Cheap Home Goods and Persistent Inequality*

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Abstract

There exists a large literature which shows that public education is favorable for growth because it increases the level of human capital and at the same time it tends to produce a more even income distribution. More egalitarian societies are also associated with less social conflicts and individuals have a lower tendency to report themselves happy when inequality is high. Therefore it is important to study the reasons of why countries do not adopt and implement a compulsory and efficient public educational system. It might be that education is related to social status and therefore the elite might oppose the development of a strong public education system or any reform that would threat their political power. We show that one of this social status is the specialization of skilled workers in high-paid jobs and the abundance of unskilled workers in the production of cheap "home goods" in the market, such as painting and cleaning a house, babysitting and/or cooking. We emphasize the role of general equilibrium price adjustments to show that depending on the level of inequality, the elite might prefer an economy with a positive and "high" cost of education than an economy where skills are freely provided.

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"...in the traditional caste system groups in the population were condemned for life, and their descendants in perpetuity after them, to such task as the cleaning of latrines and the removal of dead carcasses... Apart from slavery, it is hard to think of system with greater inequality of opportunity, and the results are also most unequal."

– Mancur Olson (1982: 156)

1 Introduction

When markets function perfectly inequality reflects differences in innate ability to acquire skills, to invest in capital, and/or to manage a labor force. This is an efficient inequality. In this case, wealth, social status, caste, and/or family connections would not affect individual outcomes. However, as argued by Banerjee (2006), markets do not *"work anywhere close to perfect"*. Empirical evidences, for instance, show that credit access and borrowing interest rates depend on wealth and social status (see Banerjee (2006) for some examples). In human capital investment, parents cannot borrow against their children's future income. Consequently, poor individuals under-invest in both physical and human capital and an inefficient inequality persists over time.[1]

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[1]There is no equality of opportunities when wealth and social status affect outcomes.

This idea is formalized well in a seminal article by Galor and Zeira (1993) who emphasize the role of credit market imperfections and non-convexity in human capital investment in the persistence of income inequality. Recent empirical evidences (see Easterly (2005)) have shown that *"inequality does cause underdevelopment."* In the model presented in Galor and Zeira (1993), for instance, it is straightforward to design a policy to reduce inequality that is Pareto improving and would also increase development.[2] Contrary to the traditional *efficiency-equity* trade-off, such policies might increase efficiency and improve income distribution. Therefore, a complementary and important question is: Why countries do not adopt policies to improve the functioning of their credit market and/or their educational system? We, however, present a model economy based on Doepke and Zilibotti (2005) to show that, depending on the level of initial inequality, it is a vested interest of skilled parents to erect barriers to the acquisition of skills in order to block poor parents to educate their children and keep the price of some home goods at low levels.

The novel about our result is the following. As in Ríos-Rull (1993), there are two differentiated consumption goods in our economy. One good (good Y) is produced in the market with skilled labor only (ex., cars, computers and others). The other one (good Z) can be produced at home or in the market with both skilled and unskilled labor (ex., babysitting, cleaning a house, painting a house, and others). Skilled and unskilled agents have similar productivity in the production of the "home" good. Parents care about the consumption of both goods and about the average discounted utility of their children. Acquisition of skills is costly and parents cannot borrow against their children's future income. When inequality is high, skilled parents educate their children, specialize in the production of good Y and do not work at home. They buy good Z in the market. On the other hand, unskilled parents do not educate their children and do not buy good Z in the market. They instead produce it at home. They will work in the market and at home. Inequality will therefore persist over time, as in Galor and Zeira (1993).

In addition, we show that depending on the initial inequality, skilled parents might prefer an economy with a positive and "high" cost of education than an economy where skills are freely provided.[3] If education is freely provided, then unskilled parents will educate their children and general equilibrium price adjustments implies that future prices of the "home good" (good Z) will increase and the skill wage premium will decrease. Since parents value the future utility of their children, the welfare of skilled parents might be reduced. Depending on the political power of skilled agents, they therefore might block any policy that decreases the cost of skill acquisition.

This paper is related to two strands of literature. The first literature focuses on the intergenerational transmission of inequality and its persistence. It emphasizes the role of credit market imperfections and some form of non-convexity in human and physical capital investment to show how inequality persists over time (e.g., Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira (1993), and Ray and Strefert (1993)).[4] Our analysis complements this literature by analyzing when it is optimal for some groups of society to block policies and

[2]Government could issue bonds to finance education. The debt would be paid by the skilled descendants of unskilled parents. This policy is Pareto improving as long as the tax paid by these agents are not higher than the difference between the skilled and unskilled wages. Alternatively, multilateral agencies might provide financial aids for developing countries to improve their public education systems to institute a free, compulsory education system or to improve the functioning of the credit market.

[3]We show that depending on the level of inequality, there is a threshold value of education where unskilled parents are just indifferent between sending or not their children to school.

[4]Interestingly, Moav (2005) shows that inequality might be persistent even when the schooling choice is convex. He differs from the above literature because in his model individual's productivity as teachers increases with their own human capital. Another related view emphasizes the tendency of the market mechanism and imperfections in the credit market to create inequality. Mookherjee and Ray (2002, 2003) show (see also Ljungqvist (1993)) that if several occupations requiring different levels of skills are necessary, wages must adjust to force separation in choices even if all individuals are *ex-ante* identical. Current individuals will have the same payoffs, but since credit market are imperfect, future generations will have different payoffs and inequality will persist.

institutions that might increase development and decrease inequality. In this respect, Acemoglu (2005), Acemoglu and Robinson (2000), Bourguignon and Verdier (2000), Doepke and Zilibotti (2005), Grossman and Kim (1999), and Parente and Prescott (1999, 2000) are closest to our work. They all emphasize the conflicts of interest among social classes in the persistence of high inequality or inefficient technology.[5] We differ from this set of papers because we explicitly consider the role of cheap home goods (i.e., general equilibrium price adjustments) in the persistence of income inequality.[6] According to our knowledge we were the first to emphasize and formalize the role of cheap home goods, that can be produced either at home or in the market, in the persistence of income inequality.[7]

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the equilibrium and derive the main results. Section 4 presents some possible extensions of our model economy. Section 5 provides some concluding remarks.

2 The model

The model economy is populated by overlapping generations of agents with differentiated skill levels. For simplicity, there are only two skill levels: unskilled and skilled, $h \in \{S, U\}$. As in Doepke and Zilibotti (2005), each household consists of one parent and her children, where the number of children depends on the parent's earlier fertility decisions. All decisions are made by adults and they are made sequentially. Soon a parent die, her children become adult. As soon as they become adult, agents make their fertility decision. There are only two family sizes, large (grande) and small (petite), $n \in \{G, P\}$. Then, agents decide on the education of their children, consumption, and labor supply.[8]

There are two consumption goods in this economy, Y and Z. Good Y is produced in the market with skilled labor only. Good Z, on the other hand, can be produced at home or in the market with both skilled and unskilled labor. Skilled and unskilled agents have similar productivity in the production technology of good Z. Children might either work at home ($e = 0$) or go to school ($e = 1$). Working children provide $l \in (0, 1)$ units of unskilled labor at home. When they become adults working children become unskilled workers, while those that went to school become skilled workers. There is an education cost ϕ per child and children that attend school do not supply any labor.

Production technologies

The production side follows closely Ríos-Rull (1993). There are two market technologies in this economy. We assume that there is a continuum of firms in each sector that are competitive in output and factor markets. Since technologies exhibit constant returns to scale profits are zero and firm ownership in unimportant.

Skilled labor sector: Technology in the sector Y uses only skilled labor. The production process in this sector is represented by:

$$Y = AL_S, \quad A > 0, \tag{1}$$

[5] Differently, Galor and Moav (2006) show that due to complementarity between physical and human capital in production, capitalists might have incentives to support public education which benefits directly the working class.

[6] Home production has been explicitly treated in general equilibrium models in the study of business cycles (e.g., Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991), and Ríos-Rull (1993)), and economic development (e.g., Parente, Rogerson, and Wright (2000) and Greenwood, J. and A. Seshadri (2005)).

[7] This does not mean that our view is superior to the existing literature. We also do not quantify how important is the role of cheap home goods in the persistency of income inequality. Our analysis is qualitative.

[8] Another way to set this is to assume that there are two periods during adulthood. One when adults (young adulthood) choose only their family size, and another (old adulthood) when they make the other relevant decisions.

where L_S corresponds to the amount of skilled labor units employed in the production of good Y, and A is a positive productive factor.

Unskilled labor sector: Good Z can be produced in the market or at home, where all agents have the same productivity. The market technology for good Z is represented by

$$Z^M = \delta B L^M, \quad \delta \in (0,1),\ A > B > 0, \tag{2}$$

where L^M corresponds to hours employed by the representative firm in the production of good Z, and B is a positive constant.

Agents are more productive at home in the production of good Z than in the market, such that

$$Z^H = B L^H, \quad A > B > 0. \tag{3}$$

The units of Z produced at home, however, cannot be transferred, or used to buy market consumption goods.

Let good Y be the numerarie, q be the market price of good Z, and w_U and w_S be the unskilled and skilled wage. All prices are in terms of the consumption good Y. Profit maximization implies that $w_S = A$, and $\frac{w_U}{q} \geq \delta B$, with equality if $Z^M > 0$. Given the linearity of the production functions, we just need one equilibrium market condition to define w_U and then q. Notice that given that skilled agents can also work in the unskilled sector, it implies that the skilled wage cannot be lower than the unskilled wage.

Households:

Let $V_{nh}(\Omega)$ denotes the utility of an adult with n children and skill h and let Ω be the aggregate state of the economy, which is explained below. Parents care about the consumption of goods Y and Z, and about the average discounted utility of their children, which is discounted by $\gamma \in (0,1)$. Let c and z be the household consumption of good Y and Z, respectively. Let a be the time spent at home in the production of good Z. The problem of an adult with n children and skill h is represented by

$$V_{nh}(\Omega) = \max_{e,c,z^M,a} \{\ln c + \alpha \ln z + \gamma[e \max_{n\in\{G,P\}} V_{nS}(\Omega') + (1-e) \max_{n\in\{G,P\}} V_{nU}(\Omega')]\} \tag{4}$$

Subject to

$$c + q(\Omega) z^M + \phi e n \leq w_h(\Omega)(1-a), \tag{5}$$

$$z = z^M + Ba + Bn(1-e)l, \tag{6}$$

$$e \in \{0,1\}, c \geq 0, z^M \geq 0, a \in [0,1], h = U, S. \tag{7}$$

It is important to highlight that fertility n does not enter directly into the utility function. As in Doepke and Zilibotti (2005), parents care only about the average utility of their children. This implies that they will have a large family size only if they expect to send their children to work. On the other hand, since education is costly, parents have a small family only if they expect to send their children to school. Therefore, the quantity-quality tradeoff here is: the higher present consumption if their children work versus the higher future average utility if their children attend school.

The budget constraint (equation (5)) states that the sum of expenditures on consumption of good Y and good Z, and on education costs cannot exceed labor income. Equation (6) states that good z can be bought in the market or can be produced at home. The last term on the right hand side of (6) implies that children who do not go to school help their parents in the production of good z at home. Equation (7) shows the constraints on choice variables. Notice that education is a non-convex choice.

Assume that initially there is a number X_{GU0} of unskilled parents with a large family size and a number X_{PS0} of skilled parents with a small family size. Therefore, initially there is neither unskilled parents with a small family size nor skilled parents with a large family size. In fact, as we will show, it is optimal for unskilled parents and skilled parents to choose a large and a small family size, respectively.

Assumption 1: $\alpha > lG$.

Assumption 1 defines an upper bound on the quantity of good Z produced by children.

Proposition 1 *Consider the problem of an unskilled parent, $h = U$, with a large family size, $n = G$. Under assumption 1:*

i. $z^M_{GU} = 0$ *and* $a_{GU} > 0$.

Proof. See appendix A ■

Proposition 1 suggests that unskilled parents with a large family size do not buy good Z in the market. They instead produce it at home. Therefore, unskilled parents work in the market either to buy consumption good Y or/and to pay for their children's education. Assumption 1 guarantees that unskilled parents with working children will also work at home. If the productivity of their children were too high ($lG > \alpha$) then only their children will produce good Z at home.

In order to investigate the problem of skilled parents, let's assume that education costs are small relatively to the skilled market wage. Otherwise, there will be no skilled agents in the economy.

Assumption 2 $A > \phi P$.

Proposition 2 *Consider the problem of a skilled parent, $h = S$, with a small family size, $n = P$. It can be show that under assumption 2.*

i. If $q(\Omega) < \frac{A}{B}$, *then* $a_{PS} = 0$, $z^M_{PS} > 0$. *Otherwise,* $a_{PS} > 0$, $z^M_{PS} = 0$.

ii. If $e_{GU} = 1$, *then* $e_{PS} = 1$.

Proof. See appendix B ■

Item (i) implies that if the market price of good Z is small relatively to the skilled wage, i.e., if $q(\Omega) < \frac{w_S}{B} = \frac{A}{B}$, then skilled parents will not produce good Z at home, they will instead buy it in the market. Therefore, in order to have market demand for good Z its price cannot be "too" high. Otherwise skilled agents will produce it at home. Notice that this also implies an upper bound for the unskilled wage, $w_U(\Omega) < \delta w_S$. The intuition is straightforward: If the price of good Z is low enough, it is optimal for skilled agents to specialize in the market production of good Y and do not produce good Z at home. Item (ii) suggests that if unskilled parents educate their children, then it is also optimal for skilled agents to educate their children. Therefore, in order to have production of good Y skilled agents must educate their children. Recall that X_{GU0} and X_{PS0} are the initially number of unskilled agents with a large family size and skilled agents with a small family size, respectively. The number of unskilled agents and the number of skilled agents evolve according to:

$$X'_{GU} = GX_{GU}(1 - e_{GU}), \tag{8}$$

$$X'_{PS} = PX_{PS} + GX_{GU}e_{GU}. \tag{9}$$

The state variable, Ω, that describes the position of the economy in each period of time corresponds to the ratio of unskilled and skilled workers, $\Omega = \frac{X_{GU}}{X_{PS}}$. As we will show shortly, the unskilled wage depends negatively on Ω. The skill wage premium $(\frac{w_S}{w_U(\Omega)})$, on the other hand, depends positively on the state variable. Therefore, we can view Ω as a measure of income inequality. From (8) and (9), we have that

$$\Omega' = \frac{X'_{GU}}{X'_{PS}} = \frac{GX_{GU}(1-e_{GU})}{PX_{PS} + GX_{GU}e_{GU}} = \frac{G(1-e_{GU})}{P + G\Omega e_{GU}}\Omega. \tag{10}$$

Given the law of motion of the aggregate state variable, we define the recursive competitive equilibrium as follows.

Definition: *A Recursive Competitive Equilibrium for this economy consists of value functions V_{nh}, policy function (e_{nh}, c_{nh}, z^M_{nh}, a_{nh}), for $nh \in \{GU, PS\}$, price function (w_S, w_U, q), and a law of motion $\Omega' = F(\Omega)$ for the aggregate state variable such that*

i. V_{nh} satisfy Bellman equation (4) and e_{nh}, c_{nh}, z^M_{nh}, a_{nh} are the associated policy functions.

ii. $w_S = A$ and $\frac{w_U}{q} \geq \delta B$. Moreover, good Y market clears

$$X_{GU}c_{GU} + X_{PS}c_{PS} + \phi G X_{GU}e_{GU} + \phi P X_{PS}e_{PS} = AX_{PS}(1-a_{PS}) \tag{11}$$

iii. Law of motion $F(\Omega)$ satisfies equation (10).

The equilibrium condition (11) states that the demand for good Y should be equal to the supply of good Y. The demand consists of total consumption of good Y and education expenditures by skilled and unskilled households. Recall that education costs are in terms of good Y.

3 Equilibrium Analysis

Now let's analyze the equilibrium with fixed policies. There might be two equilibria for this economy:[9]

i. Just skilled parents educate their children.

ii. Skilled and unskilled parents educate their children.

Just skilled parents educate their children: Let's consider the first equilibrium in which only skilled parents educate their children. See appendix C, part 1, for a fully characterization of this equilibrium path. In this equilibrium, there will be market production for good Z in all periods. Skilled workers will specialize in the production of good Y, devoting a zero fraction of their time endowment to the home production of good Z. They will instead buy good Z in the market. Using condition (11), it can be shown that the unskilled wage will be given by:[10]

$$w_U(\Omega) = \min\{\delta w_S, \frac{\alpha(A-\phi P)}{\Omega(1+lG)}\}. \tag{12}$$

The ratio of unskilled to skilled workers will grow at rate $\frac{G}{P} - 1 > 0$, i.e., $\Omega' = \frac{G}{P}\Omega$. Therefore, at some time period $t \geq 0$, the measure of unskilled agents relatively to skilled agents will be

[9] Notice that item (*ii*) of proposition 2 rules out the equilibrium in which only unskilled parents educate their children.

[10] Appendix C, part 1, shows that we can define a $\phi G < w^*_U$, such that for all $w_U(\Omega) < w^*_U$ it is optimal for an unskilled parent to not educate their children when all other unskilled parents are not sending their children to school. For some parameter values, we can also guarantee that w^*_U is finite and $w_U(\Omega) < w^*_U < \delta w_S$.

sufficiently large, i.e., $\Omega = \frac{X_{GU}}{X_{PS}} > \frac{(A-\phi P)}{\delta A} \times \frac{\alpha}{(1+\alpha)}$, such that $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$. Since $G > P$, we have that the skill wage premium $(\frac{w_S}{w_U(\Omega)})$ and therefore inequality will increase over time. The market price of good Z will, on the other hand, decrease over time. Inequality will be persistent for two reasons: (i) Unskilled parents will have a larger family size than unskilled agents; and (ii) they will not educate their descendants, increasing therefore the skill wage premium. Notice that if parents have the same family size ($G = P$), then inequality will still be persistent, but the skilled wage premium would in this case be constant. It can be shown that the growth rate of output per capita will be lower the higher is the differential in fertility $(\frac{G}{P})$ between unskilled and skilled parents,[11] which is consistent to the theory and data presented by de la Croix and Doepke (2001).

Skilled and unskilled parents educate their children: Let's consider the second equilibrium in which both skilled and unskilled agents educate their children. See appendix C, part 2, for a fully characterization of this equilibrium path. In this case, there will be market production of good Z only in the first period. From the second period on, all agents will be skilled and all will devote a positive fraction of their time endowment to the home production of good Z. It can be shown that the initial unskilled wage is:[12]

$$w_U(\Omega) = \min\{\delta w_S, \frac{\alpha(A - \phi(P + \Omega G))}{\Omega}\}. \tag{13}$$

Since from the second period on there is only skilled agents, we have that $\Omega' = 0$.[13] There will be no inequality among agents and skilled agents will have to work at home in the production of good Z. From the second period on, output per worker will be constant over time.[14] Therefore, as in Banerjee and Newman (1993) and Galor and Zeira (1993), the initial conditions determine final outcomes.

We now investigate whether it is optimal for skilled parents to erect barriers to the acquisition of skills.

Proposition 3 *Let Ω^* be the measure of unskilled to skilled agents such that*

$$\gamma \ln(\frac{\Omega}{\alpha}) = \alpha \ln(1 + lG) + \frac{\gamma}{1-\gamma} \ln(\frac{P}{G}). \tag{14}$$

Then, for every $\Omega > \Omega^*$ $\exists$ a unique $\bar{\phi}(\Omega) \in [0, \frac{A}{P + \frac{(1+\alpha)}{\alpha}\Omega G})$, such that:

i. $\phi < \bar{\phi}(\Omega) \Rightarrow e_{GU} = 1$.

ii. $\phi > \bar{\phi}(\Omega) \Rightarrow e_{GU} = 0$.

Proof. See appendix D ■

Item (*i*) and (*ii*) from proposition 3 and figure 1 illustrate the decision of unskilled parents to educate or not their children. They show that it is optimal for all unskilled parents to send

[11] Gross domestic output (GDP) per worker will be $Y + q(\Omega)Z^M = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}]\frac{1}{1+\Omega}$. Notice that here, as in the National Accounts, we are considering only the value of market production. If we also consider the value of home production, then GDP per worker will be $Y + q(\Omega)(Z^M + Z^H) = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}(1 + \frac{\alpha}{\delta})]\frac{1}{1+\Omega}$, which also decreases over time. We, however, could have assumed that the productivity factors (A and B) and education costs ϕ are all increasing at some exogenous rate μ, and then output would grow over time as in the standard neoclassical growth model. This, however, would not add any new insights in our analysis.

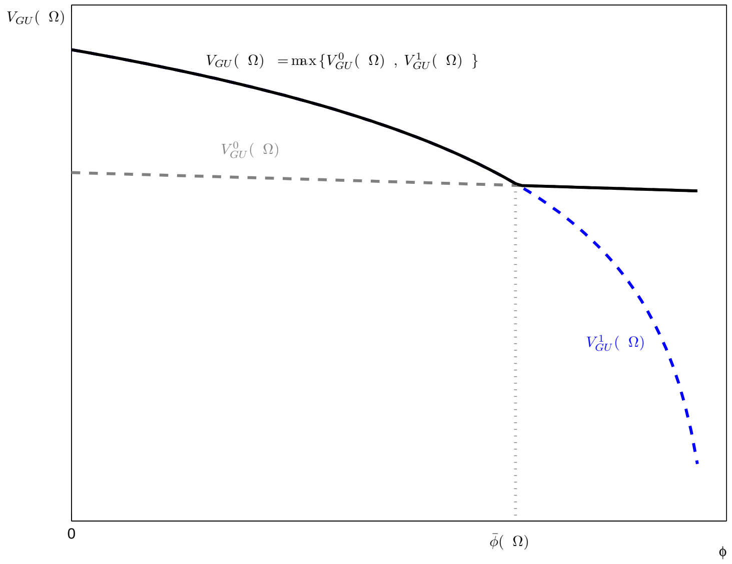
[12] Appendix C, part 2, shows that depending on parameter values there exists a $\phi G < w_U^\bullet < \delta w_S$, such that for all $w_U(\Omega) > w_U^\bullet$ it is optimal for an unskilled parents to send their children to school when all other parents are also sending their children to acquire skills.

[13] The skill wage premium will be at its lowest value $\frac{w_S}{w_U(\Omega')} = \frac{1}{\delta}$.

[14] Output per worker in the initial period is $Y + q(\Omega)Z^M = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}]\frac{1}{1+\Omega}$, while in the second period on it will be $Y = \frac{A+\alpha\phi P}{1+\alpha}$.

their children to school only if the cost of education is sufficient small ($\phi < \bar{\phi}(\Omega)$). From the point of view of a parent it is important to highlight that even if the direct cost of education is zero ($\phi = 0$), there still exists an opportunity cost (i.e., the foregone income from child labor) of sending their children to school.[15] This opportunity cost increases with the child labor relative productivity lG. In addition, the higher the fertility differential ($\frac{G}{P}$), the higher is the incentive for unskilled parents to educate their children. This is because, if they do not educate their children, then the equilibrium unskilled wage evolves according to $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$,[16] and future unskilled market income will decrease over time, as well as the future utility of their descendants.

Figure 1: Optimal schooling choice: Unskilled Parents



For skilled parents there are two opposing effects associated with an increase in education costs ϕ. First, a higher direct cost of education implies a lower present consumption and therefore lower present utility. On the other hand, a higher education cost implies a higher demand for good Y, and therefore an increase in the skill wage premium $\frac{w_S}{w_U(\Omega)}$. This allows skilled agents to purchase a higher amount of good Z in the market. When unskilled agents do not send their children to school ($\phi > \bar{\phi}(\Omega)$), the effect on the skilled wage premium is small, and the value function associated with a skilled parent $V^1_{PS}(\Omega)$ is continuous and strictly decreasing in $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$.

When unskilled parents decide to send their children to school ($\phi < \bar{\phi}(\Omega)$) this reinforces the increase in the demand for good Y and therefore the effect of ϕ on the skill wage premium. This effect is stronger when inequality (Ω) is high. This effect will, however, be present only in the first period. From the second period on, there will be only skilled agents and for future generations, utility will be strictly decreasing with education costs ϕ. Therefore, it can be shown that, in an equilibrium when both skilled and unskilled parents send their children to acquire skills, there exists a sufficient large altruism factor γ, such that the value function of skilled parents $V^{1,a'>0}_{PS}(\Omega)$ is continuous and strictly decreasing with $\phi \in [0, \bar{\phi}(\Omega))$ where $\bar{\phi}(\Omega) \in [0, \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G})$. Define $\bar{\phi}(\Omega)_{\max} = \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G}$. It can be shown that if inequality is sufficiently high, skilled agents strictly prefer an education cost $\phi = \bar{\phi}(\Omega)_{\max} > 0$, than a policy in which the acquisition of skill is freely provided ($\phi = 0$).

[15]This explains the first term on the right hand side of (14).
[16]This explains the second term on the right hand side of (14).

Proposition 4 *Let Ω^{**} be the measure of unskilled to skilled agents defined implicitly by*

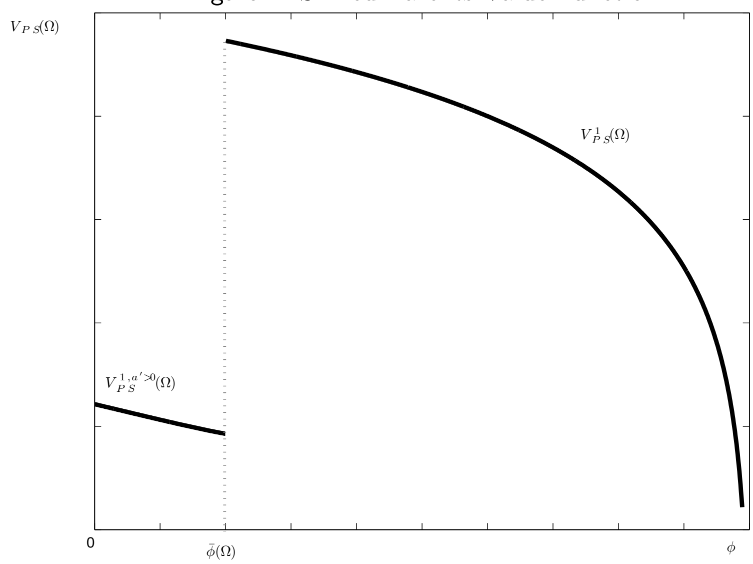
$$\alpha\gamma \ln(\frac{\delta\Omega}{\alpha}) + \alpha \ln(1 + lG) + \frac{\alpha\gamma}{1-\gamma} \ln(\frac{G}{P}) = \ln(1 + \frac{P}{\frac{(1+\alpha)}{\alpha}\Omega G}). \quad (15)$$

Then, for any $\Omega > \max\{\Omega^*, \Omega^{**}\}$ skilled agents strictly prefer education costs $\phi = \bar{\phi}(\Omega)_{\max}$ than $\phi = 0$.

Proof. See appendix D ■

Figure 2 describes well proposition 4. It shows that the value function of skilled parents is not necessarily continuous at $\bar{\phi}(\Omega)$. In addition, skilled parents might prefer an education cost $\phi > \bar{\phi}(\Omega)$, such that unskilled parents do not educate their children than a lower education cost $\phi < \bar{\phi}(\Omega)$, such that unskilled parents send their children to acquire skills. If $\phi < \bar{\phi}(\Omega)$, then future generations of skilled parents will have to work at home in the production of good Z, since future market prices of good Z will be at its maximum value (δw_S) and it is cheaper to produce good Z at home. This implies that depending on the level of inequality it is a vested interest of skilled agents to erect barriers to the acquisition of skills.

Figure 2: Skilled Parents Value Function



Consider, for instance, that multilateral agencies provide financial aids for developing countries to improve their public education systems. These financial aids might help to institute a free, compulsory education or might be in a form of subsidies to poor families to educate their children.[17] What proposition 4 and figure 2 show is that, depending on the level of inequality, this policy is not necessarily welfare improving under the Pareto criterion. General equilibrium price adjustments implies that future skill premiums would reduce as the children of unskilled parents attend schools. In addition, the future price of the "home good" (Z) will increase. Future skilled agents might have to work at home to produce good Z instead of buying it in the market. Since parents value the future utility of their children, the welfare of skilled parents might be reduced.[18]

[17] Notice that this policy does not require tax increases.

[18] Notice that with this policy the skill wage premium in the first period might increase, since, as argued before, there is a higher demand for good Y. This is because education costs ϕ is in terms of the consumption good Y. In addition, skilled parents do not have to work at home in the period. They can still buy it in the market.

Observe, however, that this policy is Pareto improving in a model similar to the one presented by Galor and Zeira (1993).[19] In their model, parents utility depends on the size of bequest and not on their offsprings' utility. The policy analyzed above is, however, also Pareto improving in the Galor and Zeira model even if parents utility depends on the utility of their children. The reason is that as long as the skilled current and future wages do not change, the policy will be Pareto improving, even though the skilled wage premium is reduced. There is no "home good" in the Galor and Zeira model. In our model, a reduced skilled wage premium implies a higher market price of good Z.

4 Extensions

In order to derive the main results of the last section, the model was kept at a very simple level. There are, however, some extensions that would enrich some of our results. Firstly, we abstract from capital accumulation. Capital accumulation is, however, key in the analysis of economic development. If, for instance, the skilled labor is more complementary to capital than the unskilled one, then the skilled wage premium would increase with capital accumulation. Skilled agents would therefore have an additional motive to block policies that provide education at low costs. Reinforcing some of our results.

We also abstract from improvements in technology in the market and at home. Improvements in the technology in the "home sector", such as the introduction and development of home appliances, that save time in household chores would increase the opportunity cost of buying the "home good" in the market.[20] This would decrease the market price of home goods and the unskilled wage. Unskilled parents will have higher incentives to send their children to school, since their future income will be lower.

As in Doepke and Zilibotti (2005), the cost of acquiring skills is exogenous in our model economy. The education cost ϕ per child could be endogenized as, for instance, in de la Croix and Doepke (2001), where this cost is defined in units of time of teachers who have the average human capital in the population. For our purpose, however, it is important to consider not only the direct cost of acquiring skills, but also some indirect costs, such as availability of schools and entrance policies in universities. Sokoloff and Engerman (2000: 230) states that *"where there existed elites who were sharply differentiated from the rest of the population on the basis of wealth, human capital, and political influence, they seem to have used their standing to restrict competition."*

Finally, the political power is also not endogenous in our model (see, for instance, Bourguignon, F. and T. Verdier (2000)). Notice that if the political power depends on the vote of the median agent, then countries would improve their public education system and inequality will decrease. However, if the political power is in the elite's hands, then improvements in the public education system will be slow and inequality will persist.

Though all extensions are important to understand the evolution of policies and institutions, they are not key to show how some cheap home goods benefits part of the society and why some individuals might block policies that are beneficial for aggregate growth and development.[21]

[19] In the model described by Doepke (2004), this policy is also not Pareto improving. In his model there is only one good which can be produced with skilled or unskilled labor with just one technology (i.e., $Y = F(L_S, L_U)$). Changes in the supply of unskilled labor will then affect the skilled wage. Notice, however, that in Doepke's model the unskilled wage might be higher than the skilled one.

[20] This is emphasized by Greenwood, Seshadri and Yorukoglu (2005) in their model of female labor force participation.

[21] Notice that the extensions suggested are important to understand why some countries started to invest in their public education, while others have a poor education system. For instance, Alston and Ferri (1993) show that the end of Paternalism in the U.S. South is associated to the mechanization of the cotton harvest in the 1950's.

5 Concluding Remarks

There exists a large literature which shows that public education is favorable for growth because it increases the level of human capital and at the same time it tends to produce a more even income distribution.[22] More egalitarian societies are also associated with less social conflicts and individuals have a lower tendency to report themselves happy when inequality is high (e.g., Alesina, DiTella, and MacCulloch (2004)). Therefore it is important to study the reasons of why countries do not adopt and implement a compulsory and efficient public educational system. It might be that education is related to social status and therefore the elite might oppose the development of a strong public education system or any reform that would threat their political power. We contribute to this literature by showing that one of this social status might be the specialization of skilled workers in high-paid jobs and the abundance of unskilled workers in the production of some cheap "home goods" in the market, such as painting and cleaning a house, babysitting and/or cooking. Unskilled workers will, however, have to work at home and in the market with low-paid jobs to acquire "market goods". We emphasize the role of general equilibrium price adjustments to show why the elite might erect barriers to policies that improve the public education system. The higher the unskilled to skilled labor ratio, the lower is the relative price of the "home good". We show that, depending on the level of inequality, the elite might oppose policies that improve the education system even if there is no tax increases to finance such policies.

We conclude by exposing some empirical narratives consistent to our results. Sokoloff and Engerman (2000) show how the "elite" of some countries protected their *status quo* by investing poorly in primary schooling or/and by erecting barriers in the right to vote and other privileges. According to them, the degree of political power of elites were indeed associated to the inequality in wealth and human capital in the society. Recently Easterly (2005), using cross-countries data, shows that not only inequality cause underdevelopment, but it is also a significant and independent barrier to high schooling. In his study of inequality and development, Mancur Olson (1982: 162-163) exemplifies well how skilled agents might erect barriers to the acquisition of skills: *"…The mine owners and management needed labor and naturally preferred to secure it at low wages rather than high wages. Since Africans had few other opportunities outside the traditional sector of African society, they were often available at low wages... European workers were employed in the mines mainly as foreman and skilled and semi-skilled laborers. It was far clear that the far-cheaper African laborers could at very little cost soon be taught the semi-skilled jobs and the employers naturally coveted the savings in labor cost that this would bring."* However, the Mines and Work Acts of 1911 and 1926 ("Colour Bar Acts") constrained employers in their use of African labor in semi-skilled and skilled jobs. *"The denial of various skilled and semi-skilled jobs to Africans not only raised the wages of the European workers, but it also crowded more labor into areas that remained open to Africans, making the wages there lower than they would otherwise be."* Another empirical evidence is the paternalism, a system of social control in the U.S. South that emerged in the late 19th century and characterized the American South in the first half of the 20th century. Alston and Ferri (1993) argue that the paternalism[23] comprised a variety of laws and practices, such as low level of expenditure on education and the exclusion of blacks and poor whites from the electoral process. Landowners also prevented the appearance of public welfare programs that could substitute this system of social control until the mechanization of the cotton harvest in the 1950's.

[22]See Galor and Zeira (1993). For a recent reference, see Doepke (2004).

[23]They define it as an implicit contract in which workers trade "dependable" labor services in exchange for housing, credit and protection.

References

Acemoglu, D. (2005). "The Form of Property Rights: Oligarchic vs. Democractic Societies," *Mimeo*, MIT.

Acemoglu, D and J. Robinson (2000). "Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective," *Quarterly Journal of Economics*, **115** (4), 1167-1199.

Aghion, P. and P. Bolton (1997), "A theory of trickle-down growth and development", *Review of Economic Studies*, **64** (2), 151-172.

Alesina, A., R. DiTella and R. MacCulloch (2004). "Inequality and Happiness: Are Europeans and Americans Different?" *Journal of Public Economics,* **88**, 2009-2042

Alston, L. and J. Ferrie (1993). "Paternalism in Agricultural Labor Contracts in the U.S. South: Implications for the Growth of the Welfare State," *American Economic Review,* **83** (4), 852-876.

Banerjee, A. and A. Newman (1993). "Occupational Choice and the Process of Development," *Journal of Political Economy,* **101**, 274-298.

Banerjee, A. (2006). "Inequality and Investment." In: *World Development Report 2006*, Eds. F. Ferreira and M. Walton, Washington DC: World Bank.

Bourguignon, F. and T. Verdier (2000). "Oligarchy, Democracy, Inequality, and Growth," *Journal of Development Economics*, **62** (2), 285-313.

De La Croix, D. and M. Doepke (2001). "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review,* **93**(4), 1091-1113.

Doepke, M. (2004). "Accounting for Fertility Decline During the Transition to Growth," *Journal of Economic Growth,* **9**, 347-383.

Doepke, M. and F. Zilibotti (2005). "The Macroeconomics of Child Labor Regulation," *American Economic Review*, **95** (5), 1492-1524.

Easterly, W. (2005). "Inequality Does Cause Underdevelopment," *Mimeo*, New York University.

Galor, O. and O. Moav (2006). "Das Human-Kapital: A Theory of the Demise of the Class Structure," *Review of Economic Studies,* **73** (1), 85-117.

Galor, O. and J. Zeira (1993). "Income Distribution and Macroeconomics," *Review of Economics Studies* **60**, 35-52.

Greenwood, J. and A. Seshadri (2005). "Technological Progress and Economic Transformation," Eds. P. Aghion and S. N. Durlauf *Handbook of Economic Growth,* Vol 1B, Amsterdam: Elsevier North-Holland, 1225-1273.

Greenwood, J., A. Seshadri and Yorukoglu, M. (2005), "Engines of liberalization," *Review of Economic Studies* **72** (1), 109133.

Grossman, H. and M. Kim (1999). "Educational Policy: Egalitarian or Elitist?," *Mimeo*, Brown University.

Ljungqvist, L. (1993). "Economic Undervelopment: The Case of Missing Market for Human Capital," *Journal of Development Economics* **40**, 219-239.

Moav, O. (2005). “Cheap Children and the Persistence of Poverty,” *Economic Journal,* **115** (1), 88-110.

Mookherjee, D. and D. Ray (2002). “Is Equality Stable?” *American Economic Review Papers and Proceedings*, **92** (2), 253-259.

Mookherjee, D. and D. Ray (2003). “Persistent Inequality,” *Review of Economic Studies*, **70**, 369-393.

Olson, M. (1982). *“The Rise and Decline of Nations: Economic Growth, Stagflation, and Social Rigidities”*, Yale University Press.

Parente, S. and E. Presott (1999). “Monopoly Rights: A Barrier to Riches,” *American Economic Review*, **89** (5), 1216-1233.

Parente, S. and E. Prescott (2000). *“Barriers to Riches”*, The MIT Press.

Parente, S., R. Rogerson and R. Wright (2000). “Homework in Development Economics: Household Production and the Wealth of Nations,” *Journal of Political Economy,* **108** (4), 680-687.

Ray, D. and P. Streufert (1993). “Dynamic Equilibria with Unemployment Due to Undernourishment,” *Economic Theory*, **3** (1), 61-85.

Ríos-Rull, J-V. (1993). “Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach,” *American Economic Review*, **83** (4), 893-907.

Sokoloff, K. and S. Engerman (2000). “Institutions, Factor Endowments, and Paths of Development in the New World,” *Journal of Economic Perspectives*, **14** (3), 217-232.

A Unskilled Households

Consider the problem of a unskilled household with a large family size.

$$\begin{aligned} V_{GU}(\Omega) &= \max_{e\in\{0,1\},z^M,a}\{\ln(w_U(\Omega)(1-a)-q(\Omega)z^M-\phi eG)+\alpha\ln(z^M+Ba+BlG(1-e)) \\ &+ \gamma[e\max_{n\in\{G,P\}}V_{nS}(\Omega')+(1-e)\max_{n\in\{G,P\}}V_{nU}(\Omega')]\}. \end{aligned}$$

The optimal conditions for a and z^M are

$$\frac{\partial V_{GU}}{\partial a} = -\frac{w_U(\Omega)}{c}+\alpha\frac{B}{z}\leq 0,\ a\geq 0,\ \frac{\partial V_{GU}}{\partial a}a=0, \tag{16}$$

$$\frac{\partial V_{GU}}{\partial z^M} = -\frac{q(\Omega)}{c}+\alpha\frac{1}{z}\leq 0,\ z^M\geq 0,\ \frac{\partial V_{GU}}{\partial z^M}z^M=0. \tag{17}$$

Notice that if $z^M>0$ this implies that $\frac{w_U(\Omega)}{q(\Omega)}\geq B$, which is a contradiction since $\frac{w_U(\Omega)}{q(\Omega)}=\delta B$ and $\delta\in(0,1)$. This implies that $z^M_{GU}=0$ and

$$a_{GU}=\max\{0,\frac{w_U(\Omega)(\alpha-lG(1-e))-\alpha\phi eG}{w_U(\Omega)(1+\alpha)}\}.$$

If $e_{GU}=0$

$$a^0_{GU}=\max\{0,\frac{\alpha-lG}{1+\alpha}\},$$

and

$$V^0_{GU}(\Omega) = \ln(w_U(\Omega)(1-a^0_U)) + \alpha \ln(Ba^0_U + BlG) + \gamma \max_{n\in\{G,P\}} V_{nU}(\Omega').$$

Assumption 1 states that $\alpha \geq lG$. Then

$$V^0_{GU}(\Omega) = \ln(\frac{w_U(\Omega)(1+lG)}{1+\alpha}) + \alpha \ln(\frac{B\alpha(1+lG)}{1+\alpha}) + \gamma \max_{n\in\{G,P\}} V_{nU}(\Omega').$$

Now, suppose that $e_{GU} = 1$, in this case:

$$a^1_{GU} = \max\{0, \frac{\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)}\}.$$

Then, as long as education costs are not too high relative to the unskilled wage, i.e., $w_U(\Omega) > \phi G$, a^1_{GU} is positive and

$$V^1_{GU}(\Omega) = \ln(\frac{w_U(\Omega) - \phi G}{1+\alpha}) + \alpha \ln(\frac{B\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)}) + \gamma \max_{n\in\{G,P\}} V_{nS}(\Omega').$$

Unskilled agents send their children to school if and if

$$V^1_{GU}(\Omega) - V^0_{GU}(\Omega) > 0.$$

This requires that

$$\gamma[\max_{n\in\{G,P\}} V_{nS}(\Omega') - \max_{n\in\{G,P\}} V_{nU}(\Omega')] > \ln(\tfrac{w_U(\Omega)(1+lG)}{1+\alpha}) + \alpha \ln(\tfrac{B\alpha(1+lG)}{1+\alpha}) - [\ln(\tfrac{w_U(\Omega)-\phi G}{1+\alpha}) + \alpha \ln(\tfrac{B\alpha(w_U(\Omega)-\phi G)}{w_U(\Omega)(1+\alpha)})].$$

Or

$$\gamma[\max_{n\in\{G,P\}} V_{nS}(\Omega') - \max_{n\in\{G,P\}} V_{nU}(\Omega')] > (1+\alpha)\ln(\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G}) \tag{18}$$

B Skilled Households with Small Family

Now let's consider the problem of a skilled parent with a small family size:

$$\begin{aligned} V_{PS}(\Omega) &= \max_{e\in\{0,1\},z^M,a} \{\ln(w_S(\Omega)(1-a) - q(\Omega)z^M - \phi eP) + \alpha \ln(z^M + Ba + BlP(1-e)) \\ &+ \gamma[e \max_{n\in\{G,P\}} V_{nS}(\Omega') + (1-e) \max_{n\in\{G,P\}} V_{nU}(\Omega')]\}. \end{aligned}$$

The optimal conditions for a and z_M are

$$\frac{\partial V_{PS}}{\partial a} = -\frac{w_S(\Omega)}{c} + \alpha\frac{B}{z} \leq 0,\ a \geq 0,\ \frac{\partial V_{PS}}{\partial a}a = 0, \tag{19}$$

$$\frac{\partial V_{PS}}{\partial z^M} = -\frac{q(\Omega)}{c} + \alpha\frac{1}{z} \leq 0,\ z^M \geq 0,\ \frac{\partial V_{PS}}{\partial z^M}z^M = 0. \tag{20}$$

It is straightforward to show that:

i. When $q(\Omega) < \frac{A}{B}$, then $a = 0$, $z^M > 0$.

ii. When $q(\Omega) = \frac{A}{B}$, then $a > 0$, $z^M > 0$.

iii. When $q(\Omega) > \frac{A}{B}$, then $a > 0$, $z^M = 0$.

Therefore, in order to have market demand for good Z its price cannot be "too" high. Otherwise agents will produce good Z at home.

Assumption 2 states that education cost is not "too" high relatively to the skilled wage, i.e., $w_S = A > \phi P$. Otherwise, skilled parents will never educate their children. Let's first assume that there is initially market production of good Z, therefore we are under case 1 above. We have that $a_{PS} = 0$. If $e_{PS} = 1$, then

$$z^M_{PS} = \frac{\alpha(w_S - \phi P)}{q(\Omega)(1+\alpha)},$$

as long as $w_S = A > \phi P$, which is true by assumption. Therefore,

$$V^1_{PS}(\Omega) \quad = \quad \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)}) + \gamma \max_{n\in\{G,P\}} V_{nS}(\Omega').$$

Now, if $e_{PS} = 0$, then

$$z^M_{PS} = \frac{\alpha w_S - q(\Omega)BlP}{(1+\alpha)q(\Omega)}.$$

Therefore,

$$V^0_{PS}(\Omega) \quad = \quad \ln(\frac{w_S + q(\Omega)BlP}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S + qBlP)}{(1+\alpha)q(\Omega)}) + \gamma \max_{n\in\{G,P\}} V_{nU}(\Omega').$$

Now, let's consider case (iii). We have that $a_{PS} > 0$ and $z^M_{PS} = 0$. If $e_{PS} = 1$, then

$$V^{1,a>0}_{PS}(\Omega) \quad = \quad \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{B\alpha(w_S - \phi P)}{(1+\alpha)w_S}) + \gamma \max_{n\in\{G,P\}} V_{nS}(\Omega').$$

Notice that under case (iii) there exists no market demand for good Z. Let's assume initially that there is market production for good Z, and the economy is under case (i). In this case, skilled parents educate their children if

$$V^1_{PS}(\Omega) - V^0_{PS}(\Omega) > 0.$$

This requires that

$$\gamma[\max_{n\in\{G,P\}} V_{nS}(\Omega') - \max_{n\in\{G,P\}} V_{nU}(\Omega')] \quad > \quad \ln(\tfrac{w_S+q(\Omega)BlP}{1+\alpha}) + \alpha\ln(\tfrac{\alpha(w_S+qBlP)}{(1+\alpha)q(\Omega)}) - [\ln(\tfrac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\tfrac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})].$$

Therefore,

$$\gamma[\max_{n\in\{G,P\}} V_{nS}(\Omega') - \max_{n\in\{G,P\}} V_{nU}(\Omega')] > (1+\alpha)\ln(\frac{w_S + q(\Omega)BlP}{w_S - \phi P}). \tag{21}$$

In order to prove that $e_{GU} = 1$ implies $e_{PS} = 1$ it is sufficient to show that the right hand side of (18) is greater than the right hand side of (21). We have to show that

$$(1+\alpha)\ln(\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G}) > (1+\alpha)\ln(\frac{w_S + q(\Omega)BlP}{w_S - \phi P}).$$

Or

$$\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G} > \frac{w_S + q(\Omega)BlP}{w_S - \phi P}.$$

Recall that $q(\Omega) = \frac{w_U(\Omega)}{\delta B}$, which implies that

$$\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G} > \frac{\delta w_S + w_U(\Omega)lP}{\delta(w_S - \phi P)}.$$

Rearranging the above equation yields

$$w_U(\Omega)[\delta w_S G - w_U(\Omega)P]l + \delta\phi[w_S G - w_U(\Omega)P] + w_U(\Omega)GlP\phi[1-\delta] > 0,$$

which is clearly positive, since $w_U(\Omega) < \delta w_S$, $G > P$, and $\delta \in (0,1)$.

C Characterization of the Steady State

We can have the following equilibrium paths:

Part 1: Just skilled parents educate their children: Let's characterize the first equilibrium in which just skilled parents educate their children. In this case there will be market production of good Z in every period and skilled parents will not work at home. Therefore,

$$V_{PS}^1(\Omega) = \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)}) + \gamma \max_{n \in \{G,P\}} V_{nS}(\Omega').$$

In this equilibrium in which skilled agents choose a small family size, educate their children and do not work at home, we have that[24]

$$V_{PS}^1(\Omega) = \frac{\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})}{1-\gamma} - \frac{\alpha\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}). \tag{22}$$

Unskilled parents will not educate their children and will also not by good Z in the market. Thus

$$V_{GU}^0(\Omega) = \ln(w_U(\Omega)(1-a_U^0)) + \alpha \ln(Ba_U^0 + BlG) + \gamma \max_{n \in \{G,P\}} V_{nU}(\Omega').$$

Since unskilled parents will not send their children to school they will not pay the cost associated to education and will receive income from child labor. The family size enters positively in the indirect utility function. Therefore, $n_U = G$ and

$$V_{GU}^0(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha \ln \alpha B + \ln w_U(\Omega)}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}). \tag{23}$$

In order to fully characterize the equilibrium it remains to find equilibrium prices. We know that $w_S = A$ and $\frac{w_U(\Omega)}{q(\Omega)} = \delta B$. Using the equilibrium in the goods market, we can show that

$$\frac{X_{GU}}{X_{PS}} c_{GU} + c_{PS} + \phi P = A,$$

and

$$w_U(\Omega) = \frac{\alpha(A - \phi P)}{\Omega(1+lG)},$$

as long as $w_U(\Omega) < \delta w_S$. Recall that $\Omega' = \frac{G}{P}\Omega$, and the initial state Ω_0 is given. It is also important to show that if a parent deviates from its optimal choice, then he gets a lower utility. Consider first an unskilled parent that chooses to deviate from this equilibrium. This parent will send their children to school when all other unskilled parents are not educating their children. Future descendants choose to have a small family size and educate their children. The utility of this parent is:

$$\begin{aligned}\tilde{V}_{GU}(\Omega) = {} & \ln(\frac{w_U(\Omega) - \phi G}{1+\alpha}) + \alpha \ln(\frac{B\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)}) \\ & + \gamma[\frac{\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)}) - \frac{\alpha}{1-\gamma}\ln(\frac{P}{G}))}{1-\gamma}].\end{aligned}$$

Therefore, as long as

$$V_{GU}^0(\Omega) - \tilde{V}_{GU}(\Omega) \geq 0,$$

[24] Notice that the size of the family enters negatively because agents have to pay education costs. They should also foregone the income from child labor.

it is optimal for an unskilled parent to not send their children to school when all other unskilled parents are not educating their children. It can be shown that this condition is satisfied when

$$\begin{aligned}(1+\alpha)\ln(w_U(\Omega)) - (1-\gamma)(1+\alpha)\ln(w_U(\Omega)-\phi G) \quad > \\ \gamma(1+\alpha)\ln(w_S-\phi P)+\gamma\alpha\ln\delta-(1+\alpha)\ln(1+lG)-\frac{(1+\alpha)\gamma}{1-\gamma}\ln(\frac{P}{G}).\end{aligned} \tag{24}$$

Since $\gamma \in (0,1)$ we have that the left hand side (LHS) of (24) is always positive. Moreover, it is continuous in $w_U(\Omega) > \phi G$ and it goes to infinity as $w_U(\Omega) \to \phi G$. The LHS of (24) is also decreasing with $w_U(\Omega)$ as long as $\frac{w_U(\Omega)}{w_U(\Omega)-\phi G} > \frac{1}{1-\gamma}$.[25] On the other hand, the right hand side (RHS) of (24) is independent of $w_U(\Omega)$. Therefore, we can define a w_U^* such that for any $w_U(\Omega) < w_U^*$, it is optimal for unskilled parents to not educate their children. Observe that the RHS of (24) might be a negative number. In this case, for any $w_U(\Omega) < \delta w_S$ it is optimal for unskilled agents to not educate their children. However, we can choose parameter values such that not only the RHS is positive, but we are under the interesting case where $\phi G < w_U^* < \delta w_S$.

Now, consider a skilled parent that choose to not educate their children when all skilled parents are sending their descendants to school. Future descendants choose to have a large family size and do not educate their children. The utility of this parent is

$$\begin{aligned}\tilde{V}_{PS}(\Omega) = (1+\alpha)\ln(\frac{w_S+q(\Omega)BlP}{1+\alpha})+\alpha\ln(\frac{\alpha}{q(\Omega)})+ \\ \frac{\gamma}{1-\gamma}[(1+\alpha)\ln(\frac{1+lG}{1+\alpha})+\ln w_U(\Omega)+\alpha\ln\alpha B+\frac{1}{1-\gamma}\ln(\frac{P}{G})].\end{aligned}$$

Therefore, as long as $V_{PS}^1(\Omega) - \tilde{V}_{PS}(\Omega) > 0$, it is optimal for a skilled parent to educate their children when all other skilled parents are also sending their children to school and unskilled parents are not educating their children. This condition is satisfied when

$$\begin{aligned}(1+\alpha)\ln(w_S-\phi P)+\gamma\alpha\ln\delta-\gamma(1+\alpha)\ln(1+lG)-\frac{(1+\alpha)\gamma}{1-\gamma}\ln(\frac{P}{G}) > \\ (1-\gamma)(1+\alpha)\ln(w_S+\frac{w_U(\Omega)}{\delta}lP)+\gamma(1+\alpha)\ln(w_U(\Omega)).\end{aligned} \tag{25}$$

The LHS of (25) is independent of $w_U(\Omega)$, while the RHS is continuous and increasing in $w_U(\Omega)$. Moreover, as $w_U(\Omega) \to 0$ the $LHS \to -\infty$, and when $w_U(\Omega) \to \infty$, then the $LHS \to \infty$. Therefore, there exists a $w_U^{**} > 0$, such that for any $w_U(\Omega) < w_U^{**}$, it is optimal for skilled parents to educate their children. Define $\bar{w}_U = \min\{w_U^*, w_U^{**}\}$. This implies that for any $w_U(\Omega) \leq \bar{w}_U$ it is optimal for unskilled parents to not send their children to school and it is optimal for skilled parents to educate their children.

Part 2: Skilled and unskilled parents educate their children: Now, let's characterize the equilibrium when skilled and unskilled parents educate their children. In this equilibrium there is market production of good Z only in the first period. The value function of a skilled worker with a small family size is

$$V_{PS}^{1,a'>0}(\Omega) = \frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha})+\alpha\ln(\alpha B)-\gamma\alpha\ln w_S}{1-\gamma}+\alpha\ln\delta-\alpha\ln w_U(\Omega). \tag{26}$$

Unskilled agents with a large family size will educate their children. Their descendants will then choose a small family size and will also educate their children. The value function of an

[25] Since $w_U(\Omega) < \delta w_S = \delta A$, we have that the left hand side decreases with the unskilled wage as long as $\frac{\delta A}{\delta A - \phi G} > \frac{1}{1-\gamma}$.

unskilled parent with a large family size is:

$$V^1_{GU}(\Omega) = (1+\alpha)\ln(\frac{w_U - \phi G}{1+\alpha}) + \frac{1}{1-\gamma}\alpha\ln(\alpha B) - \alpha\ln(w_U(\Omega)) + \gamma\frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) - \alpha\ln w_S}{1-\gamma}. \tag{27}$$

Now, it is important to show that individual deviations from the optimal policy yield a lower payoff. Given item (*ii*) of proposition 2 it is sufficient to show only the condition that guarantees that it is optimal for unskilled parents to educate their children. Suppose that an unskilled parent decides to not educate their children when all other parents (skilled and unskilled) are sending their descendants to attend school. There are two possibilities: their descendants choose to educate or not their children. Notice, however, that in the next period, their children will be the only unskilled agents in the economy. If they are a small measure of the total population, then the unskilled wage will be at its maximum value,[26] $w_U(\Omega') = \delta w_S$. Under the interesting case, where $w^*_U < \delta w_S$, their descendants will choose to educate their children. Then, the value associated with this deviation is:

$$\hat{V}_{GU}(\Omega) = (1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \ln w_U(\Omega) + \alpha\ln\alpha B + \gamma[(1+\alpha)\ln(\frac{\delta w_S - \phi G}{1+\alpha}) + \alpha\ln(\frac{\alpha B}{\delta w_S})] + \frac{\gamma^2}{1-\gamma}[(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\frac{\alpha B}{\delta w_S})]$$

As long as $V^1_{GU}(\Omega) - \hat{V}_{GU}(\Omega) > 0$ it is optimal for unskilled parents send their children to school when all other parents are also educating their descendants. This is true as long as

$$(1+\alpha)\ln(\frac{w_U(\Omega)}{w_U(\Omega)-\phi G}) < (1+\alpha)\ln(w_S - \phi P) - (1+\alpha)\ln(1+lG) - \gamma[(1+\alpha)\ln(\delta w_S - \phi G) - \alpha\ln\delta]. \tag{28}$$

The LHS of (28) is continuous and decreasing in $w_U(\Omega)$. Moreover, when $w_U(\Omega) \to \phi G$, then the $LHS \to \infty$, and when $w_U(\Omega) \to \infty$, then the $LHS \to 0$. Therefore, as long as the RHS is positive, there exists a $w^\bullet_U$, such that for any $w_U(\Omega) > w^\bullet_U$, condition (28) is satisfied.

D Main Results

Proof of Proposition 3: The equilibrium value function when unskilled parents do not educate their children is:

$$V^0_{GU}(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha\ln\alpha B + \ln w_U(\Omega)}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}).$$

In this equilibrium, the unskilled wage is:

$$w_U(\Omega) = \frac{\alpha(A-\phi P)}{\Omega(1+lG)}.$$

Notice that $V^0_{GU}(\Omega)$ is clearly continuous in $\phi \in [0, \frac{A}{P})$. Moreover, for $\phi \in [0, \frac{A}{P})$

$$\frac{\partial V^0_{GU}(\Omega)}{\partial\phi} = \frac{\frac{\partial w_U(\Omega)}{\partial\phi}}{w_U(\Omega)(1-\gamma)} < 0.$$

[26]The next period state variable will be $\Omega' = \frac{G}{G(X_{GU}-1)+PX_{PS}}$.

In addition:

$$\lim_{\phi \to \frac{A}{P}} V_{GU}^0(\Omega) = -\infty,$$

and

$$\lim_{\phi \to 0} V_{GU}^0(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha \ln \alpha B + \ln(\frac{\alpha A}{\Omega(1+lG)})}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}).$$

The equilibrium value function when unskilled parents educate their children is:

$$V_{GU}^1(\Omega) = (1+\alpha)\ln(\frac{w_U(\Omega)-\phi G}{1+\alpha}) + \frac{\alpha \ln \alpha B}{1-\gamma} - \alpha \ln w_U(\Omega) + \gamma \frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) - \alpha \ln w_S}{1-\gamma},$$

where

$$w_U(\Omega) = \frac{\alpha(A-\phi(P+\Omega G))}{\Omega}.$$

The value function $V_{GU}^1(\Omega)$ is continuous in $\phi \in [0, \frac{A}{P+\frac{1+\alpha}{\alpha}\Omega G})$ and

$$\frac{\partial V_{GU}^1(\Omega)}{\partial \phi} = \frac{\partial w_U(\Omega)}{\partial \phi}[\frac{w_U(\Omega)+\alpha\phi G}{w_U(\Omega)(w_U(\Omega)-\phi G)}] - \frac{(1+\alpha)G}{w_U(\Omega)-\phi G} - \frac{\gamma(1+\alpha)P}{(1-\gamma)(A-\phi P)} < 0.$$

We also have that:

$$\lim_{\phi \to \frac{A}{P+\frac{1+\alpha}{\alpha}\Omega G}} V_{GU}^1(\Omega) = -\infty,$$

and

$$\lim_{\phi \to 0} V_{GU}^1(\Omega) = \ln \frac{\alpha A}{\Omega} + \frac{\alpha}{1-\gamma}\ln \alpha B + \frac{\gamma}{1-\gamma}\ln A - \frac{1}{1-\gamma}(1+\alpha)\ln(1+\alpha).$$

Condition (14) guarantees that $\lim_{\phi\to 0} V_{GU}^1(\Omega) > \lim_{\phi \to 0} V_{GU}^0(\Omega)$. Since $\frac{A}{P} > \frac{A}{P+\frac{1+\alpha}{\alpha}\Omega G}$, this proves item (i) and item (ii) of proposition 3.

Proof of Proposition 4: In equilibrium, skilled parents always educate their children. Therefore:

$$V_{PS}^1(\Omega) \quad = \quad \ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)}) + \gamma \max_{n\in\{G,P\}} V_{nS}(\Omega').$$

If $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$ and $\bar{\phi}(\Omega) \in [0, \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G})$, then unskilled parents will not educate their children and skilled parents will not work at home. Then

$$V_{PS}^1(\Omega) \quad = \quad \frac{\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})}{1-\gamma} - \frac{\alpha\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}),$$

with $w_S = A$, $q(\Omega) = \frac{w_U(\Omega)}{\delta B}$ and $w_U(\Omega) = \frac{\alpha(A-\phi P)}{\Omega(1+lG)}$. It is straightforward to show that $V_{PS}^1(\Omega)$ is continuous in $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$ and

$$\frac{\partial V_{PS}^1(\Omega)}{\partial \phi} = -\frac{1}{1-\gamma}\frac{P}{A-\phi P} < 0.$$

Notice also that

$$\lim_{\phi \to \frac{A}{P}} V_{PS}^1(\Omega) = -\infty.$$

If $\phi \in [0, \bar{\phi}(\Omega))$, then unskilled parents will educate their children and there exists market production of good Z only in the first period. The value associated with a skilled parent problem is

$$V_{PS}^{1,a'>0}(\Omega) = \frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha \ln(\alpha B) - \gamma\alpha \ln w_S}{1-\gamma} + \alpha \ln \delta - \alpha \ln w_U(\Omega),$$

with $w_S = A$ and $w_U(\Omega) = \frac{\alpha(A-\phi(P+\Omega G))}{\Omega}$. Moreover:

$$\frac{\partial V_{PS}^{1,a'>0}(\Omega)}{\phi} = -\frac{1+\alpha}{1-\gamma}\frac{P}{A-\phi P} + \alpha\frac{P+\Omega G}{A-\phi(P+\Omega G)}.$$

Notice that there are two effects in opposing directions. It can be shown that for a large altruism factor $\frac{1}{1-\gamma} > \frac{\alpha}{1+\alpha}\frac{A-\phi P}{A-\phi(P+\Omega G)}\frac{P+\Omega G}{P}$, we have that $\frac{\partial V_{PS}^{1,a'>0}(\Omega)}{\phi} < 0$.

Notice that if $\phi = 0$ and $\Omega > \Omega^*$, then

$$V_{PS}^{1,a'>0}(\Omega, \phi = 0) = \frac{1+\alpha}{1-\gamma}\ln(\frac{A}{1+\alpha}) + \frac{\alpha}{1-\gamma}\ln(\alpha B) - \frac{\gamma\alpha}{1-\gamma}\ln A + \alpha\ln(\frac{\delta\Omega}{\alpha A}).$$

Recall that $\bar{\phi}(\Omega) \in [0, \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G})$. Define $\bar{\phi}(\Omega)_{\max} = \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G}$. If $\phi = \bar{\phi}(\Omega)_{\max}$ and $\Omega > \Omega^*$, then unskilled parents will not educate their children and

$$V_{PS}^{1}(\Omega, \phi = \bar{\phi}(\Omega)_{\max}) = \frac{1}{1-\gamma}\ln(\frac{A}{1+\frac{\alpha P}{(1+\alpha)\Omega G}}) + \frac{\alpha}{1-\gamma}\ln(\delta B\Omega(1+lG)) - ...$$
$$\frac{1+\alpha}{1-\gamma}\ln(1+\alpha) + \frac{\alpha\gamma}{(1-\gamma)^2}\ln\frac{G}{P}.$$

It can be shown that $V_{PS}^{1}(\Omega, \phi = \bar{\phi}(\Omega)_{\max}) > V_{PS}^{1,a'>0}(\Omega, \phi = 0)$ if and only if $\Omega > \Omega^*$ and

$$\alpha\gamma\ln(\frac{\delta\Omega}{\alpha}) + \alpha\ln(1+lG) + \frac{\alpha\gamma}{1-\gamma}\ln\frac{G}{P} > \ln(1+\frac{\alpha P}{(1+\alpha)\Omega G}). \quad (29)$$

The LHS of (29) is continuous and increasing in $\Omega > 0$. Moreover, as $\Omega \to 0$, then $LHS \to -\infty$ and as $\Omega \to \infty$, then $LHS \to \infty$. On the other hand, the RHS of (29) is continuous and decreasing in $\Omega > 0$. In addition, as $\Omega \to 0$, then $RHS \to \infty$ and as $\Omega \to \infty$, then $LHS \to 0$. Therefore, there exists an $\Omega^{**} > 0$, such that if $\Omega > \max\{\Omega^{**}, \Omega^*\}$, then skilled agents strictly prefer an education cost $\phi = \bar{\phi}(\Omega)_{\max}$ than a zero education cost ($\phi = 0$).