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COMMENTS ON "A RECONSIDERATION OF TAX SHIELD VALUATION"
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Abstract

While Arzac and Glosten (2005) affirm that “the value of tax shields depends upon the nature of the equity stochastic process, which, in turn, depends upon the free cash flow process,” I prove that the value of tax shields depends **only** upon the nature of the stochastic process of the net increase of debt. Arzac and Glosten (2005) formulate the constant leverage ratio assumption as $D_t = L \cdot E_t$. The assumption of Fernández (2004) is $E\{D_t\} = L \cdot E\{E_t\}$, where $E\{\cdot\}$ is the expected value operator, D the value of debt, E the equity value, and L a constant. The Arzac and Glosten (2005) assumption requires continuous debt rebalancing, while mine does not. Under both financial policies, the expected leverage ratio is constant, but the Arzac and Glosten (2005) assumption is too extreme.

JEL classification: G12; G13; G31; G32; G33

Keywords: Value of tax shields, APV, required return to equity, cost of capital, net increase of debt

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Introduction

Arzac and Glosten (2005) affirm that “the value of tax shields depends upon the nature of the equity stochastic process, which, in turn, depends upon the free cash flow process.” I do not agree with this conclusion because, as will be proven below in equation (16a), the value of tax shields depends **only** upon the nature of the stochastic process of the net increase of debt.

The derivation of the Miles and Ezzell (1980) formula using pricing kernels helps to consider the value of tax shields from a different perspective. Arzac and Glosten (2005) formulate the constant leverage ratio assumption as $D_t = L \cdot E_t$. The implicit assumption of Fernández (2004) is $E\{D_t\} = L \cdot E\{E_t\}$, where $E\{\cdot\}$ is the expected value operator, D the value of debt, and E the equity value. L is a constant. The Arzac and Glosten (2005) assumption requires continuous debt rebalancing, while my assumption does not. Under both financial policies, the expected leverage ratio is constant, but the Arzac and Glosten (2005) assumption is too extreme. Arzac and Glosten (2005) affirm in their conclusion that “a constant target leverage ratio... is both a realistic approximation to many real life situations and a computationally convenient assumption.” I agree that it is computationally convenient, but it is not a realistic approximation.

To assume $D_t = L \cdot E_t$ is not a realistic approximation to the debt policy of a company because:

1. If the company pays a dividend Div_t , simultaneously the company should reduce debt in an amount $\Delta D_t = -L \cdot Div_t$
2. If the equity value increases, then the company should increase its debt, while if the equity value decreases, then the company should reduce its debt. If the equity value is such that $L \cdot E_t > \text{Assets of the company}$, then the company should have excess cash only for the sake of complying with the debt policy.

1. Value of tax shields and the stochastic process of net debt increases

For simplicity, in Fernández (2004) I neglected to use expected value notation. The equations of my paper that are affected by using the expected value notation, are:

$$ECF_t = PAT_{Lt} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \tag{5a}$$

Where, PAT is Profit after Tax.

$\Delta WCR_t = WCR_t - WCR_{t-1}$. Increase of Working Capital Requirements in period t.

$\Delta NFA_t = NFA_t - NFA_{t-1}$. Increase of Net Fixed Assets in period t.

$\Delta D_t = D_t - D_{t-1}$. Increase of Debt in period t.

$$FCF_t = PAT_{u_t} - \Delta NFA_t - \Delta WCR_t \quad (7a)$$

$$Taxes_{u_t} = [T/(1+T)] PAT_{u_t} = [T/(1+T)] (FCF_t + \Delta NFA_t + \Delta WCR_t) \quad (9a)$$

$$Taxes_{L_t} = [T/(1+T)] (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \quad (12a)$$

$Taxes_{u_t}$ and $Taxes_{L_t}$ are the taxes paid by the unlevered company and those paid by the levered company.

$PV_0[\cdot]$ is the present value operator. The present value in $t=0$ of equations (9a) and (12a) is:

$$G_{u_0} = [T/(1+T)] (V_{u_0} + PV_0[\Delta NFA_t + \Delta WCR_t]) \quad (11a)$$

$$G_{L_0} = [T/(1+T)] (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \quad (14a)$$

The value of tax shields (VTS) comes from the difference between (11a) and (14a):

$$VTS_0 = G_{u_0} - G_{L_0} = [T/(1+T)] (V_{u_0} - E_0 + PV_0[\Delta D_t])$$

As, according to equation (9) of Arzac-Glosten, $V_{u_0} - E_0 = D_0 - VTS_0$, then

$VTS_0 = [T/(1+T)] (D_0 - VTS_0 + PV_0[\Delta D_t])$. And the value of tax shields is:

$$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t] \quad (16a)$$

Equation (16a) shows that the value of tax shields depends **only** upon the nature of the stochastic process of the net increase of debt. The problem of equation (16a) is how to calculate $PV_0[\Delta D_t]$, which requires knowing the appropriate discount rate to apply to the increase of debt.

Equation (16a) is equivalent to the first part of equation (12) of Arzac and Glosten (2005).

It is illustrative to apply (16a) to specific situations.

1.1. Perpetual debt

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and $VTS_0 = T \cdot D_0$

1.2. Debt of one year maturity but perpetually rolled over

As in the previous case, $E\{D_t\} = D_0$, but the debt is expected to be rolled over every year. The appropriate discount rate for the cash flows due to the existing debt is K_d .¹ Define K_{ND} as the appropriate discount rate for the new debt that must be obtained each year, then:

Present value of obtaining the new debt each year² = D_0 / K_{ND}

Present value of the principal repayments at the end of each year³ = $D_0 (1 + K_{ND}) / [(1+K_d) K_{ND}]$

$PV_0[\Delta D_t]$ is the difference of these two expressions. Therefore:

$$PV_0[\Delta D_t] = - D_0 (K_{ND} - K_d) / [(1+K_d) K_{ND}] \quad (50)$$

If $K_{ND} = K_d$, then $PV_0[\Delta D_t] = 0$

In a constant perpetuity ($E\{FCF_t\} = FCF_0$), it seems reasonable that, if we do not expect credit rationing, $K_{ND} = K_d$, which means that the risk associated with the repayment of the actual debt and interest (K_d) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time (K_{ND}).

However, Arzac and Glosten (2005) implicitly assume that $K_{ND} = K_u$, and so,

$$PV_0[\Delta D_t] = - D_0 (K_u - K_d) / [(1+K_d) K_u],$$

because substituting this expression in (16a), we get their equation (13) for $g = 0$.

1.3. Debt is proportional to the Equity value

Arzac and Glosten (2005) show that if $D_t = L \cdot E_t$, then the value of tax shields for perpetuities growing at a constant rate g is their equation (13):

$$VTS_0 = \frac{D_0 K_d T (1 + K_u)}{(K_u - g) (1 + K_d)}$$

Substituting it in (16a) we get:

$$PV_0[\Delta D_t] = D_0 \frac{(K_d - K_u) + g(1 + K_d)}{(K_u - g)(1 + K_d)}$$

Note that the constant leverage ratio ($D_t = L \cdot E_t$) assumption is equivalent to assuming that $K_{ND} = K_u$.

¹ We use K_d so as not to complicate the notation. It should be K_{d_t} , a different rate following the yield curve. Using K_d we may also think of a flat yield curve.

² Present value of obtaining the new debt every year = $D / (1+K_{ND}) + D / (1+K_{ND})^2 + D / (1+K_{ND})^3 + \dots$ because $D = E\{D_t\}$, where D_t is the new debt obtained at the end of year t (beginning of $t+1$).

³ The present value of the principal repayment at the end of year 1 is $D / (1+K_d)$
 The present value of the principal repayment at the end of year 2 is $D / [(1+K_d)(1 + K_{ND})]$
 The present value of the principal repayment at the end of year t is $D / [(1+K_d)(1 + K_{ND})^{t-1}]$
 Because $D = E\{D_t\}$, where D_t is the debt repayment at the end of year t .

1.4. Debt increases are as risky as the free cash flows

Then the correct discount rate for the expected increases of debt is K_u , the required return to the unlevered company. In the case of a constant growing perpetuity,

$$PV_0[\Delta D_t] = g \cdot D_0 / (K_u - g),$$

and the VTS is equation (28) in Fernández (2004): $VTS_0 = T \cdot K_u \cdot D_0 / (K_u - g)$.

1.5. The company expects to repay the current debt without issuing new debt

In this situation, the appropriate discount rate for the negative ΔD_t (because they are principal payments) is K_d , the required return to the debt. In this situation, Myers (1974) applies.

2. Value of net debt increases implied by the alternative theories

From equation (16a) the present value of the increases of debt is

$$PV_0[\Delta D_t] = (VTS_0 - T \cdot D_0) / T.$$

Applying this equation to some of the theories mentioned in Fernández (2004), we may construct the predictions that each of these theories have for $PV_0[\Delta D_t]$. These predictions are reported in Table 1.

Table 1. Present value of the increases of debt implicit in some formulae for calculating the value of tax shields. Constant growing perpetuities at a rate g

	$PV_0[\Delta D_t]$	VTS_0
Fernández (2004)	$g \cdot D_0 / (K_u - g)$	$PV[K_u; D \cdot T \cdot K_u]$
Myers (1974)	$g \cdot D_0 / (K_d - g)$	$PV[K_d; D \cdot T \cdot K_d]$
Miles and Ezzell (1980), Arzac and Glosten (2005)	$\frac{g \cdot D_0}{K_u - g} - \frac{D_0 (K_u - K_d)}{(K_u - g)(1 + K_d)}$	$PV[K_u; D \cdot T \cdot K_d] (1 + K_u) / (1 + K_d)$
Modigliani-Miller (1963) Sick (1990)	$g \cdot D_0 / (R_F - g)$	$PV[R_F; D \cdot T \cdot R_F]$

Fieten *et al.* (2003) (mentioned by Arzac and Glosten) conclude that the right theory is that of Sick (1990), but, obviously, it may be applied only if the debt is risk-free.

We have argued that Miles and Ezzell (1980) does not make sense because it depends on the assumption that $D_t = L \cdot E_t$ in every time t .

The two theories that have some economic sense are Myers (1974) and Fernández (2004). As we have already argued, Myers (1974) should be used when the company will not issue new debt, and Fernández (2004) when the company expects to issue new debt in the future. Both theories provide the same value for no-growth perpetuities.

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