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# Job Search with Bidder Memories

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# Job Search with Bidder Memories

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# ABSTRACT

# Job Search with Bidder Memories[*]

This paper revisits the no-recall assumption in job search models with take-it-or-leave-it offers. Workers who can recall previously encountered potential employers in order to engage them in Bertrand bidding have a distinct advantage over workers without such attachments. Firms account for this difference when hiring a worker. When a worker first meets a firm, the firm offers the worker a sufficient share of the match rents to avoid a bidding war in the future. The pair share the gains to trade. In this case, the Diamond paradox no longer holds.

JEL Classification:     J24, J42, J64

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# 1. Introduction

This paper analyzes the role of recall in job search. When a worker meets a potential employer, a wage offer from the firm is a bid for the worker's services in an auction. If workers have memories that allow them to recall previous encounters with potential employers, these workers have the capacity to alter the number of bidders for their services, induce Bertrand bidding and thereby obtain high wages. Without a recall option in bilateral matching, firms are monopsonists. As Diamond (1971) illustrates, these firms offer low wages that capture the gains to trade.

The standard search model plays down the recall option and focuses on the single bidder outcome. In this literature (e.g. McCall, 1970; Mortensen, 1970; Albrecht and Axell, 1984), if traders fail to agree to terms, they break-up, the match dissolves entirely, and potential trading partners lose all contact. They forget the match existed. As Rogerson, Shimer and Wright (2005) point out, the no-recall assumption is innocuous given that previous bids are fixed. In a stationary world, rejected offers do not become acceptable when viewed a second time around.

This argument assumes that firms will not revise their wage offer when called upon again, possibly in different circumstances. Recalled bidders, however, have an incentive to update their offers to account for the competition for the worker.[1] When there are no competing bidders, the firm finds it optimal to offer the worker's reservation wage, i.e. the wage that makes the worker indifferent between accepting and rejecting the job. In contrast, when there are competing bidders, the firm finds it optimal to offer a wage slightly higher than those offered by the other bidders. As a result, when there are competing bidders, every firm offers its own reservation wage, i.e. the wage that makes the firm indifferent between hiring and not hiring the worker.[2]

When firms are able to update their wage offers, the possibility of recall fundamentally alters the equilibrium of the economy. Without recall, it is well understood (see Diamond, 1971; Burdett and Judd, 1983; Albrecht and Axel, 1984) that every employed worker earns the monopsony wage, no matter how small the search frictions are. With recall, forward looking firms avoid a future bidding war by offering enough at the initial encounter. A worker continuing with job search has

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[1] As in models of dynamic monopoly pricing, the firm would prefer to commit to bid a fixed wage for all circumstances, provided other bidders committed as well. Of course, as with the Coase Conjecture, this strategy is not subgame perfect. Two bidders without a commitment to a single bid will not in equilibrium maintain monopoly offers.

[2] Unlike the text book Bertrand outcome, in this environment wages from auctions with competing bidders are less than marginal productivity. Because worker-firm contacts can die off, firms in a multiple bidder auction might at some time in the future become the lone contact at which point they acquire some monopsony power. Given this potential future payoff, firms in the auction will hold back to some extent and not concede all productivity to the worker. They prefer waiting over bidding up to marginal product. In this way they obtain some of the gains to trade. See Taylor (1995) for a related result.

a chance of generating a wage that is strictly greater than the monopsony wage, so the incumbent firm must offer the worker a fraction of the gains from trade to make him take the job. With recall, every employed worker earns a wage strictly greater than the monopsony wage. Moreover, this wage converges to the competitive wage as the search frictions become arbitrarily small.[3]

The intuition is straightforward. When a firm is the sole bidder for a worker, it offers a wage that makes the worker indifferent between becoming employed and continuing to search. If the worker continues to search, he may find a second bidder and engage the two firms in a bidding war. In order to convince the worker to forgo the option of searching, the firm has to offer a wage that is higher than the monopsony wage even when the firm is the sole bidder. For the worker, the value of attached search is a distinctly better position for the worker than unattached search.

The outcome of the wage auction relies on firms observing other bidders. Firms must know or at least believe there is a positive probability that other bidders are involved at some point. We therefore study two different cases. Sections 2 through 4 characterize the equilibrium of the labor market under the assumption that the number of firms that are bidding for a worker is public information. This specification seems to be appropriate for some labor markets. Academics, lawyers, and other professionals come to mind. Public information might not be plausible in other labor markets such as the market for clerical work. To address this possibility, Section 5 characterizes an equilibrium assuming that the number of bidders involved in an auction is private information of the worker.

Recognizing other bidders in the auction is critical. When a firm meets a worker, the firm must form beliefs about how many other bidders are involved in the auction. Knowing that in the equilibrium with private information all workers accept the first offer that they receive, a firm rationally believes itself to be the sole bidder when it meets a worker for the first time. The firm offers the worker's reservation wage. Off the equilibrium path, when a firm meets a worker for the second time, it knows that any other bidder involved in the auction believes itself to be a monopsonist. Therefore, the firm again finds it optimal to offer the worker's reservation wage. The worker always receives his reservation wage, hence every employed worker earns the monopsony wage no matter how small the search frictions are. If employers cannot verify the number of other bidders in an auction, the equilibrium with recall is the same as the equilibrium without recall.

The model here with recall can be related to the wage posting models of Butters (1977),

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[3]The proposed mechanism also applies outside the labor market setting into other markets with matching frictions, for example housing and durable goods where other resolutions of Diamond's paradox, for example on-the-job search or bargaining may not be appropriate.

Burdett and Judd (1983), and Burdett and Mortensen (1998). Those models, which can be interpreted as having auctions for workers with an unknown number of bidders, generate non-degenerate wage distributions in equilibrium. In this paper, dispersion does not arise without on-the-job search but like these other models, equilibrium wages depend critically on the expectation of the number of bidders found during the search process. These expectations depend on the observability of the worker's options. With observable recall, the worker can verifiably increase the number of bidders from one to two and thereby extract some match rents. With unobservable recall, the firm rationally believes itself to always be the lone bidder. The worker is unable to obtain any rents.

## 2. The Model

### 2.1. Economic Environment

A continuum of workers with measure one populate a labor market that operates in continuous time. Each worker is endowed with one unit of labor per unit of time, maximizes the expected sum of lifetime consumption, and leaves the labor market at the Poisson rate $r > 0$. The reader may interpret $r$ as the rate at which workers retire, or as the rate at which workers die. The measure of workers who participate in the labor market is constant over time because, per every unit of time of length $dt$, a continuum of workers with measure $rdt$ enter the labor market. A continuum of firms with positive measure also populates the labor market. Each firm operates a constant return to scale technology that transforms one unit of labor into $x$ units of output. Each firm maximizes the expected sum of its profits.

At any point in time, employment status and networking status characterize a worker. The worker's employment status is either unemployed or employed at the wage $w$. The worker's networking status is an integer between 0 and $N$ which represents the number of firms that are in contact with the worker. The upper bound $N$ is the largest number of firms with which the worker can have a long-distance relationship. It is important to note that $N$ does not include the current employer nor any firm that the worker might have just met. Most of the existing literature implicitly assumes that $N = 0$ (e.g. Diamond, 1971; Mortensen, 1971; Mortensen and Pissarides, 1999a,b; Burdett and Mortensen, 1998).

Consider an unemployed worker with $i$ (distant) contacts, $i = 0, 1, ..., N$. During unemployment, the worker consumes $z > 0$ units of output per unit of time. The reader may interpret $z$ as the consumption equivalent of leisure, or as an unemployment benefit. At the Poisson rate of $i\phi$, $\phi \geq 0$, the worker loses one of these contacts. At the Poisson rate of $\lambda > 0$, the worker meets

a firm. When this happens, the worker receives a take-it-or-leave-it wage offer from the just met firm, as well as from every one of the firms that are still in contact with the worker. If the worker accepts one of these $i + 1$ offers, the worker becomes employed. If the worker rejects all of these offers, the worker adds the just met firm to the list of contacts and keeps on searching.

Consider next an employed worker with $i$ contacts, excluding the current employer, $i = 0, 1, ..., N$. While employed, the worker produces $x > z$ units of output and consumes $w$ units of output per unit of time, where $w$ is the worker's wage rate. At the Poisson rate of $i\phi$, the worker loses one of these contacts. At the Poisson rate of $\delta \geq 0$, the worker is exogenously displaced from the current job. When this happens, the worker receives a take-it-or-leave-it offer from every one of the firms that are still in contact. If the worker accepts one of these $i$ offers, the worker moves from one employer to the other without an intervening spell of unemployment. If the worker rejects all of these offers (or if the worker did not have any contacts), the worker becomes unemployed.

## 2.2. The Problem of the Worker

Let $\sigma_f = (w_1, w_2, ..., w_{N+1})$ denote the strategy of a firm. The $i$-th element of $\sigma_f$ denotes the firm's bid in an auction in which there are $i$ bidders. Let $U_i$ denote the lifetime utility of a worker who is unemployed and has $i$ contacts, $i = 0, 1, ..., N$. Let $E_i(w)$ denote the lifetime utility of a worker who is employed at the wage $w$ and has $i$ contacts, $i = 0, 1, ..., N$. Let $M_i$ denote the value to the firm of participating in an auction with $i$ bidders, $i = 1, 2, ..., N + 1$. Let $C_i^u$ denote the value to the firm of being in contact with an unemployed worker who has $i - 1$ other contacts, $i = 1, 2, ..., N$. Similarly, let $C_i^e$ denote the value to the firm of being in contact with an employed worker who is in contact with $i - 1$ other firms, $i = 1, 2, ..., N$. Finally, let $J_i(w)$ denote the value to a firm from employing a worker who has $i$ contacts, $i = 0, 1, ..., N$.

A worker employed at the wage $w$ who has $i$ contacts, $i = 0, 1, ..., N$, receives flow utility equal to the wage $w$. At rate $\delta$, the worker is displaced from the current job. In this case, the worker receives the offer $w_i$ from every one of the contacts. If the worker accepts one of these offers, he remains employed but the wage goes from $w$ to $w_i$. If the worker rejects all of these offers, he becomes unemployed with $i$ contacts. At rate $i\phi$, the worker loses one of these contacts. In this case, the worker remains employed at the wage $w$, but has only $i - 1$ contacts left. Therefore, the worker's lifetime utility $E_i(w)$ is given by

$$rE_i(w) = w + \delta \left[\max\left\{E_{i-1}(w_i), U_i\right\} - E_i(w)\right] + i\phi \left[E_{i-1}(w) - E_i(w)\right]. \qquad (2.1)$$

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From equation (2.1) for $i = 0$, it follows that the value function $E_0(w)$ is strictly increasing with respect to $w$. In turn, from equation (2.1) for $i = 1, 2, ..., N$ and from the monotonicity of $E_{i-1}(w)$, it follows that the value function $E_i(w)$ is strictly increasing with respect to $w$.

An unemployed worker who has $i$ contacts, $i = 0, 1, ..., N$, receives the flow utility $z$. At rate $i\phi$, the worker loses one of these contacts. In this case, the worker remains unemployed and continues searching with $i - 1$ contacts. At rate $\lambda$, the worker meets a firm. In this case, the worker receives the wage offer $w_{i+1}$ from the just met firm as well as from every one of the $i$ contacts. If the worker accepts one of these offers, he becomes employed at the wage $w_{i+1}$. If the worker rejects all of these offers, he remains unemployed and continues searching with $\max\{i + 1, N\}$ contacts. Therefore, the worker's lifetime utility $U_i$ is given by

$$rU_i = z + \lambda \left[\max\{E_i(w_{i+1}), U_{i+1}\} - U_i\right] + i\phi \left[U_{i-1} - U_i\right], \quad \text{if } i < N,$$
$$rU_i = z + \lambda \left[\max\{E_i(w_{i+1}), U_i\} - U_i\right] + i\phi \left[U_{i-1} - U_i\right] \qquad \text{if } i = N. \tag{2.2}$$

Remember that $E_i(w)$ is strictly increasing in $w$. Therefore, equation (2.2) implies that the worker's acceptance strategy in an auction with $i + 1$ bidders has the reservation property. That is, the worker accepts the offer $w_{i+1}$ if and only if it is greater than the reservation wage $R_{i+1}$, where $R_{i+1}$ is such that $E_i(R_{i+1}) = U_{i+1}$ for $i < N$, and $E_i(R_{i+1}) = U_i$ for $i = N$. The vector $\sigma_w = (R_1, R_2, ..., R_{N+1})$ describes the strategy of the worker.

## 2.3. The Problem of the Firm

Consider first a firm that enters an auction as the sole bidder and offers the worker the wage $w$. If $w$ is weakly greater than the reservation wage $R_1$, the worker accepts the offer. In this case, the value to the firm from hiring the worker is $J_0(w)$. If $w$ is strictly smaller than the reservation wage $R_1$, the worker rejects the offer and continues searching. In this case, the value of the worker to the firm is $C_1^u$. Therefore, the value to the firm of entering an auction as the sole bidder is

$$M_1 = C_1^u + \max_w \left\{1(w \geq R_1)\left(J_0(w) - C_1^u\right)\right\}. \tag{2.3}$$

where $1(w \geq R_1)$ is an indicator function that takes the value of one if $w \geq R_1$ and zero otherwise.

Consider next a firm that enters into an auction with $i$ bidders, $i = 2, 3, ..., N + 1$, and offers the wage $w$ to the worker. Every one of the other bidders offers the wage $w_i$. Suppose that $w_i \geq R_i$. If $w$ is strictly greater than $w_i$, the worker accepts the offer of the firm. In this case, the value of the worker to the firm is $J_{i-1}(w)$. If $w$ equals $w_i$, the worker accepts the offer of the firm with probability $1/i$ and accepts the offer of one of the other bidders with probability $(1 - i)/i$. In this case the value of the worker to the firm is $J_{i-1}(w)/i + (i - 1)C_{i-1}^e/i$. Finally, if $w$

is strictly smaller than $w_i$, the worker rejects the offer of the firm. In this last case, the value of the worker to the firm is $C^e_{i-1}$. Therefore, if $w_i \geq R_i$, the value to the firm of entering an auction with $i$ bidders is

$$M_i = C^e_{i-1} + \max_w \left\{ [1(w > w_i) + 1(w = w_i)/i] \left( J_{i-1}(w) - C^e_{i-1} \right) \right\}. \tag{2.4}$$

Now, suppose that $w_i < R_i$. If $w$ is weakly greater than $R_i$, the worker accepts the offer of the firm. In this case, the value of the worker to the firm is $J_{i-1}(w)$. If $w$ is strictly smaller than $R_i$, the worker rejects the offer of the firm. In this case the value of the worker to the firm is $C^u_i$ if $i < N + 1$ and $C^u_{i-1}(i-1)/i$ if $i = N + 1$. Therefore, if $w_i < R_i$, the value to the firm of entering an auction with $i$ bidders contacts is

$$
\begin{aligned}
M_i &= C^u_i + \max_w \left\{ 1(w \geq R_i) \left( J_{i-1}(w) - C^u_i \right) \right\}, && \text{if } i < N + 1, \\
M_i &= C^u_{i-1}(i-1)/i + \max_w \left\{ 1(w \geq R_i) \left( J_{i-1}(w) - C^u_{i-1}(i-1)/i \right) \right\}, && \text{if } i = N + 1.
\end{aligned} \tag{2.5}
$$

Now, consider a firm that employs a worker with $i$ contacts, $i = 0, 1, ..., N$. The firm receives the flow profit $x - w$. At rate $\delta$, the worker is displaced from the firm. In this case, the value of the worker to the firm is zero. At rate $i\phi$, the worker loses one of the contacts. In this case, the value of the worker to the firm is $J_{i-1}(w)$. Therefore, the value to the firm of employing a worker with $i$ contacts is

$$r J_i(w) = x - w - \delta J_i(w) + i\phi \left[ J_{i-1}(w) - J_i(w) \right]. \tag{2.6}$$

From equation (2.6) for $i = 0$, it follows that the value function $J_0(w)$ is strictly decreasing with respect to $w$. In turn, from equation (2.6) for $i = 1, 2, ..., N$ and from the monotonicity of $J_{i-1}(w)$, it follows that the value function $J_i(w)$ is strictly decreasing with respect to $w$.

Finally, the value to the firm of being in contact with a worker who has $i - 1$ other contacts, $i = 1, 2, ..., N$, is such that

$$r C^u_i = \lambda(M_{i+1} - C^u_i) - \phi C^u_i + (i-1)\phi[C^u_{i-1} - C^u_i], \tag{2.7}$$

$$r C^e_i = \delta(M_i - C^e_i) - \phi C^e_i + (i-1)\phi \left[ C^e_{i-1} - C^e_i \right]. \tag{2.8}$$

## 2.4. Equilibrium

The previous paragraphs motivate the following definition of equilibrium.

DEFINITION 1: *A Symmetric Equilibrium is an acceptance strategy of the worker, $\sigma_w = (R_1, ..., R_{N+1})$, and a bidding strategy of the firm, $\sigma_f = (w_1, ...w_{N+1})$, such that: (i) For $i = 0, 1, ..., N - 1$,*

$E_i(R_{i+1}) = U_{i+1}$, and $E_N(R_{N+1}) = U_N$; (ii) For $i = 2, 3, ..., N+1$ and $w_i \geq R_i$, $w_i$ is the solution to the maximization problem (2.4); for $i = 2, 3, ..., N+1$ and $w_i < R_i$, $w_i$ is the solution to the maximization problem (2.5); and $w_1$ is the solution to the maximization problem (2.3).

## 3. Characterizing the $\delta > 0$ Case

It is useful to break down the characterization of equilibrium into two distinct cases determined by the job destruction rate $\delta$. The first case, in which the job destruction rate $\delta$ is strictly positive, is more challenging. We provide a complete characterization of equilibrium only for $N = 1$ and establish some general properties of equilibrium for $N \geq 2$. The second case, in which the job destruction rate $\delta$ equals zero, is easier. We fully characterize the equilibrium for any $N \geq 1$. This section focuses on the first case. The next section studies the second case.

### 3.1. Indifference of the Firm

Let $N = 1$ and suppose $w_i \geq R_i$, $i = 1$, 2. The value to the firm from meeting a worker without any contacts is

$$M_1 = C_1^u + \max_w \left\{ 1(w \geq R_1)(J_0(w) - C_1^u) \right\}. \tag{3.1}$$

If $J_0(R_1) < C_1^u$, the solution to the maximization problem in (3.1) is a wage offer, $w_1$, that is strictly smaller than the worker's reservation wage, $R_1$. If $J_0(R_1) \geq C_1^u$, the solution to the maximization problem in (3.1) is a wage offer, $w_1$, that equals the worker's reservation wage, $R_1$. From these observations, it follows that the conjecture $w_1 \geq R_1$ is satisfied if and only if $J_0(R_1) \geq C_1^u$. Moreover, whenever the conjecture $w_1 \geq R_1$ is satisfied, $w_1 = R_1$. That is, the outcome of an auction with one bidder is a wage that makes the worker indifferent between accepting and rejecting the job.

Given the conjecture $w_2 \geq R_2$, the value to the firm of entering an auction with two bidders is

$$M_2 = C_1^e + \max_w \left\{ [1(w > w_2) + 1(w = w_2)/2](J_1(w) - C_1^e) \right\}. \tag{3.2}$$

If $J_1(w_2) > C_1^e$, the solution to the maximization problem in (3.2) is a wage offer strictly greater than $w_2$. If $J_1(w_2) < C_1^e$, the solution is a wage offer strictly less than $w_2$. If $J_1(w_2) = C_1^e$, the solution is any wage offer less than or equal to $w_2$. In a Symmetric Equilibrium, the wage offer that solves the maximization problem (3.2) of one bidder equals the wage offer of the other bidder, $w_2$. Hence, in a Symmetric Equilibrium, $J_1(w_2) = C_1^e$ : the outcome of an auction between two firms is a wage that makes the firms indifferent between hiring and not hiring the worker. It

is important to notice that the value to the firm of not hiring the worker need not equal zero because, with some probability, the worker will lose his job and contact the firm again.

From equation (2.4) and the equilibrium condition $J_1(w_2) = C_1^e$, it follows that the value to the firm of participating in an auction with two bidders equals the value of being in contact with an employed worker, i.e. $M_2 = C_1^e$. It then follows from equation (2.7) and $M_2 = C_1^e$ that the value to the firm from being in contact with an unemployed worker is

$$C_1^{u} = \frac{\lambda}{r + \lambda + \phi} C_1^e. \tag{3.3}$$

From equation (2.3) and the conjecture that $w_1 \geq R_1$, it follows that the value to the firm from participating in an auction as the sole bidder is $M_1 = J_0(w_1)$. In turn, from equation (2.8) and $M_1 = J_0(w_1)$, it follows that the value to the firm from being in contact with an employed worker is

$$C_1^e = \frac{\delta}{r + \delta + \phi} M_1 = \frac{\delta (x - w_1)}{(r + \delta + \phi)(r + \delta)}, \tag{3.4}$$

where the second equality uses the fact that $M_1 = J_0(w_1)$ and $J_0(w_1) = (x - w_1)/(r + \delta)$.

Using (3.4) and the result that $J_1(w_2) = (x - w_2)/(r + \delta)$, we can rewrite the equilibrium condition $C_1^e = J_1(w_2)$ as

$$w_2 = \frac{r + \phi}{r + \delta + \phi} x + \frac{\delta}{r + \delta + \phi} w_1. \tag{3.5}$$

Condition (3.5) describes the wage $w_2$ that makes a firm indifferent between hiring and not hiring a worker in an auction with two bidders. Equation (3.5) implies that the wage $w_2$ is a weighted average of the productivity of an employed worker, $x$, and the equilibrium wage offered to a worker without other contacts, $w_1$. The weights on $x$ and $w_1$ are both positive. The weight on $x$ is positive because the value of hiring the worker is increasing in $x$. The weight on $w_1$ is positive because the value of not hiring the worker (and waiting for a job displacement and a subsequent wage of $w_1$) is decreasing in $w_1$. Naturally, the rate, $\phi$, at which a worker loses touch with a firm reduces the weight on $w_1$. The rate $\delta$ at which an employed worker loses the current job reduces the weight on $x$. In Figure 1, the green line represents the wages $(w_1, w_2)$ that satisfy the firm's indifference condition (3.5).

## 3.2. Indifference of the Worker

An unemployed worker who enters an auction with one bidder receives a wage offer that makes him indifferent between accepting (and becoming employed) and rejecting (and staying unemployed),

i.e. $E_0(w_1) = U_1$. Given this equilibrium property and equation (2.2), the lifetime utility of an unemployed worker without contacts is

$$U_0 = \frac{r}{r+\lambda}\frac{z}{r} + \frac{\lambda}{r+\lambda}U_1. \tag{3.6}$$

The value to the worker of being unemployed without a contact equals a weighted average of the present value of leisure, $z/r$, and the value of being unemployed with one contact, $U_1$. The rate, $\lambda$, at which a worker meets a firm decreases the weight on $z/r$ and increases the weight on $U_1$.

An unemployed worker who enters an auction with two bidders receives a wage offer, $w_2$, that is greater than $R_2$, according to our initial conjecture. Given this offer and equation (2.2), we find that the lifetime utility of an unemployed worker with one contact is

$$U_1 = \frac{r}{r+\lambda+\phi}\frac{z}{r} + \frac{\phi}{r+\lambda+\phi}U_0 + \frac{\lambda}{r+\lambda+\phi}E_1(w_2). \tag{3.7}$$

The value to the worker of being unemployed with one contact equals a weighted average between the present value of leisure, $z/r$, the value of being unemployed with no contacts, $U_0$, and the value of being employed with one contact at the wage $w_2$. The rate, $\phi$, at which a worker loses touch with a firm increases the weight on $U_0$ and decreases the weight on $z/r$ and $E_1(w_2)$. The rate, $\lambda$, at which a worker meets a firm increases the weight on $E_1(w_2)$ and decreases the weight on $z/r$ and $U_0$. Taken together, equations (3.7) and (3.6) identify two necessary and sufficient conditions for the value of unemployment to be greater than the present value of leisure. The first condition is that the probability that the worker finds himself in an auction with two bidders is positive, i.e. the rate, $\phi$, at which a worker loses a long-distance contact is finite. The second condition is that, when the worker finds himself in an auction with two bidders, he extracts some of the gains from trade, i.e. $E_1(w_2) > U_1$.

Given equation (2.1), the lifetime utility of a worker who is employed at the wage $w$ and has no contacts is

$$E_0(w) = \frac{r}{r+\delta}\frac{w}{r} + \frac{\delta}{r+\delta}U_0. \tag{3.8}$$

The value to the worker of being employed at the wage $w$ is the weighted average of the present value of the wage, $w/r$, and the value of being unemployed without contacts, $U_0$. The rate, $\delta$, at which a worker loses his job decreases the weight on $w/r$ and increases the weight on $U_0$.

Using equation (3.8) and (2.1), we further find that the lifetime utility of a worker who is employed at the wage $w_2$ and has one contact is

$$E_1(w_2) = \frac{r}{r+\delta}\frac{w_2}{r} + \frac{\delta}{r+\delta}\left[\frac{\phi}{r+\delta+\phi}U_0 + \frac{r+\delta}{r+\delta+\phi}U_1\right]. \tag{3.9}$$

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The value to the worker of being employed at a job that pays the wage $w_2$ is a weighted average of the present value of the wage $w_2$, and of the value to the worker of losing the job (the term in square brackets). The weight on the present value of the wage is decreasing in $\delta$. The value to the worker of losing the job is a weighted average of the value of being unemployed with no contacts, $U_0$, and the value of being unemployed with one contact, $U_1$. Taken together, equations (3.9) and (3.6) imply that $E_1(w_2) > U_1$ if and only if $w_2 > z$.

Solving the system of equations (3.6)-(3.9) for the values $U_0$, $U_1$, $E_0(w_1)$ and $E_1(w_2)$ gives the equilibrium condition $U_1 = E_0(w_1)$ as

$$w_1 = z + \alpha \left( w_2 - z \right), \tag{3.10}$$

where $\alpha$ is given by
$$\alpha = \frac{\lambda(r + \delta + \phi)(r + \delta + \lambda)}{(r + \delta)(r + \phi + \lambda)(r + \delta + \phi + \lambda) + \lambda^2 \phi}.$$

In Figure 1, the red line represents the wages $(w_1, w_2)$ that satisfy the worker's indifference condition (3.10). Condition (3.10) describes the wage $w_1$ that a firm needs to offer to an unemployed worker who does not have any other contact in order to make the worker indifferent between becoming employed and waiting for a second bidder. Notice that condition (3.10) implies a positive (and linear) relationship between $w_1$ and $w_2$, i.e. $\alpha > 0$. Intuitively, a higher $w_2$ increases the value to the worker from waiting for a second bidder and thereby raises the wage $w_1$ that makes him indifferent between waiting and becoming employed. Also notice that $w_1$ increases less than one-for-one with $w_2$, i.e. $\alpha < 1$. Due to time discounting, equal increases in $w_2$ and $w_1$ lead to a larger impact on the worker's value of becoming employed than on the worker's value of remaining unemployed. Finally, notice that $w_1$ is greater than $z$ if and only if $\phi$ is finite and $w_2$ is greater than $z$. The intuition for this property follows from the discussion above of the worker's value functions.

### 3.3. Equilibrium Outcomes

The solution to equilibrium conditions (3.5) and (3.10) is given by

$$
\begin{aligned}
w_1 &= \frac{(1 - \alpha)(r + \phi + \delta)}{r + \phi + (1 - \alpha)\delta} z + \frac{\alpha(r + \phi)}{r + \phi + (1 - \alpha)\delta} x, \\
w_2 &= \frac{(1 - \alpha)\delta}{r + \phi + (1 - \alpha)\delta} z + \frac{r + \phi}{r + \phi + (1 - \alpha)\delta} x.
\end{aligned}
\tag{3.11}
$$

The wage, $w_1$, offered by a firm to an unemployed worker without contacts is greater than the monopsony wage, $z$, and smaller than the wage, $w_2$, offered by a firm to an unemployed worker

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with another contact. The wage $w_2$ is smaller than the competitive wage, $x$. When the rate, $\lambda$, at which workers meet firms goes to infinity, the wages $w_1$ and $w_2$ converge to the competitive wage, $x$. When the rate, $\phi$, at which workers lose contact with firms goes to infinity, the wages $w_1$ and $w_2$ converge to the monopsony wage, $z$.

These findings are noteworthy and deserve further discussion. First, notice that a firm is willing to offer a wage $w_1$ greater than the monopsony wage $z$ to an unemployed worker without any contacts. Intuitively, even though the worker currently is not in contact with any other firm, the worker has the option to keep on searching and meet another firm. Because of this option, the firm faces (in a probabilistic sense) some competition for the worker and is willing to offer a wage greater than the monopsony wage. Second, notice that a firm offers strictly less than the competitive wage to a worker who has a contact with another firm. Intuitively, the firm that loses the auction has some positive probability of later on meeting the worker in a position of partial monopsony. Therefore, the wage that makes the firm indifferent between winning and losing the auction is lower than the competitive wage. In light of these arguments, it is straightforward to understand why $w_1$ and $w_2$ converge to the competitive wage when the meeting rate goes to infinity, and why $w_1$ and $w_2$ converge to the monopsony wage when the contact loss rate goes to infinity.

To complete the characterization of equilibrium, note that the worker's reservation wage in an auction with one bidder, $R_1$, is the unique solution for $w$ to the equation $E_0(w) = U_1$. Since the equilibrium wage $w_1$ is such that $E_0(w_1) = U_1$, it follows that $R_1$ equals $w_1$. The worker's reservation wage in an auction with two bidders, $R_2$, is the unique solution for $w$ to the equation $E_1(w) = U_1$. Having characterized the strategy of the firms, $(w_1, w_2)$, and the strategy of the workers, $(R_1, R_2)$, we can characterize the equilibrium behavior of the economy. Since $w_1$ is equal to $R_1$, an unemployed worker becomes employed as soon as he meets a firm, and remains employed until his job is exogenously destroyed. Since all workers accept the first offer they receive, all employed workers earn a wage of $w_1$.

Finally, given the equilibrium strategies $(w_1, w_2)$ and $(R_1, R_2)$, we can compute the values of the firms using the equilibrium conditions (3.1) – (3.4), as well as the values of the workers using the equilibrium conditions (3.6) – (3.9). Given these values it is straightforward to verify that the worker's reservation wage in an auction with two bidders, $R_2$, is smaller than the wage offer $w_2$. It is also straightforward to verify that the value to the firm of hiring an unemployed worker at the reservation wage $R_1$ is greater than the value of being in contact with him, i.e. $J_0(R_1) > C_1^u$. These findings prove that the conjectures made at the beginning of the sections are satisfied. Hence, the strategies $(w_1, w_2)$ and $(R_1, R_2)$, constitute a Symmetric Equilibrium of

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the model.

Theorem 1 summarizes these findings and rules out the existence of other equilibria.

THEOREM 1: *Let $\delta > 0$. For $N = 1$, there exists a unique Symmetric Equilibrium. (i) The equilibrium strategy of the firm is given by the wage offers $(w_1, w_2)$, where $w_1 < w_2$. (ii) The equilibrium wages $w_1$ and $w_2$ are strictly greater than the monopsony wage, $z$, and strictly smaller than the competitive wage, $x$. Moreover, $w_1 \to z$ and $w_2 \to z$ as $\phi \to \infty$; and $w_1 \to x$ and $w_2 \to x$ as $\lambda \to \infty$. (iii) The equilibrium strategy of the worker is given by the reservation wages $(R_1, R_2)$, where $R_1 = w_1$ and $R_2 < w_2$. (iv) All employed workers accept the first offer they receive and earn the wage $w_1$.*

PROOF: In Appendix. ■



Figure 1: N=1 Symmetric Equilibrium
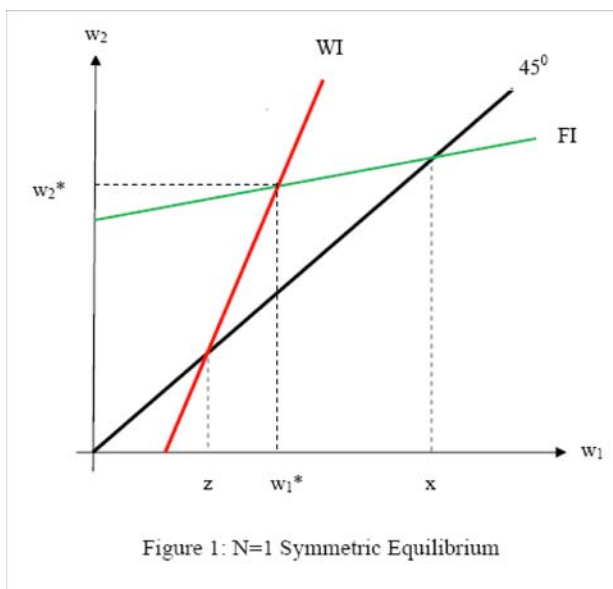
### 3.4. Equilibrium Properties for $N \geq 2$

The above exposition provides a complete characterization of equilibrium for $N = 1$. For the $N \geq 2$ case, it is substantially harder to solve the system of equations that implicitly define the worker's reservation wages and the firm's optimal bids. Suppose $N \geq 2$. If $w_1 \geq R_1$ and $w_2 \geq R_2$, the equilibrium wages are the same as in the $N = 1$. However, verifying the conjecture that $w_2 \geq R_2$, requires computing $w_3$, which in turn requires verifying conjectures about the sign of

the difference $w_i - R_i$, $i \geq 3$. This latter process becomes algebraically cumbersome as one has to solve the difference equations describing $U_i$ and $E_i(w)$.

Despite these obstacles, we can still establish that in any equilibrium with $N \geq 2$, the worker's expected value of unemployment is above $z/r$ and workers receive some wage offers that are strictly greater than the monopsony wage. The Diamond outcome, where firms extract the entire gains to trade, is not an equilibrium allocation when workers recall past contacts and firms observe the number of bidders in the auction.

THEOREM 2: *Let $\delta > 0$. For $N \geq 2$, any Symmetric Equilibrium has the following properties: (i) The lifetime utility of an unemployed worker is greater than the present value of leisure, $U_i > z/r$ for $i = 0, 1, ..., N$. (ii) There is a wage offer that is greater than the monopsony wage, $w_i > z$ for some $i = 1, 2, ..., N + 1$.*

PROOF: In Appendix. ∎

## 4. Characterizing the $\delta = 0$ Case

The previous section characterized equilibrium under the assumption that the job destruction rate is strictly positive. This section characterizes equilibrium under the simpler alternative that the job destruction rate equals zero.

### 4.1. Indifference of the Firm

Let $N \geq 1$ and suppose $w_i \geq R_i$, $i = 1, 2, ..., N + 1$. The value to the firm of entering an auction as the sole bidder is

$$M_1 = C_1^u + \max_w \left\{ 1(w \geq R_1)\left(J_0(w) - C_1^u\right) \right\}. \tag{4.1}$$

If $J_0(R_1) < C_1^u$, the solution to the maximization problem in (4.1) is a wage offer, $w_1$, that is strictly smaller than the worker's reservation wage, $R_1$. If $J_0(R_1) \geq C_1^u$, the solution to the maximization problem in (4.1) is a wage offer, $w_1$, that equals the worker's reservation wage, $R_1$. These observations imply that, in any equilibrium such that $w_1 \geq R_1$, $w_1 = R_1$. The outcome of an auction with one bidder is a wage that makes the worker indifferent between accepting and rejecting the job.

The value to the firm of entering an auction with multiple bidders is

$$M_i = C_{i-1}^e + \max_w \left\{ [1(w > w_i) + 1(w = w_i)/i]\left(J_{i-1}(w) - C_{i-1}^e\right) \right\}. \tag{4.2}$$

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If $J_{i-1}(w_i) > C_{i-1}^e$, the solution to the maximization problem in (4.2) is a wage offer strictly greater than $w_i$. If $J_{i-1}(w_i) < C_{i-1}^e$, the solution is a wage offer strictly smaller than $w_i$. If $J_{i-1}(w_i) = C_{i-1}^e$, the solution is any wage offer smaller or equal to $w_i$. In equilibrium, the wage offer that solves the maximization problem (3.2) of one bidder equals the wage offer of the other bidders, $w_i$. Hence, in equilibrium, $J_{i-1}(w_i) = C_{i-1}^e$. The outcome of an auction with multiple bidders is a wage that makes the firms indifferent between hiring and not hiring the worker.

Using equation (2.8), it can be shown that the value to the firm from being in contact with an employed worker equals zero, $C_i^e = 0$ for $i = 1, 2, ..., N$. This result is intuitive. Since the job destruction rate $\delta$ is zero, a worker will never look for another job once employed. The firm receives no payoff in the future from being in contact with an employed worker.

Using equation (2.4) and the equilibrium condition $J_{i-1}(w_i) = C_{i-1}^e$, it can be further shown that the value to the firm from entering an auction with multiple bidders equals zero, i.e. $M_i = 0$ for $i = 2, 3, ..., N + 1$. This result is likewise intuitive. In an equilibrium of an auction with multiple bidders, every firm is indifferent between hiring and not hiring the worker. Since there is no benefit from waiting and not hiring the worker, the firm again receives zero payoff from participating in an auction with multiple bidders.

Using equation (2.7) it can also be shown that the value to the firm from being in contact with an unemployed worker equals zero, $C_i^u = 0$ for $i = 1, 2, ..., N$. Intuitively, an unemployed worker recalls an old bidder only after meeting a new one. Since the firm receives zero benefit from participating in an auction with multiple bidders, there is no payoff from being in contact with an unemployed worker.

From equation (2.6), it follows that the value to the firm from employing a worker at the wage $w$ is $J_i(w) = (x - w)/r$. From the discussion in the above paragraph, the value to the firm from being in contact with an employed worker is $C_i^e = 0$. Hence, the firm's indifference condition $J_{i-1}(w_i) = C_{i-1}^e$ implies that $w_i = x$ for $i = 2, 3, ..., N + 1$. This result is again intuitive. The outcome of an auction with multiple bidders is a wage that makes all firms indifferent between hiring and not hiring the worker. Since the value of not hiring the worker is zero, the outcome of an auction with multiple bidders is a wage that exhausts the output of the match.

### 4.2. Indifference of the Worker

An unemployed worker who enters an auction with one bidder receives a wage offer that makes him indifferent between employment and unemployment, i.e. $E_0(w_1) = U_1$. In contrast, an unemployed worker who enters an auction with multiple bidders receives a wage offer that exhausts

the output of the match, i.e. $w_i = x$ for $i = 2, 3, ..., N+1$. Using these observations and equations (2.2) and (2.3), it can be established that the lifetime utility of a worker is

$$U_0 = \frac{r}{r+\lambda}\frac{z}{r} + \frac{\lambda}{r+\lambda}U_1,$$

$$U_i = \frac{r}{r+\lambda+i\phi}\frac{z}{r} + \frac{i\phi}{r+\lambda+i\phi}U_{i-1} + \frac{\lambda}{r+\lambda+i\phi}E_{i-1}(x), \quad i = 1, ..., N, \qquad (4.3)$$

$$E_i(w) = \frac{w}{r} \qquad\qquad\qquad\qquad\qquad\qquad i = 0, ..., N.$$

The first line in (4.3) states that the value to the worker of being unemployed without a contact is a weighted average of the present value of leisure, $z/r$, and the value of being unemployed with one contact, $U_1$. The second line states that the value to the worker of being unemployed with $i \geq 1$ contacts is a weighted average of the present value of leisure, $z/r$, the value of being unemployed with $i-1$ contacts, $U_{i-1}$, and the value of being employed at the wage $x$. The last line in (4.3) states that the value to the worker of being employed at the wage $w$ is equal to the present value of the wage, $w/r$.

Using (4.3), it can be shown that the worker's indifference condition $U_1 = E_1(w_1)$ implies

$$w_1 = z + \frac{\lambda(r+\phi)(r+\lambda)}{r(r+\phi+\lambda)^2 + \lambda^2\phi}\left(x - z\right). \qquad (4.4)$$

The wage, $w_1$, which is the outcome of an auction with only one bidder, is strictly greater than the monopsony wage, $z$, and strictly smaller than the competitive wage, $x$. It is also increasing with respect to the rate, $\lambda$, at which workers meet firms and converges to $x$ as $\lambda$ goes to infinity. It is decreasing with respect to the rate, $\phi$, at which workers lose contacts and it converges to $z$ as $\phi$ goes to infinity. The reasoning behind these results is the same as for the case of $\delta > 0$.

We stress that the wage $w_1$ does not depend on the number of contacts $N$ that the worker can keep. When $\delta = 0$, the firm gets no value from being in contact with an employed worker. Without a payoff down the road from staying in contact with an employed worker, two bidders are enough to drive the wage up to its competitive value.

To summarize, the equilibrium strategy of the firm is $\sigma_f = (w_1, w_2, ..., w_{N+1})$, where $w_1$ equals the right hand side of (4.4) and $w_2, w_3, ..., w_{N+1}$ all equal $x$. The equilibrium strategy of the worker is $\sigma_w = (R_1, R_2, ..., R_{N+1})$, where $R_1$ equals $w_1$, and $R_i$ equals the solution to the indifference condition $E_{i-1}(w_i) = U_i$ for $i = 2, 3, ..., N+1$. Since $w_1 = R_1$, every worker accepts the first offer received and all employed workers earn the wage $w_1$. Finally, it is straightforward to verify the conjecture $w_i \geq R_i$ for $i = 1, 2, ..., N+1$. Therefore, the strategies $(\sigma_w, \sigma_f)$ constitute a Symmetric Equilibrium. It is also straightforward to verify that there are no Symmetric Equilibria for which the condition $w_i \geq R_i$ does not hold.

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Theorem 3 summarizes these findings.

THEOREM 3: *Let $\delta = 0$. For all $N \geq 1$, there exists a unique Symmetric Equilibrium. (i) The equilibrium strategy of the firm is given by the wage offers $(w_1, w_2, ..., w_{N+1})$, where $w_1 < w_2 = ...w_{N+1}$. (ii) The equilibrium wage $w_1$ is strictly greater than the monopsony wage, $z$, and strictly smaller than the competitive wage, $x$. Moreover, $w_1 \to z$ as $\phi \to \infty$; and $w_1 \to x$ as $\lambda \to \infty$. (iii) The equilibrium wages $w_2, w_3, ..., w_{N+1}$ are equal to the competitive wage $x$. (iv) The equilibrium strategy of the worker is given by the reservation wages $(R_1, R_2, ..., R_{N+1})$, where $R_1 = w_1$ and $R_i < w_i$ for $i = 2, 3, ..., N + 1$. (v) All employed workers accept the first offer they receive and earn the wage $w_1$.*

## 5. Asymmetric Information

Sections 3 and 4 characterized the labor market equilibrium under the assumption that firms can perfectly observe the number of bidders participating in the auction for a worker. This assumption is reasonable for some labor markets, but not for all. For instance, this assumption is probably reasonable for academics, but not for clerks. For this reason, this section characterizes an equilibrium under the alternative assumption that firms cannot observe the number of bidders who participate in a labor auction. For ease of exposition the analysis assumes that a worker can keep at most one long-distance contact, i.e. $N = 1$, and that the exogenous job destruction rate equals zero, i.e. $\delta = 0$.

### 5.1. Strategies

Let $\sigma_w = (R_1, R_2, \varphi)$ be the strategy of a worker where $R_1$ denotes the worker's reservation wage in an auction with one bidder; $R_2$ denotes the worker's reservation wage in an auction with two bidders; and $\varphi$ denotes the Poisson rate at which an unemployed worker with one contact calls for an auction before meeting a second firm. In general, the worker's reservation strategy could depend on the entire history of the relationship with a bidder, e.g., the previous offers that the worker received from that bidder along with the time that has elapsed between any two auctions. Here, we restrict attention to simple equilibria in which the worker's reservation wages only depend on the number of bidders.

Let $\sigma_f = (w_n, w_r)$ be the strategy of a firm where $w_n$ denotes the firm's bid in an auction for a worker that the firm has never met before whereas the second element $w_r$ denotes the firm's bid in an auction for a worker that the firm has met and recalled from the past. Like the worker's strategy, the firm's bidding strategy could depend on the entire history of its relationship with

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the worker. Again, we restrict attention to simple firm strategies with bids that depend only on whether or not the firm met the worker before.

Finally, let $\pi = (\pi_n, \pi_r)$ denote the beliefs of the firm. In particular, in an auction for a worker that it has never met before, the firm believes that there is another bidder with probability $\pi_n$, and that there is not another bidder with probability $1 - \pi_n$. In an auction for a worker that it has met in the past, the firm believes that there is another bidder with probability $\pi_r$, and that there is not another bidder with probability $1 - \pi_r$. The strategies $\sigma_w$ and $\sigma_f$ together with the beliefs $\pi$ constitute a Perfect Bayesian Equilibrium under the following conditions:

DEFINITION 2: *The strategies $\sigma_w$ and $\sigma_f$ and the beliefs $\pi$ constitute a Perfect Bayesian Equilibrium if: (i) $\sigma_w$ is the worker's optimal strategy, given that the firm's strategy is $\sigma_f$; (ii) $\sigma_f$ is the firm's optimal strategy, given that the workers' strategy is $\sigma_w$ and the firm's beliefs are $\pi$; (iii) $\pi$ is derived from the strategies $\sigma_w$ and $\sigma_f$ through Bayes' rule (whenever possible).*

### 5.2. Existence

Let the strategy of the worker be $\sigma_w = (R_1^*, R_2^*, \varphi^*) = (z, z, 0)$. In an auction with one bidder, the worker accepts any wage offer greater than the value of leisure, $z$. In an auction with two bidders, the worker accepts the highest wage offer, as long as it is greater than $z$. If the two offers are identical and acceptable, assume that the worker accepts the offer of the firm that was met first, i.e. the recalled firm.

Let the strategy of the firm be $\sigma_f = (w_n^*, w_r^*) = (z, z)$. The firm bids $z$ both in an auction for a worker that it has never met before and in an auction for a worker that it has previously met. Let the firm's beliefs be $\pi = (\pi_n^*, \pi_r^*) = (0, 1)$. In an auction for a worker that it has never met before, the firm believes itself to be the sole bidder. In an auction for a worker that it has met in the past, the firm believes that there is a second bidder. In what follows, we establish that the strategies $(R_1^*, R_2^*, \varphi^*)$ and $(w_n^*, w_r^*)$, and the beliefs $(\pi_n^*, \pi_r^*)$ constitute a Perfect Bayesian Equilibrium.

To prove that $\sigma_w = (R_1^*, R_2^*, \varphi^*) = (z, z, 0)$ is an optimal strategy for the worker, consider an unemployed worker who does not have any contacts. The worker receives the flow utility $z$. At rate $\lambda$, the worker meets a firm for the first time and receives the wage offer $w_n^*$. If the worker accepts the offer, the worker becomes employed. If the worker rejects the offer, the worker becomes unemployed with one contact. Therefore, the worker's lifetime utility, $U_0$, is given by

$$rU_0 = z + \lambda[\max\{E_1(w_n^*), U_1\} - U_0]. \tag{5.1}$$

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Consider next an unemployed worker with one contact. The worker has the option to recall the contact and ask for an offer. If the worker exercises this option and accepts the subsequent offer, the worker becomes employed at the wage $w_r^*$. If the worker does not exercise this option, the worker receives the flow utility $z$. At rate $\lambda$, the worker meets a second firm and is offered the wage $w_n^*$ by the just met firm and the wage $w_r^*$ by the recalled firm. If the worker accepts one of these offers, the worker becomes employed. If the worker rejects these offers, the worker remains unemployed and continues searching with one contact. At rate $\phi$, the worker loses touch with the contact. Therefore, the worker's lifetime utility, $U_1$, is such that

$$rU_1 = \max\{rE_1(w_r^*), z + \lambda[\max\{E_1(w_r^*), E_1(w_n^*), U_1\} - U_1] + \phi[U_0 - U_1]\}. \qquad (5.2)$$

Finally, consider a worker employed at the wage $w$ who is in contact with $i$ other firms, $i = 0, 1$. Given the zero rate of job destruction, it follows immediately that the lifetime utility of this worker equals the present value of the current wage, i.e. $E_i(w) = w/r$ for $i = 0, 1$.

From equations (5.1), (5.2) and $w_r^* = w_n^* = z$, it follows that the lifetime utility of an unemployed worker equals the present value of leisure, i.e. $U_0 = U_1 = z/r$. From $E_0(w) = E_1(w) = w/r$ and $U_1 = z/r$, it then follows that (a) the reservation wage, $R_1^*$, of the worker in an auction with one bidder is $z$; (b) the reservation wage, $R_2^*$, of the worker in an auction with two bidders is $z$; (c) an unemployed worker with one contact does not call an auction before meeting another firm. This establishes that $(R_1^*, R_2^*, \varphi^*) = (z, z, 0)$ is an optimal strategy of the worker.

To prove that $\sigma_f = (w_n^*, w_r^*) = (z, z)$ is an optimal strategy for a firm, consider a potential employer that enters an auction without observing the number of participating bidders. In an auction for a worker that it has never met before, the firm believes that there is another bidder with probability $\pi_n^*$, and that there is not a second bidder with probability $1 - \pi_n^*$. Suppose the firm offers the wage $w$. If there happens to be a second bidder, the worker accepts $w$ if this offer is weakly greater the reservation wage $R_2^*$ and strictly greater than the offer of the second firm, $w_r^*$. If the two bids are identical and above $R_2^*$, the worker breaks a tie in favor of the first met firm, i.e. the recalled bidder. If there is no second bidder, the worker accepts $w$ as long as it is greater than the reservation wage, $R_1^*$. Therefore, the value to the firm from entering an auction for a newly met worker is

$$
\begin{aligned}
M_n = \max_w \quad & \{\pi_n^*[1(w > w_r^*)1(w \geq R_2^*)(J_1(w) - C_1^e) + C_1^e] + \\
& + (1 - \pi_n^*)[1(w \geq R_1^*)(J_0(w) - C_1^u) + C_1^u]\}.
\end{aligned} \qquad (5.3)
$$

In an auction for a previously-met worker, the firm believes that there is a second bidder with probability $\pi_r^*$, and that there is no other bidder with probability $1 - \pi_r^*$. Suppose the firm offers

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the wage $w$. If there happens to be a second bidder, the worker accepts $w$ if it is weakly greater than both the reservation wage $R_2^*$ and the offer of the just met second firm, $w_n^*$. If there is only one bidder, the worker accepts $w$ as long as it is weakly greater than the reservation wage, $R_1^*$. Therefore, the value to the firm from entering an auction for a previously-met worker is

$$
\begin{aligned}
M_r = \max_w \quad & \{\pi_r^*[1(w \geq w_n^*)1(w \geq R_2^*)(J_1(w) - C_1^e) + C_1^e] + \\
& + (1 - \pi_r^*)[1(w \geq R_1^*)(J_0(w) - C_1^u) + C_1^u]\}.
\end{aligned}
\tag{5.4}
$$

Since job displacement does not occur, the value to the firm of employing a worker at the wage $w$ is $J_1(w) = (x - w)/r$, $i = 0, 1$; the value to the firm of being in contact with an unemployed worker is $C_1^u = \lambda J_1(w_r^*)/(r + \lambda)$; and the value to the firm of being in contact with an employed worker is $C_1^e = 0$.

From $R_1^* = z$ and $w_r^* = z$, it follows that $J_0(R_1^*) > C_1^u$. From $\pi_n^* = 0$ and $J_0(R_1^*) > C_1^u$, it then follows that the solution to the maximization problem in (5.3) is a wage offer $w_n^*$ that equals the value of leisure, $z$. Further, from $R_2^* = z$ and $w_r^* = z$, it follows that $J_1(R_2^*) > C_1^e$. From $\pi_r^* = 1$ and $J_1(R_2^*) > C_1^e$, it follows that the solution to the maximization problem in (5.4) is a wage offer $w_r^*$ that equals the value of leisure, $z$. This establishes that $(w_n^*, w_r^*) = (z, z)$ is an optimal strategy for the firm.

To finish the proof, we verify that the beliefs $(\pi_n^*, \pi_r^*) = (0, 1)$ are derived from the equilibrium strategies through Bayes' rule (whenever possible). In equilibrium, every worker accepts the first offer received. Hence, if a firm meets a worker for the first time, the probability that this worker is in contact with a second firm is zero. Second, in equilibrium, a worker accepts any offer greater than $z$. Hence, if a firm meets a worker for the $n$-th time, $n = 2, 3, \ldots$, and one of its previous offers was greater than $z$, the probability that the worker is in contact with another firm is not pinned down by Bayes' rule. Third, in equilibrium, an unemployed worker with one contact does not call for an auction until the worker meets another firm. Hence, if a firm meets a worker for the $n$-th time and all of its previous offers were smaller than $z$, the probability that the worker is in contact with another firm is one. This establishes that the beliefs $(\pi_n^*, \pi_r^*) = (0, 1)$ are derived from the equilibrium strategies, and leads to the following theorem.

THEOREM 4 (Recall with Asymmetric Information): *Assume that firms cannot observe the number of bidders who participate in the auction for a worker. (i) The strategy of the firm $(w_n^*, w_r^*) = (z, z)$; the strategy of the worker $(R_1^*, R_2^*, \varphi^*) = (z, z, 0)$; and the beliefs $(\pi_n^*, \pi_r^*) = (0, 1)$ constitute a Perfect Bayesian Equilibrium. (ii) In the Perfect Bayesian Equilibrium described in part (i), all workers accept the first offer they receive and earn a wage, $w_n^*$, equal to the value of leisure, z.*

Theorem 4 implies that, when firms cannot observe the number of bidders participating in an auction, there exists an equilibrium in which workers do not appropriate any of the gains from trade. In other words, there is an equilibrium that generates the same allocation as in Diamond (1971). The intuition is clear. In the equilibrium $(\sigma_w^*, \sigma_f^*, \pi^*)$, every worker accepts the first wage offer received. Therefore, when a firm meets a worker for the first time, it reasonably believes itself to be the sole bidder in which case it is optimal to offer the monopsony wage. Moreover, when a firm meets a worker for the second time, the firm knows that the other bidder believes itself to be a monopsonist. Therefore, even in this case, the previously contacted firm finds it optimal, given that the tie breaking rule goes to the recalled firm, to offer the monopsony wage. These observations taken together imply that a worker will never get an offer higher than the monopsony wage. Hence, the monopsony wage equals the value of leisure.

It is important to stress that Theorem 4 does not rule out the existence of equilibria in which the workers appropriate some of the gains from trade. The argument above does, however, suggest that, in these other equilibria, there should be a strictly positive measure of workers who reject the first offer they receive. In these other equilibria, when a firm meets a worker for the first time, it will be uncertain about its market power and the worker will be able to extract some informational rents, as in Albrecht and Axel (1984), Burdett and Judd (1983), and Burdett and Mortensen (1998). We leave the proof of existence (or the proof of non-existence) of these equilibria for future research.

## 6. Conclusion

Economic rents arise when potential trading partners bilaterally meet each other in the presence of search frictions. The way in which traders allocate these rents has profound consequences on economic outcomes. For example, Diamond (1971) demonstrates that in a model with homogeneous goods and price setting, the selling firm obtains all of the gains to trade. In this model - one that on the surface appears to be a natural framework - buyers have no incentive to participate. Market breakdown or unravelling can occur.

This paper revisits wage setting in search models with take-it-or-leave-it offers.[4] The innovation here is to allow job seekers to recall (at least to some extent) past encounters if they

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[4]The aim of the paper is to demonstrate the importance of recall in models of job search, not to provide a resolution to the Diamond paradox. The literature has identified several resolutions that do not rely on recall. For example, Mortensen and Pissarides (1999a, 1999b) assume wages are the outcome of a bargain between firms and workers. Moen (1997), Burdett, Shi and Wright (2001), Menzio and Shi (2009) solve the paradox by assuming that search is directed rather than random. Albrecht and Axell (1984) solve the paradox by introducing heterogeneity in the workers' value of leisure.

decide to continue searching for employment opportunities. Provided recalled bidders can update their offers to take into account the number of employers pursuing the worker, such memory fundamentally alters wage determination.

The Diamond (1971) search model and those that followed specified that if the traders fail to agree to terms, potential partners lose all contact. Imposing no-recall is inconsequential if previous bids are fixed. When summoned again, inferior offers do not become acceptable given stationarity. On the other hand, with offer updating as well as recall, a job seeker can potentially improve the outcome of the auction. A worker who can recall bidders in order to engage them in Bertrand competition has a distinct advantage over workers without such attachments. A worker with two competing suitors engages them in a bidding war that results in higher wages. Firms avoid this outcome by offering enough to the worker when the worker has no other bidders. With recall, firms who encounter job seekers have to pay more than the value of unattached search to secure the worker's services. The ability to recall previously eligible bidders provides the worker with a counter weight to the firm's advantages from wage setting as highlighted by the well-known Diamond paradox. Rent sharing occurs.

Unlike other models with heterogeneous agents, the results are not sensitive to unraveling with small search costs (Albrecht and Axell, 1984; Gaumont, Schindler and Wright, 2005). The results are, however, sensitive to information verification. Bidders must recognize the existence of other bidders in the auction. Without knowing whether or not a worker truly has an alternative suitor, employers must form beliefs about their competition. If a potential employer believes that it is the lone bidder - a plausible conjecture in equilibrium - when it first meets a worker, wages revert to the monopsony outcome.

# References

[1] Albrecht, J. and B. Axell (1984) "An Equilibrium Model of Search Unemployment", *Journal of Political Economy,* 92: 824-840.

[2] Burdett, K. and K. Judd (1983) "Equilibrium Price Dispersion", *Econometrica*, 51: 955-970.

[3] Burdett, K. and D. Mortensen (1998) "Wage Differentials, Employer Size and Unemployment", *International Economic Review*, 39: 257-273.

[4] Burdett, K., S. Shi and R. Wright (2001) "Pricing and Matching with Frictions", *Journal of Political Economy,* 109: 1060-1085

[5] Butters, G., (1977) "Equilibrium Distributions of Sales and Advertising Prices", *Review of Economic Studies,* 44: 465-491.

[6] Diamond, P. (1971) "A Model of Price Adjustment", *Journal of Economic Theory*, 3:156-168.

[7] Gaumont, D., M. Schindler and R. Wright (2006) "Alternative Theories of Wage Dispersion", *European Economic Review*, 50: 831-848.

[8] McCall, J., (1970), "Economics of information and job search", *Quarterly Journal of Economics* 84: 113-126.

[9] Menzio, G. and S. Shi (2009) "Efficient Search on the Job and the Business Cycle", NBER Working Paper 14905.

[10] Moen, E. (1997), "Competitive Search Equilibrium", *Journal of Political Economy,* 105: 385-411.

[11] Mortensen, D. and C. Pissarides (1999a), "New Developments in Models of Search in the Labor Market", in *Handbook of Labor Economics*, O. Ashenfelter and D. Card (eds.), Amsterdam: North Holland.

[12] Mortensen, D. and C. Pissarides (1999b), "Job Reallocation, Employment Fluctuations, and Unemployment Differences", in M. Woodford and J. Taylor (eds.) *Handbook of Macroeconomics*, Amsterdam, North-Holland.

[13] Rogerson, R., R. Shimer and R. Wright, (2005), "Search-theoretic Models of the Labor Market: A Survey" *Journal of Economic Literature*, 43: 959-988.

[14] Taylor, C., (1995), "The Long Side of the Market and the Short End of the Stick: Bargaining Power and Price Formation in Buyers', Sellers' and Balanced Markets", *Quarterly Journal of Economics,* 110: 837-855.

# Appendix

## A. Proof of Theorem 1

The main text established the existence of a unique Symmetric Equilibrium such that $w_1 \geq R_1$ and $w_2 \geq R_2$. Moreover, the main text also proved that this equilibrium has properties (i) – (iv) listed in Theorem 1. It remains to prove that there are no Symmetric Equilibria such that either $w_1 < R_1$ or $w_2 < R_2$.

Suppose that there exists a Symmetric Equilibrium such that $w_1 < R_1$ and $w_2 < R_2$. From the inequalities $w_1 < R_1$ and $w_2 < R_2$ and the fact that the value functions $E_0(w)$ and $E_1(w)$ are strictly increasing in $w$, it follows that $E_0(w_1) < E_0(R_1) = U_1$ and $E_1(w_2) < E_1(R_2) = U_1$. The latter inequalities and equation (2.2) imply that $U_i = z/r$ for $i = 0, 1$. Finally, equation (2.1) and $U_1 = z/r$ implies that $E_0(w) = (rw + \delta z)/[r(r + \delta)]$. Hence, the solution to the reservation wage equation $E_0(w) = U_1$ is $R_1 = z$.

Equations (2.3), (2.5) and the inequalities $w_1 < R_1$ and $w_2 < R_2$ imply that $M_1 = C_1^u$ and $M_2 = C_1^u/2$. Equation (2.7) and $M_2 = C_1^u/2$ imply $C_1^u = 0$. Finally, equation (2.6), $C_1^u = 0$ and $R_1 = z$ imply that $J_0(R_1) - C_1^u > 0$. Hence, the solution to the wage offer problem in (2.3) is $w_1 = R_1$. Since $w_1 = R_1$ contradicts the assumption $w_1 < R_1$, there are no Symmetric Equilibria in which $w_1 < R_1$ and $w_2 < R_2$. A similar proof by contradiction establishes that there are no Symmetric Equilibria such that $w_1 < R_1$ and $w_2 \geq R_2$, and hence that there are no Symmetric Equilibria such that $w_1 \geq R_1$ and $w_2 < R_2$.

## B. Proof of Theorem 2

The proof of the theorem is divided into three claims.

CLAIM 1: In any Symmetric Equilibrium, $U_i \geq z/r$ for $i = 0, 1, ...N$.
PROOF: On the way to a contradiction, suppose that $U_0 < z/r$. When $U_0 < z/r$, equation (2.2) implies that $U_0 > U_1 > ...U_N$ and $\max\{E_N(w_{N+1}), U_N\} < U_N$. Since the latter inequality fails, it must be the case that $U_0 \geq z/r$. Using a similar argument, we can prove $U_i \geq z/r$ for $i = 1, 2, ...N$.

CLAIM 2: In any Symmetric Equilibrium, $U_0 > z/r$.
PROOF: On the way to a contradiction, suppose that $U_0 = z/r$. When $U_0 = z/r$, equation (2.2) implies that $U_i = z/r$ and $E_i(w_{i+1}) \leq z/r$ for $i = 0, 1, ...N$. Moreover, equation (2.1) implies $E_i(w) = (w + \delta z/r)/(r + \delta)$ for $i = 1, 2, ...N$. Hence, the solution to the reservation wage

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equation $E_{i-1}(w) = U_i$ is $R_i = z$ for $i = 1, 2, ...N + 1$. Since $R_i = z$, equations (2.3)-(2.8) imply $M_{i+1} \leq (x - z)/(r + \delta)$, $C_i^u < (x - z)/(r + \delta)$ and $C_i^e < (x - z)/(r + \delta)$ for $i = 1, 2, ...N$. Now, notice that the equilibrium bid $w_2$ is such that $J_1(w_2) = C_1^e$. Since $J_1(w) = (x - w)/(r + \delta)$ and $C_i^e < (x - z)/(r + \delta)$, $w_2$ is strictly greater than $z$ and $E_1(w_2) > z/r$. This contradicts one of the implications of equation (2.2).

CLAIM 3: In any Symmetric Equilibrium, $w_i > z$ for some $i = 1, 2, ...N + 1$.

PROOF: On the way to a contradiction, suppose that $w_i \leq z$ for $i = 1, 2, ...N + 1$. In this case, equation (2.1) and (2.2) imply $U_i = z/r$ for $i = 0, 1, ...N$. This contradicts Claim 2.