cost) firms pay a higher (smaller) tax. Hence, the abovementioned distortions on firms' output are reinforced.

Finally, the expected tax revenue in period 1 amounts to

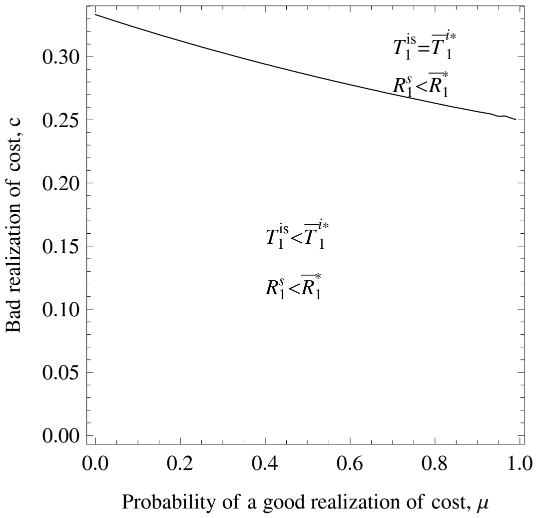
$$R_1^s=\begin{cases}\dfrac{1}{6}\left[1-(1-\mu)\dfrac{\sqrt{2c+(3-2\mu)c^2}}{3}\right]^2, \text{ if } c<\dfrac{1}{3+\mu}\\ \dfrac{1}{6}[1-(1-\mu)c]^2, \qquad \text{if } c\geq\dfrac{1}{3+\mu}\end{cases} \tag{17}$$

To sum up, we may conclude that for a sufficiently large difference in production costs as $c\geq 1/(3+\mu)$, signaling has no effect on taxes: firms' period-1 optimal outputs in case of incomplete information automatically generate the correct signals to third parties, and the tax office needs not to reduce the tax relative to that expected under symmetric information. Despite of this fact, fiscal revenue is lower than under symmetric information, as firms' productions are distorted, as stated in (15) and (16). On the other hand, when signaling is costly, which occurs when the difference in production cost is small enough as $c < 1/(3+\mu)$, the tax office is obliged to reduce tax size relative to those under symmetric information, and tax revenue fall short of that under symmetric information. Such a decrease amounts to

$$R_1^s - \overline{R}_1^* = \begin{cases} -\dfrac{1}{27}(1-\mu)[3\sqrt{2c+(3-2\mu)c^2} - (10-\mu)c + (3+7\mu-\mu^2)c^2], & \text{if } c < \dfrac{1}{3+\mu} \\ -\dfrac{1}{3}\mu(1-\mu)c^2, & \text{if } c \geq \dfrac{1}{3+\mu} \end{cases} \tag{18}$$

Figure 1 illustrates the combined result of Lemmas 1 and 6, and Expression (18).

Fig. 1. Effect of asymmetric information and signaling on taxes and fiscal revenue in oligopoly



Lemmas 3 and 6 allow us to examine the impact of market structure on both taxes and tax revenue. It is easy to check that, regardless of the fact that signaling in oligopoly can be either costless or costly, the tax office needs not to impose, in oligopoly, a per-unit tax as reduced as in monopoly. This is because competition allows the tax office to set not very low taxes to alleviate opportunistic behavior. This contrasts with the result obtained under symmetric information, where per-unit taxes are equal, regardless of the existence of a monopoly or an oligopoly. Since the per-unit tax in oligopoly is higher than in monopoly and expected output is also higher, tax revenue in oligopoly is higher than in monopoly.

Finally, (8) and (18) enable us to compare the tax office's losses due to asymmetric information in each market structure. Regarding symmetric information, private information reduces tax revenues, although such decrease is less pronounced in oligopoly than in monopoly. In a sense, the tax office reaches a trade-off between signaling costs and competition: (more) competition may lead to lower signaling cost and lower fiscal revenue reductions.

5. Welfare analysis

The consequences of the tax office behavior for expected social welfare is analyzed in this section. To this end, welfare is defined as the un-weighted sum of the expected value of consumer surplus, the expected value of firms' profits, and expected tax revenue. Given that complete information is restored in period 2, welfare comparison in asymmetric and symmetric information contexts can be restricted to that of period 1.

In monopoly, the expected consumer surplus under symmetric information amounts to $\overline{CS}_1^* = (1/32)[1-2(1-\mu)c+(1-\mu)c^2)]$, which, once the firm's profit and the fiscal revenue (see Table 1) are considered, leads to

$$\overline{W}_1^* = \frac{1}{32}[7-14(1-\mu)c+7(1-\mu)c^2] \tag{19}$$

as the value of expected welfare in period 1. On the other hand, under asymmetric information (and costly signaling) conditions, social welfare in period 1 amounts to

$$W_1^s = \frac{1}{128}[28-12(1-\mu)\sqrt{2c+c^2}-2(1-\mu)(17+3\mu-8(1+\mu)\sqrt{2c+c^2})c-(1+2\mu-3\mu^2)c^2] \tag{20}$$

Thus, despite the fact that $T_1^s < \overline{T}_1^*$, asymmetric information decreases social welfare in almost the whole (μ,c)-region of parameters. However, in the northeast of the (μ,c)-space of parameters, a very small region is defined by the following condition

$$12\sqrt{2c+c^2}-[22+16(1+\mu)\sqrt{2c+c^2}-6\mu]c+(29+3\mu)c^2<0, \tag{21}$$

for which $W_1^s > \overline{W}_1^*$. This condition, stated in (21), defines a small region where both μ, the probability of a firm becoming efficient, and c, difference in production costs, are very high ($\mu > .75$, $c > .3$), so the cost of signaling is very small and the firm, which is very likely an efficient taxpayer, produces a greater output level than under symmetric information.

Similarly, in oligopoly the expected welfare in period 1 amounts to

$$\overline{W}_1^* = \frac{1}{18}[5 - 10(1-\mu)c + (1-\mu)(5+6\mu)c^2] \tag{22}$$

under symmetric information and to

$$W_1^s = \begin{cases} \frac{1}{162}[45 - 12(1-\mu)\Delta - 2(1-\mu)(28-\mu-9(1+2\mu)\Delta)c + (1-\mu)^2(3-2\mu)c^2], & \text{if } c < \frac{1}{3+\mu} \\ \frac{1}{18}[5 - 10(1-\mu)c + (1-\mu)(5+13\mu)c^2], & \text{if } c \geq \frac{1}{3+\mu} \end{cases} \tag{23}$$

being $\Delta = \sqrt{2c + (3-2\mu)c^2}$, under asymmetric information. Comparing (22) and (23) allows us to conclude that welfare level under asymmetric information may be either lower or higher than under symmetric information. Namely,

$$W_1^s - \overline{W}_1^* = \begin{cases} -\frac{1}{62}(1-\mu)[12\Delta - 2(17+\mu+9(1+2\mu)\Delta)c + (48+49\mu+2\mu^2)c^2], & \text{if } c < \frac{1}{3+\mu} \\ \frac{1}{18}7\mu(1-\mu)c^2 & \text{if } c \geq \frac{1}{3+\mu} \end{cases} \tag{24}$$

In particular, it is higher, $W_1^s > \overline{W}_1^*$, when signaling is costless or, signaling being costly, when parameters μ and c satisfy

$$12\Delta - 2[17 + \mu + 9(1+2\mu)\Delta]c + (48 + 49\mu + 2\mu^2)c^2 < 0 \ . \tag{25}$$

All these results can be summarized as follows.

Proposition 3. *Compared to a symmetric information context, the behavior of the tax office in the signaling game increases social welfare in the following cases.*

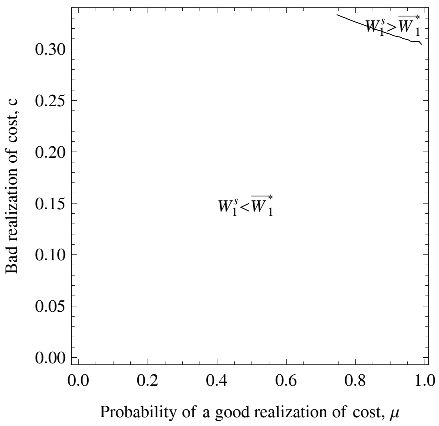
(i) *In monopoly, when parameters* μ *and c satisfy the condition given in (21).*

(ii) *In oligopoly, when signaling is costless or, being costly, when parameters* μ *and c satisfy the condition defined in (25).*

Proof. See the Appendix.

Figure 2 illustrates the content of part (i) of Proposition 3.

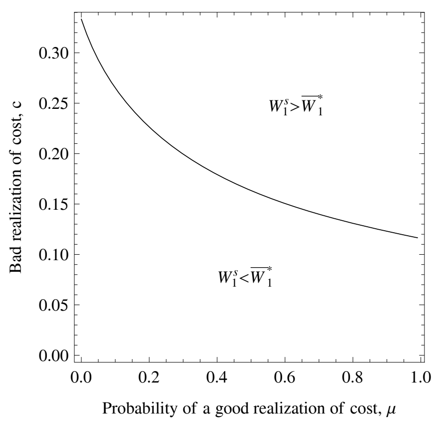
Fig. 2. Effect of asymmetric information on welfare in monopoly



The line depicted northeast the region of parameters is the (μ,c)-locus defined by condition $12\sqrt{2c+c^2} - [22+16(1+\mu)\sqrt{2c+c^2} - 6\mu]c + (29+3\mu)c^2 = 0$ and separates the (μ,c)-region of parameters where signaling improves welfare from where it does not. If the taxed firm is very likely an efficient firm and its level of efficiency is very high (relative to the inefficient one), signaling then increases social welfare, as consumers' improvement outweighs firm and tax-office worsening.

This result is achieved in oligopoly even under less demanding conditions. In fact, part (ii) can be illustrated as in Figure 3.

Fig. 3. Effect of asymmetric information on welfare in oligopoly



in which the curve is the locus defined by condition (25) as an equality.

6. Large difference in production costs

Analysis so far has assumed $c < 1/3$ to ensure that any type of firm produces a positive output level, regardless of prior beliefs of the remaining players. In this subsection, parameter c is allowed to adopt values equal or above $1/3$.

Let us start analyzing a monopoly. In this case, if $1/3 < c < 2/3$, all previously-obtained results for the (μ,c)-region of parameters in which $0 < c < 1/3$ remain valid, as the separating equilibrium is still costly. However, if $c > 2/3$, the separating equilibrium becomes costless (that is, the high-cost firm signals its true cost by simply producing output $q_{1H}^{s} \equiv q_{1H}^{m} = (1 - c - T_1)/2$ in period 1). Consequently, the tax charged in period 1 amounts to $T_1^{s} = (1/2)[1 - (1-\mu)c]$, equaling the one expectedly imposed under symmetric information. Furthermore, it remains unchanged as information evolves from incomplete to complete, $T_1^{s} = \overline{T}_2^{*}$.

In oligopoly, if the difference in efficiency levels is as large as $1/(3+\mu) < c < 1/(1+2\mu)$, the separating Perfect Bayesian-Cournot equilibrium is costless, implying a tax as $T_1^{is} = (1/2)[1-(1-\mu)c]$ in period 1, and period-1 productions given by $q_{1H}^{is} = (1/6)[1-(1+2\mu)c]$ and $q_{1L}^{is} = (1/6)[1+(1-\mu)c]$, for high- and low-cost firms, respectively. That is, both types of firms play the tax-game in period 1. However, if $c > 1/2$, a high-cost firm will not produce in period 2. Thus, if $c \geq 1/3$, the only (μ,c)-region of parameters within which both types of firms are active in both production periods is where $c < 1/(1+2\mu)$ and $c < 1/2$. In such region, the costless separating equilibrium is played in the tax-game and, consequently, all results obtained for the region in which $1/(3+\mu) < c < 1/3$ are still valid.

Finally, if the efficiency gap is as $c \geq 1/(1+2\mu)$, the costless separating equilibrium remains valid, and $T_1^{is} = (1/2)[1-(1-\mu)c]$, from which $q_{1H}^{i} = (1/6)[1-(1+2\mu)c]$ holds. That is, high-cost firms will not find it profitable to produce in (incomplete information) period 1. As a result, a tax $T_1^{is} = 1/2$ is charged in period 1, while low-cost firms' output level is

$$q_{1L}^{i} = \arg\max_{q^{i}} \mu(1 - T_1^{is} - q_{1L}^{i} - q_{1L}^{j})q_{1L}^{i} + (1-\mu)(1 - T_1^{is} - q_{1L}^{i})q_{1L}^{i} \qquad (26)$$

which affords $q_{1L}^{i} = (1 - T_1^{is} - \mu q_{1L}^{j})/2$, and bearing symmetry in mind, $q_{1L}^{i} = q_{1L}^{j}$, it renders that $q_{1L}^{i} = (1 - T_1^{is})/(2+\mu) = 1/2(2+\mu)$. Finally, the fiscal revenue amounts to $R_1^{s} = \mu/2(2+\mu)$.

7. Conclusions

Asymmetric information may induce opportunistic behavior in well informed players. Some papers have addressed how less informed players can mitigate this problem. This paper supplements literature by exploring how a tax office counteracts opportunistic behavior through the size of quantity-related taxes imposed to firms with private information on their technological attributes. With this purpose, a two-period signaling tax-game is presented, where a revenue-raising tax office imposes an output tax in each period to each producing firm. Firms' marginal costs of production—whether low or high—are privately known and can only be inferred by third parties—the tax office among them—after observing period-1 output level.

In the (asymmetric information) first period of the game, tax size is the same for all types of firms because the tax office cannot distinguish their typology. When this tax is compared to that expectedly imposed under symmetric information, result is unambiguous. In monopoly, the tax office tries to make a separating equilibrium possible, but anticipates that the low-cost firm has a (vertical) incentive to misrepresent as a high-cost firm. This incentive is so strong that signaling is always costly (a reduction on high-cost firm's period-1 output). This may lead the tax office to counteract this incentive by reducing taxes in period 1 (relative to that expectedly charged under symmetric information). That is, both per-period expected taxes and tax revenue increase along time as information evolves from asymmetric to symmetric.

On the other hand, when the tax office faces an oligopoly, findings show the emergence of a new (horizontal) incentive that leads high-cost firms to misrepresent as low-cost firms. This obviously counteracts the vertical incentive of efficient oligopolists to misrepresent as inefficient taxpayers, whereby the net incentive to misrepresent as inefficient taxpayers is less intense than in monopoly. This leads the tax office to react differently than in monopoly. In particular, for a sufficiently large difference in production costs, signaling is costless, and the tax imposed to firms in period 1 is equal to that expectedly charged under symmetric information. In a sense, competition is enough to mitigate opportunistic behavior. Furthermore, in period 2, when complete information is restored, taxes for high-cost firms are smaller than for low-cost firms. Overall, expected taxes are time-constant, in expected terms, as information evolves from incomplete to complete, whereas tax revenue to tax office is time-increasing. Contrariwise, as the difference in production costs narrows, it becomes increasingly costly for the tax office to screen for firms' costs and for firms to signal them. In this case, the tax office reacts by imposing a smaller tax than under symmetric information to mitigate opportunistic behavior. In spite of this, it is worth noting that the presence of several firms helps the tax office in alleviating opportunistic behavior, and allows it to extract information by charging a not very low per-unit tax in period 1 as in monopoly.

Finally, the paper shows that, in social terms, signaling is typically welfare-decreasing, especially in monopoly. Likewise, in oligopolistic markets, signaling increases welfare when costless or, being costly, its cost is very low. In all these cases, the positive impact of signaling on consumer surplus is so large that outweighs its negative impact on firms' profits and tax office revenue.

Appendix

Proof of Lemma 1. In monopoly, the firm's output in each period t (t=1,2) is given by $q^*_{tL} = (1-T_{tL})/2$ if efficient and $q^*_{tH} = (1-c-T_{tL})/2$ if inefficient. In the former, the problem solved by the tax office is $\max_{T_{tL}} T_{tL}(1-T_{tL})/2$, which enables $T^*_{tL} = 1/2$; in the latter it is $\max_{T_{tH}} T_{tH}(1-c-T_{tH})/2$, which leads to $T^*_{tH} = (1-c)/2$. The values in Table 1 for per-period firm's output and profit, as well as for per-period tax revenue, hold straightforwardly.

In oligopoly, the output produced by each firm i (i=A,B) in each period t is given by

$$q^i_t = \frac{1-2\tilde{c}^i - 2T^i_t + \tilde{c}^j + T^j_t}{3} = \begin{cases} \dfrac{1-2T^i_{tL} + T^j_{tL}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (0,0) \\ \dfrac{1-2T^i_{tL} + c + T^j_{tH}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (0,c) \\ \dfrac{1-2c-2T^i_{tH} + T^j_{tL}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (c,0) \\ \dfrac{1-c-2T^i_{tH} + T^j_{tH}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (c,c) \end{cases} \quad \text{(A1)}$$

Thus, the objective function of the tax office is

$$R_t = T^i_t q^i_t + T^j_t q^j_t = \begin{cases} T^i_{tL}\dfrac{1-2T^i_{tL} + T^j_{tL}}{3} + T^j_{tL}\dfrac{1-2T^j_{tL} + T^i_{tL}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (0,0) \\ T^i_{tL}\dfrac{1-2T^i_{tL} + c + T^j_{tH}}{3} + T^j_{tH}\dfrac{1-2c-2T^j_{tH} + T^i_{tL}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (0,c) \\ T^i_{tH}\dfrac{1-2c-2T^i_{tH} + T^j_{tL}}{3} + T^j_{tL}\dfrac{1-2T^j_{tL} + c + T^i_{tH}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (c,0) \\ T^i_{tH}\dfrac{1-c-2T^i_{tH} + T^j_{tH}}{3} + T^j_{tH}\dfrac{1-c-2T^j_{tH} + T^i_{tH}}{3}, \text{ if } (\tilde{c}_i, \tilde{c}_j) = (c,c) \end{cases} \quad \text{(A2)}$$

and the revenue-maximizing tax is given by $T^{i*}_{tL} = 1/2$ if the taxpayer is efficient and $T^{i*}_{tH} = (1-c)/2$ otherwise. This completes the proof of the lemma. ■

Proof of Lemma 2. Let $\pi_2(x|z)$ stand for the firm's period-2 profit if the firm represented itself in the previous period as facing cost x when facing cost z. Then $\pi_2(0|c) = (1/4)^2(1+c)^2$ and $\pi_2(c|0) = (1/4)^2(1-2c)^2$. Comparison of these profits with those presented in Table 1 shows that $\pi_2(0|c) > \pi_2(0|0)$ and $\pi_2(c|c) > \pi_2(c|0)$. This proves the lemma. ■

Proof of Lemma 3. Spelling out incentive compatibility conditions (4) and (5) yields

$$(1-c-T_1-q_{1H}^s)q_{1H}^s + \left(\frac{1-c}{4}\right)^2 - \left(\frac{1-c-T_1}{4}\right)^2 - \left(\frac{1-2c}{4}\right)^2 \geq 0 \tag{A3}$$

and

$$(1-T_1-q_{1H}^s)q_{1H}^s + \left(\frac{1+c}{4}\right)^2 - \left(\frac{1-T_1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 \leq 0\,. \tag{A4}$$

Inequalities (A3) and (A4) are simultaneously satisfied by any output level $q_{1H}^s \in [u^-, v^-]$ where u^- and v^- are the smaller roots of the quadratic equations formed by taking (A3) and (A4), respectively, as equalities. That is,

$$u^- = \frac{1-c-T_1}{2} - \frac{\sqrt{2c-3c^2}}{4} \tag{A5}$$

and

$$v^- = \frac{1-T_1}{2} - \frac{\sqrt{2c+c^2}}{4}\,. \tag{A6}$$

There is therefore a continuum of outputs within the interval $[u^-, v^-]$ that defines separating equilibria. The high-cost firm maximizes its profit in period 1 by producing $q_{1H}^m = (1-c-T_1)/2$, yet this strategy does not satisfy conditions (A3) and (A4), because $(1-c-T_1)/2 > v^-$ according to Assumption A1. Thus, the separating equilibrium of minimum cost leads the high-cost firm to produce the output given by v^-, strictly below the period-1 profit-maximizing output.

For the tax office, the expected output in period 1 is $\mu[(1-T_1)/2] + (1-\mu)[(1-T_1)/2 - \sqrt{2c+c^2}/4]$. In such period the per unit tax charged is then

$$T_1^s = \arg\max_{T_1} \mu\left(\frac{1-T_1}{2}\right) + (1-\mu)\left[\frac{1-T_1}{2} - \frac{\sqrt{2c+c^2}}{4}\right]. \qquad \text{(A7)}$$

Solving (A7) provides the result of part (i). The result stated in part (ii) follows after substituting the value of T_1^s in v^- and q_{1L}^m. ■

Proof of Lemma 4. As in Lemma 2, let $\pi_2^i(x|z;\tilde{c}^j)$ denote the period-2 profit of firm *i* if the firm represented itself as facing cost *x* when facing cost *z*, and firm *j*, *j*≠*i*, is publicly known to face cost $\tilde{c}^j \in \{0,c\}$. Then it follows that $\pi_2^i(0|c;0) = (1/6)^2(1-3c)^2$, $\pi_2(0|c;c) = (1/6)^2(1-2c)^2$, $\pi_2^i(c|0;0) = (1/6)^2(1+c)^2$ and $\pi_2^i(c|0;c) = (1/6)^2(1+2c)^2$. From here, regardless of whether firm *j* is low- or high-cost, we obtain that $\pi_2^i(c|0;\tilde{c}^j) > \pi_2^i(0|0;\tilde{c}^j)$ and $\pi_2^i(c|c;\tilde{c}^j) > \pi_2^i(0|c;\tilde{c}^j)$. This proves the lemma. ■

Proof of Lemma 5. Once particularized, conditions (8)-(11) can be written as follows:

$$\mu\left[(1-T_1^i-2q_{1L}^i)q_{1L}^i + \left(\frac{1}{6}\right)^2\right] + (1-\mu)\left[(1-T_1^i-q_{1L}^i-q_{1H}^j)q_{1L}^i + \left(\frac{1+c}{6}\right)^2\right]^2$$

$$\geq \mu\left[(1-T_1^i-q_{1H}^i-q_{1L}^j)q_{1H}^i + \left(\frac{1+c}{6}\right)^2\right] + (1-\mu)\left[(1-T_1^i-2q_{1H}^i)q_{1H}^i + \left(\frac{1+2c}{6}\right)^2\right]^2, \qquad \text{(A8)}$$

$$q_{1L}^i = \arg\max_q\left[\mu(1-T_1^i-q^i-q_{1L}^j)q^i + (1-\mu)(1-T_1^i-q^i-q_{1H}^j)q^i\right]$$

$$= \frac{1-T_1^i-(1-\mu)q_{1H}^i}{2+\mu}, \qquad \text{(A9)}$$

$$q_{1H}^{i*} = \arg\max_q\left[\mu(1-c-T_1^i-q^i-q_{1L}^j)q^i + (1-\mu)(1-c-T_1^i-q^i-q_{1H}^j)q^i\right]$$

$$= \frac{2(1-c-T_1^i)-\mu c-2(1-\mu)q_{1H}^i}{2(2+\mu)}, \qquad \text{(A10)}$$

and

$$\mu\left[(1-c-T_1^i-q_{1H}^i-q_{1L}^j)q_{1H}^i+\left(\frac{1-2c}{6}\right)^2\right]+(1-\mu)\left[(1-c-T_1^i-2q_{1H}^i)q_{1H}^i+\left(\frac{1-c}{6}\right)^2\right]^2$$

$$\geq\mu\left[(1-c-T_1^i-q_{1H}^{i*}-q_{1L}^j)q_{1H}^{i*}+\left(\frac{1-3c}{6}\right)^2\right]+(1-\mu)\left[(1-c-T_1^i-q_{1H}^{i*}-q_{1H}^j)q_{1H}^{i*}+\left(\frac{1-2c}{6}\right)^2\right]^2$$

(A11)

The two roots of condition (A8) as taken as equality are

$$q_{1H}^{is}=\frac{1-T_1^i}{3}\pm\frac{(2+\mu)\sqrt{2c+(3-2\mu)c^2}}{18} \quad \text{(A12)}$$

and those of condition (A11) are

$$q_{1H}^{is}=\frac{1-T_1^i}{3}-\frac{(2+\mu)c}{6}\pm\frac{(2+\mu)\sqrt{2c-(3-2\mu)c^2}}{18}. \quad \text{(A13)}$$

Let s^- and r^- denote the lowest roots of (A12) and (A13), respectively. Thus the continuum of outputs that constitute separating Perfect Bayesian equilibria is given by $q_{1H}^{is}\in[r^-,s^-]$. Comparing outputs s^- and $q_{1H}^{i\max\pi}=(1-T_1^i)/3-(2+\mu)c/6$, the output of each profit-maximizing high-cost firm in period 1, yields that $s^-<q_{1H}^{i\max\pi}$ as long as $c<1/(3+\mu)$. Thus, outputs in the (costly) separating Perfect Bayesian equilibrium of minimum cost are $q_{1H}^{is}=s^-$, for each high-cost firm, and q_{1L}^i, given by condition (A10), for each low-cost firm. This proves part (i) of Lemma 5. Part (ii) is proven by the fact that $s^-\geq q_{1H}^{i\max\pi}$ as long as $c\geq 1/(3+\mu)$, in which case the outputs that define (costless) separating equilibrium are $q_{1H}^{is}\geq q_{1H}^{i\max\pi}$ and q_{1L}^i, given by condition (A10), for each low-cost firm. ■

Proof of Lemma 6. For the tax office, the expected output of each firm *i* in period 1 is $\mu q_{1L}^{is}+(1-\mu)q_{1H}^{is}$. Hence, per unit tax in such period for each firm *i* is

$$T_1^{is}=\arg\max_{T_1^i} 2T_1^i[\mu q_{1L}^{is}+(1-\mu)q_{1H}^{is}]. \quad \text{(A14)}$$

The results for cases (i) and (ii) in Lemma 6 follow from the first-order conditions of the problem given in (A14) after using the results of Lemma 5 to substitute for q_{1L}^{is} and q_{1H}^{is} as appropriate for cases (i) and (ii). ■

Proof of Proposition 3. (i) In monopoly, the expected consumer surplus amounts to

$$CS_1^s = \frac{1}{128}[4(1-(1-\mu)\sqrt{2c+c^2}) + 2(1+2\mu-3\mu^2)c + (1+2\mu-3\mu^2)c^2] \tag{A15}$$

and the firm's expected profit amounts to

$$\pi_1^s = \frac{1}{64}[4(1+(1-\mu)\sqrt{2c+c^2} - 2(1-\mu)(11+\mu-4(1+\mu)\sqrt{2c+c^2})c - (3-2\mu-\mu^2)c^2]\,. \tag{A16}$$

Once (A15), (A16) and (7) are taken into account the result given in (22) follows directly. The proof of part (i) is then immediate.

(ii) In oligopoly, consumer surplus, as calculated at tax-game onset, is

$$CS_1^s = \begin{cases} \frac{1}{2}\left[\left(\frac{3+2(1-\mu)\Delta}{9}\right)^2 + (1-\mu)\left(\frac{3-(1+2\mu)\Delta}{9}\right)^2\right], & \text{if } c < \frac{1}{3+\mu} \\ \frac{1}{18}[1-2(1-\mu)c + (1-\mu)(1+8\mu)c^2], & \text{if } c \geq \frac{1}{3+\mu} \end{cases} \tag{A17}$$

where $\Delta = \sqrt{2c+(3-2\mu)c^2}$. Industry profits, on the other hand, are

$$\pi_1^s = \sum_{i=A,B} \pi_1^{is}$$

$$= \begin{cases} \frac{1}{162}[9+12(1-\mu)\Delta + 2(1-\mu)(9(1+2\mu)\Delta - 4(8+\mu))c - (1-\mu)(3-2\mu)(5+4\mu)c^2], & \text{if } c < \frac{1}{3+\mu} \\ \frac{1}{18}[1-2(1-\mu)c + (1-\mu)(1+8\mu)c^2], & \text{if } c \geq \frac{1}{3+\mu} \end{cases} \tag{A18}$$

Consideration of (A17), (A18) and R_1^s , the fiscal revenue, leads to the result given in (24). The result stated in (26) follows immediately. ■

References

Cech, P.A. (1991), Market efficient oligopoly regulation: An incentive compatible mechanism, Working Paper, Northwestern University.

Kim, J. and K. Chang (1993), An optimal tax/subsidy for output and pollution control under asymmetric information in oligopoly markets, *Journal of Regulatory Economics* 5, 183-197.

Liu, J. and Y.-H. Tan (2008), Towards asymmetric information for the G2B inter-organizational networks, *BLED Proceedings*. Paper 12, 451-463. (Available at http://aisel.aisnet.org/bled2008/12)

Loeb, M. and W.A. Magat (1979), A decentralized method for utility regulation, *Journal of Law and Economics* 22, 393-404.

EU Commission (2006), EU coherent strategy against fiscal fraud—FAQs. (Available at http://europa.Eu/rapid/pressreleasesaction.Do?Reference=memo/06/221&format=html&aged=0&language=en&guilanguage=en)