Forecasting Quarterly Brazilian GDP Growth Rate With Linear and NonLinear Diffusion Index Models

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Abstract

This paper uses linear and non-linear diffusion index models and combination of them to produce one-step-ahead forecast of quarterly Brazilian GDP growth rate. The non-linear diffusion index models are not only parsimonious ones, but they also purport to describe economic cycles through a Threshold diffusion index model and a Markov-Switching diffusion index model.

Resumo

Este trabalho usa modelos lineares e não lineares de índices de difusão para prever, em um período à frente, a taxa de crescimento trimestral do PIB brasileiro. Os modelos de índice de difusão assemelham-se aos modelos de fatores dinâmicos. Além de parcimoniosos, os modelos utilizados neste trabalho se propõem a captar as fases de recessão e expansão econômica, através de modelos não lineares do tipo Threshold Effect e Markov-Switching.

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1 Introduction

Recent lessons in economic forecasting practice have shown that lack of parsimony is an important cause of forecast failure. This should be expected because the more coefficients there are in a model, more uncertainty about the estimated parameters is introduced and this can reflect negatively on the model's forecast accuracy. Not only does this mean that some variables, which could give important information about the series to be predicted, would likely be out of the model, but also that lags of the included variables would be restricted.

Factor models for time series have been used to allow the construction of a large number of cross-sections in macro forecasting models. The main idea is that all the information included in a large number of variables could be captured by a few number of common factors among them. At least two distinct strands of literature have been using this method. One of these branches is represented by the dynamic factor models (Geweke and Singleton (1981); Engle and Watson (1981); Stock and Watson (1989); Kim and Nelson (1998)). The main characteristic of these studies is the effort to estimate the unobservable common factors among some macroeconomic variables, relying on the use of Maximum likelihood estimation(MLE), Kalman filter or both.

The other factor model approach is represented by diffusion index models (Connor and Korajczyk (1993); Geweke and Zhou (1996); Forni et al. (1998, 2000); Stock and Watson (1998, 2002); Brisson et al. (2003)), which uses principal components to estimate these common factors. This method allows a larger information set than MLE, and it seems to be more appropriate to compute factor estimates when the sample period is short but there is a moderate number of related variables in the information set.

Besides lack of parsimony, there are many other causes of forecast failure. A major one occurs when structural breaks exist in the series to be forecast. In this case a non-linear model could be tried on to improve predictions made by linear models.

There are three major classes of non-linear models – Markov Switching (MS), Threshold autoregressive (TAR) and Smooth Transition (STAR) models. These models have been used in macroeconometrics to characterize features of the business cycles such as expansions and recessions.

Chauvet et al. (2002) show that the use of non-linear models to forecast Brazilian GDP growth rate improves on linear models. In this study a linear diffusion index (DI) model was used to forecast Brazilian GDP growth rate and these predictions were compared to AR and VAR forecasts. DI forecasts were made using two kinds of data sets. In the first one, factors were estimated using current values of 72 regressors. The second data set was constructed allowing for lags [1] of these predictors. Quarterly data were used from 1975.Q1 to 2003.Q3. One step ahead forecasts were produced in a simulated real time design.

After the best linear DI model was chosen, a Time-Varying-Parameter DI model was tried. Moreover, the linear DI model was tested for the existence of a threshold effect in short and long differences of the endogenous variable and estimated following the method presented in Hansen (1997). Another non-linear model used in this work was a Markov switching DI model.

Once all these models were estimated and used to forecast, a linear combination, made up of these individual predictions, was found in an attempt to improve forecast performance.

There are at least two contributions provided by this work that are important to stress. First, it applies the DI method to forecast an important Brazilian macroeconomic variable, GDP growth rate, and this has not been done up to now in Brazil. The second, and the most important one, was the use of a Threshold and a Markov Regime Shift specification of a DI model and their predictive performances were analyzed from an empirical point of view.

Besides this introduction this study has four more sections. The first one explains the data used in this work. The second one, as usual, contains a review of the most important theoretical background of the work. Subjects such as latent variables and factor models, the estimation process and forecast environment used in this study are discussed. The third one contains the main results of the forecasting experiment. The conclusions

1 Sets with one up to three lags were applied.

and main remarks are presented in the last section.

2 The Data

The quarterly sample data used in this study cover major Brazilian macroeconomic series available from 1975.Q1 to 2003.Q3. In this study the time series to be forecast is the growth of Brazilian GDP. Traditional out-of-sample predictions are produced for the period 2002:1 to 2003:3.

The explanatory variables (x_t), which was used to compute the diffusion index used, are composed of a total of 72 national and international macroeconomic variables, selected [2] to represent categories such as real output, income and earnings; production capacity constraints; employment; real retail; credit constraints; interest rates; price index; investment; exchange rate; money aggregates; balance of payment results; international trade; economic indicators of industrialized countries and miscellaneous. These macroeconomic categories are in tune with Stock and Watson (1998), but they are not the same. First, the USA economy has a larger data set, covering variables that are not available for Brazil. Second, the Brazilian economy is very dependent on its external sector and international economic indicators. Balance of payments and international reserves have influenced Brazilian economic growth and economic policies. Thus, this study included some international macroeconomic variables to capture these external effects. The list of these variables is presented in appendix I.

The estimation and asymptotic results presented in Stock and Watson (1998) assume that all the series in the data matrix are stationary. Thus, these 72 series have been analyzed for unit roots and seasonal patterns. All the nonnegative series were expressed in logs, except for the percentage scaled ones. Nominal variables in R$ (Brazilian currency) were deflated. Seasonal adjustments were made based on the Census X-11 procedure [3].

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[2] These selection variables was chosen to represent the main macroeconomic categories with the same length of quarterly Brazilian GDP data.

[3] This procedure can be explained as follows: a) let y_t be the series to be adjusted. A centered moving average of y_t is computed and stored as x_t; b) compute $d_t = y_t - x_t$; c) the seasonal index i_q for quarter q is the average of d_t using data only of the q quarter; d) compute $s_j = i_j - \bar{i}$, where $\bar{i}$ is the index average; e) the seasonally adjusted series is obtained by taking the difference

Moreover, first and second differences were taken to achieve stationarity when needed. After these transformations the sample started at 1976.Q1. These variables and their transformations are presented in appendix I.

3 Theoretical Background

3.1 Diffusion index model

A consensual point about economic models is that a good model of the business cycle must reproduce some stylized facts. Burns and Mitchell (1946) present a statistical description of the cycle phenomenom. They claim that during an economic cycle there is a comovement between macroeconomic variables. Economists agree that a good business cycle model must reproduce this comovement among output, trade, exchange rate, employment, inflation, money aggregates and interest rate. But there is no agreement on what set of explanatory variables should be used to either explain or to forecast economic cycles.

The models used in this work to forecast Brazilian GDP growth rate are constructed considering comovement, economic phases and the possibility of the existence of structural breaks. The Diffusion Index (DI) model, following Stock and Watson (1998, 2002), is used to elaborate parsimonious models that capture the mentioned comovement. Besides the comovement, the economy would be subject to nonlinearities which can be summarized in economic cycles. Threshold autoregressive models proposed by Tong (1983) is a possibility to model these nonlinearities. Another way to do this is to follow the ideas presented by Hamilton (1989). The Markov regime shifting model proposed by Hamilton is a latent variable model that captures economic cycles.

Therefore, once the best linear DI model is selected for forecasting purposes, nonlinearities are considered through a Time Varying DI model (TVPDI); a Threshold Autoregressive DI Model (TARDI) and a Markov Shifting

$y_t - s_j$.

DI model (MSDI). Combining forecast techniques are also applied. A description of these models is presented in the next subsections.

According to Bartholomew and Knott (1999), factor models, thus DI models, are models with latent variables. This means that some variables are unobservable. Let f represent r of those variables and x to be k observable or *manifest* ones, with $r < k$. The commom factor analysis model expresses the data matrix $X_{(T\times k)}$ as a linear combination of unknown linearly independent vectors, usually called common factors, plus a unique factor. Following the ideas presented by Stock and Watson (1998), a linear diffusion index (DI) model to produce one step ahead forecasts can be represented as:

$$y_{t+1} = c + \alpha y_t + \beta' F_t + \epsilon_{t+1} \tag{1}$$

$$x_t = \Lambda F_t + e_t \tag{2}$$

Where, $x_{t=}[x_{1t}, ..., x_{kt}]'$ is a $(k \times 1)$ vector, Λ is a $(k \times r)$ matrix of factor loadings, $F_t = [f_1, ..., f_r]'$ is a $(r \times 1)$ vector, $e_t = [e_{1t}, ..., e_{kt}]'$ is a $(k \times 1)$ vector of errors component, y_{t+1} is the variable to be forecast, $\alpha = (\alpha_0, ..., \alpha_q)'$, $F_t = (f_t', ..., f_{t-q}')'$ is a $(r \times 1)$ vector with $r \leq (q+1)\bar{r}$, $\Lambda_i = (\lambda_{i0}, ..., \lambda_{iq})$ and $\beta = (\beta_0, ..., \beta_q)'$.

If the usual infinite lag assumption were applied, then this static representation of a dynamic factor model would have infinitely many factors. Furthermore, the main advantage of the last representation is to allow the estimation of factors by principal component. Stock and Watson (1998) show that factors estimated by principal components are consistent as the number of variables goes to infinity, even for a fixed time period of observations, and this is a good characteristic for empirical work when there is a reasonable number of variables but just a few observations of them.

3.2 Time varying parameter diffusion index model

The problems generated by the existence of structural breaks and shifts in the parameters of a model can be avoided if one allows these parameters to vary. The Time-Varying-Parameter (TVP) model is a special case of the general state-space (SS) model. This model can be represented as follows.

$$y_{t+1} = \beta_{1t} + \beta_{2t} F_t + \epsilon_{t+1} \tag{3}$$

Where,

$$\beta_{it} = \delta_i + \phi_i \beta_{it-1} + v_{it} \ , \ i = 1,2 \tag{4}$$

$$\epsilon_t \sim iid\ N(0, R) \tag{5}$$

$$v_{it} \ \sim iid\ N(0, Q) \tag{6}$$

$$E(\epsilon_t v_{is}) = 0 \text{ for all t and s} \tag{7}$$

The SS representation is made up of a measurement equation (3), which describes the relation between data and unobserved state variables, and a transition equation (4) used to specify the dynamics of the state variables.

In the Time-Varying-Parameter version of the linear diffusion index model (TVPDI) proposed here, the measurement and transition equations are respectively expressed as:

$$y_{t+1} = H_t \beta_t + \epsilon_{t+1}, \text{ and} \tag{8}$$

$$\beta_t = \mu + A_t \beta_{t-1} + v_t \tag{9}$$

Where $H_t = [1\ \hat{F}_t]$, $\beta_t = [\ \beta_{1t}\ \beta_{2t}]'$, $\mu = \delta_i(1-\phi_i)$, $A_t = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix}$, and $v_t = [v_{1t}\ v_{2t}]'$. Once the model is in the state-space form, an interactive procedure using the Kalman filter and MLE is available to produce estimates of the parameters and inference can be made about the unobserved state vector β_t. This estimation procedure will be described later on.

3.3 Threshold diffusion index model

Switching-regime models, such as the threshold autoregressive (TAR) model, have been used in empirical macroeconomic studies to capture expansions and recessions phases of the business cycle or any other situation that requires a split in the sample induced by different regimes. TAR models were proposed by Tong (1983). Hansen (1996, 1997, 2000) shows how to estimate and to make inference in a TAR model. A two regime threshold autoregressive diffusion index model (TARDI) can be expressed as:

$$y_{t+1} = (\alpha_0^1 + \alpha_1^1 y_t + ... + \alpha_q^1 y_{t-q} + \beta_1^1 F_t + ... + \beta_q^1 F_{t-q}) I(g_{t-1} \leq \gamma) + \quad (10)$$

$$+(\alpha_0^2 + \alpha_1^2 y_t + ... + \alpha_q^2 y_{t-q} + \beta_1^2 F_t + ... + \beta_q^2 F_{t-q}) I(g_{t-1} > \gamma) + \epsilon_{t+1}$$

$$x_t = \Lambda F_t + e_t \quad (11)$$

Where g_{t-1} is a known function of the data and $I(.)$ is the indicator function. Let $\alpha^j = (\alpha_0^j ... \alpha_q^j)'$, $\beta^j = (\beta_1^j, ..., \beta_q^j)'$ for $j = 1, 2$, and $z_t = (1\ y_t ... y_{t-q}\ F_t ... F_{t-q})'$, $\pi_j = (\ \alpha^j\ \beta^j)'$, $z_t(\gamma) = (\ z_t' I(g_{t-1} \leq \gamma)\ z_t' I(g_{t-1} > \gamma))'$ and $\theta = (\ \pi^{1'}\ \pi^{2'})'$. Then eq(10) may be written as:

$$y_{t+1} = z_t(\gamma)'\theta + \epsilon_{t+1} \quad (12)$$

3.4 Markov-switching diffusion index models

Another way to model either regime shifts or economic phases is to use models of the type proposed by Hamilton (1989, 1994), where the parameters of the model are allowed to change according to the economic regime, and this regime is treated as an unobservable variable modeled as a first order Markov-switching process. The next few equations will describe the Markov-switching diffusion index model (MSDI) used in this study.

$$y_{t+1} = c_{St} + \beta'_{St} F_t + \epsilon_{t+1} \tag{13}$$

$$x_t = \Lambda F_t + e_t \tag{14}$$

$$\epsilon_t \sim iid\ N(0, \sigma^2_{St}) \tag{15}$$

$$c_{St} = c_0(1 - S_t) + c_1 S_t \tag{16}$$

$$S_t = 0 \text{ or } 1 \tag{17}$$

$$P[S_t = 1 | S_{t-1} = 1] = p \text{ and } P[S_t = 0 | S_{t-1} = 0] = q \tag{18}$$

As one can see this model allows the coefficients of the model to change according to the unobservable economic phase. In this set up there are two possible regimes, representing respectively economic recession and expansion. The model described by the equations above also tries to capture the comovement between macroeconomic variables and the business cycle pattern, as in the TARDI model. But unlike the TARDI model, the MSDI model does not use any kind of variable to capture the cycle and to split up the sample.

3.5 Estimation, testing, forecasting and combining forecasts

3.5.1 Estimation procedure for the DI model

The estimation [4] procedure for the linear diffusion index model represented by (1) and (2) is composed of two steps. First, the exact number of factors is unknown. Thus, under the hypothesis of the existence of n ($n < k$) common factors, the observed data x_t are used to estimate these factors. The static formulation of a dynamic factor model allows the use of principal components technique to estimate the unobservable common factors. Since principal components are very sensitive to data scaling, standardized values of x_t were used. The factors estimates $\hat{F}_t$ are the eigenvectors associated with the n largest eigenvalues of the standardized $(T \times T)$ matrix $k^{-1} \sum_{i=1}^{k} \underline{x_i x_i{}'}$, where $\underline{x_i} = (x_{i1}, ..., x_{iT})$ is a $(T \times 1)$ vector.

In the second step, y_{t+1} is regressed onto a constant, $\hat{F}_t$ and y_t to obtain estimates of c, α and β. This two step estimation method was adopted in Stock and Watson (1998, 2002) [5].

Three types of panel sets were tried. The first panel set was made up of the current values of the 72 macroeconomic variables described earlier in section 2. The second and the third sets allowed for one and two lags, respectively, of these series. Thus, in the second stacked panel, the numbers of columns of x_t were 144, and in the third this number jumped to 216 series.

3.5.2 Estimation procedure for the TVPDI model

As stated before, the state space representation of TVPDI model in equations (8) and (9) can be estimated by interactive MLE and Kalman filter. The basic Kalman filter is composed of two procedures – prediction and updating. In the prediction step, an optimal prediction of y_t is made

[4] The approach is quasi-MLE, because there are some parametric assumptions. A Gauss code was used to estimate the DI model, and to produce forecasts.

[5] Theorem 1 in Stock and Watson (1998):16 shows that the estimated factors are uniformly consistent, and this result does not depend on the structure of eq(1). Moreover, they also show that if r is unknown and even if $m \geq r$ the efficient forecast MSE can be achieved.

up of all available information up to time t-1 (ψ_{t-1}). To do this, first an expectation about β_t conditional on ψ_{t-1} must be established.

Afterwards, when y_t is observed, the prediction error is computed and used to make a better inference on β_t. This is the aim of the updating step. In the next period this new expectation about β_t is used in the prediction step, and this is repeated until the end of the sample.

Let ψ denote the information set as before and consider the following definitions: $y_{t|t-1} = E[y_t|\psi_{t-1}]$; $\eta_{t|t-1} = y_t - y_{t|t-1}$ and $U_{t|t-1} = E[\eta_t^2|\psi_{t-1}]$.

Given initial values for the parameters of the model, $\beta_{0|0}$ and $P_{0|0}$, the Kalman filter produces the prediction error $\eta_{t|t-1}$ and its variance $U_{t|t-1}$. Reminding that ϵ_t and v_t are both assumed to be Gaussian, the conditional distribution of y_t on ψ_{t-1} is also Gaussian; i.e.,

$$y_t|\psi_{t-1} \sim N(y_{t|t-1}, U_{t|t-1}) \tag{19}$$

Thus, the likelihood function can be expressed as

$$lnL = -\frac{1}{2}\sum_{t=1}^{T} ln(2\pi U_{t|t-1}) - \frac{1}{2}\sum_{t=1}^{T} \eta'_{t|t-1} U_{t|t-1}^{-1} \eta_{t|t-1} \tag{20}$$

Estimates of the unknown parameters in the prediction and updating equations are obtained when the likelihood function is maximized with respect to them. A nonlinear numerical optimization procedure is used for this purpose. At each search step these prediction and updating equations are computed and the likelihood function is evaluated, until convergence is reached [6] .

[6] The E-views 3.1 software was used to estimate this model, and the Marquardt algorithm was used in the numerical optimization.

3.5.3 Estimation procedure for the TARDI model

The estimation[7] of the TARDI model will follow the ideas presented in Hansen (1997). Two kinds of functions will be used as g_{t-1}, the traditional short lag approach $(Ln(gdp_{t-1}/gdp_{t-2}))_{t-d}$, and the long difference $Ln(gdp_{t-1}/gdp_{t-d})$ where d is a positive integer called *delay lag*. Since in this case the regression equation is both nonlinear and discontinuous, the estimates of the parameters θ and γ will be obtained by sequential conditional least squares. Leting $\gamma = g_{t-1}$ and $\Gamma = [\underline{\gamma}, \overline{\gamma}]$, the LS estimate of γ can be found by a direct search of values of Γ that minimizes the residuals of the regression of y_t on $z_t(\gamma)$. In other words,

$$\widehat{\gamma} = \underset{\gamma\in\Gamma}{argmin}\frac{1}{n}\left(y_t - z_t(\gamma)'\hat{\theta}(\gamma)\right)'\left(y_t - z_t(\gamma)'\hat{\theta}(\gamma)\right) \qquad (21)$$

Where,

$$\hat{\theta}(\gamma) = \left(\sum_{t=1}^{n} z_t(\gamma)z_t(\gamma)'\right)^{-1}\left(\sum_{t=1}^{n} z_t(\gamma)y_t\right) \qquad (22)$$

After obtaining $\hat{\gamma}$ the Least Squares estimates of θ is computed as $\hat{\theta} = \hat{\theta}(\hat{\gamma})$.

3.5.4 Testing for threshold

Hansen (1996, 1997, 2000) shows how one can test the null hypothesis $H_0 : \pi^1 = \pi^2$; i.e., to test the null hypothesis of linearity against the alternative of a TAR model. Neglected heteroskedasticity in this case may cause spurious rejection of H_0. A heteroskedasticity-consistent Wald test suggested by Hansen (1997) is presented below.

[7] An adaptation of Hansen's Gauss code was used to estimate, to forecast and to test for the threshold effect.

$$W_n = \sup_{\gamma \in \Gamma} W_n(\gamma) \tag{23}$$

Where,

$$W_n(\gamma) = (R\hat{\theta}(\hat{\gamma}))'[R(M_n(\gamma)^{-1}V_n(\gamma)M_n(\gamma)^{-1})R']^{-1}(R\hat{\theta}(\hat{\gamma})) \tag{24}$$

In equation (24), $R = [I \quad -I]; M_n(\gamma) = \sum_{t=1}^{n} z_t(\gamma)z_t(\gamma)'$; $V_n(\gamma) = \sum_{t=1}^{n} z_t(\gamma)z_t(\gamma)'\hat{e}^2$ and $\hat{e}_t^2 = (y_t - z_t(\gamma)'\hat{\theta}(\gamma))^2$. As one can see, the $W_n(\gamma)$ statistic does not follow an asymptotic χ^2 distribution, thus the distribution of W_n is nonstandard. Hansen (1996) derives the asymptotic distribution and a p-value transformation of the test is presented in eq(23). The asymptotic p-value approximation is obtained by simulation (bootstrap). The bootstrap suggested by Hansen is in fact a four step procedure, as follows:

- Let u_t^* $(t = 1, ..., n)$ be *i.i.d.* $N(0, 1)$ random draws;
- Set $y_t^* = u_t^*\hat{e}$;
- Obtain $W_n^*(\gamma)$, and thus W_n^*, and
- The asymptotic p-value is computed counting the percentage of bootstrap samples in which $W_n^* > W_n$.

3.5.5 Estimation procedure for the MSDI model

The estimation[8] procedure for the Markov-switching diffusion index model is centered on the evaluation of a weighted likelihood function. The weights in this case are the filtered probabilities of each regime. The density function of y_t conditional on the past information set (ψ_{t-1}) is given by:

8 A Gauss code was used to estimate this model. The optimum command and the Broyden-Fletcher-Goldfarb-Shanno were used as the algorithm in the nonlinear optimization.

$$f(y_t|\psi_{t-1}) = \sum_{S_t=0}^{1} f(y_t, S_t|\psi_{t-1}) \tag{25}$$

$$= \sum_{S_t=0}^{1} f(y_t|S_t, \psi_{t-1}) f(S_t|\psi_{t-1})$$

$$= \frac{1}{\sqrt{2\pi}\sigma_0} exp\left[\frac{-(y_t - c_0 - \beta_0 \hat{F}_t)^2}{2\sigma_0^2}\right] P[S_t = 0|\psi_{t-1}] +$$

$$+ \frac{1}{\sqrt{2\pi}\sigma_1} exp\left[\frac{-(y_t - c_1 - \beta_1 \hat{F}_t)^2}{2\sigma_1^2}\right] P[S_t = 1|\psi_{t-1}]$$

Thus, the likelihood function can be written as

$$lnL = \sum_{t=1}^{T} ln[\sum_{S_t=0}^{1} f(y_t|S_t, \psi_{t-1}) P[S_t|\psi_{t-1}]] \tag{26}$$

Before the evaluation of the likelihood function, the weights factors $P[S_t = j|\psi_{t-1}]$ for $j = 0, 1$ must be calculated. This is accomplished following the procedure suggested by Hamilton (1989). To start the described filter, the steady-state probabilities can be used as $P[S_0 = j|\psi_0]$. A detailed derivation of these steady-state probabilities is found in Hamilton (1994, p. 683).

After $P[S_t = j|\psi_{t-1}]$ is calculated, the log likelihood function is maximized with respect to $c_0, c_1, \beta_0, \beta_1, \sigma_0^2, \sigma_1^2, p$ and q. To do this, a similar procedure as the one described in the last paragraph of the estimation procedure for the TVPDI model is used. Once p and q are estimated the expected duration of a regime can be computed. Defining D as the duration of state 1, for example, it follows that [9],

[9] The duration of state 0 is obtained changing p by q.

$$E(D) = \sum_{j=1}^{\infty} jP[D = j] = \frac{1}{1-p} \tag{27}$$

3.5.6 Forecasting

The forecasting environment used in this work is based on a common practice nowadays – simulated real-time design forecasts. The simulated real-time forecasting environment has also influenced the estimation procedure. Predictions were made in a recursive fashion, except for TVPDI model. For the DI model after each forecast, the sample was updated and the model was re-estimated, Bayesian information criterion (BIC) was again computed, and another round of forecasts was produced. Thus, as the forecast period begins at 2002.Q1 the models were estimated from 1975.Q4 up to 2001.Q4 and the first period forecast was computed. Then, actual values at 2002.Q1 of these variables were included in the estimation sample, and the model and BIC for the DI model were re-estimated from 1975.Q4 up to 2002.Q1 and a forecast of $y_{2002:Q2}$ was generated. This step was repeated until the forecast of $y_{2003:Q3}$ was produced. Another difference is the sample length of the TARDI model, which was composed by the quarters between 1976.Q2 to 2001.Q4.

The general equation used for DI models, to make one step ahead forecasts, is:

$$\hat{y}_{T+1|T} = \hat{c} + \sum_{i=1}^{q_1} \hat{\alpha}_i y_{T-i+1} + \sum_{j=1}^{q_2} \hat{\beta}_j \hat{F}_{T-j+1} \tag{28}$$

Where, $y_{t+1} = \ln(\frac{y_{t+1}}{y_t})$ and $y_t = \ln(\frac{y_t}{y_{t-1}})$. Different versions of (28) were used to forecast. As in Stock and Watson (2002), the DI model uses only the current factor to forecast. DI-AR model is the DI model plus lags of the dependent variable $[1 \leq q_1 \leq 3]$. Another DI forecasts based on these two versions were tried. The DI-Lag allowed lags on the factors $[1 \leq q_2 \leq 3]$ and the DI-AR-Lag used current and lagged factors and lags of the dependent variable. Moreover, results of these models, where the number of factors and lags were chosen by Bayesian Information Criterion(BIC), are presented as DI-BIC, DIAR-BIC, DILAG-BIC and DIARLAG-BIC, respectively.

The number of factors in a model depends on whether the model has lagged factors or not. DILAG and DIARLAG models used up to three factors, while DI and DIAR models used up to five factors.

An autoregressive (AR) and a vector autoregressive (VAR) models were used as benchmarks for DI models' performance. Forecasts from the AR model were generated disregarding the second term in the right hand side of equation (28) and allowing for lags $[1 \leq q_1 \leq 3]$ to be set by BIC. The next equation shows the VAR model used in this study.

$$Y_t = \beta_0 + \Pi_1 y_{t-1} + \varepsilon_t \tag{29}$$

The variables used in the VAR [10] were the growth rate of Brazilian GDP (y_t), the growth rate of the money aggregate $(m1_t)$, the real interest rate (r_t) and the inflation rate (i_t). The only I(0) variable in level was r_t. All the others variables were I(1) in levels, but I(0) when expressed in growth rates.

In the case of the time varying parameter diffusion index (TVPDI) model, an equation similar to equation (28) was also used to compute the one step ahead forecast. The one step ahead forecast equation of the the TARDI model is,

$$\hat{y}_{T+1|T} = \left(\hat{c}_1 + \sum_{i=1}^{q_1} \hat{\alpha}_i^1 y_{T-i+1} + \sum_{j=1}^{q_2} \hat{\beta}_j^1 \hat{F}_{T-j+1} \right) I(g_{t-1} \leqslant \hat{\gamma}) + \\ + \left(\hat{c}_2 + \sum_{i=1}^{q_1} \hat{\alpha}_i^2 y_{T-i+1} + \sum_{j=1}^{q_2} \hat{\beta}_j^2 \hat{F}_{T-j+1} \right) I(g_{t-1} > \hat{\gamma}) \tag{30}$$

[10] The selection of variables for this model was grounded on some of the ideas presented in Moreira et al. (1996)

The variables tried in the function g_{t-1} were the short and long differences of log of GDP . The *delay lag* interval search was $d = [1, ..., 4]$.

The one step ahead equation of Markov-switching diffusion index (MSDI) model is,

$$\hat{y}_{T+1|T} = E(y_{T+1|T}) = \int y_{T+1} f(y_{T+1}|\psi_T) dy_{T+1} \tag{31}$$

$$= \sum_{j=0}^{1} P[S_{T+1} = j|\psi_T] E(y_{T+1}|S_{T+1} = j, \psi_T)$$

Where,

$$E(y_{T+1}|S_{T+1} = j, \psi_T) = c_j + \beta_j \hat{F}_t \tag{32}$$

$$P[S_{T+1} = j|\psi_T] = \sum_{i=0}^{1} P[S_{T+1} = j|S_T = i] P[S_T = i|\psi_T] \tag{33}$$

In the next section the practical problems and results of the estimation and forecasting procedures will be presented. A comparison of forecast efficiency for all the models is also calculated. This comparison is made up of ratios of Mean Square Forecast Error (MSFE) and plots of actual values against the predicted ones.

3.5.7 Combining forecasts

Since Bates and Granger (1969) the practice of pooling forecasts has shown consistent evidence in the sense that combined prediction may produce a smaller mean squared forecast error than individual forecasts of the same event.

This fact is not difficult to understand. First, if each of the individual forecasts provides only partial and non-overlapping information about some future event, it is natural to expect that its combination will present a larger information set. Moreover, Newbold and Granger (1974) also show that pooling is a good practice when its components are differently biased information sets. For example, combining an upward and a downward biased forecast is expected to outperform both isolated results.

What happens if overlapping information sets were combined? Diebold (1989) shows that if fixed weights are being used in the averaging process; then pooling may produce poorer predictions. His suggestion is to test for forecast encompassing first, and then to exclude encompassed forecasts from the combination.

This picture is completely different if structural breaks are being considered. Clements and Hendry (2002) show that in the presence of structural breaks a combination with an encompassed forecast may do better than another without it. Therefore, in this kind of situation, pretest for forecast encompassing can not produce a conclusive result about the choice of the components of certain forecast combinations.

With these ideas in mind, this work will use the following combining process. Let $If_t = [if_t^1 ... if_t^n]$ be the vector of n individual forecast made at time t, and $W_t = [w_t^1 ... w_t^n]'$ to be the vector of weights used in the pooling process. Then the type of combination used in this work can be described as:

$$C_t = If_t . W_t \tag{34}$$

Five different processes were used to calculate W_t, and then $If_t . W_t$:

a) Average – C_t will be the arithmetic average of If_t;

b) Median – C_t will be the median of If_t;

c) Regression 1 – $C_t = \alpha + w_t^1 if_t^1 + ... + w_t^n if_t^n + e_t$

d) Regression 2 – $C_t = w_t^1 if_t^1 + ... + w_t^n if_t^n + e_t$, subject to $\sum_i w_t^i = 1$; and

e) Variance of forecast error – $\left[\frac{\sum_t (ef_t^i)^2}{\sum_i (\sum_t (ef_t^i)^2)}\right]^{-1}$, where ef_t^i is the forecast error of the individual forecast i at time period t.

The method (d) is called the constrained regression form [11]. In this case, if all individual forecasts are unbiased, the combination will be too. Granger and Ramanathan (1984) show that the unconstrained form (c) is expected not only to produce smaller errors than (d), but also to produce unbiased combined forecast even if the component forecasts are biased. The inverse of the variance proportion of the forecast error technique follows Bates and Granger (1969).

4 Empirical Results

The AR(1) was chosen because it generated the best forecasts among AR(p) models, for $p = 1, 2, 3$. Moreover, the AR(1) forecasts were better than the ones generated by the VAR(1) model [12]. The Mean Square Forecast Error (MSFE) between AR(1) and VAR(1)–$\frac{MSFE\ VAR(1)}{MSFE\ AR(1)}$– was around 1.03. Thus, the efficiency measure of the different prediction mechanisms used in this study was the ratio [13] of the MSFE of AR(1) to the other DI models. All the tables in the next subsections are presented in appendix II, while figures are in appendix III.

4.1 Diffusion index results

Table 1 shows the MSFE ratios of one step ahead forecast errors of AR and linear DI models. The DI one step ahead forecasts for the growth of GDP were better than the ones from the AR(1) model, except for DI-AR and for the DI-AR-Lag forecasts. One can see that the simplest DI model, with just

[11] Both (c) and (d) are estimated by OLS.

[12] It was used a VAR(1) with the variables mentioned above plus a dummy variable to capture the structural breaks.

[13] This efficiency measure is very common in empirical studies. It was also used, for instance, by Stock and Watson (1998).

one factor, could improve almost 35% on AR(1) forecasts. Moreover, model selection by BIC, in the case of pure DI models without the autoregressive part, has the same forecast efficiency as the unique fixed factor DI model. Also, allowing for factor lags does not improve on the fixed DI model.

After that, two stacked panels were used to estimate the factor loadings. They included one and two lags of all the series contained in the unstacked model, respectively. The results of stacked data were not better than the results of the unstacked panel. Indeed, some of the models did worse with stacked data. A next step was to verify if a binary panel data would predict better. Thus, the positive values of the unstacked panel were set equal to one, and the negative values were set equal to zero. The results of this procedure were very similar to the original unstacked panel.

The result that only a small set of factors could be used to forecast is in tune with other recent studies, for example Stock and Watson (1998) and Brisson et al. (2003). Indeed, the forecasts generated by DI models with one, two or three factors are so similar that their plots are indistinguishable; i.e., the plots become a thick line.

Based on that, all the analyses from now on will be concentrated on the fixed DI model with only one factor, because it is parsimonious and it was chosen by BIC criterion. All the slope parameters of this model are significant at the 5% level.

Figure A1 shows that the DI model forecast values are not only closer to actual values, but also that they predict changes of direction more accurately than the AR model. If the large shift at 2003.Q1, due to presidential election and market's negative expectations about the upcoming economic policy, were included in the model these forecasts probably would have had a better performance.

4.2 Time varying parameter diffusion index results

Some modifications of this model were made in the state equation. These charges included lag length and stationary and nonstationary autoregressive coefficients. The best forecast model was the one with the same structure as equations (3) and (4). The parameters of the model were not significant and the predictions were better than in the Autoregressive model, but worse than in the linear DI model. Its MSFE ratio compared to the AR(1) model was 0.81, meaning that this model improved on AR(1) model something around 19% in terms of predictive accuracy. But, as discussed before, DI model improved almost 35% on AR(1) model. This model predicted signs very well as table 6 shows. Figure A2 plots actual values, TVPDI and AR forecasts.

Stock and Watson (1994) show that TVP models hardly improve on recursive least squares when the goal is to produce one step ahead forecasts. This study corroborates this finding. TVPDI forecasts were not generated recursively and they were the worst among DI models.

4.3 Threshold autoregressive diffusion index results

In dealing with TAR models, it is usual to test for the existence of different regimes before forecasting. Thus, the selected linear DI model specification to forecast the growth of Brazilian GDP was tested against the alternative of a two regime TAR model. For this purpose it was used an adaptation of the GAUSS code designed by Hansen (1997). As stated before, short and long differences of GDP were tried as the threshold effect variable. The integer *delay lag* was allowed to vary in the set $d = [1, ..., 4]$.

Table 2 presents a summary of the testing results. The $p - values$ suggest that there is a significant threshold effect at less than a 5% significance level when the long difference $Ln(gdp_{t-1}/gdp_{t-3})$ is considered.

Table 3A and 3B present a summary of the estimation procedure[14]. From these tables it is possible to verify that the estimated values of the parameters were almost constant. This pattern changes substantially in 2003.Q1, when the growth of Brazilian GDP suffered a huge dive.

The residuals of the TARDI model were heteroskedasticity free. The tests to check for remaining non-linearity was not able to reject the hypothesis

[14] *SD* and *df* mean standard deviation and degrees of freedom, respectively.

of linearity. Thus a two regime TAR model is sufficient to capture the non-linear pattern in the time series under observation.

In terms of forecasting quality the ratio between the MSFE of TARDI and DI (AR) is 0.93 (0.60). Not only is the MSFE of the TARDI model smaller than the MSFE of the DI model, but also was found that the TARDI model predicted the direction more accurately than the DI model, except for the 2002.Q4 and 2003:2 values. This result is presented in table 6 and plotted in figure A3.

4.4 Markov-Switching diffusion index results

Some modifications of MSDI models were estimated and used to forecast. Among these, models allowing for changes in both their coefficients and in the variance parameter were tried. After that, models allowing for different intercepts, with equal and different variance, parameters were estimated.

When the subject is to estimate a Markov-switching (MS) model it is common to use intervention procedures such as dummy variables, and the use of different variance coefficients to capture the effects of those pulses that look like outliers in the sample. The reason for that is that without an intervention procedure the MS model only captures those larger peaks, and this may cause problems to the estimation process of the mean, p and q values.

Thus, some changes of the MSDI models with dummy variables such as $c_{St} = (c_0 + \tilde{c}_0 D)(1 - S_t) + (c_1 + \tilde{c}_1 D)S_t$ were also estimated and used to forecast.

A model allowing only the intercept to change with a single variance and without a dummy variable for the regime 0, i.e., with $\tilde{c}_0 = 0$, called here MSDI1, produced the best forecast but weird values for p and q. These results are presented in table 4.

Except for the estimates of c_0 and q all the other parameters are significant at the 5% level. This model was the one which produced the best fit of the data. But the most important result in Table 4 is the estimate for p.

This estimate means that the Markov-switching model is a reducible one and that once it reaches an expansion stage the economy will stay there forever.

On the other hand, when forecast performance is the goal, the MSDI1 model works fine. Its MSFE ratio to the AR's MSFE is only 0.53, almost equal to the TARDI model which presented a 0.60 ratio. The ratio of MSDI1's MSFE to the MSFE of the TARDI model is about 0.89. Figure A4, shows actual and forecast values from MSDI1 and AR models.

Table 6 indicates that both MSDI1 and TARDI model produced similar forecasts, specially when one observes the direction of the predicted values. Another MSDI model that deserves attention is the one estimated with $\tilde{c}_0 \neq 0$ and $\tilde{c}_1 \neq 0$, and different variance parameters for each economic regime – recession and expansion. The estimation results for the first round of estimates in 2001.Q4 of this model, which will be called from now on MSDI2, are presented in table 4.

Excluding $\hat{q}$, all the other estimates are statistically significant at the 5% significance level. MSDI2 models also fitted the data quite well, but its prediction efficiency was no better than in the linear DI model. The MSFE ratio between MSDI2 and AR was 0.88, meaning that this non-linear DI model improves on AR forecast, but it could not improve on any other model.

Chauvet et al. (2002) proposed a Hamilton type and a Lam type Markov-switching model to estimate Brazilian business cycle and to forecast quarterly Brazilian GDP growth rates. Their result was compared to an ARMA(1,1) and AR(3) forecasts. They found something similar to the results of this study. First, the best Markov-switching type model to forecast is not the same to estimate business cycle properties. Their best model to explain cycles was also a model that incorporated an intervention analysis and the model without this mechanism was the best to forecast.

Their model estimates that in recession (expansion) Brazilian GDP grows at an average rate of -1.4% (1.6%) per quarter. In this study, the MRSDI1 figures are -1.3% (0.87%) and for the MRSDI2 they are -1.3% (2.2%), respectively. In terms of duration of the economic cycle, Chauvet et al. (2002) estimate a 2-3 quarters for the recession duration and 4-5 and 6-7 for the expansion duration, with the two types of MS used – Hamilton's MS-AR(2) and Lam's MSG-AR(2). In this study the duration results for the MRSDI2 model are 1-2 quarters for the recession period and 2-3 for the expansion phase.

Chauvet, Lima and Vaquez used their estimated Hamilton's MS-AR(4) model without any type of intervation mechanism to forecast Brazilian GDP growth rate for the period 1992:2 to 2002:2, and compared it to an ARMA[15] (1,1) and AR(3). The one-step-ahead MSFE ratio amog these models was used for this purpose. Their estimated MS model improved only 2.5% upon ARMA(1,1) forecasts. In this study MRSDI1 (MRSDI2) was 47% (12%) better than an AR(1) model.

4.5 Combining forecast

The regression methods to combine forecasts, discussed above, improved on the best individual forecast mechanism – MSDI model. As expected, the unconstrained method produced the best results in terms of smallest MSFE. Table 4 shows the ratio of each pooling process MSFE compared to AR, DI and MSDI models.

The pooling technique based on the unconstrained regression (c) improves almost 84%, 75% and 69% on AR, DI and MSDI respectively. What could explain this enormous supremacy of combined forecast over individual forecast? Clements and Hendry (2001) show that when there are structural breaks in the variable to be predicted, pooling is a good technique to diminish the negative effect of these breaks on individual forecasts.

Figure A5 plots actual, AR and combined forecasts with method (c). As one can see, all these models are more useful to predict direction and signs than values. The non-linear DI models forecast direction and signs better than the linear DI model, which is better than AR. But in this type of comparison, the unconstrained technique of pooling forecasts is the best one. It missed only one direction (2002.Q3) and got all signs right.

[15] They also used an AR(3) as the benchmark model. In the case of one-step-ahead forecast horizon, the ARMA(1,1) was better than AR(3) forecasts.

5 Concluding Remarks

In order to forecast GDP growth rate, macroeconomic theory would suggest the use of a large set of financial, monetary, and other real and nominal variables to be included in a model capable to mimic some stylized facts of business cycles, such as the comovements among a set of variables.

From the point of view of economic forecasting practice, parsimonious models have a great advantage in terms of forecast performance compared to large econometric theory based models.

This work used linear and nonlinear diffusion index models (DI) to forecast quarterly Brazilian GDP growth rate. A DI model is basically a static representation of an unobservable dynamic factor model. Both models may be used to capture the comovements between variables and to reduce, at the same time, the number of parameters in the model used to forecast.

Quarterly data from 1975:1 up to 2003:3 about Brazilian GDP and another 72 macroeconomic variables, representing the external sector and the nominal and real side of the economy, were used to compute the diffusion index. The estimation period ended up in 2001:4 and forecasts were made from 2002:1 to 2003:3 in a recursive environment.

The results in terms of forecast performance were very encouraging. The linear DI model with only one factor improved 35% on an autoregressive (AR) model, when their MSFE were compared. A time varying DI model was tried and its forecast performance was better than the AR's, but not better than the simple linear DI model. This corroborates a previous result found by Stock and Watson (1994). They found out that time varying parameter models are not good to forecast instability in macroeconomic time series, something that is better accomplished with recursive forecasts. This model also predicted signs very well.

In addition, nonlinear models such as a threshold DI (TARDI) and Markov-switching DI (MSDI) model were used to forecast. This kind of model allows the parameters to change according to economic regime (recession and expansion). Not only did the TARDI model improve on the linear DI model by 7% and 40% compared to the AR, but also that the test for a threshold effect against the linear model confirmed that a nonlinear pattern in the Brazilian GDP growth rate exists.

The results concerning MSDI models are dubious. On the one hand, the MSDI1 model improved on linear DI forecast performance around 17% and 47% compared to the AR, but it was not useful to estimate the duration of economic regimes.

On the other hand, the MSDI2 explains the cycles better than does MSDI1, but it was not as good as MSDI1 in predicting. The MRDI2 model estimates a duration of 2-3 (1-2) quarters for the expansion (recession) phase and these results are close to the ones found by Chauvet et al. (2002). They estimated a duration of 4-5 and 6-7 quarters for expansion and 2-3 quarters for recession. These estimates show that Brazilian business cycles are very short.

The forecast performance of MSDI2 was only 12% better than the AR model, and worse than the linear DI model. However, this result is not too bad, specially when one takes into account that the one-step-ahead forecast for 1992:2 to 2000:2 of a MS model used by Chauvet, Lima and Vasquez improved only 2.5% upon an ARMA(1,1) model.

Combined forecasts produced the best forecast results. Their MSFE were only 16%, 25% and 31% of the MSFE of AR, DI and MSDI1 models, respectively. One possible reason for this fact is the presence of structural breaks in the variable being predicted. In this case, pooling forecasts usually produces better results than do individual ones. It is important to remember that all these linear, non-linear and combining forecast mechanisms used in this study work better predicting direction, turning points and signs than values of quarterly Brazilian GDP growth rate.

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Appendix I

List of series and transformations

BDI	Brazilian Direct Investment	**	0
CDB	Interest Rate-Bank Deposit Certificate (CDB)	*	1
DINT	Internal Debt	**	3
DINV	Direct Investment	**	0
DMF	Federal Internal Mobiliary debt	*	3
EFD	Financial Execution of National Treasury Debt	*	3
EFR	Financial Execution of National Treasury Credit	*	3
EMPG	Loans of Financial System to Private Sector	*	3
EMPH	Loans of Financial System to Private Sector-Habitation	*	3
EXPOM	Mundial Exports (index)	*	2
EXPOR	Exports (index)	*	2
EXPPI	Imports of Industrialized Countries (index)	*	2
IBOV	IBOVESPA-Index of Stock Market-Brazil	***	3
EXRDEF	Exchange Rate (R$/US$)	*	3
FDI	Foreign Direct Investment	*	0
FPI	Foreign Portfolio Investment	*	0
GDP_CA	GDP of Canada (index)	*	6
GDP_CH	GDP of China (index)	****	4
GDP_CO	GDP of Korea (index)	*	4
GDP_ES	GDP of Spain (index)	*	4
GDP_F	GDP of France (index)	*	4
GDP_G	GDP of Germany (index)	*	2
GDP_IT	GDP of Italy (index)	*	6
GDP_J	GDP of Japan (index)	*	6
GDP_UK	GDP of United Kingdom (index)	*	6
GDP_US	GDP of USA (index)	*	4
HTSP	Index of Hours Worked In Ind. Prod. of The State of Sao Paulo	*	4
IBC	Index of Industrial Production – Consumer Goods	*	4
IBI	Index of Industrial Production – Intermediate Goods	*	4
IBK	Index of Industrial Production – Capital Goods	*	4
IBNC	Index of Industrial Production – Nondurable Consumer Goods	*	4
ICD	Index of Industrial Production – Durable Consumer Goods	*	4
ICVSP	Cost of Living Index of Sao Paulo	*	6
IEM	Index of Industrial Production –Mining	*	4
IF	Index of Industrial Production – Pharmaceuticals	*	4
IG	Index of Industrial Production – General	*	4
IGPDI	General Price Index Domestic Supply	*	2
IMEC	Index of Industrial Production – Mechanics	*	4
IMELET	Index of Ind. Production – Electrical and Communications Equip.	*	4
IMETA	Index of Industrial Production –Metallurgy	*	4
IMP	Imports (index)	*	4
IMPM	Mundial Imports (index)	*	4
IMPPI	Imports of Industrialized Countries (index)	*	4
IMTRANS	Index of Industrial Production –Transport Equi.	*	4
INCC	INCC Price Index	*	2
IPALI	Index of Industrial Production –Food Products	*	4
IPAPEL	Index of Industrial Production –Paper and Cardboard	*	4
IPLAS	Index of Industrial Production –Plastics	*	4
IQ	Index of Industrial Production –Chemicals	*	4
IT	Index of Industrial Production –	*	4
ITEX	Index of Industrial Production –Textiles	*	4
IVEST	Index of Ind. Prod.–Clothing, Footwear and Leather Goods	*	4
KGIR	Interest Rate Credit Operations to Short Term Private Capital	*	1
M0	M0-Monetary Aggregate	*	4
M1	M1-Monetary Aggregate	*	4
OPB	Overall Balance of Payment Results	*	2
OVER	SELIC Interest Rate (Monetary Policy)	*	1
GDPBR	GDP of Brazil	*	5
PINV	Portfolio Investment	*	0
POSP	Index of Employed People in Ind. Prod. of State of Sao Paulo	*	2
REI	International Reserve	*	2
UCIBC	Capacity Utilization Rate-Industry-Capital Goods	*	7
UCIBI	Capacity Utilization Rate-Industry-Intermediate Goods	*	7
UCIMC	Capacity Utilization Rate-Industry-Material construction	*	7
UCIMEAN	Capacity Utilization Rate-Industry-Mean	*	7
USIR1	USA Interest Rate-Federal Funds-3-month	*	1
USIR2	USA Interest Rate-Treasury Maturities-10-years	*	1
USIR3	USA Interest Rate-Treasury Maturities-3-years	*	1
USIR4	USA Interest Rate-Prime-3-month	*	1
USIR5	USA Interest Rate-Treasury Bills-3-month	*	1
USIR6	USA Interest Rate-Treasury Bills-6-month	*	1
VNSP	Index of Nominal of The Retail Trade in Sao Paulo-Industry	*	4

Where,

(*) Data from Ipeadata; (**) Data from Central Bank of Brazil; (***) Data from Economatica; (****) IMF/IFS; [0] Growth Rates; [1] First Difference (1diff); [2] Ln+1diff; [3] Ln+Deflating+1diff; [4] Ln+Seas. Adj.+1diff; [5] Ln+deflating+seas.adj+1diff; [6] Ln+Second Difference (2diff); [7] $\Delta Ln(\frac{X_t}{100-X_t})$.

Appendix II

Table 1−MSFE Ratios of One Step Ahead Forecasts Errors of AR and Linear DI Models. Period: 2002.Q1 to 2003.Q3

	Models	Models
	DI	**DI-AR**
num. fac.		
$r = 1$	0.65	1.00
$r = 2$	0.65	1.08
$r = 3$	0.64	1.08
$r = 4$	0.81	1.78
$r = 5$	0.82	1.84
num. Lags	**DI-Lag**	**DI-AR-Lag**
$q_2 = 1$	0.65	1.50
$q_2 = 2$	0.65	1.08
$q_2 = 3$	0.64	1.08
BIC	**DI-BIC**	**DIAR-BIC**
	0.65	1.00
BIC	**DILAG-BIC**	**DIARLAG-BIC**
	0.65	1.50

* Tests of equality of MSFE's (Clements and Hendry 1998) would be desirable. Small sample size, however, would ruin their results.

Table 2−Testing for Threshold Effect:1976.Q2 to 2001.Q4

g_{t-1}	$\widehat{\gamma}$	$p - value$
$(Ln(gdp_{t-1}/gdp_{t-2}))_{t-1}$	0.093	0.507
$(Ln(gdp_{t-1}/gdp_{t-2}))_{t-2}$	-0.050	0.336
$(Ln(gdp_{t-1}/gdp_{t-2}))_{t-3}$	0.060	0.242
$(Ln(gdp_{t-1}/gdp_{t-2}))_{t-4}$	0.060	0.716
$Ln(gdp_{t-1}/gdp_{t-2})$	0.134	0.692
$Ln(gdp_{t-1}/gdp_{t-3})$	-0.029	0.026
$Ln(gdp_{t-1}/gdp_{t-4})$	0.068	0.230

Table 3A–Estimatinon Results for Regime 1: $g_{t-1} \leqslant \hat{\gamma}$

OBS	$\hat{c}_1$	$\hat{\beta}_0^1$	SD_{c1}	$SD_{\beta 1}$	$\widehat{\gamma}$	*df*
01.$Q4$	0.024	0.068	0.008	0.070	-0.029	33
02.$Q1$	0.024	0.068	0.008	0.070	-0.029	33
02.$Q2$	0.024	0.068	0.008	0.070	-0.029	33
02.$Q3$	0.024	0.068	0.008	0.070	-0.029	33
02.$Q4$	0.024	0.068	0.008	0.070	-0.029	33
03.$Q1$	0.026	0.079	0.008	0.070	-0.034	31
03.$Q2$	0.024	0.086	0.008	0.070	-0.034	32

Table 3B–Estimation Results for Regime 1: $g_{t-1} > \hat{\gamma}$

OBS	$\hat{c}_2$	$\hat{\beta}_0^2$	SD_{c2}	$SD_{\beta 2}$	$\widehat{\gamma}$	*df*
01.$Q4$	-0.006	0.239	0.006	0.104	-0.028	66
02.$Q1$	-0.006	0.239	0.006	0.104	-0.028	67
02.$Q2$	-0.007	0.238	0.006	0.104	-0.028	68
02.$Q3$	-0.007	0.238	0.006	0.103	-0.028	69
02.$Q4$	-0.007	0.235	0.006	0.103	-0.028	70
03.$Q1$	-0.008	0.225	0.006	0.103	-0.034	73
03.$Q2$	-0.008	0.225	0.006	0.103	-0.034	73

Table 4–Estimation Results for MSDI1 and MSD2: 1975.Q3 to 2001.Q4

OBS	$MSDI1$	$MSDI2$	$SD(MSDI1)$	$SD(MSDI2)$
$\hat{c}_0$	-0.013	-0.013	0.062	0.006
$\hat{c}_1$	0.009	0.022	0.004	0.009
$\hat{\beta}$	0.110	0.136	0.047	0.041
$\hat{p}$	1	0.606	0.000	0.159
$\hat{q}$	0	0.339	0.000	0.235
$\hat{\sigma}_0^2$	0.045	0.050	0.003	0.004
$\hat{\sigma}_1^2$	-	0.019	-	0.005
$\tilde{c}_0$	-	-0.229	-	0.022
$\tilde{c}_1$	-0.178	-0.066	0.027	0.037
R^2	0.41	0.293		

Table 5– MSFE Ratios for Different Pooling Procedures

Combining Methods\Denominator	**AR**	**DI**	**MSDI**
Average	0.73	1.15	1.38
Median	0.58	0.91	1.09
Unconstrained Regression	0.16	0.25	0.31
constrained Regression	0.35	0.55	0.66
Variance of Forecast Error	0.85	1.33	1.60

Table 6– Actual and Predicted Values, MSFE and RMSFE

	Actual	**AR**	**DI**	**TARDI**
2002:1	-0.013	-0.005	0.016	-0.006
2002:2	-0.000	0.007	-0.008	0.011
2002:3	-0.017	0.004	0.010	-0.022
2002:4	-0.027	0.008	0.006	0.002
2003:1	-0.077	0.010	-0.009	-0.003
2003:2	-0.014	0.020	-0.007	-0.023
2003:3	0.036	0.006	0.025	0.020
MSFE		0.00163	0.00105	0.00098
RMSFE		0.040	0.033	0.031
	Actual	**TVPDI**	**MSDI1**	**Comb Unc**
2002:1	-0.013	0.017	-0.002	-0.031
2002:2	-0.000	-0.000	0.008	-0.015
2002:3	-0.017	-0.000	-0.008	-0.001
2002:4	-0.027	-0.008	-0.005	-0.018
2003:1	-0.077	-0.015	-0.007	-0.052
2003:2	-0.014	-0.040	-0.021	-0.033
2003:3	0.036	-0.020	0.015	0.038
MSFE		0.00131	0.00087	0.00027
RMSFE		0.036	0.030	0.016

Appendix III

Figures A1 to A5 – Actual and predicted values