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Estate Taxes, Consumption Externalities, And Altruism

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Estate Taxes, Consumption Externalities, and Altruism*

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Abstract

We study how the introduction of consumption externalities affects the efficiency of the dynamic equilibrium in an economy displaying dynastic altruism. When the bequest motive is inoperative consumption externalities affect the intertemporal margin between young and old consumption and thus modify the intertemporal path of consumption and capital. The optimal tax policy that solves this intertemporal inefficiency consists of a tax on capital income and a pay-as-you-go social security system. The later solves the overaccumulation of capital due to the inoperativeness of the bequest motive and the former solves the inefficient allocation of consumption due to consumption externalities. When the bequest motive is operative consumption externalities only cause an intratemporal inefficiency that affects the allocation of consumption between the generations living in the same period but do not affect the optimality of the capital stock level. This suboptimal allocation of consumption implies in turn that the path of bequest is also suboptimal. The optimal tax policy in this case consists of an estate tax and a capital income tax. The estate tax corrects the intratemporal inefficiency but generates an intertemporal inefficiency which is corrected by means of an appropriate capital income tax.

JEL classification codes: E21, E13, E62.

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1. Introduction

In this paper we aim at analyzing the inefficiency arising from consumption externalities in an overlapping generation model (OLG) with dynastic altruism. We use this model to show how the interaction between consumption externalities and altruism affects the efficiency of the equilibrium path. In our model the consumption externality will take the form of a reference level of consumption that is used to compare the utility derived from own consumption. We will assume that this reference consumption is a weighted average of the consumption of all the agents living in the same period.

Several authors have analyzed the implications of a reference consumption due to own past consumption (or internal habits) in OLG models. Examples of this strand of the literature are the papers of Lahiri and Puhakka (1998) and Wendner (2002), who study the effect of habits on saving in a pure exchange economy; and Alonso-Carrera, et al. (2005b), de la Croix (1996) and de la Croix and Michel (1999, 2001), who analyze several related stability issues. Obviously, when the reference is the others' consumption, then inefficiency of the equilibrium path is likely to arise. In a framework with infinitely lived agents, Alonso-Carrera, et al. (2004 and 2005a), Fisher and Hof (2000), Guo (2002), Liu and Turnovsky (2005), Ljungqvist and Uhlig (2000), and Turnovsky and Monteiro (2005), among many others, have characterized the optimal tax rates that solve this kind of inefficiency. In the framework of OLG models, Abel (2005) shows that a capital income tax and a pay-as-you-go social security system constitute the optimal tax policy. We extend the latter paper by introducing consumption externalities in the dynastic altruism model of Barro (1974). Following Abel's analysis, we assume that these externalities take the form of a weighted average of the consumption of the two types of agents living in the same period. By introducing altruism and the possibility of bequests in Abel's model we can analyze new phenomena, like the potential suboptimality of bequests and the interaction between consumption externalities and the inefficiency of the bequest constrained equilibrium.

The efficiency analysis in Abel (2005) is based on the comparison between the model of Diamond (1965) with consumption externalities and the corresponding planner problem when the objective function of the planner is just the discounted sum of the utilities of all the agents in the economy. This analysis does not allow to distinguish the inefficiency due to consumption spillovers from the inefficiency due to the inoperativeness of the bequest motive. In contrast, the efficiency analysis of our paper is based on the comparison between the model of Barro (1974) with consumption externalities and the corresponding planner problem. As shown by Abel (2005), when the bequest motive is inoperative, inefficiency is due to both consumption externalities and the typical dynamic inefficiency resulting in capital overaccumulation (Cass, 1972). However, when the bequest motive is operative the only source of inefficiency are the consumption externalities. Therefore, our efficiency analysis shows how the inefficiency due to consumption externalities depends on the operativeness of the bequest motive.

When the bequest motive is operative consumption externalities only cause an intratemporal inefficiency, whereas they cause an intertemporal inefficiency when the bequest motive is inoperative. In the former case, they only affect the intratemporal margin between consumption of the two generations living in the same period. This implies that they modify the allocation of consumption between the two living generations, but do not affect the efficiency of the intertemporal paths of saving and output. In fact, this suboptimal allocation of consumption is associated with a suboptimal level of bequest. In contrast, when the bequest motive is inoperative, consumption externalities affect the intertemporal margin of consumption along the life cycle of agents. Given that consumption spillovers affect the intertemporal path of consumption, they also modify the paths of saving and production. In fact, the externality associated with young consumption reduces the stock of capital and, hence, reduces the overaccumulation of capital due to the inoperativeness of the bequest motive. Obviously, the externality associated with old consumption has the opposite effect on the stock of capital and thus increases the gap between the optimal stock of capital and the one obtained in a competitive equilibrium.

Note that the nature of the inefficiency crucially depends on the operativeness of the bequest motive and this operativeness depends in turn on the intensity of consumption externalities. In this respect we show that in the long run a stronger young consumption reference reduces the critical level of altruism above which the bequest motive is operative, whereas a larger old consumption reference rises this critical level of altruism.

The optimal values of tax rates also depend on the operativeness of the bequest motive. When the bequest motive is inoperative, the optimal tax policy consists of a tax on capital income and a pay-as-you-go social security system. The latter solves the overaccumulation of capital and the former solves the inefficient allocation of consumption due to consumption externalities. Note that the optimal capital income tax rate modifies the intertemporal path of consumption and, in this way, solves the inefficiency due to consumption spillovers. When the bequest motive is operative, the optimal tax policy consists of an estate tax and a capital income tax. On the one hand, the pay-as-you-go social security system is not required as there is no overaccumulation of capital. On the other hand, the capital income tax does not solve the inefficiency due to consumption externalities because this inefficiency is intratemporal when bequests are positive. In fact, since this inefficiency results into a suboptimal level of bequest, an estate tax allows to correct this intratemporal inefficiency. However, this estate tax causes an intertemporal inefficiency, which is solved by means of introducing an appropriate capital income tax.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the competitive equilibrium when the bequest motive is operative and when it is not. Section 4 characterizes the social planner solution. In Section 5 we conduct the efficiency analysis by comparing the solution achieved by the planner with the competitive solution. Section 6 characterizes the optimal tax rates. Section 7 concludes the paper.

2. The model

Let us consider an OLG model where N_t identical individuals are born in period t . These individuals live for two periods. Each individual has offspring at the end of the first period of his life and the number of children per parent is $n \geq 1$. As in Diamond (1965), each agent supplies inelastically one unit of labor in the first period of his life and is retired in the last period of his life. We index each generation by the period in which its members work.

Individuals are assumed to be altruistic towards their children. Let b_t be the amount of bequest that an old individual leaves to each of their children in period t . We impose the constraint that parents cannot force their descendants to give them gifts,

$$b_t \geq 0. \quad (2.1)$$

Each young individual distributes his labor income and his inheritance between consumption and saving. Therefore, the budget constraint faced by an individual during the first period of life is

$$w_t + b_t = c_t + s_t, \quad (2.2)$$

where c_t is the amount of consumption of a young agent, w_t is the labor income and s_t is the amount saved. In the second period of life individuals receive a return on the amount of their saving, which is distributed between consumption and bequests for their children. Therefore, the budget constraint of an old individual is

$$R_{t+1}s_t = x_{t+1} + nb_{t+1}, \quad (2.3)$$

where R_{t+1} is the gross rate of return on saving and x_{t+1} is the amount of consumption of an old individual.

The utility function of an individual belonging to generation t is

$$V_t = U(\hat{c}_t, \hat{x}_{t+1}) + \beta V_{t+1}, \quad (2.4)$$

where V_{t+1} represents the indirect utility of each of his descendants, the parameter $\beta \in [0, 1)$ is the altruism factor,¹ and the variables \hat{c}_t and \hat{x}_{t+1} represent the effective consumption in the first and second periods of life, respectively, of a representative individual belonging to generation t . We assume that individuals do not derive utility from the absolute level of consumption but from the comparison between their consumption and some consumption reference. In particular, we assume the following functional forms for effective consumption:

$$\hat{c}_t = c_t - \gamma v_t^y, \quad (2.5)$$

and

$$\hat{x}_{t+1} = x_{t+1} - \delta v_{t+1}^o, \quad (2.6)$$

¹Each parent cares equally about the felicity of their n children. Thus, the intercohort utility discount β could be rewritten as $\beta = n\rho\beta'$, where ρ would be the temporal discount factor and β' is the pure interpersonal (from parents to children) discount factor.

where $\gamma \in [0, 1)$ and $\delta \in [0, 1)$ provide a measure of the intensity of the consumption reference.² These consumption references are assumed to be a weighted arithmetic average of the per capita consumption of the two living generations. On the one hand, we assume that

$$v_t^y = \frac{N_t c_t + \theta^y N_{t-1} x_t}{N_t + \theta^y N_{t-1}} = \left(\frac{n}{n + \theta^y} \right) c_t + \left(\frac{\theta^y}{n + \theta^y} \right) x_t, \quad (2.7)$$

where $\theta^y \in [0, 1]$ is the weight of consumption of a representative old consumer in the specification of the reference for young consumers. On the other hand, we assume that

$$v_{t+1}^o = \frac{\theta^o N_{t+1} c_{t+1} + N_t x_{t+1}}{\theta^o N_{t+1} + N_t} = \left(\frac{\theta^o n}{\theta^o n + 1} \right) c_{t+1} + \left(\frac{1}{\theta^o n + 1} \right) x_{t+1}, \quad (2.8)$$

where $\theta^o \in [0, 1]$ is the weight of consumption of a representative young consumer in the specification of the reference for old consumers. Note that the restrictions imposed on the values of the parameters θ^y and θ^o imply that we are giving a larger weight to the average consumption of the agents belonging to the same generation. Let us define $\varepsilon^y = \frac{n}{n + \theta^y}$ and $\varepsilon^o = \frac{\theta^o n}{\theta^o n + 1}$. Then, equations (2.7) and (2.8) can be rewritten as follows:

$$v_t^y = \varepsilon^y c_t + (1 - \varepsilon^y) x_t, \quad (2.9)$$

and

$$v_{t+1}^o = \varepsilon^o c_{t+1} + (1 - \varepsilon^o) x_{t+1}. \quad (2.10)$$

As in Abel (1986) or Laitner (1988), we assume that the function $U(\cdot, \cdot)$ is twice continuously differentiable and additive in its two arguments. Therefore, we will use the following functional form:

$$U(\hat{c}_t, \hat{x}_{t+1}) = u(\hat{c}_t) + \rho u(\hat{x}_{t+1}), \quad (2.11)$$

where $\rho > 0$ is the temporal discount factor. We assume that $u' > 0$, $u'' < 0$, $\lim_{z \rightarrow 0} u'(z) = \infty$ and $\lim_{z \rightarrow \infty} u'(z) = 0$.

There is a single commodity in this economy, which can be devoted to either consumption or investment. Let us assume that this commodity is produced by means of a neoclassical net production function $F(K_t, L_t)$, where K_t is the capital stock and L_t is the amount of labor used in period t . The net production function per capita is $f(k_t)$, where k_t is the capital stock per capita. As firms behave competitively, the rental prices of the two inputs equal their marginal productivities,

$$R_t = f'(k_t) \equiv R(k_t), \quad (2.12)$$

$$w_t = f(k_t) - f'(k_t)k_t \equiv w(k_t). \quad (2.13)$$

We assume that $f'(k_t) \rightarrow -\nu$ as $k_t \rightarrow \infty$, where $\nu \in (0, 1)$ is the depreciation rate of the capital stock. In equilibrium the capital stock installed in period $t + 1$ is equal to the aggregate saving in period t and, thus, we have

$$nk_{t+1} = s_t. \quad (2.14)$$

²We assume an additive specification for effective consumption instead of the multiplicative formulation of Abel (2005) in order to guarantee concavity of the social planner's utility function (see Alonso-Carrera, et al., 2005a).

3. Competitive equilibrium

The problem faced by each individual is to maximize (2.4) with respect to $\{c_t, x_{t+1}, b_{t+1}\}$ subject to (2.1), (2.2), (2.3), (2.5) and (2.6), which is equivalent to solving the following dynamic programming problem:

$$V_t(b_t) = \max_{\{s_t, b_{t+1}\}} \{u(w_t + b_t - s_t - \gamma v_t^y) + \rho u(R_{t+1}s_t - nb_{t+1} - \delta v_{t+1}^o) + \beta V_{t+1}(b_{t+1})\}, \quad (3.1)$$

with $b_{t+1} \geq 0$, for v_t^y, v_{t+1}^o, w_t and R_{t+1} given for all t .

Using the envelope theorem we obtain,

$$\frac{\partial V_{t+1}}{\partial b_{t+1}} = u'(\hat{c}_{t+1}^1). \quad (3.2)$$

Using (3.2), we obtain the first order conditions of problem (3.1) corresponding to the derivatives with respect to s_t and b_{t+1}

$$u'(c_t - \gamma v_t^y) = \rho R_{t+1} u'(x_{t+1} - \delta v_{t+1}^o), \quad (3.3)$$

and

$$n\rho u'(x_{t+1} - \delta v_{t+1}^o) \geq \beta u'(c_{t+1} - \gamma v_{t+1}^y), \quad (3.4)$$

where the last condition holds with equality if $b_{t+1} > 0$. Equation (3.3) characterizes the optimal allocation of consumption along the lifetime of an individual. If the bequest motive is operative, then equation (3.4) characterizes the optimal allocation of consumption between two consecutive generations. This equation tells us that, when the bequest motive is operative ($b_{t+1} > 0$), the utility loss of parents arising from a larger amount of bequest must be equal to the discounted utility gain of their direct descendants.

The competitive equilibrium of this economy is a path $\{k_t, c_t, x_t, b_t\}_{t=0}^{\infty}$ that solves the system of difference equations composed of (3.3) and (3.4), together with (2.1), (2.2), (2.3), (2.9), (2.10), (2.12), (2.13), (2.14) and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t u'(\hat{c}_t^1) b_t = 0. \quad (3.5)$$

The previous transversality condition states that the present value of bequests tends to zero.

We will restrict our analysis to steady state equilibria, that is competitive equilibria where the variables k_t, c_t, x_t and b_t are all constant.³ To this end, we combine (3.3) with (2.9) and (2.10) to obtain

$$u'[c(1 - \gamma\varepsilon^y) - \gamma(1 - \varepsilon^y)x] - \rho R u'[x(1 - \delta(1 - \varepsilon^o)) - \delta\varepsilon^o c] = 0,$$

and use (2.2), (2.3), (2.12), (2.13) and (2.14) to obtain

$$h(k, b) \equiv u'[(1 - \gamma\varepsilon^y)(w(k) + b - nk) - \gamma(1 - \varepsilon^y)(R(k)nk - nb)]$$

³We will suppress the time subindex when we refer to the steady state value of a variable.

$$-\rho R(k) u' [(1 - \delta(1 - \varepsilon^o))(R(k)nk - nb) - \delta\varepsilon^o(w(k) + b - nk)] = 0. \quad (3.6)$$

Moreover, we are going to restrict the parameter space of our model in order to obtain existence, uniqueness and stability of the steady state equilibrium. We first introduce the following assumption aimed at guaranteeing both the uniqueness of the steady state when the bequest motive is inoperative and the corresponding saddle path stability when the values of the parameters δ and γ are sufficiently small:

Assumption A. The following conditions hold in the steady state:

$$R'(k)k + R(k) > 0,$$

$$R'(k)k + n > 0.$$

Lemma 3.1. $h_b \equiv \frac{\partial h}{\partial b} < 0$ and $h_k \equiv \frac{\partial h}{\partial k} > 0$.

Proof. The proof follows directly from the functional form of $h(k, b)$ and Assumption A. ■

Proposition 3.2. a) If a steady state with no bequests exists, then it is unique and solves

$$h(\bar{k}, 0) = 0,$$

$$\bar{c} = f(\bar{k}) - \bar{k}f'(\bar{k}) - n\bar{k},$$

and

$$\bar{x} = nf'(\bar{k})\bar{k},$$

where \bar{k} , \bar{c} and \bar{x} are the steady state values of capital and consumption when young and when old, respectively.

b) If a steady state with positive bequests exists, then it is unique and solves

$$f'(k^*) = \frac{n}{\beta},$$

$$h(k^*, b^*) = 0,$$

$$c^* = f(k^*) - k^*f'(k^*) - nk^* + b^*,$$

and

$$x^* = nf'(k^*)k^* - nb^*,$$

where k^* , b^* , c^* and x^* are the steady state values of capital, bequest and consumption when young and when old, respectively.

Proof. Note that $h(k, b) = 0$ holds regardless of the operativeness of the bequest motive. If the bequest motive is not operative ($b = 0$), then the steady state capital stock \bar{k} solves $h(\bar{k}, 0) = 0$. The steady state value of capital \bar{k} is then unique since $h_k > 0$.

If the bequest motive is operative ($b > 0$), then (3.4) holds with equality and the steady state is characterized by this equation and $h(k, b) = 0$. Using these two equations, it follows that the steady state capital stock k^* satisfies

$$R(k^*) = \frac{n}{\beta},$$

and the bequest level b^* in the steady state is such that $h(k^*, b^*) = 0$. Finally, note that $h_b < 0$ for all $b > 0$ implies that the relationship between k and b implied by $h(k, b) = 0$ is monotonic since $\frac{\partial b}{\partial k} = -\frac{h_k}{h_b} > 0$. Therefore, there is at a unique steady state value b^* of bequests. ■

We should mention that, whereas the uniqueness of the steady state with positive bequests does not depend on Assumption A, the uniqueness of the steady state with zero bequests is not guaranteed when this assumption does not hold.⁴

Proposition 3.3. *a). The steady state with operative bequest motive is saddle path stable.*

b). The steady state with inoperative bequest motive is saddle path stable for values of δ and γ sufficiently close to zero.

Proof. See Appendix A. ■

Whereas the steady state with positive bequests is always saddle path stable, saddle path stability of the steady state with zero bequests depends on the intensity of consumption externalities, which is summarized by the value of the parameters δ and γ . As we have already said, when the intensity of consumption externalities is sufficiently low, Assumption A guarantees saddle path stability. In contrast, when the intensity of consumption externalities is sufficiently large, the steady state with zero bequests may not be saddle path stable even though Assumption A holds.⁵ From now on, we assume that δ and γ are low enough to guarantee saddle path stability of the steady state with inoperative bequest motive. For the sake of completeness, we show in the Table 1 of Appendix B how the stability properties of a steady state with inoperative bequest motive vary with the value of the parameters δ and γ .

Proposition 3.4. *Assume that the bequests motive is operative. Then, $\frac{\partial k^*}{\partial \gamma} = 0$, $\frac{\partial k^*}{\partial \delta} = 0$, $\frac{\partial b^*}{\partial \beta} > 0$, $\frac{\partial b^*}{\partial \gamma} > 0$, $\frac{\partial b^*}{\partial \delta} < 0$, and $\frac{\partial b^*}{\partial \beta} > 0$.*

Proof. If bequests are positive, then k^* is such that $R(k^*) = \frac{n}{\beta}$ and b^* satisfies $h(k^*, b^*) = 0$. It is obvious that $\frac{\partial k^*}{\partial \gamma} = \frac{\partial k^*}{\partial \delta} = 0$ and $\frac{\partial k^*}{\partial \beta} > 0$. To see the effect on b^* , we use $h(k, b) = 0$ and Lemma 3.1. On the one hand, we obtain the following: $\frac{\partial b}{\partial \gamma} = -\frac{h_\gamma}{h_b} > 0$ and $\frac{\partial b}{\partial \delta} = -\frac{h_\delta}{h_b} < 0$, where

⁴See Thibault (2000) for a discussion on the existence of multiple steady states in a model without consumption externalities.

⁵In related OLG models where individuals preferences are not subject to consumption externalities, de la Croix and Michel (2001) and Alonso, et al. (2005b) show that the steady state with inoperative bequest is saddle path stable when $h_k > 0$. However, this condition is not sufficient to guarantee saddle path stability when consumption externalities are present.

$$h_\gamma \equiv \frac{\partial h}{\partial \gamma} = -u''(\hat{c}) \underbrace{[\varepsilon^y (w(k) + b - nk) + (1 - \varepsilon^y) (R(k) nk - nb)]}_{+} > 0$$

and

$$h_\delta \equiv \frac{\partial h}{\partial \delta} = \rho R(k) u''(\hat{x}) \underbrace{[(1 - \varepsilon^o) (R(k) nk - nb) + \varepsilon^o (w(k) + b - nk)]}_{+} < 0.$$

On the other hand, $\frac{\partial b}{\partial \beta} = -\frac{h_\beta}{h_b} > 0$, where $h_\beta = h_k \frac{\partial k^*}{\partial \beta} > 0$. ■

If the amount of bequest is positive, then consumption externalities modify the intergenerational distribution of consumption but do not affect the long run value of capital. In other words, consumption externalities neither modify the amount of saving nor the output level, but they modify the allocation of consumption between young and old generations. This change in the allocation of consumption is achieved by adjusting the amount of bequest. An increase in the value of the parameter γ raises the marginal valuation of young consumption and, as follows from (3.4), this results in an utility gain from a larger amount of inheritances. In contrast, as δ increases agents are willing to increase old consumption and this requires a reduction in the amount of bequest.

Proposition 3.5. *Assume that the bequest motive is not operative. Then, $\frac{\partial \bar{k}}{\partial \beta} = 0$, $\frac{\partial \bar{k}}{\partial \delta} > 0$, and $\frac{\partial \bar{k}}{\partial \gamma} < 0$.*

Proof. Assume that the amount of bequest is zero. Then, the steady state capital stock solves $h(\bar{k}, 0) = 0$. Using this equation, we obtain that $\frac{\partial \bar{k}}{\partial \gamma} = -\frac{h_\gamma}{h_k}$ and $\frac{\partial \bar{k}}{\partial \delta} = -\frac{h_\delta}{h_k}$, where

$$h_\gamma = -u''(\hat{c}) \underbrace{[\varepsilon^y (w(k) - nk) + (1 - \varepsilon^y) Rnk]}_{+} > 0$$

and

$$h_\delta = \rho R(k) u''(\hat{x}) \underbrace{[(1 - \varepsilon^o) R(k) nk + \varepsilon^o (w(k) - nk)]}_{+} < 0.$$

The result then follows from Lemma 3.1. ■

An increase in γ induces agents to increase young consumption, whereas an increase in δ induces agents to increase old consumption. Agents rise young (old) consumption by decreasing (increasing) the amount of saving. This explains the effect of δ and γ on the capital stock since savings coincide with the stock of capital in equilibrium.

We next discuss how the operativeness of the bequest motive is modified by these consumption spillovers. To this end, let us combine conditions (3.3), (3.4) when it

is just binding, and (2.12), all of them evaluated at the steady state, to obtain the threshold value of the altruism factor β above which bequests are positive,

$$\bar{\beta} = \frac{n}{f'(\bar{k})}. \quad (3.7)$$

Note that this condition coincides with the one obtained in Weil (1987). Note also that consumption externalities modify the value of $\bar{\beta}$ through their effect on the steady state capital stock.

Proposition 3.6. $\frac{\partial \bar{\beta}}{\partial \delta} > 0$ and $\frac{\partial \bar{\beta}}{\partial \gamma} < 0$.

Proof. Obvious from the previous arguments and Proposition 3.5. ■

Clearly, these results show that the introduction of consumption externalities affects the operativeness of the bequest motive and, thus, affects the dynamic efficiency of the equilibrium path (Cass, 1972).

4. The social planner solution

We assume that the social planner gives the same weight to all generations in his objective function. Therefore, the social planner maximizes⁶

$$\hat{U}_t = \sum_{t=0}^{\infty} \beta^t (u((1 - \gamma \varepsilon^y) c_t - \gamma(1 - \varepsilon^y) x_t) + \rho u((1 - \delta(1 - \varepsilon^o)) x_{t+1} - \delta \varepsilon^o c_{t+1})).$$

subject to the resource constraint

$$f(k_t) = c_t + \frac{x_t}{n} + nk_{t+1}. \quad (4.1)$$

Thus, the Lagrangean of the planner's maximization problem can be written as

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t (u((1 - \gamma(1 - \varepsilon^y)) c_t - \gamma \varepsilon^y x_t) + \rho u((1 - \delta(1 - \varepsilon^o)) x_{t+1} - \delta \varepsilon^o c_{t+1})) \\ & + \lambda_t \left(f(k_t) - c_t - \frac{x_t}{n} - nk_{t+1} \right). \end{aligned}$$

The corresponding first order conditions are

$$(1 - \gamma \varepsilon^y) \beta^t u'(\hat{c}_t^p) - \delta \varepsilon^o \beta^{t-1} \rho u'(\hat{x}_t^p) - \lambda_t = 0, \quad (4.2)$$

$$-\gamma(1 - \varepsilon^y) \beta^t u'(\hat{c}_t^p) + [1 - \delta(1 - \varepsilon^o)] \beta^{t-1} \rho u'(\hat{x}_t^p) - \frac{\lambda_t}{n} = 0, \quad (4.3)$$

and

$$\lambda_{t+1} f'(k_{t+1}^p) - n \lambda_t = 0, \quad (4.4)$$

⁶Note that this objective function coincides with the planner's objective function in Abel (2005).

where the superindex p denotes an optimal path. Combining (4.2) and (4.3), we obtain

$$\left(\frac{n\rho}{\beta}\right) \left(\frac{u'(\hat{x}_t^p)}{u'(\hat{c}_t^p)}\right) = I, \quad (4.5)$$

where

$$I \equiv \frac{1 - \gamma((1+n)\varepsilon^y - n)}{1 - \delta\left(1 - \left(1 + \frac{1}{n}\right)\varepsilon^o\right)}.$$

Using (4.2), (4.3) and (4.4), we get

$$\left(\frac{u'(\hat{c}_{t+1}^p)}{u'(\hat{c}_t^p)}\right) f'(k_{t+1}^p) = \frac{n}{\beta}. \quad (4.6)$$

The social planner solution is a path $\{k_t^p, c_t^p, x_t^p\}_{t=0}^\infty$ solving equations (4.1), (4.5), (4.6) and the transversality condition

$$\lim_{t \rightarrow \infty} \left[\beta^t u'(\hat{c}_t^p) (1 - \gamma\varepsilon^y) - \beta^{t-1} \rho u'(\hat{x}_t^p) \delta\varepsilon^o \right] k_{t+1} = 0.$$

Proposition 4.1. *There exists a unique steady state in the social planner problem. This steady state is given by*

$$\begin{aligned} f'(k^p) &= \frac{n}{\beta}, \\ \frac{u'(\hat{x}^p)}{u'(\hat{c}^p)} &= \frac{I\beta}{n\rho}, \end{aligned} \quad (4.7)$$

and

$$f(k^p) = c^p + \frac{x^p}{n} + nk^p, \quad (4.8)$$

where k^p , c^p and x^p are the steady state values of capital, consumption when young and when old, respectively.

Proof. The proof follows from (4.1), (4.5) and (4.6) and by noticing that (4.7) defines an increasing relation between x^p and c^p and (4.8) defines a decreasing relation between these two variables. ■

The following proposition characterizes the dynamics around the steady state of the planner's solution and its proof is omitted since is similar to that of Proposition 3.4.

Proposition 4.2. *The steady state of the social planner solution is saddle path stable.*

5. Efficiency

There are two different sources of inefficiency in our economy: the contemporaneous consumption externalities and the inoperativeness of the bequest motive. In what follows, we will analyze the effects of these different sources of inefficiency, which interact in a non-obvious way. To this end, we first compare the social planner solution with the competitive equilibrium solution when bequests are positive, and then we perform the same comparison when bequests are zero.

5.1. Inefficiency of the equilibrium path when the bequest motive is operative

When the bequest motive is operative, consumption externalities are the only source of inefficiency. To analyze the effects of this inefficiency, we compare the social planner solution, which is characterized by equations (4.1), (4.5) and (4.6), with the competitive equilibrium solution when the bequest motive is operative, which is characterized by equations (3.3), (3.4) and (4.1). We first rewrite the social planner solution by rearranging (4.5) as follows:

$$\left(\frac{u'(\hat{x}_t^p)}{u'(\hat{c}_t^p)}\right) \left(\frac{\rho}{I}\right) = \frac{\beta}{n}, \quad (5.1)$$

where the left hand side (LHS) is the social marginal rate of substitution (MRS) between young and old consumption at period t and the right hand side (RHS) is the social marginal rate of transformation (MRT). Thus, equation (5.1) drives the optimal intratemporal allocation of consumption between agents belonging to different generations. Rearranging (4.6), we obtain

$$\left(\frac{u'(\hat{c}_{t+1}^p)}{u'(\hat{c}_t^p)}\right) \left(\frac{\beta}{n}\right) = \frac{1}{f'(k_{t+1}^p)}, \quad (5.2)$$

where the LHS is the social MRS between current and next period consumption of young individuals and the RHS is the corresponding social MRT. It then follows that equation (5.2) determines the optimal intertemporal allocation of consumption. This equation is the Keynes-Ramsey equation that, given an initial condition on the stock of capital, characterizes the optimal path of saving and thus the optimal path of the capital stock and output. Therefore, (5.1) simply determines the optimal allocation of consumption between the two generations living in the same period.

We follow a similar procedure with the equilibrium solution. We thus rewrite (3.4) as follows

$$\left(\frac{u'(\hat{x}_t)}{u'(\hat{c}_t)}\right) \rho = \frac{\beta}{n}, \quad (5.3)$$

where the LHS is the private MRS and the RHS is the MRT. Equation (5.3) drives the intratemporal allocation of consumption between generations. Combining (3.3) with (3.4), we obtain

$$\left(\frac{u'(\hat{c}_{t+1})}{u'(\hat{c}_t)}\right) \left(\frac{\beta}{n}\right) = \frac{1}{f'(k_{t+1})}, \quad (5.4)$$

where the LHS is the MRS and the RHS is the MRT. Equation (5.4) determines the intertemporal allocation of consumption between agents along the equilibrium path.

From the comparison of (5.1) and (5.2) with (5.3) and (5.4), it follows that consumption externalities affect the intratemporal allocation of consumption between generations, as the private and social MRS do not coincide, but do not affect the intertemporal allocation of consumption. It follows then that consumption externalities do not affect the optimality of the capital path. In fact, if the initial condition on the stock of capital of the planned economy coincides with the initial condition of the competitive economy, then the path of capital and output of the

two economies will coincide in each period. Thus, consumption externalities do not cause any inefficiency in terms of production. However, the intratemporal allocation of consumption between young and old individuals is suboptimal, as follows from the comparison between (5.1) and (5.3). Given that production and savings are at its optimal level, this suboptimal allocation of consumption between young and old agents arises because the path of bequest is also suboptimal.

Proposition 5.1. *If $I > (<) 1$ then bequests are suboptimally small (large).*

Proof. See Appendix A. ■

The size of the difference between the private and social MRS is defined by the variable I , which provides a measure of the inefficiency due to consumption externalities. Note that $I > (<) 1$ when

$$\frac{\gamma}{\delta} < (>) \frac{1 - \left(1 + \frac{1}{n}\right) \varepsilon^o}{(1+n) \varepsilon^y - n}.$$

Thus, the amount of bequest is suboptimally small (large) when the externalities make agents value old (young) consumption in excess to young (old) consumption. Note also that the two consumption externalities result in opposite effects and, in fact, when $I = 1$ the competitive equilibrium is efficient even though consumption externalities are introduced. Table 2 in Appendix B illustrates the previous discussion since it shows that, as δ increases, I rises and the ratio between the long run value of the amount of bequest at the competitive equilibrium and its optimal long run value decreases. It also shows that an increase in γ has the opposite effects.

5.2. Inefficiency of the equilibrium path when the bequest motive is inoperative

When the amount of bequest is zero, the equilibrium is characterized by equations (2.3), (3.3) and (4.1). We proceed to compare these equations with those characterizing the social planner solution. First, we combine (4.5) with (4.6) to obtain

$$\left(\frac{u'(\hat{x}_{t+1}^p)}{u'(\hat{c}_t^p)} \right) \left(\frac{\rho}{I} \right) = \frac{1}{f'(k_{t+1}^p)},$$

where the LHS is the social MRS between consumption when young and when old and the RHS is the corresponding social MRT. We can compare this equation with the equilibrium equation (3.3), which can be rewritten as

$$\left(\frac{u'(\hat{x}_{t+1})}{u'(\hat{c}_t)} \right) \rho = \frac{1}{f'(k_{t+1})},$$

where the LHS is the private MRS and the RHS is the corresponding private MRT. This equation drives the intertemporal margin between consumption when young and when old along an equilibrium path with inoperative bequest motive. Note that this equilibrium margin differs from the optimal margin in both the MRS and in the MRT. The difference between the social and the private MRS is due to

consumption externalities and the difference between the social and the private MRT arises because the competitive stock of capital differs from its planned counterpart when the bequest motive is not operative. To see this, note that (4.6) does not hold along an equilibrium path when bequests are zero. Moreover, as follows from (2.3) and (3.3), consumption spillovers modify the equilibrium stock of capital and, hence, the private MRT when bequests are zero. Thus, when the bequest motive is not operative, consumption externalities affect both the MRS and the MRT and thus give rise to both a suboptimal level of production and a suboptimal allocation of consumption.

Given that there are two different sources of inefficiency, the effects of this inefficiency in the equilibrium paths of consumption and capital are ambiguous and may change along the transition. In what follows, we limit our analysis to the steady state. We first show how the inefficiency affects the long run level of production and then we analyze how the inefficiency affects the consumption levels of the two generations.

The long run value of the stock of capital is inefficient since the optimal long run value of the stock of capital k^p is obtained from (4.6), whereas the long run value \bar{k} of the stock of capital in the competitive equilibrium comes from the equation $h(\bar{k}, 0) = 0$. As follows from the condition (3.7) for the operativeness of the bequest motive, $\bar{k} > k^p$. Thus, the inoperativeness of the bequest motive implies a suboptimally large capital stock. In Proposition 3.5 it is shown that the steady state stock of capital rises with δ and decreases with γ . This means that the young consumption reference reduces the level of overaccumulation of capital, whereas the old consumption reference increases this level. Finally, the strength of altruism rises the optimal stock of capital but does not affect the equilibrium stock of capital when the bequest motive is inoperative. Thus, a larger degree of altruism reduces the magnitude of the production inefficiency. The following result shows how the parameters β , δ and γ affect the size of the production inefficiency, where this inefficiency is measured by the difference between the long run competitive stock of capital and the long run stock of capital achieved by the social planner.

Proposition 5.2. *The production inefficiency decreases with the altruism factor β and with the intensity of the externality due to the young consumption reference γ , whereas it increases with the intensity of the externality due to the old consumption reference δ .*

Proof. Obvious from the results in Proposition 3.5. ■

The equilibrium stock of capital can be expressed as a function of γ and δ , $\bar{k}(\gamma, \delta)$, whereas the optimal stock of capital can be written as a function of the altruism factor, $k^p(\beta)$. Note that the inefficiency due to the inoperativeness of the bequest motive can be defined as $\bar{k}(0, 0) - k^p(\beta)$, which decreases as β increases. Note also that the inefficiency due to consumption externalities is $\bar{k}(\gamma, \delta) - \bar{k}(0, 0)$, which decreases with γ and increases with δ . Finally, the overall inefficiency is $\bar{k}(\gamma, \delta) - k^p(\beta)$.

The effect of the inefficiency on the allocation of consumption is far from obvious. On the one hand, the overaccumulation of capital implies that the social MRT is

larger than the private MRT. This introduces a wealth and a substitution effect that results into an ambiguous effect on the allocation of consumption. On the other hand, the effect of consumption externalities on the distribution of consumption among agents of different ages depends on the relative intensity of the two consumption references. Therefore, the effect of the inefficiency on the allocation of consumption is potentially ambiguous and depends on the assumptions made on the functional forms of the production and utility functions and on the value of the parameters of the model. Table 3 of Appendix B shows how the level of consumption can be suboptimally large or small depending on the value of the parameters γ and δ . This table shows that a rise in γ (δ) reduces (increases) the production inefficiency, which is measured in the table by the ratio between the optimal stock of capital and the competitive stock of capital. Concerning the long run value of consumption, a rise in γ (δ) reduces (increases) the difference between the optimal and equilibrium values of young consumption, whereas the effect on the difference between the optimal and equilibrium values of old consumption is ambiguous.

6. Optimal taxes

In this section we proceed to characterize the optimal tax rates that restore the efficiency when consumption externalities interact with altruism. We consider a tax policy consisting of an estate tax, a capital income tax and a system of lump-sum taxes. Concerning the lump-sum taxes, we assume that young agents pay a lump-sum tax τ_t^y and the revenues are devoted to finance a lump-sum subsidy to the old $-\tau_t^o$. Therefore, these two tax rates are related by the following balanced budget constraint:

$$\tau_t^o = -n\tau_t^y. \quad (6.1)$$

We also assume that young agents pay an estate tax on the inheritance they receive and that the revenues accruing from these taxes are returned to the same young agents by means of a lump-sum subsidy. Thus, there is a second government budget constraint, which is

$$\phi_t^y = \tau_t^b b_t, \quad (6.2)$$

where ϕ_t^y is a lump-sum subsidy and τ_t^b is the estate tax rate. Finally, we assume that old agents pay a capital income tax on the returns of savings and that these revenues are returned to these agents by means of a lump-sum subsidy. Therefore, another government budget constraint is

$$\phi_t^o = \tau_t^k s_t f'(k_{t+1}), \quad (6.3)$$

where ϕ_t^o is a lump-sum subsidy and τ_t^k is the capital income tax rate.

Consider the individual maximization of (2.4) with respect to $\{c_t, x_{t+1}, b_{t+1}\}$ subject to (2.1) and the following constraints:

$$c_t = w_t - \tau_t^y + \phi_t^y + (1 - \tau_t^b) b_t - s_t, \quad (6.4)$$

$$x_{t+1} = (1 - \tau_{t+1}^k) R_{t+1} s_t + \phi_{t+1}^o - n b_{t+1} - \tau_{t+1}^o, \quad (6.5)$$

which amounts to solve the following dynamic programming problem:

$$V_t(b_t) = \max_{\{s_t, b_{t+1}\}} \left\{ u \left(w_t - \tau_t^y + \phi_t^y + (1 - \tau_t^b) b_t - s_t - \gamma v_t^y \right) \right. \\ \left. + \rho u \left((1 - \tau_{t+1}^k) R_{t+1} s_t + \phi_{t+1}^o - n b_{t+1} - \tau_{t+1}^o - \delta v_{t+1}^o \right) + \beta V_{t+1}(b_{t+1}) \right\},$$

with $b_{t+1} \geq 0$, for v_t^y , v_{t+1}^o , w_t and R_{t+1} given for all t . The first order conditions of the maximization problem are

$$u'(c_t - \gamma v_t^y) = \rho \left(1 - \tau_{t+1}^k \right) R_{t+1} u'(x_{t+1} - \delta v_{t+1}^o), \quad (6.6)$$

and

$$n \rho u'(x_{t+1} - \delta v_{t+1}^o) \geq \left(1 - \tau_{t+1}^b \right) \beta u'(c_{t+1} - \gamma v_{t+1}^y), \quad (6.7)$$

where the last condition holds with equality if $b_{t+1} > 0$.

The competitive equilibrium of this economy is a path $\{k_t, c_t, x_t, b_t, \tau_t^y, \tau_t^o, \tau_t^b, \tau_t^k, \phi_t^y, \phi_t^o\}_{t=0}^\infty$ that solves the system of difference equations composed of (6.6) and (6.7), together with (2.12), (2.13), (2.14), (6.1), (6.2), (6.3), (6.4), (6.5) and the transversality condition (3.5). Let us denote the path of the optimal tax rates as $\{\tilde{\tau}_t^y, \tilde{\tau}_t^o, \tilde{\tau}_t^b, \tilde{\tau}_t^k\}_{t=0}^\infty$. These optimal tax rates are such that make the equilibrium path of $\{k_t, c_t, x_t\}$ coincide with the optimal path $\{k_t^p, c_t^p, x_t^p\}$, characterized by (4.1), (4.5) and (4.6). By using (2.3), we obtain that the optimal path of bequest implied by the planners' solution is given by

$$b_{t+1}^p = f'(k_{t+1}^p) k_{t+1}^p - \frac{x_{t+1}^p}{n}. \quad (6.8)$$

A positive optimal amount of bequest implies that the bequest motive is operative along an equilibrium path that attains the first best solution and a negative optimal amount of bequest implies that the bequest motive is inoperative along this equilibrium path. To characterize the optimal tax rates, we must distinguish between these two cases.

When the optimal amount of bequest is positive, the equilibrium path associated to the optimal tax rates is characterized by (4.1), (6.5) and the following two equations, which are obtained from combining (6.6) and (6.7):

$$R(k_{t+1}) \left(\frac{u'(\hat{c}_{t+1})}{u'(\hat{c}_t)} \right) = \left(\frac{n}{\beta} \right) \left(\frac{1}{(1 - \tilde{\tau}_{t+1}^b)(1 - \tilde{\tau}_{t+1}^k)} \right), \quad (6.9)$$

and

$$\left(\frac{n\rho}{\beta} \right) \left(\frac{u'(\hat{x}_t)}{u'(\hat{c}_t)} \right) = 1 - \tilde{\tau}_t^b. \quad (6.10)$$

From the comparison among (6.9) and (6.10) with (4.5) and (4.6), we obtain the following result:

Proposition 6.1. *When the optimal amount of bequest is positive, the optimal tax rates are $\tilde{\tau}_t^b = 1 - I$, $\tilde{\tau}_t^k = 1 - \frac{1}{I}$, $\tilde{\tau}_t^y = 0$ and $\tilde{\tau}_t^o = 0$.*

If the optimal amount of bequest is positive and consumption externalities are the only source of inefficiency, the optimal lump-sum tax rates are zero. In this case, the capital income tax rate does not solve the inefficiency due to consumption externalities. This occurs because bequests are suboptimal and an estate tax is necessary to solve the inefficiency due to consumption externalities. In Proposition 5.1 we showed that, if $I > (<) 1$ then bequests are suboptimal small (large) and, as shown in Proposition 6.1, the optimal estate tax is negative (positive) so that the inefficiency is solved by raising (reducing) the amount of bequest. Therefore, the optimal estate tax solves the intratemporal inefficiency due to consumption externalities. However, as follows from (6.9), this tax introduces an intertemporal inefficiency, which is corrected by means of the capital income tax. In this respect note that the sign of $\tilde{\tau}_t^b$ is the opposite of that of $\tilde{\tau}_t^k$.

When the optimal amount of bequest is negative, the equilibrium path associated with the optimal tax rates is characterized by equations $b_{t+1} = 0$, (4.1), (6.5) and (6.6). We proceed to characterize the optimal tax rates. We first combine (4.5) with (4.6) to obtain

$$R(k_{t+1}^p) \rho \left(\frac{u'(\tilde{x}_{t+1}^p)}{u'(\tilde{c}_t^p)} \right) = I. \quad (6.11)$$

From the comparison between (6.6) and (6.11), it follows that the optimal capital income tax rate is $\tilde{\tau}_t^k = 1 - \frac{1}{I}$. This tax solves the inefficiency due to consumption externalities, whereas the inefficiency due to the inoperativeness of the bequest motive is thus solved through lump-sum taxes. To obtain the latter optimal taxes, we use (2.12), (2.14), (6.3) and (6.5) to obtain

$$x_{t+1} = n f'(k_{t+1}) k_{t+1} - \tau_{t+1}^o. \quad (6.12)$$

From the comparison between (6.8) and (6.12), we obtain that $\tilde{\tau}_t^o = n b_t^p$ and, using (6.1), we get $\tilde{\tau}_t^y = -b_t^p$, where b_t^p is defined in (6.8). Given that $b_t^p < 0$, we obtain that $\tilde{\tau}_t^o < 0$ and $\tilde{\tau}_t^y > 0$. Thus, the optimal system of lump-sum taxes corrects the overaccumulation of capital by means of reducing the income of the young agents and increasing the income of the old agents. In this sense, the optimal lump-sum taxes are a pay-as-you-go social security system. These results are summarized in the following proposition:

Proposition 6.2. *When the optimal amount of bequest is negative, the optimal tax rates are $\tilde{\tau}_t^k = 1 - \frac{1}{I}$, $\tilde{\tau}_t^o = n b_t^p$ and $\tilde{\tau}_t^y = -b_t^p$.*

We have thus shown that if agents are altruistic the optimal tax policy depends on the operativeness of the bequest motive along an optimal path. On the one hand, if bequests are zero then, as in Abel (2005), the optimal tax policy consists of a tax on capital income and a pay-as-you go social security system. The latter solves the overaccumulation of capital due to the inoperativeness of the bequest motive and the former solves the inefficient allocation of consumption due to consumption externalities. Note that the optimal capital income tax rate will be positive or negative, depending on the relative intensity of the two externalities. Thus, this optimal tax is constructed in a way that stimulates or deters saving depending on the overall effect of externalities on the allocation of consumption.

On the other hand, when the bequest motive is operative, the only source of inefficiency are the consumption spillovers. In this case, the optimal lump-sum taxes are zero and the optimal tax policy consists of an estate tax and a capital income tax. Thus, when generations are effectively linked by means of bequests, the capital income tax does not solve the inefficiency due to consumption externalities since this inefficiency does not affect the level of saving but that of bequests, which in turn determines both the allocation of consumption between old and young individuals and the external reference levels of consumption. Thus, when bequests are positive, both an estate tax and a capital income tax must be introduced in order to solve the inefficiency due to consumption externalities. The former brings bequests at their optimal level and the latter induces the optimality of the amount of saving. To see how this tax policy works, assume that $I > 0$. As shown in Proposition 5.1, this inefficiency implies that bequests are suboptimally low. In order to correct this inefficiency, the optimal estate tax is a subsidy. As it is shown in equation (6.9), this subsidy affects the intertemporal allocation of consumption and, hence, it would result into a suboptimally large level of saving, i.e., into capital over-accumulation. In order to correct this, a positive income tax rate must be introduced in order to offset the inefficiency brought about by the optimal estate tax.

Finally, note that the optimal level of the tax rate on capital income is the same in the two scenarios we have just considered even though it is aimed at fixing inefficiencies coming through different channels.

7. Conclusion

In this paper we have analyzed the inefficiency due to the existence of consumption externalities in a dynastic altruism model. We have shown that the operativeness of the bequest motive changes the nature of the inefficiency due to consumption externalities. When bequests are positive, consumption spillovers only give raise to an intratemporal inefficiency, whereas they are a source of intertemporal inefficiency when bequests are zero. In the former case, consumption externalities only affect the intratemporal margin between consumption of the two generations living in the same period. This implies that consumption externalities only affect the efficiency of the allocation of consumption between the two living generations, which is associated with a suboptimal level of bequest. Thus, the intertemporal paths of consumption, saving and output remain at their optimal levels. In contrast, when bequests are zero, consumption externalities affect the intertemporal margin between consumption when young and when old. Given that consumption externalities affect the intertemporal path of consumption, they also modify the path of capital and production. In fact, the externality associated with the young consumption reference reduces the stock of capital and thus reduces the overaccumulation of capital. Obviously, the externality associated with the old consumption reference has the opposite effect on the stock of capital and thus rises the overall long run inefficiency.

We have characterized the optimal tax rates and we have shown that they depend on the operativeness of the bequest motive. On the one hand, when the bequest motive is inoperative, the optimal tax policy consists of a tax on capital income and

a pay-as-you go social security system. On the other hand, when the bequest motive is operative, optimal lump-sum taxes should be set to zero and the optimal tax policy consists of an estate tax and a capital income tax. In this case the estate tax corrects the intratemporal inefficiency but gives rise to an intertemporal inefficiency, which is solved by means of introducing a capital income tax. Therefore, consumption externalities justify the introduction of estate taxes.

We have shown that the optimal capital income and estate tax rates are constant along the transition. This is simply a consequence of the fact that externalities accrue from contemporaneous consumption. If we had assumed instead that the reference levels of consumption are related to the past average consumption in the economy (external habits or aspirations), then the optimal tax rates would not be constant along the transition. This seems a promising line of future research as the absolute separation between intratemporal and intertemporal inefficiency will not hold when externalities accruing from past consumption are introduced. Finally, let us mention that other tax instruments can also solve the inefficiency due to consumption externalities. In particular, taxes on consumption that discriminate between generations may solve this inefficiency. The characterization of optimal consumption taxes is thus another interesting extension even though the results would be critically dependent on the functional forms of the utility and production functions.

References

- [1] Abel A. (1986), Capital Accumulation and Uncertain Lifetimes with Adverse Selection, *Econometrica* 54, 1079-1097.
- [2] Abel, A. (2005), Optimal Taxation when Consumers Have Endogenous Benchmark Levels of Consumption, *Review of Economic Studies* 75, 21-42.
- [3] Alonso-Carrera, J., Caballé, J., and Raurich X. (2004), Consumption Externalities, Habit Formation and Equilibrium Efficiency, *Scandinavian Journal of Economics*, 106, 231-251.
- [4] Alonso-Carrera, J., Caballé, J., and Raurich X. (2005a), Growth, Habit Formation, and Catching Up with the Joneses, *European Economic Review*, 49, 1665-1691.
- [5] Alonso-Carrera, J., Caballé, J. and X. Raurich. (2005b), Aspirations, Habit Formation and Bequest Motive, *Barcelona Economics Working paper* 136.
- [6] Barro, R.J. (1974), Are Government Bonds Net Wealth? *Journal of Political Economy* 82, 1095-1117.
- [7] Cass, D. (1972), On Capital Overaccumulation in the Aggregate, Neoclassical Model of Economic Growth: A Complete Characterization, *Journal of Economic Theory* 4, 200-223.
- [8] de la Croix, D. (1996), The Dynamics of Bequeathed Tastes, *Economics Letters* 51, 89-96.
- [9] de la Croix, D. and Michel P. (1999), Optimal Growth when Tastes Are Inherited, *Journal of Economic Dynamics and Control* 23, 519-537.
- [10] de la Croix, D. and Michel P. (2001), Altruism and Self-Restraint, *Annales d'Economie et de Statistique* 63-64, 233-260.
- [11] Diamond, P.A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review* 55, 1126-1150.
- [12] Fisher, W.H. and Hof, F.X. (2000), Relative Consumption, Economic Growth, and Taxation, *Journal of Economics* 72, 241-262.
- [13] Guo, J.T. (2002), Tax Policy under Keeping Up with the Joneses and Imperfectly Competitive Product Markets, University of California, Mimeo.
- [14] Lahiri, A. and Puhakka, P. (1998), Habit Persistence in Overlapping Generations Economies under Pure Exchange, *Journal of Economic Theory* 78, 176- 188.
- [15] Laitner, J.P. (1988), Bequests, Gifts and Social Security, *Review of Economic Studies* 55, 275-299.
- [16] Liu, W.F. and Turnovsky, S. (2005), Consumption Externalities, Production Externalities, and Long Run Macroeconomics Efficiency", *Journal of Public Economics*, 89, 1098-1129.

- [17] Ljungqvist, L. and Uhlig, H. (2000), Tax Policy and Aggregate Demand Management under Catching Up with the Joneses, *American Economic Review* 90, 356-366.
- [18] Thibault, E. (2000), Existence of Equilibrium in an OLG Model with Production and Altruistic Preferences, *Economic Theory* 15, 709-715.
- [19] Turnovsky, S. and G. Monteiro (2005), Consumption Externalities, Production Externalities, and Efficient Capital Accumulation under Time Non-separable Preferences, *Working paper University of York*, 2005/08.
- [20] Weil, P. (1987), Love thy Children: Reflections on the Barro Debt Neutrality Theorem, *Journal of Monetary Economics* 19, 377-391.
- [21] Wendner, R. (2002), Capital Accumulation and Habit Formation, *Economics Bulletin* 4, 1-10.

Appendix

A. Proofs

Proof of Proposition 3.3.

a) Stability when the bequest motive is not operative

We first derive conditions aimed at guaranteeing the stability of the steady state with zero bequests. To this end, we assume that along the transition path the following inequality is satisfied:

$$n\rho u'(x_{t+1} - \delta v_{t+1}^o) > \beta u'(c_{t+1} - \gamma v_{t+1}^y),$$

so that $b_{t+1} = 0$ for all t . Then, the transitional dynamics is characterized by (3.3), which can be rewritten as

$$h_t \equiv u'(c_t - \gamma v_t^y) - \rho R_{t+1} u'(x_{t+1} - \delta v_{t+1}^o) = 0.$$

By using (2.2), (2.3), (2.9), (2.10) and (2.12)-(2.14), it is immediate to see that we can obtain the relationship $h_t = h(k_t, k_{t+1}, k_{t+2})$. Next, we obtain the following derivatives evaluated at the steady state:

$$\begin{aligned} \frac{\partial h_t}{\partial k_t} &= u'(\hat{c}) \left(-\sigma(\hat{c}) \frac{\frac{\partial \hat{c}_t}{\partial k_t}}{\hat{c}} + \pi(\hat{x}) \frac{\frac{\partial \hat{x}_{t+1}}{\partial k_t}}{\hat{x}} \right), \\ \frac{\partial h_t}{\partial k_{t+1}} &= u'(\hat{c}) \left(-\sigma(\hat{c}) \frac{\frac{\partial \hat{c}_t}{\partial k_{t+1}}}{\hat{c}} + \pi(\hat{x}) \frac{\frac{\partial \hat{x}_{t+1}}{\partial k_{t+1}}}{\hat{x}} + \alpha(k) \right), \\ \frac{\partial h_t}{\partial k_{t+2}} &= \pi(\hat{x}) u'(\hat{c}) \left(\frac{\frac{\partial \hat{x}_{t+1}}{\partial k_{t+2}}}{\hat{x}} \right), \end{aligned}$$

where $\sigma(\hat{c}) = -\frac{\hat{c}u''(\hat{c})}{u'(\hat{c})} > 0$, $\pi(\hat{x}) = -\frac{\hat{x}u''(\hat{x})}{u'(\hat{x})} > 0$ and $\alpha(k) = -\frac{R'(k)}{R(k)} > 0$. The stability can be characterized by using the characteristic polynomial

$$P(\lambda) = \frac{\partial h_t}{\partial k_{t+2}} \lambda^2 + \frac{\partial h_t}{\partial k_{t+1}} \lambda + \frac{\partial h_t}{\partial k_t}.$$

Saddle path stability implies that only one of the two characteristic roots must belong to the unit circle. Note also that $P(1) = h_k > 0$, as follows from Lemma 3.2. Then, saddle path stability requires that $P(-1) < 0$, where

$$P(-1) = u'(\hat{c}) \left[\left(\frac{\pi(\hat{x})}{\hat{x}} \right) \left(\frac{\partial \hat{x}_{t+1}}{\partial k_{t+2}} + \frac{\partial \hat{x}_{t+1}}{\partial k_t} - \frac{\partial \hat{x}_{t+1}}{\partial k_{t+1}} \right) + \left(\frac{\sigma(\hat{c})}{\hat{c}} \right) \left(\frac{\partial \hat{c}_t}{\partial k_{t+1}} - \frac{\partial \hat{c}_t}{\partial k_t} \right) - \alpha(k) \right].$$

By using (2.2), (2.3), (2.5) and (2.6), $P(-1)$ can be rewritten as

$$P(-1) = u'(\hat{c}) \left[\left(\frac{\pi(\hat{x})}{\hat{x}} \right) [\delta \varepsilon^o (n - kR'(k)) - (1 - \delta(1 - \varepsilon^o)) n (R(k) + kR'(k))] \right]$$

$$+ \left(\frac{\sigma(\hat{c})}{\hat{c}} \right) [\gamma \varepsilon^y n (R(k) + kR'(k)) - (1 - \gamma(1 - \varepsilon^y)) (n - kR'(k))] - \alpha(k) \Big].$$

Assumption A implies that $P(-1) < 0$ when $\delta = \gamma = 0$. Thus, by continuity, $P(-1) < 0$ for values of δ and γ sufficiently close to zero. ■

b) Stability when the bequest motive is operative

Note that (3.4) implicitly defines $\hat{x}_t = \phi(\hat{c}_t)$, where the derivative of this implicit function is

$$\phi' = \left(\frac{u'(\hat{x}_t)}{u''(\hat{x}_t)} \right) \left(\frac{u''(\hat{c}_t)}{u'(\hat{c}_t)} \right) > 0.$$

Next, combine

$$\hat{c}_t = (1 - \gamma(1 - \varepsilon^y)) c_t - \gamma \varepsilon^y x_t,$$

and

$$\hat{x}_t = (1 - \delta(1 - \varepsilon^o)) x_t - \delta \varepsilon^o c_t,$$

to obtain

$$c_t = \frac{(1 - \delta(1 - \varepsilon^o)) \hat{c}_t + \gamma \varepsilon^y \hat{x}_t}{(1 - \gamma(1 - \varepsilon^y))(1 - \delta(1 - \varepsilon^o)) - \delta \varepsilon^o \gamma \varepsilon^y},$$

and

$$x_t = \frac{(1 - \gamma(1 - \varepsilon^y)) \hat{x}_t + \delta \varepsilon^o \hat{c}_t}{(1 - \gamma(1 - \varepsilon^y))(1 - \delta(1 - \varepsilon^o)) - \delta \varepsilon^o \gamma \varepsilon^y}.$$

Using the previous equations, the resource constraint

$$f(k_t) = c_t + \frac{x_t}{n} + nk_{t+1}$$

can be rewritten as

$$k_{t+1} = \frac{f(k_t)}{n} - \left(\frac{[n(1 - \delta(1 - \varepsilon^o)) + \delta \varepsilon^o] \hat{c}_t + (1 - \gamma(1 - \varepsilon^y) + n\gamma \varepsilon^y) \phi(\hat{c}_t)}{(1 - \gamma(1 - \varepsilon^y))(1 - \delta(1 - \varepsilon^o)) - \delta \varepsilon^o \gamma \varepsilon^y} \right) \left(\frac{1}{n^2} \right). \quad (\text{A.1})$$

Finally, combining (3.3) and (3.4) we obtain

$$u'(\hat{c}_{t+1}) = \frac{nu'(\hat{c}_t)}{\beta R(k_{t+1})},$$

which implicitly defines the relationship

$$\hat{c}_{t+1} = \kappa(\hat{c}_t, k_{t+1}). \quad (\text{A.2})$$

Note that (A.1) and (A.2) form the system of difference equations that completely drives the transition of the equilibrium path. The elements of the Jacobian matrix of this system are

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{f'(k)}{n} = \frac{1}{\beta} > 0,$$

$$\frac{\partial k_{t+1}}{\partial \hat{c}_t} = - \left(\frac{[n(1 - \delta(1 - \varepsilon^o)) + \delta \varepsilon^o] + (1 - \gamma(1 - \varepsilon^y) + n\gamma \varepsilon^y) \phi'(\hat{c}_t)}{(1 - \gamma(1 - \varepsilon^y))(1 - \delta(1 - \varepsilon^o)) - \delta \varepsilon^o \gamma \varepsilon^y} \right) \left(\frac{1}{n^2} \right) < 0,$$

$$\begin{aligned}\frac{\partial \hat{c}_{t+1}}{\partial k_t} &= - \left(\frac{nu'(\hat{c}) R'(k)}{u''(\hat{c}) \beta (R(k))^2} \right) \left(\frac{\partial k_{t+1}}{\partial k_t} \right) \\ &= - \left(\frac{u'(\hat{c})}{u''(\hat{c})} \right) \left(\frac{f''(k)}{f'(k)} \right) \left(\frac{\partial k_{t+1}}{\partial k_t} \right) < 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \hat{c}_{t+1}}{\partial \hat{c}_t} &= \frac{nu''(\hat{c})}{u''(\hat{c}) \beta R(k)} - \left(\frac{nu'(\hat{c}) R'(k)}{u''(\hat{c}) \beta (R(k))^2} \right) \left(\frac{\partial k_{t+1}}{\partial \hat{c}_t} \right) \\ &= 1 - \left(\frac{u'(\hat{c})}{u''(\hat{c})} \right) \left(\frac{f''(k)}{f'(k)} \right) \left(\frac{\partial k_{t+1}}{\partial \hat{c}_t} \right).\end{aligned}$$

Define the characteristic polynomial as

$$Q(\lambda) = \lambda^2 - \left(\frac{1}{\beta} + 1 - \left(\frac{u'(\hat{c})}{u''(\hat{c})} \right) \left(\frac{f''(k)}{f'(k)} \right) \left(\frac{\partial k_{t+1}}{\partial c_t} \right) \right) \lambda + \frac{1}{\beta}.$$

Note that

$$Q(1) = \left(\frac{u'(\hat{c})}{u''(\hat{c})} \right) \left(\frac{f''(k)}{f'(k)} \right) \left(\frac{\partial k_{t+1}}{\partial c_t} \right) < 0,$$

and

$$Q(-1) = \left(1 + \frac{1}{\beta} \right) 2 - \left(\frac{u'(\hat{c})}{u''(\hat{c})} \right) \left(\frac{f''(k)}{f'(k)} \right) \left(\frac{\partial k_{t+1}}{\partial c_t} \right) > 0.$$

This implies that there is a unique root within the unit circle, which proves the desired saddle path stability. ■

Proof of Proposition 5.1.

By combining (4.5) and (5.3), we obtain

$$\left(\frac{u'(\hat{x}_t^p)}{u'(\hat{c}_t^p)} \right) \left(\frac{1}{I} \right) = \frac{u'(\hat{x}_t)}{u'(\hat{c}_t)},$$

which implies that

$$\left(\frac{u'(\hat{x}_t^p)}{u'(\hat{c}_t^p)} \right) > (<) \frac{u'(\hat{x}_t)}{u'(\hat{c}_t)},$$

whenever $I > (<) 1$. Using (2.2), (2.3), (2.5), (2.6), (2.9), (2.10) and (2.12)-(2.14), the previous inequality can be rewritten as

$$\Psi(k_t^p, k_{t+1}^p, b_t^p) > (<) \Psi(k_t, k_{t+1}, b_t),$$

where

$$\Psi(k_t, k_{t+1}, b_t) = \frac{u'((1 - \delta(1 - \varepsilon^o))(nR(k_t)k_t - nb_t) - \delta\varepsilon^o(w(k_t) + b_t - nk_{t+1}))}{u'((1 - \gamma\varepsilon^y)(w(k_t) + b_t - nk_{t+1}) - \gamma(1 - \varepsilon^y)(nR(k_t)k_t - nb_t))}.$$

Given that along an equilibrium path with positive bequests $k_t = k_t^p$ for all t and $\frac{\partial \Psi(k_t, b_t)}{\partial b_t} > 0$, then $b_t^p > (<) b_t$ when $I > (<) 1$. ■

B. Numerical example and tables

Assume instantaneous utility function with constant elasticity of substitution

$$u(z_t) = \frac{z_t^{1-\sigma}}{1-\sigma},$$

and a Cobb-Douglas production function

$$f(k) = Ak^\mu.$$

Concerning the value of the parameters, we assume that $\sigma = 1$, $\rho = 0.045$, $\mu = 0.35$, $n = m^{35}$, with $m = 1.01$, and $A = 1$. We also give equal weight to the two living agents, i.e., $\theta^o = \theta^y$.

Table 1. Region of saddle path stability

	$\theta^y = \theta^o = 0$	$\theta^y = \theta^o = 0.5$	$\theta^y = \theta^o = 1$
$\delta = 0$	$\gamma \in [0, 1]$	$\gamma \in [0, 0.993)$	$\gamma \in [0, 0.989)$
$\delta = 0.5$	$\gamma \in [0, 1]$	$\gamma \in [0, 0.992)$	$\gamma \in [0, 0.986)$
$\delta = 0.75$	$\gamma \in [0, 1]$	$\gamma \in [0, 0.987)$	$\gamma \in [0, 0.965)$
$\delta = 0.85$	$\gamma \in [0, 1]$	$\gamma \in [0, 0.943)$	$\gamma \in [0, 0.575)$
$\delta = 1$	$\gamma \in [0, 0.999)$	ϕ	ϕ

This table shows the combinations of parameter values for which the equilibrium exhibits saddle path stability.

Table 2. Efficiency of the equilibrium with operative bequest motive

$\left(I, \frac{b^*}{b^p}\right)$	$\delta = 0$	$\delta = 0.25$	$\delta = 0.75$
$\gamma = 0$	(1, 1)	(1.08, 0.98)	(1.28, 0.88)
$\gamma = 0.25$	(0.91, 1.01)	(0.97, 1.003)	(1.16, 0.94)
$\gamma = 0.75$	(0.72, 1.028)	(0.78, 1.023)	(0.926, 1.012)

We assume that $\theta^o = \theta^y = 0.5$ and $\beta = 0.6$. This table shows how the inefficiency of the equilibrium path with operative bequest motive changes as δ and γ increase. Two measures of inefficiency are provided in each cell: the first component of the vector is I and the second is the ratio $\frac{b^*}{b^p}$.

Table 3. Inefficiency of the equilibrium with inoperative bequest motive

$\left(\frac{k^p}{k}, \frac{c^p}{c}, \frac{x^p}{x}\right)$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
$\delta = 0$	(0.48, 0.64, 1.03)	—	—
$\delta = 0.5$	(0.16, 0.40, 0.83)	(0.38, 0.63, 0.92)	—
$\delta = 1$	(0.02, 0.18, 0.47)	(0.022, 0.24, 0.43)	(0.03, 0.36, 0.36)

We assume that $\theta^o = \theta^y = 0.5$ and $\beta = 0.05$. This table shows how the discrepancy between the equilibrium steady state and the optimal steady state changes when δ and γ increase. Three measures of inefficiency are provided: $\frac{k^p}{k}$, $\frac{c^p}{c}$ and $\frac{x^p}{x}$. The empty cells correspond to situations where the bequest motive is operative.