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Environmental tax reform with irreversible investment, technological progress and unemployment

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March 1997

ENVIRONMENTAL TAX REFORM WITH IRREVERSIBLE INVESTMENT, TECHNOLOGICAL PROGRESS AND UNEMPLOYMENT

by

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Abstract

This paper analyzes if unemployment can be reduced through labor tax cuts that are financed in a revenue neutral way through energy tax increases. In contrast to other papers on this topic we consider investment behavior of firms in energy saving technologies, irreversibilities, embodied technological progress and involuntary unemployment. Arguments are presented that reducing the sunk costs instead of the labor tax seems to be the better instrument to reduce energy input and unemployment since this puts more pressure on firms that are using old technologies to adopt a more efficient energy saving technology.

JEL Classification-Code: E60, H32, J3, J5

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List of Symbols

Uppercase Roman letters

$A(t)$ output of a production unit; represents also the technology that was the leading technology at time t

B constant technology parameter

C updating cost for a production unit

$E(t)$ energy demand of the economy

$\tilde{E}(t)$ energy demand of a production unit

H number of hires

I installation cost for energy saving technology

J value of a job to a firm

K setup cost for a new production unit

L labor force

$Q(t)$ output of the economy

S social surplus that is generated by a match between a worker and a firm

T scrapping age of an old technology

T_d destruction age of an old production unit

T_i age, when old production unit is updated with latest technology

U value of unemployment to a worker

W value of a job to a worker

Lowercase Roman letters

a age of a production unit

c_0 constant

e Euler's e

f cross section density of production units

g growth rate of technological progress , equal to steady state growth rate of energy price

$\tilde{g}$ government expenditure deflated with the growth rate of technological progress

k number of events in a Poisson distribution

m arrival parameter of a Poisson distribution that gives the probability that an unemployed worker finds a job

n number of times that an old production unit has been updated

q arrival parameter of a Poisson distribution that gives the probability that a vacant job is filled with a worker

r interest rate

s index for time

t time

u unemployment rate

v rate of vacant jobs

w wage

x ratio of hired to unemployed workers

Greek letters

β bargaining power of the worker

$\tilde{\beta}$ short writing for $\frac{(1+\tau_L)\beta}{(1+\beta\tau_L)}$

δ	arrival parameter of a Poisson distribution that gives the probability that a production unit is destroyed by an exogenous shock
λ	arrival parameter of Poisson distribution
π	Poisson distribution
$\mu(t)$	energy saving technology installed at date t
σ	elasticity of energy demand with respect to energy saving technology
τ_L	labor tax reate
τ_E	energy tax rate
θ	ratio of vacant jobs to unemployed workers

1. Introduction

Is it possible to improve environmental quality and, at the same time, reduce unemployment with an environmental tax reform? This is the hope of the supporters of an environmental tax reform. Since raising a tax on polluting activities creates income for the government, the government has the possibility to lower other taxes. Therefore, given the political importance of the unemployment problem, it is quite natural to ask whether this additional income can be used to create better conditions for the production factor labor. In Germany the public and academic debate focuses on the question whether it is possible to reduce unemployment through adjusted cuts in firms contribution to social insurances that work like a tax on labor. This paper tries to analyze if unemployment can be reduced through labor tax cuts that are financed in a revenue neutral way through energy tax increases.

Ecological tax reforms that improve next to environmental quality also a second welfare indicator are said to yield a double dividend. The double dividend gains importance once it is realized that the magnitude of environmental benefits is largely unknown due to missing markets for environmental quality (See Goulder (1995)). Since raising an environmental tax might cause costs in form of the reduction of other welfare indicators there is no guarantee that the environmental benefits outweigh the economic cost it creates. Therefore, the net welfare effect of an environmental tax reform might be negative even if environmental quality is improved. Therefore, in order to guarantee positive net welfare effects, an environmental tax reform must yield a double dividend. Then there is also no need for a country to coordinate its policy with other countries, since the welfare benefits of the environmental tax reform are guaranteed to supersede the cost.

The question of environmental tax reforms and employment has been addressed in a number of papers. In Bovenberg, de Mooij (1994), and Bovenberg, van der Ploeg (1994) employment always decreases in a revenue neutral environmental tax reform that raises an environmental tax and reduces the tax on labor in exchange. Koskela, Schöb (1996) analyze a revenue neutral environmental tax reform and its consequences on unemployment. However, in these papers pollution is caused by households and not by a production sector, as is more common in many cases. Bovenberg, de Mooij (1995) and Bovenberg, van der Ploeg (1995) analyze environmental tax reforms in a framework where pollution is caused by the supply side of the economy. A shortcoming of these papers is that investment behavior of firms in energy saving technologies is completely neglected, although an increasing number of studies [see e.g. Enquete Kommission des Deutschen Bundestages zum „Schutz der Erdatmosphäre" (1995), Frisch (1991), or Kreuzberg (1996)] point at the important relationship between a firm's energy demand and its investment in energy saving technologies. Especially, Klodt (1994) found that industrial energy saving in West Germany after the oil price shocks was mainly achieved by improved technology and not by structural shifts. Another extreme assumption of the mentioned papers is that investment behavior of firms is a reversible decision. Physical capital can be costlessly shifted from one application to another. In many cases this is unrealistic. Typically investments are irreversible. Once physical capital is locked into an application it cannot be costlessly shifted to another application. This has important consequences for the effects of an environmental tax reform. Once an investment into a new technology has occurred a firm is unable to reduce its energy input. In order to significantly change energy input the firm has to invest into new, more energy efficient technology. But since an investment is connected to sunk costs a firm might consider it optimal not to react to a price change right away, since reaction means new sunk costs, which might supersede the gains from an investment. On the other hand an installed capital unit or technology is not used for an infinite time. Due to technological progress and depreciation there is a continuous process of sorting out technologies that are relatively unproductive and hence, cannot pay equilibrium factor prices. Therefore, there comes the day for every production unit when it is too far away from the productivity frontier. Then it has to be decided whether this production unit is updated with the latest technology and new sunk costs are incurred or whether updating is too expensive and it is decided to scrap the production unit. The irreversibility of the investment decision and the ensuing sunk costs play a decisive role for unemployment in our model. Following Caballero, Hammour (1994, 1996) the assumption that capital cannot be shifted costlessly between applications gives rise to a bilateral bargaining situation between firm and worker that raises the wage above its efficient level and is responsible for unemployment.

Regarding the updating decision of production units, modern economic theory distinguishes two points of view. In schumpeterian theories technological progress is destructive. The arrival of a new technology destroys an old production unit, but creates a new one. According to this view it is too expensive to update an old production unit with the latest technology. As a consequence technological progress leads to a permanent reallocation of labor. Based on empirical evidence by Davis, Haltiwanger (1990) recent papers of Aghion, Howitt (1994), Caballero, Hammour (1994), and Mortensen, Pissarides (1995) introduce a friction in form of a labor matching function into this process of labor reallocation, such that the process of creative destruction leads to unemployment.[1] Mortensen, Pissarides (1995) also point at another popular view of technological progress in modern economic theory. Solovian or neoclassical technological progress does not lead to creative destruction, because existing production units are continually updated with the latest technology. This is only possible if the updating cost is sufficiently small. As a consequence of this assumption technological progress does not lead to labor reallocation and technological progress does not have this negative consequence for unemployment.

The aspect of whether to update or not to update existing production units with the latest technology and the significance of sunk costs is an important characteristic of investment decisions that has to be considered by environmental policymakers, who want to use environmental tax reforms to fight unemployment. Environmental policy

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1 From a theoretical point of view these models are an extension of Pissarides (1990).

has the aim to reduce the energy input of firms and to induce an investment decision of firms into energy saving technologies. Investment into energy saving technologies increases the sunk cost that arises, when a new job is created or updated. Therefore, environmental policy has important consequences for the firm decision to create a new job and to update old jobs. In this paper, we concentrate on the effects of an environmental tax reform under explicit consideration of the investment decision of firms. The environmental tax reform is modeled as raising the energy tax and an adjusted change of a labor tax so that the government budget is balanced. We consider two benchmark models that follow from the prevailing division of technological progress in economic theory in schumpeterian and neoclassical technological progress. This affects the investment decision of firms and the level of unemployment.

In this paper it is shown that a revenue neutral environmental tax reform that uses the additional revenue resulting from an increase of the energy tax to cut down the labor tax can possibly reduce unemployment if the cost of updating old jobs with a new technology is sufficiently low and the tax on labor is not too high. The environmental tax reform achieves the opposite if the updating cost is too high, because in this case the environmental tax reform will lead to job destruction. Therefore, reducing the labor tax in exchange for raising the energy tax might not have the desired effects. Also we find in numerical examples that the effects of environmental tax reforms as described above on unemployment turn out to be small. We draw the conclusion that if the government wants firms to change their production technology and at the same time increase employment the government might do better if it facilitates the implementation of new technologies for firms.

The rest of the paper is structured as follows: In section 2 we introduce the structure of the economy. In section 3 we introduce the first benchmark model with schumpeterian technological progress. It is assumed that the cost of updating old technologies is too high such that a firm always exits from the market when it is too far away from the productivity frontier and hence, unable to pay the equilibrium factor prices. Then we analyze the effects of tax changes and of a revenue neutral environmental tax reform. It is shown that policy makers cannot expect to reap a double dividend when the revenue from an energy tax increase is used to cut the labor tax. The proposed environmental tax reform leads to more unemployment since the rents from creating jobs are diminished. In section 4 we introduce our second benchmark model with neoclassical technological progress. The updating cost is sufficiently low such that technological progress does not lead to job destruction. In this case the government can possibly reap a double dividend. In all models only steady states are compared and the transitional dynamics are completely neglected. In section 5 we analyze possible short term effects of an environmental tax reform. Section 6 concludes.

2. The economy

The small open economy trades two goods with the rest of the world at world market prices: a produced good whose aggregate output at time t is $Q(t)$, and energy. Consequently, the prices of energy and the aggregate output are exogenous. The productive structure is made up of many production units that combine in fixed proportions a unit of labor and an efficiency unit of energy. Exogenous technological progress is embodied in production units and drives the continuous process of their creation and destruction. A production unit that was created or updated at time t produces $A(t)$ units of output. $A(t)$ represents also the technology that was the leading technology at time t. It is assumed that the leading technology grows at the exogenous rate $g > 0$, which is also the steady state growth rate of the model.

The efficiency unit of energy is related to the installed energy saving technology $\mu(t)$ in the following way: $\tilde{E}(t+s)\dfrac{\mu(t)^{\sigma}}{B} = 1$, where $\tilde{E}(t)$ denotes energy demand of a production unit, $-\sigma < 0$ denotes the elasticity of energy demand with respect to energy saving technology and B denotes a constant technology parameter and where s lies between zero and the scrapping age, T, i.e. $0 \le s \le T$. We assume that a firm can choose $\mu(t)$ from a set of energy saving technologies. The set of energy saving technologies is a public good, i.e. the choice of $\mu(t)$ is free, but the firms pay for the installation of $\mu(t)$. The larger $\mu(t)$ the lower the energy demand $E(t)$ of a production unit. Energy demand of a production unit that was created or updated at time t can be written as $\tilde{E}(t+s) = B\mu(t)^{-\sigma}$. Note that $\tilde{E}_{\mu}(t) < 0$, $\tilde{E}_{\mu\mu}(t) > 0$, i.e. the marginal energy savings of an additional unit $\mu(t)$ are always positive but decreasing.

From these assumptions it follows that new production units hav a higer labor productivity and energy productivity than production units from an older vintage. Schurr et al. (1990) presents empirical evidence for the US economy that justifies these assumptions. Especially, Schurr et al. (1990) show that times of high labor productivity increases were also times of high energy productivity increases. Varying over time is the ratio of energy input per unit of labor. This feature is captured in the endogenous choice of $\mu(t)$.

Creating a production unit is costly. The creation cost consists of two parts. First, a firm that creates a production unit has to pay a setup cost $KA(t)$ that is paid when the worker arrives. The second part is the installation cost for the energy saving technology, $A(t)I(\mu(t))$. It is assumed that in steady state creation costs grow over time at the rate of technological progress. This assumption is necessary assure the existence of a steady state. The creation cost deflated with the rate of technological progress is $I(\mu(t))+K$. The term $I(\mu(t))$ describes the relationship between the energy saving technology $\mu(t)$ and the deflated creation cost. We assume that $I_{\mu}(\mu(t)) > 0$ and $I_{\mu\mu}(\mu(t)) \geq 0$.[2] Note that the assumptions about $I_{\mu}(\mu(t))$, $I_{\mu\mu}(\mu(t))$, $E_{\mu}(t)$, and $E_{\mu\mu}(t)$ imply that energy can be substituted through capital, but that the lower the energy demand, i.e. the higher $\mu(t)$, the more difficult it is to substitute energy through capital.

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[2] Note that due to the endogenous choice of $\mu(t)$ aggregate capital intensity is an endogenous variable, too, if we consider the aggregate net of energy output as a function of labor and physical capital.

Each production unit employs one worker. Therefore, the number of production units gives the number of employed workers. Production units were created at different times and, thus, also have different productivities. New technologies emerge continually and each technology eventually reaches its scrapping age T when it will be put into obsolescence, hence, a vintage of technology at any time t can be described by its age a, that will range between 0 and T. The cross section density of production units aged a at date t is

$$f(a,t), \qquad 0 \leq a \leq T(t)$$

Therefore, with a fixed and normalized labor force $L=1$, employment can be given as the difference between labor force L and unemployed uL. Keeping in mind that each production unit employs only one worker, we have

$$1-u=\int_0^T f(a,t)da$$

Similarly, we can describe the ecnomy's output and energy demand

$$Q(t)=\int_0^T A(t-a)f(a;t)da$$

$$E(t)=\int_0^T B\mu(t-a)^{-\sigma}f(a;t)da$$

A firm is engaged in a labor market matching process. At each unit of time H matches take place or equivalently, H workers are hired from the unemployment pool. The number of matches or hires H is a linearly homogenous function of the number of unemployed uL and of the number of vacancies vL, $H=m(uL;vL)$. In addition the function m is concave and increasing in both arguments. The fraction of vacant jobs that is filled in a unit of time is therefore $q(\theta)=\frac{m(uL;vL)}{vL}=\frac{m(1;\theta)}{\theta}$, with $\theta=v/u$.

Therefore, the process that changes the state of a vacant job can be expressed as a stochastic Poisson process. The Poisson distribution is defined as $\pi(k;\lambda t_0)=e^{-\lambda t_0}\frac{(\lambda t_0)^k}{k!}$. $\pi(k;\lambda t_0)$ gives the probability that a certain event (filling a vacant job) takes place k times in a time interval $[0;t_0]$. The probability that a certain event occurs exactly 0 times in $[0;t_0]$ is $p(0;\lambda t_0)=e^{-\lambda t_0}$. Therefore, the probability that an event occurs at least once in $[0;t_0]$ is $1-e^{-\lambda t_0}=\int_0^{t_0}\lambda e^{-\lambda t}dt$. The parameter λ is called the arrival parameter and gives the expected number of events that take place in the unit time interval $[0;1]$. Therefore, $m(\theta;1)$ and $q(\theta)$ can be interpreted as arrival parameters of a Poisson distribution.

As will be shown in detail, for firm will arrive at the date where it has to decide whether to update an existing production unit with a new technology or to let the production unit turn obsolete such that the incumbent worker has to get fired. Similar to Mortensen, Pissarides (1995) it is assumed that the decision of whether to update or not depends on the updating cost $A(t)[I(\mu(t))+C]$. In the first model we assume that technological progress is schumpeterian. Technological progress is destructive, such that each production unit is only on the market for a finite time. Due to technological progress it will shift further and further away from the productivity frontier such that it is eventually not profitable anymore. Once a production unit is unprofitable it will be destroyed. Therefore, schumpeterian technological progress implies that the updating cost C is so high that firms will never update old production units. In this case technological progress implies also a permanent reallocation of labor and the scrapping age T does not denote only the scrapping of the technology in use, but also the destruction of the whole production unit. In section 4 we assume the other extreme case that technological progress does not imply reallocation of labor. This implies that the updating cost C is so low that all firms choose to update existing production units with the latest technology. In this case T denotes the scrapping of the old technology in use and the implementation of a new technology. The main difference between section 3 and 4 is that we assume in section 3 that the updating cost is always higher than the setup cost and vice versa in section 4. This difference in assumptions makes sense if schumpeterian technological progress is understood in form of new products. In this case it seems easy to imagine that the updating cost might higher than the setup cost of a new production unit. In the neoclassical case technological progress is best understood in form of product or process improvements. Then it seems plausible to assume that the updating cost lies below the setup cost of a new production unit.

3. The decentralized economy with creative destruction in the long run

In this section we assume schumpeterian technological progress and thus, that the updating cost C is so high that firms will never update old production units. The variable T denotes destruction of a production unit and is therefore marked with the subscript d.

3.1. The firm

The firm can be in one of two states: idle or producing. In the idle state the firm has to decide whether to remain idle or to create a vacancy and to employ a worker. If the firm decides to create a vacancy it also has to decide which technology to install that it will use until the production unit is destroyed. Regarding the technology choice the firm has to choose the productivity level and the level of energy consumption. Profit maximization will lead every firm that creates a job to choose the highest productivity level currently available. The choice of energy consumption depends on the energy saving technology. Here the firm has to consider the investment cost and the benefits that occur from the energy saving technology in form of a smaller energy cost. In the active state the firm is producing, and in each period deciding whether to continue producing or scrap the production unit and go into the idle state.

The expected value of creating a production unit at date t to a firm equals the expected value of a producing production unit minus the expected cost that it takes to setup the production unit

$$(1) \qquad V(t) = \max_{\mu(s)\geq 0} \int_t^\infty q(\theta(s))\big[J(s;0;\mu(s)) - A(s)\big(K + I(\mu(s))\big)\big] e^{-\int_t^s (r+q(\theta(\tilde{s})))d\tilde{s}}\, ds$$

where s denotes a time index. $J(s;0;\mu(s))$ denotes the value of a job that has the age $a=0$ at date t.[3]

The value of a vacancy equals the expected value of a filled job minus the expected cost of filling the vacancy. The expression $\int_t^\infty q(\theta(s))J(s;0;\mu(s))e^{-\int_t^s(r+q(\theta(\tilde{s})))d\tilde{s}}ds$ gives the discounted value of a filled job multiplied with the probability that the job is filled. The expression $\int_t^\infty q(\theta(s))A(s)\left(K+I(\mu(s))\right)e^{-\int_t^s(r+q(\theta(\tilde{s})))d\tilde{s}}ds$ gives the expected present value of the cost to setup the production unit. From equation (1) it becomes clear that the setup cost must be paid when the worker is found who is going to occupy the production unit. This is expressed through $q(\theta)$ in (1). Assuming free entry of firms, in equilibrium the value of a vacant job must be zero, hence for date t the following equilibrium conditions must hold

(2) $$J(t;0;\mu(t))=A(t)\left[K+I(\mu(t))\right]$$

(3) $$J_\mu(t;0;\mu(t))=A(t)I_\mu(\mu(t))$$

The value of a filled job with age a is

(4) $$J(t;a;\mu(t))=\max_{T_d}\int_a^{T_d}\Big[A(t)-(1+\tau_E)p(s)B\mu(t)^{-\sigma} -(1+\tau_L)w(t;s;\mu(t))\Big]e^{-(r+\delta)(s-a)}ds$$

Equation (4) says that the value of a job with age a that was opened at date t equals the discounted sum of expected profits. Profit in period s equals output that was produced with a technology that was installed at time t, $A(t)$, minus expenditure for energy, $(1+\tau_E)p(s)B\mu(t)^{-\sigma}$, and labor, $(1+\tau_L)w(t;s;\mu(t))$. τ_E and τ_L denote the tax rates on energy and labor, respectively. Note that energy expenditure of a firm in period s depends on the energy saving technology that was installed at time t and the producer price of energy for period s. As is shown below the wage $w(t;s;\mu(t))$

[3] In our model search costs of the firm are completely neglected. Caballero, Hammour (1994), p. 5 argue „that shifting the emphasis to specific investment costs [...] is a more promising avenue in providing satisfactory interpretation of the facts."

depends on the chosen energy saving technology $\mu(t)$ and the date t at which the job was equipped with $\mu(t)$ and the latest production technology. With the choice of T the firm determines the exit age for the technology in use, which is finite as we will demonstrate below.

In order to assure the existence of a steady state we have to make the assumption $p(s) = pe^{g(s-t)}A(t)$, i.e. the world market energy price grows at the rate steady state growth rate g. This assumption does not influence the main results of this model and is needed to ensure the existence of a steady state. If the energy price remained constant technological progress would let the share of energy in firm expenditure converge to zero and the incentive to save energy would vanish.

The optimization problem of the firm is solved as follows: First the exit age is determined, given the energy saving technology $\mu(t)$. From (4) follows

(5) $$A(t) - (1 + \tau_E)pA(t)e^{gT_d}B\mu(t)^{-\sigma} - (1 + \tau_L)w(t; T_d; \mu(t)) = 0$$

This exit condition states that the firm chooses to exit the market and join the idle state, if the cash flow for the firm equals zero.

The technology $\mu(t)$ is chosen according to equation (3). From (3) and (4) follows the first order condition for the choice of $\mu(t)$.

(6)

$$A(t)I_\mu(\mu(t)) = \int_0^{T_d}\left[\sigma(1 + \tau_E)p(s)B\mu(t)^{-(1+\sigma)} - (1 + \tau_L)w_\mu(t; s; \mu(t))\right]e^{-(r+\delta)s}ds$$

This condition states that the optimal investment level in energy saving technology is achieved, when the marginal investment cost equals the marginal sum of expected and discounted return from this investment. As will be shown below, the wage depends on the choice of $\mu(t)$.

In the next section we briefly describe the worker and the Nash bargaining process between a worker and a firm. For a more detailed description the reader is referred to Pissarides (1990) and Mortensen, Pissarides (1995).

3.2. The worker

The value of a job to a worker is

$$W(t;a) = \int_a^{T_d} w(t;s;\mu(t))e^{-(r+\delta)(s-a)} + \delta U(t+s)e^{-(r+\delta)(s-a)}ds \tag{7}$$

Where $\int_a^{T_d} w(t;s;\mu(t))e^{-(r+\delta)(s-a)}ds$ describes the expected discounted sum of wage payments from a match that was created at time t and equipped with technology $\mu(t)$. The term $e^{-\delta(s-a)}$ captures the probability that the match is not destroyed. The second term, $\int_a^{T_d} \delta U(t+s)e^{-(r+\delta)(s-a)}ds$, describes the value of unemployment, where $\delta e^{-\delta(s-a)}$ captures the probabilty of remaining unemployed.

The value of being unemployed equals

$$U(t+a) = \int_{t+a}^{\infty} m(1;\theta(s))W(t+s)e^{-\int_{t+a}^{s}(r+m(1;\theta(\tilde{s})))d\tilde{s}}ds \tag{8}$$

We assume that there is no value of leisure or unemployment transfer so that the only benefit of unemployment is derived from the prospect of finding a new job.

3.3. Wage bargaining

Wages are determined in a Nash bargaining process between firms and workers. The Nash product to be maximized is

$$\max_{J;W} J(t)^{1-\beta}(W(t)-U(t))^{\beta} \text{ s.t } S(t) = J(t) + W(t)$$

β denotes the bargaining power of the worker and $1-\beta$ the bargaining power of the firm. Additionally it is assumed that $0<\beta<1$. $S(t)$ denotes the surplus that is generated from a match between a worker and a firm. From Nash bargaining results

$$(1-\beta)(W(t;a)-U(t+a)) = \beta J(t;a) \tag{9}$$

Differentiation with respect to age a gives

$$(1-\beta)(W_a(t;a)-U_{t+a}(t+a)) = \beta J_a(t;a)$$

From this last equation and equations (4)-(9) we can derive the following wage equation as is shown in appendix A

$$w(t;a;\mu(t)) = \frac{\beta}{(1+\beta\tau_L)}\left[A(t) - (1+\tau_E)p(a)B\mu(t)^{-\sigma}\right] + \frac{\beta}{(1+\beta\tau_L)}m(1;\theta(t))J(t+a;0;\mu(t+a)) \tag{10}$$

The wage at date $t+a$ of a worker that works with a technology that was leading at date t consists of a share of the cash flow his job generates plus the opportunity cost that the worker faces when he stays with the technology that is not up to date at date $t+a$. The opportunity cost is the same for all employed workers and does not depend on the age of their job. The share, however, that they receive from their current match depends on the age of their job and due to technological progress older technologies create a smaller surplus than new technologies.The opportunity cost equals the share that the worker would get at the most modern job multiplied with the probability to get matched with the new technology. This wage equation shows that also the wage of old technologies profit from technological progress since the outside option, that is the wage that the worker would receive at a new production unit, grows at the rate of technological progress. If the wage of an old production unit would not grow, the worker would be better off quitting his job join the unemployment pool and look for a new job that pays a higher wage. The opportunity cost does not depend on technological progress alone. The term $m(1;\theta(t))$ captures the current labor market situation. The longer the worker has to expect to stay unemployed if he quits his job, the lower his opportunity cost to stay with the old technology. It can also be seen that the most modern technologies pay the highest wage and the oldest technologies pay the lowest wage.

Only the cost of shifting capital from one application to another, which is infinite in our model, gives bargaining power to the worker which allows him to set the wage above the efficient level. Unemployment has now the role to limit the bargaining power of the worker by reducing his threat point $U(t)$, since $m(1;\theta(t))$ equals the ratio of hires and unemployment.

3.4. The steady state equilibrium

Inserting the wage equation (10) into (4) and making use of the exit condition (5) gives the following entry condition

$$(11)\qquad J(t;0;\mu(t)) = \int_0^{T_d} \Big[\big(1-\tilde{\beta}(\tau_L)\big)\big[A(t)-(1+\tau_E)pe^{gs}A(t)B\mu(t)^{-\sigma}\big] - \tilde{\beta}(\tau_L)m(1;\theta(s))J(t+s;0;\mu(t+s))\Big]e^{-(r+\delta)s}ds$$

where $\tilde{\beta}(\tau_L)$ summarizes $\tilde{\beta}(\tau_L)=\dfrac{(1+\tau_L)\beta}{(1+\beta\tau_L)}$, $\tilde{\beta}'>0$. Equation (11) is also derived in appendix B.

Since in a steady state equilibrium the variables θ and $\mu(t)$ have to remain constant, we can conclude from (2) that in steady state $J=\dfrac{J(t+a;0;\mu(t+a))}{A(t+a)}=\dfrac{J(t;0;\mu(t))}{A(t)}$ is constant in steady state. Therefore, from (11) and (2) we can derive the following steady state entry condition

$$(12)\qquad J=\int_0^{T_d}\Big[\big(1-\tilde{\beta}(\tau_L)\big)\big[1-(1+\tau_E)pe^{gs}B\mu^{-\sigma}\big]-\tilde{\beta}(\tau_L)m(1;\theta)Je^{gs}\Big]e^{-(r+\delta)s}ds$$

with $J=[K+I(\mu)]$.[4]

In appendix B we show how to derive the following equation from (6) and (10), which gives the first order condition for technology choice

$$(13)\qquad I_\mu(\mu)=\int_0^{T_d}\big(1-\tilde{\beta}(\tau_L)\big)\sigma(1+\tau_E)pe^{gs}B\mu^{-(1+\sigma)}e^{-(r+\delta)s}ds$$

This condition states that the optimal investment level in energy saving technology is achieved, when the marginal investment cost equals the marginal sum of expected and discounted return from this investment. The return from the investment equals the cost

[4] Since we deal with steady state equilibria, where the endogenous variables are the same for all vintages, we drop the variable t.

savings due to the energy saving technology multiplied with the share that the firm receives from the match with the worker. Since the firm has to carry the investment cost alone there occurs a distortion in the optimal choice of energy saving technology. This distortion results from the fact that the worker benefits from the investment in energy saving technology in form of an increased cash flow, but does not participate in the increased investment cost. Therefore, the investment decision is distorted since actually the marginal match surplus should equal the marginal investment cost, but only the marginal benefit that occurs to the firm is equalized with the marginal investment cost. Hence, investment in energy saving technology is too low compared to the socially optimal level. This distortion results from the positive externality of the investment decision on the wage of the worker. From equation (13) it is easy to derive the following results for the choice of energy saving technology.

$$\frac{d\mu}{d\tau_E} = \frac{\int_0^{T_d}\left(1-\tilde{\beta}(\tau_L)\right)\sigma B\mu^{-\sigma} p e^{-(r+\delta-g)s} ds}{I_{\mu\mu}\mu + (1+\sigma)I_\mu} > 0$$

$$(14): \quad \frac{d\mu}{dT_d} = \frac{\left(1-\tilde{\beta}(\tau_L)\right)\sigma B\mu^{-\sigma}(1+\tau_E) p e^{-(r+\delta-g)T_d}}{I_{\mu\mu}\mu + (1+\sigma)I_\mu} > 0$$

$$\frac{d\mu}{d\tau_L} = -\frac{\int_0^{T_d}\tilde{\beta}' B\mu^{-\sigma}\sigma(1+\tau_E) p e^{-(r+\delta-g)s} ds}{I_{\mu\mu}\mu + (1+\sigma)I_\mu} < 0$$

An increase in the energy tax will lead to the installation of better energy saving technologies. Since an increase in the energy tax increases the expenditure for energy it will make the installation of energy saving technologies more profitable. Also an increase of the scrapping age T will lead to more energy saving since there is more time for the firm to recover the investment cost. A higher labor tax τ_L has a negative impact on the choice of the energy saving technology. To understand this result better, a look at equation (12) might be helpful. From the perspective of the firm a higher labor tax has the same effect as an increase in the workers bargaining power. Therefore, a higher labor tax will decrease the cash flow that results from an investment. This will lead the firm to reduce its investment cost, which results in the choice of a less expensive energy saving technology.

The exit condition (5) can be transformed with the wage equation (10) to

$$(15) \quad \left(1-\tilde{\beta}(\tau_L)\right)\left[A(t)-(1+\tau_E)pA(t)e^{gT_d}B\mu(t)^{-\sigma}\right] = \tilde{\beta}(\tau_L)m(1;\theta)J\left(t+T_d;0;\mu(t+T_d)\right)$$

Equations (12), (13), and (15) describe the equilibrium of the model. With the redefinition of variables $x = m(1;\theta)$ and the help of (14), we have the following system of equations, as is shown in appendix C

$$(16a) \quad \frac{\left[K + I\left(\mu(T_d;\tau_E;\tau_L)\right)\right]}{\left(1-\tilde{\beta}(\tau_L)\right)} = \int_0^{T_d}\left[1 - e^{gs}e^{-gT_d}\right]e^{-(r+\delta)s}\,ds$$

$$(16b) \quad \frac{\tilde{\beta}(\tau_L)}{\left(1-\tilde{\beta}(\tau_L)\right)}x\left[K + I\left(\mu(T_d;\tau_E;\tau_L)\right)\right]e^{gT_d} = \left[1-(1+\tau_E)pe^{gT_d}B\mu(T_d;\tau_E;\tau_L)^{-\sigma}\right]$$

Equations (16a) and (16b) describe the steady state equilibrium of the economy. (16a) gives the entry condition for firms and determines the scrapping age T_d. (16b) gives the exit condition and for a given value of x it can be used to calculate the exit age T_d. Usually one would expect that the relationship between x and T_d in (16a) is increasing. An increasing scrapping age T_d should stimulate entry of firms, since there is more time to recover the sunk cost. At the same time the relationship between T_d and x in (16b) is expected to be decreasing. This is so, because an increasing x raises the opportunity cost for workers to stay with an old technology, since the probability of finding a new job after quitting the old job increases. Hence, the wages paid by old technologies should be increasing prompting earlier exit of firms i.e. decreasing scrapping age. Differentiation of (16a) with respect to x and T_d, however, gives

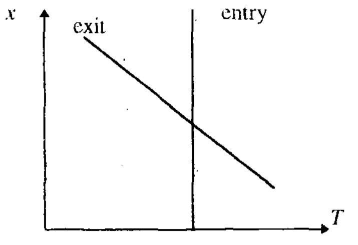
$$(17a)\qquad \left(\frac{dx}{dT_d}\right)^{entry} = \left(1-\tilde{\beta}\right)\frac{ge^{-gT_d}\int_0^{T_d} e^{-(r+\delta-g)s}ds - \frac{I_\mu}{1-\tilde{\beta}}\frac{\partial\mu}{\partial T_d}}{0} = \infty$$

Although intuition suggests that $\left(dx/dT_d\right)^{entry}$ should be positive, theoretically it is possible that this relationship is negative. This is the case if an increasing T_d leads to such a strong increase in the installation cost of energy saving technology that the gain in the present value of the cash flow is overcompensated. Referring to the discussion of equation (14), however, we know that a better energy saving technology is chosen only, because there is more time to recover the increased investment cost. This means in (17a) the negative effect is a direct consequence of the first effect. Therefore, it seems reasonable to assume that the first effect is stronger than the second effect and hence that $\left(dx/dT_d\right)^{entry}$ is positive.

Differentiation of (16b) with respect to x and T_d, gives

$$(17b)\qquad \left(\frac{dT_d}{dx}\right)^{exit} = \frac{\frac{\tilde{\beta}}{1-\tilde{\beta}}\left[K + I\left(\mu\left(T_d;\tau_E;\tau_L\right)\right)\right]}{-ge^{-gT_d} + \left(1+\tau_E\right)p\sigma B\mu^{-(1+\sigma)}\frac{\partial\mu}{\partial T_d} - \frac{\tilde{\beta}}{1-\tilde{\beta}}xI_\mu\frac{\partial\mu}{\partial T_d}} < 0$$

Also here the counterintuitive case, a positive value for $\left(dT_d/dx\right)^{exit}$, is possible. As was pointed out above an increasing x raises the opportunity cost of workers to stay with old technologies, hence, old technologies have to pay a higher wage to prevent workers from quitting their job. This effect will lead to a decrease in the scrapping age. Old and new firms anticipate the increase in x and base their investment decision on new expected lifetime T_d. The new choice T_d is based on three considerations. The first effect origins from the fact that a new exit age is also connected to a new cash flow at the date of exit. If the firms decide to extend T_d they have to consider that because of growth, the energy price and the wage will be higher. This effect is reinforced by the fact that if old firms planned with a longer T_d also new firms plan with a longer T_d. This leads to more investment into energy saving technology. The higher investment level of new firms makes their matches more valuable and, therefore, increases the wage for old firms. Old firms however, also invested more into energy saving technology. This effect increases the cash flow of old firms at the date of exit. The investment decision of old firms might lead to a positive $(dT_d/dx)^{exit}$, if the investment reaction is so strong that the cash flow of old firms, that originally wanted to exit at a given date, becomes positive at that given date, so that they choose a later exit date. In what follows we assume that the impact on the investment decision of the old firms is weaker than the other three effects. This assumption is satisfied for sufficiently small values of σ or B. Hence, we assume that $(dx/dT_d)^{entry}$ in (17a) is positive and $(dT_d/dx)^{exit}$ in (17b) is negative. These results can be presented in the following diagram:



Determination of T_d and x

Now only the equilibrium condition for unemployment has to be determined. In equilibrium the flow into unemployment is equal to job destruction. Job destruction equals the sum of the jobs that are hit by an exogenous shock and the number of jobs that reach their scrapping age. Hence the flow into unemployment equals $\delta(1-u)+m(1;\theta)ue^{-\delta T_d}$. The flow out of unemployment equals job creation which is $um(1;\theta)$. In steady state equilibrium the flow into unemployment equals the flow out of unemployment. Hence, with $x = m(1;\theta)$ we have in equilibrium

$$(18) \qquad u = \frac{\delta}{\delta + \left(1 - e^{-\delta T_d}\right)x}$$

With $\frac{\partial u}{\partial x} < 0$ and $\frac{\partial u}{\partial T_d} < 0$. Note that unemployment increases with decreasing T_d. A decreasing T_d means a faster job turnover, such that unemployment increases.

In the next sections we consider the effects of tax policy on exit, entry, energy demand and unemployment.

3.5. Comparative static analysis

3.5.1. A labor tax increase

First we consider the effects of a labor tax change. From (16a) we can derive

$$\left(\frac{dT_d}{d\tau_L}\right)^{entry} = \frac{\left[\frac{I_\mu \frac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}\right)} + \frac{\tilde{\beta}'\left[K + I(\mu)\right]}{\left(1-\tilde{\beta}\right)^2}\right]}{ge^{-gT_d}\int_0^{T_d} e^{-(r+\delta-g)s}ds - \frac{I_\mu \frac{\partial\mu}{\partial T_d}}{1-\tilde{\beta}}} > 0$$

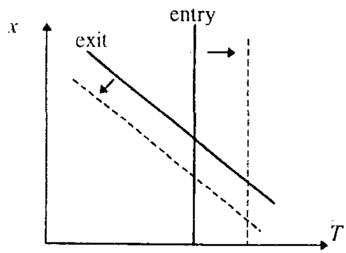
This expression gives the reaction of entry to an increase in the labor tax. There are two opposing effects. First an increase in the labor tax is tantamount to a reduction of the expected present value of an investment. This effect affects entry negatively. In reaction to a labor tax increase firms also choose a less expensive energy saving technology which reduces the cost of entry. This will have a positive effect on entry. The second effect however is caused by the first effect. This can be seen clearly from equation (14). Therefore, it seems reasonable to assume that the first effect is stronger than the second effect, hence $\left(\frac{dT_d}{d\tau_L}\right)^{entry} > 0$, in order to prevent x from going to zero which cannot be an equilibrium, since u would equal zero.

Differentiating the exit condition (16b) with respect to T_d and τ_L yields

$$\left(\frac{dT_d}{d\tau_L}\right)^{exit} = \frac{-\left(x\left[\tilde{\beta}\frac{I_\mu \frac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}\right)} + \frac{\left[K + I(\mu)\right]\tilde{\beta}'}{\left(1-\tilde{\beta}\right)^2}\right] - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\frac{\partial\mu}{\partial\tau_L}\right)}{\left(\frac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial T_d} + ge^{-gT_d}}$$

There are two main opposing effects at work. The increase in the gross wage payments of old firms certainly has a positive effect on an early exit age. But since an increase in the labor tax also decreases investment expenditure of entering firms the values of the new matches decrease. This will reduce the opportunity cost of workers to stay with old technologies and hence represents a downward pressure on net wage payments of existing firms. The effect that emerges from the existing firms' choice of another technology, which raises the energy expenditure of the firm works in the same direction as the first effect. Usually one would expect that after a labor tax increase the gross wage payment of firms increase and from the assumption about the denominator and the $\left(\frac{dT_d}{d\tau_L}\right)^{entry}$, it follows that the first effect that works in the direction of decreasing the exit age is stronger than the second effect, hence $\left(\frac{dT_d}{d\tau_L}\right)^{exit} < 0$.

In the diagram the effects of a labor tax increase can be demonstrated as follows



The effects of a labor tax increase

An increase in the labor tax unambiguously reduces the hiring unemployed ratio and increases the destruction age of production units. This result might be surprising, since the labor tax increase causes two effects on unemployment which are opposite in direction.

$$\frac{du}{d\tau_L} = -\frac{\delta\left(1-e^{-\delta T_d}\right)}{\left[\delta+\left(1-e^{-\delta T_d}\right)x\right]^2}\frac{dx}{d\tau_L} - \frac{\delta^2 e^{-\delta T_d}x}{\left[\delta+\left(1-e^{-\delta T_d}\right)x\right]^2}\frac{dT_d}{d\tau_L}.$$

This expression is equivalent to the following:

$$\frac{du}{d\tau_L} = -u^2\left[\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\frac{dx}{d\tau_L} + xe^{-\delta T_d}\frac{dT_d}{d\tau_L}\right].$$

We have not been able to derive the sign of this expression generally. In appendix D we show that a sufficient condition for a positive value of $\frac{du}{d\tau_L}$ is

$$\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\left(\frac{dx}{dT_d}\right)^{exit} + xe^{-\delta T_d} \leq 0.$$

In numerical simulations we have not found any parameter constellation that leads to less unemployment. Similar results are reported by Caballero, Hammour (1994).

3.5.2. An energy tax increase

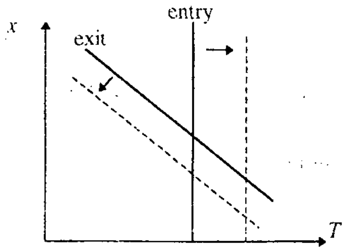
Consider now an increase in the energy tax. From (16a) we can derive

$$\left(\frac{dT_d}{d\tau_E}\right)^{entry} = \frac{\dfrac{I_\mu \dfrac{\partial\mu}{\partial\tau_E}}{\left(1-\tilde{\beta}\right)}}{ge^{-gT_d}\int_0^{T_d} e^{-(r+\delta-g)s}ds - \dfrac{I_\mu \dfrac{\partial\mu}{\partial T_d}}{1-\tilde{\beta}}} > 0$$

From the discussion of (17a) we know that this expression is positive. An increase in τ_E makes entry more expensive. In order to prevent x from going to zero, we need $\left(\frac{dT_d}{d\tau_E}\right)^{entry} > 0$. From the exit condition we obtain after differentiating with respect to T_d and τ_E:

$$\left(\frac{dT_d}{d\tau_E}\right)^{exit} = \frac{-\left(\dfrac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial\tau_E} - pB\mu^{-\sigma}}{\left(\dfrac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial T_d} + ge^{-gT_d}}$$

Intuitively, one would guess that raising any tax would decrease the scrapping age, because the cash flow of a match is reduced. In the case of the energy tax increase there are three effects. An increase of the energy tax raises the energy expenditure of existing firms and reduces the surplus. This leads to a faster arrival of the destruction date. This effect will lead to an earlier exit of the firm. This effect is reinforced by the second effect. The increased investment spending of new firms which make new matches more valuable increases the workers opportunity cost to stay with an old technology. The third effect results from the old firms change of behavior. The increase in the energy tax leads the firms at the time when they had to decide over their energy saving technology level also to an increased investment level. This had the effect to reduce their energy expenditure. Only when this effect is stronger than the two preceding effects, it is possible that the exit age increases as a consequence of an increase in the energy tax. This would mean that the actual energy expenditure of old firms decreases after the tax raise. This would require an unrealistically high elasticity of energy demand. Therefore, we assume $\left(\frac{dT_d}{d\tau_E}\right)^{exit} < 0$. From the last two expressions we can derive the folloing diagram:



The effects of an energy tax increase

Also in the case of the energy tax increase the effects on unemployment are twofold and opposite in direction. Therefore, it is important two find out which one of the two opposing effects dominates. In appendix D, we show that a sufficient condition for a positive value of $\frac{du}{d\tau_E}$ is

$$\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\left(\frac{dx}{dT_d}\right)^{exit} + xe^{-\delta T_d} \leq 0$$

Also here we have not found any numerical example in which $\frac{du}{d\tau_E}$ was negative.

A conclusion that can be drawn from this and the last section is that an increase in the energy tax and a decrease in the labor tax could be able to reduce unemployment. Whether this will be the case depends on two important issues. First, which tax has a stronger impact on unemployment and second, to which extent is the government able to reduce the labor tax, if it has to consider its budget restriction.

3.6. Environmental tax reform

The question whether an environmental tax reform is able to yield an improvement of environmental quality and at the same time to reduce unemployment can already be answered in parts. Economy wide demand for energy is given by

$$E(t) = \int_0^{T_d} B\mu(t-a)^{-\sigma} f(a;t)da$$

The number of jobs at date t, with age a , $f(a;t)$, equals the sum of hires $H(t-a)$ multiplied with the probability that no event has destroyed jobs that were created at time t-a, $e^{-\delta a}$. Therefore, energy demand can be rewritten in steady state, where $H(t-a)$ and $\mu(t-a)$ is the same for all vintages, as

$$(19) \qquad E(t) = \frac{x\delta B\mu(T_d;\tau_E;\tau_L)^{-\sigma}}{\delta + \left(1-e^{-\delta T_d}\right)x} \int_0^{T_d} e^{-\delta a}da$$

An increase in τ_E and a decrease in τ_L affect the steady state value of energy saving technology in the same direction. The direct effect of the tax changes on energy demand leads to less energy consumption, since the tax reform stimulates investment into energy saving technologies. Unclear are the effects of a tax reform on the steady state values of x and T_d, since the proposed tax changes affect T_d and x in opposite directions. The increase in τ_E leads to a higher value of T_d and a lower value of x. A decrease in τ_L has the opposite effects. The extent of these two opposite effects depends on the magnitude of the labor tax decrease relatively to the energy tax increase. This magnitude in turn depends on the government's ability to use the revenue created by an energy tax increase to cut the labor tax rate. The government's ability to cut the labor tax is largely determined by its budget restriction. Therefore, we need to specify the government budget restriction. An exogenous level of government expenditure $\tilde{g}A(t)$ has to be financed each period through the revenue created from the taxation of labor and energy. The revenue from the labor tax, which can be derived with the help of the wage equation (10) equals

(20)

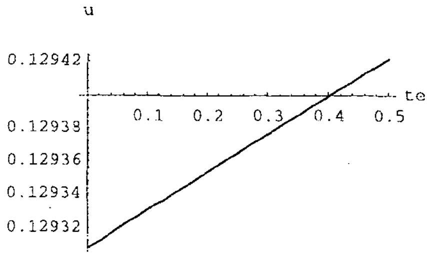
$$\begin{aligned}\tau_L\int_0^{T_d} H(t-a)w(t-a;a)e^{-\delta a}da &= \frac{\tau_L\beta H}{(1+\beta\tau_L)}\int_0^{T_d}\left[A(t-a)-(1+\tau_E)p(t)B\mu(t)^{-\sigma}\right]e^{-\delta a}da \\ &+\frac{\tau_L\beta H}{(1+\beta\tau_L)}\int_0^{T_d} xJ(t;0;\mu(t))e^{-\delta a}da \\ &=\frac{\tau_L\beta x}{(1+\beta\tau_L)}\frac{\delta A(t)}{\delta+\left(1-e^{-\delta T_d}\right)x}\int_0^{T_d} e^{-(\delta+g)a}da \\ &-\frac{\tau_L\beta x}{(1+\beta\tau_L)}\frac{\delta A(t)}{\delta+\left(1-e^{-\delta T_d}\right)x}(1+\tau_E)pB\mu(T_d;\tau_E;\tau_L)^{-\sigma}\int_0^{T_d} e^{-\delta a}da \\ &+\frac{\tau_L\beta x}{(1+\beta\tau_L)}\frac{\delta\left[K+I(\mu(T_d;\tau_E;\tau_L))\right]A(t)}{\delta+\left(1-e^{-\delta T_d}\right)x}\int_0^{T_d} e^{-\delta a}da\end{aligned}$$

The government budget restriction is

$$(21)\qquad \tilde{g}A(t)=\tau_E pA(t)E(t)+\tau_L\int_0^{T_d} H(t-a)w(t-a;a)e^{-\delta a}da$$

The assumption that the government expenditure grows at the steady state growth rate g assures the existence of a steady state.

With the equations (16a), (16b) and (18)-(21) it is possible to determine the equilibrium values of x, T_d and τ_L, when $\tilde{g}$ and τ_E are exogenous. In order to determine the effects of an environmental tax reform, one has to calculate the comparative statics with an exogenous change of τ_E in the equation system (16a), (16b) and (18)-(21). Unfortunately, we were unable to determine the signs of the effects in general. Therefore, we had to run some numerical examples. In all numerical examples that we calculated with a linear investment function of the type $I(\mu)=c_0\mu$, we found that energy consumption of firms decreases.[5] Hence, there is an environmental benefit. We also found in all numerical examples that unemployment increases slightly as a consequence of the environmental tax reform, although it is possible to cut the labor tax.



The effects of an environmental tax reform on unemployment with parameters:

$c_0 = 0.1$; $\beta = 0.5$; p=0.01; g=0.01; r=0.07; δ =0.01; σ=0.01; B=0.1; K=3; $\tilde{g}$ =0.29;

The results displayed in the diagram hold also for small values of $\tilde{g}$. Therefore, there is no double dividend, when technological progress is destructive, although the effects on unemployment are small. The economic intuition lies in what Bovenberg, de Mooij (1994) call the tax base erosion effect. An increase in the energy tax causes increased investment in energy saving technology which is a policy aim. The subsequent decrease in energy consumption erodes the tax base of the energy tax and limits the ability of the government to cut the labor tax sufficiently. The labor tax cut even reinforces the tax base erosion effect, because the labor tax cut also leads to a decrease in energy consumption. Therefore, if the government is restricted by its budget constraint, it is impossible to compensate the firms sufficiently for the energy tax increase if the compensation takes place in form of labor tax cuts. The increased tax burden of the firms leads to less entry and more exit and as a consequence to more unemployment.

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[5] All numerical examples mentioned in this paper were calculated with Mathematica 2.2. The file and the results are available from the author on request.

4. The decentralized economy with neoclassical technological progress in the long run

In the preceding section we made the extreme assumption that the cost of updating old jobs with new technologies was high compared with the job creation cost such that firms chose to keep a job open as long as it generated a positive cash flow and then destroy it. This assumption resulted from the assumption of schumpeterian technological progress. The alternative is to assume neoclassical technological progress. At the other extreme is the assumption of zero implementation cost such that existing jobs are continuously updated with the latest technology. The scrapping of an old ptechnology is not connected anymore with reallocation of labor. Therefore, we denote T with a subscript i for implementation. Following Mortensen, Pissarides (1995), we now consider the case where the updating cost or implementation cost $C + I(\mu)$ is sufficiently low such that a firm always updates an existing job with the latest technology. The implementation cost consists of the updating cost and the investment cost in energy saving technology. We also assume that installation of the latest technology always requires the choice of a new energy saving technology. This assumption can be interpreted as that a certain energy saving technology that is matched with a given technology is destroyed with the scrapping of an old technology or incompatible with the new technology. We also show that there exists a unique level of implementation cost $C + I(\mu)$ where the decision of a firm to update switches to the decision of destroying an existing job. In what follows, we assume that the installation of energy saving technology causes the same costs as when the producion unit is created and leave the interesting and possibly realistic extension of the case where the installation of energy saving technology is different when a production unit is updated open for future research.

4.1. The steady state equilibrium

Since most of the arguments of the preceding model can be applied also to the model of this section we only sketch the new equations.

The value of a filled job with age a is

$$J(t;a;\mu(t)) = \max_{T_i} \int_a^{T_i} \left[A(t) - (1+\tau_E)p(s)B\mu(t)^{-\sigma} - (1+\tau_L)w(t;s)\right] e^{-(r+\delta)(s-a)} ds + e^{-(r+\delta)(T_i - a)} \max_{\mu(t+T_i)} \left[J(t+T_i;0;\mu(t)) - A(t+T_i)\left[C + I(\mu(t+T_i))\right]\right] \tag{22}$$

The variable T_i reflects the scrapping age and the subscript i indicates that the scrapping of an old technology is not connected to the destruction of the production unit, but to the implementation of a new technology. The implementation date gives the age that a job reaches before it is updated with the latest technology. Compared to section 3, the value of a job includes only one additional term that reflects the fact that the updating decision gives additional value to a job, because after scrapping the old technology production continues with the new technology. This aspect is also reflected in the value of a job for an employed worker

$$W(t;a) = \int_a^{T_i} w(t;s;\mu(t))e^{-(r+\delta)(s-a)} + \delta U(t+s)e^{-(r+\delta)(s-a)} ds + e^{-(r+\delta)(T_i - a)} W(t+T_i;0) \tag{23}$$

The value of a job is higher for a worker than in section 3, because after updating, his share of the match surplus increases due to the increased productivity level and the possibly adjusted choice of the energy savings technology μ.

From equations (22) and (23) one can calculate the following steady state entry and exit condition, as shown in appendix E:

$$\frac{C + I(\mu(T_i;\tau_E;\tau_L))}{\left(1 - \tilde{\beta}(\tau_L)\right)} = \int_0^{T_i} \left[1 - e^{gs}e^{-gT_i}\right] e^{-(r+\delta)s} ds \tag{24a}$$

$$e^{-gT_i} - (1+\tau_E) pB\mu(T_i;\tau_E;\tau_L)^{-\sigma}$$

(24b)
$$= \frac{\tilde{\beta}(\tau_L)}{\left(1-\tilde{\beta}(\tau_L)\right)} x\left[K + I\left(\mu(T_i;\tau_E;\tau_L)\right)\right] + \frac{(r+\delta-g)}{\left(1-\tilde{\beta}(\tau_L)\right)} K$$

From the exit condition (24b) it can be seen that the scrapping age is smaller when the firms update old jobs, i.e. $T_i < T_d$. This fact is reflected in the additional term $\frac{(r+\delta-g)}{\left(1-\tilde{\beta}(\tau_L)\right)} K$, which was missing in (16b). This term represents the capitalization effect. Since a firm has the chance to update existing jobs it does not have to cover the installation cost with one installation only. Updating gives the firm the possibility to allocate the installation cost over a much longer horizon and hence, effectively make entry cheaper. Since the only kind of implementation cost that arises is in the form of investment cost for the energy saving technology the entry condition (24a) gives us the relationship between the implementation horizon T_i and the implementation cost $C + I(\mu)$.

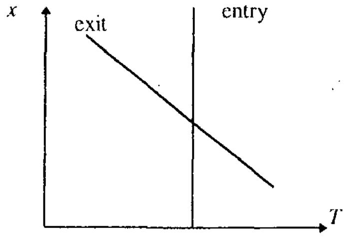
The firm chooses to implement, when the destruction horizon is at least as long as the implementation horizon. Comparing (24a) with (16a) shows that because of $\frac{d}{dT_i}\int_0^{T_i}\left[1 - e^{gs}e^{-gT_i}\right]e^{-(r+\delta)s}ds > 0$, implementation occurs only if

$$\frac{C + I\left(\mu(T_i;\tau_E;\tau_L)\right)}{\left(1-\tilde{\beta}(\tau_L)\right)} = \int_0^{T_i}\left[1 - e^{gs}e^{-gT_i}\right]e^{-(r+\delta)s}ds$$

$$< \frac{\left[K + I\left(\mu(T_d;\tau_E;\tau_L)\right)\right]}{\left(1-\tilde{\beta}(\tau_L)\right)} = \int_0^{T_d}\left[1 - e^{gs}e^{-gT_d}\right]e^{-(r+\delta)s}ds$$

This is always the case if $C \le K$. Therefore, the difference between the model of section 3 and this section is that in section 3 we implicitly assumed that the updating cost is higher than the setup cost for a new production unit.

With the same arguments and assumptions as in section 3, we can derive the following diagram



Determination of T_i and x

The only equilibrium condition that is missing is the unemployment equation. Job destruction equals the sum of the jobs that are hit by an exogenous shock. Hence the flow into unemployment equals $\delta(1-u)$. Scrapping does not influence unemployment any longer, since it is assumed that old jobs are updated. The flow out of unemployment equals job creation which is $um(1;\theta)$. In equilibrium the flow into unemployment equals the flow out of unemployment. Hence, with $x = m(\theta;1)$ we have in equilibrium

$$(25) \qquad u = \frac{\delta}{\delta + x}$$

In the next sections we consider the effects of tax policy on exit, entry, energy demand and unemployment.

4.2. Comparative static analysis

4.2.1. A labor tax increase

First we consider the effects of a labor tax change. Differentiation of (24a) with respect to T_i and τ_L yields

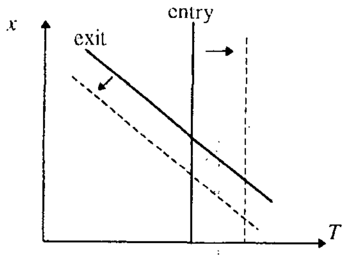
$$\left(\frac{dT_i}{d\tau_L}\right)^{entry} = \frac{\dfrac{I_\mu \dfrac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}(\tau_L)\right)} + \dfrac{C + I\left(\mu(T_i;\tau_E;\tau_L)\right)}{\left(1-\tilde{\beta}(\tau_L)\right)^2}\tilde{\beta}'}{ge^{-gT_i}\int_0^{T_i} e^{(r+\delta-g)s}ds - \dfrac{I_\mu \dfrac{\partial\mu}{\partial T_i}}{\left(1-\tilde{\beta}(\tau_L)\right)}} > 0$$

The interpretation of section 3 applies also here. First an increase in the labor tax is tantamount to a reduction of the expected present value of an investment. This effect affects entry negatively. In reaction to a labor tax increase firms also choose a less expensive energy saving technology which reduces the cost of entry. This will have a positive effect on entry. The second effect however is caused by the first effect. This can be seen clearly from equation (14). Therefore, it seems reasonable to assume that the first effect is stronger than the second effect, hence $\left(\frac{dT_i}{d\tau_L}\right)^{entry} > 0$, in order to prevent x from going to zero which cannot be an equilibrium, since u would equal zero.

Differentiation of (24b) with respect to T_i and τ_L yields

$$\left(\frac{dT_i}{d\tau_L}\right)^{exit} = -\frac{\left(x\left[\tilde{\beta}\dfrac{I_\mu \dfrac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}\right)} + \dfrac{\left[K + I(\mu)\right]\tilde{\beta}'}{\left(1-\tilde{\beta}\right)^2}\right] - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\dfrac{\partial\mu}{\partial\tau_L} + \dfrac{(r+\delta-g)K\tilde{\beta}'}{\left(1-\tilde{\beta}(\tau_L)\right)^2}\right)}{\left(\dfrac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial T_i} + ge^{-gT_i}} < 0$$

From the last two exressions we derive the following diagram:



The effects of a labor tax increase

Since x decreases it is straightforward to see that unemployment increases as a consequence of a labor tax increase.

4.2.2. An energy tax increase

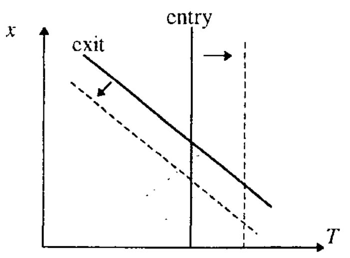
Consider now an increase in the energy tax. From (24a) we can derive

$$\left(\frac{dT_i}{d\tau_E}\right)^{entry} = \frac{\dfrac{I_\mu \dfrac{\partial\mu}{\partial\tau_E}}{\left(1-\tilde{\beta}(\tau_L)\right)}}{ge^{-gT_i}\int_0^{T_i} e^{(r+\delta-g)s}ds - \dfrac{I_\mu \dfrac{\partial\mu}{\partial T_i}}{\left(1-\tilde{\beta}(\tau_L)\right)}} > 0$$

This is the same expression as in section 3, therefore, the same interpretation applies here. From (24b) we can derive

$$\left(\frac{dT_i}{d\tau_E}\right)^{exit} = \frac{-\left(\dfrac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial\tau_E} - pB\mu^{-\sigma}}{\left(\dfrac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)} - \sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial T_i} + ge^{-gT_i}} < 0$$

Also this expression is not new. Since an energy tax increase shifts the entry curve to the right and the exit curve to the left, we can display the effects of an energy tax increase in the following diagram



The effects of an energy tax increase

Also here the effects on unemployment are clear. Since T_i does not influence the unemployment rate, unemployment will increase as a consequence of an energy tax increase.

4.3. Environmental tax reform

We first explain the channels through which a revenue neutral tax reform that increases τ_E and adjusts τ_L affects energy demand. In appendix F we show that aggregate energy demand can be written as

$$(26) \qquad E(t) = \frac{x}{\delta + x} B\mu(T_i; \tau_E; \tau_L)^{-\sigma}$$

An increase in the energy tax and a decrease in the labor tax have a positive effect on the energy saving technology. Hence, energy consumption is reduced through this channel. Unclear is the effect of the proposed tax reform on the variables x and T_i. An increase in x raises the number of firms and hence, increases energy demand. Since at the same time a higher value of x means less unemployment there is an objective conflict operating through x in environmental tax reforms that have the aim of reducing energy consumption and unemployment. If firms destroy old jobs after scrapping old technologies this objective conflict also operates through T_i. If firms update old technologies T_i has only the qualitative effect on energy demand working through μ. Since the decision whether to update or not depends only on the sunk implementation cost, we have another reason why sunk cost might play an important role in the evaluation of the cost of environmental policy.

The revenue from the labor tax can be written as

(27)

$$\tau_L \int_0^{T_i} f(a;t)w(t-a;a)da = \tau_L \frac{x}{\left(1-e^{-\delta T_i}\right)} \frac{\delta}{\delta + x} \int_0^{T_i} w(t-a;a)e^{-\delta a} da$$

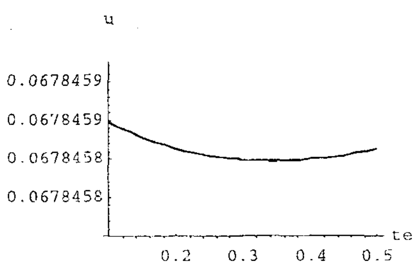
$$= \frac{\tau_L x}{\left(1-e^{-\delta T_i}\right)} \frac{\delta}{\delta + x} \frac{\beta A(t)}{(1+\beta\tau_L)} \int_0^{T_i} e^{-(\delta+g)a} da$$

$$-\frac{\tau_L x}{\delta + x} \frac{\beta(1+\tau_E)pA(t)B\mu(T_i;\tau_E;\tau_L)^{-\sigma}}{(1+\beta\tau_L)} + \frac{\tau_L x}{\delta + x} \frac{\beta x\left[K + I(\mu(T_i;\tau_E;\tau_L))\right]A(t)}{(1+\beta\tau_L)}$$

The government budget restriction is

$$\text{(28)} \qquad \tilde{g}A(t) = \tau_E pA(t)E(t) + \tau_L \int_0^{T_i} f(t;a)w(t-a;a)da$$

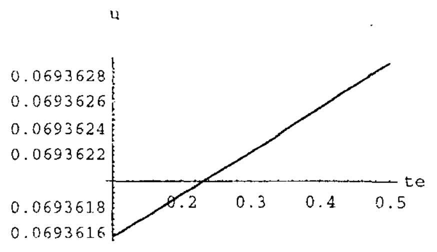
Also here we were not able to derive general results. Therefore, also in this section we run numerical examples. Here, we found mixed results. A double dividend is possible, if the labor tax is initially low and energy demand is inelastic, i. e. σ is small.



The effects of an environmental tax reform on unemployment with parameters:

β=0.5; p=0.01; r=0.07; δ=0.01; B=0.1; g=0.01, c_0=0.1, C=0.5, K=3, σ=0.01, $\tilde{g}$=0.001

Notice that the value for $\tilde{g}$ is extremely small. A ceteris paribus increase to $\tilde{g}$=0.01 leads to the following diagram:



The effects of an environmental tax reform on unemployment

In all numerical examples we found a double dividend only for the case that the initial labor tax, or equivalently $\tilde{g}$, is small.

5. Effects of an environmental tax reform in the short run

In this section we discuss the short term effects of an environmental tax reform. First we consider the case of destructive technological progress.

At each moment in time, exit is determined by

(29)

$$A(t - T_d) - (1 + \tau_E)p(t)B\mu(t - T_d)^{-\sigma} - (1 + \tau_L)w(t - T_d; T_d; \mu(t - T_d)) = 0$$

In order to derive equation (30) we solve the integral in equation (11) and subsequently solve for the term $\tilde{\beta}(\tau_L)x(t)JA(t)$ and substitute this expression in the wage equation (10) and successively the wage in (29). Then we arrive at

$$\left(1 - \tilde{\beta}(\tau_L)\right)\left[A(t)\frac{1 - e^{-(r+\delta)T_d}}{r + \delta} - A(t - T_d)\right]$$

$$(30) \qquad +\left(1 - \tilde{\beta}(\tau_L)\right)\int_0^{T_d} e^{-(r+\delta)s}(1 + \tau_E)pe^{gs}A(t)B\left(\mu(t - T_d)^{-\sigma} - \mu(t)^{-\sigma}\right)ds$$

$$= \left[K + I(\mu(t))\right]A(t)$$

The first term on the left hand side describes the productivity advantage of the new vintage to the oldest vintage currently in usage. The productivity advantage is always positive due to technological progress. The second term describes the advantage of the new vintage in energy consumption to the exiting vintage. The right hand side describes the installation cost of the new vintage.

In equilibrium the sum of the advantages has to equal the installation cost. Equation (30) makes clear that it is not enough for the new vintage to be more productive and to consume less energy than the oldest vintage. The advantages of the new vintage must be large enough to justify the installation cost of the new vintage. Equation (30) gives the minimum advantage that is necessary to justify the installation cost of the new vintage. We consider an environmental tax reform that increases τ_E and decreases τ_L in a revenue neutral way between t-T_d and t. We assume that the scrapping age T_d remains constant for simplicity. The environmental tax reform has the following effects:

After the energy tax τ_E increases, the new vintages are equipped with a better energy saving technology so $\mu(t)$ increases. This raises the energy saving advantage of the new vintage. At the same time the installation cost of the new vintage increases as well. The first effect makes installation of the new vintage more profitable and the second effect makes the new vintage less profitable. The larger the first effect and the smaller the second effect the faster the replacement of the old vintage through the new vintage.

The decrease in τ_L will increase the first and the second effect since also a decrease in τ_L increases $\mu(t)$. In addition to these two effects the change in τ_L also decreases $\tilde{\beta}(\tau_L)$. This effect reduces the advantage of the new vintage, since it reduces the wage payments not only of the new vintage but also of the old vintage. This will raise the advantage that is necessary to justify the installation cost of the new vintage.

Since these effects are opposing in sign it is possible that the environmental tax reform actually protects older production units from an earlier exit and prevents the new, more energy saving, vintage from an earlier entry. Therefore, a decrease in τ_L might actually insulate the old vintage from changes. Two effects are inevitable. An increase in $\mu(t)$

will always have two opposing effects, namely an increase in the advantage of the new vintage and an increase in the installation cost of the new vintage. This effect is necessary in order to decrease aggregate energy demand. The decrease in $\tilde{\beta}(\tau_L)$, however, can be avoided, if instead of τ_L the installation cost of the new vintage is reduced. E. g. a cut in K reduces the necessary advantage of the new vintage without having any negative effects. A cut in $I(\mu(t))$ leads to a larger $\mu(t)$ and also increases the advantage of the new vintage. At the same time both policies compensate for the increase in installation cost.

Similar arguments will hold when technological progress is neoclassical. However, the short term effects on unemployment are different in the creative destruction and the neoclassical approach. In the model with creative destruction early exit of an old vintage also means more unemployment, while in the neoclassical model exit of the old vintage has no consequences for unemployment. In the model of creative destruction there is a short term trade off between energy saving and unemployment. A cut of the labor tax might compensate the negative effects short term effects on unemployment caused by an energy tax increase. But this will happen at the cost of a smaller environmental effect. In the short term it might be even possible that no reaction will take place after the environmental tax reform since the advantage of the new vintage does not justify its installation cost. In this case only in the long term will the environmental tax reform lead to less energy consumption of the production sector.

6. Conclusions

A government that wants to reduce energy input and reduce unemployment at the same time can only mean to prompt installation of new technologies and to avoid that implementation of new technologies leads to job destruction. An environmental tax reform that increases the energy tax and adjusts the labor tax in a revenue neutral way might have the desired effects if the updating cost of old production units and the initial tax on labor are sufficiently small. However, in numerical examples we found only very small decreases of unemployment. For reasonable parameter values we found that unemployment slightly increases in both benchmark models in the long run. We conclude from this that policymakers cannot expect an environmental tax reform, as specified above, to reduce unemployment, i.e. to yield a double dividend.

Another result is that in the short run the described environmental tax reform might slow down the installation of new technologies and help to keep alive old production units with high energy demand. This is so, because also old production units benefit from the labor tax cuts. This increases the necessary advantage in productivity and energy efficiency to justify their installation or updating cost. This insulation effect of old production units can be avoided if instead of the labor tax the installation cost of new production units or the updating cost of old production units are reduced. In this case the installation of more energy saving technologies would take place at a faster pace. In the case of creative destruction this would lead to more unemployment in the short run. Reducing the updating cost might also cause a switch for some production units from the first benchmark model to the second benchmark model. In the first benchmark model the unemployment rate is always higher than in the second benchmark model. A switch from the first to the second benchmark model, might also cut the unemployment level. This switch could be achieved through a decrease in the updating cost, In this case technological progress might loose its destructive effects. Therefore, a government should make updating possible for as many production units as possible through a cut in the updating and installation cost.

References

Aghion, Philippe and Howitt, Peter, (1994), Growth and Unemployment, Review of Economic Studies, (61), 2, pp. 477-494.

Bovenberg, A. Lans and de Mooij, Ruud A., (1994), Environmental Levies and Distortionary Taxation, American Economic Review (Papers and Proceedings), (94), 4, pp. 1085-1089.

Bovenberg, A. Lans and de Mooij, Ruud A., (1995), Environmental Taxation and the Double-Dividend: The Role of Factor Substitution and Capital Mobility, Discussionpaper prepared for the SFB-Workshop 1995: Environmental Policy in Open Economies, June 5-7, 1995, in Konstanz , pp. 29.

Bovenberg, A. Lans and van der Ploeg, Frederick, (1994), Environmental Policy, Public Finance and the Labour Market in a Second-Best World, Journal of Public Economics, (55), pp. 349-390.

Bovenberg, A. Lans and van der Ploeg, Frederick, (1995), Tax Reform, Structural Unemployment and the Environment, Fondazione Eni Enrico Mattei. (Nota di Lavoro 6.95), pp. 18.

Caballero, Ricardo J. and Hammour, Mohamad L., (1994), On the Timing and Efficiency of Creative Destruction, NBER Working Paper, (No. 4768), pp. 49.

Caballero, Ricardo J. and Hammour, Mohamad L., (1995), On the Ills of Adjustment, NBER Working Paper, (No. 5198), pp. 34.

Caballero, Ricardo J. and Hammour, Mohamad L., (1996), The "Fundamental Transformation" in Macroeconomics, NBER Working Paper, (No. 5471), pp. 12.

Davis, Steven J. and Haltiwanger, John, (1990), Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications, in Blanchard, Olivier J. and Fischer, Stanley, (Ed.), NBER Macroeconomics Annual, Cambridge, pp. 123-186.

Enquete Kommission des Deutschen Bundestages zum Schutz des Menschen und der Umwelt, (1994), Bewertungskriterien und Perspektiven für umweltverträgliche Stoffkreisläufe in der Industriegesellschaft, Bonn: Deutscher Bundestag.

Frisch, Helmut, (1991), Wirtschaftswachstum und Energieversorgung, Wirtschaftspolitische Blätter, (38), 2, pp. 146-154.

Goulder, Lawrence H., (1995), Environmental Taxation and the Double Dividend: A Reader's Guide, International Tax and Public Finance, (2), 2, pp. 157-184.

Klodt, Henning, (1994), Energy Consumption and Structural Adjustment: The Case of West Germany, Structural Change and Economic Dynamics (5), 1, pp. 177-185.

Koskela, Erkki and Schöb, Ronnie, (1996), Alleviating Unemployment: The Case for Green Tax Reforms, Working Paper of the Center of Economic Studies, Munich, (106), pp. 38.

Kreuzberg, Peter, (1996), Energieeinsparung als Versicherung gegen Marktrisiken, München: R. Oldenbourg Verlag, München.

Mortensen, Dale T. and Pissarides, Christopher A., (1995), Technological Progress, Job Creation and Job Destruction, Discussion Paper Series of the Centre for Economic Performance , (No. 264), pp. 28.

Pearce, David, (1991), The Role of Carbon Taxes in Adjusting to Global Warming, Economic Journal, (101), pp. 938-948.

Pissarides, Christopher A., (1990), Equilibrium Unemployment Theory, Oxford: Basil Blackwell.

Schurr, Sam H., Burwell, Calvin C., Devine, Jr. Warren D., and Sonenblum, Sidney, (1990), Electricity in the American Economy. Agent of Technological Progress, New York: Greenwood Press.

Appendix A

In this appendix we derive the wage equation. Differentiating (4) with respect to a gives

$$J_a(t;a;\mu(t)) = -\left[A(t)-(1+\tau_E)p(a)B\mu(t)^{-\sigma}-(1+\tau_L)w(t;a;\mu(t))\right] + (r+\delta)J(t;a;\mu(t)) \quad (31)$$

Differentiating (7) and (8) with respect to a gives

$$W_a(t;a) = -\left[w(t;a;\mu(t))+\delta U(t+a)\right]+(r+\delta)W(t;a) \quad (32)$$

The value of being unemployed equals

$$U_a(t+a) = -\left[m(1;\theta)W(t+a)\right]+(r+m(1;\theta))U(t+a) \quad (33)$$

From these three equations we obtain the following two equations

$$(r+\delta)J(t;a;\mu(t)) = \left[A(t)-(1+\tau_E)p(a)B\mu(t)^{-\sigma}-(1+\tau_L)w(t;a;\mu(t))\right] + J_a(t;a;\mu(t))$$

and

$$(r+\delta)(W(t;a)-U(t+a)) = w(t;a;\mu(t))$$
$$+\left(W_a(t;a)-U_a(t+a)\right) - m(1;\theta)(W(t+a)-U(t+a))$$

Inserting the equations into

$$(9) \qquad (1-\beta)(W(t;a)-U(t+a)) = \beta J(t;a;\mu(t))$$

where we consider equation (10) gives:

$$(1-\beta)\left(w(t;a;\mu(t)) - m(1;\theta)(W(t+a)-U(t+a))\right) =$$
$$\beta\left(\left[A(t)-(1+\tau_E)p(a)B\mu(t)^{-\sigma} - (1+\tau_L)w(t;a;\mu(t))\right]\right)$$

Solving for $w(t;a;\mu(t))$ gives

$$w(t;a;\mu(t)) = \frac{\beta}{(1+\beta\tau_L)}\left[A(t)-(1+\tau_E)p(a)B\mu(t)^{-\sigma}\right]$$
$$+\frac{(1-\beta)m(1;\theta)(W(t+a)-U(t+a))}{(1+\beta\tau_L)}$$

Substituting (9) yields

$$w(t;a;\mu(t)) = \frac{\beta}{(1+\beta\tau_L)}\left[A(t)-(1+\tau_E)p(a)B\mu(t)^{-\sigma}\right]$$
$$+\frac{\beta}{(1+\beta\tau_L)}m(1;\theta)J(t+a;0;\mu(t+a))$$

Appendix B

In this appendix we derive equation (11). Inserting equation (5) into (4) gives for *a=0*:

$$(34) \qquad J(t;0;\mu(t)) = \int_0^{T_d}\Big[(1+\tau_E)pA(t)B\mu(t)^{-\sigma}\left[e^{gT_d}-e^{gs}\right]$$
$$+(1+\tau_L)\left[w(t;T_d;\mu(t)) - w(t;s;\mu(t))\right]\Big]e^{-(r+\delta)s}ds$$

Inserting equation

$$w(t;a;\mu(t)) = \frac{\beta}{(1+\beta\tau_L)}\left[A(t) - (1+\tau_E)p(a)B\mu(t)^{-\sigma}\right] \tag{10}$$
$$+\frac{\beta}{(1+\beta\tau_L)}m(1;\theta)J(t+a;0;\mu(t+a))$$

into equation

$$J(t;0;\mu(t)) = \int_0^{T_d}\left[(1+\tau_E)pA(t)B\mu(t)^{-\sigma}\left[e^{gT_d} - e^{gs}\right]\right. \tag{34}$$
$$\left.+(1+\tau_L)\left[w(t;T_d;\mu(t)) - w(t;s;\mu(t))\right]\right]e^{-(r+\delta)s}ds$$

gives

$$J(t;0;\mu(t)) = \int_0^{T_d}\left[(1+\tau_E)pA(t)B\mu(t)^{-\sigma}\left[e^{gT_d} - e^{gs}\right]\right.$$
$$-\frac{(1+\tau_L)\beta}{(1+\beta\tau_L)}\left[A(t) - (1+\tau_E)pe^{gs}A(t)B\mu(t)^{-\sigma}\right] \tag{35}$$
$$\left.+(1+\tau_L)w(t;T_d;\mu(t)) - \frac{(1+\tau_L)\beta}{(1+\beta\tau_L)}m(1;\theta)J(t+s;0;\mu(t+s))\right]e^{-(r+\delta)s}ds$$

Solving

$$A(t) - (1+\tau_E)pA(t)e^{gT_d}B\mu(t)^{-\sigma} - (1+\tau_L)w(t;T_d;\mu(t)) = 0 \tag{5}$$

for $(1+\tau_L)w(t;T_d;\mu(t))$ and substituting the result in (35) gives

(11)

$$J(t;0;\mu(t)) = \int_0^{T_d}\left[\left(1 - \frac{(1+\tau_L)\beta}{(1+\beta\tau_L)}\right)\left[A(t) - (1+\tau_E)pe^{gs}A(t)B\mu(t)^{-\sigma}\right]\right.$$
$$\left.-\frac{(1+\tau_L)\beta}{(1+\beta\tau_L)}m(1;\theta)J(t+s;0;\mu(t+s))\right]e^{-(r+\delta)s}ds$$

Now we derive equation (13). From (10) follows

$$w_\mu(t;s;\mu(t)) = \frac{\beta}{(1+\beta\tau_L)}\sigma(1+\tau_E)pe^{gs}A(t)B\mu(t)^{-(1+\sigma)}$$

Inserting this expression into

$$A(t)I_\mu = \int_0^{T_d}\left[\sigma(1+\tau_E)p(s)B\mu(t)^{-(1+\sigma)} - (1+\tau_L)w_\mu(t;s;\mu(t))\right]e^{-(r+\delta)(s-a)}ds \tag{6}$$

yields

$$(13) \qquad I_\mu = \int_0^{T_d} \left(1-\tilde{\beta}\right)\sigma\left(1+\tau_E\right) p e^{gs} B\mu(t)^{-(1+\sigma)} e^{-(r+\delta)(s-a)} ds$$

Appendix C

Inserting the exit condition in steady state form

$$(15) \qquad \left(1-\tilde{\beta}\right)\left[1-\left(1+\tau_E\right) p e^{gT_d} B\mu(t)^{-\sigma}\right] = \tilde{\beta} m(1;\theta) J e^{gT_d}$$

into the entry condition

$$(12) \qquad J = \int_0^{T_d} \left[\left(1-\tilde{\beta}\right)\left[1-\left(1+\tau_E\right) p e^{gs} B\mu(t)^{-\sigma}\right] - \tilde{\beta} m(1;\theta) J e^{gs}\right] e^{-(r+\delta)s} ds$$

gives

$$\frac{J}{\left(1-\tilde{\beta}\right)} = \int_0^{T_d} \left[1-e^{gs}e^{-gT_d}\right] e^{-(r+\delta)s} ds$$

Under consideration of (14) we end with

$$(16a) \qquad \frac{\left[K + I\left(\mu\left(T_d;\tau_E;\tau_L\right)\right)\right]}{\left(1-\tilde{\beta}\right)} = \int_0^{T_d} \left[1-e^{gs}e^{-gT_d}\right] e^{-(r+\delta)s} ds$$

and from (15)

$$(16b) \qquad \left[1-\left(1+\tau_E\right) p e^{gT_d} B\mu\left(T_d;\tau_E;\tau_L\right)^{-\sigma}\right] = \frac{\tilde{\beta}}{\left(1-\tilde{\beta}\right)} x \left[K + I\left(\mu\left(T_d;\tau_E;\tau_L\right)\right)\right] e^{gT_d}$$

Appendix D

Differentiating

$$(16a) \qquad \frac{\left[K + I\left(\mu\left(T_d;\tau_E;\tau_L\right)\right)\right]}{\left(1-\tilde{\beta}(\tau_L)\right)} = \int_0^{T_d} \left[1-e^{gs}e^{-gT_d}\right] e^{-(r+\delta)s} ds$$

$$(16b) \qquad \frac{\tilde{\beta}(\tau_L)}{\left(1-\tilde{\beta}(\tau_L)\right)} x \left[K + I\left(\mu\left(T_d;\tau_E;\tau_L\right)\right)\right] e^{gT_d} = \left[1-\left(1+\tau_E\right) p e^{gT_d} B\mu\left(T_d;\tau_E;\tau_L\right)^{-\sigma}\right]$$

The differentiated equation system (16) takes the following form

$$\begin{pmatrix} 0 & -\left[\int_0^{T_d} g e^{-gT_d} e^{-(r+\delta-g)s} ds - \dfrac{I_\mu \frac{\partial\mu}{\partial T_d}}{(1-\tilde{\beta})}\right] \\ \tilde{\beta}\dfrac{[K+I(\mu)]}{(1-\tilde{\beta})} & \left[\left(\dfrac{\tilde{\beta} x I_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial T_d} + g e^{-gT_d}\right] \end{pmatrix} \begin{pmatrix} dx \\ dT_d \end{pmatrix} =$$

$$\begin{pmatrix} -\dfrac{I_\mu \frac{\partial\mu}{\partial \tau_E}}{(1-\tilde{\beta})} d\tau_E - \left[\dfrac{I_\mu \frac{\partial\mu}{\partial \tau_L}}{(1-\tilde{\beta})} + \dfrac{\tilde{\beta}'[K+I(\mu)]}{(1-\tilde{\beta})^2}\right] d\tau_L \\ -\left(\left(\dfrac{\tilde{\beta} x I_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial \tau_E} + p\mu^{-\sigma}\right) d\tau_E - \left(x\left(\tilde{\beta}\dfrac{x I_\mu \frac{\partial\mu}{\partial \tau_L}}{(1-\tilde{\beta})} + \dfrac{[K+I(\mu)]\tilde{\beta}'}{(1-\tilde{\beta})^2}\right) - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\dfrac{\partial\mu}{\partial \tau_L}\right) d\tau_L \end{pmatrix}$$

In this case we have

$$Det = \left[\int_0^{T_d} g e^{-gT_d} e^{-(r+\delta-g)s} ds - \frac{I_\mu \frac{\partial\mu}{\partial T_d}}{(1-\tilde{\beta})}\right] \tilde{\beta}\frac{[K+I(\mu)]}{(1-\tilde{\beta})} > 0$$

$$\frac{dx}{d\tau_L} = -\frac{\left[\dfrac{I_\mu \frac{\partial\mu}{\partial \tau_L}}{(1-\tilde{\beta})} + \dfrac{\tilde{\beta}'[K+I(\mu)]}{(1-\tilde{\beta})^2}\right]}{\left[\int_0^{T_d} g e^{-gT_d} e^{-(r+\delta-g)s} ds - \dfrac{I_\mu \frac{\partial\mu}{\partial T_d}}{(1-\tilde{\beta})}\right]} \frac{\left[\left(\dfrac{\tilde{\beta} x I_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\dfrac{\partial\mu}{\partial T_d} + g e^{-gT_d}\right]}{\tilde{\beta}\dfrac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

$$-\frac{\left(x\left(\tilde{\beta}\dfrac{x I_\mu \frac{\partial\mu}{\partial \tau_L}}{(1-\tilde{\beta})} + \dfrac{[K+I(\mu)]\tilde{\beta}'}{(1-\tilde{\beta})^2}\right) - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\dfrac{\partial\mu}{\partial \tau_L}\right)}{\tilde{\beta}\dfrac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

$$\frac{dT_d}{d\tau_L}=\frac{\left[\frac{I_\mu \frac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}\right)}+\frac{\tilde{\beta}'\left[K+I(\mu)\right]}{\left(1-\tilde{\beta}\right)^2}\right]}{\left[\int_0^{T_d} ge^{-gT_d}e^{-(r+\delta-g)s}ds-\frac{I_\mu \frac{\partial\mu}{\partial T_d}}{\left(1-\tilde{\beta}\right)}\right]}$$

$$\frac{dx}{d\tau_E}=-\frac{\frac{I_\mu \frac{\partial\mu}{\partial\tau_E}}{\left(1-\tilde{\beta}\right)}}{\left[\int_0^{T_d} ge^{-gT_d}e^{-(r+\delta-g)s}ds-\frac{I_\mu \frac{\partial\mu}{\partial T_d}}{\left(1-\tilde{\beta}\right)}\right]}\frac{\left[\left(\frac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)}-\sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial T_d}+ge^{-gT_d}\right]}{\tilde{\beta}\frac{\left[K+I(\mu)\right]}{\left(1-\tilde{\beta}\right)}}$$

$$-\frac{\left(\left(\frac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)}-\sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial\tau_E}+p\mu^{-\sigma}\right)}{\tilde{\beta}\frac{\left[K+I(\mu)\right]}{\left(1-\tilde{\beta}\right)}}$$

$$\frac{dT_d}{d\tau_E}=\frac{\frac{I_\mu \frac{\partial\mu}{\partial\tau_E}}{\left(1-\tilde{\beta}\right)}}{\left[\int_0^{T_d} ge^{-gT_d}e^{-(r+\delta-g)s}ds-\frac{I_\mu \frac{\partial\mu}{\partial T_d}}{\left(1-\tilde{\beta}\right)}\right]}$$

The expressions $\frac{dx}{d\tau_L}$ and $\frac{dx}{d\tau_E}$ can be rearranged with the help of $\frac{dT_d}{d\tau_L}$ and $\frac{dT_d}{d\tau_E}$ to give

$$\frac{dx}{d\tau_L} = -\frac{dT_d}{d\tau_L}\frac{\left[\left(\frac{\tilde{\beta}xI_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial T_d} + ge^{-gT_d}\right]}{\tilde{\beta}\frac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

$$-\frac{\left(x\left(\tilde{\beta}\frac{xI_\mu\frac{\partial\mu}{\partial\tau_L}}{(1-\tilde{\beta})} + \frac{[K+I(\mu)]\tilde{\beta}'}{(1-\tilde{\beta})^2}\right) - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\frac{\partial\mu}{\partial\tau_L}\right)}{\tilde{\beta}\frac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

$$\frac{dx}{d\tau_E} = -\frac{dT_d}{d\tau_E}\frac{\left[\left(\frac{\tilde{\beta}xI_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial T_d} + ge^{-gT_d}\right]}{\tilde{\beta}\frac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

$$-\frac{\left(\left(\frac{\tilde{\beta}xI_\mu}{(1-\tilde{\beta})} - \sigma(1+\tau_E)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial\tau_E} + p\mu^{-\sigma}\right)}{\tilde{\beta}\frac{[K+I(\mu)]}{(1-\tilde{\beta})}}$$

Differentiating the unemployment equation (18) with respect to τ_L and τ_E, respectively yields

$$\frac{du}{d\tau_L} = -u^2\left[\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\frac{dx}{d\tau_L} + xe^{-\delta T_d}\frac{dT_d}{d\tau_L}\right]$$

and

$$\frac{du}{d\tau_E} = -u^2\left[\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\frac{dx}{d\tau_E} + xe^{-\delta T_d}\frac{dT_d}{d\tau_E}\right]$$

Substituting the above expressions and considering (17b) gives

$$\frac{du}{d\tau_L}=-u^2\left[\frac{dT_d}{d\tau_L}\left(\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\left(\frac{dx}{dT_d}\right)^{exit}+xe^{-\delta T_d}\right)\right.$$

$$\left.-\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\frac{\left(x\left(\tilde{\beta}\frac{xI_\mu\frac{\partial\mu}{\partial\tau_L}}{\left(1-\tilde{\beta}\right)}+\frac{\left[K+I(\mu)\right]\tilde{\beta}'}{\left(1-\tilde{\beta}\right)^2}\right)-\sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\frac{\partial\mu}{\partial\tau_L}\right)}{\tilde{\beta}\frac{\left[K+I(\mu)\right]}{\left(1-\tilde{\beta}\right)}}\right]$$

and

$$\frac{du}{d\tau_E}=-u^2\left[\frac{dT_d}{d\tau_E}\left(\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\left(\frac{dx}{dT_d}\right)^{exit}+xe^{-\delta T_d}\right)\right.$$

$$\left.-\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\frac{\left(\left(\frac{\tilde{\beta}xI_\mu}{\left(1-\tilde{\beta}\right)}-\sigma\left(1+\tau_E\right)pB\mu^{-(1+\sigma)}\right)\frac{\partial\mu}{\partial\tau_E}+p\mu^{-\sigma}\right)}{\tilde{\beta}\frac{\left[K+I(\mu)\right]}{\left(1-\tilde{\beta}\right)}}\right]$$

From these last two expressions we can see that a sufficient condition for a negative effect of the labor tax and energy tax, respectively on unemployment, $\frac{\left(1-e^{-\delta T_d}\right)}{\delta}\left(\frac{dx}{dT_d}\right)^{exit}+e^{-\delta T_d}x\leq 0$, are the same.

Appendix E

The optimal investment decision in energy saving technology is determined as in section 3. Therefore, condition (13) is also valid in section 4 and we can drop the notation $\max_{\mu}$. The wage equation which does not change can be calculated analogously to the preceding section. Inserting the wage equation (10) into (22) gives

$$J(t;a;\mu(t)) = \max_{T_i} \int_a^{T_i} \Big[\big(1-\tilde{\beta}(\tau_L)\big)\big[A(t)-(1+\tau_E)p(s)B\mu(t)^{-\sigma}\big]$$

$$(36) \qquad -\tilde{\beta}(\tau_L)m(1;\theta)J(t+a;0;\mu(t+a))\Big]e^{-(r+\delta)(s-a)}ds$$

$$+e^{-(r+\delta)(T_i-a)} \max_{\mu(t+T_i)}\Big[J\big(t+T_i;0;\mu(t)\big)-A\big(t+T_i\big)\big[C+I\big(\mu\big(t+T_i\big)\big)\big]\Big]$$

and in steady state with constant θ and μ

(37)

$$\frac{J}{\big(1-\tilde{\beta}(\tau_L)\big)} = \max_{T_i}\int_0^{T_i}\left[\big[1-(1+\tau_E)pe^{gs}B\mu(t)^{-\sigma}\big]-\frac{\tilde{\beta}(\tau_L)}{\big(1-\tilde{\beta}(\tau_L)\big)}xe^{gs}J\right]e^{-(r+\delta)s}ds$$

$$+\frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}e^{-(r+\delta-g)T_i}$$

with $J=\big[K+I(\mu)\big]$.

Under consideration of

$$\int_0^{T_i}\frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}(r+\delta-g)e^{-(r+\delta-g)s}ds = \frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}-\frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}e^{-(r+\delta-g)T_i} \Leftrightarrow$$

$$\frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}e^{-(r+\delta-g)T_i} = \frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}-\int_0^{T_i}\frac{\big[J-C-I(\mu)\big]}{\big(1-\tilde{\beta}(\tau_L)\big)}(r+\delta-g)e^{-(r+\delta-g)s}ds$$

equation (37) can be rearranged to

$$\frac{C+I(\mu)}{\big(1-\tilde{\beta}(\tau_L)\big)} = \max_{T_i}\int_0^{T_i}\Bigg[\big[1-(1+\tau_E)pe^{gs}B\mu(t)^{-\sigma}\big]$$

$$(38) \qquad -\left[\frac{\tilde{\beta}(\tau_L)}{\big(1-\tilde{\beta}(\tau_L)\big)}xJ+\frac{(r+\delta-g)}{\big(1-\tilde{\beta}(\tau_L)\big)}\big[J-I(\mu)\big]\right]e^{gs}\Bigg]e^{-(r+\delta)s}ds$$

From (38) we derive the following steady state exit condition

$$e^{-gT_i}-(1+\tau_E)pB\mu(T_i;\tau_L;\tau_E)^{-\sigma}$$

$$(24b) \qquad = \frac{\tilde{\beta}(\tau_L)}{\big(1-\tilde{\beta}(\tau_L)\big)}x\big[K+I\big(\mu(T_i;\tau_L;\tau_E)\big)\big]+\frac{(r+\delta-g)}{\big(1-\tilde{\beta}(\tau_L)\big)}K$$

Substituting (24b) into (38) yields the steady state entry condition

$$\text{(24a)} \qquad \frac{C + I(\mu(T_i;\tau_L;\tau_E))}{(1-\tilde{\beta}(\tau_L))} = \int_0^{T_i}\left[1 - e^{gs}e^{-gT_i}\right]e^{-(r+\delta)s}ds$$

Appendix F

Aggregate energy demand is

$$E(t) = \int_0^{T_i} B\mu(t-a)^{-\sigma} f(a;t)da$$

with $f(a;t)$ describing the number of firms employing at date t a technology of age a. $f(a;t)$ equals the number of hires at date t-a plus the number of firms that had at date t-a a technology of scrapping age T_i:

$$f(a;t) = H(t-a)e^{-\delta a} + f(T_i;t-a)e^{-\delta a}$$

where

$$f(T_i;t-a) = H(t-a-T_i)e^{-\delta T_i} + f(T_i;t-a-T_i)e^{-\delta T_i}$$

Hence, $f(a;t)$ can be expressed as

$$f(a;t) = e^{-\delta a}\sum_{n=0}^{\infty} H(t-a-T_i n)e^{-\delta n T_i}$$

If the economy has been in a steady state for a long time, we can approximate the number of hires that have been done out of steady state through the steady state value $H(t-a)$. Since these values are discounted with $e^{-\delta n T_i}$, the differences are negligibly small. Hence,

$$f(a;t) = e^{-\delta a}H(t-a)\sum_{n=0}^{\infty} e^{-\delta n T_i} = \frac{e^{-\delta a}}{(1-e^{-\delta T_i})}H(t-a)$$

Therefore, steady state energy demand is

$$E(t) = B\mu^{-\sigma}\frac{H}{(1-e^{-\delta T_i})}\int_0^{T_i} e^{-\delta a}da$$

Substituting $H = xu = \frac{x\delta}{\delta + x}$, we get

$$E(t) = \frac{x\delta}{\delta + x} \frac{B\mu(T_i;\tau_E;\tau_L)^{-\sigma}}{\left(1 - e^{-\delta T_i}\right)} \int_0^{T_i} e^{-\delta a} da \Leftrightarrow$$

(26) $$E(t) = \frac{x}{\delta + x} B\mu(T_i;\tau_E;\tau_L)^{-\sigma}$$