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EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

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EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION (1)

Abstract

This paper shows that **the three valuation methods** (if used correctly) **always** yield the **same result**.

The **most striking result of this paper** is that the Net Present Value of the tax shield due to interest payments (in the APV approach) should be calculated as follows in order to derive an accurate result:

$$\textbf{NPV OF INTEREST TAX SHIELDS} = \sum_{t=1}^{\infty} \frac{D_{t-1}\, Ku_t\, T}{\prod_{t=1}^{t} (1 + Ku_t)}$$

T = Corporate tax rate
$\mathbf{Ku_t}$ = Cost of unlevered equity in period t
$\mathbf{D_{t-1}}$ = Value of debt in period t-1

At first, it would appear that this formula implies that debt has a cost of Ku , and that the interest tax shields are discounted at Ku , but this is not the case. The Net Present Value of interest tax shields **is not** (and this is the main error in previous papers about this topic) the NPV of a unique flow, **but the difference of two NPVs** of two flows with **different risks**: the NPV of the taxes paid in the unlevered firm and the NPV of taxes paid in the levered firm. Our formula is the difference of these two NPVs. Obviously, the flow of taxes paid in the levered firm is smaller but riskier than the flow of taxes paid in the unlevered firm.

We will show that, if used correctly, these three approaches to firm valuation will yield the same result. We will apply these valuation procedures to perpetuities, to growing companies (at a constant rate g) and, finally, to any company.

The main objective of this paper is to show that the three valuation methods (see formulas [1], [2] and [3]) always yield the same result. The paper also helps to think more about the meaning of the formulas and their relationships.

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

1. Valuation formulas (2)

Dictionary:

T = Corporate tax rate; **FCF** = Free cash flow; **CFacc** = available Cash flow for shareholders (3);
I = interest paid; $\mathbf{K_u}$ = Cost of unlevered equity (required return of unlevered equity);
Ke = Cost of levered equity (required return of levered equity); **Kd** = Required return of debt;
$\mathbf{K_{Tu}}$ = Appropriate discount rate for tax flow of unlevered firm; $\mathbf{R_F}$ = Risk free rate;
$\mathbf{K_{T_L}}$ = Appropriate discount rate for tax flow of levered firm; **C** = Value of shares at t = 0;
D = $\mathbf{D_0}$ = Value of debt at t = 0; $\mathbf{V_U}$ = Value of shares in the unlevered company at t = 0;
r = Cost of debt;
WACC = weighted average cost of capital; $\mathbf{N_t}$ = Nominal amount of debt repaid in year t.;
$\mathbf{b_d}$ = Beta of debt; $\mathbf{b_U}$ = Beta of unlevered equity = beta of assets;
$\mathbf{b_L}$ = Beta of levered equity; $\mathbf{P_M}$ = Market premium = E (R_M - R_F)
$\mathbf{Taxes_U}$ = Taxes paid by the unlevered company; $\mathbf{Taxes_L}$ = Taxes paid by the levered company
$\mathbf{GOV_U}$ =NPV of **TaxesU** = portion of the value of the unlevered company that belongs to the government.
$\mathbf{GOV_L}$ =NPV of **TaxesL** = portion of the value of the levered company that belongs to the government.

The WACC approach to firm valuation:

[1] $$D + C = \sum_{t=1}^{\infty} \frac{FCF_t}{\prod_{1}^{t} (1 + WACC_t)}$$

The flows to equity approach to equity valuation:

[2] $$C = \sum_{t=1}^{\infty} \frac{CFacc_t}{\prod_{1}^{t} (1 + Ke_t)}$$

The APV formula:

[3] $$D + C = \sum_{t=1}^{\infty} \frac{FCF_t}{\prod_{1}^{t} (1 + Ku_t)} + \text{NPV of interest tax shields}$$

Value of debt (4):

[4] $$D_0 = \sum_{t=1}^{\infty} \frac{I_t + N_t}{\prod_{1}^{t} (1 + Kd_t)}$$

From CAPM:

[5] $Ku = R_F + b_U P_M$; **[6]** $Ke = R_F + b_L P_M$; **[7]** $Kd = R_F + b_d P_M$

2. Perpetuities (No growth)

For perpetuities without growth, [1], [2] and [3] can be written as [1p], [2p] and [3p]:

[1p] $C = FCF / WACC - D$; $D = I/Kd$

[2p] $C = CFacc / Ke$

[3p] $C=FCF/Ku + \text{NPV of interest tax shields} - D$

The relationship between FCF and CFacc is:

[4p] $CFacc = FCF - I (1 - T) = FCF - D\,Kd\,(1-T)$

2.1 Relationships implied by the valuation formulas

Because [1p] and [2p] must yield the same result, using [4p], we get:

$$\frac{FCF}{WACC} - D = \frac{FCF - D\,Kd\,(1 - T)}{Ke}$$

Some algebra and we get the definition of WACC:

[8] $$\mathbf{WACC = \frac{C\,Ke + D\,Kd\,(1 - T)}{C + D}}$$

For a perpetuity, profit after tax (PAT) is equal to the available cash-flow for the shareholders (5) (CFacc): PAT = CFacc

FCF_0 is the free cash flow of the company without taxes (6): FCF = FCF_0 (1- T). For the unlevered company (D = 0) : $Taxes_U$ = T PBT_U = T FCF_0. From this equation it should be clear that the taxes paid by the unlevered company have the same risk as FCF_0, and their required rate of return is Ku.

For the levered company: $Taxes_L$ = T PBT_L = T PAT_L / (1 - T) = T CFacc / (1 - T) From this equation it should be clear that the taxes paid by the levered company have the same risk as CFacc, and their required rate of return is Ke.

The NPV of taxes paid by the levered company, which is the portion of the value of the company that belongs to the government (GOV_L) is (7):

GOV_L = T PBT / Ke = T PAT_L / [(1 - T) Ke] = T CFacc / [(1 - T) Ke]

The NPV of taxes paid by the unlevered company, which is the portion of the value of the unlevered company that belongs to the government (GOV_U) is

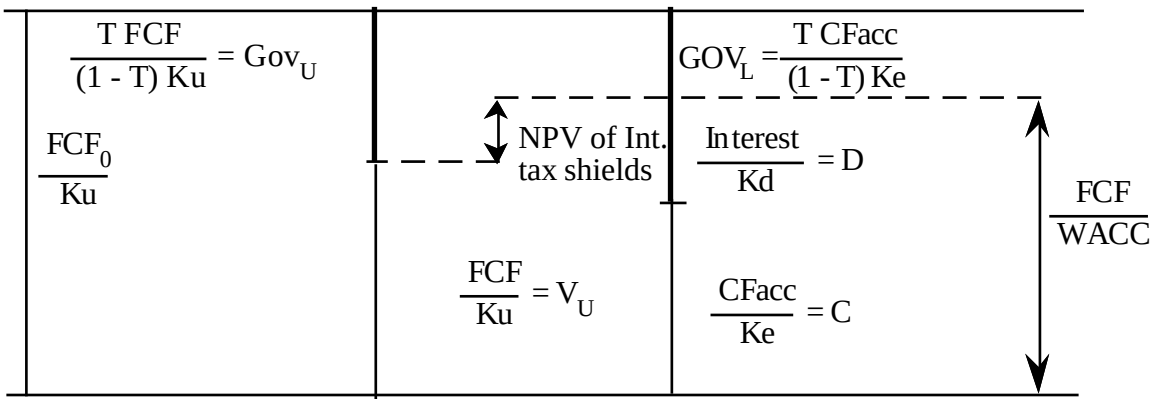
GOV_U = T FCF / [(1 - T) Ku]

The NPV of interest tax shield is: GOV_U - GOV_L = [T/1-T)] [FCF/Ku - CFacc/Ke]

Using [2p] and [3p]:

[9] NPV of interest tax shield = DT

As we will see, formula [9] only holds for constant perpetuities (without growth).



Because [2p] and [3p] must yield the same result (8), using [4p], we get:

[10]

$$\mathbf{Ku = \frac{C\ Ke + D\ Kd\ (1 - T)}{C + D\ (1 - T)}}$$

From [10], and using [5], [6] and [7], we get

[11] $$\beta_L = \frac{\beta_U[C + D(1 - T)] - \beta_d D(1 - T)}{C}$$

Because [1p] and [3p] must yield the same result, we get:

[12] $$\mathbf{WACC = Ku \frac{C + D\,(1 - T)}{C + D} = Ku \left(1 - \frac{D\,T}{C + D} \right)}$$

Formula [12] means that (if T>0) WACC is smaller (9) than Ku.

From [3p] we find another formula to value the equity:

$$C = \frac{FCF}{Ku} + DT - D = \frac{CFacc + D\,Kd\,(1 - T)}{Ku} + \frac{D\,Ku\,T}{Ku} - D$$

[13] $$\mathbf{C = \frac{CFacc}{Ku} - \frac{D\,(1 - T)\,(Ku - Kd)}{Ku}}$$

[14] $$\mathbf{K_{Tu} = K_u;} \qquad \mathbf{K_{TL} = Ke}$$

2.3 Examples of companies without growth

Table 1 shows the valuation of 4 companies without growth. The four companies have different debt and tax rates. Company 1 is a company without debt and T=0. Company 2 is a company without debt and T=35%. Company 3 is a company with debt = 1000 million and T=0. Company 4 is a company with debt = 1000 million and T=35%. We calculate the value of the equity using the three valuation formulas ([1p], [2p] and [3p]) and we get the same result (see lines 20, 23 and 26).

Table 1. Perpetuities without growth

		D = 0 T = 0% g=0%	D = 0 T = 35% g=0%	D = 1000 **T = 0%** Kd = 13% g=0%	D = 1000 T = 35% Kd = 13% g=0%
		Company 1 **[1]**	**Company 2** **[2]**	**Company 3** **[3]**	**Company 4** **[4]**
1	P.B.I.T. (profit before interest and taxes)	1000	1000	1000	1000
2	Interest	0	0	130	130
3	PBT	1000	1000	870	870
4	Taxes	0	350	0	304.5
5	PAT	1000	650	870	565.5
6	+ Depreciation	200	200	200	200
7	- Payment of fixed assets	-200	-200	-200	-200
8	**CFacc**	**1000**	**650**	**870**	**565.5**
9	**FCF**	**1000**	**650**	**1000**	**650**
10	ßu	1.00	1.00	1.00	1.00
11	Rf	12.00%	12.00%	12.00%	12.00%
12	E (Rm - Rf) = market premium	8.00%	8.00%	8.00%	8.00%
13	Ku	20.00%	20.00%	20.00%	20.00%
14	**Vu**	**5,000**	**3,250**	**5,000**	**3,250**
15	D	0	0	1,000	1,000
16	Kd			13.00%	13.00%
17	Beta d (ßd)			0.125	0.125
18	NPV of interest tax shields = DT	0	0	0	350
19	NPV of interest tax shields + Vu	5,000	3,250	5,000	3,600
20	- D = **C**	**5,000**	**3,250**	**4,000**	**2,600**
21	Beta levered (ßL)	1.000000	1.000000	1.218750	1.218750
22	Ke	20.00%	20.00%	21.75%	21.75%
23	**C = CF / Ke**	**5,000**	**3,250**	**4,000**	**2,600**
24	WACC	20.0000%	20.0000%	20.0000%	18.0556%
25	FCF / WACC	5,000	3,250	5,000	3,600
26	**C** = (FCF / WACC) - D	**5,000**	**3,250**	**4,000**	**2,600**
27	**Value of EQUITY (C)**	**5,000**	**3,250**	**4,000**	**2,600**
28	**Value of DEBT (D)**	**0**	**0**	**1,000**	**1,000**
29	**NPV of taxes (GOV)**	**0**	**1,750**	**0**	**1,400**
30	**SUM (27 + 28 + 29)**	**5,000**	**5,000**	**5,000**	**5,000**
31	**FCFo**	**1,000**	**1,000**	**1,000**	**1,000**

Lines 1 to 5 show the P&L of the four companies. **Line 8** shows the available cash flow for shareholders (CFacc). **Line 9** shows the Free Cash Flow (FCF). **Line 10.** We assume the unlevered beta = ßu = 1. **Line 11.** We assume the risk-free rate = RF = 12%. **Line 12**. Market premium = E (Rm - RF) = 8%. **Line 13**. Cost of unlevered equity = Ku = RF + ßu E(Rm - Rf) =20%. **Line 14**. Value of unlevered company (Vu = FCF/Ku) = 5,000 millions when T=0 and 3,250 millions when T= 35%. **Line 15.** Value of debt. **Line 16** is the cost of debt (Kd). **Line 17.** Beta that corresponds to debt according to [7]. **Line 18.** NPV of interest tax shields, which for **these companies** (perpetuities without growth) is DT. **Lines 19 and 20**. Use of formula [3p]. **Line 21.** Leveraged beta according to formula [11]. **Line 22.** Required return of leveraged equity according to [6]. **Line 23.** Value of equity according to formula [2p]. **Line 24**. WACC according to formula [8]. **Lines 25 and 26.** Value of equity according to formula [1p].

3. Companies with constant growth

For companies growing at a constant rate g, formulas [1], [2] and [3] can be written as [1c], [2c] and [3c]:

[1c] $$C = \frac{FCF_1}{WACC - g} - D$$

[2c] $$C = \frac{CFacc_1}{Ke - g}$$

[3c] $$C = \frac{FCF_1}{K_u - g} + \text{NPV of interest tax shields} - D$$

The relationship between FCF and CFacc:

[4c] $CFacc = FCF - I\,(1 - T) + \Delta D$; as $I = D\,Kd$; and $\Delta D = g\,D$,
$CFacc_1 = FCF_1 - Do\,[Kd\,(1 - T) - g]$

3.1 Relationships implied by the valuation formulas

Because [1c] and [2c] must yield the same result, using [4c], we find again formula [8]:

[8] $$WACC = \frac{C\,Ke + D\,Kd\,(1-T)}{C + D}$$

Because [1c] and [3c] must yield the same result, we get:

$(C + D)\,(WACC - g) = (C + D - \text{NPVint tax shields})\,(Ku - g)$ then:

[15] NPV interest tax shields = $(C + D)\,(Ku - WACC) / (Ku - g)$

FCF_0 is the free cash flow of the company without taxes.
By definition: $FCF_0 = FCF + Taxes_U$
$Taxes_U$ are the taxes paid by the unlevered company (D = 0) : $Taxes_U = T\,PBT_U$
For constant growth companies (10), $PBT_U \neq FCF0$. We define a parameter H that takes into account this difference (normally due to increases in working capital requirements and net fixed assets): $PBT_U = FCF_0 + H$. Then: $Taxes_U = T\,(FCF_0 + H)$. And also (11):
$FCF_0 = (FCF + T\,H) / (1-T)$.
For the levered company:
$Taxes_L = [FCF_0 + H - Kd\,D]\,T$
It is clear that the taxes paid by the levered company **do not have** the same risk as CFacc.

The NPV of taxes paid by the unlevered company, which is the portion of the value of the unlevered company that belongs to the government (GOV_U), is

$GOV_U = Taxes_U / (Ku - g) = T (FCF0 + H) / (Ku - g)$

The NPV of taxes paid by the levered company, which is the portion of the value of the company that belongs to the government (GOV_L), is (12):

$GOV_L = Taxes_L / (K_{TL} - g) = (FCF_0 + H - Kd\ D)\ T / (K_{TL} - g)$

The NPV of the interest tax shield is:

$$GOV_U - GOV_L = [T (FCF_0 + H) / (Ku - g)] - [T (FCF_0 + H - Kd\ D) / (K_{TL} - g)] =$$

$$= [T / (1 - T)] [[(FCF + H) / (Ku - g)] - [(CFacc+ H - gD) / (K_{TL} - g)]]$$

Taking into account [10] and [15], we find [16] and [17]:

$$(C + D) (Ku - g) - DT\ Ku = C (Ke - g) + D[Kd (1 - T) - g], \qquad [10]$$

$$\text{NPV of interest tax shields} = \frac{C + D}{Ku - g}\left[Ku - Ku\frac{C + D(1 - T)}{C + D}\right] = \frac{DTKu}{Ku - g}$$

[16] NPV of interest tax shields = DT Ku / (Ku-g)

At first, it would appear that this formula implies that debt has a cost of Ku , and that the interest tax shields are discounted at Ku , but this is not the case (13). The NPV of interest tax shields **is not** (and this is the main error in previous papers on this topic) the NPV of a unique flow, **but the difference of two NPVs** of two flows with **different risk**: the NPV of the taxes paid in the unlevered firm and the NPV of taxes paid in the levered firm. Our formula is the difference of the two NPV. Obviously, the flow of taxes paid by the levered firm is smaller, but riskier than the flow of taxes paid by the unlevered firm.

[17] $$K_{T_L} - g = \frac{CKe - g(C + D) + H}{CKu - g(C + D) + H}(Ku - g)$$

[17] can also be written as (14):

$$K_{T_L} = Ku + \frac{D(1 - T)\ (Ku - Kd)}{C + \dfrac{H - gD}{Ku - g}}$$

Comparing [17] with [10], it is clear that $K_{TL} > Ke$ when $H < g\ D$, and $K_{TL} < Ke$ when $H > g\ D$.

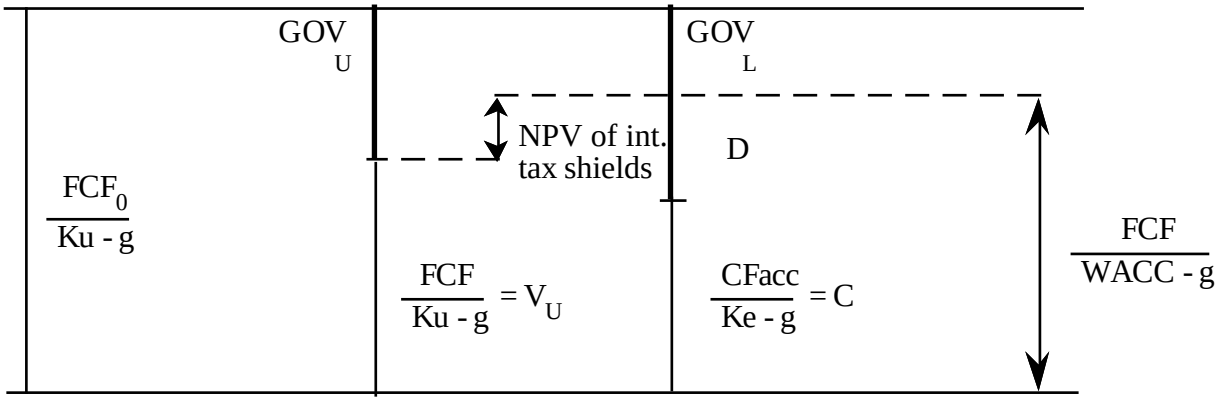
In Exhibits 1, 2, 3 and 4, the difference between PBT and FCF is only due to the increase in working capital requirements: H = g WCR. In Exhibit 1**, the difference is also due to increases in net fixed assets: H = g (WCR + NFA).

Because [2c] and [3c] must yield the same result, using [4p], we get:

$$(C + D)\, Ku - DT\, Ku = C\, Ke + D\, Kd\, (1 - T)\,, \qquad [10]$$

Value of the company that corresponds to shares (C), debt (D) and government (GOV) in three different scenarios.

We consider that FCF and FCFo have the same risk



D=0 **D = 0** **Levered company**
T=0

From [3c], using [15] and [4c], we get [18], which is an alternative formula (a kind of certainty equivalent) to [2c]: this provides a way to value the equity without calculating Ke.

$$C = \frac{FCF}{Ku - g} + \frac{D\, Ku\, T}{Ku - g} - D = \frac{CFacc + D\, Kd\, (1 - T) - g\, D + D\, Ku\, T - D\, Ku + g\, D}{Ku - g}$$

[18]
$$C = \frac{CFacc}{Ku - g} - \frac{D\, (Ku - Kd)\, (1 - T)}{Ku - g}$$

The «maximum theoretical debt» that the company can support, that is, debt when C = 0, is (when CFacc = 0) (15):

[19]
$$D_0 = \frac{FCF}{Kd\, (1 - T) - g}$$

In this situation, as [18] holds, Kd = Ku, then from [4c]:

$$D_0 = \frac{FCF_1}{Ku\, (1 - T) - g} = \frac{V_U\ (Ku - g)}{Ku\, (1 - T) - g}$$

3.2 Examples of companies with constant growth

Exhibits 1, 1**, 2, 3 and 4 show the valuation of five different companies with constant growth of 5%. Exhibit 1** differs from Exhibit 1 only in net fixed assets: Exhibit 1 has constant net fixed assets (as do Exhibits 2, 3 and 4), but in Exhibit 1** net fixed assets also grow 5%. The companies all have different Debt and Tax rates. Exhibits 1 and continuation show two companies with an initial Debt = 500 and T=35%. Exhibit 2 shows a company with an initial Debt=500 and T=0 (no taxes). Exhibit 3 shows an unlevered company (no debt) with T=0. Exhibit 4 shows an unlevered company (no debt) with T=35%. We calculate the value of the equity using the three valuation formulas ([1c], [2c] and [3c]) and we get the same result (see lines 53, 56 and 61). Table 2 shows some of the results of these five exhibits and provides explanations of the lines.

Table 2. Cash-flows, discount rates and values of companies with constant growth (g = 5%)
Numbers come from Exhibits 1 to 4

	Exhibits that correspond to these values				
	[3]	***[2]***	***[4]***	***[1]***	***[1**]***
Cash-flows of year 1					
CFacc (line 27)	1,000	950	632.5	608.75	558.75
Taxes (line 21)	—	—	367.5	341.25	341.25
Debt flow (D Kd - ΔD) (line 19 - line 24)	—	50	—	50	50
SUM = FCFo (line 43)	1,000	1,000	1,000	1,000	950
Ke (line 55)	20%	20.405%	20%	20.411%	20.45%
Kd (line 49)	—	15%	—	15%	15%
KTax	—	—	20%	20.395%	20.395%
Value of Shares (C), Debt (D) and NPV of taxes paid (GOV) in t=0					
C (lines 53, 56 and 61)	6,667	6,167	4,217	3,950	3,617
GOV = Taxes/(KTax-g)	—	—	2,450	2,217	2,217
D = Debt flow/(Kd-g) (line 48)	—	500	—	500	500
SUM (lines 44 and 47)	6,667	6,667	6,667	6,667	6,334
Two ways of calculating the NPV of interest tax shields in t=0:					
Correct: DTKu/(Ku-g) (line 51)				**233.33**	**233.33**
Wrong: DTKd/(Kd-g) (line 62)				**262.50**	**262.50**

Notes to Exhibits 1, 1, 2, 3 and 4**
Lines 1 to 7 show the Assets of the companies. **Lines 8 to 11** show the Liabilities and Net Worth of the companies. **Line 12** shows the Working Capital Requirements (WCR). **Lines 14 to 22**: P&L of the companies. **Lines 23 to 27**: available cash flow for shareholders (CFacc). **Line 28** shows the Free Cash Flow (FCF). **Line 37.** We assume the unlevered beta = ßu = 1. **Line 38.** We assume the risk free rate = R_F = 12%. **Line 39**. Market premium = E (R_m-R_F) = 8%. **Line 40**. Cost of unlevered equity = Ku = R_F + $ß_u$ E(Rm - Rf) =20%. **Line 41**. Value of unlevered company [Vu = FCF/(Ku-g)]. **Line 43** shows the Free cash flow of the company without taxes (FCFo). **Line 44**. Value of unlevered company without taxes [Vu = FCFo/(Ku-g)]. **Line 45**: Taxes paid by the unlevered company ($Taxes_u$). **Line 46**: NPV of Taxes paid by the unlevered company (GOV_u). **Line 47**: GOV_u + Vu (line 46 + line 41 = line 44). **Line 48.** Value of debt. **Line 49** is the cost of debt (Kd). **Line 50.** Beta that corresponds to debt according to [7]. **Line 51. NPV of interest tax shields = DTKu/(Ku-g)**. **Lines 52 and 53**. Value of equity according to APV, using formula [3c]. **Line 54.** Leveraged beta according to formula [11]. **Line 55.** Required return of leveraged equity according to [6]. **Line 56.** Value of equity according to formula [2c]. **Line 59**. WACC according to formula [8]. **Lines 60 and 61.** Value of equity according to formula [1c]. **Line 62 =** «traditional way» of calculating NPV of interest tax shields = DTKd/(Kd-g).

4. General case

4.1 Relationships implied by the formulas

We can rewrite [1], [2] and [3] as

[1*] $D_1 + C_1 = (D_0 + C_0)(1 + WACC_1) - FCF_1$

[2*] $C_1 = C_0 (1 + Ke_1) - CFacc_1$

[3*] $D_1 + C_1 = (D_0 + C_0)(1 + Ku_1) - FCF_1 - D_0 Ku_1 T$

The relationship between CFacc and FCF is:

[4*] $CFacc_1 = FCF_1 + D_1 - D_0 - D_0 Kd_1 (1 - T)$

From [1*], [2*], [3*] and [4*], and following the same procedure as in previous sections, we also get:

$C_0 Ke_1 + D_0 Kd_1 (1 - T) = (D_0 + C_0) WACC_1$, equivalent to **[8]**

$C_0 Ke_1 + D_0 Kd_1 (1 - T) = [C_0 + D_0 (1 - T)] Ku_1$, equivalent to **[9]**

$[C_0 + D_0 (1 - T)] Ku_1 = (D_0 + C_0) WACC_1$, equivalent to **[12]**

and the most noticeable:

[20] **NPV OF INTEREST TAX SHIELDS** $= \sum_{t=1}^{\infty} \frac{D_{t-1} Ku_t T}{\prod_{t=1}^{t} (1 + Ku_t)}$

The following relationships are also important:

$K_{TU} = Ku$

$CFacc_t = FCF_t + \Delta D_t - I_t (1 - T)$

$\Delta D_t = D_t - D_t\text{-}1$

$I_t = D_t\text{-}1 \, Kd_t$

KT_{L1} = required rate for the $Taxes_{L1} = Ku_1 + T D_0 (Ku_1 - Kd_1) / GOV_{L0}$

4.2. An example of valuation

Exhibit 5 shows the valuation of a company (16) that grows (but the pattern of growth is not constant) until year 9. After year 10, we assume a constant growth rate of 5%.

We calculate the value of the equity using the three valuation formulas ([1], [2] and [3]) and we find the same result (see lines 53, 56 and 61): 506 million in t=0.

The most important results of Exhibit 5 are:

1) We get the same value of equity (in t=0 and in future years) using the valuation formulas [1], [2] and [3] (see lines 53, 56 and 61): 506 million.

2) The net present value of interest tax shields (using [20]) is 626.72 million (line 51).

3) If we calculate the net present value of the interest tax shields as the NPV of D Kd T discounted at Kd (see line 62), we obtain a different value: 621.93 million. This therefore leads to a different value from formula [3] in comparison with formulas [1] and [2].

Table 3 shows the most important results of Exhibit 5.

Table 3. Cash-flows, discount rates and values of the company analyzed in Exhibit 5

Year	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CFacc (line 27)	.	87	19.5	20.75	38.25	25.13	35	31.65	78.65	171.02	463.42	486.59	510.92
FCF (line 28)		262.5	-305	245	512.5	475	310.5	447.4	470.02	488.02	510.92	536.47	563.29
D (line 9)	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
Kd (line 49)	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%
C (lines 53, 56, 57 and 61)	**506**	579	734	935	1158	1431	1741	2113	2504	2873	3016	3167	3326
Net Worth (line 10)	500	530	660	740	770	1000	1290	1610	1930	2209	2243	2280	2318
ßu Beta unlevered (line 37)	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ßL Beta levered (line 54)	2.44	2.26	2.27	2.00	1.72	1.51	1.40	1.28	1.19	1.14	1.14	1.14	1.14
Ku (line 40)	**20%**	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
Ke (line 55)	31.55%	30.10%	30.18%	28.00%	25.75%	24.09%	23.17%	22.23%	21.56%	21.13%	21.13%	21.13%	21.13%
Unlevered company without taxes:													
Vu without taxes (line 44)	2917.1	3080.6	3826.7	4172.0	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2
Unlevered company with taxes = 35%:													
Vu (line 41)	1679.6	1753.1	2408.7	2645.4	2662.0	2719.4	2952.8	3096.0	3245.1	3406.1	3576.5	3755.3	3943.0
GOVu (line 42)	1237.5	1327.5	1418.0	1526.6	1674.4	1764.3	1847.6	1938.5	2035.5	2137.3	2244.0	2356.3	2474.2
SUM	2917.1	3080.6	3826.7	4172.0	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2
Levered company with taxes = 35%:													
C (lines 53, 56, 57 and 61)	506.3	579.2	734.0	934.8	1158.2	1431.4	1741.1	2113.0	2504.0	2872.8	3016.4	3167.3	3325.7
D (line 9)	1800.0	1800.0	2300.0	2300.0	2050.0	1800.0	1700.0	1450.0	1200.0	1000.0	1050.0	1102.5	1157.6
GOV_L (line 58)	610.8	701.4	792.7	937.2	1128.2	1252.3	1359.3	1471.5	1576.6	1670.6	1754.1	1841.8	1933.9
SUM	2917.1	3080.6	3826.7	4172.0	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2

Table 4 shows a sensitivity analysis for the value of the shares (C) when t = 0.

Table 4. Sensitivity analysis for the value of the shares (C) in t = 0
Value of the shares (C) in t = 0, changing one assumption in Exhibit 5

Value of shares from Exhibit 5	**506 millions**
Tax rate = 30% (instead of 35%)	594
Risk-free rate (RF) = 11% (instead of 12%)	653
Risk premium (PM) = 7% (instead of 8%)	653
bu = 0.9 (instead of 1.0)	622
Terminal growth (after year 10) = 6% (instead of 5%)	546

5. Formulas when the nominal value of debt (n) is not equal to the «market value» (D)

In previous sections we have assumed that r = Kd, but if r ≠ Kd then the value of debt (D) will be different from its nominal amount (N).

N is the nominal value of debt (the money that the company has received), **r** is the interest rate paid and **Nr** the annual interest paid.

Kd is the required return of the debt: the «reasonable» rate that would be required by the bondholders or the bank (depending on the business risk and the amount of the debt) (17) if they had the same information that the shareholders have and if they did not expect any agency costs.

5.1. Perpetuities

For perpetuities, **Nr = DKd**, and it can be shown that formulas [1p], [2p], [3p] and [4p] hold. We only need to take into account that **D = Nr / Kd.**

It is obvious that [4p] holds:

[4p] $CFacc = FCF - Nr\,(1 - T) = FCF - D\,Kd\,(1 - T)$

5.2. Companies with constant growth

For companies with constant growth g:

$$\mathbf{D = \frac{r\,N - g\,N}{Kd - g} = N\,\frac{r - g}{Kd - g}}$$

$D\,Kd - Nr = g\,(D - N)$ if debt grows (the net revenues from banks or bond issues) in year 1 $\Delta N_1 = g\,N_0$.

For these companies, we have to introduce some changes in the equations that we developed in the previous sections because

[4c'] $CFacc = FCF - Nr(1 - T) + gN = FCF - D(Kd - g) + NrT$

It is clear that if $r \neq Kd$, then [4c'] is different from [4c].

Substituting [4c'] and [2c] in [1c], we get a new formulation of WACC:

$$C + D = \frac{CFacc + D(Kd - g) - NrT}{WACC - g} = \frac{C(Ke - g) + D(Kd - g) - NrT}{WACC - g}$$

[8'] $$\mathbf{WACC = \frac{C\,Ke + D\,Kd - Nr\,T}{C + D}}$$

Note that formula [8'] is equal to formula [8] when N = D, which means r = Kd.

As a result of these changes, formula [16] will also change to [16']. Formula [16'] is equal to formula [16] when N = D, which means r = Kd.

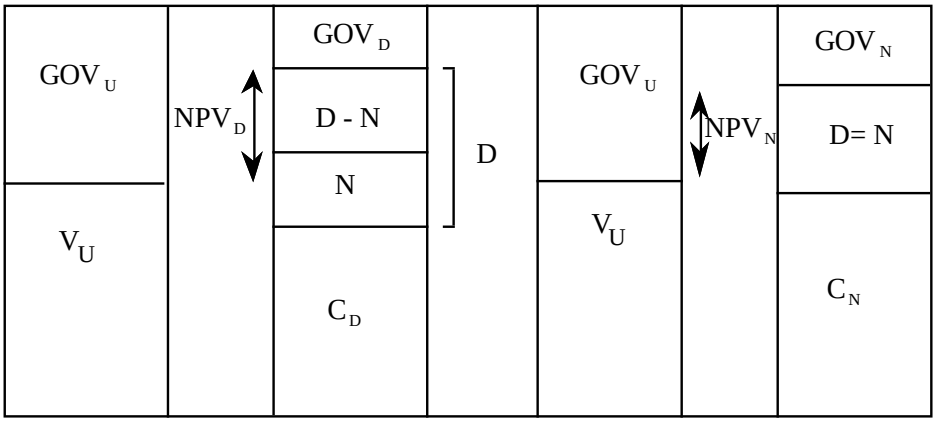
[16'] $$\text{NPV of interest tax shields} = \frac{DTKu + T[Nr - D\,Kd]}{Ku - g}$$

Some other interesting formulas are:

$$\Delta D - \Delta N = g\,D\left[1 - \frac{Kd - g}{r - g}\right] = g\,D\left[\frac{r - Kd}{r - g}\right]$$

[16´´] $$\text{NPV of interest tax shields} = \frac{DTKu}{Ku - g} - \frac{Tg(D - N)}{Ku - g}$$

Figure 1. Difference of two companies: one with r > Kd; the other with r = Kd.



Company with D > N r > Kd; D > N

Company with D = N r = Kd; D = N

Figure 1 allows us to write the following identities:

$C_D + D + GOV_D = C_N + N + GOV_N$;

$NPV_D - NPV_N = GOV_N - GOV_D$;

Using [16´´] and [16]

$$NPV_D - NPV_N = \frac{DTKu}{Ku - g} - \frac{Tg(D-N)}{Ku - g} - \frac{NTKu}{Ku - g} = \frac{T}{Ku - g}\left[(D-N)Ku - g(D-N)\right] =$$

$NPV_D - NPV_N = GOV_N - GOV_D = T\,(D - N)$

$C_N - C_D = (D\text{-}N)\,(1\text{-}T)$

5.3. General case

$$D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1}\, r_t - (N_t - N_{t-1})}{\prod_{1}^{t} (1 + Kd_t)}$$

$$D_1 = \sum_{t=2}^{\infty} \frac{N_{t-1}\, r_t - (N_t - N_{t-1})}{\prod_{2}^{t} (1 + Kd_t)}\;;\; \frac{D_1}{1 + Kd_1} = \sum_{t=2}^{\infty} \frac{N_{t-1}\, r_t - (N_t - N_{t-1})}{\prod_{1}^{t} (1 + Kd_t)}$$

$$D_0 - \frac{D_1}{1 + Kd_1} = \frac{N_0\, r_1 - (N_1 - N_0)}{1 + Kd_1}$$

$$D_1 = D_0\,(1 + Kd_1) - N_0\,(1 + r_1) + N_1$$

$$D_1 - D_0 = N_1 - N_0 + D_0\, Kd_1 - N_0\, r_1$$

5.4. A formula for Kd

We still do not have a formula for Kd, the «reasonable» rate that would be required by the bondholders or the bank. We can also think of Kd as the «reasonable» required rate of the debt flows for a person who thinks that the «reasonable» required rate for the FCF is Ku. Another way of thinking about Kd is to assume that debt and equity belong to the same person: if this person thinks that Ku is the «reasonable» required rate for the FCF, Kd is the «reasonable» required rate for the debt flows.

Formula [10] tells us the relationship among Ku, Ke and Kd for any level of debt, but so far we do not have any formula to calculate Kd given (Ku) and the debt level.

From formulas [13] or [19], we can calculate the «maximum theoretical debt» that the company can support, that is, debt when C = CFacc = 0. With this leverage, Kd = Ku because the debt supports the same risk as the unlevered equity.

On the other hand, the required rate of return for a minimum debt (think of a debt of $1) should be RF. A formula for Kd that fulfills both requirements is [21]:

[21] ; $$Kd = R_F + \frac{D(1 - T)}{D(1 - T) + C}\left(Ku - R_F\right)$$

which (18) means:

[22] $$\beta_d = \frac{D(1 - T)}{D(1 - T) + C}\beta_U$$

[21] and [22] imply **$Ke - Kd = Ku - RF = b_U\, PM$**

Other interesting relationships are:

$$Ke - Kd = \frac{D\,(1 - T) + C}{C}\,(Ku - Kd)\ ;\ Ke - Ku = \frac{D\,(1 - T)}{C}\,(Ku - Kd)$$

Note that in this paper we are considering FCF and Ku independent of leverage.

5.5. Impact on the valuation

Exhibits 6 and 7 provide a comparison with the firm reflected in Exhibit 5 when D is different from N. To value (D), we use formula [21] in Exhibits 6 and 7. Exhibit 7 differs from Exhibit 6 in the interest rate paid by the debt: r= 17% instead of 15%. Table 5 shows the differences between Exhibits 5, 6 and 7.

Table 5. Comparison of Exhibits 5, 6 and 7

(millions)	*Exhibit 5*	*Exhibit 6*	*Exhibit 7*
N (Nominal value of debt)	1,800	1,800	1,800
r	15%	15%	17%
Debt value D	**1,800**	**1,705**	**1,882**
Value of shares C	**506**	**568**	**453**
NPV of taxes paid (GOV_L)	**611**	**644**	**582**
SUM	**2,917**	**2,917**	**2,917**

Table 6 has more information about the impact of further changes in r (in Exhibits 6 and 7) on the value of the different stakeholders of the company. To value D, we get Kd using formula [21].

Table 6. Influence of r on the value of shares (C), debt (D) and NPV of taxes paid (GOV_L)
Changes in the values of Exhibits 6 and 7

r	**(in t=0) Kd**	**Face value of debt N**	**Value of debt D**	**Value of shares C**	**NPV of taxes paid (GOV_L)**
14%	17.00%	1800	1612	628	677
15% (Exhibit 6)	17.29%	1800	1705	568	644
16%	17.57%	1800	1794	510	613
17% (Exhibit 7)	17.84%	1800	1882	453	582
18%	18.11%	1800	1969	397	551
19%	18.37%	1800	2053	342	522
20%	18.63%	1800	2136	288	493
21%	18.88%	1800	2217	235	465

(1) I would like to thank CIIF (International Center for Finance Research) and its director Natalia Centenera for their financial support and for their encouragement. Professor Rafael Termes gave me the initial push to think about these valuation issues.

I also would like to thank Prof. Carliss Baldwin (my main professor of corporate finance and the director of my dissertation committee at Harvard), Prof. Timothy Luehrman and Prof. Scott Mason, who taught me the correct way to approach the issues in this paper.

Finally, I would like to acknowledge the help of my MBA and PhD students at IESE, who encouraged me to think about the valuation formulas that appear in this paper. I wish to thank specially Christopher Golembiewski for his suggestions.

(2) This paper is based on a 90-page note that I have in Spanish for my second-year MBA students. The Spanish version includes more examples and comments; the impact of using reduced formulas for $ß_L$; charts, etc.

(3) Note that FCF= CFacc when the company has no debt

(4) We assume that Kd (required return for debt) is equal to the interest rate that the company pays for the debt (r). In this situation, D (value of debt) is equal to par value. In section 5 we will show how formulas change when Kd ≠ r, and will provide a tentative formula for Kd.

(5) This equality only holds for perpetuities with no growth. For other companies see sections 3 and 4 of this paper.

(6) Note that FCF and FCFo have the same risk. Their required rate of return is Ku.

(7) PAT = PBT (1 - T). PBT is the profit before taxes.

(8) Note that [3p] assumes Modigliani-Miller. It also assumes that FCF is independent of leverage. We are not considering bankruptcy costs.

(9) Another consequence of [12] is that if T is large enough, WACC can be smaller than R_F.

(10) In Exhibits 1 to 5 it will be clear that, in general, $PBT_U \neq FCF_0$.

(11) For simplicity, we will assume that H is proportional to FCFo, then FCF and FCFo have the same risk: their required rate of return would be Ku. But this assumption is not necessary for the conclusions of this paper.

(12) K_{TL} is the required return for the taxes paid by the levered company.

(13) In Appendix 1 we show what happens if we consider the «traditional» (and wrong) formula that says: NPV of interest tax shields = DT Kd / (Kd-g). We show that if we use this formula, then [10] does not apply; $ß_L$ and Ke have to be dependent on g; and other inconsistencies. All these inconsistencies come from considering the «NPV of interest tax shields» as the NPV of a flow (DT Kd), **when it is the difference of two NPVs** of two flows with **different risk**: the NPV of the taxes paid in the unlevered firm and the NPV of taxes paid in the levered firm.

(14) Note that Ke = K_{TL} if H = gD

(15) In this situation, C=CFacc=0. It seems that the full cash-flow (FCFo) generated by the company goes to the debt, but $Taxes_L$ = T (H-gDo)/(1-T); GOV_L= [T/(1-T)] [(H-gDo)/(Ku-g)]. Also Kd = K_{TL} = Ku.

(16) The three valuation methods give the same value for any company. Also, when RF changes over time and when Ku changes over time.

(17) We can also think of Kd as the «reasonable» required rate of the debt flows for a person who believes the «reasonable» required rate for the FCF is Ku.

(18) An alternative formula could be Kd = R_F + (Ku - R_F) D / (D + C), but then (Ke - Kd) decreases with leverage, which is not very sensible.

Exhibit 1

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

D = 500; Growth = 5.00%; T = 35.00%; Δ Net fixed assets = 0.

			0	*1*	*2*	*3*	*4*	*g*
1		Required cash	100	105	110.25	115.76	121.55	5.00%
2		Receivables	900	945	992.25	1,041.86	1,093.96	5.00%
3		Stocks	240	252	264.60	277.83	291.72	5.00%
4		Gross fixed assets	1200	1410	1,630.50	1,862.03	2,105.13	13.06%
5		- accum. depreciation	200	410	630.50	862.03	1,105.13	28.20%
6		Net fixed assets	1000	1000	1,000.00	1,000.00	1,000.00	0.00%
7		**TOTAL ASSETS**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**	**2,95%**
8		Acc. payable	240	252	264.60	277.83	291.72	5.00%
9		Debt	500	525	551.25	578.81	607.75	5.00%
10		Net Worth (book value)	1500	1525	1,551.25	1,578.81	1,607.75	1.83%
11		**TOTAL LIAB. & NW**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**	**2.95%**
12		WCR	1,000	1,050	1,102.50	1,157.63	1,215.51	5.00%
14	***P&L***	Sales	3,000	3150	3,307.50	3,472.88	3,646.52	5.00%
15		Cost of sales	1,200	1260	1,323.00	1,389.15	1,458.61	5.00%
16		General, selling & adm. expen.	600	630	661.50	694.58	729.30	5.00%
17		Depreciation	200	210	220.50	231.53	243.10	5.00%
18		PBIT	1000	1050	1,102.50	1,157.63	1,215.51	5.00%
19		Interest	75	75	78.75	82.69	86.82	5.00%
20		PBT	925	975	1,023.75	1,074.94	1,128.68	5.00%
21		Taxes (35%)	323.75	341.25	358.31	376.23	395.04	5.00%
22		**PAT**	**601.25**	**633.75**	**665.44**	**698.71**	**733.64**	5.00%
23		+ Depreciation	200	210	220.50	231.53	243.10	5.00%
24		+ Δ Debt		25	26.25	27.56	28.94	5.00%
25		- Δ WCR		-50	-52.50	-55.13	-57.88	5.00%
26		- Payments of fixed assets		-210	-220.50	-231.53	-243.10	5.00%
27		**CFacc**		**608.75**	**639.19**	**671.15**	**704.70**	5.00%
28		**FCF**		**632.5**	**664.13**	**697.33**	**732.20**	5.00%
37		Beta U	1.000000	1.000000	1.000000	1.000000	1.000000	
38		Rf	12.00%	12.00%	12.00%	12.00%	12.00%	
39		Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	
40		Ku	20.00%	20.00%	20.00%	20.00%	20.00%	
41		**Vu = FCF/(Ku - g)**	**4,216.67**	**4,427.50**	**4,648.87**	**4,881.32**	**5,125.38**	
43		**FCFo = FCF without taxes**		1,000.00	1,050.00	1,102.50	1,157.63	5.00%
44		**Vu without taxes**	**6,666.67**	7,000.00	7,350.00	7,717.50	8,103.37	
45		$Taxes_u$		367.50	385.88	405.17	425.43	5.00%
46		$GOV_u = Taxes_u/(Ku - g)$	2,450.00	2,572.50	2,701.12	2,836.18	2,977.99	5.00%
47		$Vu + GOV_u$	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	5.00%
48		D	500.00	525.00	551.25	578.81	607.75	5.00%
49		Kd	15.00%	15.00%	15.00%	15.00%	15.00%	
50		Beta d	0.375000	0.375000	0.375000	0.375000	0.375000	
51		DTKu/(Ku-g) = NPV int. tax shield.	233.33	245.00	257.25	270.11	283.62	5.00%
52		NPV int. tax shield. + Vu	4,450.00	4,672.50	4,906.12	5,151.43	5,409.00	5.00%
53	**- D =**	**C**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**	**5.00%**
54		Beta L	1.051424	1.051424	1.051424	1.051424	1.051424	0.00%
55		Ke	20.41%	20.41%	20.41%	20.41%	20.41%	0.00%
56		**C = CFacc / (Ke - g)**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**	**5.00%**
59		WACC	19.2135%	19.2135%	19.2135%	19,2135%	19.2135%	0.00%
60	D + E =	FCF / (WACC-g)	4,450.00	4,672.50	4,906.13	5,151.43	5,409.00	5.00%
61	**- D =**	**C**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**	**5.00%**
		Two ways of calculating the NPV of interest tax shields:						
51		**DTKu/(Ku-g)**	**233.33**	**245.00**	**257.25**	**270.11**	**283.62**	**5.00%**
62		**DTKd/(Kd-g)**	**262.50**	**275.63**	**289.41**	**303.88**	**319.07**	**5.00%**

Exhibit 1 (continued)

D = 500; Growth = 5.00%; T = 35.00%; Δ Net fixed assets = 5%.

			0	*1*	*2*	*3*	*4*	*g*
1		Required cash	100	105	110.25	115.76	121.55	5.00%
2		Receivables	900	945	992.25	1,041.86	1,093.96	5.00%
3		Stocks	240	252	264.60	277.83	291.72	5.00%
4		Gross fixed assets	1200	1460	1733	2019.65	2320.6325	14.90%
5		- accum. depreciation	200	410	630.50	862.03	1,105.13	28.20%
6		Net fixed assets	1000	1050	1102.5	1157.625	1215.5063	5.00%
7		**TOTAL ASSETS**	**2,240**	**2,352**	**2,469.60**	**2,593.08**	**2,722.73**	**5.00%**
8		Acc. payable	240	252	264.60	277.83	291.72	5.00%
9		Debt	500	525	551.25	578.81	607.75	5.00%
10		Net Worth (book value)	1500	1575	1,653.75	1,736.44	1,823.26	5.00%
11		**TOTAL LIAB. & NW**	**2,240**	**2,352**	**2,469.60**	**2,593.08**	**2,722.73**	**5.00%**
12		WCR	1,000	1,050	1,102.50	1,157.63	1,215.51	5.00%
14	***P&L***	Sales	3000	3150	3,307.50	3,472.88	3,646.52	5.00%
15		Cost of sales	1200	1260	1,323.00	1,389.15	1,458.61	5.00%
16		General, selling & adm. expen.	600	630	661.50	694.58	729.30	5.00%
17		Deprecitation	200	210	220.50	231.53	243.10	5.00%
18		PBIT	1000	1050	1,102.50	1,157.63	1,215.51	5.00%
19		Interest	75	75	78.75	82.69	86.82	5.00%
20		PBT	925	975	1,023.75	1,074.94	1,128.68	5.00%
21		Taxes	323.75	341.25	358.31	376.23	395.04	5.00%
22		**PAT**	**601.25**	**633.75**	**665.44**	**698.71**	**733.64**	**5.00%**
23		+ Depreciation	200	210	220.50	231.53	243.10	5.00%
24		+ Δ Debt		25	26.25	27.56	28.94	5.00%
25		- Δ WCR		-50	-52.50	-55.13	-57.88	5.00%
26		- Payments of fixed assets		-260	-273.00	-286.65	-300.98	5.00%
27		**CFacc**		**558.75**	**586.69**	**616.02**	**646.82**	**5.00%**
28		**FCF**		**582.5**	**611.63**	**642.21**	**674.32**	**5.00%**
37		Beta U	1.000000	1.000000	1.000000	1.000000	1.000000	
38		Rf	12.00%	12.00%	12.00%	12.00%	12.00%	
39		Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	
40		Ku	20.00%	20.00%	20.00%	20.00%	20.00%	
41		Vu = FCF/(Ku - g)	3,888.33	4,077.50	4,281.37	4,495.44	4,720.22	
43		**FCFo= FCF without taxes**		950.00	997.50	1,047.38	1,099.74	5.00%
44		**Vu without taxes**	6,333.33	6,650.00	6,982.50	7,331.63	7,698.21	5.00%
45		Taxes u		367.50	385.88	405.17	425.43	5.00%
46		GOV u = IMPu/(Ku - g)	2,450.00	2,572.50	2,701.12	2,836.18	2,977.99	5.00%
47		Vu + GOV u	6,333.33	6,650.00	6,982.50	7,331.63	7,698.21	5.00%
48		D	500.00	525.00	551.25	578.81	607.75	5.00%
49		Kd	15.00%	15.00%	15.00%	15.00%	15.00%	
50		Beta d	0.375000	0.375000	0.375000	0.375000	0.375000	
51		DTKu/(Ku-g) = NPV int. tax shield	233.33	245.00	257.25	270.11	283.62	5.00%
52		NPV int. tax shield + Vu	4,116.67	4,322.50	4,538.62	4,765.56	5,003.83	5.00%
53	**- D =**	**C**	**3,617**	**3,797**	**3,987**	**4,187**	**4,396**	**5.00%**
54		Beta E	1.056164	1.056164	1.056164	1.056164	1.056164	
55		Ke	20.45%	20.45%	20.45%	20.45%	20.45%	
56		**C = CFacc / (Ke - g)**	**3,617**	**3,798**	**3,987**	**4,187**	**4,396**	**5.00%**
59		WACC	19.1498%	19.1498%	19.1498%	19.1498%	19.1498%	
60	D + E =	FCF / (WACC-g)	4,116.67	4.322,50	4,538.62	4,765.56	5,003.83	5.00%
61	**- D =**	**C**	**3,617**	**3,798**	**3,987**	**4,187**	**4,396**	**5.00%**
		Two ways of calculating the NPV of interest tax shields:						
51		**DTKu/(Ku-g)**	**233.33**	**245.00**	**257.25**	**270.11**	**283.62**	**5.00%**
62		**DTKd/(Kd-g)**	**262.50**	**275.63**	**289.41**	**303.88**	**319.07**	**5.00%**

Exhibit 2

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

D = 500; Growth = 5.00%; T = 0%; Δ Net fixed assets = 0.

			0	*1*	*2*	*3*	*4*	*5*
1		Required cash	100	105	110.25	115.76	121.55	127.63
2		Receivables	900	945	992.25	1,041.86	1,093.96	1,148.65
3		Stocks	240	252	264.60	277.83	291.72	306.31
4		Gross fixed assets	1200	1410	1,630.50	1,862.03	2,105.13	2,360.38
5		- accum. depreciation	200	410	630.50	862.03	1,105.13	1,360.38
6		Net fixed assets	1000	1000	1,000.00	1,000.00	1,000.00	1,000.00
7		**TOTAL ASSETS**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**	**2,582.59**
8		Acc. payable	240	252	264.60	277.83	291.72	306.31
9		Debt	500	525	551.25	578.81	607.75	638.14
10		Net Worth (book value)	1500	1525	1,551.25	1,578.81	1,607.75	1,638.14
11		**TOTAL LIAB. & NW**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**	**2,582.59**
12		WCR	1,000	1,050	1,102.50	1,157.63	1,215.51	1,276.28
14	***P&L***	Sales	3000	3150	3,307.50	3,472.88	3,646.52	3,828.84
15		Cost of sales	1200	1260	1,323.00	1,389.15	1,458.61	1,531.54
16		General, selling & adm. expen.	600	630	661.50	694.58	729.30	765.77
17		Depreciation	200	210	220.50	231.53	243.10	255.26
18		PBIT	1000	1050	1,102.50	1,157.63	1,215.51	1,276.28
19		Interest	75	75	78.75	82.69	86.82	91.16
20		PBT	925	975	1,023.75	1,074.94	1,128.68	1,185.12
21		Taxes	0	0	0.00	0.00	0.00	0.00
22		**PAT**	**925**	**975**	**1,023.75**	**1,074.94**	**1,128.68**	**1,185.12**
23		+ Depreciation	200	210	220.50	231.53	243.10	255.26
24		+ Δ Debt		25	26.25	27.56	28.94	30.39
25		- Δ WCR		-50	-52.50	-55.13	-57.88	-60.78
26		- Payments of fixed assets		-210	-220.50	-231.53	-243.10	-255.26
27		**CFacc**		**950**	**997.50**	**1,047.38**	**1,099.74**	**1,154.73**
28		**FCF**		**1000**	**1,050.00**	**1,102.50**	**1,157.63**	**1,215.51**
37		Beta U	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
38		Rf	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39		Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40		Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41		Vu = FCF/(Ku - g)	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
43		**FCFo= FCF without taxes**		1,000.00	1,050.00	1,102.50	1,157.63	1,215.51
44		Vu without taxes	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
45		Taxes u		0.00	0.00	0.00	0.00	0.00
46		GOV u = IMPu/(Ku - g)	0.00	0.00	0.00	0.00	0.00	0.00
47		Vu + GOV u	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
48		D	500.00	525.00	551.25	578.81	607.75	638.14
49		Kd	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
50		Beta d	0.375000	0.375000	0.375000	0.375000	0.375000	0.375000
51		DTKu/(Ku-g) = NPV int. tax shield	0.00	0.00	0.00	0.00	0.00	0.00
52		NPV int. tax shield + Vu	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
53	**- D =**	**C**	**6,167**	**6,475**	**6,799**	**7,139**	**7,496**	**7,870**
54		Beta E	1.050676	1.050676	1.050676	1.050676	1.050676	1.050676
55		Ke	20.41%	20.41%	20.41%	20.41%	20.41%	20.41%
56		**C = CFacc / (Ke - g)**	**6,167**	**6,475**	**6,799**	**7,139**	**7,496**	**7,870**
59		WACC	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%
60	D + E =	FCF / (WACC-g)	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
61	**- D =**	**C**	**6,167**	**6,475**	**6,799**	**7,139**	**7,496**	**7,870**

Exhibit 3

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

D = 0; Growth = 5.00%; T = 0%; Δ Net fixed assets = 0.

			0	*1*	*2*	*3*	*4*	*5*
1		Required cash	100	105	110.25	115.76	121.55	127.63
2		Receivables	900	945	992.25	1,041.86	1,093.96	1,148.65
3		Stocks	240	252	264.60	277.83	291.72	306.31
4		Gross fixed assets	1200	1410	1,630.50	1,862.03	2,105.13	2,360.38
5		- accum. depreciation	200	410	630.50	862.03	1,105.13	1,360.38
6		Net fixed assets	1000	1000	1,000.00	1,000.00	1,000.00	1,000.00
7		TOTAL ASSETS	2,240	2,302	2,367.10	2,435.46	2,507.23	2,582.59
8		Acc. payable	240	252	264.60	277.83	291.72	306.31
9		Debt	0	0	0.00	0.00	0.00	0.00
10		Net Worth (book value)	2000	2050	2,102.50	2,157.63	2,215.51	2,276.28
11		TOTAL LIAB. & NW	2,240	2,302	2,367.10	2,435.46	2,507.23	2,582.59
12		WCR	1,000	1,050	1,102.50	1,157.63	1,215.51	1,276.28
14	***P&L***	Sales	3000	3150	3,307.50	3,472.88	3,646.52	3,828.84
15		Cost of sales	1200	1260	1,323.00	1,389.15	1,458.61	1,531.54
16		General, selling & adm. expen.	600	630	661.50	694.58	729.30	765.77
17		Depreciation	200	210	220.50	231.53	243.10	255.26
18		PBIT	1000	1050	1,102.50	1,157.63	1,215.51	1,276.28
19		Interest	0	0	0.00	0.00	0.00	0.00
20		PBT	1000	1050	1,102.50	1,157.63	1,215.51	1,276.28
21		Taxes	0	0	0.00	0.00	0.00	0.00
22		**PAT**	**1000**	**1050**	**1,102.50**	**1,157.63**	**1,215.51**	**1,276.28**
23		+ Depreciation	200	210	220.50	231.53	243.10	255.26
24		+ Δ Debt		0	0.00	0.00	0.00	0.00
25		- Δ WCR		-50	-52.50	-55.13	-57.88	-60.78
26		- Payments of fixed assets		-210	-220.50	-231.53	-243.10	-255.26
27		**CFacc**		**1000**	**1,050.00**	**1,102.50**	**1,157.63**	**1,215.51**
28		**FCF**		**1000**	**1,050.00**	**1,102.50**	**1,157.63**	**1,215.51**
37		Beta U	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
38		Rf	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39		Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40		Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41		Vu = FCF/(Ku - g)	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
43		**FCFo= FCF without taxes**		1,000.00	1,050.00	1,102.50	1,157.63	1,215.51
44		Vu without taxes	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
45		Taxes u		0.00	0.00	0.00	0.00	0.00
46		GOV u = IMPu/(Ku - g)	0.00	0.00	0.00	0.00	0.00	0.00
47		Vu + GOV u	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
48		D	0.00	0.00	0.00	0.00	0.00	0.00
49		Kd	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
50		Beta d	0.375000	0.375000	0.375000	0.375000	0.375000	0.375000
51		DTKu/(Ku-g) = NPV int. tax shield	0.00	0.00	0.00	0.00	0.00	0.00
52		NPV int. tax shield + Vu	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
53	**- D =**	**C**	**6,667**	**7,000**	**7,350**	**7,718**	**8,103**	**8,509**
54		Beta E	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
55		Ke	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
56		**C = CFacc / (Ke - g)**	**6,667**	**7,000**	**7,350**	**7,718**	**8,103**	**8,509**
59		WACC	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%
60	D + E =	FCF / (WACC-g)	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
61	**- D =**	**C**	**6,667**	**7,000**	**7,350**	**7,718**	**8,103**	**8,509**

Exhibit 4

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

D = 0; Growth = 5.00%; T = 35%; Δ Net fixed assets = 0.

			0	*1*	*2*	*3*	*4*	*5*
1		Required cash	100	105	110.25	115.76	121.55	127.63
2		Receivables	900	945	992.25	1,041.86	1,093.96	1,148.65
3		Stocks	240	252	264.60	277.83	291.72	306.31
4		Gross fixed assets	1200	1410	1.630.50	1,862.03	2,105.13	2,360.38
5		- accum. depreciation	200	410	630.50	862.03	1,105.13	1,360.38
6		Net fixed assets	1000	1000	1.000.00	1,000.00	1,000.00	1,000.00
7		TOTAL ASSETS	2,240	2,302	2,367.10	2,435.46	2,507.23	2,582.59
8		Acc. payable	240	252	264.60	277.83	291.72	306.31
9		Debt	0	0	0.00	0.00	0.00	0.00
10		Net Worth (book value)	2000	2050	2,102.50	2,157.63	2,215.51	2,276.28
11		**TOTAL LIAB. & NW**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**	**2,582.59**
12		WCR	1,000	1,050	1,102.50	1,157.63	1,215.51	1,276.28
14	***P&L***	Sales	3000	3150	3,307.50	3,472.88	3,646.52	3,828.84
15		Cost of sales	1200	1260	1,323.00	1,389.15	1,458.61	1,531.54
16		General, selling & adm. expen.	600	630	661.50	694.58	729.30	765.77
17		Depreciation	200	210	220.50	231.53	243.10	255.26
18		PBIT	1000	1050	1,102.50	1,157.63	1,215.51	1,276.28
19		Interest	0	0	0.00	0.00	0.00	0.00
20		PBT	1000	1050	1,102.50	1,157.63	1,215.51	1,276.28
21		Taxes	350	367.5	385.88	405.17	425.43	446.70
22		**PAT**	**650**	**682.5**	**716.63**	**752.46**	**790.08**	**829.58**
23		+ Depreciation	200	210	220.50	231.53	243.10	255.26
24		+ Δ Debt		0	0.00	0.00	0.00	0.00
25		- Δ WCR		-50	-52.50	-55.13	-57.88	-60.78
26		- Payment of fixed assets		-210	-220.50	-231.53	-243.10	-255.26
27		**CFacc**		**632.5**	**664.13**	**697.33**	**732.20**	**768.81**
28		**FCF**		**632.5**	**664.13**	**697.33**	**732.20**	**768.81**
37		Beta U	1.000000	1.000000	1000000	1.000000	1.000000	1.000000
38		Rf	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39		Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40		Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41		**Vu = FCF/(Ku - g)**	**4,216.67**	**4,427.50**	**4,648.87**	**4,881.32**	**5,125.38**	**5,381.65**
43		**FCFo= FCF without taxes**		1,000.00	1,050.00	1,102.50	1,157.63	1,215.51
44		**Vu without taxes**	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
45		Taxes u		367.50	385.88	405.17	425.43	446.70
46		GOV u = IMPu/(Ku - g)	2,450.00	2,572.50	2,701.12	2,836.18	2,977.99	3.126.89
47		Vu + GOV u	6,666.67	7,000.00	7,350.00	7,717.50	8,103.37	8,508.54
48		D	0.00	0.00	0.00	0.00	0.00	0.00
49		Kd	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
50		Beta d	0.375000	0.375000	0.375000	0.375000	0.375000	0.375000
51		DTKu/(Ku-g) = NPV int. tax shield	0.00	0.00	0.00	0.00	0.00	0.00
52		NPV int. tax shield + Vu	4,216.67	4,427.50	4,648.87	4,881.32	5,125.38	5,381.65
53	**- D =**	**C**	**4,217**	**4,428**	**4,649**	**4,881**	**5,125**	**5,382**
54		Beta E	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
55		Ke	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
56		**C = CF / (Ke - g)**	**4,217**	**4,428**	**4,649**	**4,881**	**5,125**	**5,382**
59		WACC	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%	20.0000%
60	D + E =	FCF / (WACC-g)	4,216.67	4,427.50	4,648.87	4,881.32	5,125.38	5,381.65
61	**- D =**	**C**	**4,217**	**4,428**	**4,649**	**4,881**	**5,125**	**5,382**

Exhibit 5

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

Kd= 15 %; T=35%

											g=5%		
		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
1	Required cash	100	120	140	160	180	200	210	220	230	240	252	264.6	277.83
2	Receivables	900	960	1020	1080	1140	1200	1260	1320	1380	1449	1521.45	1597.52	1677.4
3	Stocks	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
4	Gross fixed assets	1500	1800	2700	3100	3300	3500	3900	4204	4523.2	4858.36	5210.28	5579.79	5967.78
5	- accum. depreciation	200	550	900	1300	1800	2100	2380	2684	3003.2	3338.36	3690.28	4059.79	4447.78
6	Net fixed assets	1300	1250	1800	1800	1500	1400	1520	1520	1520	1520	1520	1520	1520
7	**TOTAL ASSETS**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
8	Acc. payable	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
9	Debt	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
10	Net Worth (book value)	500	530	660	740	770	1000	1290	1610	1930	2209	2243.45	2279.62	2317.6
11	**TOTAL LIAB. & NW**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
12	Working Capital Requirements	1000	1080	1160	1240	1320	1400	1470	1540	1610	1689	1773.45	1862.12	1955.23
13	Δ WCR		8.00%	7.41%	6.90%	6.45%	6.06%	5.00%	4.76%	4.55%	4.91%	5.00%	5.00%	5.00%
	P&L (millions)													
14	Sales		3200	3400	3600	3800	4000	4200	4400	4600	4830	5071.5	5325.08	5591.33
15	Cost of sales		1600	1700	1800	1900	2000	2100	2200	2300	2415	2535.75	2662.54	2795.66
16	General, selling & adm. expenses		800	850	900	950	1000	1050	1100	1150	1207.5	1267.88	1331.27	1397.83
17	Depreciation		350	350	400	500	300	280	304	319.2	335.16	351.92	369.51	387.99
18	PBIT		450	500	500	450	700	770	796	830.8	872.34	915.96	961.75	1009.84
19	Interest		270	270	345	345	308	270	255	218	180	150	158	165
20	PBT		180	230	155	105	392.5	500	541	613.3	692.34	765.96	804.25	844.47
21	Taxes		63	80.5	54.25	36.75	137.38	175	189.35	214.66	242.32	268.08	281.49	295.56
22	**Profit after taxes (PAT)**		**117**	**149.5**	**100.75**	**68.25**	**255.13**	**325**	**351.65**	**398.65**	**450.02**	**497.87**	**522.77**	**548.9**
23	+ Depreciation		350	350	400	500	300	280	304	319.2	335.16	351.92	369.51	387.99
24	+ Δ Debt		0	500	0	-250	-250	-100	-250	-250	-200	50	52.5	55.13
25	- Δ WCR		-80	-80	-80	-80	-80	-70	-70	-70	-79	-84.45	-88.67	-93.11
26	- Payments of fixed assets		-300	-900	-400	-200	-200	-400	-304	-319.2	-335.16	-351.92	-369.51	-387.99
27	**CF shares**		**87**	**19.5**	**20.75**	**38.25**	**25.13**	**35**	**31.65**	**78.65**	**171.02**	**463.42**	**486.59**	**510.92**
28	**FCF**		**262.5**	**-305**	**245**	**512.5**	**475**	**310.5**	**447.4**	**470.02**	**488.02**	**510.92**	**536.47**	**563.29**

Notes to Exhibit 5

Lines 1 to 7 show the Assets of the companies. Lines 8 to 11 show the Liabilities and Net Worth of the companies. Line 12 shows the Working Capital Requirements (WCR). Lines 14 to 22: P&L of the companies. Lines 23 to 27: available cash flow for shareholders (CFacc). Line 28 shows the Free cash flow (FCF)

Exhibit 5 (continued)

Kd= 15 %; T=35%

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
37	Beta Unlevered	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
38	Rf (risk free rate)	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39	E (Rm - Rf) = Market premium	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40	Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41	Vu = NPV (Ku;FCF)	1679.6	1753.1	2408.7	2645.4	2662.0	2719.4	2952.8	3096.0	3245.1	3406.1	3576.5	3755.3	3943.0
42	GOVu	1237.5	1327.5	1418.0	1526.6	1674.4	1764.3	1847.6	1938.5	2035.5	2137.3	2244.0	2356.3	2474.2
	WITHOUT TAXES													
43	FCFo = FCF without taxes		420	-130	420	670	720	580	726	760.8	793.34	831.51	873.08	916.74
44	Vu without taxes (Ku)	2917.1	3080.6	3826.7	4172.0	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2
49	Kd	15.00%	15.00%	15.00%	15,00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
50	Beta d	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375
	D T Ku		126	126	161	161	143.5	126	119	101.5	84	70	73.5	77.18
51	NPV(Ku;DTKu) = NPV tax savings	626.72	626.06	625.28	589.33	546.2	511.94	488.33	466.99	458.89	466.67	490	514.5	540.23
52	NPV tax savings + Vu	2306.37	2379.14	3033.97	3234.76	3208.22	3231.36	3441.13	3562.96	3704.03	3872.81	4066.45	4269.78	4483.27
53	[52] - D = C	506	579	734	935	1158	1431	1741	2113	2504	2873	3016	3167	3326
54	Beta Equity (levered)	2.444109	2.262645	2.273043	1.999584	1.719048	1.510878	1.396653	1.278786	1.194686	1.141412	1.141412	1.141412	1.141412
55	Ke	31.55%	30.10%	30.18%	28.00%	25.75%	24.09%	23.17%	22.23%	21.56%	21.13%	21.13%	21.13%	21.13%
56	C = NPV(Ke;CFacc)	506	579	734	935	1158	1431	1741	2113	2504	2873	3016	3167	3326
57	Ct = C t-1 * (1+Ke) - CFacc	506	579	734	935	1158	1431	1741	2113	2504	2873	3016	3167	3326
58	GOV L = GOV u - NPV	610.8	701.4	792.7	937.2	1128.2	1252.3	1359.3	1471.5	1576.6	1670.6	1754.1	1841.8	1933.9
59	WACC	14.54%	14.70%	14.69%	15.02%	15.53%	16.10%	16.54%	17.15%	17.73%	18.19%	18.19%	18.19%	18.19%
60	D + C = NPV(WACC;FCF)	2306.37	2379.14	3033.97	3234.76	3208.22	3231.36	3441.13	3562.96	3704.03	3872.81	4066.45	4269.78	4483.27
61	[60] - D = C	506	579	734	935	1158	1431	1741	2113	2504	2873	3016	3167	3326
	Two ways of calculating the NPV of interest tax shields:													
51	**CORRECT: NPV(Ku;DTKu)**	626.72	626.06	625.28	589.33	546.2	511.94	488.33	466.99	458.89	466.67	490	514.5	540.23
62	**WRONG: NPV(Kd;DTKd)**	621.93	620.71	619.32	591.47	559.44	535.73	521.59	510.58	511.04	524,.7	550.9	578.41	607.29

Notes to Exhibit 5

Line 37. We assume the unlevered beta = ßu = 1. Line 38. We assume the risk free rate = RF = 12%. Line 39. Market premium = E (Rm - RF) = 8%.
Line 40. Cost of unlevered equity = Ku = Rf + ßu E(Rm - Rf) =20%. Line 41. Value of unlevered company [Vu = NPV(Ku;FCF)]. Line 42. GOVu = NPV(Ku;TAXESu)
Line 43: Free cash flow of the company with no taxes (FCFo). Line 44. Value of unlevered company with no taxes [Vu = NPV(Ku;FCFo)].
Line 49 is the cost of debt (Kd). Line 50. Beta that corresponds to debt according to [7]. Line 51. NPV of interest tax shields = NPV(Ku;DTKu). (formula [20])
Lines 52 and 53. Value of equity according to APV, using [3]. Line 54. Leveraged beta according to [11]. Line 55. Required return of leveraged equity according to [6].
Line 56. Value of equity according to formula [2]. Line 58. Net present value of taxes paid by the levered firm: GOV L = NPV (KTL; TAXES L) = [42] - [51] = [44] - [9] - [56]

Exhibit 6

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

Kd = Rf +(Ku - Rf) * D(1-T) / [D(1-T) + E]
r = 15%; T = 35%

											g=5%			
		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
1	Required cash	100	120	140	160	180	200	210	220	230	240	252	264.6	277.83
2	Receivables	900	960	1020	1080	1140	1200	1260	1320	1380	1449	1521.45	1597.52	1677.4
3	Stocks	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
6	Net fixed assets	1300	1250	1800	1800	1500	1400	1520	1520	1520	1520	1520	1520	1520
7	**TOTAL ASSETS**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
8	Acc. payable	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
9	Debt	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
10	Net Worth (book value)	500	530	660	740	770	1000	1290	1610	1930	2209	2243.45	2279.62	2317.6
11	**TOTAL LIAB. & NW**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
18	PBIT		450	500	500	450	700	770	796	830.8	872.34	915.96	961.75	1009.84
19	Interest		270	270	345	345	308	270	255	218	180	150	158	165
20	PBT		180	230	155	105	392.5	500	541	613.3	692.34	765.96	804.25	844.47
21	Taxes		63	80.5	54.25	36.75	137.8	175	189.35	214.66	242.32	268.08	281.49	295.56
22	**Profit after taxes (PAT)**		**117**	**149.5**	**100.75**	**68.25**	**255.13**	**325**	**351.65**	**398.65**	**450.02**	**497.87**	**522.77**	**548.9**
23	+ Depreciation		350	350	400	500	300	280	304	319.2	335.16	351.92	369.51	387.99
24	+ Δ Debt		0	500	0	-250	-250	-100	-250	-250	-200	50	52.5	55.13
25	- Δ WCR		-80	-80	-80	-80	-80	-70	-70	-70	-79	-84.45	-88.67	-93.11
26	- Payments of fixed assets		-300	-900	-400	-200	-200	-400	-304	-319.2	-335.16	-351.92	-369.51	-387.99
27	**CF shares**		**87**	**19.5**	**20.75**	**38.25**	**25.13**	**35**	**31.65**	**78.65**	**171.02**	**463.42**	**486.59**	**510.92**
28	**FCF**		**262.5**	**-305**	**245**	**512.5**	**475**	**310,5**	**447.4**	**470.02**	**488.02**	**510.92**	**536.47**	**563.29**
37	Beta U	1	1	1	1	1	1	1	1	1	1	1	1	1
38	Rf	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39	Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40	Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41	Vu = NPV (Ku;FCF)	1679.6	1753.1	2408.7	2645.4	2662	2719.4	2952.8	3096	3245.1	3406.1	3576.5	3755.3	3943
	WITHOUT TAXES													
43	FCFo = FCF without taxes		420	-130	420	670	720	580	726	760.8	793.34	831.51	873.08	916.74
44	Vu without taxes = NPV (FCFo;Ku)	2917.1	3080.6	3826.7	4172	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2
48	N	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
49	r	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
100	**D**	**1704.42**	**1729.06**	**2255.43**	**2299.76**	**2093.91**	**1879.22**	**1805.32**	**1576.51**	**1340.48**	**1149.79**	**1207.28**	**1267.65**	**1331.03**
101	**Kd**	**17.29%**	**17.14%**	**17.26%**	**16.92%**	**16.37%**	**15.76%**	**15.30%**	**14.68%**	**14.12%**	**13.70%**	**13.70%**	**13.70%**	**13.70%**
102	Beta d	0.6609	0.6425	0.6577	0.6152	0.5464	0.4696	0.4123	0.3354	0.2653	0.2122	0.2122	0.2122	0.2122
103	D/N	0.9469	0.9606	0.9806	0.9999	1.0214	1.044	1.062	1.0872	1.1171	1.1498	1.1498	1.1498	1.1498
104	Nr-DKd		-24.6432	-26.3667	-44.3261	-44.1592	-35.3068	-26.0991	-21.1851	-13.9785	-9.3106	-7.4897	-7.8642	-8.2574
105	Ke - Kd	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
	D T Ku + (Nr-DKd)*T		110.68	111.81	142.37	145.53	134.22	122.41	118.96	105.46	90.58	77.86	81.76	85.85
51	NPV(Ku;DtKu) = NPV int. tax shields	593.27	601.24	609.68	589.25	561.57	539.67	525.19	511.27	508.06	519.09	545.05	572.3	600.92
52	NPV int. tax shields + Vu	2272.91	2354.31	3018.37	3234.68	3223.59	3259.09	3477.99	3607.23	3753.2	3925.24	4121.5	4327.58	4543.96
53	**- D = C**	**568**	**625**	**763**	**935**	**1130**	**1380**	**1673**	**2031**	**2413**	**2775**	**2914**	**3060**	**3213**
54	Beta Equity	1.660877	1.642539	1.657716	1.615221	1.546447	1.46956	1.412299	1.335377	1.265319	1.21215	1.21215	1.21215	1.21215
55	Ke	25.29%	25.14%	25.26%	24.92%	24.37%	23.76%	23.30%	22.68%	22.12%	21.70%	21.70%	21.70%	21.70%
56	**C = NPV(Ke;CFacc)**	**568**	**625**	**763**	**935**	**1130**	**1380**	**1673**	**2031**	**2413**	**2775**	**2914**	**3060**	**3213**
59	WACC reformed	15.13%	15.25%	15.28%	15.50%	15.84%	1624%	16.58%	17.08%	17.59%	18.02%	18.02%	18.02%	18.02%
60	D + E = NPV(WACC;FCF)	2272.91	2354.31	3018.37	3234.68	3223.59	3259.09	3477.99	3607.23	3753.2	3925.24	4121.5	4327.58	4543.96
61	**- D = C**	**568**	**625**	**763**	**935**	**1130**	**1380**	**1673**	**2031**	**2413**	**2775**	**2914**	**3060**	**3213**

Exhibit 7

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

Kd = Rf +(Ku - Rf) * D(1-T) / [D(1-T) + E]

r = 17%; T = 35%

											g = 5%		
		0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
1	Required cash	100	120	140	160	180	200	210	220	230	240	252	264.6	277.83
2	Receivables	900	960	1020	1080	1140	1200	1260	1320	1380	1449	1521.45	1597.52	1677.4
3	Stocks	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
6	Net fixed assets	1300	1250	1800	1800	1500	1400	1520	1520	1520	1520	1520	1520	1520
7	**TOTAL ASSETS**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
8	Acc. payable	300	320	340	360	380	400	420	440	460	483	507.15	532.51	559.13
9	Debt	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
10	Net Worth (book value)	500	530	660	740	770	1000	1290	1610	1930	2209	2243.45	2279.62	2317.6
11	**TOTAL LIAB. & NW**	**2600**	**2650**	**3300**	**3400**	**3200**	**3200**	**3410**	**3500**	**3590**	**3692**	**3800.6**	**3914.63**	**4034.36**
18	PBIT		450	500	500	450	700	770	796	830.8	872.34	915.96	961.75	1009.84
19	Interest		306	306	391	391	349	306	289	247	204	170	179	187
20	PBT		144	194	109	59	351.5	464	507	584.3	668.34	745.96	783.25	822.42
21	Taxes		50.4	67.9	38.15	20.65	123.03	162.4	177.45	204.51	233.92	261.08	274.14	287.85
22	**Profit after taxes (PAT)**		**93.6**	**126.1**	**70.85**	**38.35**	**228.48**	**301.6**	**329.55**	**379.8**	**434.42**	**484.87**	**509.12**	**534.57**
23	+ Depreciation		350	350	400	500	300	280	304	319.2	335.16	351.92	369.51	387.99
24	+ Δ Debt		0	500	0	-250	-250	-100	-250	-250	-200	50	52.5	55.13
25	- Δ WCR		-80	-80	-80	-80	-80	-70	-70	-70	-79	-84.45	-88.67	-93.11
26	- Payments of fixed assets		-300	-900	-400	-200	-200	-400	-304	-319.2	-335.16	-351.92	-369.51	-387.99
27	**CF shares**		**63.6**	**-3.9**	**-9.15**	**8.35**	**-1.52**	**11.6**	**9.55**	**59.8**	**155.42**	**450.42**	**472.94**	**496.59**
28	**FCF**		**262.5**	**-305**	**245**	**512.5**	**475**	**310.5**	**447.4**	**470.02**	**488.02**	**510.92**	**536.47**	**563.29**
37	Beta U	1	1	1	1	1	1	1	1	1	1	1	1	1
38	Rf	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
39	Rm - Rf	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
40	Ku	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
41	Vu = NPV (Ku;FCF)	1679.6	1753.1	2408.7	2645.4	2662	2719.4	2952.8	3096	3245.1	3406.1	3576.5	3755.3	3943
	WITHOUT TAXES													
43	FCFo = FCF without taxes		420	-130	420	670	720	580	726	760.8	793.34	831.51	873.08	916.74
44	Vu without taxes = NPV (FCFo;Ku)	2917.1	3080.6	3826.7	4172	4336.4	4483.7	4800.4	5034.5	5280.6	5543.4	5820.5	6111.6	6417.2
48	N	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050	1102.5	1157.63
49	**r**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**	**17.00%**
100	**D**	**1882.46**	**1912.27**	**2444.46**	**2486.19**	**2275.82**	**2058.55**	**1984.28**	**1755.33**	**1521.91**	**1337.22**	**1404.08**	**1474.28**	**1548**
101	**Kd**	**17.84%**	**17.69%**	**17.70%**	**17.32%**	**16.75%**	**16.11%**	**15.63%**	**14.99%**	**14.41%**	**13.97%**	**13.97%**	**13.97%**	**13.97%**
102	Beta d	0.7299	0.7106	0.7128	0.6651	0.5939	0.5144	0.4532	0.3734	0.3012	0.2467	0.2467	0.2467	0.2467
103	D/N	1.0458	1.0624	1.0628	1.081	1.1102	1.1436	1.1672	1.2106	1.2683	1.3372	1.3372	1.3372	1.3372
104	Nr-DKd		-29.8163	-32.1846	-41.7351	-39.6273	-32.7303	-25.734	-21.0515	-16.5782	-15.305	-16.8609	-17.7039	-18.5891
105	Ke - Kd	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
	D T Ku + (Nr-DKd)*T		121.34	122.59	156.5	160.16	147.85	135,09	131.53	117.07	101.18	87.7	92.09	96.69
51	NPV(Ku;DtKu) = NPV int. tax shields	655.58	665.36	675.84	654.5	625.23	602.43	587,82	573.86	571.56	584.69	613.93	644.62	676.86
52	NPV int. tax shields + Vu	2335.23	2418.44	3084.53	3299.93	3287.25	3321.85	3540,63	3669.82	3816.7	3990.84	4190.38	4399.9	4619.9
53	**-D = C**	**453**	**506**	**640**	**814**	**1011**	**1263**	**1556**	**1914**	**2295**	**2654**	**2786**	**2926**	3072
54	Beta Equity	1.729909	1.710621	1.712839	1.665095	1.593918	1.514368	1,453171	1.373419	1.301229	1.246732	1.246732	1.246732	1.246732
55	Ke	25.84%	25.68%	25.70%	25.32%	24.75%	24.11%	23,63%	22.99%	22.41%	21.97%	21.97%	21.97%	21.97%
56	**C = NPV(Ke;CFacc)**	**453**	**506**	**640**	**814**	**1011**	**1263**	**1556**	**1914**	**2295**	**2654**	**2786**	**2926**	**3072**
59	WACC reformed	14.80%	14.93%	14.93%	15.15%	15.50%	15.93%	16,29%	16.81%	17.35%	17.80%	17.80%	17.80%	17.80%
60	D + E = NPV(WACC;FCF)	2335.23	2418.44	3084.53	3299.93	3287.25	3321.85	3540,63	3669.82	3816.7	3990.84	4190.38	4399.9	4619.9
61	**-D = C**	**453**	**506**	**640**	**814**	**1011**	**1263**	**1556**	**1914**	**2295**	**2654**	**2786**	**2926**	**3072**

Appendix 1

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

Alternative formula for companies with constant growth «g»

Many authors and professors maintain that

[24] $$\text{NPV of interest tax shields} = \frac{DTKd}{Kd - g}$$

The intuition behind this formula considers that the NPV of interest tax shields is the NPV of the tax shield (DT Kd = $Taxes_U$ - $Taxes_L$) discounted at the cost of the debt.

The first problem with this formula, as we have already said, is that it discounts a flow (DT Kd) at the debt rate, even though DT Kd is the difference of 2 flows ($Taxes_U$ and $Taxes_L$) with different risk.

The second problem is that the three valuation formulas (formulas [1], [2] and [3]) give different values, unless we redefine several formulas. If we want to maintain [24], and have the same value with formulas [1], [2] and [3] (**Ke_W** = Required return of levered equity if [24] holds; **C_W** = Value of shares at t = 0 if [24] holds), we get the following relationships:

Instead of formula [10], we get [10w]

[10w] $$Ku = \frac{Kd\left[C_w Ke_w + DKd(1-T)\right] - g(C_w Ke_w + DKd)}{Kd\left[C_w + D(1-T)\right] - g(C_w + D)} =$$

$$= \frac{C_w Ke_w + DKd\dfrac{\left[Kd(1-T) - g\right]}{Kd - g}}{C_w + D\dfrac{\left[Kd(1-T) - g\right]}{Kd - g}}$$

$$Ke_w = Ku + \frac{D}{C_w}\left[\frac{Kd(1-T) - g}{Kd - g}\right](Ku - Kd)$$

We can see that if g > Kd (1 - T), then Ke < Ku, which makes no sense.

Appendix 1 (continued)

And instead of [11], we now have [11w]

[11] $$\beta_U = \beta_L \frac{C}{C + D(1 - T)} + \beta_D \frac{D(1 - T)}{C + D(1 - T)}$$

[11w] $$\beta_U = \beta_{LW} \frac{C_w}{C_w + D\left[\frac{Kd(1-T)-g}{Kd-g}\right]} + \beta_{LW} \frac{D\left[\frac{Kd(1-T)-g}{Kd-g}\right]}{C_w + D\left[\frac{Kd(1-T)-g}{Kd-g}\right]}$$

$C_W - C = NPV_W - NPV = DT\,[Kd/(Kd-g) - Ku/(Ku-g)] > 0$ if $Kd<Ku$

If $g > Kd(1-T)$, then $C_W > Vu$, which is, obviously, inconsistent.

Appendix 2

EQUIVALENCE OF THE APV, WACC AND FLOWS TO EQUITY APPROACHES TO FIRM VALUATION

Most important valuation formulas

General formulas:

[1] $$D + C = \sum_{t=1}^{\infty} \frac{FCF_t}{\prod_{1}^{t} (1 + WACC_t)}$$

[2] $$C = \sum_{t=1}^{\infty} \frac{CFacc_t}{\prod_{1}^{t} (1 + Ke_t)}$$

[3] $$D + C = \sum_{t=1}^{\infty} \frac{FCF_t}{\prod_{1}^{t} (1 + Ku_t)} + \text{NPV of interest tax shields}$$

[4] $$D_0 = \sum_{t=1}^{\infty} \frac{I_t + N_t}{\prod_{1}^{t} (1 + Kd_t)}$$

[20] NPV OF INTEREST TAX SHIELDS $= \sum_{t=1}^{\infty} \frac{D_{t-1}\, Ku_t\, T}{\prod_{t=1}^{t} (1 + Ku_t)}$

[5] $Ku = R_F + ß_U P_M$ [6] $Ke = R_F + ß_L P_M$ [7] $Kd = R_F + ß_d P_M$

$K_{TU} = Ku$ KT_{L1} = required rate for the $Taxes_{L1} = Ku_1 + T D_0 (Ku_1 - Kd_1)/ GOV_{L0}$

Perpetuities without growth:

[1p] $C = FCF / WACC - D$; **[2p]** $C = CFacc/Ke$ **[14]** $K_{Tu} = Ku$; $K_{TL} = Ke$

[3p] $C=FCF/Ku$ + NPV of interest tax shields - D **[4p]** $CFacc = FCF - I (1 - T) = FCF - D Kd (1-T)$

[8] $$\mathbf{WACC} = \frac{\mathbf{C\ Ke + D\ Kd\ (1 - T)}}{\mathbf{C + D}}$$ **[9] NPV of interest tax shield = DT**

Appendix 2 (continued)

[10] $\mathbf{Ku = \frac{C\ Ke + D\ Kd\ (1 - T)}{C + D\ (1 - T)}}$ **[11]** $\beta_L = \frac{\beta_U[C + D(1-T)] - \beta_d D(1-T)}{C}$

[12] $\mathbf{WACC = Ku\ \frac{C + D\ (1 - T)}{C + D} = Ku\left(1 - \frac{D\ T}{C + D}\right)}$ **[13]** $\mathbf{C = \frac{CFacc}{Ku} - \frac{D\ (1 - T)\ (Ku - Kd)}{Ku}}$

Perpetuities with constant growth g:

[1c] $C = \frac{FCF_1}{WACC - g} - D$ **[2c]** $C = \frac{CFacc_1}{Ke - g}$ **[3c]** $C = \frac{FCF_1}{K_u - g}$ + NPV of interest tax shields-D

[4c] $CFacc_1 = FCF_1 - Do\ [Kd\ (1 - T) - g]$ **[8]** $WACC = \frac{C\ Ke + D\ Kd\ (1-T)}{C + D}$

[15] NPV int tax shields = (C + D) (Ku - WACC) / (Ku - g) **[16]** NPV int tax shields = DT Ku / (Ku-g)

[17] $K_{T_L} - g = \frac{CKe - g(C + D) + H}{CKu - g(C + D) + H}(Ku - g)$ **[18]** $C = \frac{CFacc}{Ku - g} - \frac{D\ (Ku - Kd)\ (1 - T)}{Ku - g}$

Perpetuities when r ≠ Kd. D = Nr / Kd.

Companies with constant growth when r ≠ Kd.
[4c'] CFacc = FCF - Nr (1 - T) + gN = FCF - D (Kd - g) + NrT

[8'] $\mathbf{WACC = \frac{C\ Ke + D\ Kd - Nr\ T}{C + D}}$ **[16´´]** NPV of interest tax shields $= \frac{DTKu}{Ku - g} - \frac{Tg(D - N)}{Ku - g}$

A formula for Kd **[21]** $Kd = R_F + \frac{D(1 - T)}{D(1 - T) + C}(Ku - R_F)$ **[22]** $\beta_d = \frac{D(1 - T)}{D(1 - T) + C}\beta_U$

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