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Discussion Paper 10-01

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Abstract

This paper analyzes an endogenous choice problem with regard to tax instruments in a capital tax competition model. Considering a symmetric and two-region model of tax competition, where each region is allowed to choose either unit or ad valorem tax, we show that selecting unit tax as a policy instrument is the dominant strategy of governments. An interpretation of this result is clearly explained by the properties of the best response curves.

JEL Classification: H20, H21, H77

Keywords: Tax competition, Unit tax, Ad valorem tax

1 Introduction

As surveyed by Wilson (1999), there are many literatures on tax competition. However, attention about whether it matters if the taxes are unit or ad valorem is not so popular as the topic. In almost literatures in the context of the Zodrow-Mieszkowski-Wilson (ZMW) model, the tax is levied per unit of capital (i.e., it is a unit tax), whereas in reality, taxes are on capital income and are therefore always ad valorem (i.e., a proportion of income from the capital).

The pioneering work on this problem is Lockwood (2004), which shows that the Nash equilibrium in unit taxes is generally different from the Nash equilibrium in ad valorem taxes.

"If countries are symmetric, and both private and public goods are normal, then (i) the symmetric Nash equilibrium in taxes exists and is unique in each case; and (ii) equilibrium taxes and public good provision are always lower when countries compete with ad valorem taxes." (Lockwood, 2004)

Given this interesting result, one simple straightforward extension of this model is to introduce a stage for the selection of the tax system, a unit tax or an ad valorem tax, before they compete with each other. Even if the equilibrium with a unit tax is superior, this system may not be selected in the dynamic two-stage strategic game. Therefore, it is valuable to explore this strategic behavior of the government in a dynamic game with the strategic selection of tax instruments.

This paper analyzes an endogenous choice problem of tax instruments, unit or ad valorem, in a capital tax competition model. Considering a symmetric and two-region model where each region maximizing its tax revenue is allowed to choose either a unit tax or an ad valorem tax,[1] we show that both regions choose unit taxes in a Nash equilibrium.

In order to interpret this result, we can point out the following two mechanisms. First, the reason for the difference in Nash equilibrium outcomes with unit and ad valorem taxation. Second, the reason for the adoption of unit tax in the dynamic game with strategic selection of tax instruments.

First, as clearly explained in Lockwood (2004), the difference in Nash equilibrium outcomes with unit and ad valorem taxation depends on the elasticity of tax increase on capital. The elasticity in the case where ad valorem taxation is adopted in another region is greater in absolute value than that in the case with unit tax.

The simple intuition for the greater elasticity when competing with the region under ad valorem taxation comes from how the effective tax rate in another region is changed with an inflow of capital. When an ad valorem tax is adopted

[1] Our model is the same as the Zodrow-Mieszkowski-Wilson (ZMW) model, except for the number of regions and the revenue-maximizing governments.

in another region, any outflow of capital by the tax increase lowers the effective unit tax in another region, given that the ad valorem tax in region 2 is fixed, because this effective rate is the fixed ad valorem tax multiplied by the marginal product of capital that falls with an inflow of capital. The lower effective tax rate in another region induces more inflow of capital into another region, which means that the capital elasticity by the tax increase becomes greater.

Second, we consider why the unit tax is selected in the dynamic game with the strategic selection of tax instruments. As long as the effective tax rate is the same, the own reaction curve remained unchanged, given the tax instrument in another region. This means that the reaction curve becomes the same in both cases with unit and ad valorem taxation, given the tax instrument in another region. The crucial fact is that the selection of the own tax instruments changes the reaction of another region through the change of the tax elasticity on capital in another region. The change of the reaction curve affects the equilibrium. When the own tax instrument is unit, the effective tax rate in another region becomes higher, which is better for the own region because its capital becomes relatively large through the lower tax elasticity in another region. As a result, both governments adopt unit tax and obtain a larger revenue as compared with ad valorem taxation.

This paper is organized as follows. The model is explained in the next section, and the main proposition is derived in Section 3. Section 4 concludes the paper.

2 The model

We consider a symmetric and two-region (regions 1 and 2) model.[2] Region i is populated by L_i identical residents, and has K_i endowments of capital. Labor is immobile, whereas capital is mobile, involving no transportation costs between the two regions. Firms in each region are perfectly competitive and produce an identical product. The profit of each firm is zero in equilibrium owing to free entry. The output of region i is given by $F^i(L_i, K_i)$, which has technology involving constant returns to scale. The symmetric assumption implies that $L_1 = L_2$ and $K_1 = K_2$. We set $L_i = 1$ without loss of generality and conduct the analysis by using the production function per capita, $f^i(k_i)$, where $k_i \equiv K_i/L_i$.

Governments of both regions impose taxes on the return on capital, and can make a choice between a unit tax and an ad valorem tax. Their aims are to maximize tax revenue, and they behave in a Nash manner.[3] The game played

[2] Our model is the same as the Zodrow-Mieszkowski-Wilson (ZMW) model except for the number of regions and the revenue-maximizing governments.

[3] This paper deals with a government that aims to maximize tax revenue, and hence does not describe the behaviors of consumers and the uses of tax revenue. This assumption could be justified, as Kanbur and Keen (1993), Janeba (2000), Kothenburger (2005), Keen and Ligthart

between the two governments is constructed by two-stage decision making.[4] In the first stage, they choose the tax instruments; they choose either unit or ad valorem taxation to maximize the tax revenue. Observing the tax instrument, the regions choose the rate of tax in the second stage.

Let us denote the unit tax of region i by T_i and the ad valorem tax by t_i. Noting that the two regions face the same post-tax rate of return on capital (r) as a result of the perfect mobility of capital, the market equilibrium conditions of capital are described by

$$\begin{cases} r = f_k^i - T_i, & \text{unit tax,} \\ r = f_k^i(1 - t_i), & \text{ad valorem tax,} \end{cases} \tag{1}$$

$$k = k_1 + k_2, \tag{2}$$

where $f_k^i (\equiv \partial f^i / \partial k_i)$ is the marginal product of capital and k is the total amount of capital in the economy.

The tax revenue (g_i) of region i is

$$\begin{cases} g_i = T_i k_i, & \text{unit tax,} \\ g_i = t_i f_k^i k_i, & \text{ad valorem tax.} \end{cases} \tag{3}$$

3 The competition in tax instruments and tax levels

This section analyzes a tax competition when there is a choice of tax instruments. To avoid analytical complexity and derive solutions explicitly, we adopt the following quadratic function for capital, $f^i(k_i) = [a_i - (b_i k_i/2)]\, k_i,$ where $a_i > 0$ and $b_i > 0$. This function is often used in the literature on this field, for example, Wildasin (1991), Brueckner (2004), and Itaya, Okamura and Yamaguchi (2008). The marginal product of capital is $f_k^i = a_i - b_i k_i$. From the assumption of the symmetric regions, $a_1 = a_2 (\equiv a)$ and $b_1 = b_2 (\equiv b)$.

We consider a two-stage game of tax competition. In stage 1 of the game, the governments of the two regions noncooperatively and simultaneously choose

(2006), and among others, if we assume a Leviathan-type of government. Alternatively, the governments' objective of revenue maximization can be justified when the tax-competing governments face severe revenue shortfalls, such that tax revenue is sufficiently more important than private good consumption. This setting, admittedly, is made only for technical reasons and makes it possible for us to analytically solve the equilibrium when two governments impose a different type of tax on capital.

[4] Wildasin (1991) is the pioneering research that has applied this type of two-stage game to a tax competition analysis.

the tax instruments: either unit or ad valorem taxation. In the second stage, observing the tax instruments, they determine the levels of tax rates. Taking tax instruments and tax levels of the two regions, firms maximize their profits. We derive a tax instrument in the subgame perfect Nash equilibrium by using a backwards induction.

3.1 Tax level (stage 2)

This section examines the tax rates determined by the two regions in stage 2, given the tax instruments of the regions: (i) both regions choose unit tax; (ii) both regions choose ad valorem tax; and (iii) one region chooses unit tax, while the other chooses ad valorem tax. We show the equilibrium tax rates in each case by the following:

Lemma 1 *The equilibrium tax rates in Stage 2 are as follows. (i) When both regions choose unit tax,*

$$T_i = bk, \qquad i = 1, 2,$$

(ii) when both regions choose ad valorem tax,

$$t_i = \frac{bk}{a}, \qquad i = 1, 2,$$

(iii) when region i chooses unit tax, while region j chooses ad valorem tax,

$$T_i = \frac{bk}{2} + \frac{H}{4}, \text{ and } t_j = \frac{H}{2(a - bk)}, \qquad i, j = 1, 2 \text{ and } i \neq j,$$

where $H \equiv 6a - bk - \sqrt{25b^2k^2 + 36a^2 - 36abk}$.

Proof. See Appendix A. ■

To make our solution meaningful (for capital demands and the post-tax rate of return on capital to be positive in all cases), we make the following assumption.

Assumption $2a > 3bk$.

This guarantees that tax rates and tax revenues are positive in all cases (see Appendix A).

The conversion of unit tax into the effective ad valorem rate enables the comparison of the tax rates between different tax instruments; the best response of unit tax is converted into ad valorem tax by using $t_i = T_i/f_k^i$. Let us define t_i^{mh} as the effective ad valorem tax rate of region i in the Nash equilibrium when region i chooses tax instrument m for $m = U, A$ and the other region chooses tax instrument h for $h = U, A$, where U indicates the choice of unit tax and A, that of the ad valorem tax. The effective ad valorem tax rates in the Nash equilibrium are given by

$$t_i^{UU} = \frac{2bk}{2a - bk}, \qquad i = 1, 2, \tag{4}$$

$$t_i^{AA} = \frac{bk}{a}, \qquad i = 1, 2. \tag{5}$$

$$t_i^{UA} = \frac{(2bk + H)\left[4(a - bk) - H\right]}{4a\left[4(a - bk) - H\right] - 2(H + 2bk)(a - bk)}, \qquad i = 1, 2, \tag{6}$$

$$t_i^{AU} = \frac{H}{2(a - bk)}, \qquad i = 1, 2, \tag{7}$$

Note that (5) is given in Lemma 1-(ii) and that (7) is given in Lemma 1-(iii). See Appendix B for the derivation of t_i^{UU} and t_i^{UA}. From these expressions, we provide the following proposition.

Proposition 1 *The equilibrium taxes evaluated in terms of the effective ad valorem rates in Stage 2 satisfy* $t_i^{AA} < t_i^{UA} < t_i^{AU} < t_i^{UU}$.

Proof. See Appendix C. ■

We characterize the equilibrium tax rate in Proposition 1 by using the governments' reaction curves evaluated in terms of the effective ad valorem tax rate. Such reaction curves are obtained by using $t_i = T_i/f_k^i$. (i) When both regions choose unit tax, the best response evaluated in terms of the effective ad valorem tax rate of each region is given by

$$t_i = \frac{2t_j(a - bk) + 2bk}{4a - bk - t_j(2a - bk)}, \qquad i, j = 1, 2 \text{ and } i \neq j, \tag{8}$$

(ii) when both regions choose ad valorem tax, it is given by

$$t_i = \frac{(2 - t_j)[bk + (a - bk)t_j]}{4a - bk - (3a - bk)t_j}, \qquad i, j = 1, 2 \text{ and } i \neq j, \tag{9}$$

(iii) when region i chooses unit tax, while region j chooses ad valorem tax, the best responses of regions i and j are, respectively

$$t_i = \frac{(2 - t_j)[bk + (a - bk)t_j]}{4a - bk - (3a - bk)t_j}, \text{ and} \tag{10}$$

$$t_j = \frac{2(a - bk)t_i + 2bk}{4a - bk - (2a - bk)t_i}, \qquad i, j = 1, 2 \text{ and } i \neq j.$$

See Appendix D for the derivation of these equations. In Figure 1, the ad valorem tax rate of government i, $t_i \in (0,1)$, is taken on the axis. Government i's reaction function when both governments employ the ad valorem tax system is plotted by the curve R_i^{AA}, and the stable equilibrium is given by point A. The reaction function when both governments employ the unit tax system can by plotted in this figure since we can replace the unit tax by the effective ad valorem tax. The curve R_i^{UU} represents government i's reaction function when both governments employ the unit tax system. Point B shows the (converted) ad valorem tax rates in the equilibrium when both governments employ unit tax. Furthermore, government i's reaction function when it employs the ad valorem tax system and government j employs the unit tax system is plotted by the curve R_i^{AU}. Symmetrically, the curve R_i^{UA} represents government i's reaction curve when it employs unit tax but government j employs ad valorem tax. The points C and D are the equilibria obtained in the tax competition in which one government employs unit tax, while the other government employs ad valorem tax.

Two features of reaction curves should be mentioned in Figure 1. First, government i's reaction curve is unchanged when it changes its tax instrument from unit to ad valorem tax, and vice versa. Second, government i's reaction curves when government j employs ad valorem tax is located further upwards from its reaction curve when government j employs unit tax. Here, we give a proof of the identity of R_i^{AA} and R_i^{UA}. When government i chooses the unit tax and government j chooses the ad valorem tax, from (1), we have $f_k^i - T_i = (1-t_j)f_k^j$, which yields,

$$\frac{dk_i}{dT_i} = \frac{1}{f_{kk}^i + (1-t_j)f_{kk}^j}, \tag{11}$$

where $f_{kk}^i \equiv \partial f_k^i / \partial k_i$. When both governments choose ad valorem tax, from (1), we have $(1-t_i)f_k^i = (1-t_j)f_k^j$, which yields

$$\frac{dk_i}{dt_i} = \frac{f_k^i}{(1-t_i)f_{kk}^i + (1-t_j)f_{kk}^j}. \tag{12}$$

When government i employs ad valorem tax, from $g_i = t_i f_k^i k_i$, the first-order condition for revenue maximization is expressed as

$$0 = f_k^i k_i + t_i k_i f_{kk}^i \frac{dk_i}{dt_i} + t_i f_k^i \frac{dk_i}{dt_i}. \tag{13}$$

Substituting (12) for dk_i/dt_i in (13) yields

$$\left[f_{kk}^i + (1-t_j)f_{kk}^j\right] k_i + t_i f_k^i = 0. \tag{14}$$

When government i employs the unit tax, from $g_i = T_i k_i$ the first-order condition for revenue maximization is expressed as

$$0 = k_i + T_i \frac{dk_i}{dT_i}. \tag{15}$$

Substituting (11) for dk_i/dT_i in (15) yields

$$\left[f^i_{kk} + (1 - t_j) f^j_{kk}\right] k_i + T_i = 0. \tag{16}$$

The effective unit tax rate can be described by the ad valorem tax rate as

$$T_i = t_i f^i_k. \tag{17}$$

From (16) and (17) we obtain

$$\left[f^i_{kk} + (1 - t_j) f^j_{kk}\right] k_i + t_i f^i_k = 0, \tag{18}$$

Eq. (18) is identical to (14). Therefore, we find that R_i^{AA} is identical to R_i^{UA}. Furthermore, since this argument can be applied for any tax instrument government j employs, we prove that R_i^{UU} is identical to R_i^{AU}.

We now turn to the consideration of the geometric positioning of the reaction curves. The intuition for the positioning of the reaction curve follows from the consideration of the welfare cost of increasing the tax rate in the jurisdiction. Taking government 1, we first consider the positioning of R_1^{UU} and R_1^{UA}. Let us assume that both governments employ unit tax. Government 1 expects some outflow of capital to occur if it increases its unit tax, while the other keeps its unit tax rate fixed. This capital outflow is recognized as the cost of raising unit tax. If government 2 employs ad valorem tax, government 1 expects an even larger outflow as it raises its unit tax rate and thereby the cost of raising the unit tax. This is because the other jurisdiction, 2, lowers the effective unit tax rate in response to the tax increase in region 1, and this magnifies the flow of capital from region 1 to 2. As a description of this point, remember that the effective unit tax in region 2 is equivalent to the ad valorem tax rate multiplied by the marginal product of capital. Although the unit tax rate of government 2 is given when government 1 increases its unit tax rate, the capital inflow in region 2, accompanied by the tax increase in region 1, reduces the marginal productivity of capital; hence, the effective unit tax rate will be lowered. The decrease in the effective unit tax rate in region 2 leads to further capital outflow from region 1, so that the tax base is more elastic for government 1 when the other employs the ad valorem tax system. Therefore, the curve R_1^{UU} lies on the upper side of the curve R_1^{UA}. The same explanation will be applicable to the positioning of R_2^{UU} and R_2^{UA}. These properties of the reaction curves give rise to the pattern of equilibrium outcome in Figure 1.

We now return to Proposition 1, in which the inequalities $t_i^{UA} < t_i^{UU}$ and $t_i^{AA} < t_i^{AU}$ show that the tax rates of region i when the partner region (region j) uses ad valorem tax are lower than those when region j chooses unit tax regardless of whether the tax instrument of region i is unit or ad valorem. The inequalities $t_i^{AA} < t_i^{UA}$ and $t_i^{AU} < t_i^{UU}$ show that the tax rates of region i when it chooses ad valorem tax are lower than those when it chooses unit tax regardless of whether the tax instrument of region j is unit or ad valorem. These findings can be interpreted by considering the case of $i = 1$ and $j = 2$. When region 2 changes tax instruments from unit to ad valorem, the reaction curve of region 1 shifts downward and that of region 2 does not, so that the equilibrium tax rate of region 1 decreases. When region 1 changes tax instruments from unit to ad valorem, the reaction curve of region 2 shifts upward and that of region 1 does not, so that the equilibrium tax rate of region 2 decreases.

3.2 Tax instrument (stage 1)

This section solves the equilibrium tax instruments in Stage 1. We use g_i^{mh} to denote the tax revenue of region i when region i chooses tax instrument m for $m = U, A$ and the other region chooses tax instrument h for $h = U, A$, where U indicates the choice of unit tax and A, that of the ad valorem tax.

Lemma 2 *(i) When both regions choose unit tax, the tax revenues of both region are*

$$g_i^{UU} = \frac{bk^2}{2}, \qquad i = 1, 2, \tag{19}$$

(ii) when both regions choose ad valorem tax, these are

$$g_i^{AA} = \frac{(2a - bk)bk^2}{4a}, \qquad i = 1, 2, \tag{20}$$

(iii) when region i chooses unit tax, while region j chooses ad valorem tax, the tax revenues of regions i and j are, respectively

$$g_i^{UA} = \frac{(H + 2bk)^2(a - bk)}{8b[4(a - bk) - H]}, \text{ and} \tag{21}$$

$$g_j^{AU} = \frac{H[(a - bk)(6bk - H) - 2bkH][2(4a - 3bk) - H]}{8b\,[4(a - bk) - H]^2}, \qquad i, j = 1, 2 \text{ and } i \neq j.$$

Proof. See Appendix E. ■

Table 1 shows the payoff matrix in the first stage, in which g_i^{mh} is given by (19)-(21). Lockwood (2004) simply compares the symmetric equilibrium between

g_i^{UU} and g_i^{AA} and shows $g_i^{UU} > g_i^{AA}$. This result is easily confirmed in Table 1 since $g_i^{UU} - g_i^{AA} = b^2k^3/4a > 0$. The characteristic feature is that in the case where the two regions choose different tax instruments, the tax revenue is not equal between the two regions ($g_1^{AU} \neq g_2^{UA}$ and $g_1^{UA} \neq g_2^{AU}$).

We provide the following proposition.

Proposition 2 *Choosing unit tax is a dominant strategy; two regions choose unit tax as a policy instrument in a Nash equilibrium.*

Proof. See Appendix F. ■

This result shows that unit tax is selected when the tax instrument can be selected before the governments compete with regard to the tax rate. Given the result from Lockwood (2004) that the equilibrium with unit tax is superior, this proposition suggests that this desirable equilibrium with unit tax is derived even if we allow for endogenous choice on tax instruments.

Region 1 tries to make region 2 set a higher tax rate in order to attract capital to region 1. To make region 2 set a high tax rate, it is an effective strategy for region 1 to employ unit tax. This is because while the employment of ad valorem tax produces a side effect that gives region 2 an incentive to lower the tax rate, the employment of unit tax does not have such an unfavorable effect. To demonstrate this, we consider the tax setting of region 2. When region 1 employs unit tax in the first stage, given region 1's unit tax rate in the second stage, region 2 sets its tax rate to equate the marginal benefits of tax increase (an increase in tax revenue) with marginal cost (a decrease in regional output accompanied by tax outflow). When region 1 employs ad valorem tax, region 2 sets its tax rate to equate the marginal benefit and marginal cost. In this case, however, the marginal cost region 2 faces is greater than that it faces when region 1 employs unit tax. Since the effective (unit) tax rate of region 1 is given by $T_1 = t_1 f_k^1(k_1)$, the tax increase in region 2 increases k_1 and decreases f_k^1, which lowers the effective tax rate of region 1 even if t_1 is given. This will cause further capital outflow from region 2, resulting in region 2 choosing a lower tax rate. Owing to the presence of the unfavorable side effect in the case of ad valorem tax, region 1 commits itself to employing unit tax.

4 Conclusion

Extending the result obtained from Lockwood (2004), we have shown in this paper that a superior equilibrium can be achieved with the selection of the unit tax, even in the case where the tax instrument can be selected before tax competition. In this paper, we focus on which tax instruments, unit or ad valorem, are selected in the capital tax competition model. This analysis can be applied to various models with tax instruments, for example, tax exporting competition or tariff competition.

Our model, which is in the line of a basic one in the literature on tax competition, generates a tax-cutting war as each region avoids the outflow of capital to the partner's country. As explained below Proposition 2, the outflow effect of capital owing to the imposition of tax can be repressed when the partner's region employs unit tax as compared to ad valorem tax. Therefore, the use of unit tax leads to a higher effective tax rate than ad valorem tax, and hence, unit taxes are beneficial to both countries. However, in a optimal tariff framework with two symmetric countries, Lockwood and Wong (2000) showed that ad valorem tariffs lead to less tariff-induced distortion in Nash equilibrium, and both countries choose the ad valorem tariffs. The choice of tax instruments, unit or ad valorem forms, depends on the structure of a model.

Finally, it should also be noted that the results are derived within the context of a model that is very general in some respects, but they obviously depend on other less general assumptions. For example, the model qualifies the governments as Leviathans, but one could easily imagine that the governments are benevolent and compete for mobile capital so as to maximize the utility of residents, or more specifically, they could be moderate Leviathans so that they take both resident's welfare and budget size into account. Furthermore, the production function is specified by the quadratic form. On the one hand, these assumptions clear the way for analytical examination, on the other they leave room for additional generalization for future research.

Appendix A: the proof of Lemma 1.

(i) Unit vs. Unit

Noting the symmetry ($a_1 = a_2 (\equiv a)$ and $b_1 = b_2 (\equiv b)$), from a quadratic-type production technology and (1), we obtain

$$r = a - bk_i - T_i, \qquad i = 1, 2. \tag{A-1}$$

The revenue maximization problem for government i is given by

$$\max_{T_i} \; g_i = T_i k_i, \qquad i = 1, 2.$$

The first-order condition gives

$$\frac{\partial g_i}{\partial T_i} = k_i + T_i \frac{\partial k_i}{\partial T_i} = 0, \qquad i = 1, 2. \tag{A-2}$$

From (2) and (A-1), we obtain

$$k_i = \frac{bk + T_j - T_i}{2b}, \qquad i = 1, 2. \tag{A-3}$$

which yields

$$\frac{\partial k_i}{\partial T_i} = -\frac{1}{2b}, \qquad i = 1, 2. \tag{A-4}$$

Using (A-2)-(A-4), we obtain the best response of region i:

$$T_i = \frac{bk + T_j}{2}, \qquad i, j = 1, 2 \text{ and } i \neq j, \tag{A-5}$$

which yields the tax rate in the Nash equilibrium:

$$T_1 = T_2 = bk. \tag{A-6}$$

From the symmetry, $k_1 = k_2 = k/2$ in the equilibrium. From this, (A-1), and (A-6), the post-tax rate of return on capital in the equilibrium is given by

$$r = \frac{2a - 3bk}{2} > 0,$$

where the inequality follows from the Assumption.

(ii) Ad valorem vs. ad valorem

In this case, it follows from (1) that

$$r = (a - bk_i)(1 - t_i), \qquad i = 1, 2. \tag{A-7}$$

The revenue maximization problem for government i is given by

$$\max_{t_i} \; g_i = t_i(a - bk_i)k_i, \qquad i = 1, 2.$$

The first-order condition gives

$$\frac{\partial g_i}{\partial t_i} = k_i(a - bk_i) + t_i(a - 2bk_i)\frac{\partial k_i}{\partial t_i} = 0, \qquad i = 1, 2. \tag{A-8}$$

Using (2) and (A-7), we obtain

$$\frac{\partial k_i}{\partial t_i} = -\frac{a - bk_i}{b(2 - t_i - t_j)}, \qquad i, j = 1, 2 \text{ and } i \neq j. \tag{A-9}$$

From (2), (A-8), and (A-9), we obtain the best response of region i:

$$t_i = \frac{(2-t_j)\left[bk+(a-bk)t_j\right]}{4a-bk-(3a-bk)t_j}, \qquad i,j=1,2 \text{ and } i \neq j. \tag{A-10}$$

which yields the tax rate in the Nash equilibrium:

$$t_1 = t_2 = \frac{bk}{a}. \tag{A-11}$$

Note that $t_i < 1$ from the Assumption.

From the symmetry, $k_1 = k_2 = k/2$ in the equilibrium. From this, (1), and (A-11), the post-tax rate of return on capital in the equilibrium is given by

$$r = \frac{(2a-bk)(a-bk)}{2a} > 0,$$

where the inequality follows from the Assumption.

(iii) Unit vs. ad valorem

Without loss of generality, we consider the case where region 1 chooses unit tax and region 2 chooses ad valorem tax. In this case,

$$r = a - bk_1 - T_1 = (a-bk_2)(1-t_2). \tag{A-12}$$

The revenue maximization problem in each region is given by

$$\max_{T_1}\ g_1 = T_1 k_1,$$
$$\max_{t_2}\ g_2 = t_2(a-bk_2)k_2.$$

The first-order conditions are

$$\frac{\partial g_1}{\partial T_1} = k_1 + T_1\frac{\partial k_1}{\partial T_1} = 0, \tag{A-13}$$

$$\frac{\partial g_2}{\partial t_2} = k_2(a-bk_2) + t_2(a-2bk_2)\frac{\partial k_2}{\partial t_2} = 0. \tag{A-14}$$

From (2) and (A-12), we have

$$k_1 = \frac{at_2 + (1-t_2)bk - T_1}{b(2-t_2)}, \tag{A-15}$$

$$k_2 = \frac{-at_2 + T_1 + bk}{b(2-t_2)}. \tag{A-16}$$

These show that

$$\frac{\partial k_1}{\partial T_1} = -\frac{1}{b(2-t_2)}, \text{ and } \frac{\partial k_2}{\partial t_2} = -\frac{a-bk_2}{b(2-t_2)}. \tag{A-17}$$

From these, (A-13), (A-14), and (A-17), we obtain the best responses:

$$T_1 = \frac{at_2 + (1-t_2)bk}{2}, \tag{A-18}$$

$$t_2 = \frac{2\,(T_1 + bk)}{(4a - T_1 - bk)}. \tag{A-19}$$

By using these, the tax rates in the Nash equilibrium are given by

$$T_1 = \frac{bk}{2} + \frac{H}{4}, \tag{A-20}$$

$$t_2 = \frac{H}{2(a-bk)}. \tag{A-21}$$

To confirm that $T_1 > 0$ and $t_2 > 0$, we prove that $H > 0$. By noting that $6a - bk > 0$ and $25b^2k^2 + 36a^2 - 36abk > 0$ from the Assumption, $H > 0$ if and only if $(6a-bk)^2 - (25b^2k^2 + 36a^2 - 36abk) > 0$. It follows that

$$(6a-bk)^2 - \left(25b^2k^2 + 36a^2 - 36abk\right) = 24bk(a-bk),$$

which shows that $H > 0$ from the Assumption. Thus, (A-20) and (A-21) show that $T_1 > 0$ and $t_2 > 0$.

Next, we confirm that $k_1 > 0$, $k_2 > 0$ and $r > 0$. From (A-12), (A-15), (A-16), (A-18), and (A-19), the capital demand in each region and the post-tax rate of return on capital in the equilibrium are given by

$$k_1 = \frac{(H+2bk)(a-bk)}{2b\,[4(a-bk) - H]}, \tag{A-22}$$

$$k_2 = \frac{(a-bk)(6bk - H) - 2bkH}{2b\,[4(a-bk) - H]}, \tag{A-23}$$

$$r = \frac{[2(4a-3bk) - H]\,[2(a-bk) - H]}{4\,[4(a-bk) - H]}. \tag{A-24}$$

Let us prove that $2(a-bk) - H > 0$ under the Assumption. It follows that

$$2(a-bk) - H = -(4a+bk) + \sqrt{25b^2k^2 + 36a^2 - 36abk}.$$

Note that $2(a-bk) - H > 0$ if and only if

$$25b^2k^2 + 36a^2 - 36abk - (4a+bk)^2 > 0.$$

We obtain

$$25b^2k^2 + 36a^2 - 36abk - (4a + bk)^2 = (10a - 7bk)(2a - 3bk) + 3b^2k^2 > 0,$$

where the inequality immediately follows from the Assumption. Thus,

$$2(a - bk) - H > 0. \tag{A-25}$$

This and the Assumption imply that

$$2(4a - 3bk) - H > 0, \text{ and } 4(a - bk) - H > 0. \tag{A-26}$$

From the Assumption, $H > 0$, (A-22) and (A-24), to (A-26), we obtain $k_1 > 0$ and $r > 0$.

Next, we define

$$(a - bk)(6bk - H) - 2bkH = M_1 - M_2,$$

$$M_1 \equiv (a + bk)\sqrt{25b^2k^2 + 36a^2 - 36abk)},$$
$$M_2 \equiv 5b^2k^2 + 6a^2 - abk.$$

As $M_1 > 0$ and $M_2 > 0$, $(a - bk)(6bk - H) - 2bkH > 0$ if $M_1^2 - M_2^2$. It follows that

$$M_1^2 - M_2^2 = 8abk\,(3a - bk)\,(2a - 3bk) + 15a^2(bk)^2 > 0,$$

where the inequality follows from the Assumption. Thus, $(a - bk)(6bk - H) - 2bkH > 0$. From this and (A-23), $k_2 > 0$.

Appendix B: the derivations of (4) and (6)

From $f_k^i = a - bk_i$ and $t_i = T_i/f_k^i$, we have

$$t_i = \frac{T_i}{a - bk_i}. \tag{B-1}$$

We obtain (4) from (A-6), (B-1) and $k_i = k/2$, and (6) from (A-20), (A-22), and (B-1).

Appendix C: the proof of Proposition 1

To prove Proposition 1, we show that (i) $t_i^{AA} < t_i^{UA}$, (ii) $t_i^{UA} < t_i^{AU}$ and (iii) $t_i^{AU} < t_i^{UU}$.

(i) the proof of $t_i^{AA} < t_i^{UA}$

From (5) and (6),

$$t_i^{UA} - t_i^{AA} = \frac{(2bk + H)\,[4(a - bk) - H]}{4a\,[4(a - bk) - H] - 2(H + 2bk)(a - bk)} - \frac{bk}{a}.$$

Rearranging this yields

$$a\,(O_1 - O_2)\left(t_i^{UA} - t_i^{AA}\right) = (O_3 - O_4)\,, \tag{C-1}$$

where

$$O_1 \equiv (3a - bk)\sqrt{25b^2k^2 + 36a^2 - 36abk} > 0,$$
$$O_2 \equiv abk + 10a^2 - b^2k^2 > 0,$$
$$O_3 \equiv \left(4a^2 - abk + b^2k^2\right)\sqrt{25b^2k^2 + 36a^2 - 36abk} > 0,$$
$$O_4 \equiv 24a^3 - 18a^2bk + b^3k^3 + 13ab^2k^2 > 0,$$

where the ineqaulities follows from the Assumption.

It follows that

$$O_1^2 - O_2^2 = 8b^2k^2(a - bk)\left[8a(a - bk)^2 + 3b^2k^2(a - bk) + 2a^2bk\right] > 0,$$
$$O_3^2 - O_4^2 = 8b^2k^2(a - bk)\left[8a(a - bk)^2 + 2a^2bk + 3b^3k^3(a - bk)\right] > 0,$$

where the inequalities follow from the Assumption. These show that $O_1 - O_2 > 0$ and $O_3 - O_4 > 0$. From this and (C-1), we have $0 < t_i^{UA} - t_i^{AA}$.

(ii) the proof of $t_i^{UA} < t_i^{AU}$

From (6) and (7),

$$t_i^{AU} - t_i^{UA} = \frac{H}{2(a - bk)} - \frac{(2bk + H)\,[4(a - bk) - H]}{4a\,[4(a - bk) - H] - 2(H + 2bk)(a - bk)}.$$

This yields

$$\frac{(a - bk)\,(O_1 - O_2)\left(t_i^{AU} - t_i^{UA}\right)}{2} = N_1 - N_2, \tag{C-2}$$

where

$$N_1 \equiv \left(5a^2 - abk + b^2k^2\right)\sqrt{25b^2k^2 + 36a^2 - 36abk} > 0,$$
$$N_2 \equiv 30a^3 + 15ab^2k^2 - 21a^2bk + b^3k^3 > 0,$$

where the inequalities follow from the Assumption.

It follows that

$$N_1^2 - N_2^2 = 4b^2k^2(a - bk)^2(5a - 6bk)(2a - bk) > 0,$$

in which the inequality follows from the Assumption. This shows that $N_1 > N_2$. Since $O_1 - O_2 > 0$ and $N_1 - N_2 > 0$, (C-2) shows that $t_i^{AU} > t_i^{UA}$.

(iii) the proof of $t_i^{AU} < t_i^{UU}$

From (4) and (7),

$$t_i^{UU} - t_i^{AU} = \frac{2bk}{2a - bk} - \frac{H}{2(a - bk)}.$$

This yields

$$2(a - bk)(2a - bk)\left(t_i^{UU} - t_i^{AU}\right) = Q_1 - Q_2,$$

where

$$Q_1 \equiv (2a - bk)\sqrt{25b^2k^2 + 36a^2 - 36abk},$$

$$Q_2 \equiv -12abk + 5b^2k^2 + 12a^2.$$

From the Assumption $(a - bk)(2a - bk) > 0$, $Q_1 > 0$ and $Q_2 > 0$.

It follows that

$$Q_1^2 - Q_2^2 = 16ab^2k^2(a - bk) > 0,$$

in which the inequality follows from the Assumption. Therefore, $t_i^{UU} > t_i^{AU}$.

Appendix D: the derivations of (8), (9), and (10)

From (2), (A-1), and $T_i = (a - bk_i)t_i (= f_k^i t_i)$, we have

$$(a - bk_i)(1 - t_i) = [a - b(k - k_i)(1 - t_j)], \qquad i, j = 1, 2 \text{ and } i \neq j.$$

From this, we obtain

$$k_i = \frac{at_i - at_j - bk + bkt_j}{-2b + bt_i + bt_j}, \qquad i, j = 1, 2 \text{ and } i \neq j.$$

Substituting this for k_i in $T_i = (a - bk_i)t_i$ yields

$$T_i = \frac{(2a - bk)(t_j - 1)t_i}{t_i + t_j - 2}, \qquad i, j = 1, 2 \text{ and } i \neq j.$$

From this and (A-5), we obtain (8).

Expression (9) is given by (A-9).

From (2), (A-12), and $T_1 = (a - bk_1)t_1$, we have

$$k_i = \frac{at_i - at_j - bk + bkt_j}{-2b + bt_i + bt_j}, \qquad i, j = 1, 2 \text{ and } i \neq j. \tag{D-1}$$

From (A-13), (A-17), and (D-1), we obtain the first equation in (10). From (A-14), (A-17), and (D-1), we obtain the second equation in (10).

Appendix E: the proof of Lemma 2

Expression (19) is derived from (3), (A-6) and $k_i = k/2$, and (20) from (3), (A-11), and $k_i = k/2$. In the case of (iii), without loss of generality, we assume that region 1 chooses unit tax and region 2 chooses ad valorem tax. We obtain the first equation in (21) from (3), (A-20), and (A-22) and the second equation from (3), (A-21), and (A-23).

Appendix F: the proof of Proposition 2

To prove Proposition 2, we show that $g_i^{UU} > g_i^{AU}$ and $g_i^{UA} > g_i^{AA}$.

(i) The proof of $g_i^{UU} > g_i^{AU}$

From Lemma 2, we obtain

$$g_i^{UU} - g_i^{AU} = \frac{bk^2}{2} - \frac{H\left[(a-bk)(6bk-H) - 2bkH\right]\left[2(4a-3bk) - H\right]}{8b\left[4(a-bk) - H\right]^2}. \quad \text{(F-1)}$$

Let us define $s \equiv bk/a$ and $h \equiv H/a (= 6 - s - \sqrt{25s^2 - 36s + 36})$. Note that $s < 1$ from the Assumption. Using this definition, (F-1) can be rewritten as

$$\begin{aligned} \Im\left(g_i^{UU} - g_i^{AU}\right) &= 4s^2[4(1-s) - h]^2 - h[(1-s)(6s-h) - 2sh][2(4-3s) - h] \\ &= V_1 - V_2, \end{aligned}$$

where

$$\Im \equiv 8b\left[4(a-bk) - H\right]^2 / a^4 > 0$$

$$V_1 \equiv (6 + 5s + 2s^2 + 2s^3)\sqrt{36 - 36s + 25s^2} > 0,$$

$$V_2 \equiv (36 + 12s + 5s^2 + 14s^3 + 8s^4) > 0,$$

in which $V_1 > 0$ follows from $s < 1$. It is easily shown from $1 - s > 0$ that

$$V_1^2 - V_2^2 = 12s^3(3s+4)(1-s)^2(2-s)^2 > 0.$$

This shows that $V_1 - V_2 > 0$ and, hence, $g_i^{UU} - g_i^{AU} > 0$.

(ii) The proof of $g_i^{UA} > g_i^{AA}$

From Lemma 2, we obtain

$$g_i^{UA} - g_i^{AA} = \frac{(H + 2bk)^2(a-bk)}{8b[4(a-bk) - H]} - \frac{(2a-bk)bk^2}{4a}. \quad \text{(F-2)}$$

Using the definition of $s \equiv bk/a$ and $h \equiv H/a$, (F-2) can be rewritten as

$$\begin{aligned}\Re\left(g_i^{UA} - g_i^{AA}\right) &= (h+2s)^2(1-s) - 2\left[4(1-s)-h\right](2-s)s^2 \\ &= (36 - 48s + 29s^2 - 9s^3 - 3s^4) \\ &\quad -(6-5s+s^2-s^3)\sqrt{36-36s+25s^2},\end{aligned}$$

where $\Re \equiv 8b[4(a-bk) - H]/a^3 > 0$, where the inequality follows from (A-25) and $a - bk > 0$.

Let us define the following:

$$\begin{aligned}W_1 &\equiv (36 - 48s + 29s^2 - 9s^3 - 3s^4) \\ &= 3s^2(4+s)(1-s) + 17\left(s - \frac{24}{17}\right)^2 + \frac{36}{17} > 0,\end{aligned}$$

$$\begin{aligned}W_2 &\equiv (6-5s+s^2-s^3)\sqrt{36-36s+25s^2} \\ &= \left[1+(1-s)(5+s^2)\right]\sqrt{36-36s+25s^2} > 0,\end{aligned}$$

where the inequalities follow from $1 > s$. It follows from $1 - s > 0$ that

$$W_1^2 - W_1^2 = 4s^3(1-s)[(1-s)(11-4s)+1](2-s)^2 > 0.$$

This shows that $W_1 - W_1 > 0$ and, hence, $g_i^{UU} - g_i^{AU} > 0$.

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	Unit tax	Ad valorem tax
Unit tax	g_1^{UU}, g_2^{UU}	g_1^{UA}, g_2^{AU}
Ad valorem tax	g_1^{AU}, g_2^{UA}	g_1^{AA}, g_2^{AA}

Table 1.

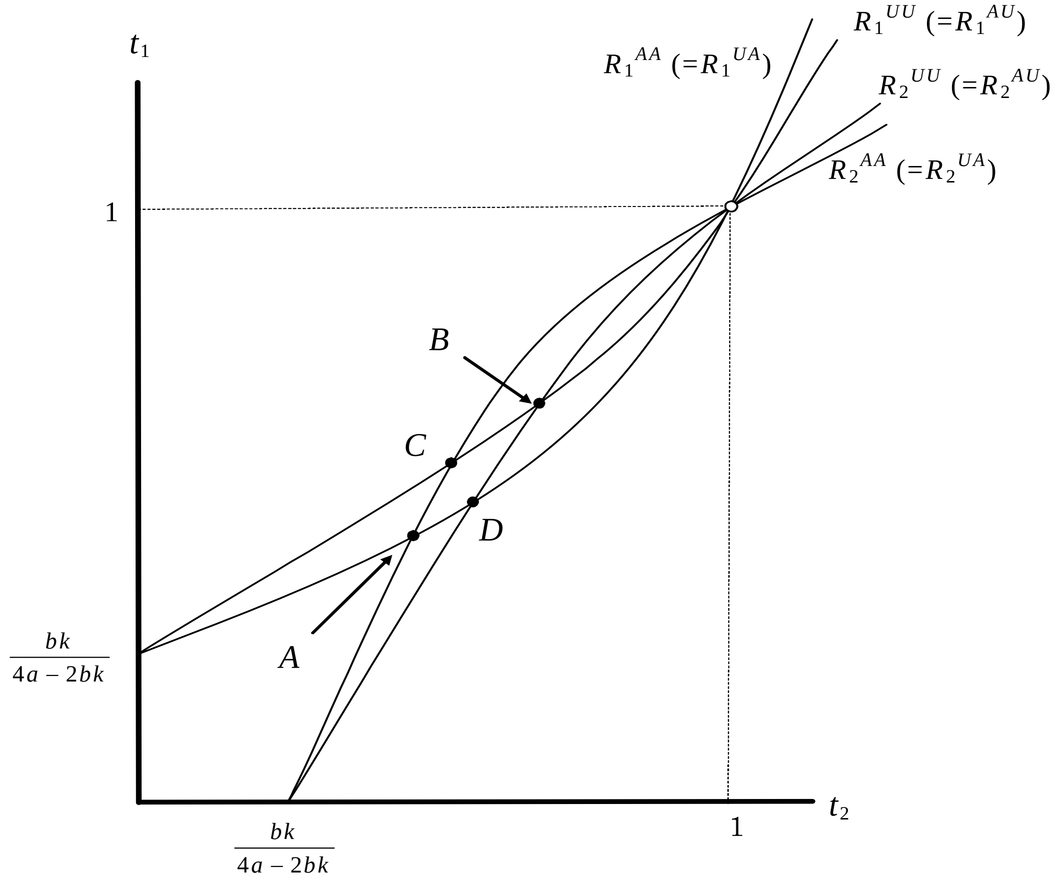

Figure 1. Reaction Curves