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Working Paper

A structural policy model for the Federal Republic of Germany

Kiel Working Papers, No. 240

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Kiel Working Papers

Working Paper No. 240

A Structural Policy Model for the Federal Republic of Germany*

by

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ISSN 0342 - 0787

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October 1985

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ISSN 0342 - 0787

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I. Introduction

In the German public debate about economic policy options structural effects are considered important. Researchers, consequently, are often asked to project the medium- and long-run impact of disputed interventions on variables like employment and income by production branch, region, occupation, household income class or some other distinction. Such requests are made for a wide array of policies including, i.a., agricultural price guarantees, steel production subsidies, textile import quotas, regional tax preferences, public spending programmes or the dismantling of any of these or other interventions. To meet the manifold demand for their services researchers need to integrate the available theoretical and empirical knowledge in a computable, economy-wide and disaggregated model which is flexible enough to allow for the simulation of numerous policies.

In search for such a model for the Federal Republic several authors have disaggregated business-cycle models and have extended them to the long-run.[1] The impressive work on these dynamic disequilibrium structural forecasting systems, most notably Kiy (1984), follows the tradition of the Wharton School which in turn has its origin in Klein's seminal monograph on economic fluctuations in the United States 1921-1941 (Klein, 1950). For each endogenous variable, the systems simulate a sequence of quarterly results. The legacy of the short-run macro forecasting model makes itself felt in a strong domination by the categories of final demand. Relative prices play

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* The Structural Policy Model for the Federal Republic of Germany was constructed and tested by Egbert Gerken, Martin Groß and Ulrich Lächler. These authors closely followed the lines of earlier research undertaken in the Department IV of the Kiel Institute of World Economics to which David Vincent made an essential contribution. In describing model theory and solution procedures the authors of this report borrowed heavily from a Working Paper (Vincent, 1981) and an unpublished consultant report by David Vincent. For a recent application in a study on subsidies commissioned by the WIRTSCHAFTSWOCHE see Gerken, Jüttemeier, Schatz and Schmidt (1985).

1 See Erber, Haas and Kiy (1984) for a short and enlightening discussion of these disaggregated long-term forecasting systems and also for the exposition of a comprehensive new attempt currently undertaken.

a minor role by comparison with "activity variables". Lags are freely used to come to grips with expectations and frictions in the system (Powell, 1981). These traits which are somewhat disquieting for a long-term analysis account for some of the reservation with which the models were initially received in part of the profession.

In a new development model builders go a long way to meet the critique by adding supply constraints and by more strictly observing microeconomic theory. One reservation, however, remains also with respect to the new model as described by Erber, Haas and Kiy (1984). The disequilibrium time sequence may or may not approach long-run equilibrium. Any simulation result about the impact of a certain policy change is, therefore, difficult to interpret. The researcher as the model user cannot rest assured that a result would hold when the model was modified so as to secure, say, the return to a zero balance of official settlements.

The researcher might want to concentrate on the equilibrium solution. He or she could then do with a comparative-static general equilibrium model. Such a model can be constructed in a way as to allow for short- as well as long-run solutions. Notwithstanding, it always has the disadvantage of not allowing for a precise time interpretation of results. The user receives no information on what happens between the date of policy intervention and the completion of the adjustment process. In their daily work, of course, researchers need to know about both the equilibrium solution and the disequilibrium path leading to it. Ideally, therefore, model builders would first explore the steady-state properties of their system and then estimate from time-series data the behavioural equations with these properties imposed. This is an extremely difficult and resource-using task which is unlikely to be accomplished in the near future.[1] Meanwhile, the researcher must make a choice. In the case of structural policies, we believe, it is rather important to be precise about the equilibrium towards which the economy is heading.

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[1] So far, only Kirkpatrick's aggregate cyclical growth model of the Federal Republic of Germany has been constructed and estimated in this way (Kirkpatrick, 1984).

In this paper we present a structural policy model of the comparative-static general equilibrium type. Our model for the Federal Republic of Germany closely resembles the Australian ORANI model (Dixon et al., 1982) which in turn has its origin in Johansen's pioneering work for Norway (Johansen, 1960). An attractive trademark of Johansen-models is that they are written as a set of structural equations which are linear in all growth rates. Exogenous and endogenous variables can be exchanged easily and solutions require no more than simple matrix operations. This makes for a highly flexible instrument of policy analysis. A consequence of linearisation, of course, is that model solutions provide only for linear approximations. However, a method has been developed to correct for the "linearisation error" in case of large policy changes.

The model theory is well advanced making it fairly easy to construct new members of the Johansen-class (Vincent, 1981). The model for the Federal Republic was developed at the Kiel Institute of World Economics and has, with modifications, been used by staff members on various occasions (Groß, 1984; Gerken, Groß and Lächler, 1984; Gerken, Jüttemeier, Schatz and Schmidt, 1985). The purpose of this paper is to document the equation system and the data base, to demonstrate closure options and to discuss the solution procedure. Results are not reported.

II. The Input-Output Base

The backbone of a structural model is provided by an Input-Output (IO) base. Note from the schematic presentation in Figure 1 that the base facilitates the inclusion of many types of commodity and factor flows, e.g., commodity flows from domestic and imported sources to current production, capital creation, households, government and exports, and factor flows (labour by occupation, fixed capital, industry-specific factors) to industries for use in current production. It thus allows the model user to trace, in considerable detail, the effects of alternative policies on the pattern as well as the level of domestic economic activity.

Figure 1 is shown as distinguishing g domestic commodities, g import commodities, r labour occupations and h domestic industries. The dimensions of g, r and h, that is, the degree of disaggregation can be chosen within the limits of the available IO table and further statistics. As yet, the model has been specified with the 1978 IO table of the Federal Statistical Office. The table was aggregated to 13 industries and 13 commodities. The Office's Fachserie 16 (Reihen 2.1 and 2.2) was used to disaggregate labour into two occupations along levels of skill. Figure 1 has no regional dimension as regional IO tables are not available for West Germany. As will be shown later, further disaggregation is fairly easy on the side of model technique. The binding constraint is the availability of data.

The first matrix $\tilde{A}$ of Figure 1 shows the flows of domestically produced commodities into the production processes of domestic industries. Matrix $\tilde{B}$ shows the flows of domestic commodities into capital formation, and column vectors $\tilde{C}$, $\tilde{D}$ and $\tilde{E}$ show the flows into household, export and other (mainly government) demands. The matrices $\tilde{F}$ and $\tilde{G}$ and the vectors $\tilde{H}$ and $\tilde{I}$ in the second row of Figure 1 contain the corresponding flows for imported commodities.[1]

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[1] Note that reexports of imports, without domestic processing, are not permitted.

Figure 1: Schematic Input Output Data Base

		Industries (producing current goods) h	Industries (capital formation) h	Household consumption 1	Exports 1	Other demands (govt.) 1	Trade taxes
Domestic commodities	g	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$	$\tilde{E}$	$\tilde{D}(S)$
Import commodities	g	$\tilde{F}$	$\tilde{G}$	$\tilde{H}$		$\tilde{I}$	$-\tilde{Z}$
Indirect taxes	1	$\tilde{N}$	$\tilde{B}\tilde{G}(S)$	$\tilde{W}$			
Subsidies on production	1	$-\tilde{N}(S)$					
Subsidies on intermediate inputs	1	$-\tilde{A}\tilde{F}(S)$					
Labour occupations	r	$\tilde{K}$					
Fixed capital	1	$\tilde{L}$					
Industry-specific factor	1	$\tilde{M}$					
Subsidies on use of labour by occupation	r	$\tilde{K}(S)$					
Subsidies on use of fixed capital	1	$\tilde{L}(S)$					
Subsidies on use of ind.-spec. factors	1	$\tilde{M}(S)$					
Commodity supplies	g	$\tilde{O}$					

The final column vectors in both rows show, respectively, the subsidy ($\tilde{D}(S)$) received for exports and the negative of the import duty ($-\tilde{Z}$) paid on imports. The row vectors $\tilde{N}$ and $\tilde{B}\tilde{G}(S)$ show, respectively, the production tax and the investment subsidy by industry, whereas the scalar $\tilde{W}$ shows the non-redeemable turnover tax which is treated as a household consumption tax. The row vectors ($-\tilde{N}(S)$) and ($-\tilde{A}\tilde{F}(S)$) contain the negative of the subsidies on, respectively, current production and the use of intermediate inputs in current production.

The matrix $\tilde{K}$ and the row vectors $\tilde{L}$ and $\tilde{M}$ show the payments of industries to, respectively, the various labour occupations, fixed capital and each industry's specific factor. The matrix $\tilde{K}(S)$ and the row vectors $\tilde{L}(S)$ and $\tilde{M}(S)$ correspondingly show the subsidies on factor payments. It is easy to see that $\tilde{K}+\tilde{K}(S)$, $\tilde{L}+\tilde{L}(S)$ and $\tilde{M}+\tilde{M}(S)$ contain the information on earnings received by factors. The system of $\tilde{N}+(-\tilde{N}(S)) + (-\tilde{A}\tilde{F}(S)) + \tilde{K}+\tilde{L}+\tilde{M}$ provides a breakdown of gross value added by industry. Figure 1 provides for more detail in subsidies than the underlying IO table. The additional data were taken from the Kiel Institute of World Economics' file on subsidies (Jüttemeier, 1985).

The final matrix in Figure 1, $\tilde{O}$, shows the commodity composition of each industry's output. The model system does not necessarily impose a 1:1 mapping between commodity rows and industry columns. In applied research, however, this is relevant only for the agricultural sector in which a wide selection of commodities is produced in a joint production process.

The domestic outputs of each industry in base year value units are represented by the column sums of $\tilde{A}+\tilde{F}+\tilde{N}+(-\tilde{N}(S))+(-\tilde{A}\tilde{F}(S)) + \tilde{K}+\tilde{L}+\tilde{M}$, whereas the base year values of domestic commodities are represented by the row sums of $\tilde{A}+\tilde{B}+\tilde{C}+\tilde{D}+\tilde{E}$. Alternatively, domestic industry outputs can be obtained as the column sums of $\tilde{O}$ and domestic commodity outputs as the row sums of $\tilde{O}$. The row sums of $\tilde{F}+\tilde{G}+\tilde{H}+\tilde{I}+(-\tilde{Z})$ represent the c.i.f. value of imports.

Finally, note that Figure 1 provides no explicit treatment of the demands for margins services to facilitate the flows of goods in the domestic economy. The recognition of margins dramatically increases the size of a model, yet is only of secondary importance for most policy simulations. It was therefore decided to postpone the modeling of margins to a second phase in which the model is to be refined and extended.

III. The Model Theory

In this chapter we present the theory necessary to explain all the flows in Figure 1. We first outline the production technology available to domestic industries before turning to the component parts of the general equilibrium system. Some comments on the notational conventions followed are in order. We use lower case letters to indicate the percentage change in the corresponding upper case variables.[1] That is, the percentage change in any variable V is represented by v where $v = \frac{dV}{V} 100$. Also used is an extensive system of superscripts and subscripts to distinguish different variables. For example, $X^{(k)}_{(is)j}$ is used to denote the demand by using industry j for input i of type s for purpose k. The letter i refers to commodities. Possible values for k are 1 (current production), 2 (capital creation), 3 (household consumption), 4 (exports) and 5 (other demands). Possible values for s are 1 (domestically produced) and 2 (imported). Thus $X^{(2)}_{(i2)j}$ would denote the demand for imported good i into industry j for capital creation. Note from Figure 1 that not all combinations of i,s,j,k are possible. Thus for example, $X^{(4)}_{(i1)}$ would signify the demand for domestic good i for export. In this case the j subscript is redundant while s would always be 1. Further examples are X^P_{vj} which denotes the input of primary factor designated X^P, of type v into industry j (v = 1 denotes aggregate labour, v = 2 denotes fixed capital and v = 3 denotes land) and $X^P_{1,q,j}$ which denotes the input of labour of occupation type q into industry j.

1. Production Technology for Current Goods

Following Dixon et al. (1982) we describe the production technology available to each of the h industries in each country in two parts, (i) the relationship between an industry's inputs and its activity

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[1] The reason why we present the model system in percentage change form is because this is the form in which the model is solved. See chapter V.

level, and (ii) the relationship between its activity level and its commodity outputs. On the input side we assume that industry production functions exhibit constant returns to scale (CRTS) and are of a three level or nested form. At the first level is the Leontief assumption of no substitution between the inputs of the IO commodity groups or between them and an aggregate of the primary factor inputs. At the second level are CES functions describing substitution possibilities between imported and domestic goods of the same type. At this level we also have CRESH[1] functions describing substitution possibilities between the three groups of primary factors, labour, fixed capital and land. At the third level are CRESH functions describing substitution prospects between the r labour occupations within the aggregate labour input category.[2]

On the output side, producers in each industry can produce a combination of commodities with the aggregation of commodities to the

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1 CRESH (Constant Ratio Elasticities of Substitution Homothetic) functions were introduced by Hanoch (1971). Under CRESH, the aggregation of primary factors X_1, X_2, X_3 to a composite X is written $\sum_{v=1}^{3} (X_v/X)^{hv} Q_v/h_v = \kappa_1$ (i) where $h_v \leq 1$ (but not equal to zero), $Q_v \geq 0$ and the Q_v's and κ_1 are normalised so that $\sum_v Q_v = 1$. The partial elasticities of substitution between factors 1 and 2 (σ_{12}) is given by $\sigma_{12} = (1/1-h_2)(1/1-h_2)(1/\sum_{u=1}^{3} \hat{S}_v)$. $\hat{S}_v = S_v/1-h_v$, with h_v being the CRESH parameter for factor v, and S_v the share of total primary factor costs accounted for by factor v. The advantage of CRESH over CES is that it allows σ_{12}, σ_{13} and σ_{23} to differ. Thus CRESH provides additional flexiblity when more than two factors are involved. Note that if all h_v share a common value of CRESH collapses to CES with substitution elasticity $\sigma = 1/1-h$.

2 The three level specification of production technology on the input side represents a reasonable tradeoff between the desire to provide a comprehensive treatment of input substitution prospects on the one hand and the availability of estimates of the relevant substitution parameters on the other. While production technologies allowing for a greater range of substitution prospects could easily be included, the microeconomic evidence to support their use is lacking.

industry activity level described by CRETH[1] functions. These allow us to capture the idea of imperfect transformation between commodities that constitute an industry's output according to changes in relative commodity prices and the ease of transformation between commodities.

2. Commodity and Primary Factor Input Demands for Current Production

Demand functions for the various types of inputs into current production are derived under the assumption that producers minimize their costs of producing a given output level subject to the constraints imposed by the nested production functions described above. That is, the typical producer in industry j must choose the input levels

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1 A summary of the properties of CRETH (Constant Ratio Elasticity of Transformation Homothetic) functions and an illustration of their use in commodity supply analysis is given in Vincent, Dixon and Powell (1980). Under CRETH, the aggregation of the i industry products, Y_i, to an index of industry activity Z is given by $\sum_i (Y_i/Z)^{k_i} Q_i/k_i = \kappa_2$

(ii) where $k_i > 1$ and the Q_i's and κ_2 are normalised to that $\sum_i Q_i = 1$. Thus apart from restrictions on the parameters (which in CRETH ensure product-product transformation surfaces that are convex to the origin compared with input substitution isoquants which are concave to the origin in CRESH), CRESH and CRETH are analogous. The partial elastiicity of transformation between commodities 1 and 2 (τ_{12}) in the set of i competing commodities is given by $\tau_{12} = -(1/k_1-1)(1/k_2-1)(1/\sum_i \hat{S}_i)$ where k_i is the CRETH transformation parameter for commodity i and $\hat{S}_i = S_i/k_{i-1}$, S_i being the share of the total output of the industry represented by the output of commodity i. Note that CRETH allows the partial elasticities of transformation to differ between pairs of products.

$X_{ij}^{(1)}$ $i = 1,\ldots,g$ "effective"[1] intermediate inputs,

X_j^P "effective"[2] primary input,

$X_{(is)j}^{(1)}$ $i = 1,\ldots,g$, $s = 1,2$ intermediate inputs from domestic and imported sources,

$X_{v,j}^P$ $v = 1,2,3$ aggregate labour[3], fixed capital and land inputs

$X_{1,q,j}^P$ $q = 1,\ldots,r$ input of labour of occupation type q

to minimise

$$\sum_{i=1}^{g} \sum_{s=1}^{2} P_{(is)}.\,(1-V_j^{(1)})\, X_{(is)j}^{(1)} + \sum_{v=2}^{3} P_{vj}^P\, (1-V_{vj}^P)\, X_{vj}^P + \sum_{q=1}^{r} P_{1,q,j}^P\, (1-V_{1,q,j}^P)\, X_{1,q,j}^P \qquad (1)$$

subject to Leontief[4]

$$i = 1,\ldots,g \qquad \left\{ \frac{X_{ij}^{(1)}}{A_{ij}^{(1)}} , \frac{X_j^P}{A_j^P} \right\} = Z_j \;, \qquad (2)$$

---

1 The concept of "effective" intermediate input is defined by (3).

2 The concept of "effective" primary inputs is defined by (4).

3 The labour input aggregation is given by (5).

4 In (2), Leontief $\{f_i\} = \underset{i=1,\ldots,r}{\text{minimum}} \{f_1,f_2,\ldots,f_r\}$.

$$X_{ij}^{(1)} = \underset{s=1,2}{\text{CES}} \; X_{(is)j}^{(1)} \;, \qquad (3)[1]$$

$$X_j^P = \underset{v=1,2,3}{\text{CRESH}} \; X_{vj}^P \;, \qquad (4)[2]$$

and $$X_{1j}^P = \underset{q=1,\dots,r}{\text{CRESH}} \; X_{1,q,j}^P \;, \qquad (5)[3]$$

where Z_j denotes industry j's activity level, the P's denote the respective prices of the X's and the V's are ad valorem rates of subsidies on the use of the X's in current production. (From the point of view of the producer the Z, P's and V's are treated as being exogenous). Thus $P_{(is)}$. is the price of good i from source s to industry j for current production. In the absence of margins on commodity flows the price of a given commodity will be the same to all end users, hence the omission of the (1) superscript and the j subscript. Similarly, $P_{1,q,j}^P$ is the price to industry j of a unit of labour of occupation q and the P_{vj}^P , v = 2,3 are the rental costs to industry j of capital and the industry-specific factor. As an example for V's take $V_{1,q,j}^P$. It is the ad valorem rate of a wage cost subsidy to industry j of a unit of labour of occupation q. The unit cost of employing labour of occupation q in industry j is thus $P_{1,q,j}^P(1-V_{1,q,j}^P)$. In the same manner we model the unit costs of using other primary factors and intermediate inputs in current production. Finally, the A's are a set of Leontief IO coefficients. $A_{ij}^{(1)}$ for example represents the minimum amount of "effective" input of good i to support a unit of activity in industry j.

---

1 Equation (3) assumes that, in order to capture the idea of imperfect substitutability between domestic and imported commodities of the same category, these commodities are combined to provide a unit of effective input according to the well known CES function.

2 Equations (4) and (5) indicate that X_{vj}^P and $X_{1,q,j}^P$ are aggregated according to the CRESH functional form given in chapter III.1.

3 Equations (4) and (5) indicate that X_{vj}^P and $X_{1,q,j}^P$ are aggregated according to the CRESH functional form given in chapter III.1.

The solution to the above cost minimising problem[1] yields input demand equations of the form

$$x_{(is)j}^{(1)} = z_j - \sigma_{ij}^{(1)} (p_{is} - \sum_{s=1}^{2} S_{(is)j}^{(1)} p_{is}) \tag{6}$$

$$+ \sigma_{ij}^{(1)} \frac{V_j^{(1)}}{1-V_j^{(1)}} (v_j^{(1)} - \sum_{s=1}^{2} S_{(is)j}^{(1)} v_j^{(1)})$$

$$i = 1,\ldots,g \quad s = 1,2 \quad j = 1,\ldots,h$$

$$x_{vj}^{P} = z_j - \sigma_{vj}^{P} (p_{vj}^{P} - \sum_{v=1}^{3} S_{vj}^{*} p_{vj}^{P}) \tag{7}$$

$$+ \sigma_{vj}^{P} \frac{V_{vj}^{P}}{1-V_{vj}^{P}} (v_{vj}^{P} - \sum_{v=1}^{3} S_{vj}^{*} v_{vj}^{P})$$

$$v = 1,2,3 \quad j = 1,\ldots,h$$

$$x_{1,q,j}^{P} = x_{1,j}^{P} - \sigma_{1,q,j}^{P} (p_{1,q,j}^{P} - \sum_{q=1}^{r} S_{1,q,j}^{*} p_{1,q,j}^{P}) \tag{8}$$

$$+ \sigma_{1,q,j}^{P} \frac{V_{1,q,j}^{P}}{1-V_{1,q,j}^{P}} (v_{1,q,j}^{P} - \sum_{q=1}^{r} S_{1,q,j}^{*} v_{1,q,j}^{P})$$

$$q = 1,\ldots,r \quad j = 1,\ldots,h$$

where

$$p_{1j}^{P} = \sum_{q=1}^{r} p_{1,q,j}^{P} S_{1,q,j} \tag{9}$$

$$v_{1j}^{P} = \sum_{q=1}^{r} v_{1,q,j}^{P} S_{1,q,j} \tag{10}$$

---

1 See Dixon et al. (1982) for a complete algebraic derivation of the solution to this type of problem.

In equation (6), $\sigma^{(1)}_{ij}$ is the CES substitution elasticity (between domestic and imported sources) for commodity i used as a current input into industry j while $S^{(1)}_{(is)j}$ denotes the share of good i from source s in the total costs of input i into industry j for current production. If there are no changes in the relative prices of good i from alternative sources then a 1 per cent increase in Z_j leads to a 1 per cent increase in each of $X^{(1)}_{(i1)j}$ and $X^{(1)}_{(i2)j}$. If however the price of domestic good i rises relative to the price of imported good i then there will be substitution against the domestic source of good i in favour of imports. The strength of this substitution effect is governed by the size of the substitution parameter $\sigma^{(1)}_{ij}$.

Equations (7) and (8) have a similar interpretation to (6). In (7) which specifies the demand functions for primary factors, σ^{P}_{vj} ($v = 1,2,3$) are the CRESH substitution parameters for each of the primary factors and S^{*}_{vj} is the "modified" primary factor cost share.[1] In (8), $\sigma^{P}_{1,q,j}$, $q = 1,\ldots,r$, are the CRESH substitution parameters for each labour occupation in industry j and $S^{*}_{1,q,j}$ is the CRESH "modified" cost share of labour of type q in the total labour costs of industry j.[2] In equation (7) p^{P}_{1j} is the price of labour in general, defined via (9) as a share weighted average of the prices of each of the labour occupations. Likewise v^{P}_{1j} is the ad valorem rate of subsidy on labour use in general which is defined via (10). Equation (7) implies that, in the absence of factor cost changes, a one per cent increase in j's activity level requires a one per cent increase in j's requirements for labour in general, capital and the specific factor. However, increases in the cost to industry j of any particular factor relative to a weighted average to the costs of the three factors leads to

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1 In terms of the equation defining the CRESH function (see footnote in chapter III.1) $\sigma^{P}_{vj} = (1/1-h_{vj})$ and $S^{*}_{vj} = \sigma^{P}_{vj} S_{vj} / \sum_{v=1}^{3} \sigma^{P}_{vj} S_{vj}$ where S_{vj} is the share of primary factor v in the total primary factor cost of industry j.

2 $\sigma^{P}_{1,q,j} = (1/1-h_{1,q,j})$ where $h_{1,q,j}$ is the "h" parameter from the CRESH function aggregating occupational labour inputs and $S^{*}_{1,q,j} = \sigma^{P}_{1,q,j}\ S_{1,q,j} / \sum_{q=1}^{r} \sigma^{P}_{1,q,j}\ S_{1,q,j}$ where $S_{1,q,j}$ is the cost share of labour of occupation type q in the total labour costs of industry j (see the footnote on CRESH functions in chapter III.1).

substitution away from that factor towards the other two. Similarly (8) indicates that if there is no change in the relative costs of the different types of labour then the occupational composition of industry j's workforce is unchanged. However, if $p^{P}_{1,q,j}$ increases and $v^{P}_{1,q,j}$ falls relative to a weighted average of all the occupational wage and subsidy rates in industry j then j's use of labour of type q will increase more slowly than j's use of labour in general.

3. Demands for Inputs for the Production of Fixed Capital

We assume that a unit of capital for use in industry j can be created according to the production function

$$Y_j = \underset{i=1,\ldots,g}{\text{Leontief}} \quad \frac{X^{(2)}_{ij}}{A^{(2)}_{ij}} \qquad (11)$$

where

$$X^{(2)}_{ij} = \underset{s=1,2}{\text{CES}} \; X^{(2)}_{(is)j} \qquad (12)$$

In (11) and (12), Y_j denotes the number of units of capital created for industry j, $X^{(2)}_{(is)j}$ the input of good i from domestic and imported sources (s = 1 and 2) for the production of capital for industry j, and the A's a set of Leontief IO coefficients. Equation (13) indicates that, as with current production, domestic and imported goods are allowed to be imperfect substitutes when they are used for capital creation.

Input prices are assumed to be beyond the control of producers who, for a given level of capital creation, Y_j, choose $X^{(2)}_{(is)j}$ to minimise

$$\sum_{s=1}^{2} \sum_{i=1}^{g} P_{is} \, X^{(2)}_{(is)j}$$

subject to (12) and (13). The solution to this problem[1] yields a set of demand functions for goods for capital creation of the form

$$x^{(2)}_{(is)j} = y_j - \sigma^{(2)}_{ij} (p_{is} - \sum_{s=1}^{2} S^{(2)}_{(is)j}\, p_{is}) \qquad (13)$$

$$i = 1,\ldots,g$$
$$s = 1,2$$
$$j = 1,\ldots,h$$

where $S^{(2)}_{(is)j}$ is the share of good i from source s in the total cost of good i used for creation of capital in industry j and $\sigma^{(2)}_{ij}$ is the elasticity of substitution between imported and domestic good i as inputs for creation of capital of type j. The above specification allows for the commodity composition of capital to vary across industries. Hence we can recognize that, for example, a given unit increase in investment in agriculture brings forth a greater increase in demand for tractors than say a similar unit increase in investment in the textile processing industry.

4. Household Demands

These are explained by the conventional utility maximising framework. Letting Q be the number of households we assume that the consumption bundle of effective inputs $(X^{(3)}_i/Q)$ for the average household is chosen to maximise the utility function denoted by

$$U(X^{(3)}_i/Q)$$

subject to

$$X^{(3)}_i = \underset{s=1,2}{\text{CES}}\; X^{(3)}_{is} \quad \text{and} \qquad (14)$$

$$\sum_{s=1}^{2} \sum_{i=1}^{g} P_{is}\, X^{(3)}_{is} = C \qquad (15)$$

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1 See Dixon et al. (1982).

where $X_{is}^{(3)}$ is the demand for good i from source s by households and C is the aggregate consumer budget. Hence in consumption, as well as in the production of current and capital goods, the model system allows for imperfect substitution between imported and domestic goods according to CES functions. The solution to the above utility maximising problem[1] yields consumer demand functions of the form

$$x_{is}^{(3)} = x_i^{(3)} - \sigma_i^{(3)} (p_{is} - \sum_{s=1}^{2} S_{is}^{(3)} p_{is}) \qquad i = 1,\ldots,g \quad s = 1,2 \qquad (16)$$

and

$$x_i^{(3)} = q + \varepsilon_i (c - q) + \sum_{k=1}^{g} \eta_{ik} p_k^{(3)} \qquad i = 1,\ldots,g \qquad (17)$$

$$p_k^{(3)} = \sum_{s=1}^{2} S_{ks}^{(3)} p_{ks} \qquad k = 1,\ldots,g \qquad (18)$$

In the above, $\sigma_i^{(3)}$ is the elasticity of substitution between domestic and imported sources of good i in consumption, $S_{is}^{(3)}$ is the share of total consumer spending on good i which is devoted to good i from source s, $p_k^{(3)}$ is the percentage change in the price of composite good k in the consumption and the ε_i and η_{ik} are expenditure elasticities and own and cross price elasticities of consumption respectively.

5. Export Demands

Export demand functions for a country's commodities by the rest of the world are written as

$$P_{i1}^{e} = X_{i1}^{(4)^{-\gamma_i}} F_{i1}^{(4)} \qquad (19)$$

where P_{i1}^{e} is the foreign currency price of domestic good i, γ_i is a positive parameter (the reciprocal of the foreign elasticity of demand for good i) and $F_{i1}^{(4)}$ is a shift variable which will increase if there is an increase in foreign demand for domestic good i. In percentage change form (19) becomes

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[1] See Dixon et al. (1982).

$$p^e_{i1} = -\gamma_i x^{(4)}_{i1} + f^{(4)}_{i1}. \qquad (20)$$

The parameter γ_i governs the slope of the foreign demand curve for a particular country's exports of good i. A γ_i value approaching zero depicts the small country assumption - exports from that country are not able to influence the world price.

6. Other Demands

These consist of government purchases (plus changes in inventories).[1] No formal theory is presented. We simply write that

$$x^{(5)}_{is} = g_R \qquad \begin{array}{l} i = 1,\ldots,g \\ s = 1,2 \end{array} \qquad (21)$$

where g_R is the percentage change in aggregate real government expenditure. The government is thus viewed as buying goods and services in constant proportions. We define g_R as

$$g_R = g - \varepsilon^{(5)} \qquad (22)$$

where g is the percentage change in aggregate government expenditure in money terms and $\varepsilon^{(5)}$ is an appropriately constructed price index defined by

$$\varepsilon^{(5)} = \sum_{i=1}^{g} \sum_{s=1}^{2} W^{(5)}_{is} p_{is} \qquad (23)$$

where $W^{(5)}_{is}$ represents the share of aggregate government spending devoted to good i from source s.

7. Commodity Supplies

Commodity supply equations are derived assuming that at a given activity level, Z_j, producers in industry j choose the commodity output combination to maximise their revenue. That is, we assume

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[1] Changes in inventories, as in evidence in the base period IO table, are for convenience lumped with government demands to form the "other demand" category. It is difficult to incorporate such changes into a model framework which emphasises equilibrium conditions.

that for each industry j

$X_{(i1)j}$ $\qquad$ i = 1,...,g (outputs of commodities)

are chosen to maximise

$$\sum_{i=1}^{g} P_{i1} X_{(i1)j}$$

subject to

$$\text{CRETH}^{1} \; X_{(i1)j} = Z_j \qquad (24)$$

$$i = 1,\ldots,g$$

where the P's and Z's are treated as exogenous.

The solution to the above revenue maximising problem[2] yields supply equations of the form

$$x_{(i1)j} = z_j + \sigma^T_{(i1)j} \left(p_{i1} - \sum_{i=1}^{g} C^*_{(i1)j} \, p_{i1}\right) \qquad (25)$$

$$i = 1,\ldots,g$$
$$j = 1,\ldots,h.$$

Equation (25) relates each industry's supplies of commodities to the industry's overall activity level and to the relative prices of the various commodities produced by that industry. If there are no relative commodity price changes, then a one per cent increase in industry j's activity level generates a one per cent increase in the supplies of the commodities it produces. If, however, the price of domestic commodity i increases relative to a weighted average of the prices of all the commodities produced by industry j then this industry transforms the commodity composition of its output in favour of commodity i and away from the other commodities. The strength of this transformation effect is governed by the transformation parameter $\sigma^T_{(i1)j}$.[3] The $C^*_{(i1)j}$ are the "modified"

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1 See footnote on CRETH functions in chapter III.1.

2 See Dixon et al. (1982).

3 $\sigma^T_{(i1)j} = (1/k_{(i1)j} - 1)$. See footnote on CRETH functions in chapter III.1.

revenue shares of commodity i in the total commodity revenue of industry j.[1] While (25) allows for all industries to produce all products, in reality, multiproduct outputs are relevant only for the agricultural sector. Thus the matrix $C^*_{(i1)j}$ has mainly zero entries. Where the industry produces only the commodity of the same label, then $x_{(i1)j} = z_j$, $i = j$. Hence there is no need for separate commodity supply equations for such industries.

8. The Price System

Because of the absence of a treatment of margins the model uses only one set of domestic prices. These are assumed to be the same to each end user in each industry. Our theory assumes that there are no pure profits[2] in each of the activities recognized; the production of current goods, the production of capital goods, importing, and exporting. Hence we write that;

Domestic Production

$$\sum_{i=1}^{g} P_{i1} X_{(i1)j} = \sum_{i=1}^{g} \sum_{s=1}^{2} P_{is} (1-V^{(1)}_{(is)j}) X^{(1)}_{(is)j} + \sum_{q=1}^{r} P^{P}_{1,q,j}(1-V^{P}_{1,q,j}) X^{P}_{1,q,j} + \sum_{v=2}^{3} P^{P}_{vj}(1-V^{P}_{vj}) X^{P}_{vj} + (T_j-V_j) \sum_{i=1}^{g} P_{i1}X_{(i1)j}$$

$$i = 1,\ldots,h \qquad (26)$$

where T_j and V_j respectively denote the ad valorem rate of production tax and subsidy. The left hand side of (26) is the value of the output of industry j and the right hand side is the total payment for inputs (intermediate input costs, labour costs, capital plus industry-specific factor costs) net of input subsidies and for taxes net of subsidies on production. The equality is implied by the assumption of no pure profits.

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1 $C^*_{(i1)j} = C_{(i1)j}\, \sigma^T_{(i1)j} / \sum_{i=1}^{g} C_{(i1)j}\, \sigma^T_{(i1)j}$ where $C_{(i1)j}$ is the revenue share of commodity i in industry j's output.

2 That is, profits accrue to factors of production. This follows from the assumption of constant returns to scale and competitive behaviour.

In percentage change form (26) becomes;

$$\sum_{i=1}^{g} p_{i1}\, C_{(i1)j}\, (1-T_j+V_j) = \sum_{i=1}^{g}\sum_{s=1}^{2} (p_{is}(1-V^{(1)}_{(is)j}) - v^{(1)}_{(is)j}\, V^{(1)}_{(is)j}) H^{(1)}_{(is)j}$$

$$+ \sum_{q=1}^{r} (p^P_{1,q,j}\, (1-V^P_{1,q,j}) - v^P_{1,q,j}\, V^P_{1,q,j})\, H^P_{1,q,j}$$

$$+ \sum_{v=2}^{3} p^P_{vj}\, (1-V^P_{vj}) - v^P_{vj}\, V^P_{vj})\, H^P_{vj} + t_j T_j - v_j V_j \qquad (27)$$

$$j = 1,\ldots,h$$

where the C's are revenue shares and the H's are cost shares. Thus $C_{(i1)j}$ is the revenue share of commodity i in the output of industry j while $H^{(1)}_{(is)j}$ for example is the share of industry j's current production costs accounted for by the cost of its inputs of good i from source s.

Capital Creation

$$\Pi_j Y_j = \sum_{i=1}^{g}\sum_{s=1}^{2} P_{is}\, X^{(2)}_{(is)j} \qquad j = 1,\ldots,h \qquad (28)$$

where Π_j is the price of a unit of capital in industry j.[1] Equation (28) imposes the condition that the value of new capital in industry j equals the cost of its production. In percentage change form (28) becomes

$$\pi_j = \sum_{i=1}^{g}\sum_{s=1}^{2} p_{is}\, H^{(2)}_{(is)j} \qquad (29)$$

where $H^{(2)}_{(is)j}$ is the cost share of good i from source s in the total cost of constructing a unit of capital for industry j.

Importing

$$P_{i2} = P^m_{i2}\, T^m_i\, \Phi \qquad i = 1,\ldots,g \qquad (30)$$

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[1] Note the distinction between Π_j in (28) and P^P_{2j} first introduced in (1). Π_j represents the cost of producing a unit of capital for industry j whereas P^P_{2j} is the cost of using or renting a unit of capital for industry j.

where P_{i2} is the basic price of imported good i (the price received by importers), P^m_{i2} is its foreign currency c.i.f. price, Φ is the exchange rate (domestic currency units per unit of foreign exchange) and T^m_i is one plus the ad valorem tariff (or tariff equivalent) rate on imports of good i. In (30) the price received by importers (which is also the domestic selling price) is equated with the cost of importing, i.e., the foreign currency import price expressed in domestic currency plus the tariff component. In percentage change terms (30) becomes

$$p_{i2} = p^m_{i2} + t^m_i + \phi \qquad i = 1,...,g. \qquad (31)$$

Exporting

Our final set of zero pure profits conditions equates the revenue from exporting to the relevant costs. That is,

$$P^e_{i1}\, V^e_i\, \Phi = P_{i1} \qquad i = 1,...,g \qquad (32)$$

where P^e_{i1} is the foreign currency price of domestic good i f.o.b. and V^e_i is one plus the ad valorem rate of export subsidy. Thus on the left of (32) we have the value in domestic currency units of exporting a unit of commodity i1 and on the right we have the cost of doing so, that is, the domestic price of a unit of i1. In percentage change form (32) becomes

$$p^e_{i1} + v^e_i + \phi = p_{i1} \qquad i = 1,...,g \qquad (33)$$

9. Market Clearing

In this section equations are specified which ensure that demand equals supply for domestically produced commodities and for the primary factors of production, labour, capital and land. The equations are

$$X_{i1} = \sum_{j=1}^{h} X^{(1)}_{(i1)j} + \sum_{j=1}^{h} X^{(2)}_{(i1)j} + X^{(3)}_{i1} + X^{(4)}_{i1} + X^{(5)}_{i1} \qquad (34)$$

$$i = 1,...,g$$

where

$$X_{i1} = \sum_{j=1}^{h} X_{(i1)j} \qquad i = 1,\dots,g. \qquad (35)$$

$$L_q = \sum_{j=1}^{h} X^P_{1,q,j} \qquad q = 1,\dots,r, \qquad (36)$$

$$K_j = X^P_{2j} \qquad j = 1,\dots,h, \qquad (37)$$

$$N_j = X^P_{3j} \qquad j = 1,\dots,h. \qquad (38)$$

In (34) supply is equated with demand for domestically produced goods. Total domestically produced supply for a particular commodity is determined via (35) as the sum of the outputs of that commodity over the j industries in which it is produced. Total demand is composed of intermediate input demand, demand for inputs into the production of capital equipment, household consumption demand, export demand, and "other" demand. In (36) labour supply in each occupation is equated to the demand for it. It implies that occupational labour is shiftable between industries. Note, however, that (36) does not necessarily imply a situation of full employment. It merely requires that occupational labour demands be satisfied.[1] In (37) and in (38) supply is equated with demand for capital and for the specific factor in each industry. Whereas the specific factor is always assumed to be non-shiftable between industries, capital may or may not be mobile depending on the model closure.[2]

Expressing (34)-(38) in percentage changes gives

$$x_{i1} = \sum_{j=1}^{h} x^{(1)}_{(i1)j}\, B^{(1)}_{(i1)j} + \sum_{j=1}^{h} x^{(2)}_{(i1)j}\, B^{(2)}_{(i1)j} + x^{(3)}_{i1} B^{(3)}_{i1} +$$

$$x^{(4)}_{i1} B^{(4)}_{i1} + x^{(5)}_{i1} B^{(5)}_{i1} \qquad i = 1,\dots,g, \qquad (39)$$

$$x_{i1} = \sum_{j=1}^{h} x_{(i1)j}\, D_{(i1)j} \qquad i = 1,\dots,g, \qquad (40)$$

---

1 Full employment could, however, be imposed by setting L_q at their full employment levels.

2 See chapter V.

$$\ell_q = \sum_{j=1}^{h} x^P_{1,q,j}\, B_{1,q,j} \qquad q = 1,\ldots,r, \qquad (41)$$

$$k_j = x^P_{2j} \qquad j = 1,\ldots,h, \qquad (42)$$

$$n_j = x^P_{3j} \qquad j = 1,\ldots,h. \qquad (43)$$

The B's in (39) refer to the shares of the sales of domestically produced goods which are absorbed by the various types of demands identified on the right hand side. For example $B^{(2)}_{(i1)j}$ refers to the share of total sales of domestic good i absorbed by sales to industry j for capital creation. In (40) the D's are production shares. $D_{(i1)j}$ is the share of industry j in the economy's output of good i. In (41) $B^P_{1,q,j}$ is the share of the total employment of labour of type q which is accounted for by industry j.

10. The Allocation of Investment Across Industries

In section 3 demand functions for inputs to capital creation in each industry were specified. Here we present a theory describing how many units of capital will be created in each industry (the Y_j).

Five steps are involved.

(i) The current rate of return on capital in industry j, R_j is defined as

$$R_j = \frac{P^P_{2j}}{\Pi_j(1-V^{(2)}_j)} - d_j \qquad j = 1,\ldots,h \qquad (44)$$

where d_j is the rate of depreciation in industry j (assumed constant), $V^{(2)}_j$ is the ad valorem rate of investment subsidy in industry j and P^P_{2j} and Π_j were previously defined as the rental rate on capital in industry j and the cost of producing a unit of capital in industry j respectively.

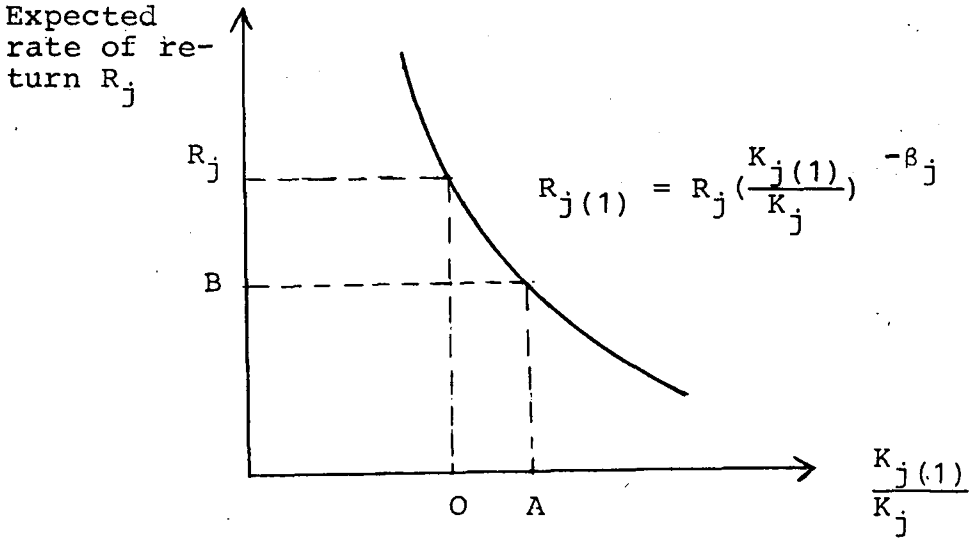
(ii) Capital is assumed to take one period to install.

(iii) Investors are assumed to be cautious in assessing the effects of expanding the capital stock in industry j. They behave as if they expect that industry j's rate of return schedule in one period's time will have the form

$$R_{j(1)} = R_j\left(\frac{K_{j(1)}}{K_j}\right)^{-\beta_j} \qquad (45)$$

where β_j is a positive parameter, K_j is the current level of capital stock in industry j and $K_{j(1)}$ is the level at the end of one period. The situation described in (45) is illustrated in Figure 2.

Figure 2 Expected Rate of Return Schedule for Industry j



The horizontal axis measures the ratio of next period's capital stock to current capital stock and the vertical axis measures the expected rate of return. If the capital stock were maintained at the existing level O, then the expected rate of return is the current rate R_j. However, if investment plans were set so that

$K_{j(1)}/K_j$ would reach A then industry entrepreneurs would behave as if they expected the rate of return to fall to B.

(iv) Total investment, I, is assumed to be allocated across industries so as to equate expected rates of return. This implies that there exists some rate of return Λ such that

$$\left(\frac{K_{j(1)}}{K_j}\right)^{-\beta_j} R_j = \Lambda \tag{46}$$

(v) Equations are defined for $K_{j(1)}$ and I. These are

$$K_{j(1)} = K_j(1 - d_j) + Y_j \qquad j = 1,...,h, \tag{47}$$

$$I = \sum_{j=1}^{h} \Pi_j Y_j \tag{48}$$

Equation (48) assumes that the effects of past investment decisions are fully incorporated in the current capital stock, with the only variables influencing capital stock at the end of one period being current capital stock and current investment. Equation (48) simply defines aggregate investment spending.

Expressing (44), (46)-(48) in percentage change form gives

$$r_j = Q_j \left(p^P_{2j} - \pi_j + v_j^{(2)} \frac{V_j^{(2)}}{1-V_j^{(2)}}\right) \qquad j = 1,...,h \tag{49}$$

$$-\beta_j (k_{j(1)} - k_j) + r_j = \lambda \qquad j = 1,...,h, \tag{50}$$

$$k_{j(1)} = k_j(1 - G_j) + y_j G_j \qquad j = 1,...,h, \tag{51}$$

$$\sum_{j=1}^{h} (\pi_j + y_j) T_j = i \tag{52}$$

where $Q_j = (R_j + d_j)/R_j$ i.e., the ratio of the gross rate of return in industry j to the net rate of return, $G_j = y_j/K_{j(1)}$ i.e., the ratio of gross investment in industry j to its future capital stock and T_j is the share of total aggregate fixed investment accounted for by industry j.

Equations (49)-(52) effectively endogenise investment allocation across industries. Suppose for example that the government raises subsidies for steel production. This would tend to increase the demand for capital required by the steel industry leading initially to an increase in the rental rate on capital and hence the rate of return in the steel industry relative to other rates. Equations (49)-(52) will ensure that industries for which the upward movements in their rate of return schedules are most pronounced will receive an increased share of the investment budget.

11. The Balance of Payments

Our treatment of the balance of payments is rudimentary. As yet we have not incorporated a financial sector, including a service account, in our model. The capital account, therefore, plays no active role. The balance on capital account either moves to compensate for any change in the balance on current account while the exchange rate is set exogenously or it is kept constant while the exchange rate moves to allow for an offsetting balance on current account. Furthermore, we offer no theory on the non-trade components of the current account, that is unrequited transfers, investment income, earnings from work and government transactions n.i.e. Their consolidated balance is just assumed constant. We write

$$\Delta B_C + \Delta B_O = \Delta B_K \tag{53}$$

$$100\Delta B_C = Ee - Mm \tag{54}$$

where

$$E = \sum_{i=1}^{g} P^{e}_{i1} X^{(4)}_{i1} \tag{55}$$

$$M = \sum_{i=1}^{g} P^{m}_{i2} X_{i2} \tag{56}$$

$$X_{i2} = \sum_{j=1}^{h} X^{(1)}_{(i2)j} + \sum_{j=1}^{h} X^{(2)}_{(i2)j} + X^{(3)}_{i2} + X^{(5)}_{i2} \tag{57}$$

Equations (53) and (54) define, respectively, balance of payments equilibrium and the trade balance. Aggregate export receipts and import expenditures in foreign currency values are, respectively, defined in equations (55) and (56). Aggregate demand for import of good i in equation (57) is equated to the sum of its demands over all intermediate and end uses.

In percentage change form (55)-(57) become

$$e = \sum_{i=1}^{g} (p^{e}_{i1} + x^{(4)}_{i1})\ E_{i} \qquad (58)$$

$$m = \sum_{i=1}^{g} (p^{m}_{i2} + x_{i2})\ M_{i2} \qquad (59)$$

$$x_{i2} = \sum_{j=1}^{h} x^{(1)}_{(i2)j}\ B^{(1)}_{(i2)j} + \sum_{j=1}^{h} x^{(2)}_{(i2)j}\ B^{(2)}_{(i2)j} \qquad (60)$$

$$+ x^{(3)}_{i2}\ B^{(3)}_{i2} + x^{(5)}_{i2}\ B^{(5)}_{i2} \qquad i = 1,\dots,g$$

where E_{i1} and M_{i2} respectively are commodity i1's share of export receipts and commodity i2's share of import expenditure. The B's are shares of total import flows. For example, $B^{(1)}_{(i2)j}$ denotes the share of total imports of good i which is absorbed by industry j for current production.

12. The Government Budget

Government expenditures on goods and services and on subsidies, in the model, are equated with revenues from direct and indirect taxes and other sources of finance. No theory is offered on the other sources, the amount is just kept constant. In percentage form we therefore write

$$g = t^{D}\ T^{D} + t^{I}\ T^{I}_{G} - v\ V_{G} \qquad (61)$$

where the rates of change of aggregate revenues from direct and indirect taxes and of aggregate expenditure on subsidies are respectively denoted by t^{D}, t^{I} and v, whereas T^{D}_{G} , T^{I}_{G} and V_{G}

denote the ratios between these aggregates and total government expenditure on goods and services.

The treatment of taxes is as yet rudimentary - only ad valorem tax rates are taken into account. To facilitate tax policy experiments the system would have to be refined. Total direct tax revenue is equated to the sum of tax revenue from factor earnings, that is,

$$T^D = \sum_{j=1}^{h} \sum_{q=1}^{r} T^P_{1,q} \; P^P_{1,q,j} \; X^P_{1,q,j} + \sum_{j=1}^{h} \sum_{v=2}^{3} T^P_v \; P^P_{vj} \; X^P_{vj} \tag{62}$$

where the $T_{1,q}$ and T_v denote the average tax rates on earnings from labour in different occupations and from the other primary factors of production respectively. Total indirect tax revenue is equated to the sum of revenue from production taxes, import taxes (tariffs and border taxes) and the non-redeemable turnover-tax which is treated as a household consumption tax, that is

$$T^I = \sum_{j=1}^{h} \sum_{i=1}^{g} T_j \; P_{i1} \; X_{(i1)j} + \sum_{i=1}^{g} (T^m_i - 1) P^m_{i2} \; \phi \; X_{i2} + T^C \; C \tag{63}$$

where T^C is the ad valorem consumption tax rate. Finally, the sum of subsidies on production, on the use of intermediates and factor services in production, on investment and on exporting is collected in equation (64), namely

$$\begin{aligned} V = & \sum_{j=1}^{h} \sum_{i=1}^{g} V_j \; P_{i1} \; X_{(i1)j} + \sum_{j=1}^{h} \sum_{i=1}^{g} \sum_{s=1}^{2} V^{(1)}_j \; P_{is} \; X^{(1)}_{(is)j} \\ & + \sum_{j=1}^{h} \sum_{q=1}^{r} V^P_{1,q,j} \; P^P_{1,q,j} \; X^P_{1,q,j} + \sum_{j=1}^{h} \sum_{v=2}^{3} V^P_{vj} \; P^P_{vj} \; X^P_{vj} \\ & + \sum_{j=1}^{h} V^{(2)}_j \Pi_j \; Y_j + \sum_{i=1}^{g} (V^e_i - 1) \; P^e_{i1} \; \phi \; X^{(4)}_{i1} \end{aligned} \tag{64}$$

In percentage change form equations (62)-(64) become

$$\begin{aligned} t^D = & \sum_{q=1}^{r} t^P_{1,q} \sum_{j=1}^{h} H^D_{1,q,j} + \sum_{j=1}^{h} \sum_{q=1}^{r} (p^P_{1,q,j} + x^P_{1,q,j}) \; H^D_{1,q,j} \\ & + \sum_{v=2}^{3} t^P_v \sum_{j=1}^{h} H^D_{vj} + \sum_{j=1}^{h} \sum_{v=2}^{3} (p^P_{vj} + x^P_{vj}) \; H^D_{vj} \end{aligned} \tag{65}$$

$$t^I = \sum_{j=1}^{h} t_j \sum_{i=1}^{g} H^I_{(i1)j} + \sum_{i=1}^{g} p_{i1} \sum_{j=1}^{h} H^I_{(i1)j} + \sum_{j=1}^{h} \sum_{i=1}^{g} x_{(i1)j} H^I_{(i1)j}$$

$$+ \sum_{i=1}^{g} (p_{i2} + x_{i2}) H^I_{i2} + \sum_{i=1}^{g} t^m_i H^I_{i2} (T^m_i - 1)^{-1}$$

$$+ (t^c + c) H^I_C \qquad (66)$$

$$v = \sum_{j=1}^{h} v_j \sum_{i=1}^{g} H^V_{(i1)j} + \sum_{i=1}^{g} p_{i1} \sum_{j=1}^{h} H^V_{(i1)j} + \sum_{j=1}^{h} \sum_{i=1}^{g} x_{(i1)j} H^V_{(i1)j}$$

$$+ \sum v^{(1)}_j H^{(1)V}_j + \sum_{i=1}^{g} \sum_{s=1}^{2} p_{is} \sum_{j=1}^{h} H^{(1)V}_j + \sum_{j=1}^{h} \sum_{i=1}^{g} \sum_{s=1}^{2} x^{(1)}_{(is)j} H^{(1)V}_j$$

$$+ \sum_{j=1}^{h} \sum_{q=1}^{r} (v^P_{1,q,j} + p^P_{1,q,j} + x^P_{1,q,j}) H^V_{1,q,j}$$

$$+ \sum_{j=1}^{h} \sum_{v=2}^{3} (v^P_{vj} + p^P_{vj} + x^P_{vj}) H^V_{vj} + \sum_{j=1}^{h} (v^{(2)}_j + \pi_j + y_j) H^{(2)V}_j$$

$$+ \sum_{i=1}^{g} (p_{i1} + x^{(4)}_{i1}) H^V_{i1} + \sum_{i=1}^{g} v^e_i H^v_{i1} (V^e_i - 1)^{-1} \qquad (67)$$

where the H^D's, H^I's and H^V's denote the shares of specific items of direct tax revenues, indirect tax revenues and subsidy expenditures in their respective totals.

13. Miscellaneous Equations

Apart from the structural equations central to the general equilibrium system, the model contains additional equations whose role it is to define useful summary variables, describe the behaviour of macro-economic aggregates and specify indexing relationships.

The first group of equations relate to the percentage change in real domestic absorption (rda) and its components real household consumption (c_R), real investment (i_R) and real government consumption (g_R)

$$rda = S_c c_R + S_i i_R + S_g g_R \qquad (68)$$

where

$$c_R = c - \varepsilon^{(3)} \tag{69}$$

$$i_R = i - \varepsilon^{(2)} \tag{70}$$

$$\varepsilon^{(3)} = \sum_{i=1}^{g} \sum_{s=1}^{2} p_{is} W_{is}^{(3)} \tag{71}$$

$$\varepsilon^{(2)} = \sum_{j=1}^{h} \pi_j T_j \tag{72}$$

The S's denote shares in domestic absorption, $\varepsilon^{(3)}$ and $\varepsilon^{(2)}$ denote respectively the price indices of household consumption and investment, $W_{is}^{(3)}$ stands for the weight of commodity i of origin s in the households' consumption basket whereas T_j, the share of total aggregate fixed investment accounted for by sector j, was already encountered in equation (52).

Real gross domestic product is defined via the expenditure side[1]

$$gdp_R = S_{rda}\, rda + S_e e - S_m m \tag{73}$$

where the S's denote the shares in gross domestic product. Note that aggregate export receipts and import expenditures have been defined in foreign currency terms. Equation (72) thus encompasses the impact of Terms-of-Trade changes, that is it describes the percentage change in the real value of GDP. We include an equation

$$z = \sum_{j=1}^{h} z_j L_j^z \tag{74}$$

to define a summary variable for aggregate economic activity which corresponds to a measure of aggregate gross value added at constant base year prices.[2]

Our treatment of the behaviour of the components of real domestic absorption is very simple. Real government consumption was already defined through the equations (22) and (61). We now add the equation

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[1] Note that with the explicit recognition of balance of payments equilibrium in equations (53) and (54) the Walrasian condition was imposed on the model. This implicitly equates the income to the expenditure side of gross domestic product.

[2] Note that equation (2) provides for a Leontief relation between output and demand for intermediate inputs.

$$i_R = c_R + f_R \tag{75}$$

where f_R is an exogenous shift variable whose role is to fix the relationship between movements in real aggregate consumption and investment. If for example f_R were set exogenously to zero then this would imply that the ratio of aggregate investment to aggregate consumption was constant, i.e., invariant to the experiment under consideration.

Next we add equations to define aggregate employment and the aggregate capital stock. These are

$$\ell = \sum_{q=1}^{r} \ell_q \, \psi_{1q} \tag{76}$$

$$k = \sum_{j=1}^{h} k_j \, B_{2j}^P \tag{77}$$

where ℓ and k are the percentage changes in aggregate employment and the aggregate capital stock, respectively, whereas ψ_{1q} is the share of employment of occupation q in total employment and B_{2j}^P is industry j's share in the economy-wide capital stock.

Next we define several price indexing equations to increase the flexibility of operation of the model. These are

$$p_{1,q,j,R} = p_{1,q,j} - \varepsilon^{(3)} \qquad j = 1,\ldots,h \quad q = 1,\ldots,r \tag{78}$$

$$p_{1,q,j,R}^{AT} = p_{1,q,j,R} - t_{1,q}^P \frac{T_{1,q}^P}{1-T_{1,q}^P} + f_{1,q} \qquad j = 1,\ldots,h \quad q = 1,\ldots,r \tag{79}$$

$$r_j = r + f_j^R \qquad j = 1,\ldots,h \tag{80}$$

$$r^{AT} = r_j - t_2^P \frac{T_2^P}{1-T_2^P} \qquad j = 1,\ldots,h \tag{81}$$

$$t_{1,q}^P = f_{1,q}^T + f^T \qquad q = 1,\ldots,r \tag{82}$$

$$t_v^P = f_v^T + f^T \qquad v = 2,3 \tag{83}$$

In (78) and (79) real wage rates before and after wage tax are defined. The $f^{P}_{1,q}$ are shift variables, their role is to allow for the exogenous setting of wage rates as negotiated between unions and associations. In (80) r is a scalar representing the percentage change in the absolute rate of return to capital and f^{R}_{j} is an industry length vector depicting percentage changes in relative rates of return. (81) defines the industry rate of return after capital gains tax. In (82) and (83) f^{T} as well as $f^{T}_{1,q}$ and f^{T}_{v} are shift variables. These equations allow for the exogenous setting of various direct tax rates and the relationship between them.

IV. The Complete Model

The complete set of model equations is given in Table 1. All model variables are defined in Table 2 and parameters in Table 3. As Table 3 shows, many model parameters are coefficients directly obtainable from an IO table. Others such as the substitution elasticities between imported and domestic commodities in various end uses, the substitution parameters between various occupations and between primary factors, the parameters that specify consumer demand, investment demand and export demand behaviour must be obtained from other sources. These are indicated in Table 3.

From the bottom lines of Tables 1 and 2 we see that there are fewer equations than variables. Hence to close the model requires exogenously setting the values for sufficient variables. It is to model closure options that we turn next.

Table 1 Model Equation System

Identifier	Equation	Subscript Range	Number of Equations	Description
(6)	$x^{(1)}_{(is)j} = z_j - \sigma^{(1)}_{ij} \left(p_{is} - \sum_{s=1}^{2} S^{(1)}_{(is)j}\, p_{is}\right) + \sigma^{(1)}_{ij} \frac{V_j^{(1)}}{1-V_j^{(1)}} \left(v_j^{(1)} - \sum_{s=1}^{2} S^{(1)}_{(is)j}\, v_j^{(1)}\right)$	i=1,...,g s=1,2 j=1,...,h	2gh	Demands for intermediate inputs
(7)	$x^P_{vj} = z_j - \sigma^P_{vj} \left(p^P_{vj} - \sum_{v=1}^{3} S^*_{vj}\, p^P_{vj}\right) + \sigma^P_{vj} \frac{V^P_{vj}}{1-V^P_{vj}} \left(v^P_{vj} - \sum_{v=1}^{3} S^*_{vj}\, v^P_{vj}\right)$	v=1,2,3 j=1,...,h	3h	Demands for labour in general, fixed capital and industry-specific factor
(8)	$x^P_{1,q,j} = x^P_{1,j} - \sigma^P_{1,q,j} \left(p^P_{1,q,j} - \sum_{q=1}^{r} S^*_{1,q,j}\, p^P_{1,q,j}\right) + \sigma^P_{1,q,j} \frac{V^P_{1,q,j}}{1-V^P_{1,q,j}} \left(v^P_{1,q,j} - \sum_{q=1}^{r} S^*_{1,q,j}\, v^P_{1,q,j}\right)$	q=1,...,r j=1,...,h	rh	Demands for labour of each occupation
(9)	$p^P_{1j} = \sum_{q=1}^{r} p^P_{1,q,j}\, S_{1,q,j}$	j=1,...,h	h	Prices of labour in general by industry

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(10)	$v^{P}_{1j} = \sum_{q=1}^{r} v^{P}_{1,q,j} S_{1,q,j}$	j=1,...,h	h	Ad valorem rate of subsidy on labour use in general by industry
(13)	$x^{(2)}_{(is)j} = y_j - \sigma^{(2)}_{ij} (p_{is} - \sum_{s=1}^{2} S^{(2)}_{(is)j} p_{is})$	i=1,...,g s=1,2 j=1,...,h	2gh	Demands for inputs to capital creation
(16)	$x^{(3)}_{is} = x^{(3)}_{i} - \sigma^{(3)}_{i} (p_{is} - \sum_{s=1}^{2} S^{(3)}_{is} p_{is})$	i=1,...,g s=1,2	2g	Household demands for commodities by source
(17)	$x^{(3)}_{i} = q + \varepsilon_i (c - q) + \sum_{k=1}^{g} \eta_{ik} p^{(3)}_{k}$	i=1,...,g	g	Household demands for commodities undif-ferentiated by source
(18)	$p^{(3)}_{k} = \sum_{s=1}^{2} S^{(3)}_{ks} p_{ks}$	k=1,...,g	g	General price of goods to households
(20)	$p^{e}_{i1} = -\gamma_i x^{(4)}_{i1} + f^{(4)}_{i1}$	i=1,...,g	g	Export demands
(21)	$x^{(5)}_{is} = g_R$	i=1,...,g s=1,2	2g	Other demands
(22)	$g_R = g - \varepsilon^{(5)}$		1	Real government expenditure

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(23)	$\varepsilon^{(5)} = \sum_{i=1}^{g} \sum_{s=1}^{2} W_{is}^{(5)} \; p_{is}$		1	Price index for government purchases
(25)	$x_{(i1)j} = z_j + \sigma^{T}_{(i1)j} \; (p_{i1} - \sum_{i=1}^{g} C^{*}_{(i1)j} \; p_{i1})$	i=1,...,g j=1,...,h	gh	Commodity supplies by industry
(27)	$\sum_{i=1}^{g} p_{i1} \; C_{(i1)j} \; (1-T_j+V_j) = \sum_{i=1}^{g} \sum_{s=1}^{2} (p_{is}(1-V^{(1)}_{(is)j})$ $- v^{(1)}_{(is)j} \; V^{(1)}_{(is)j} \; H^{(1)}_{(is)j} + \sum_{q=1}^{r} (p^{P}_{1,q,j} \; (1-V^{P}_{1,q,j})$ $- v^{P}_{1,q,j} \; V^{P}_{1,q,j}) \; H^{P}_{1,q,j} + \sum_{v=2}^{3} p^{P}_{vj}(1-V^{P}_{vj})$ $- v^{P}_{vj} \; V^{P}_{vj}) \; H^{P}_{vj} + t_j T_j - v_j V_j$	j=1,...,h	h	Zero pure profits in production
(29)	$\pi_j = \sum_{i=1}^{g} \sum_{s=1}^{2} p_{is} \; H^{(2)}_{(is)j}$	j=1,...,h	h	Zero pure profits in capital creation
(31)	$p_{i2} = p^{m}_{i2} + t^{m}_{i} + \phi$	i=1,...,g	g	Zero pure profits in importing

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(33)	$p^e_{i1} + v^e_i + \phi = p_{i1}$	i=1,...,g	g	Zero pure profits in exporting
(39)	$x_{i1} = \sum_{j=1}^{h} x^{(1)}_{(i1)j} B^{(1)}_{(i1)j} + \sum_{j=1}^{h} x^{(2)}_{(i1)j} B^{(2)}_{(i1)j} + x^{(3)}_{i1} B^{(3)}_{i1} + x^{(4)}_{i1} B^{(4)}_{i1} + x^{(5)}_{i1} B^{(5)}_{i1}$	i=1,...,g	g	Supply equals demand for domestically produced commodities
(40)	$x_{i1} = \sum_{j=1}^{h} x_{(i1)j} D_{(i1)j}$	i=1,...,g	g	Total output of good i
(41)	$\ell_q = \sum_{j=1}^{h} x^P_{1,q,j} B_{1,q,j}$	q=1,...,r	r	Supply equals demand for labour of each occupation
(42)	$k_j = x^P_{2j}$	j=1,...,h	h	Supply equals demand for capital
(43)	$n_j = x^P_{3j}$	j=1,...,h	h	Supply equals demand for industry-specific factor

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(49)	$r_j = Q_j \left(p^P_{2j} - \pi_j + v_j^{(2)} \frac{V_j^{(2)}}{1-V_j^{(2)}}\right)$	j=1,...,h	h	Rate of return on capital
(50)	$-\beta_j (k_{j(1)} - k_j) + r_j = \lambda$	j=1,...,h	h	Equality of rates of return
(51)	$k_{j(1)} = k_j(1 - G_j) + y_j G_j$	j=1,...,h	h	Capital accumulation
(52)	$\sum_{j=1}^{h} (\pi_j + y_j) T_j = i$		1	Investment budget
(53)	$\Delta B_C + \Delta B_O = \Delta B_K$		1	Balance of payments equilibrium
(54)	$100 \Delta B_C = Ee - Mm$		1	Trade balance
(58)	$e = \sum_{i=1}^{g} (p^e_{i1} + x^{(4)}_{i1}) E_i$		1	Aggregate foreign currency value of exports
(59)	$m = \sum_{i=1}^{g} (p^m_{i2} + x_{i2}) M_{i2}$		1	Aggregate foreign currency value of imports

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(60)	$x_{i2} = \sum_{j=1}^{h} x^{(1)}_{(i2)j} B^{(1)}_{(i2)j} + \sum_{j=1}^{h} x^{(2)}_{(i2)j} B^{(2)}_{(i2)j} + x^{(3)}_{i2} B^{(3)}_{i2} + x^{(5)}_{i2} B^{(5)}_{i2}$	i=1,...,g	g	Import volume
(61)	$g = t^D T^D + t^I T^I_G - v V_G$		1	Government budget
(65)	$t^D = \sum_{q=1}^{r} t^P_{1,q} \sum_{j=1}^{h} H^D_{1,q,j} + \sum_{j=1}^{h} \sum_{q=1}^{r} (p^P_{1,q,j} + x^P_{1,q,j}) H^D_{1,q,j} + \sum_{v=2}^{3} t^P_v \sum_{j=1}^{h} H^D_{vj} + \sum_{j=1}^{h} \sum_{v=2}^{3} (p^P_{vj} + x^P_{vj}) H^D_{vj}$		1	Direct tax revenue
(66)	$t^I = \sum_{j=1}^{h} t_j \sum_{i=1}^{g} H^I_{(i2)j} + \sum_{i=1}^{g} p_{i1} \sum_{j=1}^{h} H^I_{(i1)j} + \sum_{j=1}^{h} \sum_{i=1}^{g} x_{(i1)j} H^I_{(i1)j} + \sum_{i=1}^{g} (p_{i2} + x_{i2}) H^I_{i2} + \sum_{i=1}^{g} t^m_i H^I_{i2} (T^m_i - 1)^{-1} + (t^c + c) H^I_C$		1	Indirect tax revenue

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(67)	$v = \sum_{j=1}^{h} v_j \sum_{i=1}^{g} H^V_{(i1)j} + \sum_{i=1}^{g} p_{i1} \sum_{j=1}^{h} H^V_{(i1)j}$ $+ \sum_{j=1}^{h} \sum_{i=1}^{g} x_{(i1)j} H^V_{(i1)j} + \Sigma v_j^{(1)} H_j^{(1)}$ $+ \sum_{i=1}^{g} \sum_{s=1}^{2} p_{is} \sum_{j=1}^{h} H_j^{(1)V} + \sum_{j=1}^{h} \sum_{i=1}^{g} \sum_{s=1}^{2} x^{(1)}_{(is)j} H_j^{(1)V}$ $+ \sum_{j=1}^{h} \sum_{q=1}^{r} (v^P_{1,q,j} + p^P_{1,q,j} + x^P_{1,q,j}) H^V_{1,q,j}$ $+ \sum_{j=1}^{h} \sum_{v=2}^{3} (v^P_{vj} + p^P_{vj} + x^P_{vj}) H^V_{vj}$ $+ \sum_{j=1}^{h} (v_j^{(2)} + \pi_j + y_j) H_j^{(2)V} + \sum_{i=1}^{g} (p_{i1} + x^{(4)}_{i1}) H^V_{i1}$ $+ \sum_{i=1}^{g} v_i^e H^v_{i1} (V_i^e - 1)^{-1}$		1	Expenditure on subsidies
(68)	$rda = S_c c_R + S_i i_R + S_g g_R$		1	Real domestic absorption
(69)	$c_R = c - \varepsilon^{(3)}$		1	Aggregate real consumption
(70)	$i_R = i - \varepsilon^{(2)}$		1	Aggregate real investment

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(71)	$\varepsilon^{(3)} = \sum_{i=1}^{g} \sum_{s=1}^{2} p_{is} W_{is}^{(3)}$		1	Consumer price index
(72)	$\varepsilon^{(2)} = \sum_{j=1}^{h} \pi_j T_j$		1	Capital goods price index
(73)	$gdp_R = S_{rda} rda + S_e e - S_m m$		1	Defines real value of GDP
(74)	$z = \sum_{j=1}^{h} z_j L_j^z$		1	Aggregate gross value added at base year prices
(75)	$i_R = c_R + f_R$		1	Relationship between real consumption and real investment
(76)	$\ell = \sum_{q=1}^{r} \ell_q \psi_{1q}$		1	Aggregate employment
(77)	$k = \sum_{j=1}^{h} k_j B_{2j}^P$		1	Aggregate capital stock
(78)	$p_{1,q,j,R} = p_{1,q,j} - \varepsilon^{(3)}$	j=1,...,h q=1,...,r	rh	Real wage by industry for each occupation

Table 1 Continued

Identifier	Equation	Subscript Range	Number of Equations	Description
(79)	$p^{AT}_{1,q,j,R} = p_{1,q,j,R} - t^{P}_{1,q} \frac{T^{P}_{1,q}}{1-T^{P}_{1,q}} + f_{1,q}$	j=1,...,h q=1,...,r	rh	After tax real wage by industry for each occupation
(80)	$r_j = r + f^{R}_{j}$	j=1,...,h	h	Relative rates of re-turn on capital
(81)	$r^{AT} = r_j - t^{P}_{2} \frac{T^{P}_{2}}{1-T^{P}_{2}}$	j=1,...,h	h	After tax rates of return on capital by industry
(82)	$t^{P}_{1,q} = f^{T}_{1,q} + f^{T}$	q=1,...,r	r	Relative rates of wage taxes
(83)	$t^{P}_{v} = f^{T}_{v} + f^{T}$	v=2,3	2	Relative rates of taxes on gains from fixed capital and industry-specific factor
Total equations: 5gh + 3rh + 14h + 12g + 2r + 23				

Table 2 Model Variable List

First Appearance (Equation No.)	Variable	Number	Description[a]
(6)	$x^{(1)}_{(is)j}$	2gh	Demands for inputs (domestic and imported) for current production
(6)	z_j	h	Industry activity levels
(6)	p_{is}	2g	Price of goods (domestically produced and imported)
(6)	$v^{(1)}_j$	h	Ad valorem rates of subsidy on the use of intermediate inputs for current production
(7)	x^P_{vj}	3h	Industry demands for primary factors (labour in general, fixed capital, and industry-specific factors)
(7)	p^P_{vj}	3h	Rental prices of primary production factors by industry
(7)	v^P_{vj}	3h	Ad valorem rates of subsidy on the use of primary production factors in each industry
(8)	$x^P_{1,q,j}$	rh	Industry demands for labour by occupation
(8)	$p^P_{1,q,j}$	rh	Price of labour by occupation and industry
(8)	$v^P_{1,q,j}$	rh	Ad valorem rates of subsidy on the employment of labour by occupation and industry
(13)	$x^{(2)}_{(is)j}$	2gh	Demands for inputs (domestic and imported) for capital creation
(13)	y_j	h	Capital creation by using industry
(16)	$x^{(3)}_{is}$	2g	Household demands for domestic and imported goods
(16)	$x^{(3)}_i$	g	Household demands for goods undifferentiated by source
(17)	q	1	Number of households
(17)	c	1	Aggregate nominal consumption expenditure
(17)	$p^{(3)}_k$	g	Price of consumer goods by type but not by source

Table 2 Continued

First Appearance (Equation No.)	Variable	Number	Description [a]
(20)	p^e_{i1}	g	F.o.b. foreign currency export prices
(20)	$x^{(4)}_{i1}$	g	Export demands
(20)	$f^{(4)}_{i1}$	g	Export demand shift terms
(21)	$x^{(5)}_{is}$	2g	Other (mainly government) demands for domestic and imported goods
(21)	g_R	1	Real government expenditure
(22)	g	1	Nominal government expenditure
(22)	$\varepsilon^{(5)}$	1	Price index for government purchases
(25)	$x_{(i1)j}$	gh	Supplies of commodities by industry
(27)	t_j	h	Ad valorem rates of production tax by industry
(27)	v_j	h	Ad valorem rates of production subsidy by industry
(29)	π_j	h	Costs of units of capital by using industry
(31)	p^m_{i2}	g	C.i.f. foreign currency prices for imports
(31)	t^m_i	g	One plus the ad valorem rate of tariff on imports
(31)	ϕ	1	Exchange rate (DM/foreign currency)
(33)	v^e_i	g	One plus the ad valorem rate of subsidy for exports
(39)	x_{i1}	g	Total supplies of domestically produced goods
(41)	ℓ_q	r	Employment of labour by occupation

Table 2 Continued

First Appearance (Equation No.)	Variable	Number	Description[a]
(42)	k_j	h	Current capital stock by industry
(43)	n_j	h	Supply of specific factors by industry
(49)	r_j	h	Industry rates of return to capital
(49)	$v_j^{(2)}$	h	Ad valorem rates of subsidy on capital creation
(50)	$k_{j(1)}$	h	Future capital stock by industry
(50)	λ	1	Economy-wide expected rate of return to capital
(52)	i	1	Aggregate nominal investment
(53)	ΔB_C	1	Trade balance
(53)	ΔB_O	1	Consolidated balance on non-trade components of the current account
(54)	ΔB_K	1	Balance on capital account
(54)	e	1	Foreign currency value of exports
(54)	m	1	Foreign currency value of imports
(59)	x_{i2}	g	Import volumes
(61)	t^D	1	Direct tax revenue
(61)	t^I	1	Indirect tax revenue
(61)	v	1	Total expenditure on subsidies
(65)	$t^P_{1,q}$	r	Ad valorem rates of wage tax by occupation
(65)	t^P_v	2	Ad valorem rate of tax on capital gains and on industry-specific factor rents

Table 2 Continued

First appearance (Equation No.)	Variable	Number	Description[a]
(66)	t^C	1	Ad valorem rate of household consumption tax
(68)	rda	1	Aggregate real domestic absorption
(68)	c_R	1	Aggregate real household expenditure
(68)	i_R	1	Aggregate real investment expenditure
(69)	$\varepsilon^{(3)}$	1	Consumer price index
(70)	$\varepsilon^{(2)}$	1	Capital goods price index
(73)	gdp_R	1	Real value of GDP
(74)	z	1	Aggregate gross value added at base year prices
(75)	f_R	1	Shift term to set relationship between aggregate consumption and aggregate investment
(76)	ℓ	1	Aggregate employment
(77)	k	1	Aggregate capital stock
(78)	$p^P_{1,q,j,R}$	rh	Real wages by industry and occupation
(79)	$f^P_{1,q}$	r	Shift term for occupational wages
(79)	$p^{AT}_{1,q,j,R}$	rh	After-tax real wages by occupation and industry
(80)	r	1	Absolute rate of return to capital
(80)	f^R_j	h	Relative rates of return to capital

Table 2 Continued

First Appearance (Equation No.)	Variable	Number	Description[a]
(81)	r_j^{AT}	h	After-tax relative rates of return to capital
(82)	$f_{1,q}^T$	r	Shift terms for wage tax rates
(82)	f^T	1	Shift term for direct tax rate in general
(83)	f_v^T	2	Shift term for rates of taxes on gains from capital and industry-specific factors

Total variables: 5gh + 5rh + 22h + 16g + 4r + 33

[a] All variables are in percentage changes with the exception of ΔB_C and ΔB_K which are in first differences.

Table 3 Model Parameter List

First Appearance (Equation No.)	Parameter	Description	Source[a]
(6)	$\sigma^{(1)}_{ij}$	Elasticity of substitution between domestic and foreign sources of good i for use as an input in production in industry j.	Econometric (Lächler, 1984).
(6)	$S^{(1)}_{(is)j}$	Share of good i from source s (domestic or imported) in industry j's purchases of i for inputs to current production.	IO. $S^{(1)}_{(i2)j}$ is the jth element of $\tilde{A}$ divided by the sum of the ij elements of $\tilde{A} + \tilde{F}$. $S^{(1)}_{(i2)j}$ is $1-S^{(1)}_{(i1)j}$.
(6)	$V^{(1)}_{j}$	Ad valorem rate of subsidy for the use of intermediate inputs in industry j for current production.	IO. $V^{(1)}_{j}$ is the jth element of $(-\tilde{A}\tilde{F}(S))$ divided by the jth column total of $(\tilde{A} + \tilde{F})$.
(7)	σ^{P}_{vj}	Substitution parameter for primary factor v in industry j.	Econometric. Set to 1.0 in the experiments. Casual experiments showed no marked sensitivity of the results with respect to reasonable variations in these parameters.
(7)	S^{*}_{vj}	Modified cost share of primary factor v (v=1 labour in general, v=2 fixed capital, v=3 land) in total primary factor costs in industry j.	IO and Econometric. $S^{*}_{vj} = S_{vj}\, \sigma^{P}_{vj} / \sum_{v=1}^{3} S_{vj}\sigma^{P}_{vj}$. We denote by $\tilde{Q}$ a row vector the jth element of which consists of the jth column total of $\tilde{K} + \tilde{L} + \tilde{M}$. S_{1j}, then, is the sum of the r elements of the jth column of $\tilde{K}$ divided by the jth element of $\tilde{Q}$. S_{2j} and S_{3j} are, respectively, the jth element of $\tilde{L}$ and $\tilde{M}$ divided by the jth element of $\tilde{Q}$.
(7)	V^{P}_{vj}	Ad valorem rate of subsidy for the use of primary factor v in industry j.	IO. V^{P}_{1j} is the jth column total of $\tilde{K}(S)$ divided by the sum of the jth column totals of $(\tilde{K} + \tilde{K}(S))$. V^{P}_{2j} and V^{P}_{3j} are, respectively, the jth elements of $\tilde{L}(S)$ and $\tilde{M}(S)$ divided by the sum of the jth elements of $(\tilde{L} + \tilde{L}(S))$ and $(\tilde{M} + \tilde{M}(S))$.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(8)	$\sigma^{P}_{1,q,j}$	Substitution parameter for labour of occupation q in industry j.	Econometric (Pusse, 1980).
(8)	$S^{*}_{1,q,j}$	Modified cost share of labour of occupation q in total labour costs of industry j.	IO and econometric. $S^{*}_{1,q,j} = S_{1,q,j}\,\sigma^{P}_{1,q,j} / \sum_{q=1}^{r} S_{1,q,j}\sigma^{P}_{1,q,j}$. $S_{1,q,j}$ is the qjth element of $\tilde{K}$ divided by the jth column total of $\tilde{K}$.
(8)	$V^{P}_{1,q,j}$	Ad valorem rate of subsidy for the employment of labour of occupation q in industry j.	IO. $V^{P}_{1,q,j}$ is the qjth element of $\tilde{K}(S)$ divided by the sum of the qjth elements of $(\tilde{K} + \tilde{K}(S))$.
(9)	$S_{1,q,j}$	Cost share of labour of occupation q in total labour costs of industry j.	IO. $S_{1,q,j}$ is qjth element of $\tilde{K}$ divided by the jth column total of $\tilde{K}$.
(13)	$\sigma^{(2)}_{ij}$	Elasticity of substitution between domestic and imported sources of good i when used as an input to capital formation in industry j.	Econometric (Lächler, 1984).
(13)	$S^{(2)}_{(is)j}$	Share of good i from source s in industry j's total purchases of i for inputs to capital creation.	IO. $S^{(2)}_{(is)j}$ is the ijth element of $\tilde{B}$ divided by the sum of the ijth elements of $\tilde{B} + \tilde{G}$. $S^{(2)}_{(i2)j}$ is $1-S^{(2)}_{(i1)j}$.
(16)	$\sigma^{(3)}_{i}$	Elasticity of substitution between domestic and imported sources of good i when used for household consumption.	Econometric (Lächler, 1984).
(16)	$S^{(3)}_{is}$	Share of the value of good i from source s in the total purchases of good i by households.	IO. $S^{(3)}_{i1}$ is the ith element of $\tilde{C}$ divided by the sum of the ith elements of $\tilde{C} + \tilde{H}$. $S^{(3)}_{i2}$ is $1-S^{(3)}_{i1}$.
(17)	ε_{i}	Household expenditure elasticity of good i (from domestic or imported sources).	Econometric.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(17)	η_{ik}	Household cross price elasticities of demand for good i in general with respect to changes in the general price of good k.	Econometric. Since the assumed underlying household behaviour is reflected by the linear expenditure system, the matrix of uncompensated own price and cross price elasticities can be derived via the Frisch formula, using the household expenditure elasticities, the budget shares (α_i) and the Frisch parameter (w). $\eta_{ii} = \frac{\varepsilon_i}{w} - \varepsilon_i \alpha_i \ (1 + \frac{\varepsilon_i}{w})$ $\eta_{ij} = \varepsilon_i \alpha_j \ (1 + \frac{\varepsilon_j}{w}) \quad i \neq j$ The estimates are based on Lluch, Powell, Williams (1977).
(18)	$S_{ks}^{(3)}$	Defined in (16).	
(20)	γ_i	Reciprocal of the foreign elasticity of demand for good i.	Econometric; estimates based on comparable results in Winters, 1981.
(23)	$W_{is}^{(5)}$	Weight of good i from source s in the price index for government purchases	IO. $W_{i1}^{(5)}$ is the ith element of $\tilde{E}$ divided by the column total of $(\tilde{E} + \tilde{I})$. $W_{i2}^{(5)}$ is $1 - W_{i1}^{(5)}$.
(25)	$\sigma^T_{(i1)j}$	Transformation parameter for commodity i produced in the multiproduct bundle of industry j.	Econometric. As yet, the model has been specified with a 1:1 mapping between commodities and industries.
(25)	$C^*_{(i1)j}$	Modified revenue share of commodity i in the total revenue of industry j.	IO and econometric. $C_{(i1)j} = C_{(i1)j}\,\sigma^T_{(i1)j} / \sum_{i=1}^{g} C_{(i1)j}\,\sigma^T_{(i1)j}$. $C_{(i1)j}$ is the ijth element of $\tilde{O}$ divided by the jth column sum of O. In case of a 1:1 mapping all off-diagonal elements of $\tilde{O}$ are zero.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(27)	$C_{(i1)j}$	Defined in (25).	
(27)	T_j	Ad valorem rate of production tax.	IO. T_j is the jth element of $\tilde{N}$ divided by the total costs of industry.
(27)	V_j	Ad valorem rate of production subsidy.	IO. V_j is the jth element of $(-\tilde{N})$ divided by the total costs of industry j.
(27)	$H^{(1)}_{(is)j}$	Cost share of good i from source s in the total costs of industry j.	IO. $H^{(1)}_{(i1)j}$ is the ijth element of $\tilde{A}$ divided by the total costs of industry j. These can be computed as the jth column sum of $\tilde{A} + \tilde{F} + \tilde{K} + \tilde{L} + \tilde{M} + \tilde{N} + (-\tilde{N}(S)) + (-\tilde{A}\tilde{F}(S))$ but are also represented by the jth element of $\tilde{O}$. $H^{(1)}_{(i2)j}$ is the ijth element of $\tilde{F}$ divided by the total costs of industry j.
(27)	$H^{P}_{1,q,j}$	Costs share of labour of occupation q in the total costs of industry j.	IO. $H^{P}_{1,q,j}$ is the qjth element of K divided by the total costs of industry j.
(27)	H^{P}_{vj}	Cost share of primary factor v in the total costs of industry j (v=2,3).	IO. H^{P}_{2j} is the jth element of $\tilde{L}$ divided by the total costs of industry j. H^{P}_{3j} is the jth element of $\tilde{M}$ divided by the total costs of industry j.
(29)	$H^{(2)}_{(is)j}$	Share of good i from source s in the total costs of capital creation in industry j.	IO. $H^{(2)}_{(i1)j}$ is the ijth element of B divided by the sum of the jth column elements of $(\tilde{B} + \tilde{G})$. $H^{2}_{(i2)j}$ is the ijth element of $\tilde{G}$ divided by the sum of the jth column elements of $(\tilde{B} + \tilde{G})$.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(39)	$B^{(1)}_{(i1)j}$	Share of the total sales of domestic good i which is absorbed by industry j as an input into current production.	IO. $B^{(1)}_{(i1)j}$ is the ijth element of A divided by the total sales of domestic good i, i.e., the sum over the ith row of $\tilde{A} + \tilde{B} + \tilde{C} + \tilde{D} + \tilde{E}$.
(39)	$B^{(2)}_{(i1)j}$	Share of the total sales of domestic good i which is absorbed by industry j as an input into capital creation.	IO. $B^{(2)}_{(i1)j}$ is the ijth element of $\tilde{B}$ divided by the total sales of domestic good i.
(39)	$B^{(3)}_{i1}$	Share of the total sales of domestic good i which is absorbed by household consumption.	IO. $B^{(3)}_{i1}$ is the ith element of $\tilde{C}$ divided by the total sales of domestic good i.
(39)	$B^{(4)}_{i1}$	Share of the total sales of domestic good i which is absorbed by exports.	IO. $B^{(4)}_{i1}$ is the ith element of $\tilde{D}$ divided by the total sales of domestic good i.
(39)	$B^{(5)}_{i1}$	Share of the total sales of domestic good i which is absorbed by other demands.	IO. $B^{(5)}_{i1}$ is the ith element of $\tilde{E}$ divided by the total sales of domestic good i.
(40)	$D_{(i1)j}$	Share of the total output of domestic commodity i which is produced in industry j.	$D_{(i1)j}$ is the ijth element of $\tilde{O}$ divided by the sum of the elements in the ith row of $\tilde{O}$. In case of a 1:1 mapping of commodities and industries $D_{(i1)j} = 1$ when $i=j$, $D_{(i1)j} = 0$ when $i \neq j$.
(41)	$B^{P}_{1,q,j}$	Share of the economy-wide employment in occupation q which is accounted for by industry j.	$B^{P}_{1,q,j}$ is the qjth element of $\tilde{K}$ divided by the qth row sum of $\tilde{K}$.
(49)	Q_j	Ratio of gross (before depreciation) to net (after depreciation) rate of return in industry j.	Econometric. Computations based on data in Schmidt (1982).
(49)	$V^{(2)}_j$	Ad valorem rate of subsidy on capital creation.	IO. $V^{(2)}_j$ is the jth element of $\tilde{B}\tilde{G}(S)$ divided by the jth column total of $(\tilde{B} + \tilde{G})$.
(50)	β_j	Elasticity of the expected rate of return schedule in industry j with respect to increases in the planned capital stock in industry j.	Econometric. Set to 2.0 in the experiments.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(51)	G_j	Ratio of industry j's gross investment to its following year capital stock.	Econometric. Computations based on data in Schmidt (1982).
(52)	T_j	Share of total investment accounted for by investment in industry j.	IO. First sum the column elements of $(\tilde{B} + \tilde{G})$. T_j is the jth element in the array of the column sums of $(\tilde{B} + \tilde{G})$ divided by the sum of the elements in the array.
(54)	E	Aggregate foreign currency value of exports.	IO. E is the sum of the elements in $\tilde{D}$.
(54)	M	Aggregate foreign currency value of imports.	IO. M is the sum of all elements in $\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I} + (-\tilde{Z})$.
(58)	E_i	Share of total export earnings accounted for by exports of good i.	IO. E_i is the ith element of $\tilde{D}$ divided by the total of all elements in $\tilde{D}$.
(59)	M_{i2}	Share of the total foreign currency cost of imports accounted for by imports of competing good i.	IO. M_{i2} is the ith row sum of $\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I} + (-\tilde{Z})$ divided by the total foreign currency cost of imports, i.e., the sum of all elements in $\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I} + (-\tilde{Z})$.
(60)	$B^{(1)}_{(i2)j}$	Share of the total sales of imported good i which is absorbed by sales to industry j for current production.	IO. $B^{(1)}_{(i2)j}$ is the ijth element of $\tilde{F}$ divided by the total sales of imported good i, i.e., the ith row sum of $\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I}$.
(60)	$B^{(2)}_{(i2)j}$	Share of the total sales of imported good i absorbed for capital creation in industry j.	IO. $B^{(2)}_{(i2)j}$ is the ijth element of $\tilde{G}$ divided by the total sales of imported good i.
(60)	$B^{(3)}_{i2}$	Share of the total sales of imported good i absorbed by household consumption.	IO. $B^{(3)}_{i2}$ is the ith element of $\tilde{H}$ divided by the total sales of imported good i.
(60)	$B^{(5)}_{i2}$	Share of the total sales of imported good i absorbed by other demands.	IO. $B^{(5)}_{i2}$ is the ith element of $\tilde{I}$ divided by the total sales of imported good i.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(61)	T_G^D	Share of government expenditures financed by direct tax revenues.	IO and Statistisches Bundesam (1979b). T_G^D is the direct tax income divided by total government expenditure on goods and services, i.e. the sum of all elements in $\tilde{E}$ and $\tilde{I}$ less aggregate net addition to stocks. The direct tax income is the sum of all elements in $\tilde{K}$, $\tilde{L}$, $\tilde{M}$ multiplied with the appropriate tax rate.
(61)	T_G^I	Share of government expenditures financed by indirect tax revenues.	IO. T_G^I is the sum of all elements in $\tilde{Z}$, $\tilde{N}$ and of $\tilde{W}$ divided by total government expenditure on goods and services.
(61)	V_G	Share of expenditures on subsidies in total government expenditure.	IO. Sum of all elements in $\tilde{D}(S)$, $\tilde{BG}(S)$, $\tilde{N}(S)$, $\tilde{AF}(S)$, $\tilde{K}(S)$, $\tilde{L}(S)$ and $\tilde{M}(S)$ divided by total government expenditure on goods and services.
(65)	$H_{1,q,j}^D$	Share of wage tax revenue in industry j in total direct tax revenue.	IO. $H_{1,q,j}^D$ is the qjth element of $\tilde{K}$ multiplied by the appropriate tax rate divided by total direct tax revenue.
(65)	H_{vj}^D	Share of tax revenue from gains on fixed capital (v=2) and industry-specifc factors (v=3) in industry j in total direct tax revenue.	IO. H_{vj}^D is the jth element of $\tilde{L}$ (v=2) or $\tilde{M}$ (v=3) multiplied by appropriate tax rate divided by the total direct tax revenue.
(66)	$H_{(i1)j}^I$	Share of tax revenue from the production of commodity i in industry j in total indirect tax revenue.	IO. In case of a 1:1 mapping of commodities and industries $H_{(i1)j}^I = 0$ when $i \neq j$ and H_j^I when i=j. H^I is the jth element of $\tilde{N}$ divided by the total indirect tax revenue as covered in (61).
(66)	H_{i2}^I	Share of tariff revenue from the import of commodity i in total indirect tax revenue.	IO. H_{i2}^I is the ith element of $\tilde{Z}$ divided by the total direct tax income.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(66)	T_i^m	One plus ad valorem rate of tariff on imported commodity i.	IO. T_i^m is the ith row total of $(\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I})$ divided by the ith row total of $(\tilde{F} + \tilde{G} + \tilde{H} + \tilde{I} + (-\tilde{Z}))$.
(66)	H_C^I	Share of tax revenue from household consumption in total indirect tax revenue.	IO. H_C^I is the scalar $\tilde{W}$ divided by the total indirect tax income.
(67)	$H_{(i1)j}^V$	Share of expenditure on subsidies for the production of commodity i in industry j in total expenditure on subsidies.	IO. In case of a 1:1 mapping of commodities and industries $H_{(i1)j}^V = 0$ when $i \neq j$ and $H_{(i1)j}^V = H_j^V$ when $i=j$. H_j^V is the jth element of $\tilde{N}(S)$ divided by total expenditure subsidies as covered in (61).
(67)	$H_j^{(1)V}$	Share of expenditure on subsidies for the use of intermediates by industry j in total expenditure on subsidies.	IO. $H_j^{(1)V}$ is the jth element of $\tilde{A}\tilde{F}(S)$ divided by total expenditure on subsidies.
(67)	$H_{1,q,j}^V$	Share of expenditure on subsidies for employment of occupation q in industry j in total expenditure on subsidies.	IO. $H_{1,q,j}^V$ is the qjth element of $\tilde{K}(S)$ divided by total expenditure on subsidies.
(67)	H_{vj}^V	Share of expenditure on subsidies for the use of fixed capital (v=2) and industry-specific factor (v=3) in industry j in total expenditure on subsidies.	IO. H_{vj}^V is the jth element of $\tilde{L}(S)$ and $\tilde{M}(S)$, respectively, divided by total expenditure on subsidies.
(67)	$H_j^{(2)V}$	Share of expenditure on subsidies for capital creation in industry j in total expenditure on subsidies.	IO. $H_j^{(2)V}$ is the jth element of $\tilde{B}\tilde{G}(S)$ divided by total expenditure on subsidies.
(67)	H_{i1}^V	Share of expenditure on subsidies for the export of commodity i in total expenditure on subsidies.	IO. H_{i1}^V is the ith element of $\tilde{D}(S)$ divided by total expenditure on subsidies.

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(68)	S_c	Share of aggregate real household consumption in aggregate real domestic absorption.	IO. S_c is the sum of all elements in $(\tilde{C} + \tilde{H})$ divided by aggregate real domestic absorption, i.e., the sum of all elements in $(\tilde{C} + \tilde{H} + \tilde{B} + \tilde{G} + \tilde{E} + \tilde{I})$.
(68)	S_i	Share of aggregate real investment in aggregate real domestic absorption.	IO. S_i is the sum of all elements in $(\tilde{B} + \tilde{G})$ divided by aggregate real domestic absorption.
(68)	S_g	Share of aggregate real other demand in aggregate real domestic absorption.	IO. S_g is the sum of all elements in $(\tilde{E} + \tilde{I})$ divided by real domestic absorption.
(71)	$W_{is}^{(3)}$	Weight of good i from source s in the consumer price index.	IO. $W_{i1}^{(3)}$ is the ith element of $\tilde{C}$ divided by the sum of all elements in $(\tilde{C} + \tilde{H})$. $W_{i2}^{(3)}$ is the ith element of $\tilde{H}$ divided by the sum of all elements in $(\tilde{C} + \tilde{H})$.
(72)	T_j	Defined in (52).	
(73)	S_{rda} S_{ℓ} S_m	Respectively, the shares of the real value of gross domestic product accounted for by aggregate domestic absorption, export and import demand (in foreign currency values).	IO. Sum of all shares is unity with S_m being negative.
(74)	L_j^z	Share of sector j in aggregate gross value added.	IO. First sum the column elements of $(\tilde{K} + \tilde{L} + \tilde{M} + \tilde{N} + (-\tilde{N}(S) - \tilde{A}\tilde{F}(S)))$. L_j^z is the jth element in the array of the column sums divided by the sum of the elements in the array.
(76)	ψ_{1q}	Share of aggregate employment accounted for by employment of occupation q.	IO.
(77)	B_{2j}^P	Share of total capital stock accounted for by industry j's capital stock.	Computed from data in Schmidt (1982).

Table 3 Continued

First Appearance (Equation No.)	Parameter	Description	Source[a]
(78)	$T^{P}_{1,q}$	Rate of wage tax on occupation q.	Econometric.
(80)	T^{P}_{2}	Rate of tax on capital gains.	Econometric.

[a] Parameter sources are the input-output flows matrix assembled in Figure 1 (denoted IO) or alternative sources (indicated in the source column).

V. The Solution Procedure

1. Solution Algorithm

The equation system can be represented by

(i) $Az = 0$

where A is an mxn matrix of coefficients and z is an nx1 vector of variables. From Tables 1 and 2 it can be seen that

$$m = 5gh + 3rh + 14h + 12g + 2r + 23$$

$$n = 5gh + 5rh + 22h + 16g + 4r + 33$$

Thus, to solve this model

$$n-m = 2rh + 8h + 4g + 2r + 10$$

variables must be declared exogenous.

Once the choice of exogenous variables has been made, (i) is rewritten as

(ii) $A_1y + A_2x = 0$

where A_1 is the mxn matrix formed by the m columns of A corresponding to the endogenous variables and A_2 is the mx(n-m) matrix formed by the n-m columns of A corresponding to the exogenous variables. y and x are, respectively, the mx1 and (n-m)x1 vectors of endogenous and exogenous variables.

The model user has considerable freedom in partitioning the variables into the exogenous and endogenous categories. However, the user must make sure that A_1 is invertible. No formal theory is offered for this, yet the user will not fail when he or she observes two rules. The first says that the absolute price level must be determined exogenously since a Walrasian system solves for relative prices only. The user must, therefore, select a numeraire, that is, set one price variable to unity. To allow for meaningful interpretations of results, it is advisable to use either the exchange rate, a general price index like the consumer price index or a general wage index for this purpose. The second rule says that, whenever a price appears on the exogenous list, then a corresponding quantity should be on the endogenous list and vice versa. If, for example, wages are exogenous, then employment should be endogenous; if industry capital stocks are exogenous, then industry rates of return to capital should be endogenous. With A_1 invertible we can proceed from (ii) to the solution

(iii) $y = -A_1^{-1} A_2 x$

Equation (iii) expresses the percentage change in each endogenous variable as a linear function of the percentage changes in the exogenous variables. Note that $(-A_1^{-1}A_2)_{ij}$ is the elasticity of the ith endogenous variable with respect to the jth exogenous variable - for example, the percentage change in employment in the textile industry arising from a 1 percent increase in the production subsidy to the agricultural sector. If this elasticity had the value -0.1, say, this would be interpreted as meaning that a 1 percent increase in the agricultural production subsidy would cause employment in the textile industry to be 0.1 percent lower than it otherwise would have been.

2. Closure Options

In Table 4 we have set out one possible selection of exogenous variables.[1] In working through this list we draw attention to some alternative selections.

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[1] This selection was used in the first of the four model experiments reported in Gerken, Jüttemeier, Schatz and Schmidt, 1985.

Table 4 A Possible List of Exogenous Variables

Variable	Number	Description
p^{m}_{i2}	g	C.i.f. foreign currency prices for imports
$f^{(4)}_{(i1)}$	g	Export demand shift terms
t^{m}_{i}	g	One plus a.v.r. of import tariff
v^{e}_{i}	g	One plus a.v.r. of export subsidy
$p^{AT}_{1,a,j,R}$	rh	After-tax real wages by occupation and industry
$f^{R}_{1,q}$	r	Shift terms for occupational wages
r	1	Absolute rate of return to capital
r^{AT}_{j}	h	After-tax relative rates of return to capital
n_{j}	h	Supply of industry-specific factors
$v^{P}_{1,q,j}$	rh	A.v.r. of wage subsidy by occupation and industry
v^{P}_{vj} $(v=2,3)$	2h	A.v.r. of subsidy on the use of fixed capital and of industry-specific factors
t_{j}	h	A.v.r. of production tax
v_{j}	h	A.v.r. of production subsidy
$v^{(1)}_{j}$	h	A.v.r. of subsidy on use of intermediate inputs
$v^{(2)}_{j}$	h	A.v.r. of investment subsidy
ΔB_{K}	1	Balance on capital account
ΔB_{O}	1	Balance of the non-trade components of current account

Table 4 Continued

Variable	Number	Description
f_R	1	Shift term to set relationship between real aggregate consumption and real aggregate investment
g_R	1	Real government expenditure
$f^T_{1,q}$	r	Shift terms for a.v.r. of wage tax by occupation
f^T_v	2	Shift terms for a.v.r. of taxes on gains from fixed capital and industry-specific factors
t^C	1	A.v.r. of consumption tax
q	1	Number of households
$\varepsilon^{(3)}$	1	Consumer price index

Total variables: 2rh + 8h + 4g + 2r + 10

The first four groups of variables relate to international trade in goods. The country policy model contains no equations describing foreign supply conditions or foreign demand curves for local products. The c.i.f. foreign currency prices for imports and the export demand shift terms are, therefore, always on the exogenous list. This allows for questions of the form: What would be the effects of a change in the import price of, for example, crude oil or in the world demand for German automobiles? Import tariffs and export subsidies have been placed on the exogenous list because the Federal Republic does not entertain a system of variable import levies and export restitutions. In a policy model of the European Community, one would instead select the local prices of imported and domestically produced agricultural commodities. Tariff and subsidy rates would then be determined endogenously so as to fill the gap between local and foreign prices.

The next two groups of variables relate to the labour market. As is well known, unions and associations negotiate about nominal wages of each occupation industry by industry. By placing the real after-tax wage rates and the wage shift variables on the exogenous list, it is assumed that nominal wage settlements do reflect the negotiators' concern with both the level and the structure of real net wage incomes. Changes in the consumer price index,in wage tax rates and in the desired relation between wages of different occupations and in different industries, therefore,lead to changes in nominal wage rates. Many alternatives to this treatment are possible. For example, one might wish to solve the model for the wage changes necessary to achieve desired levels of employment. These levels would then be placed on the exogenous list and the model would simulate the required changes in absolute and relative wages. Clearly, each model user has first to clarify his or her view of the labour market.

The next two groups are the absolute and the relative rates of return to capital after-tax. By placing them on the exogenous list, they are in effect assumed to reflect foreign rates. This is the appropriate assumption for the long-run in a country with open and competitive asset markets. A change in policies or some other external event will initially change industry rates of return. By allowing for capital depletion or by expanding industry capital stocks, however, investors will restore rates of return to their equilibrium level. In the open economy, this level is determined by the foreign rate. The industry capital stocks as well as the economy's aggregate capital stock are, therefore, endogenous in the long-run. For a short-run closure, obviously, one would treat the industry capital stocks as exogenous and place rates of return on the endogenous list. Note, however, that the model determines investment endogenously, but is not equipped with capital formation equations. That is, investment plans initiated by any exogenous shock are allowed to affect the demand faced by industries producing capital goods but are not allowed to augment the existing capital stock. Hence the short-run cannot exceed the gestation lag on new investment. The long-run, on the other hand, must exceed the period necessary for stock adjustment after which investment demand is again determined in the way described by the model. No solution is offered for the path leading from short to long-run.

Industry-specific factor variables must of course be set exogenous. The model determines the corresponding rental prices.

These are followed by two groups of variables representing subsidies on the use of labour by occupation and industry and on the use of the other primary inputs (fixed capital, industry-specific factors) by industry. These are characteristic policy variables. A model user asking for policy effects will assign them to the exogenous list along with either the levels of factor use or the levels of wages and returns to capital and industry-specific factor. When asking for the intervention level necessary for reaching a desired level of employment or other factor use at a given wage or factor rental price, however, the user will instead treat these levels as exogenous and place the policy variables on the endogenous list.

The next four groups of variables relate to further policy interventions - production taxes, production subsidies and subsidies on the use of intermediate inputs and on investment. Again, these are to be placed on the exogenous or the endogenous list depending on whether the question is for the policy effects or for the intervention levels necessary to secure certain target levels.

Next in Table 4 are the balance on capital account and the balance of the non-trade components of the current account. Setting these exogenous is unavoidable since we have as yet not incorporated a financial sector. The consequence is that all factors impinging on the wealth position of domestic residents are captured internally, rather than becoming in part reflected in the asset-liability positions vis-à-vis foreigners. Further quantitative research work is required to remedy this somewhat unsatisfactory state of affairs.

Real government expenditure is treated as exogenous. Since the model has a budget equation not allowing for changes in the deficit, one component of revenue must adjust. In the specification described in Table 4 these are the direct tax rates. Note that by including the shift terms $f^T_{1,q}$ and f^T_v in the exogenous list, all direct tax rates are forced to vary proportionally. Obviously, many alternative specifications are possible. For example, one might wish the household consumption tax to do the adjustment, i.e., one would take this tax rate off the exogenous list and instead include the direct tax rate. One could also investigate the implications of a rule by which all tax rates are fixed and the government spends whatever revenues flow in.

The other components of final demand are determined endogenously. The relation between aggregate real investment and aggregate real consumption, however, is fixed by setting the shift variable f_R exogenous. Whereas the model is very detailed with respect to the distribution of investment demand, the treatment of aggregate investment must remain rudimentary in a comparative-static model.

The next variable in Table 4 is the number of households. This is always exogenous in our framework as we do not have to offer a theory of household formation.

Finally, there is the choice of the numeraire. A wage index as numeraire would make the interpretation of results rather cumbersome. This leaves us with the choice between the consumer price index and the exchange rate. Note that there is no foreign exchange market in the model. The exchange rate is just a conversion factor between foreign and domestic prices, not the price of foreign currency. Assigning unity to the exchange rate instead of the consumer price index, therefore, in effect means that the absolute price level is determined by foreign instead of domestic prices. However, as long as all nominal prices are endogenous, the choice makes no difference whatsoever. In both cases all real endogenous variables are homogeneous of degree zero with respect to either the exchange rate or the consumer price index and all domestic prices are homogeneous of degree one.[1] In the experiment described by the selection of exogenous variables in Table 4 we took care not to fix any nominal price. However, in short run applications the user might want to experiment with Keynesian closures, that is allow for nominal price rigidities. The domestic price index should then be placed on the endogenous list.

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1 This property provides a reasonably powerful check on the accuracy of the computer programmes describing the model equations.

3. On the Linear Solution Method

The linear solution procedure presented in equations (i)-(iii) has one well recognised disadvantage. Because the A matrix is assumed fixed, equation (iii) provides only a local representation of the structural equation system. That is, solutions are strictly valid only for small changes in exogenous variables. Before describing a technique to cope with the linearisation error occurring in the simulation of fairly large policy and other exogenous changes, we first look at alternatives to the linear approximation method. These include

- methods which deduce a solution to the complete model from the solution of a suitable chosen constrained maximisation programming problem and its dual,[1] and
- excess demand function methods applied directly to the equations written in the levels.[2]

With the first approach, it is essential that the mathematical programming problem be kept to a reasonable size to avoid prohibitive computing costs. In this regard, Dixon et al. (1982, p. 48) note, "limiting the size of the constrained maximisation problem without reducing the model's economic detail becomes very difficult, especially when it is recognized that non-linearities in the initial specification of the model must be handled by piecewise linear approximations involving large numbers of additional variables and constraints."

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1 See, for example, Goreux and Manne (1973) and Ginsburgh and Waelbroeck (1976).

2 See, for example, Whalley (1980) and De Melo, Dervis and Robinson (1982).

While the second approach, involving the application of tâtonnement procedures to the excess demand functions of the system, looks more promising, it still requires the writing of detailed, problem-specific solution algorithms. Hence small changes in model closure to allow for the simulation of alternative policies necessitate the rewriting of these algorithms.

The linear approximation method is superior with respect to flexibility. The many closure options some which were discussed here can all be implemented within the existing solution algorithm. The method is, for example, well suited to handling problems involving the design of multi-instrument policy packages to achieve specific economic targets. Target dimensions can be varied and new policy packages computed without rerunning the model. The linear method, furthermore, places virtually no restrictions on model size. Although the number of equations may be too large to permit easy inversion of A_1 with a researcher's local computer matrix inversion package available, this number can easily be reduced by substitution. Rather, the size constraint relates to the availability of sufficiently disaggregated data for enlarging the dimensions of Figure 1 and enabling the estimation of the various behavioural parameters. Finally, the linear framework facilitates model revisions. Equation additions and modifications are handled by appropriate changes in the IO data base of Figure 1 and the parameter file, and by simply rerunning the programme which forms the A matrix. There is no rewriting of solution algorithms.

Dixon et al. (1982) provide a detailed evaluation of the linearisation error in the context of large policy shocks with the Australian ORANI model, the system on which ours is based. Recall that the elements of the A matrix are functions of the various elasticity parameters and cost and sales shares, the latter being computed from the base period IO data of Figure 1. These shares are likely to change when prices and quantities of commodities and production factors which are identified as variables in the model change in reaction to policy shocks. By assuming the shares constant the Johansen method produces an inaccurate solution. In contrast, an n-step Euler procedure comes arbitrarily close to the true solution as n is increased - with n approaching infinity Euler's method can be shown to produce the exact solution.[1] The procedure involves dividing the exogenous shock into n equally small components and computing a series of solutions for these small changes, at each step updating the A matrix, i.e., reevaluating its components on the basis of the newly computed cost and sales shares from the endogenous projections of prices and quantities.

Dixon et al. (1982) have compared Johansen solutions with up to 16 step-Euler solutions for large policy shocks. Their results suggest that the linearisation errors can be safely ignored for practical purposes. Nevertheless they developed a surprisingly simple extrapolation method based on a two-step procedure. Applying the method almost entirely eliminates the linearisation error, thus avoiding the multi-step update procedure the routine application of which would be prohibitively expensive for anything but a very small model. The true solution is approximately

(iv) $\Delta Y \approx \Delta Y_2 + (\Delta Y_2 - \Delta Y_1)$

In (iv) ΔY denotes the vector of the true changes in endogenous variables, whereas ΔY_1 and ΔY_2 denote the results of, respectively, the first and the second step.

The results obtained by Dixon et al. (1982), in our view, have firmly established the linear procedure as the superior method of solving quantitative general equilibrium models for all practical purposes.

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[1] The relevant theorem is proved by Dixon et al. (1982), section 35.

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