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CHOICE RULES WITH SIZE CONSTRAINTS
FOR MULTIPLE CRITERIA DECISION MAKING

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ABSTRACT :

In outranking methods for Multiple Criteria Decision Making (MCDM), pair-wise comparisons of alternatives are often summarized through a fuzzy preference relation.

In this paper, the binary preference relation is extended to pairs of subsets of alternatives in order to define on this basis a scoring function over subsets.

A choice rule based on maximizing score under size constraint is studied, which turns to formulate as solving a sequence of classical location problems. For comparison with the kernel approach, the interior stability property of the selected subset is discussed and analyzed.

Key-Words :

- Combinatorial optimization
- Fuzzy preferences
- Integer Programming
- Location
- Multiple Criteria Decision Aid

RESUME :

Dans les méthodes de surclassement pour l'Aide Multicritère à la Décision, la comparaison par paires des actions potentielles aboutit le plus souvent à la construction d'une relation floue de préférence.

Dans ce papier, nous proposons d'étendre la relation binaire de préférence aux sous-ensembles d'actions, ce qui nous conduit à déterminer une fonction de score associée à chaque sous-ensemble.

Une procédure de choix basée sur la maximisation du score sous contrainte de cardinalité est étudiée. Nous montrons que cette procédure de choix équivaut à résoudre de façon séquentielle des problèmes combinatoires de localisation bien connus. Afin de comparer cette nouvelle approche à la méthode classique de recherche de noyaux, nous analysons la propriété de stabilité du sous-ensemble choisi et discutons de la pertinence de cette contrainte.

Mots-clés :

- Aide multicritère à la décision
- Localisation
- Optimisation combinatoire
- Préférences floues
- Programmation en Nombres Entiers

Jel Classification : C44

1 Introduction

In a Multiple Criteria Decision Making (MCDM) problem, we are given a finite set of alternatives $X = \{a_1, \dots, a_n\}$ evaluated along a finite set \mathcal{C} of m criteria, and weights summing over one associated with criteria. Under frequently-met conditions on the type of criteria, choosing or ranking alternatives can hardly rely on the computation of a single criterion that would aggregate all criteria on a unique scale [16]. Such conditions justify the use of other methods known as *Outranking Methods* [16, 19]. These methods generally proceed in two steps: a construction step and an exploitation step. In the construction step, pairwise comparisons of the alternatives of X are summarized using a binary preference relation R , whereby for $(a_i, a_j) \in X \times X$, $R(a_i, a_j)$ indicates credibility of proposition ' a_i is at least as good as a_j ', denoted by $a_i \succeq a_j$. R is said to be *crisp* (resp., *valued* or *fuzzy*) if $R(a_i, a_j) \in \{0, 1\}$ (resp. $R(a_i, a_j) \in [0, 1]$) for all $(a_i, a_j) \in X \times X$. When R is a crisp relation, like for the Electre Is method [16], $R(a_i, a_j) = 1$ is stated when:

- a majority of criteria agrees with proposition $a_i \succeq a_j$, (*concordance* principle)
- no criterion vetoes proposition $a_i \succeq a_j$, i.e. no criterion is so favorable to a_j in the comparison with a_i that the gap exceeds a prefixed threshold (*discordance* principle).

When R is a valued relation, like for Electre III [17] or Promethee [6] methods, $R(a_i, a_j)$ is the total weight of criteria supporting proposition $a_i \succeq a_j$, possibly decreased by the product of m *non-discordance* coefficients ranging from 1 (no opposition to proposition $a_i \succeq a_j$) to 0 (full opposition to proposition $a_i \succeq a_j$).

In the whole paper, R is a reflexive valued relation and is assumed to be calculated already, so that we focus on the exploitation step. A choice rule is a function C associating with set X and relation R a subset $C(X, R) \subseteq X$ such that $C(X, R) \neq \emptyset$. The scope of the paper is to restrict to crisp choice rules for valued relations (fuzzy choice, meaning that an alternative belongs to the choice set with a certain degree of credibility, will not be studied; we refer to [13, 15, 3] for fuzzy choice rules).

A number of papers in the literature propose crisp choice rules for the exploitation step, called selection step in this case. Two main families of methods may be distinguished. The first type of methods, say, ensemble methods, consist in searching for subsets with appropriate properties. For crisp outranking relations, the kernel method looks for a subset of alternatives that is both absorbing and interior stable (kernels were originally studied by von Neumann and Morgenstern in game theory; see [16] for applications to MCDM). Extensions to quasi-kernels were studied in [12, 18]. For valued relations, characterizations of kernels were also proposed in [13, 15] and will be discussed in the paper, given their close connexions with our approach. Beside, scoring methods are based on a scoring function $score : X \rightarrow \mathfrak{R}$ so that the alternatives with highest score are selected (contrary to ensemble methods, no constraints link selected alterna-

tives together). The scoring function associated with $a_i \in X$ can be defined in a number of ways; we refer to [1] for a thorough analysis of a sample of score functions. When R is a valued relation, the 'Min in Favor' scoring function deserves special interest. It is defined by the minimum degree of credibility with which a_i is considered at least as good as any other alternative of X and can be interpreted as a security level. The associated choice rule was shown to combine three advantages : ordinality, continuity and 'greatest faith' (see [4] for definition of these terms and detailed axiomatic results).

The method developed in this paper is an ensemble method even though it uses a scoring function. It combines in some way the Min in favor score and the absorption property of kernels. The choice problem studied here is a particular issue in the sense that the selected set, which is generally required to be 'as small as possible' in the literature, is explicitly constrained to contain no more than a prefixed number p of alternatives. Selection of a fixed number of alternatives is indeed a widespread issue in real applications (for example, selection of investment projects with limited budget, or prechoice of p tender offers from architect offices for architectural projects). Nevertheless, ensemble methods like kernel methods do not generally enable the decision-maker to formally limit the size of the set, which can be a constraint of crucial importance in various decision contexts.

The paper is organized as follows. Section 2 introduces an extension of the binary valued relation R to pairs of subsets of X . This extended relation is used in section 3 for defining a scoring function over subsets, which turns to formulate as the absorbing credibility degree of a set. The associated choice rule of at most p alternatives is shown to be equivalent to solving location covering models. Section 4 discusses the links between the approach developed in the paper and the kernel approach with a focus on the interior stability property of the chosen set. Section 5 concludes the paper and lays the foundations for future work.

2 Extended preference relation

As a choice rule consists of finding a subset that is at least as good as all other subsets of X , we propose to characterize the formal preference relation between subsets underlying the scoring function that will be used. Before introducing this extended binary relation, we briefly recall some basic rules of credibility calculus. Following the terminology used in [2], let \mathcal{P} denote a set of atomic propositions above which a credibility function $r : \mathcal{P} \rightarrow [0, 1]$ is defined, i.e., $r(x)$ is the degree of credibility of proposition $x \in \mathcal{P}$. Let \mathcal{E} denote the superset constructed from \mathcal{P} such that all $x \in \mathcal{P}$ also belong to \mathcal{E} , and

$$\forall x, y \in \mathcal{E}, \neg x \in \mathcal{E}, x \wedge y \in \mathcal{E}, x \vee y \in \mathcal{E}$$

By extending r from \mathcal{P} to \mathcal{E} , we have

$$\forall x \in \mathcal{E}, r(\neg x) = 1 - r(x) \quad (1)$$

$$\forall x, y \in \mathcal{E}, r(x \wedge y) = \min(r(x), r(y)) \quad (2)$$

$$\forall x, y \in \mathcal{E}, r(x \vee y) = \max(r(x), r(y)) \quad (3)$$

Moreover, for a set of propositions $x_1, \dots, x_n \in \mathcal{E}$, let

$$\bigvee_{i=1}^n x_i = x_1 \vee x_2 \vee \dots \vee x_n \quad \text{and} \quad \bigwedge_{i=1}^n x_i = x_1 \wedge x_2 \wedge \dots \wedge x_n$$

denote the classical conjunctions and disjunctions of propositions. With the above terminology, we have for our MCDM problem $\mathcal{P} = \{(a_i \succeq a_j) : a_i, a_j \in X\}$ and $r(a_i \succeq a_j) = R(a_i, a_j)$.

Definition 1. A set $A \subseteq X$ is said to *absorb* a set $B \subseteq X$ if $\forall a_j \in B, \exists a_i \in A, a_i \succeq a_j$. It is denoted by $A \succeq B$.

Definition 2. The extended preference relation $R : 2^X \times 2^X \rightarrow [0, 1]$ is defined by $R(A, B) = r(A \succeq B)$ for $A, B \subseteq X$.

For the sake of simplicity, we use notation $R(A, a_j)$ instead of $R(A, \{a_j\})$ and $R(a_i, B)$ instead of $R(\{a_i\}, B)$. The following proposition provides characterizations of extended relation R , adapting for item (ii) a classical result usually stated for an absorbing set, i.e., for $B = X \setminus A$.

Proposition 1. *The following three propositions are equivalent:*

$$(i) \quad R(A, B) = r(A \succeq B),$$

(ii) $R(A, B)$ is analytically defined by:

$$R(A, B) = \min_{a_j \in B} \max_{a_i \in A} R(a_i, a_j) \quad (4)$$

(iii) $R(A, B)$ verifies the following properties:

$$(Coherence) \quad \forall A, B \subseteq X, \min_{a_j \in B} R(A, a_j) \leq R(A, B) \leq \max_{a_i \in A} R(a_i, B) \quad (5)$$

$$(A - monotony) \quad \forall A' \subseteq A \subseteq X, \forall B \subseteq X, \quad R(A, B) \geq R(A', B), \quad (6)$$

$$(B - monotony) \quad \forall A \subseteq X, \forall B' \subseteq B \subseteq X, \quad R(A, B) \leq R(A, B') \quad (7)$$

Proof. (i) \Leftrightarrow (ii) We have

$$\begin{aligned}
r(A \succeq B) &= r(\forall a_j \in B, \exists a_i \in A, a_i \succeq a_j) \\
&= r\left(\bigwedge_{a_j \in B} \bigvee_{a_i \in A} a_i \succeq a_j\right) \\
&= \min_{a_j \in B} r\left(\bigvee_{a_i \in A} a_i \succeq a_j\right) \quad \text{by (2)} \\
&= \min_{a_j \in B} \max_{a_i \in A} r(a_i \succeq a_j) \quad \text{by (3)} \\
&= \min_{a_j \in B} \max_{a_i \in A} R(a_i, a_j)
\end{aligned}$$

(ii) \Leftrightarrow (iii) Proving that if $R(A, B)$ is expressed by (4) then R satisfies (5)-(7) is straightforward. Conversely, assume that (5)-(7) are verified. Then,

$$\begin{aligned}
&R(A, a_j) \geq R(a_i, a_j) \text{ for } (a_i, a_j) \in A \times B \quad \text{by (6)} \\
\Rightarrow &R(A, a_j) \geq \max_{a_i \in A} R(a_i, a_j) \text{ for } a_j \in B \\
\Rightarrow &\min_{a_j \in B} R(A, a_j) \geq \min_{a_j \in B} \max_{a_i \in A} R(a_i, a_j) \\
\Rightarrow &R(A, B) \geq \min_{a_j \in B} \max_{a_i \in A} R(a_i, a_j) \quad \text{by (5)}
\end{aligned}$$

On the other side,

$$\begin{aligned}
&R(A, B) \leq R(A, a_j) \text{ for } a_j \in B \quad \text{by (7)} \\
\Rightarrow &R(A, B) \leq \min_{a_j \in B} R(A, a_j) \\
\Rightarrow &R(A, B) \leq \min_{a_j \in B} \max_{a_i \in A} R(a_i, a_j) \quad \text{by (5)}
\end{aligned}$$

and the proof is complete. \square

The min-max form of the extended binary relation R captures the monotony properties of the relation. An additional obvious property of extended relation R is reflexivity, as $\forall a_i \in X, R(a_i, a_i) = 1 \Leftrightarrow \forall A \subseteq X, R(A, A) = 1$. Transitivity of the extended relation also directly depends of the transitivity of the initial relation:

$$\begin{aligned}
&(\forall a_i, a_j, a_k \in X, R(a_i, a_j) = 1 \wedge R(a_j, a_k) = 1 \Rightarrow R(a_i, a_k) = 1) \\
\Leftrightarrow &(\forall A, B, C \subseteq X, R(A, B) = 1 \wedge R(B, C) = 1 \Rightarrow R(A, C) = 1)
\end{aligned}$$

In next subsection the extended relation R is used for scoring subsets of alternatives.

3 Scoring function and choice rule

3.1 Scoring subsets of alternatives

The aim of a choice rule is to select a set that is at least as good as others. If for subsets of alternatives the outranking binary relation expresses in terms of absorption, then the

choice can be made on the basis of the credibility with which a subset absorbs all subsets of X .

Definition 3. $score : 2^X \rightarrow [0, 1]$ is the application defined by

$$\forall A \subseteq X, score(A) = r\left(\bigwedge_{B \subseteq X} A \succeq B\right) \quad (8)$$

On the basis of the extended preference relation, expression (8) leads to score a subset A by the classical credibility of A being an absorbing set.

Proposition 2. $score(A) = R(A, X) = R(A, \bar{A})$, where $\bar{A} = X \setminus A$.

Proof. From expression (8) we have

$$\begin{aligned} score(A) &= r\left(\bigwedge_{B \subseteq X} A \succeq B\right) \\ &= \min_{B \subseteq X} r(A \succeq B) \quad \text{by (2)} \\ &= \min_{B \subseteq X} R(A, B) \\ &= R(A, X) \quad \text{by (7)} \\ &= \min(R(A, A), R(A, \bar{A})) \\ &= R(A, \bar{A}) \quad \text{as } R(A, A) = 1 \quad \square \end{aligned}$$

Given the min-max form (4) of $R(A, B)$, the scoring function is defined by

$$score(A) = \min_{a_j \in \bar{A}} \max_{a_i \in A} R(a_i, a_j) \quad (9)$$

It can interpret as a security level and extends the Min in Favor rule to $R : 2^X \times 2^X \rightarrow [0, 1]$.

Remark 1. The absorbing property of a set has already been studied in the literature from both crisp and fuzzy points of view (cf. the so-called 'exterior stability' Δ_2 rule, deeply analysed by Kitainik [13] when combined with both or any of the two classical rules Δ_1 (GOTCHA) and Δ_3 (interior stability)). However, it has not been taken directly as a scoring function. Here, converting absorbing credibility into score follows in a straightforward way from the preference relation between subsets stated in section 2 which is, in our opinion, an innovative and interesting way of structuring and analysing MCDM problems. The reason why such possible additional property as interior stability is dropped will be discussed in section 4.

3.2 Choice rule

Since $score(X) = 1$, choice rules should express additional constraints in order to obtain a set of limited size. This is the case for the kernel approach through the interior stability (Δ_3) constraint, as we shall see in section 4. In this paper, we achieve a straightforward trade-off between the score of the selected set $score(A)$ and its cardinality $|A|$, by maximizing score under size constraint. Hence, for an input $p \in \{1, \dots, n\}$, we wish to find

$$\sigma(p) = \max_{\substack{A \subset X \\ |A| \leq p}} score(A) \quad (10)$$

Remark 2. The iterated ranking rule associated with the Min in favor scoring function, analysed in [4], is a greedy approximation heuristic for $\sigma(p)$. At step one, the heuristic picks an alternative with highest score $score(a_i) = R(a_i, X \setminus \{a_i\})$ (from the latter expression one can check that our scoring function is indeed an extension of the Min in favor choice rule as claimed in the introduction). This alternative, say, α_1 , is removed from X and the process is iterated again at step two on $X \leftarrow X \setminus \{\alpha_1\}$ and generally speaking, on $X \leftarrow X \setminus \{\alpha_1, \dots, \alpha_t\}$ at step $t+1$, where α_t is the alternative found at step t . The resulting subset $\{\alpha_1, \dots, \alpha_p\}$ is a feasible though not optimal solution for the problem of choosing a set of p alternatives with highest score. When the ranking rule is not iterated, i.e., one selects in one shot the alternatives that rank from 1 to p according to their score, the security level obtained this way is generally not optimal either. The problem complexity will be discussed later on.

Function σ of (10) is clearly non-decreasing but generally not strictly non-decreasing. In order to avoid selecting bad alternatives, i.e., alternatives that do not help to absorb the rejected set, an additional requirement should express minimality of the selected subset: a smaller subset should be of lower score, i.e., $\sigma(|A| - 1) < \sigma(p)$, otherwise the chosen set could even contain alternatives that are dominated by all other alternatives of X . Moreover, all minimal subsets $A \subset X$ verifying $score(A) = \sigma(p)$ are not equivalent. For $n = 5$ and $p = 2$, consider a minimal set A such that $|A| = 2$, $score(A) = \sigma(2) = 0.6$, and the three rejected alternatives are absorbed with credibility degrees respectively equal to 0.6, 0.6 and 0.7. If another minimal subset A' also verifying $|A'| = 2$ and $score(A') = \sigma(2) = 0.6$ is such that the rejected alternatives are absorbed with credibility degrees respectively equal to 0.6, 0.7 and 0.8, one can consider that A' should be preferred to A . Hence, we select only one of those sets such that the total (or average) credibility of considering each rejected alternative absorbed by A is maximum. The choice rule finally expresses as follows.

Definition 4. The choice rule $C_p(X, R)$ is to find a subset $A^* \subset X$ such that

$$|A^*| \leq p \quad (11)$$

$$\text{score}(A^*) = \sigma(p) \quad (12)$$

$$\sigma(|A^*| - 1) < \sigma(p) \quad (13)$$

$$\Sigma_R(A^*) = \max\{\Sigma_R(A) : |A| \leq p, \text{score}(A) = \sigma(p) > \sigma(|A| - 1)\} \quad (14)$$

where $\Sigma_R(A) = \sum_{a_j \in \bar{A}} R(A, a_j)$.

The rest of the section is devoted to problem solving.

3.3 Solving via location models

We show that the choice rule of definition 4 is equivalent to solve a sequence of classical combinatorial location problems, namely, p -center, Set Covering and p -median. The p -center problem was originally introduced by Hakimi ([10], see also [11] for a review). It consists in choosing p nodes of a network for locating facilities or public services, such that the farthest demand node is as close as possible to a facility. The p -center objective is typically designed for the location of emergency services like hospitals, care centers or fire stations, for which social equity or security considerations lead to prefer small maximal distance rather than small average distance to customers. The problem is stated as follows. Let $D = \{1, \dots, n\}$ represent a set of demand nodes and $L = \{1, \dots, m\}$ be the set of all possible locations for a facility. For $(i, j) \in L \times D$, d_{ij} is the distance or time required to reach j from i . The mixed-integer LP model associated with the p -center is:

$$\text{Minimize } \delta \quad (15)$$

$$\text{s.t.} \quad \sum_{i \in L} y_i \leq p \quad (16)$$

$$\sum_{i \in L} x_{ij} = 1 \quad \text{for } j \in D \quad (17)$$

$$y_i \geq x_{ij} \quad \text{for } (i, j) \in L \times D \quad (18)$$

$$\sum_{i \in L} d_{ij} x_{ij} \leq \delta \quad \text{for } j \in D \quad (19)$$

$$y_i, x_{ij} \in \{0, 1\}, \delta \geq 0 \quad (20)$$

where binary variables y_i indicate whether $i \in L$ is selected or not as location for a facility, and binary variables x_{ij} indicate whether demand $j \in D$ is assigned to location $i \in L$ or not.

The p -median problem consists in finding p locations so that the average (or total) distance between the n customers and the p centers is minimum. When an upper bound

δ on the maximal distance is specified, setting $LD(\delta) = \{(i, j) \in L \times D : d_{ij} \leq \delta\}$, the problem formulates as follows:

$$\text{Minimize } \sum_{(i,j) \in LD(\delta)} d_{ij} x_{ij} \quad (21)$$

$$\text{s.t. } \sum_{i \in L} y_i = p \quad (22)$$

$$\sum_{(i,j) \in LD(\delta)} x_{ij} = 1 \quad \text{for } j \in D \quad (23)$$

$$y_i \geq x_{ij} \quad \text{for } i \in L, (i, j) \in LD(\delta) \quad (24)$$

$$y_i, x_{ij} \in \{0, 1\} \quad (25)$$

By constraint (23) and objective (21), $x_{ij} = 1$ if and only if $i \in L$ is the closest facility to $j \in D$ in the optimal solution.

Proposition 3. $A^* = C_p(X, R)$ can be computed by the following process:

1. solve the p -center (15)-(20) with $L = D = X$ and $d_{ij} = 1 - R(a_i, a_j)$. Let $\delta(p)$ be the optimal value.
2. solve the Set Covering problem of minimizing $\sum_{j \in L} y_j$ submitted to constraints (17)-(20) with $\delta = \delta(p)$ in (19). Let p^* be the optimal value.
3. solve the p -median (21)-(25) with $p = p^*$ in (22) and $\delta = \delta(p)$.
 $A^* = \{a_i \in X : y_i = 1\}$.

Proof. By $R(a_i, a_i) = 1$ we have $y_i = x_{ii}$ and $d_{ii} = 0$ for all $a_i \in X$. With these settings, the optimal value $\delta(p)$ of the p -center (step 1) is

$$\begin{aligned} \delta(p) &= \min_{\substack{A \subset X, \\ |A| \leq p}} \left\{ \max_{a_j \in X} \left[\min_{a_i \in A} d_{ij} \right] \right\} \\ &= 1 - \max_{\substack{A \subset X, \\ |A| \leq p}} \left\{ \min_{a_j \in A} \left[\max_{a_i \in A} R(a_i, a_j) \right] \right\} \\ &= 1 - \max_{\substack{A \subset X, \\ |A| \leq p}} \text{score}(A) \\ &= 1 - \sigma(p) \end{aligned}$$

Step 2 simply ensures to get a subset A minimizing $|A|$ under constraint

$$\max_{a_j \in X} \min_{a_i \in A} d_{ij} \leq \delta(p) \Leftrightarrow \text{score}(A) \geq \sigma(p)$$

i.e., such that $\sigma(|A| - 1) < \text{score}(A) = \sigma(p)$. Finally, at step 3,

$$\begin{aligned}
\sum_{j \in D} \sum_{i \in L} d_{ij} x_{ij} &= \sum_{j \in D} \min_{i \in L: y_i = 1} d_{ij} \\
&= \sum_{a_j \in X} \min_{a_i \in A} (1 - R(a_i, a_j)) \quad \text{with } A = \{a_i : y_i = 1\} \\
&= |X| - \sum_{a_j \in X} \max_{a_i \in A} R(a_i, a_j) \\
&= |X| - (|A| + \sum_{a_j \in \bar{A}} \max_{a_i \in A} R(a_i, a_j)) \quad \text{as } R(a_i, a_i) = 1 \\
&= (n - p^*) - \Sigma_R(A)
\end{aligned}$$

So, $\Sigma_R(A)$ is maximized under constraints $|A| = p^*$ and $\text{score}(A) = \sigma(p)$, as required in the choice rule of definition 4. \square

The p -center, Set Covering and p -median problems are all NP-hard [9]. When p is a constant independent of n , p -center and p -median problems can be solved in polynomial time by enumerating (possibly implicitly) all $\binom{p}{n} = O(n^p)$ combinations of p centers and assigning each customer to its nearest center. As covering location problems are not the main purpose of the paper we will not provide a comprehensive list of methods for these problems, which comprises exact branching methods, column generation, metaheuristics, greedy, local search and relaxation-based heuristics. The reader is referred to the detailed surveys of [7, 11, 14]. All these methods can be applied in a straightforward way to the MCDM problematic developed in this paper, which is not inconsiderable asset of the approach.

Most methods for location problems are known to provide better gaps to optimality when distances satisfy the triangle inequalities, which turn to be equivalent to the Lukasiewicz valued transitivity property for the MCDM associated problem:

$$\forall a_i, a_j, a_k \in X, \quad d_{ij} + d_{jk} \geq d_{ik} \iff R(a_i, a_k) \geq \max(R(a_i, a_j) + R(a_j, a_k) - 1, 0)$$

Unfortunately, well-known discordance and threshold effects generally prevent relation R from being transitive in most MCDM outranking methods [16].

4 On the interior stability of the selected set

4.1 Comparison with kernels

In this section, we discuss the interior stability property of the choiced subset for comparison with kernel methods. We first recall the kernel approach for a crisp relation

$R' : X \rightarrow \{0, 1\}$. This crisp relation R' is generally obtained from a valued relation R through a so-called λ -cut, where $R'(a_i, a_j) = 1$ if and only if $R(a_i, a_j) \geq \lambda$, with $\lambda \geq 0.5$ (see [16]). A *kernel* is a subset $K \subset X$ such that:

$$\begin{aligned} \forall a_j \notin K, \exists a_i \in K, R'(a_i, a_j) &= 1 && (\text{K is absorbing: } \Delta_2 \text{ rule}) \\ \forall a_i, a_j \in K, R'(a_i, a_j) &= 0 && (\text{K is interior stable: } \Delta_3 \text{ rule}) \end{aligned}$$

One can note that dropping the Δ_3 rule in the above kernel method and searching for the minimum absorbing set amounts to solving the 'dual' problem associated with the choice problem of this paper, i.e., score of (9) is constrained and size is minimized. Addition of the Δ_3 rule has two positive effects : it prevents selecting an alternative which would be outranked by another alternative of the set, meaning structural incomparability within the selected set, and it indirectly reduces the size of the set, without constraining it however by an upper bound as required in our choice rule of p alternatives. Several drawbacks may be outlined concerning the above kernel method. First, the existence of a kernel is not guaranteed (cf. the alternative proposition of quasi-kernels [12, 18]). Second, the λ -cut requires to choose an arbitrary score level $\lambda \geq 0.5$ above which fuzzy preference gets crisp, causing a loss of information then. Third, the defuzzification step $R(a_i, a_j) < \lambda \Leftrightarrow R'(a_i, a_j) = 0$ presents some problems of logical coherence related to an asymmetric treatment of truthfulness versus falseness, as evidenced by Bisdorff [2]. Some of these effects have been somewhat cancelled or reduced via the introduction by Kitainik [13] of kernels based on valued relations, avoiding defuzzification. This alternative Kernel rule, denoted by C_K , leads to search for a subset $K \subset X$ such that the minimum between its absorbing (Δ_2) credibility and its interior stability (Δ_3) credibility is maximum, i.e.,

$$\begin{aligned} C_K(X, R) &= \arg \max_{K \subset X} \left\{ r_{23}(K) = \min \left(\min_{a_j \in K} \max_{a_i \in K} R(a_i, a_j) ; \min_{(a_i, a_j) \in K^2} 1 - R(a_i, a_j) \right) \right\} \\ &= \arg \max_{K \subset X} \{ r_{23}(K) = \min(\text{score}(K), \text{stability}(K)) \} \end{aligned} \quad (26)$$

This approach also presents some inconvenients due to the Δ_3 rule. Just like the existence problem for the former kernel approach on crisp relations, the existence of a kernel K with credibility degree $r_{23}(K) \geq 0.5$ is not guaranteed though only credibility degrees exceeding 0.5 could pretend to a truthfulness interpretation (see [2]). It is easy however to find situations for which the C_p choice rule with a small p also provides an optimal score beyond 0.5, but addition of the Δ_3 rule makes these situations more likely. We show furthermore that, in some simple realistic MCDM situations, the C_K rule may lead to choose sets of alternatives with undesirable properties, namely, dominated alternatives.

Definition 5. An alternative $a_i \in X$ is *dominated* by a_j (noted $a_j \Delta a_i$) iff

$$\forall c \in \mathcal{C}, g_c(a_j) \geq g_c(a_i) \text{ and } \exists c' \in \mathcal{C} : g_{c'}(a_j) > g_{c'}(a_i)$$

where $g_c(a_i)$ denotes the performance of $a_i \in X$ on criterion $c \in \mathcal{C}$. We note $\mathcal{ND} = \{a_i \in X : \nexists a_j \in X, a_j \Delta a_i\}$ (resp. $\mathcal{D} = X \setminus \mathcal{ND}$) the set of non-dominated (resp., dominated) alternatives.

Definition 6. Let \mathcal{R} denote the set of valued relations $R : X \times X \rightarrow \{0, 1\}$ such that

$$a_i \in \mathcal{ND} \Rightarrow \forall a_l \in X \setminus \{a_i\}, R(a_l, a_i) < 1 \quad (27)$$

A choice rule C is Δ -consistent if $\forall R \in \mathcal{R}, \forall A = C(X, R), A \cap \mathcal{D} = \emptyset$.

Proposition 5.

(i) *The Kernel choice rule expressed in (26) is not Δ -consistent. Moreover, there exist MCDM problems such that $K = C_K(X, R)$ is unique, verifies $r_{23}^*(K) > 0.5$ and $K = \mathcal{D}$.*

(ii) *The choice rule $C_p(X, R)$ is Δ -consistent.*

Proof. (i) We construct an MCDM instance with a set X of six alternatives evaluated along five equally weighted criteria. The data are given in figure 1. Performances range from 1 (poorest performance) to 3 (top performance) on each criterion. The indifference, preference and veto thresholds are respectively set to 0, 1 and 2 for all five criteria. The performance on the fourth (resp., fifth) criteria is equal for all alternatives to an arbitrary number u (resp., v). We have $a_i \Delta a_{i+3}$ for $i = 1, 2, 3$, $\mathcal{ND} = \{a_1, a_2, a_3\}$ and $\mathcal{D} = \{a_4, a_5, a_6\}$. Application of the Kernel choice rule of (26) provides a unique kernel $K = \mathcal{D}$ with credibility degree $r_{23}^*(\mathcal{D}) = 0.80 > 0.5$. Indeed, $\text{stability}(\mathcal{D}) = 1$ as $R(a_i, a_j) = 0$ for all $i, j = 4, 5, 6, i \neq j$, and $\text{score}(\mathcal{D}) = 0.8$ as $R(a_4, a_1) = R(a_5, a_2) = R(a_6, a_3) = 0.8$. Hence, the Kernel choice rule is not Δ -consistent since $K = \mathcal{D}$, which is the worst possible case: all selected alternatives are dominated ones and all efficient alternatives are rejected.

(ii) Let $A = C_p(X, R)$ and suppose that $A \cap \mathcal{D} \neq \emptyset$. Consider an alternative $a_i \in A \cap \{\mathcal{D}\}$ dominated by an alternative a_j . We show that $a_j \in A$ (case a) or $a_j \in \bar{A}$ (case b) both lead to a contradiction. If $a_j \in A$ (case a), then for all $a_k \in \bar{A}$, $R(a_i, a_k) \leq R(a_j, a_k)$ so $R(A \setminus \{a_i\}, \bar{A}) = R(A, \bar{A})$, and we deduce from this equality and $R(a_j, a_i) = 1$ that $R(A \setminus \{a_i\}, \bar{A} \cup \{a_i\}) = R(A, \bar{A})$, i.e., $\text{score}(A \setminus \{a_i\}) = \text{score}(A)$. This contradicts (13). If $a_j \in \bar{A}$ (case b), we have two subcases: $a_j \in \mathcal{D}$, and $a_j \in \mathcal{ND}$. If $a_j \in \mathcal{D}$, then there is an $a_l \in A$ which dominates both a_j and a_i since relation Δ is transitive. So,

c	1	2	3	4	5	R	a_1	a_2	a_3	a_4	a_5	a_6
a_1	3	1	2	u	v	a_1	1	0	0.6	1	0	0.8
a_2	2	3	1	u	v	a_2	0.6	1	0	0.8	1	0
a_3	1	2	3	u	v	a_3	0	0.6	1	0	0.8	1
a_4	3	1	1	u	v	a_4	0.8	0	0	1	0	0
a_5	1	3	1	u	v	a_5	0	0.8	0	0	1	0
a_6	1	1	3	u	v	a_6	0	0	0.8	0	0	1

Figure 1: performance table and R -matrix. Equal weights ; veto threshold = 2

by removing a_i from A we reduce the size of A without compromising score as seen for case a, contradicting (13). Finally, if $a_j \in \mathcal{ND}$, then

$$\begin{aligned}
\Sigma_R((A \setminus \{a_i\}) \cup \{a_j\}) - \Sigma_R(A) &\geq R((A \setminus \{a_i\}) \cup \{a_j\}, a_i) - R(A, a_j) \\
&= 1 - R(A, a_j) \text{ since } a_j \Delta a_i \\
&> 0 \text{ by (27)}
\end{aligned}$$

So $\Sigma_R((A \setminus \{a_i\}) \cup \{a_j\}) > \Sigma_R(A)$, which contradicts (14) and ends the proof of item (ii). \square

Let us re-examine the example of figure 1 comparing C_K and C_p choice rules. We have seen in the proof of proposition 5 that the Kernel choice rule $C_K(X, R)$ leads to select the unique kernel $K = \mathcal{D} = \{a_4, a_5, a_6\}$ with $score(\mathcal{D}) = 0.8$ and $stability(\mathcal{D}) = 1$. The efficient set \mathcal{ND} verifies $score(\mathcal{ND}) = 1$ but $stability(\mathcal{ND}) = 1 - 0.6 = 0.4 < 0.8$, justifying rejection for C_K but selection for C_p with $p \geq 3$. Let us remark that even if the valued relation is defuzzified to form a crisp relation through a λ -cut, $K = \mathcal{D}$ remains an admissible kernel for $\lambda = 0.8$. It sounds natural however to claim that any rational decision-maker would prefer the efficient set \mathcal{ND} rather than the dominated set \mathcal{D} , and score should not be compromised. In a sense, absorption and stability are measures of exterior and interior preference, respectively, but only the former enables to justify rejection of non chosen alternatives, which is the aim of a choice rule.

Moreover, the optimal credibility degrees for C_K and C_p can diverge in a drastic way. Consider the example of figure 2, with four alternatives evaluated along an arbitrary number m of equally weighted criteria. As in figure 1, indifference, preference and veto thresholds are set to 0, 1 and 2, respectively. We set $\epsilon = 1/m$. The unique kernel with the C_K rule of (26) is $K = \{a_3, a_4\}$, with $score(K) = 2\epsilon$, $stability(K) = 1$ and $r_{23}^*(K) = score(K) = 2\epsilon$. Set $\{a_1, a_2\}$ may be perceived as much better than set K as $score(\{a_1, a_2\}) = 1 - \epsilon$, but it is penalized for the C_K rule by its interior stability since $stability(\{a_1, a_2\}) = \epsilon < r_{23}^*(K)$. The latter credibility degree tends to 0 when m is large, whereas the absorption credibility (or score) of the set $\{a_1, a_2\}$ tends to one.

c	1	2	3	4	5	...	m	R	a_1	a_2	a_3	a_4
a_1	2	1	3	3	3	...	3	a_1	1	$1-\epsilon$	$1-\epsilon$	0
a_2	1	2	3	3	3	...	3	a_2	$1-\epsilon$	1	0	$1-\epsilon$
a_3	3	0	3	2	2	...	2	a_3	2ϵ	0	1	0
a_4	0	3	3	2	2	...	2	a_4	0	2ϵ	0	1

Figure 2: performance table and R -matrix. Equal weights = ϵ ; veto threshold = 2

4.2 Sufficient conditions for interior stability in the $\sigma(p)$ -cut

For ending the section, we consider again the kernel approach on crisp relations built through λ -cuts, like Electre 1 and 1s methods. We focus on the case $\lambda = \sigma(p)$.

Definition 7. $A = C_p(X, R)$ is called *interior stable in the $\sigma(p)$ -cut* if $\forall a_i, a_j \in A, a_i \neq a_j, R(a_i, a_j) < \sigma(p)$.

We study whether for the defuzzification approach, transitivity of relation R is a sufficient condition to make A an interior stable set, i.e., a kernel in the $\sigma(p)$ -cut, or not. Transitivity for a crisp relation is stated by logical expression $R(a_i, a_j) \wedge R(a_j, a_k) \Rightarrow R(a_i, a_k)$ for $a_i, a_j, a_k \in X$. There are several ways of turning this logical expression to an equivalent inequality valid for the valued case, for example, min-transitivity or Lukasiewicz-transitivity:

$$\begin{aligned}
R(a_i, a_k) &\geq \min(R(a_i, a_j); R(a_j, a_k)) && \text{(min-transitivity)} \\
R(a_i, a_k) &\geq \max(R(a_i, a_j) + R(a_j, a_k) - 1, 0) && \text{(L-transitivity)}
\end{aligned}$$

We first show that min-transitivity is a sufficient condition for making $A = C_p(X, R)$ a kernel in the $\sigma(p)$ -cut, using the following useful proposition.

Proposition 6. *For all $a_k \in A = C_p(X, R)$, at least one of the following two propositions is true:*

- (i) $\forall a_i \in A \setminus \{a_k\}, R(a_i, a_k) < \sigma(p)$.
- (ii) $\exists a_j \in \bar{A}, \forall a_i \in A \setminus \{a_k\}, R(a_k, a_j) \geq \sigma(p) > R(a_i, a_j)$.

Proof. Assume that there is an $a_k \in A$ such that (i) and (ii) are both false, i.e.,
 $(\neg i) \exists a_i \in A \setminus \{a_k\}, R(a_i, a_k) \geq \sigma(p)$, and
 $(\neg ii) \forall a_j \in \bar{A}, \exists a_i \in A \setminus \{a_k\}, R(a_k, a_j) \leq R(a_i, a_j)$.

This can be re-formulated as

$$\begin{aligned}
(\neg i) \quad & R(A \setminus \{a_k\}, a_k) \geq \sigma(p), \text{ and} \\
(\neg ii) \quad & R(A \setminus \{a_k\}, \bar{A}) \geq R(a_k, \bar{A})
\end{aligned}$$

We have

$$\begin{aligned}
score(A) &= R(A, \bar{A}) \\
&= \min_{a_j \in \bar{A}} \max_{a_i \in A} R(a_i, a_j) \\
&= \min_{a_j \in \bar{A}} \max \left(\max_{a_i \in A \setminus \{a_k\}} R(a_i, a_j); R(a_k, a_j) \right) \\
&= \min_{a_j \in \bar{A}} \max_{a_i \in A \setminus \{a_k\}} R(a_i, a_j) \quad \text{by } (\neg ii) \\
&= R(A \setminus \{a_k\}, \bar{A})
\end{aligned}$$

Hence, as $score(A) = \sigma(p)$,

$$R(A \setminus \{a_k\}, \bar{A}) = \sigma(p) \quad (28)$$

On the other side, we have

$$\begin{aligned}
score(A \setminus \{a_k\}) &= R(A \setminus \{a_k\}, \bar{A} \cup \{a_k\}) \\
&= \min_{a_j \in \bar{A} \cup \{a_k\}} \max_{a_i \in A \setminus \{a_k\}} R(a_i, a_j) \\
&= \min \left(\min_{a_j \in \bar{A}} \max_{a_i \in A \setminus \{a_k\}} R(a_i, a_k); \max_{a_i \in A \setminus \{a_k\}} R(a_i, a_k) \right) \\
&= \min (R(A \setminus \{a_k\}, \bar{A}); R(A \setminus \{a_k\}, a_k)) \\
&= \min (\sigma(p); R(A \setminus \{a_k\}, a_k)) \quad \text{by (28)} \\
&= \sigma(p) \quad \text{by } (\neg i)
\end{aligned}$$

Thus we showed that if (i) and (ii) are both false, then $\sigma(p) = score(A \setminus \{a_k\}) \leq \sigma(|A| - 1)$, which contradicts item (13) of definition 4 and ends the proof of proposition 6. \square

Proposition 7. *If R is min-transitive, then $A = C_p(X, R)$ is interior stable in the $\sigma(p)$ -cut.*

Proof. Assume that R is min-transitive and consider an arbitrary $a_k \in A$. We show that $R(a_i, a_k) < \sigma(p)$ for all $a_i \in A \setminus \{a_k\}$. If item (i) of proposition 4 is true, then it is done. Otherwise, there is an $a_j \in \bar{A}$ such that for all $a_i \in A \setminus \{a_k\}$, we have

$$\begin{aligned}
R(a_k, a_j) \geq \sigma(p) &> R(a_i, a_j) \quad (\text{by part (ii) of proposition 6}) \\
&\geq \min (R(a_i, a_k); R(a_k, a_j)) \quad (\text{by min-transitivity}) \\
&= R(a_i, a_k)
\end{aligned}$$

as $\min(x, y) < y \Rightarrow \min(x, y) = x$. Hence, $R(a_i, a_k) < \sigma(p)$ for all $a_i \in A \setminus \{a_k\}$. Since this proposition holds for any $a_k \in A$, the proof is complete. \square

R	a_1	a_2	a_3	a_4	a_5
a_1	1	γ	γ	r_{14}	r_{15}
a_2	$1-\gamma$	1	r_{23}	γ	γ
a_3	$1-\gamma$	$1-r_{23}$	1	r_{34}	r_{35}
a_4	$1-r_{14}$	$1-\gamma$	$1-r_{34}$	1	r_{45}
a_5	$1-r_{15}$	$1-\gamma$	$1-r_{35}$	$1-r_{45}$	1

Figure 3: R -matrix for proposition 8.

The reverse is not true: there is no need that R be min-transitive for making $A = C_p(X, R)$ interior stable in the $\sigma(p)$ -cut (the proof is obvious). Beside, no equivalent result holds for L-transitivity as claimed in the following proposition.

Proposition 8. *There exist MCDM instances such that R is L-transitive and $A = C_p(X, R)$ is not interior stable in the $\sigma(p)$ -cut.*

Proof. We exhibit an instance with a set of five alternatives $X = \{a_1, \dots, a_5\}$ comparing in such a way that :

$$\begin{aligned} R(a_i, a_j) + R(a_j, a_k) &\leq 1 + R(a_i, a_k) \\ R(a_i, a_j) &= 1 - R(a_j, a_i) \end{aligned}$$

The R -matrix is given in figure 3, with $\gamma \in (1/2, 2/3]$, and $r_{14}, r_{15}, r_{23}, r_{34}, r_{35}, r_{45} \in (1 - \gamma, \gamma)$.

Bouyssou showed in [5] that for *any* reflexive valued relation R , there is an Electre III situation such that the construction step of the method leads to an outranking relation identical to R . Hence, the valued relation R of figure 3 can be obtained as the result of an MCDM construction technique. This relation is L-transitive indeed as

$$R(a_i, a_j) + R(a_j, a_k) - R(a_i, a_k) \leq \gamma + \gamma - (1 - \gamma) = 3\gamma - 1 \leq 1$$

using $\gamma \leq 2/3$. The C_p choice rule associated with $p = 2$ leads to select

$$A = C_2(X, R) = \{a_1, a_2\}$$

with $\sigma(2) = \gamma$. Since $R(a_1, a_2) = \sigma(2)$, A is not interior stable in the $\sigma(p)$ -cut. \square

We deduce from proposition 8 that when R is L-transitive and $\sigma(p) > 0.5$, $A = C_p(X, R)$ is generally not interior stable either when interior stability is defined by $stability(A) = \min\{1 - R(a_i, a_j) : a_i, a_j \in A, a_i \neq a_j\}$ as in (26), since $1 - \sigma(p) < \sigma(p)$.

5 Conclusion

In this paper, we have proposed an innovative choice rule which allows the decision-maker, generally confronted with hard budget constraints, to specify an input upper bound p on the number of selected alternatives. The choice rule, named C_p , is shown to be equivalent to solving a sequence of p -center, set covering and p -median location problems, and thus benefits from all the abundant literature on this topic. Axiomatic analysis of the underlying preference relation between subsets and comparison with alternative kernel methods are provided for ensuring confidence in the model. In particular, it is shown that some negative effects induced by the interior stability requirement of the kernel approach disappear when applying the C_p choice rule. Just like kernel methods, unicity of the selected set is not systematic for C_p , although deciding between equally-scored subsets through the p -median formulation reduces the search space. Further work should analyse conditions on which the choice rule C_p is inclusive, in the sense that whenever $\exists A = C_p(X, R)$ such that $a_i \in A$, then $\exists A' = C_{p+1}(X, R)$ such that $a_i \in A'$. Also, the axiomatic study of the properties of the extended preference relation can be deepened for re-use in other choice rules or even ranking rules based on the absorption concept.

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