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Nicolas Cœurdacier
Stéphane Guibaud

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Abstract:

The goal of this paper is to analyze the determination of countries equity portfolios and countries stock returns behavior in the context of imperfectly integrated financial markets. We build a continuous-time equilibrium model of a two-country endowment economy in which the level of financial integration is simply captured by with holding taxes on foreign dividends. Despite the heterogeneity among investors induced by these taxes, we obtain approximate closed-form expressions for asset prices and we characterize equity holdings and national assets returns behavior in equilibrium. The existence of a friction akin to a with holding tax on foreign dividends has two opposite effects on portfolios: the first mechanical effect is to reduce foreign holdings by reducing expected returns on foreign assets; but there is a second effect, which is to reduce endogenously the correlation between national asset returns, thus increasing the willingness to diversify internationally. Quantitatively, we show that the direct effect dwarfs the indirect effect and we find that, for a reasonably high level of substituability between national assets, small frictions on equity markets can generate a large home bias in portfolios. Empirically, our model is consistent with a broad range of findings on international financial integration. Moreover, we provide an explanation for the puzzling positive relationship that has been found in the data between bilateral equity holdings and bilateral stock returns correlations.

Keywords: Asset Pricing with Heterogeneous Investors, Financial Integration, Home Bias in Portfolio, International Stock Returns Correlations, Stochastic Pareto-Negishi Weight

Résumé:

Ce papier analyse les portefeuilles d'actions internationaux et les prix des actifs financiers lorsque les marchés financiers sont imparfaitement intégrés. Dans un modèle d'équilibre général dynamique à deux pays, nous modélisons une intégration financière imparfaite par une taxe au rapatriement des dividendes étrangers. Nous dérivons des formes fermées approximées pour les portefeuilles, les prix des actifs et leurs moments de second ordre (volatilité et corrélation). Nos prédictions théoriques sont en ligne avec un certain nombre de faits empiriques concernant l'intégration financière.

Mots-clés: Valorisation d'actifs avec agents hétérogènes, Biais domestique dans les portefeuilles, Corrélation des rendements boursiers

JEL Classification: F30, G11, G12

A Dynamic Equilibrium Model of Imperfectly Integrated Financial Markets *

Nicolas Coeurdacier[†] **Stéphane Guibaud**[‡]

Abstract

The goal of this paper is to analyze the determination of countries equity portfolios and countries stock returns behavior in the context of imperfectly integrated financial markets. We build a continuous-time equilibrium model of a two-country endowment economy in which the level of financial integration is simply captured by withholding taxes on foreign dividends. Despite the heterogeneity among investors induced by these taxes, we obtain approximate closed-form expressions for asset prices and we characterize equity holdings and national assets returns behavior in equilibrium. The existence of a friction akin to a withholding tax on foreign dividends has two opposite effects on portfolios: the first mechanical effect is to reduce foreign holdings by reducing expected returns on foreign assets; but there is a second effect, which is to reduce endogenously the correlation between national asset returns, thus increasing the willingness to diversify internationally. Quantitatively, we show that the direct effect dwarfs the indirect effect and we find that, for a reasonably high level of substituability between national assets, small frictions on equity markets can generate a large home bias in portfolios. Empirically, our model is consistent with a broad range of findings on international financial integration. Moreover, we provide an explanation for the puzzling positive relationship that has been found in the data between bilateral equity holdings and bilateral stock returns correlations.

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[†]ESSEC Business School, Av. Bernard Hirsh BP105 95021 Cergy-Pontoise, France . E-mail: coeurdacier@essec.fr

[‡]London School of Economics, Department of Accounting and Finance, Houghton Street, London WC2A 2AE, United Kingdom. E-mail: s.guibaud@lse.ac.uk

1 Introduction

In this paper, we analyze the workings of international financial markets in between the polar cases of perfect financial integration and complete segmentation. We consider a two-country endowment economy with one non-storable good, one "Lucas tree" in each country and corresponding claims on national equity. The friction which induces equity markets to be partially segmented takes the form of a proportional cost that shareholders have to pay on the dividends earned abroad. Naturally, the size of the home bias in portfolios depends on the size of the friction on equity markets, but it also depends on the international correlation of returns which makes national risky assets more or less substitutes. At the same time, this correlation is affected by cross-border equity holdings, since portfolio rebalancing effects can generate comovements in asset prices. Our main achievement is to determine endogenously both assets substituability and portfolios composition in equilibrium for various levels of financial integration. We believe our setting is appropriate to make sense of i) the extent of international portfolio diversification, ii) national asset prices joint behavior and iii) how they are affected by the process of financial integration.

Over the last decades most equity markets over the world have been liberalized[1] and cross-border equity holdings have surged (Lane et Milesi-Ferreti [2003]). But a number of frictions remain on international equity markets: transaction costs, currency risk, international capital taxation, differences in accounting standards and in shareholder protection systems, not to mention informational and agency problems, still act as impediments to cross-border investment. In a sense, the mere existence of a home bias in portfolios (initially documented by French and Poterba [1991]) indicates that some frictions are at play[2] . Thus, as a big picture, it is probably fair to describe international equity markets today as neither perfectly integrated nor autarkic. Our goal is to explore this intermediate case.

Though it is appealling for its realism and for the insights it can give on the actual determination of international asset prices and international equity holdings, thinking about imperfectly integrated financial markets is technically challenging. The difficulty stems from the fact that financial segmentation necessarily goes with heterogeneous investors, and this feature can make the pricing of assets

[1] Quinn [1997] provides a direct institutional measure of financial openness, as do Kaminsky and Schmuckler [2003] and Bekaert and Harvey [2000].

[2] If markets were *perfectly* integrated, all investors would hold the "world market portfolio", independently of their country. One should certainly keep in mind though that this proposition fails to be true if national investors face unhedgeable idiosyncratic shocks or in presence of information asymetries. Deviations from purchasing power parity (possibly related to trade costs) constitute another source of departure from the benchmark (Adler and Dumas [1983]).

fairly complicated. We manage to keep the problem tractable by capturing in a simple way the partial segmentation of international financial markets. Our friction essentially acts as a withholding tax on foreign dividends. Despite the relative simplicity of this friction, the asset pricing problem that we face remains non trivial. Indeed, since each investor has a specific "after-tax" investment opportunity set, the equilibrium allocation resulting from trade in assets is not Pareto efficient, risk-sharing is imperfect, and we cannot use the pricing kernel of a single representative investor holding the world market portfolio and consuming the aggregate endowment at each instant to price assets. In order to characterize the equilibrium of our imperfectly integrated financial markets, we need to introduce an extra state variable (a time-varying Pareto-Negishi weight) to keep track of the time-varying distribution of wealth. Working under the assumption of logarithmic utility and lognormal dividend processes[3] , we finally obtain expressions for asset prices as functions of three state variables: the aggregate dividend, the relative size of the two economies and the relative Pareto-Negishi weight which fluctuates endogenously. Then we derive returns joint behavior and equity portfolios. This allows us to analyze how these variables are affected by a variation in the size of the impediments to foreign equity holdings[4] .

The "tax-like" cost that we consider in our model provides a meaningful metric to assess quantitatively the structural level of integration of international financial markets. In a calibration exercise, we find that small frictions akin to withholding taxes of the order of 10 to 15% can generate a level of domestic exposure close to 90%, matching the observed home bias for the US economy. This finding is driven partly by a high level of correlation of economic fundamentals and by a high elasticity of asset demand. The idea that small frictions on cross-border holdings combined with a high level of assets substituability can result in substantial portfolio home bias is reminiscent of Cole and Obstfeld [1991]. But while the correlation among national assets in their two-good setting is driven by terms of trade fluctuations, the substituability between national assets in our model is driven by common shocks affecting national economic fundamentals and by portfolio rebalancing.

The intuition for the portfolio rebalancing mechanism that induces the correlation of two assets returns to be higher than their "fundamental" correlation solely because some investors hold both assets is the

[3] Our analysis could be extended to other cash flow assumptions, such as those in Menzly, Santos and Veronesi [2004].

[4] We assume that fundamentals are not affected by the integration process – as could be the case if access to new risk-sharing opportunities and new sources of finance induced inter-sectoral reallocations (cf. Obstfeld [1994], for instance). Empirically, Imbs [2004] finds a positive impact of financial integration on GDP synchronization.

following. When there is a good shock on domestic dividends, this drives the price of the domestic asset up and increases its share in investors portfolios. When financial markets are integrated, investors increase their demand for the foreign asset in order to keep the composition of their portfolios constant, which drives the price of the foreign asset up. Another way to put it is that as the share of the domestic asset in the world market portfolio increases, the required return on the foreign asset decreases because its diversification properties become more valuable. In financial autarky by contrast, a good shock on an asset drives its price up without affecting the price of the other asset and the correlation of asset returns is equal to the correlation of economic fundamentals. In-between complete segmentation and perfect integration, the lower the frictions between two markets, the higher the comovements of their stock prices, for a given level of fundamental correlation[5] . We shall insist on the fact that this portfolio rebalancing effect, though spectacular for low levels of fundamental correlation and no friction on financial markets, is quantitatively small for a realistic calibration of the model. This result is interesting when one wants to think about the home bias from a general equilibrium perspective. The point is that any cost associated to foreign equity holdings has two opposite effects on portfolios. The first direct effect is to reduce cross-border holdings by reducing expected returns on foreign assets. But there is also this endogenous indirect effect, which is to reduce the substituability between national assets by reducing the correlation of their returns, thus increasing the willingness to diversify internationally. The overall quantitative impact of a friction depends on the relative size of the two effects[6] , and the fact that the indirect effect is of small magnitude plays in favor of the result that small frictions can generate a large home bias.

Our analysis also allows us to derive broader qualitative and quantitative results on the impacts of financial integration, which in the context of our model means a decrease in the withholding tax on foreign dividends. As the friction on international equity markets decreases, asset prices increase, international returns correlation and cross-country equity holdings both also increase (the latter being a first-order effect, while the former is a second-order effect) and asset returns volatility diminishes (also a second-

[5] One might prefer to think in terms of stochastic discount factors (SDF). The two agents have perfectly correlated SDF in the perfectly integrated case, so that the two assets are discounted the same way, which increases their correlation compared to the extreme case of complete segmentation where each asset is priced using the corresponding autarkic SDF. As financial integration increases, the discount factors that are applied to national assets become closer to each other, which increases the correlation of their returns.

[6] Of course, the direction of the overall impact itself is unambiguous.

order effect). The overall impact of financial integration on the cost of funds is not clear-cut, depending on the respective size of the increase in the riskfree rate (due to lower precautionary saving) and of the decrease in risk premium (which shows up in an extra term in a modified version of the CCAPM, where the level of friction is interacted with the relative wealth of countries). Also, as a by-product of our analysis, we derive a gravity equation for international trade in financial assets, giving theoretical foundations to the use of gravity equation regressions in recent empirical papers on cross-border asset holdings (following Portes and Rey [2005]).

Finally, our analysis yields an insight on the correlation puzzle in international equity holdings, by which we refer to the empirical finding of a robust positive relationship between bilateral equity holdings and bilateral stock returns correlations (see Lane and Milesi-Ferreti [2004], Portes and Rey [2005], Chan *et al.* [2005] and Aviat and Coeurdacier [2005]). As the level of financial integration between two countries affects positively both their cross-border holdings and the correlation of their returns, it could be that the correlation puzzle is just driven by the variations in the level of financial integration across pairs of countries. In an empirical companion paper (Coeurdacier and Guibaud [2005]), we show that once this endogeneity issue is taken into account the correlation puzzle indeed disappears.

Related literature

In the context of perfectly integrated financial markets, Dumas, Harvey and Ruiz [2003] and Cochrane, Longstaff and Santa-Clara [2005] analyzed the endogenous determination of asset returns correlation[7] . Our paper completes their work by extending the analysis to partially integrated markets. This allows us to sketch the joint determination of country portfolios and national stock returns comovements.

This paper also contributes to a literature that attempts to modelize imperfectly integrated financial markets. Martin and Rey [2004] build a model featuring a transaction cost on international trade in assets. This generates the home bias in their model – the size of the bias depending on the elasticity of the demand for foreign assets. They relate this elasticity to investors risk aversion (an effect which shows up in our model, for a given risk aversion, through the impact of volatility). But in their static model, they do not explore issues related to asset returns correlations. Bhamra [2002] has a full-fledged dynamic equilibrium model of partially segmented financial markets, but he imposes constraints directly

[7] The implications of portfolio rebalancing for the joint behavior of asset returns and the exchange rate is explored in Hau and Rey [2004].

on the amount of wealth that can be invested abroad. We get the home bias in a more endogenous way by relating it to small frictions characterizing the market environment[8] .

Technically, our paper is close to Basak and Gallmeyer [2003]. They consider a dynamic asset pricing model with asymmetric taxation. Since there is a single risky asset in their model, nothing can be said about portfolio composition or assets returns correlation. But on the methodological side, we follow these authors in the way they deal with investors heterogeneity by introducing a time-varying Pareto-Negishi weight. This stochastic weight is reminiscent of equilibrium with incomplete markets, like in Cuoco and He [1994]. But in our setup like in Basak and Gallmeyer [2003], markets are dynamically complete. The deviation from Pareto optimality only comes from differential taxation.

In the CAPM literature, Black [1974], Errunza and Losq [1985, 1989], Eun and Jarakiramanan [1986], have analyzed the impact of international financial barriers on porfolio holdings and asset pricing in a static mean-variance framework, leaving no room to an endogenous determination of returns correlation through portfolio rebalancing effects. In a spirit close to their work though, we derive a modified version of the CCAPM in our dynamic asset pricing model.

Our theoretical predictions concerning the impact of financial integration on asset prices behavior relate to some empirical contributions on this subject. Henry [2000] and Chari and Henry [2004] document a positive impact of financial integration on asset prices and Bekaert and Harvey [2000] and Walti [2004] find evidence of a positive relationship between the level of financial market integration and stock returns correlations. Our results are consistent with this set of findings.

We shall add that by focusing on small frictions on financial markets, we depart from a literature which tries to relate the observed segmentation of financial markets to the imperfect integration of markets for goods and services. Obstfeld and Rogoff [2000] argued that, in the presence of trade costs, agents would tilt their portfolios towards domestic assets in order to best hedge the fluctuations of their own consumption price index. They consider a static model with trading in a complete set of contingent markets. For specific parameter values implying a risk aversion below one, they show that the presence of trade costs generates a home bias. But for a risk aversion above one, trade costs generate a "reverse" bias. The intuition is that a good supply shock abroad induces an increase in the relative price of

[8] It should be noticed that the friction we consider is by nature different from a transaction cost à la Constantinides [1986]: it does not bear on transactions but instead reduces cash-flows during the holding period.

domestic goods (a scarcity effect), so that in equilibrium the returns on the foreign asset are high when the home real exchange rate appreciates, which makes the foreign asset safer for home investors. This mechanism plays in Uppal [1993], who concludes that trade costs do not in general lead to equity home bias. In another strand of the literature, Baxter, Jermann and King [1998], building on Stockman and Dellas [1989], show that models with non-tradable goods have the counterfactual prediction that agents portfolios should be perfectly diversified internationally in the tradable sector. Serrat [2001] happens to argue that the home bias could be explained by the existence of non-tradables, but Kollman [2005] points out the flaws in his analysis. We therefore believe that though frictions on markets for goods do certainly explain some patterns of trade in assets, most of the action leading to the home bias takes place on financial markets.

Finally, though we capture international financial frictions in a quite abstract way for the sake of tractability, we certainly believe that approaches to the home bias sketching more explicitly the role of informational processing (Van Nieuweburgh and Veldkamp [2005]) and of agency costs due to moral hazard on cross-border investment (Stulz [2005]) are very much required to refine our comprehension of international equity holdings patterns.

The rest of the paper is organized as follows. In section 2, we lay out the model. In section 3, we emphasize the economics of the model and show how to solve it by taking Taylor expansions around the frictionless case. The implications of imperfect market integration for asset prices, asset returns and portfolios are derived in section 4. Section 5 is devoted to discussions and comments and section 6 concludes. The proofs of the main propositions are relegated in a separate appendix in section 7.

2 The model

2.1 Setup

We consider a continuous time economy with an infinite horizon. There are two countries, home (H) and foreign (F), and a single non-storable good. Each country has a representative agent with utility functional

$$U_{it} = E_t \left[\int_t^{+\infty} e^{-\rho(s-t)} \log(c_{is}) ds \right] \tag{1}$$

where c_{is} is the consumption rate in country $i \in \{H, F\}$ and ρ is the common rate of time preference.

Endowments. There is a Lucas tree in each country. We assume the real endowments (dividends) follow geometric brownian motions, meaning that their instantaneous growth rate are independently and identically (normally) distributed:

$$\frac{dD_i(t)}{D_i(t)} = \mu_{D_i} dt + \boldsymbol{\sigma}_{D_i}^T d\mathbf{W}(t) \qquad i \in \{H, F\} \tag{2}$$

All uncertainty is generated by the 2-dimensional standard Wiener process $\mathbf{W}(t)$. We call η the instantaneous correlation of the two dividend growth rates, which we henceforth refer to as the "fundamental" correlation[9] . Throughout, we use bold cases for vectors and matrices and $\mathbf{A}^T$ to denote the transpose of $\mathbf{A}$.

From (2), the world endowment $D \equiv D_H + D_F$ follows a diffusion process whose drift and diffusion coefficients are weighted averages of those of D_H and D_F, with a time-varying weight depending on the size of each economy's endowment relative to the world endowment. We can write

$$\frac{dD(t)}{D(t)} = \underbrace{[\delta(t)\mu_{D_H} + (1-\delta(t))\mu_{D_F}]}_{\equiv \mu_D(t)} dt + \underbrace{[\delta(t)\boldsymbol{\sigma}_{D_H}^T + (1-\delta(t))\boldsymbol{\sigma}_{D_F}^T]}_{\equiv \boldsymbol{\sigma}_D^T(t)} d\mathbf{W}(t) \tag{3}$$

where $\delta(t) \equiv D_H(t)/\left(D_H(t) + D_F(t)\right)$ captures the relative size of the domestic economy. This variable δ will be an important state-variable in the model.

Menu of assets. The menu of financial assets consists of stocks that are claims on the two Lucas trees (each stock being in constant net supply normalized to one) and a frictionless international bank deposit (in zero net supply). We will note S_H and S_F the two stock prices and r the riskfree interest rate. The interest rate process as well as the time-varying drift and diffusion coefficients for asset prices will be determined in equilibrium.

Frictions on equity markets. We assume investors have to pay a proportional cost $\tau \in (0, 1)$ on the dividends they earn abroad[10] . For instance, a domestic agent who holds a unit of foreign stock receives the instantaneous dividend $(1-\tau)D_F$. No cost is paid on the domestic dividends.

One way to think about this τ is that it captures literally differences in fiscal treatment of dividend income for domestic and foreign shareholders. Such kind of fiscal discrimination is a real world feature (cf. Gordon and Hines [2002]): it can be due to withholding taxes on foreign dividends[11] , or to tax credits

[9] With $\sigma_{D_{ij}}$ the loading on the j^{th} component of $d\mathbf{W}$ in country i dividend process, η is given by $\frac{\sigma_{D_{H1}}\sigma_{D_{F1}} + \sigma_{D_{H2}}\sigma_{D_{F2}}}{\sqrt{(\sigma_{D_{H1}}^2 + \sigma_{D_{H2}}^2)(\sigma_{D_{F1}}^2 + \sigma_{D_{F2}}^2)}}$.

[10] Our analysis could easily be extended to the case where these costs differ between countries.

[11] In some cases, it is true that the payment of these taxes to foreign fiscal authorities gives a right to tax credits at home. But for tax-exempt investors like pension funds, withholding taxes constitute a real cost.

that are extended to domestic shareholders to avoid the double taxation of dividends at the corporate and at the personal level. These "dividend imputation schemes" are quite common and they provide a strong incentive to stay invested domestically[12] . But our proportional cost could be given other interpretations: τ can capture for instance higher fees required by mutual funds investing in international stocks, or it could be micro-founded as an agency cost in a model with moral hazard on cross-border investment. In what follows though, we shall often refer to τ as a tax. When $\tau = 0$, financial markets are perfectly integrated.

For tractability, we assume that taxes are redistributed in the economy as lump sum transfers, each agent continuously receiving transfers $e_i(t)dt$. This assumption allows us to write the market clearing condition for goods in a simple way, keeping the aggregate consumption equal to aggregate dividend at each instant. The particular redistribution scheme under consideration does not matter much for our results. One could assume for instance that each agent receives the taxes paid by the other investor[13] . In that case,

$$\begin{aligned} e_H(t) &= \tau\alpha_{FH}(t)D_H(t) \qquad (4) \\ e_F(t) &= \tau\alpha_{HF}(t)D_F(t) \end{aligned}$$

where α_{ij} denotes the quantity of claim on country j output held by the representative investor in country i.

2.2 Individual optimization and definition of equilibrium

Investor i is endowed with an initial share $\alpha_{ij}(0)$ of each stock j. At each point in time, given the price processes S_H, S_F and r, her wealth X_i and a transfer process e_i, she chooses consumption c_i and asset holdings $\boldsymbol{\alpha}_i = (\alpha_{iH}, \alpha_{iF})^T$ in order to maximize her intertemporal utility (1). The induced process for financial wealth X_i is given by

$$dX_i(t) = [r(t)X_i(t) + \boldsymbol{\alpha}_i^T(t)\mathbf{I}_S(t)(\boldsymbol{\mu}_i(t) - \mathbf{r}(t)) + e_i(t) - c_i(t)]dt + \boldsymbol{\alpha}_i^T(t)\mathbf{I}_S(t)\boldsymbol{\sigma}^T(t)d\mathbf{W}(t) \qquad (5)$$

[12] Until a recent reform, this was the case with the so-called "avoir fiscal" in France. In the context of tax-exempt "Equity Saving Plans", this "avoir fiscal" (amounting to 50% of received dividends) came in compensation of no tax! Only domestic stocks were eligible to be included in such equity saving plans, which created a powerful incentive to invest domestically.

[13] We assume all investors act competitively. Therefore, the redistribution of taxes does not give rise to any kind of strategic behavior.

with $\mathbf{I}_S$ a diagonal matrix that has S_H and S_F as coefficients, and $\boldsymbol{\sigma}$ the diffusion matrix of stock prices to be defined shortly.

Competitive equilibrium. Given preferences, initial endowments and a tax reallocation rule, an equilibrium is a set of adapted processes for asset prices, consumption c_i and asset holdings $\boldsymbol{\alpha}_i$ such that $(c_i, \boldsymbol{\alpha}_i)$ is a solution to investor i's optimization problem, and all markets clear at all dates, i.e. for all $t \geq 0$

– market for good

$$c_H(t) + c_F(t) = D_H(t) + D_F(t) = D(t)$$

– equity markets

$$\boldsymbol{\alpha}_H(t) + \boldsymbol{\alpha}_F(t) = \mathbf{1}$$

– bank deposit

$$X_H(t) + X_F(t) = S_H(t) + S_F(t),$$

the constraint that the aggregate position on the bank deposit be zero implying that the aggregate financial wealth be equal to the world market capitalization.

3 Equilibrium

In this section, we will show how to solve for asset prices in our setting. The main difficulty consists in dealing with the heterogeneity among investors caused by the friction on equity markets. We tackle this issue in sections 3.2 and 3.3. Eventually, in section 3.5, we will be able to write approximate formulas for asset prices in our economy as functions of three state variables. To start with, in section 3.1, we will briefly review the equilibrium of the model in the benchmark case of perfectly integrated financial markets. This is useful to understand by contrast what difference introducing a friction makes. Moreover, we will use the closed-form expression obtained for frictionless asset prices later in our approximations.

3.1 Benchmark case without frictions

When $\tau = 0$, investors face the same opportunity set. Since they have identical preferences, they choose the same portfolio composition – everybody holds the world market portfolio. In this case, one can use the pricing kernel of a logarithmic representative agent consuming the world endowment at every instant to price each asset as the expected present value of appropriately discounted future dividends :

$$S_i(t) = E_t \left[\int_t^{+\infty} e^{-\rho(s-t)} \frac{D(t)}{D(s)} D_i(s) ds \right] \qquad i \in \{H, F\} \tag{6}$$

From (6), we can rewrite the price of each stock by using the definition of δ :

$$\begin{aligned} S_H(t) &= D(t) E_t \left[\int_t^{+\infty} e^{-\rho(s-t)} \delta(s) ds \right] = D(t) y(\delta(t)) \\ S_F(t) &= D(t) E_t \left[\int_t^{+\infty} e^{-\rho(s-t)} (1 - \delta(s)) ds \right] = D(t) \left(\frac{1}{\rho} - y(\delta(t)) \right) \end{aligned}$$

with

$$y(\delta) = E \left[\int_t^{+\infty} e^{-\rho(s-t)} \delta(s) ds \middle| \delta(t) = \delta \right]$$

The equation for $S_H(t)$ says that the price of the home country asset at time t is equal to the the world endowment at time t, $D(t)$, times the conditional expectation at time t of the discounted future values of δ. Since this conditional expectation is $\delta(t)$-measurable, it can be written as a function y of $\delta(t)$. The expression for S_F is similar, so that both stock prices are functions of only two state variables: D and δ. The nice thing about function y is that it is known in closed-form. As pointed out by Cochrane, Longstaff and Santa-Clara [2005], y turns out to be the standard hypergeometric function (more details are given in appendix 7.1).

The consumption equilibrium allocation in the benchmark case is straightforward. The relative consumption ratio is constant along time, each agent consuming a constant fraction of the world endowment according to the relative wealth ratio. There is perfect risk sharing. Besides, due to the logarithmic utility assumption, both agents consumption wealth ratios are constant, equal to the rate of time preference: $c_i(t) = \rho X_i(t)\ \forall t,\ \forall i \in \{H, F\}$.

3.2 Heterogeneity and imperfect risk-sharing

Introducing "tax-like" costs on foreign dividends makes a big difference with the benchmark case. The reason is that agents now have different opportunity sets since they do not face the same "after-tax" returns. Therefore, we have a model of asset pricing with heterogenous investors.

Wedge in perceived expected returns. We will now pin down precisely the heterogeneity among investors, taking the returns on asset H as an example. The total instantaneous expected payoff (decomposed into a dividend flow and a capital gain) from the domestic asset for domestic and foreign investors are respectively

$$D_H(t)dt + E_t dS_H(t)$$

and

$$(1-\tau)D_H(t)dt + E_t dS_H(t)$$

The difference in the expected payoff on asset H for home and foreign investors comes from the dividends, which are lower for the foreign investor because of the tax. From this, we can define the total instantaneous expected rates of return on asset H, which we respectively note μ_H for the home investor and μ_H^F for the foreign investor :

$$\mu_H(t)dt = E_t\left[\frac{D_H(t)dt + dS_H(t)}{S_H(t)}\right] \qquad \mu_H^F(t)dt = E_t\left[\frac{(1-\tau)D_H(t)dt + dS_H(t)}{S_H(t)}\right]$$

μ_H is obviously greater than μ_H^F, and the wedge between the two is equal to the tax rate τ times the dividend-price ratio of asset H :

$$\mu_H(t) - \mu_H^F(t) = \tau\frac{D_H(t)}{S_H(t)} \tag{7}$$

Analogously, we get

$$\mu_F(t) - \mu_F^H(t) = \tau\frac{D_F(t)}{S_F(t)} \tag{8}$$

where μ_F^H and μ_F respectively denote the total instantaneous expected rates of return on asset F for home and foreign investors. These expressions for the wedges characterize tightly the heterogeneity induced by taxes.

Investor-specific state prices and static formulation of individual optimization problems. Investors being heterogenous, we have to solve their individual optimization problems separately. Since both investors face dynamically complete markets, we use the solution technique of Cox and Huang [1989]. Therefore, we introduce the investor-specific (after-tax) market prices of risk, which we note $\boldsymbol{\theta}_i$, $i \in \{H, F\}$

$$\boldsymbol{\theta}_H(t) \equiv \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1}\begin{pmatrix}\mu_H(t) - r(t)\\ \mu_F^H(t) - r(t)\end{pmatrix} \qquad \boldsymbol{\theta}_F(t) \equiv \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1}\begin{pmatrix}\mu_H^F(t) - r(t)\\ \mu_F(t) - r(t)\end{pmatrix} \tag{9}$$

with $\boldsymbol{\sigma}(t) \equiv (\boldsymbol{\sigma}_H(t) \quad \boldsymbol{\sigma}_F(t))$ a 2-by-2 matrix composed of the diffusion loadings of stock prices processes. It should be noticed that the difference between the market prices of risk relevant for the two representative agents follows directly from the wedges characterized in equations (7) and (8):

$$\boldsymbol{\theta}_H(t) - \boldsymbol{\theta}_F(t) = \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1} \begin{pmatrix} \tau \frac{D_H(t)}{S_H(t)} \\ \\ -\tau \frac{D_F(t)}{S_F(t)} \end{pmatrix} \tag{10}$$

From these market prices of risk, we define two investor-specific (after-tax) state-price deflators ξ_i

$$\xi_i(t) = \exp(-\int_0^t r(s)ds) \exp(-\int_0^t \boldsymbol{\theta}_i(s)d\mathbf{W}(s) - \frac{1}{2}\int_0^t \boldsymbol{\theta}_i^T(s)\boldsymbol{\theta}_i(s)ds) \qquad i \in \{H, F\}$$

$\xi_i(\omega, t)$ is to be understood as the price (faced by agent i) of an Arrow-Debreu security paying at time t in state ω. Each ξ_i satisfies the following stochastic differential equation:

$$\frac{d\xi_i(t)}{\xi_i(t)} = -r(t)dt - \boldsymbol{\theta}_i^T(t)d\mathbf{W}(t) \tag{11}$$

Finally, using these state prices, each individual dynamic optimization problem can be restated as a static problem, consisting in choosing a vector of contingent consumption rates under a single budget constraint:

$$\max_{\{c_i(t)\}} E\left[\int_0^{+\infty} e^{-\rho t}\log(c_i(t))dt\right]$$
$$s.t.\ E\left[\int_0^{+\infty} \xi_i(t)c_i(t)dt\right] \leq X_i(0) + E\left[\int_0^{+\infty} \xi_i(t)e_i(t)dt\right] \tag{Ψ_i}$$

where the initial wealth $X_i(0)$ depend on the initial distribution of property rights on the equity claims.

Imperfect risk sharing. The first-order conditions can be stated as

$$e^{-\rho t}\frac{1}{c_i(t)} = \Psi_i \xi_i(t) \quad \forall t,\ \forall i \in \{H, F\}$$

with Ψ_i the Lagrange multiplier on the budget constraint for investor i. Then, the FOC imply

$$\frac{c_F(t)}{c_H(t)} = \frac{\Psi_H \xi_H(t)}{\Psi_F \xi_F(t)} \equiv \lambda(t) \quad \forall t \tag{12}$$

This is a key equation. From equations (10) and (11), we know ξ_H and ξ_F follow different dynamics, which implies that the consumption ratio c_F/c_H is not constant. Using the definition of λ and the market clearing condition on the goods market, we can write

$$c_H(t) = \frac{1}{1+\lambda(t)}D(t) \qquad\qquad c_F(t) = \frac{\lambda(t)}{1+\lambda(t)}D(t) \tag{13}$$

The consumption of each agent is a function of the total endowment D and of λ. The sharing rule depends on λ, which plays as a time-varying relative Pareto-Negishi weight for agent F. This is reminiscent of equilibria with incomplete markets *à la* Cuoco and He [1994]. In our case, markets are complete but the deviation from the Pareto efficient allocation is induced by asymmetric taxation.

These results have to be contrasted with the case where $\tau = 0$. In a frictionless environment, the two investors face the same state prices, ξ_H/ξ_F is constant, the relative consumption ratio is constant and each agent consumes a constant fraction of the world endowment. In that case, λ is exactly equal to the constant wealth ratio X_F/X_H. When it comes to asset prices, the impact of the deviation from perfect risk sharing which materializes in the time-varying relative weight λ is to increase the volatility of asset returns by adding a source of volatility in the stochastic discount factors and to decrease the correlation between asset returns. The reason for this latter effect is that in the frictionless case, both assets are priced by a same SDF, whereas when $\tau \neq 0$, the effective SDFs underlying the pricing of each asset (which can be thought of as linear combinations of the intertemporal rate of substitutions of the two investors, with a weight depending on the size of their asset holdings) are no longer the same.

3.3 Additional state variable

When $\tau \neq 0$, the distribution of wealth captured by the stochastic Pareto-Negishi weight λ plays as a state variable in addition to D and δ. From the expressions for individual consumption given in equation (13), we can get the pricing kernels of both agents and use them to price the two assets:

$$\begin{aligned} S_H(t) &= E_t\left[\int_t^{+\infty} e^{-\rho(s-t)} \frac{c_H(t)}{c_H(s)} D_H(s)ds\right] = \frac{D(t)}{1+\lambda(t)} E_t\left[\int_t^{+\infty} e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds\right] \\ S_F(t) &= E_t\left[\int_t^{+\infty} e^{-\rho(s-t)} \frac{c_F(t)}{c_F(s)} D_F(s)ds\right] = \frac{\lambda(t)D(t)}{1+\lambda(t)} E_t\left[\int_t^{+\infty} e^{-\rho(s-t)} \frac{1+\lambda(s)}{\lambda(s)}(1-\delta(s))ds\right] \end{aligned}$$

The conditional expectations that appear in these two equations can be written as two functions h and f of $\delta(t)$ and $\lambda(t)$, so that the stock prices become

$$S_H(t) = D(t)\frac{1}{1+\lambda(t)}h(\delta(t),\lambda(t)) \tag{14}$$

$$S_F(t) = D(t)\frac{\lambda(t)}{1+\lambda(t)}f(\delta(t),\lambda(t)) \tag{15}$$

with

$$\begin{aligned} h(\delta,\lambda) &\equiv E\left[\int_t^{+\infty} e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds \,\middle|\, \delta(t)=\delta, \lambda(t)=\lambda\right] \\ f(\delta,\lambda) &\equiv E\left[\int_t^{+\infty} e^{-\rho(s-t)}\frac{1+\lambda(s)}{\lambda(s)}(1-\delta(s))ds \,\middle|\, \delta(t)=\delta, \lambda(t)=\lambda\right] \end{aligned}$$

There is nothing mysterious about the fact that the stock prices at time t can be written as functions of $D(t)$, $\delta(t)$ and $\lambda(t)$: this is enough information to form expectations on the future dividends of both assets and on the pricing kernels of both agents. Nonetheless, we shall comment on the noticeable fact that, though they do not share the same pricing kernels (because risk sharing is imperfect), the two investors agree on asset prices. What makes it possible is the fact that they do not face the same assets! Indeed, the dividend flows *net of taxes* are different for the two investors. Another way to put it is that investors have different perceptions both of dividends and risk : for a given investor, the bad characteristic of an investment abroad in terms of expected returns is exactly compensated by the good diversification property of such an investment.

3.4 Technical step towards the solution

The next step towards the complete characterization of equilibrium is to be more explicit about functions h and f. These conditional expectation functions involve future values of δ and λ. We therefore need to look at the dynamics of δ and λ. The process for δ is given by the fundamentals. Using the dynamics of D_H and D_F and applying Ito's lemma, one can write

$$\frac{d\delta(t)}{\delta(t)} = \mu_\delta(t)dt + \boldsymbol{\sigma}_\delta^T(t)d\mathbf{W}(t)$$

with

$$\begin{aligned}
\mu_\delta(t) &\equiv (1-\delta(t))\left[\mu_{D_H} - \mu_{D_F} - \delta(t)\boldsymbol{\sigma}_{D_H}^T\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}^T\boldsymbol{\sigma}_{D_F} + (2\delta(t)-1)\boldsymbol{\sigma}_{D_H}^T\boldsymbol{\sigma}_{D_F}\right] \\
\boldsymbol{\sigma}_\delta(t) &\equiv (1-\delta(t))(\boldsymbol{\sigma}_{D_H} - \boldsymbol{\sigma}_{D_F})
\end{aligned}$$

The dynamics of λ is endogenous. From the definition of λ given in (12) and from the stochastic differential equations for the ξ_is given in (11)

$$\frac{d\lambda(t)}{\lambda(t)} = (\boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t))^{\mathbf{T}}\boldsymbol{\theta}_F(t)dt + (\boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t))^{\mathbf{T}}d\mathbf{W}(t)$$

The drift and diffusion coefficients driving the dynamics of λ only depend on the market prices of risk. Using the market clearing condition for goods, one can derive an equilibrium restriction on the investor-specific market prices of risk, which is stated in the following lemma.

Lemma 1 *The after-tax market prices of risk, as perceived by home and foreign investors, are* *respectively given by*

$$
\begin{aligned}
\boldsymbol{\theta}_H(t) &= \boldsymbol{\sigma}_D(t) + \tau \frac{\lambda(t)}{1+\lambda(t)} \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1} \begin{pmatrix} \frac{D_H(t)}{S_H(t)} \\ -\frac{D_F(t)}{S_F(t)} \end{pmatrix} \\
\boldsymbol{\theta}_F(t) &= \boldsymbol{\sigma}_D(t) + \tau \frac{1}{1+\lambda(t)} \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1} \begin{pmatrix} -\frac{D_H(t)}{S_H(t)} \\ \frac{D_F(t)}{S_F(t)} \end{pmatrix}
\end{aligned}
$$

In these expressions, the first term corresponds to the market prices of risk in the frictionless world. When $\tau = 0$, investors face the same market prices of risk, which are equal to $\boldsymbol{\sigma}_D$, the vector of diffusion loadings in the process for the growth rate of the world endowment. The second term captures the impact of taxes, interacted with the dividend price ratios. Using these expressions, the drift and diffusion coefficients of λ in $d\lambda(t)/\lambda(t) = \mu_\lambda(t)dt + \boldsymbol{\sigma}_\lambda^T(t)d\mathbf{W}(t)$ are

$$
\mu_\lambda(t) = \tau[-\frac{D_H(t)}{S_H(t)}\ \frac{D_F(t)}{S_F(t)}]\boldsymbol{\sigma}^{-1}(t)\boldsymbol{\sigma}_D(t) + \tau^2 \frac{1}{1+\lambda(t)}[-\frac{D_H(t)}{S_H(t)}\ \frac{D_F(t)}{S_F(t)}]\left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\boldsymbol{\sigma}(t)\right)^{-1} \begin{pmatrix} -\frac{D_H(t)}{S_H(t)} \\ \frac{D_F(t)}{S_F(t)} \end{pmatrix} \quad (16)
$$

$$
\boldsymbol{\sigma}_\lambda(t) = \tau \left(\boldsymbol{\sigma}^{\mathbf{T}}(t)\right)^{-1} \begin{pmatrix} -\frac{D_H(t)}{S_H(t)} \\ \frac{D_F(t)}{S_F(t)} \end{pmatrix} \quad (17)
$$

3.5 Approximation strategy

In our economy, asset prices depend on the weighting process λ and the process followed by the distribution of wealth itself depends on asset prices. This makes the problem we face highly complex. Our trick is to consider μ_λ and $\boldsymbol{\sigma}_\lambda$ as functions of τ, which they are both directly and through the impact of τ on price-dividend ratios and on $\boldsymbol{\sigma}$. We can therefore write $\mu_\lambda(D,\delta,\lambda;\tau)$ and $\boldsymbol{\sigma}_\lambda(D,\delta,\lambda;\tau)$. Then, we can take advantage of the fact that in the benchmark frictionless case, λ is constant, so that: $\mu_\lambda(D,\delta,\lambda;0) = 0$ and $\boldsymbol{\sigma}_\lambda(D,\delta,\lambda;0) = \mathbf{0}$, $\forall (D,\delta,\lambda)$. For τ close to zero, this allows us to take the following Taylor expansions:

$$
\begin{aligned}
\mu_\lambda(D,\delta,\lambda;\tau) &= \sum_{k=1}^{n} \tau^k \frac{1}{k!} \frac{\partial^k \mu_\lambda(D,\delta,\lambda;0)}{\partial \tau^k} + o(\tau^n) \\
\boldsymbol{\sigma}_\lambda(D,\delta,\lambda;\tau) &= \sum_{k=1}^{n} \tau^k \frac{1}{k!} \frac{\partial^k \boldsymbol{\sigma}_\lambda(D,\delta,\lambda;0)}{\partial \tau^k} + o(\tau^n) \qquad \forall (D,\delta,\lambda)
\end{aligned}
$$

In particular, the first order Taylor expansions of μ_λ and $\boldsymbol{\sigma}_\lambda$ are given by

$$\begin{aligned}
\mu_\lambda(D,\delta,\lambda;\tau) &= \tau\left[-\frac{D_H(D,\delta)}{S_{H0}(D,\delta)}\ \frac{D_F(D,\delta)}{S_{F0}(D,\delta)}\right]\boldsymbol{\sigma}_0^{-1}(\delta)\boldsymbol{\sigma}_D(\delta)+o(\tau)\\
\boldsymbol{\sigma}_\lambda(D,\delta,\lambda;\tau) &= \tau{\boldsymbol{\sigma}_0}^{-1}(\delta)\begin{pmatrix} -\frac{D_H(D,\delta)}{S_{H0}(D,\delta)} \\ \frac{D_F(D,\delta)}{S_{F0}(D,\delta)}\end{pmatrix}+o(\tau)
\end{aligned}$$

where subscripts 0 refer to values prevailing when $\tau = 0$, computed from the hypergeometric function given by Cochrane, Longstaff and Santa-Clara [2005]. In the appendix, we show how one can use these approximate expressions for μ_λ and $\boldsymbol{\sigma}_\lambda$ to derive Taylor approximations for functions h and f which were introduced in section 3.3.

4 Results

In this section, we give a full description of international financial markets equilibrium in the neighborhood of the frictionless case. Section 4.1 gives first and second order approximations for asset prices. Section 4.2 explores asset returns volatility and cross-country returns correlations. Section 4.3 gives expressions for risk premia and the riskfree rate. Finally, we display results on the composition of portfolios in section 4.4. All symbols with subscript 0 will denote values computed in the frictionless case. For notational convenience, we will refer to the function y that was introduced in section 3.1 as y_H, and we define y_F such that $y_F(\delta) = \frac{1}{\rho} - y_H(\delta)\ \forall\delta$, so that

$$\begin{aligned}
S_{H0}(D,\delta) &= Dy_H(\delta)\\
S_{F0}(D,\delta) &= Dy_F(\delta)
\end{aligned}$$

4.1 Asset prices

Proposition 1 *To a first order, S_H and S_F are given by*

$$\begin{aligned}
S_H(D,\delta,\lambda;\tau) &= \left[1-\tau\frac{\lambda}{1+\lambda}\right]S_{H0}(D,\delta)+o(\tau)\\
S_F(D,\delta,\lambda;\tau) &= \left[1-\tau\frac{1}{1+\lambda}\right]S_{F0}(D,\delta)+o(\tau)
\end{aligned}$$

The first-order effect of imperfect market integration is to reduce equilibrium asset prices: frictions on financial markets translate into lower prices by reducing expected income streams on domestic shares received by foreigners. Note that the decrease in domestic asset prices is higher when λ is higher. This makes sense since λ is a proxy for the relative wealth of the foreign investors: as λ increases, the relative influence of foreign investors in the pricing of assets becomes higher, which has a negative impact on the domestic asset price, since foreigners are willing to pay a lower price because of the price they pay on dividends.

Proposition 2 *To a second order, with* $\mathbf{\Omega}_0 \equiv (\sigma_0^T)^{-1} \begin{pmatrix} -\frac{D_H}{S_{H0}} \\ \\ \frac{D_F}{S_{F0}} \end{pmatrix}$, S_H *and* S_F *are given by*

$$
\begin{aligned}
S_H(D,\delta,\lambda;\tau) &= \left[1-\tau\frac{\lambda}{1+\lambda}\right] S_{H0}(D,\delta) + \tau^2 \frac{\lambda}{(1+\lambda)^2} D\left[y_H(\delta)+h_2(\delta)\right] + o(\tau^2) \\
S_F(D,\delta,\lambda;\tau) &= \left[1-\tau\frac{1}{1+\lambda}\right] S_{F0}(D,\delta) + \tau^2 \frac{\lambda}{(1+\lambda)^2} D[y_F(\delta)+f_2(\delta)] + o(\tau^2)
\end{aligned}
$$

where h_2 *and* f_2 *are solutions of the following ODE*[14]

$$
\begin{aligned}
\rho h_2(\delta) - \delta\mu_\delta(\delta)h_2'(\delta) - \frac{1}{2}\delta^2 \boldsymbol{\sigma}_\delta^T(\delta)\boldsymbol{\sigma}_\delta(\delta)h_2''(\delta) &= \mathbf{\Omega}_0^T(\delta)\mathbf{\Omega}_0(\delta)y_H(\delta) \\
\rho f_2(\delta) - \delta\mu_\delta(\delta)f_2'(\delta) - \frac{1}{2}\delta^2 \boldsymbol{\sigma}_\delta^T(\delta)\boldsymbol{\sigma}_\delta(\delta)f_2''(\delta) &= \mathbf{\Omega}_0^T(\delta)\mathbf{\Omega}_0(\delta)y_F(\delta)
\end{aligned}
$$

with boundary conditions

$$
\begin{cases}
h_2(0) = 0 \\
h_2(1) = \lim_{\delta\to 1} \frac{1}{\rho^2}\mathbf{\Omega}_0^T(\delta)\mathbf{\Omega}_0(\delta)
\end{cases}
$$

$$
\begin{cases}
f_2(0) = \lim_{\delta\to 0} \frac{1}{\rho^2}\mathbf{\Omega}_0^T(\delta)\mathbf{\Omega}_0(\delta) \\
f_2(1) = 0
\end{cases}
$$

Making sense of the second order price effects of integration requires to understand its impacts on the riskless rate and on the variance-covariance matrix of returns. We will see below that to a second order, the riskless rate and the return correlation decrease with τ, both effects having a positive impact on asset prices through the risk-adjusted discount factor.

4.2 Volatility and correlation of asset returns

Applying Ito's lemma to asset prices second-order approximations, we can derive second-order expansions for asset prices diffusion loadings $\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$.

Proposition 3

$$
\boldsymbol{\sigma}_H(\delta,\lambda) = \boldsymbol{\sigma}_{H0}(\delta) + \tau^2\frac{\lambda}{(1+\lambda)^2}\left\{-\mathbf{\Omega}_0(\delta) + \left[\frac{h_2'(\delta)}{y_H(\delta)} - \lambda\frac{y_H'(\delta)}{y_H(\delta)} - \frac{h_2(\delta)y_H'(\delta)}{y_H^2(\delta)}\right]\delta\boldsymbol{\sigma}_\delta(\delta)\right\} + o(\tau^2) \quad (18)
$$

$$
\boldsymbol{\sigma}_F(\delta,\lambda) = \boldsymbol{\sigma}_{F0}(\delta) + \tau^2\frac{\lambda}{(1+\lambda)^2}\left\{\mathbf{\Omega}_0(\delta) + \left[\frac{f_2'(\delta)}{y_F(\delta)} - \frac{1}{\lambda}\frac{y_F'(\delta)}{y_F(\delta)} - \frac{f_2(\delta)y_F'(\delta)}{y_F^2(\delta)}\right]\delta\boldsymbol{\sigma}_\delta(\delta)\right\} + o(\tau^2) \quad (19)
$$

[14] We solve these boundary value problems numerically, using Chebychev polynomial approximations.

From (18) and (19), we obtain assets returns volatility and correlations. A conspicuous feature of the expressions for $\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$ is that withholding taxes will have no first order impact on asset returns second-order moments.

Parameter values. In order to illustrate our results, we will assume symmetric fundamentals, taking the following parameters: $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.145$ and $\sigma_{D_H,2} = \sigma_{D_F,1} = 0.039$[15] . This calibration is meant to match US stock market data: on an annual basis, the S&P500 volatility after World War II is 0.15 and the dividend yield (equal to ρ is the symmetric case of perfect integration) is around 0.04. Our fundamental correlation η is equal to 0.5, which is consistent with the empirical stock returns correlation of 0.58 between the US and a non-US synthetic world index over the period 1980-2000[16] . τ is a free parameter, the impact of which we are interested in.

Impacts of financial integration on returns volatility and cross-correlation. As illustrated respectively in figure 1 and figure 2, we find that returns volatility decreases with financial integration, while the instantaneous correlation between returns increases. These effects are found to be small though. In order to understand the impact of the degree of market integration on the equilibrium correlation of returns, one can first consider the case of perfect integration as opposed to the case of full segmentation. When markets are fully segmented, a good shock on the dividends of an asset in one country has no impact on the price of assets in another country. But it is different when investors can hold assets everywhere without any obstacle. The reason is that following the rise in the domestic price due to the good domestic shock, the share of asset H in the "world market portfolio" increases, making country F asset more appealling because the diversification opportunities it offers are suddenly more cherished. The required excess return on asset F decreases and its price increases to restore equilibrium on the asset market[17] . When $\tau > 0$, the same sort of mechanism is at work but dampened due to investors heterogeneity. Indeed, a good D_H affects each investor differently since they share risk imperfectly: the home investor is the most affected since his portfolio is biased towards home assets – and he is reluctant to rebalance his portfolio towards foreign assets. This attenuates the increase in S_F compared to the case

[15] This corresponds to $\sigma_D = 0.15$ and to a fundamental correlation $\eta = 0.5$. This calibration allow us to match the moments of stock returns in the US at the expense of the moments observed for the fundamentals. It is well known that the volatility of stock markets is well above the volatility of GDP.

[16] The empirical stock returns correlation is calculated using monthly returns of both indices in US$.

[17] And the increase of S_H is also lower than under full segmentation. This reasoning holds when the market shares of H is not "too small" to start with.

of perfect risk-sharing.

Our result that when cross-border impediments to foreign equity holdings are relaxed, stock returns correlations between countries get higher, is consistent with the empirical findings of Bekaert and Harvey [2000] who showed that following episodes of equity market liberalization in emerging markets, the stock indices of these countries became more correlated with a world aggregate index.

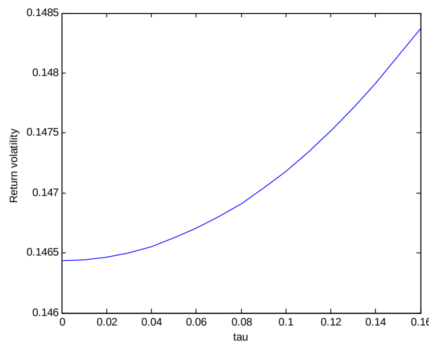

Figure 1: Stock returns volatility in the symmetric case as a function of τ (calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.145$, $\sigma_{D_H,2} = \sigma_{D_F,1} = 0.039$).

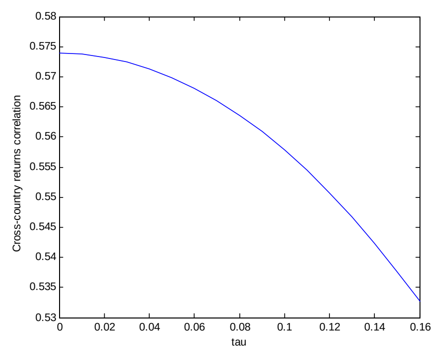

Figure 2: Stock returns correlation in the symmetric case as a function of τ (same calibration).

Sensitivity analysis. Table 1 shows the magnitude of assets returns correlation η_S conditional on three structural parameters: the degree of market integration (inversely related to τ), the level of fundamental correlation η and the rate of time preference ρ. For given η and ρ, the correlation of asset returns is always monotonously decreasing in τ. It should be noticed that for a higher level of fundamental correlation, the equilibrium correlation of asset returns η_S is closer to its fundamental value η, meaning that endogenous comovements of asset prices are less important: this is because when the fundamental correlation is higher, high dividends in one country are often accompanied by high dividends in the other country, reducing the incentives to rebalance the portfolio. Finally, we find that the impact of financial integration on the equilibrium returns correlation is much higher when the rate of time preference is low. The intuition for this effect is not obvious, though it is clear that in the limit case of complete myopia the optimal portfolio rebalancing behaviour that induces endogenous comovements of asset prices is killed.

		η_S		
		$\rho = 0.1$	$\rho = 0.05$	$\rho = 0.01$
$\eta = 0$	$\tau = 0$	0.086	0.147	0.394
	$\tau = 5\%$	0.080	0.143	0.393
	$\tau = 10\%$	0.060	0.130	0.391
$\eta = 0.25$	$\tau = 0$	0.313	0.358	0.544
	$\tau = 5\%$	0.305	0.353	0.542
	$\tau = 10\%$	0.282	0.340	0.538
$\eta = 0.5$	$\tau = 0$	0.535	0.562	0.679
	$\tau = 5\%$	0.526	0.557	0.677
	$\tau = 10\%$	0.501	0.543	0.672

Table 1: Stock returns correlation η_S as a function of the fundamental correlation η and obstacles to international investment τ (for a given volatility of fundamentals $\sigma_D = 0.15$)

4.3 Risk premia and riskfree rate

Proposition 4 (Risk premia) *Required "before-tax" excess returns for assets H and F are*

$$\begin{aligned} \mu_H - r &= \boldsymbol{\sigma}_{H0}.\boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \frac{D_H}{S_{H0}} + o(\tau) \\ \mu_F - r &= \boldsymbol{\sigma}_{F0}.\boldsymbol{\sigma}_D + \frac{\tau}{1+\lambda} \frac{D_F}{S_{F0}} + o(\tau) \end{aligned}$$

Proposition 4 is a modified version of the continuous-time consumption-based CAPM. With logarithmic utility, in the benchmark case without taxes, we would get the vector of expected excess returns for the two assets given by $\boldsymbol{\sigma}^{\mathbf{T}}\boldsymbol{\sigma}_D$: the risk premia are equal to the covariance of asset returns with aggregate consumption growth[18] . The first-order impact of τ is to drive the risk premia above their benchmark level. This is because both assets are partly held by taxed investors who require a higher pre-tax excess

[18] Our model obviously does a poor job at matching the observed equity premia. Another feature of our model is that there is predictability in returns because of time variability in the covariance term (cf. Cochrane et al. [2005]).

return to compensate for taxation[19] . The prediction that an increase in financial markets integration (a decrease in τ) reduces the required excess return is consistent with the empirical evidence (Bekaert and Harvey [2000], Henry [2000], Chari and Henry [2004]). The term in τ that appears in proposition 4 is interacted with the dividend-price ratio and the relative wealth of countries. This suggests a potential way of testing our international version of the CCAPM, by testing for the significance of this term in the pricing equation.

When we go to the second order, we have two additionnal effects on the risk premia, coming through asset prices levels and asset returns volatilities. First, since dividend-price ratios are higher under imperfect integration, this amplifies the effect of the friction on the risk premium by increasing the return on home assets required by the foreigners. This effect can also be related to the fact that a decrease in τ fosters risk-sharing, which causes a decrease in the required excess return. The decrease in the correlation of stock returns with aggregate output plays in the opposite direction, driving the risk premium down.

Proposition 5 (Riskfree rate) *The second-order approximation of the riskless rate is given by*

$$r = \rho + \mu_D - \boldsymbol{\sigma}_D^T \boldsymbol{\sigma}_D - \tau^2 \frac{\lambda}{(1+\lambda)^2} [-\frac{D_H}{S_{H0}} \ \frac{D_F}{S_{F0}}] \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0\right)^{-1} \begin{pmatrix} -\frac{D_H}{S_{H0}} \\ \\ \frac{D_F}{S_{F0}} \end{pmatrix} + o(\tau^2)$$

In the fully integrated case ($\tau = 0$), we get the standard interest rate formula: with logarithmic utility, when perfect risk-sharing prevails, the interest rate is determined by the rate of time preference and the mean and variance of aggregate consumption growth. When markets are imperfectly integrated, the interest rate is below its level of perfect integration. This can be seen from the fact that $(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0)^{-1}$ is definite positive and this is to be interpreted as an effect of higher savings for precautionary motive, due to the fact that because of taxes investors hold less diversified portfolios and have greater exposure to their domestic risk.

Total cost of capital. In our model a decrease in τ causes both an increase in the riskless rate and a decrease in the equilibrium excess returns. Therefore, the overall impact of financial integration on the cost of capital is not clear-cut, depending on the relative strenght of these two effects. Our numerical computations suggest there could be non monotonous effects.

[19] Looking at $\mu_H^F - r$ and $\mu_F^H - r$, it is straightforward to see that overall the presence of taxes lowers the "after-tax" risk premium for investors abroad.

4.4 Portfolios

In what follows, we will focus on the extent of international portfolio diversification in our imperfectly integrated financial markets. For that matter, we shall introduce $\pi_{ij} \equiv \frac{\alpha_{ij} S_j}{X_i}$, the share of equity j in the financial wealth of investor i.

Proposition 6 *To a first order, portfolio shares are given by*

$$\begin{bmatrix} \pi_{HH} \\ \pi_{HF} \end{bmatrix} = \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 \right)^{-1} \begin{bmatrix} \frac{D_H}{S_{H0}} \\ -\frac{D_F}{S_{F0}} \end{bmatrix} + \boldsymbol{\epsilon}_H + \mathbf{o}(\tau)$$

$$\begin{bmatrix} \pi_{FH} \\ \pi_{FF} \end{bmatrix} = \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D + \tau \frac{1}{1+\lambda} \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 \right)^{-1} \begin{bmatrix} -\frac{D_H}{S_{H0}} \\ \frac{D_F}{S_{F0}} \end{bmatrix} + \boldsymbol{\epsilon}_F + \mathbf{o}(\tau)$$

Portfolios can be decomposed into three components. In each expression, the first two terms correspond to $\boldsymbol{\sigma}^{-1} \boldsymbol{\theta}_i = \left(\boldsymbol{\sigma}^{\mathbf{T}} \boldsymbol{\sigma} \right)^{-1} [\boldsymbol{\mu}_i - \mathbf{r}]$, which is the standard portfolio composition of a logarithmic investor in complete markets with purely financial wealth. $\boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D$ is the world market portfolio, which is held by both investors when $\tau = 0$. For an investor in country H, τ reduces the demand for foreign stocks by reducing after-tax expected returns on these stocks. Symmetrically, due to market clearing, τ increases the domestic demand for domestic shares to compensate for the lower demand by foreign investors[20] . The third term $\boldsymbol{\epsilon}_i$ comes from the redistribution of taxes: for instance, if e_H is positively correlated with D_H, this will create a demand for foreign shares in order to hedge this additionnal income risk. However, this term is found to be quantitatively small when the two countries are not too asymmetric[21] and it depends very much on the assumed redistribution scheme. Therefore we will neglect it henceforth – but none of the following results rely on this approximation.

Introducing the following notations for the elements of the instantaneous variance-covariance matrix for stock prices,

$$\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 = \begin{pmatrix} \sigma_{S_H}^2 & \eta_S \sigma_{S_H} \sigma_{S_F} \\ \eta_S \sigma_{S_H} \sigma_{S_F} & \sigma_{S_F}^2 \end{pmatrix}$$

[20] This general equilibrium effect is relevant empirically. Chan et al. [2005] find that countries imposing high withholding taxes to foreign shareholders exhibit a higher home bias.

[21] In the appendix, we show that to a first order $\boldsymbol{\epsilon}_H = \tau\lambda(y_H y_F/(y_H + y_F)^2)[-1 \quad 1]^T$ when $e_i = \tau\alpha_{ji} D_i$.

one can rewrite π_{HH} and π_{HF} as follows

$$\pi_{HH} \simeq \frac{S_{H0}}{S_{H0}+S_{F0}} + \tau \frac{\eta_S}{1-\eta_S^2} \frac{\lambda}{1+\lambda} \frac{1}{\sigma_{S_H}\sigma_{S_F}} \frac{D_F}{S_{F0}} + \frac{\tau}{1-\eta_S^2} \frac{\lambda}{1+\lambda} \frac{1}{\sigma_{S_H}^2} \frac{D_H}{S_{H0}}$$

$$\pi_{HF} \simeq \frac{S_{F0}}{S_{H0}+S_{F0}} - \frac{\tau}{1-\eta_S^2} \frac{\lambda}{1+\lambda} \frac{1}{\sigma_{S_F}^2} \frac{D_F}{S_{F0}} - \tau \frac{\eta_S}{1-\eta_S^2} \frac{\lambda}{1+\lambda} \frac{1}{\sigma_{S_H}\sigma_{S_F}} \frac{D_H}{S_{H0}}$$

This expression makes explicit the composition of the world market portfolio, and it clearly shows the impact of the friction on portfolios going through expected returns, both directly and indirectly *via* equity markets clearing. The size of the bias in portfolios is proportional to $1/(1-\eta_S^2)$, where η_S denotes the correlation between assets: when assets are close substitutes, the effect of the friction on equity holdings is amplified.

Comparative statics in a simple symmetric case. In the symmetric case where $\mu_{D_H} = \mu_{D_F}$, $\sigma_{S_H} = \sigma_{S_F} = \sigma_S$, and $\delta = \frac{1}{2}$, one can easily prove that $D_H/S_{H0} = D_F/S_{F0} = \rho$, so that

$$\pi_{HH} = \frac{1}{2} + \tau \frac{\lambda}{1+\lambda} \frac{\rho}{\sigma_S^2(1-\eta_S)} \qquad \pi_{HF} = \frac{1}{2} - \tau \frac{\lambda}{1+\lambda} \frac{\rho}{\sigma_S^2(1-\eta_S)}$$

From these formulas for portfolio weights, one gets

$$\frac{\partial \pi_{HF}}{\partial \tau} = -\frac{\lambda}{1+\lambda} \frac{\rho}{\sigma_S^2(1-\eta_S)} < 0$$

$$\frac{\partial \pi_{HF}}{\partial \eta_S} = -\tau \frac{\lambda}{1+\lambda} \frac{\rho}{\sigma_S^2(1-\eta_S)^2} < 0 \quad (\text{for } \tau > 0)$$

and $\left|\frac{\partial \pi_{HF}}{\partial \tau}\right|$ is increasing in η_S. These expressions capture the impact of frictions, assets substituability and the interaction of the two on the extent of portfolio diversification. Besides, when investments are riskier (higher σ_S), holdings of foreign assets increase as the motive for risk-sharing increases:

$$\frac{\partial \pi_{HF}}{\partial \sigma_S^2} = \tau \frac{\lambda}{1+\lambda} \frac{\rho}{\sigma_S^4(1-\eta_S)} > 0 \quad (\text{for } \tau > 0)$$

Finally

$$\frac{\partial \pi_{HH}}{\partial \lambda} = \tau \frac{1}{(1+\lambda)^2} \frac{\rho}{\sigma^2(1-\eta_S)^2} > 0 \quad (\text{for } \tau > 0)$$

A high λ means the relative wealth of foreign investors is high, which strenghtens their influence in the pricing of assets and increases the negative impact of the friction on the price of the domestic asset. As a consequence, the larger λ, the lower the price of the domestic asset and the higher the incentive for domestic investors to stay invested domestically[22] .

[22] This prediction of our model that the home bias in portfolios should be larger in countries whose relative wealth is smaller is consistent with scarce evidence in Chan *et al.* [2005]. The lowest three values taken by their measure of home bias are for US, UK and Japan, and the highest four are for New Zealand, Norway, Portugal and Greece.

Matching the home bias. For symmetric fundamentals, $\delta = 0.5$ and $\lambda = 1$, figure 3 illustrates the share of wealth invested abroad as a function of τ and as a function of the fundamental correlation η, taking into account the endogeneity of stock returns first and second moments. For $\tau = 13\%$ and $\eta = 0.65$, we get $\pi_{HF} = 11\%$: a reasonable level of friction on cross-border equity holdings, coupled with a high level of assets substituability, can generate a domestic exposure of 89%.

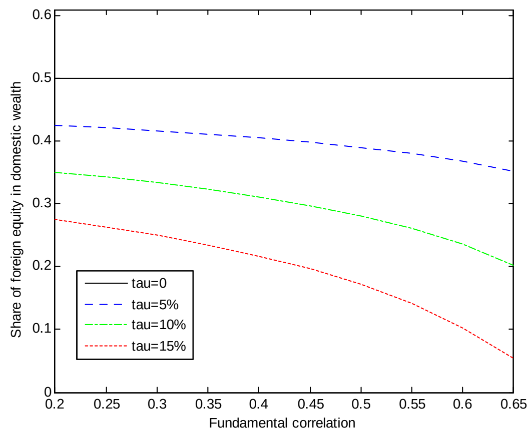

Figure 3: Share of domestic wealth invested abroad as a function of fundamental correlation, for various τ (Calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.15$, $\delta = 0.5$, $\lambda = 1$).

A gravity equation for bilateral equity holdings. Our model can also be used to give theoretical foundations to the use of gravity equations in empirical work on bilateral equity holdings. Indeed, when we turn from portfolio shares to the value of equity holdings, we have :

$$\log(\alpha_{HF} S_F) = \log X_H + \log\left(\rho y_F(\delta)\right) - \tau \frac{1}{1-\eta_S^2} \frac{\lambda}{1+\lambda} \frac{S_H + S_F}{\sigma_F S_F} \left(\frac{1}{\sigma_F} \frac{D_F}{S_F} + \eta_S \frac{1}{\sigma_H} \frac{D_H}{S_H} \right)$$

where $\log X_H$ and $\log(\rho y_F(\delta))$ are the mass terms in the gravity equation[23] . As shown by Portes and Rey [2005], gravity equations perform well in describing international asset allocations. In their work, they use the market capitalizations of origin and destination countries as proxies for the mass terms of the equation. Our model clarifies which variables should be used: for the origin country, one should use the aggregate wealth (X_H) of the country and market capitalization might be an imperfect proxy of it, whereas for the destination country, the market capitalization is certainly more appropriate as a proxy for the present value of current and future foreign dividend streams ($\rho y_F(\delta)$). Moreover, Portes and Rey [2005] propose to interact variables capturing financial frictions between countries with the

[23] In this expression, $y_F(\delta) = E_t \left[\int_t^\infty e^{-\rho(s-t)} \left(1 - \delta(s)\right) ds \right]$ is the present value of current and future contributions of country F in world production.

degree of substituability between assets (measured here by $1/\left(1-\eta_S^2\right)$) : our model provides theoretical foundations for this procedure.

5 Comments

5.1 Beyond logarithmic utility

It could be argued that by assuming logarithmic utility we tackle the case most favorable to getting home bias: indeed a low level of risk aversion implies a high elasticity of asset demand to expected returns. But as is well known, assuming power utility with relative risk aversion higher than one would have two effects. For given η_S, a higher risk aversion implies more willingness to diversify, thus reducing home bias. But at the same time, decreasing the elasticity of intertemporal substitution would increase η_S for given η, by increasing the volatility of the riskfree rate, thus creating more common discount factor shocks on both assets (Dumas, Harvey and Ruiz [2003] point to the elasticity of intertemporal substitution as the key preference parameter driving stock return correlations). The increase in returns correlation would dampen the direct effect of higher risk aversion on the extent of portfolio diversification. The two effects could be disentangled by introducing Epstein-Zin preferences.

5.2 Imperfect substituability between home and foreign goods

International asset pricing models typically restrict the commodity market to a single tradable good, and our model is no exception. In other words, it is assumed that home and foreign goods are perfect substitutes. Relaxing this assumption would not change the overall message of this paper, but it would lead to a new component driving asset prices correlations: a "terms of trade effect" (this effect appears in Pavlova and Rigobon [2004]).

Indeed, assuming perfect goods substituability and no frictions on the international goods markets implies that the terms of trade and the real exchange rate must be constant and equal to one. But as soon as goods produced at home and abroad are imperfect substitutes, the relative price of domestic and foreign goods is affected by the relative scarcity of each type of goods: the relative price of a good is negatively related to its abundance. This "terms of trade effect" would play in case of endowment shocks, a good dividend shock being accompanied by a counteracting relative price change, which would make asset prices evolutions more connected.

The strength of this effect decreases with goods substituability. For an elasticity of substitution below one, the effect is so large that a good shock in the home country reduces domestic asset prices and increases foreign asset prices, leading actually to a divergence in returns! In the special case of an elasticity of substitution equal to one (Cobb-Douglas preferences), the "terms of trade effect" exactly cancels out the initial effect of the rise in profits on asset prices, making domestic and foreign assets perfect substitutes. This is exactly what happens in Cole and Obstfeld [1991]: financial diversification is pointless since perfect risk-sharing is achieved through terms of trade movements. In the frictionless case, the substituability between assets (i.e. their returns correlation) is decreasing with respect to the substituability between goods[24] . In particular, this means that we would get the same level of assets returns correlation for a level of fundamental correlation lower than the one we used in our calibration. We leave a full characterization of the equilibrium with differentiated goods and frictions for future research.

5.3 Financial frictions vs. trade cost

Can we interpret our tax on the repatriation of dividends as a trade cost, i.e. as a cost associated with the shipping of goods? First, it is important to notice that if τ were to be interpreted as a shipping cost, it could not be an iceberg cost given our redistribution assumption (our friction does not cause any real loss in the aggregate). But even abstracting from the redistribution of taxes, a model with a tax on dividend repatriation and a model with trade costs (Dumas [1992], Uppal [1993], Sercu, Uppal and Van Hulle [2002]) are not equivalent: indeed, if domestic residents have to pay a trade cost τ when shipping goods from abroad, they can save on these costs by exchanging the goods they own abroad against domestic goods owned by foreigners at the equilibrium relative price, the real exchange rate: no shipping costs will be paid as long as foreign and domestic productions are not too asymmetric (or equivalently as long as the real exchange rate is between $1-\tau$ and $\frac{1}{1-\tau}$). This is a key difference with our setup, in which investors have no other option than repatriating their dividends and paying taxes.

A model with transportation costs could lead to an equilibrium closer to the one we get if an additional friction was introduced in the goods market. Indeed, in Dumas [1992] and the papers that followed, the goods market is perfectly competitive and agents are price-takers. We could relax this assumption and say that domestic agents who own goods abroad (in quantity q) can either ship the goods by themselves, with proportional costs T, or exchange them against home goods with a price-maker retailer at a relative

[24] This result holds for an elasticity of substitution between home and foreign goods larger than one. A proof is available on request.

price $\frac{1}{1-\tau}$. As long as $\tau < T$, the domestic resident will choose to sell his goods to the retailer, so that the final quantity of home goods that he can consume from his claim on foreign output is $(1-\tau)q$. In this modified setting with an imperfectly competitive goods market, agents would always have to pay the trade cost τ per unit of goods "shipped"[25] , and the equilibrium portfolios and asset prices would be in line with those that we found above. Frictions on the goods markets would then be equivalent to frictions on financial markets: in both cases, foreign dividend streams would be less valuable because associated with systematically paid costs τ.

6 Conclusion

This paper provides a complete description of the competitive equilibrium prevailing in a stylized model of imperfectly integrated financial markets. We find our setting appealing as it is all at once simple, empirically relevant and able of accounting for various dimensions of the data.

The technical challenge that we dealt with consists in solving for equilibrium with heterogeneous agents, the source of heterogeneity being a tax-like cost paid on dividends earned abroad, leading investors to face different opportunity sets. Our markets are complete, therefore we could use Cox and Huang [1989] to restate the partial equilibrium optimization problems. But our equilibrium outcome looks as though markets were incomplete. This is why we refer to Cuoco and He [1994], rather than resorting to a representative agent with state-independent utility. But in our case, the departure from perfect risk-sharing, which materializes in our time-varying relative weight, comes from the existence of a cost on foreign equity holdings.

In the end, our model is successful at making sense of many aspects of international financial markets and their evolution[26] . We capture the effect of integration (understood as a decrease in τ) on asset prices, we show how the CCAPM is modified relative to the fully-integrated case and how the impact of integration on the cost of capital depends on the respective size of opposite effects on the riskless rate and on the risk premium. We got a second-order effect of integration on return volatility and on the correlation of returns, this effect being due to the fact that impediments to cross-border equity holdings

[25] Note that τ is not completely disconnected from the effective transport cost T since $\frac{1}{1-T}$ is the maximum relative price that the retailer can charge. There is an optimal level of τ that retailers would charge since when τ is getting to high, either domestic residents just consume their own production or prefer shipping goods by themselves, which drives retailers profits to zero.

[26] Obviously, our assessment of the impacts of financial integration does not take into account many imperfections that are of high relevance in the real world.

prevent portfolio rebalancing and dampen comovements of the pricing kernels relevant for each asset. We shall insist on the fact that our specification provides a *lower bound* on the ability of the model to generate high returns correlation. Higher returns correlation could be obtained for given fundamental correlation by decreasing the substituability between home and foreign goods and/or by decreasing the elasticity of intertemporal substitution.

We believe our model is instrumental in understanding what "financial integration" means – and in making sense of the paradox associated with its measurement. The paradox comes from the fact that attempting to assess the degree of integration does not convey the same impression along every dimensions: portfolio biases point to segmentation, whereas large capital flows point to a high degree of integration[27] . And even though some arbitrage opportunities may still be found, assets are priced internationally. These different "sides" of world financial markets show up in our model.

The relationship between return correlation and the degree of financial integration that we emphasized is a point relevant for any empirical work looking at the impact of the correlation structure of asset returns on international portfolio allocation. Since the integration of financial markets leads simultaneously to higher comovements of stock prices and to higher levels of cross-border equity holdings, one should be careful in interpreting the impact of the correlation of stock returns on cross-border equity holdings without controlling adequately for the degree of integration between countries. In a companion paper (see Coeurdacier and Guibaud [2005]), we show that taking into account this endogeneity issue can alter dramatically the conclusions of tests of international portfolio diversification[28] .

In section 5.3, we gave some insights on the link between our setup and asset pricing models featuring frictions on *goods markets*. Having such frictions is important to get a realistic behavior of the terms of trade and of the real exchange rate, and it certainly affects portfolio choice, as originally shown in Adler and Dumas [1983], since investors facing different consumption price indices do not face the same real returns distribution for a given menu of nominal assets. Frictions on financial markets and frictions on goods markets are definitely related as emphasized by Obstfeld and Rogoff [2000], though they are not totally equivalent[29] . We sketched how multiple frictions on the goods markets could generate the effects

[27] Our model implies large flows of trade in assets (which we did not emphasize), all the more so that our friction is not a *transaction* cost.

[28] In our context, it leads the sign of the relationship between bilateral equity holdings and bilateral stock indices correlations to switch from (puzzling) positive to negative.

[29] In particular, Uppal [1993] and Sercu, Uppal and van Hulle [2002] show that, in the presence of positive but finite

on portfolio composition and asset prices that we naturally obtain in our setup. More work is needed to determine exactly the respective implications of frictions on financial markets and on goods markets and how they do interact.

iceberg costs (and a perfectly competitive goods market), portfolio holdings do not exhibit any home bias in the logarithmic utility case, and even show *reverse bias* with power utility and risk aversion higher than one.

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7 Appendix

7.1 Hypergeometric functions

Throughout the appendix, we note y_H the function that we call y in section 3.1, and y_F is such that $y_F(\delta) = \frac{1}{\rho} - y_H(\delta)\ \forall \delta$, so that asset prices without frictions are

$$
\begin{aligned}
S_{H0}(t) &= D(t) y_H(\delta(t)) \\
S_{F0}(t) &= D(t) y_F(\delta(t))
\end{aligned}
$$

Cochrane, Longstaff and Santa Clara [2005] have shown that

$$
\begin{aligned}
y_H(\delta) &\equiv E\left[\int_0^\infty e^{-\rho(s-t)}\delta(s)ds\,|\delta(0)=\delta\right] \\
&= \frac{1}{\psi(1-\gamma)}\left(\frac{\delta}{1-\delta}\right)F\left(1,1-\gamma;2-\gamma;\frac{\delta}{\delta-1}\right)+\frac{1}{\psi\theta}F\left(1,\theta;1+\theta;\frac{\delta-1}{\delta}\right)
\end{aligned}
$$

with F the standard (2,1)-hypergeometric function and

$$
\begin{aligned}
\psi &= \sqrt{\nu^2+2\rho\chi^2} \\
\gamma &= \frac{\nu-\psi}{\chi^2} \\
\theta &= \frac{\nu+\psi}{\chi^2}
\end{aligned}
$$

where

$$
\begin{aligned}
\nu &= \mu_{D_F}-\mu_{D_H}-\frac{\sigma^2_{D_{F,1}}+\sigma^2_{D_{F,2}}}{2}+\frac{\sigma^2_{D_{H,1}}+\sigma^2_{D_{H,2}}}{2} \\
\chi^2 &= \left(\sigma^2_{D_{H,1}}+\sigma^2_{D_{H,2}}\right)+\left(\sigma^2_{D_{F,1}}+\sigma^2_{D_{F,2}}\right)-2(\sigma_{D_{H,1}}\sigma_{D_{F,1}}+\sigma_{D_{H,2}}\sigma_{D_{F,2}})
\end{aligned}
$$

And

$$
\begin{aligned}
y_F(\delta) &\equiv E\left[\int_0^\infty e^{-\rho(s-t)}\left(1-\delta(s)\right)ds\,|\delta(0)=\delta\right] \\
&= \frac{1}{\psi(1+\theta)}\left(\frac{1-\delta}{\delta}\right)F\left(1,1+\theta;2+\theta;\frac{\delta-1}{\delta}\right)-\frac{1}{\psi\gamma}F\left(1,-\gamma;1-\gamma;\frac{\delta}{\delta-1}\right)
\end{aligned}
$$

7.2 Two useful results

Lemma A1 *The functions h and f defined in section 3.3 are solutions of the following PDEs*

$$
\rho h = (1+\lambda)\,\delta+\delta\mu_\delta h_\delta+\lambda\mu_\lambda h_\lambda+\frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)h_{\delta\delta}+\frac{1}{2}\lambda^2(\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda)h_{\lambda\lambda}+\delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)h_{\delta\lambda} \tag{20}
$$

$$
\rho f = \frac{1+\lambda}{\lambda}(1-\delta)+\delta\mu_\delta f_\delta+\lambda\mu_\lambda f_\lambda+\frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)f_{\delta\delta}+\frac{1}{2}\lambda^2(\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda)f_{\lambda\lambda}+\delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)f_{\delta\lambda} \tag{21}
$$

with μ_δ, σ_δ, μ_λ and σ_λ defined in the text.

Proof: Apply the Feynmac-Kac formula to h and f .

Lemma A2 $\boldsymbol{\sigma}_H$ *and* $\boldsymbol{\sigma}_F$ *must verify*

$$h\boldsymbol{\sigma}_H = h\boldsymbol{\sigma}_D + \lambda\left(h_\lambda - \frac{h}{1+\lambda}\right)\boldsymbol{\sigma}_\lambda + \delta h_\delta \boldsymbol{\sigma}_\delta \tag{22}$$

$$f\boldsymbol{\sigma}_F = f\boldsymbol{\sigma}_D + \lambda\left(f_\lambda + \frac{f}{\lambda(1+\lambda)}\right)\boldsymbol{\sigma}_\lambda + \delta f_\delta \boldsymbol{\sigma}_\delta \tag{23}$$

Proof : Applying Ito's lemma to $S_H(t) = \frac{D(t)}{1+\lambda(t)} h(\delta(t), \lambda(t))$ and focusing on the diffusion term gives:

$S_H \boldsymbol{\sigma}_H^T = \frac{1}{1+\lambda}\left[h \quad D\left(h_\lambda - \frac{h}{1+\lambda}\right) \quad Dh_\delta\right][D\boldsymbol{\sigma}_D^T \quad \lambda\boldsymbol{\sigma}_\lambda^T \quad \delta\boldsymbol{\sigma}_\delta^T]^T$

$\Rightarrow \frac{D(t)}{1+\lambda(t)} h\boldsymbol{\sigma}_H^T = \frac{1}{1+\lambda}\left[h \quad D\left(h_\lambda - \frac{h}{1+\lambda}\right) \quad Dh_\delta\right][D\boldsymbol{\sigma}_D^T \quad \lambda\boldsymbol{\sigma}_\lambda^T \quad \delta\boldsymbol{\sigma}_\delta^T]^T$

$\Rightarrow h\boldsymbol{\sigma}_H^T = \left[\frac{h}{D} \quad \left(h_\lambda - \frac{h}{1+\lambda}\right) \quad h_\delta\right]\begin{bmatrix} D\boldsymbol{\sigma}_D^T \\ \lambda\boldsymbol{\sigma}_\lambda^T \\ \delta\boldsymbol{\sigma}_\delta^T \end{bmatrix} = h\boldsymbol{\sigma}_D^T + \lambda\left(h_\lambda - \frac{h}{1+\lambda}\right)\boldsymbol{\sigma}_\lambda^T + \delta h_\delta \boldsymbol{\sigma}_\delta^T$

Idem for $S_F(t)$:

$S_F \boldsymbol{\sigma}_F^T = \frac{1}{1+\lambda}\left[\lambda f \quad D\left(\lambda f_\lambda + \frac{f}{1+\lambda}\right) \quad \lambda D f_\delta\right][D\boldsymbol{\sigma}_D^T \quad \lambda\boldsymbol{\sigma}_\lambda^T \quad \delta\boldsymbol{\sigma}_\delta^T]^T$

$\Rightarrow \frac{\lambda(t)D(t)}{1+\lambda(t)} f\boldsymbol{\sigma}_F^T = \frac{1}{1+\lambda}\left[\lambda f \quad D\left(\lambda f_\lambda + \frac{f}{1+\lambda}\right) \quad \lambda D f_\delta\right][D\boldsymbol{\sigma}_D^T \quad \lambda\boldsymbol{\sigma}_\lambda^T \quad \delta\boldsymbol{\sigma}_\delta^T]^T$

$\Rightarrow f\boldsymbol{\sigma}_F^T = \left[\frac{f}{D} \quad \left(f_\lambda + \frac{f}{\lambda(1+\lambda)}\right) \quad f_\delta\right]\begin{bmatrix} D\boldsymbol{\sigma}_D^T \\ \lambda\boldsymbol{\sigma}_\lambda^T \\ \delta\boldsymbol{\sigma}_\delta^T \end{bmatrix} = f\boldsymbol{\sigma}_D^T + \lambda\left(f_\lambda + \frac{f}{\lambda(1+\lambda)}\right)\boldsymbol{\sigma}_\lambda^T + \delta f_\delta \boldsymbol{\sigma}_\delta^T \quad \blacksquare$

7.3 Proof of lemma 1 (market prices of risk)

The outline of the proof is the following: start from first-order conditions, apply Ito's lemma to both terms and identify diffusion terms, then use market clearing.

The first-order condition is:

$e^{-\rho t}\frac{1}{c_H(t)} = \Psi_H \xi_H(t)$

$\Rightarrow -\rho e^{-\rho t}\frac{1}{c_H(t)}dt - e^{-\rho t}\frac{1}{c_H(t)^2}dc_H + e^{-\rho t}\frac{1}{c_H(t)^3}dc_H^2 = -\Psi_H \xi_H(t)[r(t)dt + \boldsymbol{\theta}_H^T(t)d\mathbf{W}(t)]$

We will use the following notations

$$dC_i = \mu_{C_i}()dt + \boldsymbol{\sigma}_{C_i}^T()d\mathbf{W} \qquad i = H, F$$

Identifying diffusion terms implies:

$-e^{-\rho t}\frac{1}{c_H(t)^2}\boldsymbol{\sigma}_{c_H}(t) = -\Psi_H \xi_H(t)\boldsymbol{\theta}_H(t)$

$\Rightarrow -e^{-\rho t}\frac{1}{c_H(t)^2}\boldsymbol{\sigma}_{c_H}(t) = -e^{-\rho t}\frac{1}{c_H(t)}\boldsymbol{\theta}_H(t)$, using $e^{-\rho t}\frac{1}{c_H(t)} = \Psi_H \xi_H(t)$

$\Rightarrow \boldsymbol{\sigma}_{c_H}(t) = c_H(t)\boldsymbol{\theta}_H(t)$

In the same way, we get $\boldsymbol{\sigma}_{c_F}(t) = c_F(t)\boldsymbol{\theta}_F(t)$

Besides, market clearing implies $\boldsymbol{\sigma}_{C_H}() + \boldsymbol{\sigma}_{C_F}() = D\boldsymbol{\sigma}_D = D\left[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}\right]$.

So, plugging the expressions for $\boldsymbol{\sigma}_{C_i}$: $c_H(t)\boldsymbol{\theta}_H(t) + c_F(t)\boldsymbol{\theta}_F(t) = \left[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}\right] D$.

We also use equation 10 : $\boldsymbol{\theta}_H - \boldsymbol{\theta}_F = (\boldsymbol{\sigma}^T)^{-1}\begin{bmatrix} \tau\frac{D_H}{S_H} \\ -\tau\frac{D_F}{S_F}\end{bmatrix}$ and substitute for $\boldsymbol{\theta}_F$ to get:

$$c_H(t)\boldsymbol{\theta}_H(t) + c_F(t)\boldsymbol{\theta}_H - c_F(t)(\boldsymbol{\sigma}^T)^{-1}\begin{bmatrix} \tau\frac{D_H}{S_H} \\ -\tau\frac{D_F}{S_F}\end{bmatrix} = D\left[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}\right]$$

i.e. (using $\frac{c_F}{D} = \frac{\lambda}{1+\lambda}$)

$$\boldsymbol{\theta}_H(t) = \left[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}\right] + \tau\frac{\lambda(t)}{1+\lambda(t)}(\boldsymbol{\sigma}^T)^{-1}\begin{bmatrix} \frac{D_H}{S_H} \\ -\frac{D_F}{S_F}\end{bmatrix}$$

The formula for $\boldsymbol{\theta}_F(t)$ follows, using $\boldsymbol{\theta}_H$ and the formula for $\boldsymbol{\theta}_H - \boldsymbol{\theta}_F$.

Remark: the drift and diffusion in the dynamics of λ, $\frac{d\lambda}{\lambda} = \mu_\lambda dt + \boldsymbol{\sigma}_\lambda^T d\mathbf{W}$, can be reexpressed:

$$\boldsymbol{\sigma}_\lambda = \underset{(2,1)}{\boldsymbol{\theta}_F - \boldsymbol{\theta}_H} = \tau\left(\boldsymbol{\sigma}^{\mathbf{T}}\right)^{-1}\begin{bmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F}\end{bmatrix}$$

$$\mu_\lambda = (\boldsymbol{\theta}_F - \boldsymbol{\theta}_H)^T\boldsymbol{\theta}_F = \left[-\tau\frac{D_H}{S_H} \quad \tau\frac{D_F}{S_F}\right]\left[\left(\boldsymbol{\sigma}^{\mathbf{T}}\right)^{-1}\right]^T\left\{\underbrace{\boldsymbol{\sigma}_D + \frac{1}{1+\lambda(t)}\left(\boldsymbol{\sigma}^{\mathbf{T}}\right)^{-1}\begin{pmatrix} -\tau\frac{D_H}{S_H} \\ \tau\frac{D_F}{S_F}\end{pmatrix}}_{\boldsymbol{\theta}_F}\right\}$$

$$= \tau\left[-\frac{D_H}{S_H} \quad \frac{D_F}{S_F}\right]\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau^2\frac{1}{1+\lambda(t)}\left[-\frac{D_H}{S_H} \quad \frac{D_F}{S_F}\right]\left(\boldsymbol{\sigma}^{\mathbf{T}}\boldsymbol{\sigma}\right)^{-1}\left[-\frac{D_H}{S_H} \quad \frac{D_F}{S_F}\right]^T$$

It is immediate that first-order Taylor expansions of expressions for $\boldsymbol{\sigma}_\lambda$ and μ_λ around $\tau = 0$ are given by :

$$\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Omega}_0(\delta) + o(\tau)$$

and

$$\mu_\lambda = \tau \mathbf{\Omega}_0^T \boldsymbol{\sigma}_D + o(\tau)$$

where $\mathbf{\Omega}_0(\delta) \equiv (\boldsymbol{\sigma}_0^T)^{-1} \begin{bmatrix} -\left(\frac{D_H}{S_H}\right)_0 \\ \left(\frac{D_F}{S_F}\right)_0 \end{bmatrix}$ can be computed from the hypergeometric function y.

7.4 Proof of proposition 1 (first-order approximation for asset prices)

In section 3.3, we wrote $S_H(t) = \frac{D(t)}{1+\lambda(t)} h(\delta(t), \lambda(t))$ with

$$h(\delta(t), \lambda(t)) = E\left[\int_t^{+\infty} e^{-\rho(s-t)} \left[1 + \lambda(s)\right] \delta(s) ds \,|\delta(t), \lambda(t)\right]$$

Since $\frac{d\lambda}{\lambda} = \mu_\lambda dt + \boldsymbol{\sigma}'_\lambda dW$, for $s > t_0$

$$\lambda(s) = \lambda(t_0) \exp\left\{\int_{t_0}^s \left[\mu_\lambda - \frac{1}{2}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda\right] dt + \int_{t_0}^s \boldsymbol{\sigma}_\lambda^T d\mathbf{W}_t\right\}$$

Besides, we know that

$$\begin{aligned} \mu_\lambda &= \tau \mathbf{\Omega}_0^T(\delta) \boldsymbol{\sigma}_D(\delta) + o(\tau) \\ \boldsymbol{\sigma}_\lambda &= \tau \mathbf{\Omega}_0(\delta) + o(\tau) \end{aligned}$$

where $\mathbf{\Omega}_0(\delta)$ is computed from the hypergeometric function.

Lemma A3 :

$$\mathbf{\Omega}_0^T(\delta) \boldsymbol{\sigma}_D(\delta) = \rho(1 - 2\delta)$$

Proof :

Substituting the definition of $\mathbf{\Omega}_0$, we have

$$\mathbf{\Omega}_0^T \boldsymbol{\sigma}_D = \left[-\left(\frac{D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right] (\boldsymbol{\sigma}_0)^{-1} \boldsymbol{\sigma}_D$$

which implies

$$\mathbf{\Omega}_0^T \boldsymbol{\sigma}_D = \left[-\left(\frac{D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right] \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix}$$

because $(\boldsymbol{\sigma}_0)^{-1} \boldsymbol{\sigma}_D$ is exactly the vector of stock holdings of a representative agent in an equilibrium without frictions, which in turn must be equal to the market portfolio. Then, using $(S_H + S_F)_0 = (X_H + X_F)_0 = \frac{D}{\rho}$, we get $\mathbf{\Omega}_0^T \boldsymbol{\sigma}_D = \rho(1 - 2\delta)$. ■

Therefore, introducing $\gamma_0(\delta) = \rho(1-2\delta)$, we can write

$$\lambda(s) = \lambda(t_0)\exp\left\{\tau\left[\int_{t_0}^{s}\gamma_0(\delta_t)dt + \int_{t_0}^{s}\mathbf{\Omega}_0^T(\delta_t)d\mathbf{W}_t\right] + o(\tau)\right\}$$

$$\Rightarrow \lambda(s) = \lambda(t_0)\left[1 + \tau\int_{t_0}^{s}\gamma_0(\delta_t)dt + \tau\int_{t_0}^{s}\mathbf{\Omega}_0^T(\delta_t)d\mathbf{W}_t\right] + o(\tau)$$

and

$$h(\delta(t),\lambda(t)) = E_t\left\{\int_t^{+\infty} e^{-\rho(s-t)}\left[1 + \lambda(t) + \tau\lambda(t)\int_t^s\gamma_0(\delta_{t'})dt' + \tau\lambda(t)\int_t^s\mathbf{\Omega}_0^T(\delta_{t'})d\mathbf{W}_{t'} + o(\tau)\right]\delta(s)ds\right\}$$

$$\begin{aligned}
\Rightarrow \quad & h(\delta(t),\lambda(t)) = (1+\lambda(t))E_t\left[\int_t^{+\infty} e^{-\rho(s-t)}\delta(s)ds\right] \\
& +\tau\lambda(t)\underbrace{E_t\left[\int_t^{+\infty} e^{-\rho(s-t)}\left[\int_t^s\gamma_0(\delta_{t'})dt' + \int_t^s\mathbf{\Omega}_0^T(\delta_{t'})d\mathbf{W}_{t'}\right]\delta(s)ds\right]}_{\equiv -H(\delta(t))} + o(\tau)
\end{aligned}$$

$$\Rightarrow h(\delta(t),\lambda(t)) = (1+\lambda(t))y_H(\delta(t)) - \tau\lambda(t)H(\delta(t)) + o(\tau) \tag{24}$$

Then S_H is given by

$$S_H(D_t,\delta_t,\lambda_t;\tau) = D_t\left[y_H(\delta_t) - \tau\frac{\lambda_t}{1+\lambda_t}H(\delta_t)\right] + o(\tau)$$

And in the same way we get S_F as

$$S_F(D_t,\delta_t,\lambda_t;\tau) = D_t\left[y_F(\delta_t) - \tau\frac{1}{1+\lambda_t}F(\delta_t)\right] + o(\tau)$$

Lemma A4 *The functions H and F must satisfy the following boundary value problem*

$$\begin{cases}
\rho H - \delta\mu_\delta H' - \frac{1}{2}\delta^2\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta H'' = -\left[\rho(1-2\delta)y_H + \left(\boldsymbol{\sigma}_\delta^T\mathbf{\Omega}_0\right)y_H'\right] = \delta \\
H(0) = 0 \\
H(1) = \frac{1}{\rho}
\end{cases}$$

$$\begin{cases}
\rho F - \delta\mu_\delta F' - \frac{1}{2}\delta^2\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta F'' = \rho(1-2\delta)y_F + \left[\boldsymbol{\sigma}_\delta^T\mathbf{\Omega}_0\right]y_F' = 1-\delta \\
F(0) = \frac{1}{\rho} \\
F(1) = 0
\end{cases}$$

Proof : We can rewrite the PDE for h (lemma A1) by using equation (24) and by applying Feynmac-Kac to $y_H(.)$ (which implies $\rho y_H = (1+\lambda)\delta + \delta\mu_\delta y_H' + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)y_H^{''}$). We get:

$$\rho H(\delta) = \delta\mu_\delta H'(\delta) + \frac{1}{2}\delta^2 \boldsymbol{\sigma}_\delta^T \boldsymbol{\sigma}_\delta H''(\delta) - \rho(1-2\delta)y_H(\delta) - \delta \left[\boldsymbol{\sigma}_\delta^T(\delta)\boldsymbol{\Omega}_0(\delta)\right] y'_H(\delta)$$

The first boundary condition follows from the fact that given the nature of the dividend process

$$S_H(D, 0, \lambda) = 0$$

The necessity of the second boundary condition can be seen from the fact that it must be the case that

$$\lim_{\lambda\to\infty} S_H(D, 1, \lambda) = \frac{(1-\tau)D}{\rho}$$

Indeed, when δ goes to 1 and λ goes to infinity, the economy tends to an economy with one tree only ($D = D_H$) and one investor located in the foreign country, thus facing an after-tax dividend stream $(1-\tau)D$.

In the same way, we characterize the foreign asset price through a function F solution of a PDE with analogous boundary conditions (see in the text).

We now prove that the non homogenous terms in the PDEs can be rewritten:

$$\begin{aligned} \rho(1-2\delta)y_H + \delta\left(\boldsymbol{\sigma}_\delta^T \boldsymbol{\Omega}_0\right) y'_H &= -\delta \\ \rho(1-2\delta)y_F + \delta\left(\boldsymbol{\sigma}_\delta^T \boldsymbol{\Omega}_0\right) y'_F &= 1-\delta \end{aligned}$$

To do that we use the fact that in the equilibrium without frictions, the restriction on price diffusion components (cf. lemma A2) takes the following form:

$$\begin{aligned} \boldsymbol{\sigma}_{H0} &= \boldsymbol{\sigma}_D + \frac{y'_H}{y_H}\delta\boldsymbol{\sigma}_\delta \\ \boldsymbol{\sigma}_{F0} &= \boldsymbol{\sigma}_D + \frac{y'_F}{y_F}\delta\boldsymbol{\sigma}_\delta \end{aligned}$$

Then :

$$\begin{aligned} \boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_{H0} &= \boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_D + \delta\frac{y'_H}{y_H}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta \\ &= \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \delta\frac{y'_H}{y_H}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta \end{aligned}$$

where the second equality follows from the fact that in the equilibrium without frictions ${\boldsymbol{\sigma}_0}^{-1}\boldsymbol{\sigma}_D$ is exactly the vector of stock holdings of a representative agent, which must be equal to the market portfolio.

Symmetrically,

$$\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_{F0} = \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \delta\frac{y_F'}{y_F}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta$$

Then, since $\boldsymbol{\sigma}_0 = (\boldsymbol{\sigma}_{H0} \quad \boldsymbol{\sigma}_{F0})$ we have :

$$\mathbf{I}_2 = \boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_0 = \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 & \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 & \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \left[\delta\frac{y_H'}{y_H}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta \quad \delta\frac{y_F'}{y_F}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta\right]$$

$$\Rightarrow \left[-\left(\frac{D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right] = [\rho(1-2\delta) \quad \rho(1-2\delta)] + \left[\delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0\right)^T\frac{y_H'}{y_H} \quad \delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0\right)^T\frac{y_F'}{y_F}\right]$$

$$\begin{aligned}\Rightarrow \left[-\left(\frac{D_H}{S_H}y_H\right)_0 \quad \left(\frac{D_F}{S_F}y_F\right)_0\right] &= [-\delta \quad (1-\delta)] \\ &= \left[\rho(1-2\delta)y_H + \delta\left(\sigma_\delta^T\Omega_0\right)y_H' \quad \rho(1-2\delta)y_F + \delta\left(\sigma_\delta^T\Omega_0\right)y_F'\right] \blacksquare\end{aligned}$$

It is immediate that y_H and y_F are solutions of the boundary value problems above (by definition of y_H and y_F). We therefore get the first-order expansion for S_H and S_F :

$$\begin{aligned}S_H(D,\delta,\lambda;\tau) &= D_t\left[1-\tau\frac{\lambda}{1+\lambda}\right]y_H(\delta)+o(\tau) \\ S_F(D,\delta,\lambda;\tau) &= D_t\left[1-\tau\frac{1}{1+\lambda}\right]y_F(\delta)+o(\tau)\end{aligned}$$

7.5 Proof of proposition 2 (second order approximation of asset prices)

First step : getting the second-order expansions of μ_λ and σ_λ

We can easily prove that

$$\left(\boldsymbol{\sigma}^T\right)^{-1} = \left(\boldsymbol{\sigma}_0^T\right)^{-1} + o(1)$$

We can also write

$$\begin{aligned}\frac{D_H}{S_H} &= \left(\frac{D_H}{S_H}\right)_0\left[1+\tau\frac{\lambda}{1+\lambda}\right]+o(\tau) \\ \frac{D_F}{S_F} &= \left(\frac{D_F}{S_F}\right)_0\left[1+\frac{\tau}{1+\lambda}\right]+o(\tau)\end{aligned}$$

Indeed

$$\begin{aligned}\frac{D_H}{S_H} &= \frac{D_H}{D}\frac{D}{S_H} \\ &= \frac{D_H}{D}\frac{1}{y_H(\delta)\left(1-\tau\frac{\lambda}{1+\lambda}\right)+o(\tau)} \\ &= \left(\frac{D_H}{S_H}\right)_0 + \tau\frac{\lambda}{1+\lambda}\left(\frac{D_H}{S_H}\right)_0 + o(\tau)\end{aligned}$$

and

$$\frac{D_F}{S_F} = \frac{D_F}{D}\left[y_F(\delta)(1-\tau\frac{1}{1+\lambda})\right]^{-1}$$
$$= \left(\frac{D_F}{S_F}\right)_0 + \tau\frac{1}{1+\lambda}\left(\frac{D_F}{S_F}\right)_0 + o(\tau)$$

Then we get the second-order approximations of μ_λ and $\boldsymbol{\sigma}_\lambda$:

$$\boldsymbol{\sigma}_\lambda = \tau\left(\boldsymbol{\sigma}_0^{\mathbf{T}}\right)^{-1}\begin{pmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F}\end{pmatrix}$$
$$= \tau\boldsymbol{\Omega}_0 + \tau^2\boldsymbol{\Omega}_1 + o(\tau^2)$$

with

$$\boldsymbol{\Omega}_1 = \frac{1}{1+\lambda}\left(\boldsymbol{\sigma}_0^{\mathbf{T}}\right)^{-1}\begin{pmatrix} -\lambda\left(\frac{D_H}{S_H}\right)_0 \\ \left(\frac{D_F}{S_F}\right)_0\end{pmatrix} \tag{25}$$

and

$$\mu_\lambda = \boldsymbol{\sigma}_\lambda^T\boldsymbol{\sigma}_D + \frac{1}{1+\lambda(t)}\boldsymbol{\sigma}_\lambda^T\boldsymbol{\sigma}_\lambda$$
$$= \tau\boldsymbol{\Omega}_0^T\boldsymbol{\sigma}_D + \tau^2\left(\boldsymbol{\Omega}_1^T\boldsymbol{\sigma}_D\right) + \tau^2\frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right) + o(\tau^2)$$
$$= \tau\rho(1-2\delta) + \tau^2\left(\boldsymbol{\Omega}_1^T\boldsymbol{\sigma}_D\right) + \tau^2\frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right) + o(\tau^2)$$
$$= \tau\rho(1-2\delta) + \tau^2\left[\frac{\rho}{1+\lambda} - \rho\delta + \frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right)\right] + o(\tau^2)$$

In order to get the final expression for μ_λ above, we used:

$$\Omega_1^T\boldsymbol{\sigma}_D = \frac{1}{1+\lambda}\begin{pmatrix} -\lambda\left(\frac{D_H}{S_H}\right)_0 & \left(\frac{D_F}{S_F}\right)_0\end{pmatrix}\left(\boldsymbol{\sigma}_0\right)^{-1}\boldsymbol{\sigma}_D$$
$$= \frac{\rho}{1+\lambda}\left(-\lambda\delta + 1 - \delta\right)$$
$$= \frac{\rho}{1+\lambda} - \rho\delta \qquad \blacksquare$$

Henceforth, we work on $h(\delta,\lambda) \equiv E\left[\int_t^{+\infty} e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds\,|\delta(t)=\delta,\lambda(t)=\lambda\right]$. Like in the proof of proposition 1, it is easy to show that the second-order approximation of h can be written as

$$h(\delta,\lambda;\tau) = (1+\lambda)\,y_H(\delta) - \tau\lambda y_H(\delta) + \tau^2\lambda H_2(\delta,\lambda) + o(\tau^2) \tag{26}$$

Using this expression into the PDE for h (lemma A1) and identifying second-order terms (i.e. terms in τ^2), we get the following PDE for H_2

$$\rho H_2 = \delta\mu_\delta \frac{\partial H_2}{\partial \delta} + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)\frac{\partial^2 H_2}{\partial \delta^2} + \left[\frac{\rho}{1+\lambda} - \rho\delta + \frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right) - \rho(1-2\delta)\right] y_H + \delta(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1 - \boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0)y'_H \tag{27}$$

We now want to simplify the expression for the non-homogenous term. We already know that :

$$-\left[\rho(1-2\delta)y_H + \delta\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0 y'_{H0}\right] = \delta$$

The following lemma points to another simplification.

Lemma A5 :

$$\left(\frac{\rho}{1+\lambda} - \rho\delta\right) y_H + \delta(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1)y'_H = -\frac{\lambda}{1+\lambda}\delta$$

Proof : we use the same reasoning as for the first-order approximation :

$$\mathbf{I}_2 = \boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_0 = \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 & \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 & \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \left[\delta\frac{y'_H}{y_H}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta \quad \delta\frac{y'_F}{y_F}\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_\delta\right]$$

$$\Rightarrow \left[-\left(\frac{\lambda D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right] = \left[\rho(1-(1+\lambda)\delta) \quad \rho(1-(1+\lambda)\delta)\right] + \left[\delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T\frac{y'_H}{y_H} \quad \delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T\frac{y'_F}{y_F}\right]$$

$$\Rightarrow \left[-\left(\frac{\lambda D_H}{S_H}y_H\right)_0 \quad \left(\frac{D_F}{S_F}y_F\right)_0\right] = \left[\rho(1-(1+\lambda)\delta)y_H \quad \rho(1-(1+\lambda)\delta)y_F\right] + \left[\delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T y'_H \quad \delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T y'_F\right]$$

$$\Rightarrow \frac{1}{1+\lambda}\left[-\lambda\delta \quad 1-\delta\right] = \frac{1}{1+\lambda}\left[\rho(1-(1+\lambda)\delta)y_H + \delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T y'_H \quad \rho(1-(1+\lambda)\delta)y_F + \delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T y'_F\right] \quad \blacksquare$$

Hence, by rewriting the non-homogenous term equation 27, we get :

$$\rho H_2 = \delta\mu_\delta\frac{\partial H_2}{\partial\delta} + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)\frac{\partial^2 H_2}{\partial\delta^2} + \frac{1}{1+\lambda}\delta + \frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right)y_H$$

Then, using the fact that $\rho y_H = \delta\mu_\delta y'_H + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)y''_H + \delta$, we can show that there exists a function h_2 such that

$$H_2(\delta,\lambda) = \frac{1}{1+\lambda}\left[y_H(\delta) + h_2(\delta)\right]$$

and h_2 verifies:

$$\rho h_2 = \delta\mu_\delta h'_2 + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)h''_2 + \left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right)y_H$$

Respectively for f, we can show that

$$f(\delta,\lambda;\tau)=\frac{1+\lambda}{\lambda}y_F(\delta)-\frac{\tau}{\lambda}y_F(\delta)+\frac{\tau^2}{\lambda}F_2(\delta,\lambda)+o(\tau^2) \tag{28}$$

with $F_2(\delta,\lambda)$ satisfying the following differential equation

$$\rho F_2=\delta\mu_\delta\frac{\partial F_2}{\partial\delta}+\frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)\frac{\partial^2 F_2}{\partial\delta^2}+\rho(1-2\delta)y_F-\left[\frac{\rho}{1+\lambda}-\rho\delta\right]y_F+\delta(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0)y_F'-\delta(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1)y_F'+\frac{\lambda}{1+\lambda}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right)y_F$$

And we can rewrite the non-homogenous term in this PDE using

$$\rho(1-2\delta)y_F+\delta(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_0)y_F'=1-\delta$$

and

$$\frac{1}{1+\lambda}\left[\rho(1-(1+\lambda)\delta)y_F\right]+\frac{1}{1+\lambda}\delta\left(\boldsymbol{\sigma}_\delta^T\boldsymbol{\Omega}_1\right)^T y_F'=\frac{1-\delta}{1+\lambda}$$

to obtain

$$\rho F_2=\delta\mu_\delta\frac{\partial F_2}{\partial\delta}+\frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)\frac{\partial^2F_2}{\partial\delta^2}+(1-\delta)\frac{\lambda}{1+\lambda}+\frac{\lambda}{(1+\lambda)}\left(\boldsymbol{\Omega}_0^T\boldsymbol{\Omega}_0\right)y_F$$

We then introduce the function f_2 such that $F_2(\delta,\lambda)=\frac{\lambda}{1+\lambda}\left[y_F(\delta)+f_2(\delta)\right]$ and show that it is solution of the ODE given in the text.

Boundary Conditions

At this stage, we know that

$$\begin{aligned}S_H(D,\delta,\lambda;\tau)&=D\left[y_H(\delta)\left(1-\tau\frac{\lambda}{1+\lambda}+\tau^2\frac{\lambda}{(1+\lambda)^2}\right)+\tau^2\frac{\lambda}{(1+\lambda)^2}h_2(\delta)\right]+o(\tau^2)\\ S_F(D,\delta,\lambda;\tau)&=D\left[y_F(\delta)\left(1-\tau\frac{1}{1+\lambda}+\tau^2\frac{\lambda}{(1+\lambda)^2}\right)+\tau^2\frac{\lambda}{(1+\lambda)^2}f_2(\delta)\right]+o(\tau^2)\end{aligned}$$

The conditions $h_2(0)=f_2(1)=0$ are required since the price of assets yielding zero payoff must be null. The derivation of the other two boundary conditions (on $h_2(1)$ and $f_2(0)$), to which we now turn, is more tricky.

When $\delta\to 1$, $S_H(t)$ tends to

$$\frac{D_H(t)}{1+\lambda_t}E\left[\int_t^{+\infty}e^{-\rho(s-t)}\left[1+\lambda(s)\right]ds\,|\lambda(t)\right]$$

Let us define $\phi(\lambda_t;\tau)\equiv E\left[\int_t^{+\infty}e^{-\rho(s-t)}\left[1+\lambda(s)\right]ds\,|\lambda(t)\right]$, so that $\lim_{\delta\to1}S_H(D,\delta,\lambda;\tau)=\frac{D}{1+\lambda}\phi(\lambda_t;\tau)$. Using the Feynman-Kac formula :

$$\rho\phi(\lambda)=(1+\lambda)+\lambda\bar{\mu}_\lambda\phi'(\lambda)+\frac{1}{2}\lambda^2\bar{\boldsymbol{\sigma}}_\lambda.\bar{\boldsymbol{\sigma}}_\lambda\phi''(\lambda) \tag{29}$$

where $\bar{\mu}_\lambda = \lim_{\delta\to 1}(\mu_\lambda)$ and $\bar{\boldsymbol{\sigma}}_\lambda = \lim_{\delta\to 1}(\boldsymbol{\sigma}_\lambda)$, i.e.

$$\begin{aligned}\bar{\boldsymbol{\sigma}}_\lambda &= \tau\boldsymbol{\Omega}_0(1) + \tau^2\boldsymbol{\Omega}_1(1)\\ \bar{\mu}_\lambda &= -\tau\rho + \tau^2\left[-\frac{\lambda\rho}{1+\lambda} + \frac{1}{1+\lambda}\left(\boldsymbol{\Omega}_0^T(1)\boldsymbol{\Omega}_0(1)\right)\right]\end{aligned}$$

Besides, we know that h_2 is such that at the second-order in τ:

$$h(\delta,\lambda) = (1+\lambda)\,y_H(\delta) - \tau\lambda y_H(\delta) + \tau^2\frac{\lambda}{1+\lambda}\left[y_H(\delta) + h_2(\delta)\right]$$

Taking the limit when δ goes to 1, we get

$$\lim_{\delta\to 1} h(\delta,\lambda) = \phi(\lambda) = \frac{1}{\rho}\left[1+\lambda-\tau\lambda+\tau^2\frac{\lambda}{(1+\lambda)} + \tau^2\frac{\lambda}{(1+\lambda)}\rho h_2(1)\right]$$

From this, we can compute $\phi'(\lambda)$ and $\phi''(\lambda)$ and plug the expressions for ϕ and its derivatives in equation (29). Then, identifying terms in τ^2 in the differential equation, we get :

$$\frac{\lambda}{1+\lambda}\rho h_2(1) = \frac{1}{\rho}\frac{\lambda}{1+\lambda}\left(\boldsymbol{\Omega}_0^T(1)\boldsymbol{\Omega}_0(1)\right)$$

$$\Rightarrow h_2(1) = \frac{1}{\rho^2}\boldsymbol{\Omega}_0^T(1)\boldsymbol{\Omega}_0(1)$$

Symmetrically :

$$f_2(0) = \frac{1}{\rho}(1 + \frac{1}{\rho}\boldsymbol{\Omega}_0^T(0)\boldsymbol{\Omega}_0(0))$$

7.6 Proof of proposition 3 (second order approximation of price diffusions)

We start from lemma A2

$$\boldsymbol{\sigma}_H = \boldsymbol{\sigma}_D + \delta\frac{h_\delta}{h}\boldsymbol{\sigma}_\delta + \lambda\left(\frac{h_\lambda}{h} - \frac{1}{1+\lambda}\right)\boldsymbol{\sigma}_\lambda$$

From $h(\delta,\lambda) = (1+\lambda)\,y_H(\delta) - \tau\lambda y_H(\delta) + \tau^2\frac{\lambda}{1+\lambda}\left(y_H(\delta) + h_2(\delta)\right)$, we get the following second order approximations:

$$\begin{aligned}\frac{1}{h} &= \frac{1}{(1+\lambda)y_H}\left[1+\tau\frac{\lambda}{1+\lambda} - \tau^2\frac{\lambda}{(1+\lambda)^2}\left(1+\frac{h_2}{y_H}\right)\right]\\ h_\delta &= (1+\lambda)\,y_H' - \tau\lambda y_H' + \tau^2\frac{\lambda}{1+\lambda}\left(y_H^{'} + h_2'\right)\\ h_\lambda &= y_H - \tau y_H + \tau^2\frac{h_2+y_H}{(1+\lambda)^2}\end{aligned}$$

Then, using $\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Omega}_0 + \tau^2\boldsymbol{\Omega}_1$, we compute $\boldsymbol{\sigma}_H$ by keeping only terms of order less or equal to 2:

$$\boldsymbol{\sigma}_H = \boldsymbol{\sigma}_{H0} + \tau^2\frac{\lambda}{(1+\lambda)^2}\left\{-\boldsymbol{\Omega}_0 + \left[\frac{h_2'}{y_H} - \lambda\frac{y_H'}{y_H} - \frac{h_2 y_H'}{(y_H)^2}\right]\delta\boldsymbol{\sigma}_\delta\right\}$$

In the same way, starting from $\boldsymbol{\sigma}_F = \boldsymbol{\sigma}_D + \delta \frac{f_\delta}{f} \boldsymbol{\sigma}_\delta + \lambda \left(\frac{f_\lambda}{f} + \frac{1}{\lambda(1+\lambda)} \right) \boldsymbol{\sigma}_\lambda$, we get

$$\boldsymbol{\sigma}_F = \boldsymbol{\sigma}_{F0} + \tau^2 \frac{\lambda}{(1+\lambda)^2} \left\{ \boldsymbol{\Omega}_0 + \left[\frac{f_2'}{y_F} - \frac{1}{\lambda} \frac{y_F'}{y_F} - \frac{f_2 y_F'}{(y_F)^2} \right] \delta \boldsymbol{\sigma}_\delta \right\}$$

7.7 Proof of proposition 4 (risk premia)

Using lemma 1 and the definition of $\boldsymbol{\theta}_i$ (such that $\boldsymbol{\mu}_i - \mathbf{r} = \boldsymbol{\sigma}^{\mathbf{T}} \boldsymbol{\theta}_i$), the after-tax expected excess returns, respectively for investors in country H and in country F, are given by:

$$\begin{aligned} \boldsymbol{\mu}_H - \mathbf{r} &= \boldsymbol{\sigma}^{\mathbf{T}} \boldsymbol{\sigma}_D + \tau \frac{\lambda(t)}{1+\lambda(t)} \begin{pmatrix} \frac{D_H}{S_H} \\ -\frac{D_F}{S_F} \end{pmatrix} \\ \boldsymbol{\mu}_F - \mathbf{r} &= \boldsymbol{\sigma}^{\mathbf{T}} \boldsymbol{\sigma}_D + \tau \frac{1}{1+\lambda(t)} \begin{pmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F} \end{pmatrix} \end{aligned}$$

The before-tax risk premia are given by the upper element of $\boldsymbol{\mu}_H - \mathbf{r}$ and by the lower element of $\boldsymbol{\mu}_F - \mathbf{r}$. The Taylor expansions follow straightforwardly.

7.8 Proof of proposition 5 (riskless rate)

Start from FOC :

$e^{-\rho t} \frac{1}{c_H(t)} = \Psi_H \xi_H(t)$

$\Rightarrow -\rho e^{-\rho t} \frac{1}{c_H(t)} dt - e^{-\rho t} \frac{1}{c_H(t)^2} dc_H + e^{-\rho t} \frac{1}{c_H(t)^3} dc_H^2 = -\Psi_H \xi_H(t) [r(t) dt + \boldsymbol{\theta}_H^T(t) d\mathbf{W}(t)]$ (Ito)

$\Rightarrow -\rho \frac{1}{c_H(t)} - \frac{1}{c_H(t)^2} \mu_{C_H} + \frac{1}{c_H(t)^3} \boldsymbol{\sigma}_{c_H}^T(t) \boldsymbol{\sigma}_{c_H}(t) = -\frac{1}{c_H(t)} r(t)$ (identification of drift terms)

$\Rightarrow r(t) = \rho + \frac{\mu_{C_H}(t)}{c_H(t)} - \frac{1}{c_H(t)^2} \boldsymbol{\sigma}_{c_H}^T(t) \boldsymbol{\sigma}_{c_H}(t) = \rho + \frac{\mu_{C_H}(t)}{c_H(t)} - \boldsymbol{\theta}_H^T(t) \boldsymbol{\theta}_H(t)$

where we used $\boldsymbol{\sigma}_{c_H} = c_H \boldsymbol{\theta}_H$ to get the last equation.

In the same way, we get: $r(t) = \rho + \frac{\mu_{C_F}(t)}{c_F(t)} - \frac{1}{c_F(t)^2} \boldsymbol{\sigma}_{c_F}^T(t) \boldsymbol{\sigma}_{c_F}(t) = \rho + \frac{\mu_{C_F}(t)}{c_F(t)} - \boldsymbol{\theta}_F^T(t) \boldsymbol{\theta}_F(t)$

Summing the two expressions for $r(t)$, we get :

$$r(t) = \rho + \frac{1}{2} \left(\frac{\mu_{C_H}(t)}{c_H(t)} + \frac{\mu_{C_F}(t)}{c_F(t)} \right) - \frac{1}{2} \left(\boldsymbol{\theta}_H(t).\boldsymbol{\theta}_H(t) + \boldsymbol{\theta}_F(t).\boldsymbol{\theta}_F(t) \right)$$

Then, using market clearing (which implies $\mu_{C_H}() + \mu_{C_F}() = \mu_D D$) and applying Ito's lemma on $D/(1+\lambda)$ to get μ_{C_H}, the term $\frac{\mu_{C_H}(t)}{c_H(t)} + \frac{\mu_{C_F}(t)}{c_F(t)}$ can be shown (after a bit of algebra) to be equal to

$$2\mu_D + \frac{\lambda - 1}{1+\lambda} \left(-\mu_\lambda + \frac{\lambda}{1+\lambda} \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda - \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D \right)$$

so that the riskless rate can be written

$$r(t) = \rho + \mu_D + \frac{1}{2}\frac{\lambda - 1}{1+\lambda}\left(-\mu_\lambda + \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda - \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D\right) - \frac{1}{2}\left(\boldsymbol{\theta}_H(t).\boldsymbol{\theta}_H(t) + \boldsymbol{\theta}_F(t).\boldsymbol{\theta}_F(t)\right)$$

By lemma 1, we further know that $\boldsymbol{\theta}_H(t) = \boldsymbol{\sigma}_D + \tau\frac{\lambda(t)}{1+\lambda(t)}(\boldsymbol{\sigma}^T)^{-1}[\frac{D_H}{S_H} \quad -\frac{D_F}{S_F}]^T$, so that

$$\begin{aligned}\boldsymbol{\theta}_H(t).\boldsymbol{\theta}_H(t) &= \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D + \tau\tfrac{\lambda(t)}{1+\lambda(t)}\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right]\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau\tfrac{\lambda(t)}{1+\lambda(t)}\boldsymbol{\sigma}_D^T(\boldsymbol{\sigma}^T)^{-1}[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}]^T \\ &\quad + \tau^2\left(\tfrac{\lambda(t)}{1+\lambda(t)}\right)^2\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right](\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}]^T\end{aligned}$$

And symmetrically $\boldsymbol{\theta}_F = \boldsymbol{\sigma}_D - \tau\frac{1}{1+\lambda(t)}\left(\boldsymbol{\sigma}^{\mathbf{T}}\right)^{-1}[\frac{D_H}{S_H} \quad -\frac{D_F}{S_F}]^T$, so that

$$\begin{aligned}\boldsymbol{\theta}_F(t).\boldsymbol{\theta}_F(t) &= \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D - \tau\tfrac{1}{1+\lambda(t)}\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right]\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D - \tau\tfrac{1}{1+\lambda(t)}\boldsymbol{\sigma}_D^T(\boldsymbol{\sigma}^T)^{-1}[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}]^T \\ &\quad + \tau^2\left(\tfrac{1}{1+\lambda(t)}\right)^2\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right](\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}]^T\end{aligned}$$

Putting the pieces together, we get:

$$\begin{aligned}r(t) &= \rho + \mu_D - \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D + \tfrac{1}{2}\tfrac{\lambda-1}{1+\lambda}\left(-\mu_\lambda + \tfrac{\lambda}{1+\lambda}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda - \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D\right) \\ &\quad -\tau\tfrac{\lambda(t)-1}{1+\lambda(t)}\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right]\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D - \tfrac{1}{2}\tau^2\tfrac{1+\lambda^2(t)}{(1+\lambda(t))^2}\left[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}\right](\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\tfrac{D_H}{S_H} \quad -\tfrac{D_F}{S_F}]^T\end{aligned}$$

After a bit of algebra (using the expressions for μ_λ and $\boldsymbol{\sigma}_\lambda$ given in equations 16 and 17, this expression simplifies to

$$r(t) = \rho + \mu_D - \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D - \tau^2\frac{\lambda}{(1+\lambda)^2}\left[\frac{D_H}{S_H} \quad -\frac{D_F}{S_F}\right](\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}\begin{bmatrix}\frac{D_H}{S_H} \\ -\frac{D_F}{S_F}\end{bmatrix}$$

The Taylor expansion follows straightforwardly.

7.9 Proof of proposition 6 (portfolio choice)

Lemma A6 :

$$\begin{aligned}\begin{bmatrix}\frac{\alpha_{HH}S_H}{X_H} \\ \frac{\alpha_{HF}S_F}{X_H}\end{bmatrix} &= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau\frac{\lambda(t)}{1+\lambda(t)}\left(\boldsymbol{\sigma}^{\mathbf{T}}\boldsymbol{\sigma}\right)^{-1}\begin{pmatrix}\frac{D_H}{S_H} \\ -\frac{D_F}{S_F}\end{pmatrix} + \boldsymbol{\epsilon}_H \\ \begin{bmatrix}\frac{\alpha_{FH}S_H}{X_F} \\ \frac{\alpha_{FF}S_F}{X_F}\end{bmatrix} &= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau\frac{1}{1+\lambda(t)}\left(\boldsymbol{\sigma}^{\mathbf{T}}\boldsymbol{\sigma}\right)^{-1}\begin{pmatrix}-\frac{D_H}{S_H} \\ \frac{D_F}{S_F}\end{pmatrix} + \boldsymbol{\epsilon}_F\end{aligned}$$

Proof: We start from the intertemporal budget constraint (dropping subscripts, as the expressions are valid for both investors)

$$\xi(t)X(t) = E_t\left[\int_t^\infty \xi(s)(c(s) - e(s))ds\right]$$

$$
\begin{aligned}
\Rightarrow X(t) &= E_t\left[\int_t^\infty \frac{\xi(s)}{\xi(t)}(c(s)-e(s))ds\right] \\
&= E_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c(t)}{c(s)}(c(s)-e(s))ds\right] \\
&= c(t)E_t\left[\int_t^\infty e^{-\rho(s-t)}\left(1-\frac{e(s)}{c(s)}\right)ds\right] \\
&= c(t)\left[\frac{1}{\rho}-E_t\int_t^\infty e^{-\rho(s-t)}\frac{e(s)}{c(s)}ds\right]
\end{aligned}
$$

Since $e_H = \tau\alpha_{FH}D_H$, we can introduce the notation u_H and rewrite :

$$
\begin{aligned}
X_H(t) &= c_H(t)\left[\frac{1}{\rho}-\tau u_H(t)\right] \\
&= \frac{1}{\rho}\frac{D(t)}{1+\lambda(t)}\left[1-\tau\rho u_H(t)\right]
\end{aligned}
$$

From this expression, Ito's lemma implies that in $dX_H = \mu_{X_H}X_H dt + X_H\boldsymbol{\sigma}_{X_H}d\mathbf{W}$

$$
\boldsymbol{\sigma}_{X_H} = \boldsymbol{\sigma}_D - \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}_\lambda + \tau\boldsymbol{\sigma}_e
$$

where $\boldsymbol{\sigma}_e$ is related to the endowment term u_H.

Applying the martingale representation theorem like Cox and Huang [1989], we identify diffusion terms in equation 5 and get the following expressions for the domestic home investor's portfolios:

$$
\begin{aligned}
\begin{bmatrix} \frac{\alpha_{HH}S_H}{X_H} \\ \frac{\alpha_{HF}S_F}{X_H} \end{bmatrix} &= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D - \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_\lambda + \underbrace{\tau\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_e}_{\equiv\boldsymbol{\epsilon}_H} \\
&= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau\frac{\lambda}{1+\lambda}\left(\boldsymbol{\sigma}^T\boldsymbol{\sigma}\right)^{-1}\begin{bmatrix} \frac{D_H}{S_H} \\ -\frac{D_F}{S_F} \end{bmatrix} + \boldsymbol{\epsilon}_H \\
&= \boldsymbol{\sigma}^{-1}\boldsymbol{\theta}_H + \boldsymbol{\epsilon}_H
\end{aligned}
$$

Approximation of hedging term $\boldsymbol{\epsilon}_H$

$$
\begin{aligned}
e_H &= \tau\alpha_{FH}D_H \\
&= \tau\frac{D_H}{S_H}\alpha_{FH}S_H
\end{aligned}
$$

$\alpha_{FH}S_H$ is the amount that foreign investors invest in the domestic asset. At the order zero, investors hold the world market portfolio, therefore:

$$\begin{aligned} e_H &= \tau \frac{D_H}{S_{H0}} \frac{S_{H0}}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \frac{D_H}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \frac{D_H}{D} \frac{D}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \delta \rho X_F + o(\tau) \end{aligned}$$

$$\begin{aligned} \Rightarrow X_H(t) &= c_H(t) \left[\frac{1}{\rho} - E_t \int_t^{\infty} e^{-\rho(s-t)} \frac{e_H(s)}{c_H(s)} ds \right] \\ &= c_H(t) \left[\frac{1}{\rho} - \tau E_t \int_t^{\infty} e^{-\rho(s-t)} \delta(s) \frac{\rho X_F(s)}{c_H(s)} ds \right] + o(\tau) \end{aligned}$$

Furthermore, $X_F = \lambda X_H + o(1)$ and $c_H = \rho X_H + 0(1)$ imply

$$\begin{aligned} \frac{\rho X_F}{c_H} &= \rho \frac{\lambda X_H}{c_H} + o(1) \\ &= \lambda + o(1) \end{aligned}$$

Since for $s > t$, $\lambda(s) = \lambda(t) + o(1)$, we get

$$X_H(t) = c_H(t) \left[\frac{1}{\rho} - \tau \lambda_t \underbrace{E_t \int_t^{\infty} e^{-\rho(s-t)} \delta(s) ds}_{\equiv y_H(\delta(t))} \right] + o(\tau)$$

Using $c_H = \frac{D}{1+\lambda}$, applying Ito's lemma and identifying diffusion terms in (5), we derive

$$\begin{bmatrix} \frac{S_H \alpha_{HH}}{X_H} \\ \frac{S_F \alpha_{HF}}{X_H} \end{bmatrix} = \boldsymbol{\sigma}^{-1} \boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \left(\boldsymbol{\sigma}^T \boldsymbol{\sigma} \right)^{-1} \begin{bmatrix} \frac{D_H}{S_H} \\ -\frac{D_F}{S_F} \end{bmatrix} + \boldsymbol{\epsilon}_H + o(\tau)$$

where: $\boldsymbol{\epsilon}_H = \tau \lambda \frac{y_H y_F}{(y_H + y_F)^2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and respectively $\boldsymbol{\epsilon}_F = \tau \frac{1}{\lambda} \frac{y_H y_F}{(y_H + y_F)^2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

The expression given in proposition 6 follows immediately using $\frac{D_i}{S_i} = \frac{D_i}{S_{i0}} + o(1)$ and $\left(\boldsymbol{\sigma}^T \right)^{-1} = \left(\boldsymbol{\sigma}_0^T \right)^{-1} + o(\tau)$ (from proposition 3).

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CENTRE DE RECHERCHE / RESEARCH CENTER
AVENUE BERNARD HIRSCH
BP 50105 CERGY
95021 CERGY PONTOISE CEDEX
FRANCE
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