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Departamento de Estadística Universidad Carlos III de Madrid Calle Madrid, 126 28903 Getafe (Spain) Fax (34) 91 624-98-49

A METHODOLOGY FOR POPULATION PROJECTIONS: AN APPLICATION TO SPAIN

Andrés M. Alonso^{*}, Daniel Peña[†], and Julio Rodríguez[‡]

Abstract_

This paper looks at projections for the Spanish population by sex and age for the period of 2005 to 2050. These were carried out using forecasts for birth and mortality rates, and migration. These rates are calculated using two main sources of information. First, a multivariate time series model was applied for the series of variables from the 1970 to 2001 period. Second a model was estimated for life expectancy and for a synthetic fertility index. Both sources of information were combined to obtain the forecasts for the rates. Immigration rates are predicted by assuming three possible scenarios based on the maximum proportion that immigrants will represent in the Spanish population. With these variables a structure of ages and sex for the Spanish population is estimated using a cohort component model.

Keywords: Population projections, time series, factorial model, bootstrap.

JEL Classification: C32, C53 and J11

^{*} Departamento de Estadística. Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), e-mail: <u>andres.alonso@uc3m.es</u>

[†] Departamento de Estadística. Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), e-mail: <u>daniel.pena@uc3m.es</u>

[‡] Departamento de Análisis Económico: Economía Cuantitativa. Universidad Autónoma de Madrid, Campus de Cantoblanco, 28049 Madrid, e-mail: jr.puerta@uam.es

1 Introduction.

Studies on population forecasts are a high priority for national statistical organizations (in Spain, the National Statistical Institute (NSI)) or international organizations such as the United Nations or the World Bank. Population projections differ by geographic area, time horizons and type of use. Thus, for example, in regions, communities or autonomous states, forecast horizons to the nearest decade are usually used (see, IEA (1995) and IECM (2004)) whereas in projections on a national level the horizons are extended to several decades (see García-Ferrer and del Hoyo (1991), USCB (2000), INE (2001, 2004) and Hyndman and Booth (2008)). On the other hand, the demand for these types of long term projections is quite diverse: human resource planning; energy resource planning; health and pension system planning. This last element in particular has been widely studied in the Spain (see Herce et al (1995, 1996), Diez (2000), Jimeno (2002) and Ahn et al (2005) among others), mainly in relation to the hypothesis that the decrease in fertility and increase in life expectancy will lead to a reduction in the active population which must support the cost of Social Security benefits.

One aspect which has been less studied is that of population forecasting in different educational cycles. One study which focused on this area was carried out by the Statistical Institute of Andalusia (Spain) for the period of 1998 - 2016 (see INE (2000)). Recently, Alonso-Meseguer and Sosvilla-Rivero (2004) looked at the gross enrolment rate and carried out projections until the year 2050 on registration and educational system expenses under different scenarios of migration flow and the integration of immigrant populations. These scenarios are based on different hypotheses of the volume of future migratory flow which were done by the National Statistical Institute in its revision of Spanish population forecasts, using the 2001 Census. Alonso et al (2007) provided the demand projections for Spain's official educational stages in period 2005 - 2050.

The population forecasts can be obtained using a wide range of procedures and models: the cohort component method; aggregate time series analysis; micro simulation; structural models and functional data modelling (see Booth (2006) for a review on stochastic population forecasting). In this work a cohort component method and time series analysis were combined as proposed by Lee and Tuljapurkar (1994), and shown by Keilman et al (2002) and Wilson and Bell (2004) for the United States, Norway and Australia respectively. Specifically, we use a dynamic factor model similar to that proposed by Lee and Carter (1992) and Lee and Tuljapurkar (1994), but with the introduction of restrictions in the common factor in order to model the mortality and fertility rates by age groups. The component method requires the establishment of future paths for the three basic components of the population changes: mortality, fertility and migration. The prediction of these components is based on the sieve bootstrap procedure proposed by Alonso et al (2002, 2004). A major advantage of this combined approach is that we obtain distribution functions of the forecasts for mortality and fertility rates for each single age and sex. Regarding net migration rates, we center our attention in immigration and we propose an analysis of immigration using different growth models. Using these models gives us future growth of the immigrant population if we make assumptions regarding the upper bounds in the percentage of immigrants out of the total population.

The rest of this paper is divided into four sections. Section 2 presents the projection methodology of the populations by single-year age and sex. In Section 3 this methodology is applied to the Spanish data for period 1970 - 2001. Also forecasts for mortality, fertility and migration rates are generated from 2005 to 2050. Finally, Section 4 describes the results of the population projections and compares them to those published by the Spanish National Statistical Institute (INE, 2004).

2 General Forecasting Methodology

2.1 Introduction

The cohort component method is widely used by official organisms for population projections. This method is based on dividing initial population numbers into cohorts defined by age and sex, and on updating each new population period for each age and sex group keeping in mind the components of the demographic change: fertility, mortality and migration. The following system of equations defines the cohort component method:

$$P_{0,t}^{(s)} = B_t^{(s)} - D_{0,t}^{(s)} + M_{0,t}^{(s)},$$

$$P_{e,t}^{(s)} = P_{e-1,t-1}^{(s)} - D_{e,t}^{(s)} + M_{e,t}^{(s)}, \text{ with } e \in \{1, 2, \dots, e_{max}\},$$
(1)

where P denotes the population at time t, B, D and M denote births, deaths and net migration in the period (t-1,t), respectively; e denotes age and s denotes sex and takes the values F, M, i.e. female and male. This system is completed with the following equations for the components of fertility, mortality and net migration:

$$B_{t}^{(s)} = \sum_{e} FR_{e,t}^{(s)}P_{e,t-1}^{(F)}, \text{ with } e \in \{<15, 15, \dots, 49, \ge 50\},\$$

$$D_{e,t}^{(s)} = MR_{e,t}(s)P_{e,t-1}^{(s)},$$

$$M_{e,t}^{(s)} = I_{e,t}(s) - ER_{e,t}(s)P_{e,t-1}^{(s)},$$
(2)

where I, FR, MR, and ER denote the immigration and the rates of fertility, mortality and emigration, respectively.

In order for the system (1) - (2) to provide population forecasts by age and sex it is necessary to have the future values of fertility, mortality and immigration rates, as well as the number of immigrants by age and sex. There are two principal sources of information for these rates. The first is historic information on specific rates, and the second is information about aggregates obtained from those rates. We will analyze the use of both these sources below.

It is possible to model and directly project each specific rate individually, but as Gutiérrez de Mesa (2003) points out, "absurd" results can appear because of the dependence of the synthetic indices in the specific rates. Nevertheless, direct estimation implies the modelling of hundreds of time series which, in general, are not independent.

One method for predicting mortality curves is that proposed by Heligman and Pollard

(1980). These authors use models of mortality curves by ages, such as the following:

$$MR_e = A_t^{(e+B_t)^{C_t}} + D_t \exp\left(-E_t (\ln e - \ln F_t)^2\right) + \frac{G_t H_t^e}{1 + G_t H_y^e},\tag{3}$$

where A_t , B_t , C_t , D_t , E_t , F_t , G_t and H_t are the parameters to be estimated. The parameters are adjusted so that life expectancies are similar to those being projected. This method is used by IECM (2004) for the Madrid Autonomous Community.

An alternative to direct projection of mortality rates by age is to obtain time series models for the eight parameters of the mortality curve from Heligman and Pollard which were estimated previously in the available years. We see that this methodology reduces the number of time series to be analyzed, from 86 series (in our case) per sex to only eight series. Using ARIMA models, McNown and Rogers (1989) carried out mortality projections in the United States to the year 2000. Felipe et al (2002) used a similar procedure to model changes in mortality in Spain between 1973 and 1993, and carried out projections for the period 1994 - 2010.

Finally, a method which considers the dependence between the time series proposed by Lee and Carter (1992) is to relate mortality rates by age to a single non-observable factor:

$$\ln(MR_{e,t}^{(s)}) = a_e^{(s)} + b_e^{(s)}k_t^{(s)} + \varepsilon_{e,t}^{(s)},$$

$$k_t^{(s)} = c^{(s)} + k_{t-1}^{(s)} + \eta_t^{(s)},$$
(4)

where $a_e^{(s)}$ and $b_e^{(s)}$ are parameters which depend on age, e; $k_t^{(s)}$ is the non-observable factor which includes the general characteristics of mortality in year t and is called the mortality index, and $\varepsilon_{e,t}^{(s)}$ is the error term and includes the characteristics pertaining to each age ewhich are not picked up by the model. The second equation in (4) establishes that the factor k_t follows an ARIMA(0, 1, 0) with a non-null constant. Recently, Hyndman and Booth (2008) proposes a functional version of Lee-Carter approach and they obtain mortality, fertility and migration forecasts for Australia.

Both with McNown and Roger's (1989) procedure as well as with that of Lee and Carter (1992) it is possible to construct prediction intervals for the mortality rates using the rela-

tionship between these rates and the forecast distributions of the parameters of Heligman and Pollard's curve or of the mortality factor, respectively.

As in the case of mortality rates, a factorial model can be written for fertility rates (see Lee and Tuljapurkar (1994)):

$$FR_{e,t}^{(s)} = c_e^{(s)} + d_e^{(s)} f_t^{(s)} + v_{e,t}^{(s)},$$

$$f_t^{(s)} = f_0^{(s)} + \phi f_{t-1}^{(s)} + \nu_t^{(s)} + \theta^{(s)} \nu_{t-1}^{(s)},$$
(5)

where $c_e^{(s)}$ and $d_e^{(s)}$ are parameters which depend on age, e; $f_t^{(s)}$ is the non-observable factor which includes general characteristics of fertility in year t and is called the fertility index, and $v_{e,t}^{(s)}$ is the error term which, as before, includes the characteristics pertaining to each age e which are not captured by the model. In this case, the superscript s refers to the fertility rate of male or female offspring. The second equation in (5) establishes that the factor $f_t^{(s)}$ follows the ARMA (1,1) model with a non-null constant. In Lee and Tuljapurkar (1994), in the second equation the value of the constant is set so that the mean value of the synthetic fertility index, $c + E[f_t]$ is equal to 2.1, where $c = \sum_e c_e$.

The second source of data is that of synthetic indices associated with each type of rate. In this case the indices are modelled and specific rates are obtained indirectly. With mortality rates, for example, some authors have proposed projecting female life expectancy by specifying a value for the last year of the forecasting horizon. For the remaining years an interpolation based on the logistic function was used. This procedure is the one used in the projections carried out by the Statistical Institute of Andalusia (IEA, 1995) and the Madrid Autonomous Community (IECM, 2004), where the values 84.0 in 2025 and 86.3 were established as the upper "bound" for female life expectancy. In both studies, male life expectancy was handled using an adjustment which takes into account the higher male mortality rate.

Establishing an upper bound for life expectancy is no simple matter. We take, for example, the predictions for life expectancy in women for the year 2025 which were carried out by the National Statistical Institute: 85.44 in "Proyecciones de población calculadas a partir del Censo de Población de 1991. Evaluación y Revisión" (Population Forecasts taken from the 1991 Census: Evaluation and Review) published in 2001 and which used available data until 1996; and 86.48 in "Proyecciones de población calculadas a partir del Censo de Población de 2001" (Population Forecasts taken from the 2001 Census: Evaluation and Review) published in 2004. In these publications it is assumed that life expectancy will remain constant from 2026 and from 2031, respectively. There is no consensus in the demographic literature either on biological limits which influence the aging process itself. These elements are the principal reason for the methodology which we propose in Section 3 due to uncertainty about the value of this limit.

In the case of fertility rates by single-year ages, the forecast of the synthetic fertility index was used: $SFI_t = \sum_e FR_e$ which is complemented by the projection of mean age of the woman at time of children's birth, MAF_t , and its variability, VAF_t . Subsequently a theoretical model was used for the fertility curve. Specifically, in the IEA (1995) and IECM (2004) a gamma type function was used:

$$FR_{e,t} = \frac{A_t B_t^{C_t} (e - 15)^{C_t - 1} \exp(-B_t (e - 15))}{\Gamma(C_t)},$$
(6)

where $A_t = SFU_t$, $B = \frac{MAF_t}{VAF_t}$ and $C = \frac{MAF_t^2}{VAF_t}$.

Finally, we would like to point out that a weakness in the usual forecasting methods is that they use a single future path commonly interpreted as the mean or most "likely". Frequently, official organisms establish other future scenarios for fertility, mortality and migration around these mean values. Generally, between three and seven scenarios are reported, with three being the most often used. The terms high, medium and low hypothesis are commonly used in these cases (see, e.g., Lee and Tuljapurkar (1994)). There are several inconveniences associated with this method of scenarios (see comments in Keilman et al (2002)), for example, the "intervals" defined by the low and high hypotheses do not have an error measurement associated with them. In the following sections we use a methodology which allows us to resolve these inconveniences by modifying the procedures of Lee and Carter (1992) and Lee and Tuljapurkar (1994).

2.2 General Methodology for Forecasting Mortality and Fertility

In this section we develop a procedure for modelling life rates, separated into different groups, which allows us to obtain the forecast distributions of the future values of these rates for a pre-established horizon. Here we mention life rates in order to include both mortality rates and fertility rates in the presentation, since as we will see in the proof, both model (4) for mortality rates as well as model (5) for fertility rates, are special cases of the dynamic factor model. This model has been studied by Geweke (1977), Peña and Box (1987), Tiao and Tsay (1989), Gonzalo and Granger (1995) and Peña and Poncela (2004), among others. Ortega and Poncela (2005) used this model for the analysis of Southern European fertility rates. We also refer to rates in certain groups and in such cases we talk of groups of individuals who share one or more characteristic such as same age and/or same sex. This allows us to make a general formulation which in the next section will be specified for the corresponding rates and groups.

2.2.1 Dynamic Factor Model

In this section we follow the presentation of the dynamic factor model carried out by Peña and Poncela (2004). Let $\{y_t\}_{t\in\mathbb{Z}}$ be a vector series of dimension m, for example, male mortality rates of m age groups. The dynamic factor model assumes that the components of the vector series, y_t , can be written as a linear combination of r common factors plus an error term:

$$\boldsymbol{y}_t = \boldsymbol{P} \quad \boldsymbol{f}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{m} \times 1 \quad \boldsymbol{m} \times \boldsymbol{r} \ \boldsymbol{r} \times 1 \quad \boldsymbol{m} \times 1$$

$$(7)$$

where f_t is the *r*-dimensional vector of common factors, P is the weight matrix of factors, and ε_t is the specific factors vector or error term. Additionally, it is assumed that the vector of common factors follows a VARIMA(p, d, q) model defined by:

1

$$\boldsymbol{\Phi}(B) \quad f_t = \boldsymbol{\Theta}(B) \quad \boldsymbol{v}_t \\ r \times r \ r \times 1 \qquad r \times r \ r \times 1 ,$$
(8)

where *B* is the backward shift operator, $\mathbf{\Phi}(B) = \mathbf{I} - \mathbf{\Phi}_1 B - \dots - \mathbf{\Phi}_p B^p$ and $\mathbf{\Theta}(B) = \mathbf{I} - \mathbf{\Theta}_1 B - \dots - \mathbf{\Theta}_q B^q$ are polynomial matrices satisfying the roots of $|\mathbf{\Phi}(B)| = 0$ and of $|\mathbf{\Theta}(B)| = 0$ are not found within the unit circle. We assume that the innovations \mathbf{v}_t are uncorrelated for all lags, i.e., $E(\mathbf{v}_t \mathbf{v}'_{t+h}) = \mathbf{0}$ for $h \neq 0$ and also they are non-correlated with the specific factors, i.e., $E(\mathbf{v}_t \mathbf{\varepsilon}'_{t+h}) = \mathbf{0}$ for all *h*. In Peña and Poncela (2004) it is supposed that the innovations and specific factors are distributed as a $\mathcal{N}_m(\mathbf{0}, \mathbf{\Sigma}_{\varepsilon})$ and $\mathcal{N}_r(\mathbf{0}, \mathbf{\Sigma}_v)$, respectively. In this paper specific factors are allowed to follow stationary univariate models.

The factorial model defined by (7) and (8) is not identified since for any non-singular matrix of dimension $r \times r$ it is possible to express the vector series, \boldsymbol{y}_i , as a new set of factors and weights. Many restrictions have been proposed to solve the problem of identification, e.g., $\boldsymbol{\Sigma}_v = \boldsymbol{I}$ or $\boldsymbol{P}'\boldsymbol{P} = \boldsymbol{I}$ (see, for example, Peña and Poncela (2004)) and $\boldsymbol{P} = [p_{i,j}]$ with $p_{i,j} = 0$ for j > i (see Harvey (1989)). In this paper we use the restriction and assume that the factors are orthogonal, i.e. $f_{\cdot,i} \perp f_{\cdot,j}$ for $i \neq j$, as in Lee and Carter (1992) and Lee and Tuljapurkar (1994).

2.2.2 Bootstrap Procedure for Forecasting

In this section we present a bootstrap procedure for constructing forecasting intervals based on a modification of the procedure proposed by Alonso et al (2002, 2004).

- (1) The factorial model defined by (7) is estimated using the singular values decomposition as in Lee and Carter (1992). From this, the estimations for the r common factors $\hat{f}_t = (\hat{f}_{t,1}, \hat{f}_{t,2}, \dots, \hat{f}_{t,r})'$ and the weight matrix \hat{P} are obtained.
- (2) The residuals of the factorial model are calculated:

$$\widehat{\boldsymbol{\varepsilon}}_t = \boldsymbol{y}_t - \widehat{\boldsymbol{P}} \, \widehat{\boldsymbol{f}}_t. \tag{9}$$

(3) An AR(p_e) model is chosen for $\hat{\varepsilon}_{e,\cdot}$ with $e \in \{1, 2, ..., m\}$, using the BIC criteria. An ARI (p_s, d_s) model is chosen for common factors with $s \in \{1, 2, ..., r\}$. As a result we obtain estimations of the autoregressive parameters, the order of differences, and the residuals of the models AR and ARI.

(4) The empirical distribution function is obtained for the centered residuals of the AR and ARI models:

$$\widehat{F}_{\widetilde{\epsilon}_e}(x) = (n - p_e)^{-1} \sum_{t=p_e+1}^n I\left(\widetilde{\epsilon}_{e,t} \le x\right),$$
(10)

and

$$\widehat{F}_{\widetilde{v}_s}(x) = (n - p_s - d_s)^{-1} \sum_{t = p_s + d_s + 1}^n I\left(\widetilde{v}_{s,t} \le x\right),$$
(11)

where $\tilde{\epsilon}_{e,t} = \hat{\epsilon}_{e,t} - \hat{\epsilon}_{e}^{(\cdot)}$ and $\hat{\epsilon}_{e}^{(\cdot)} = (n - p_e)^{-1} \sum_{t=p_e+1}^{n} \hat{\epsilon}_{e,t}$ with $e \in \{1, 2, \dots, m\}$; $\tilde{v}_{s,t} = \hat{v}_{s,t} - \hat{v}_{s}^{(\cdot)}$; and $\hat{v}_{s}^{(\cdot)} = (n - p_s - d_s)^{-1} \sum_{t=p_s+d_s+1}^{n} \hat{v}_{s,t}$ with $s \in \{1, 2, \dots, r\}$

(5) A resample ϵ_t^* of i.i.d. observation from $\widehat{F}_{\widetilde{\epsilon}}$ and a resample v_t^* of i.i.d. observation from $\widehat{F}_{\widetilde{v}}$ were selected.

The following are the forecasting steps. The final p_e and $d_s + p_s$ observations are fixed from the AR and ARI models, respectively.

(6) The future bootstrap observations are calculated for common and specific factors using the relations:

$$\varepsilon_{e,T+h}^* = -\sum_{j=1}^{p_e} \widehat{\phi}_{e,j} (\varepsilon_{e,T+h-j}^* - \bar{\varepsilon}_e) + \epsilon_{e,T+h}^*, \qquad (12)$$

and

$$f_{s,T+h}^* = \sum_{j=1}^{p_s+d_s} \hat{\phi}_{s,j} f_{s,T+h-j}^* + v_{s,T+h}^*, \tag{13}$$

where h > 0, $\varepsilon_{e,t}^* = \hat{\varepsilon}_{e,t}$ for $t \leq T$ and $f_{s,t}^* = \hat{f}_{s,t}$ for $t \leq T$, with T being the last available year.

(7) The future bootstrap observations are calculated for vector \boldsymbol{y} using the relation:

$$\boldsymbol{y}_{T+h}^* = \boldsymbol{P} \boldsymbol{f}_{T+h}^* + \boldsymbol{\varepsilon}_{T+h}^*, \qquad (14)$$

where
$$\mathbf{f}_{T+h}^* = (f_{1,T+h}^*, f_{2,T+h}^*, \dots, f_{r,T+h}^*)'$$
 and $\boldsymbol{\varepsilon}_{T+h}^* = (\varepsilon_{1,T+h}^*, \varepsilon_{2,T+h}^*, \dots, \varepsilon_{m,T+h}^*)'$.

Finally, $F_{y_{s,T+h}^*}^*$, the bootstrap distribution function of $y_{s,T+h}^*$ is used as the estimator of the conditional distribution of $y_{s,T+h}$ given the sample. As usual, the $\hat{F}_{y_{s,T+h}^*}^*$ estimation is obtained by repeating *B* times the steps 5 to 7. A $(1 - \alpha)\%$ forecast interval for $y_{s,T+h}$ is the following:

$$[Q^*(\alpha/2), Q^*(1 - \alpha/2)], \qquad (15)$$

where $Q^*(\cdot) = \hat{F}_{y^*_{s,T+h}}^{*-1}(\cdot)$ are the quantiles of the estimated bootstrap distribution.

In the demographic data that we analyze in the following sections, we see that there is a high correlation between the first factor of model (7) and a synthetic rate indicator, i.e. with a function of \boldsymbol{y}_t . Specifically, in the case of mortality rates this correlation is given with the life expectancy at birth, and in the case of fertility rates, with the synthetic fertility index. This allows us to establish a simple model between the first factor, $f_{1,t}$, and the synthetic index, i_t :

$$f_{1,t} = \alpha_0 + \alpha_1 i_t + \iota_t, \tag{16}$$

where ι_t assumes that it follows an AR(p_ι) model. The previous model together with a specific modelling of the synthetic index (which is found in sections 3.1.1. and 3.2.1 for life expectancy at birth and synthetic fertility index, respectively) allow us to make forecasts for future values of this factor.

3 Application of Spanish Data

3.1 Application of Spanish Data: Mortality

In this section we obtain the forecasts for mortality rates by age and sex for the Spanish population using a dynamic factor model which utilizes the historical series of these rates in Spain during the period 1970 to 2001. At first, the unifactorial model used by Lee and Carter (1992) was considered, and its goodness of fit was analyzed for the Spanish data. Next, the bifactorial model was considered. With both models we detected an effect of high male mortality rates which some authors have attributed to deaths in traffic accidents or deaths associated to AIDS (see, for example, Felipe et al (2002) and IECM (2004)). Both causes have subsided in the last years of the available data; therefore for a long term projection it is not desirable to introduce this transitory effect. Thus, we propose a procedure for eliminating this effect from the forecasts (see detailed study in Alonso et al (2005)).

On the other hand, as with Lee and Carter (1992), we showed that the first factor of the model for mortality rates is highly correlated with life expectancy at birth. This motivated

the study of a model for life expectancy at birth in which we impose the existence of an upper limit. Finally, the existing relationship between the first factor and life expectancy allowed us to introduce restrictions in the forecasts of mortality rates by single-year ages and sex.

3.1.1 Life Expectancy Analysis

In this section we propose a method for establishing an upper bound for life expectancy or, more specifically, for establishing a distribution function for this bound. It is important to point out the close relationship between life expectancy and the first factor of mortality; the correlation between both is: -0.9855 in men and -0.9976 in women. This allows us to establish restrictions on the mortality factor through restrictions on life expectancy. The procedure is as follows:

• The following transformation is considered for life expectancy at birth, LEB_t :

$$Y_{t,A} = \ln \frac{LEB_t}{A - LEB_t},\tag{17}$$

where A is the upper bound for life expectancy, which we assume in the interval $\mathcal{A} =$ [76, 100] in men and $\mathcal{A} =$ [84, 105] in women. The lower bounds for these intervals take into account the most recent available data. As opposed to the procedures described in IEA (1995) and IECM (2004), no single value was set for A, instead, A is considered a parameter of the model.

- For each value of parameter A, a different series, Y_{t,A}, is obtained and a model ARIMA(p̂_A, 1,0) is selected using the BIC criteria. Next, forecasts were carried out with this model using a modification of the sieve bootstrap procedure proposed by Alonso et al (2004) which allows for taking into account the uncertainty associated in the selection of p̂_A.
- Forecasting the *LEB* is based on a combination of the forecasts of the $Y_{t,A}$. Thus, the distribution of future values for life expectancy are obtained using:

$$F_{LEB_{t+h}}(x) = \int_{a \in \mathcal{A}} F_{LEB_{t+h},a}(x) f_{A|\boldsymbol{LEB}}(a) \,\mathrm{d}a, \tag{18}$$

where $F_{LEB_{t+h},a}(\cdot)$ is the distribution function of LEB_{t+h} calculated assuming that the bound for life expectancy is a, and $f_{A|LEB}(\cdot)$ is the density function of A conditional to the observed sample, LEB.

Expression (18) is no more than a formulation of Bayesian Model Averaging with the introduction of uncertainty regarding the two unknown elements in the model: A and \hat{p}_A . Similar approach had been used in prediction using polynomial regression of unknown degree (see Guttman et al. 2005) and using nonparametric regression (see Peña and Redondas, 2006). One advantage of this procedure is that it allows us to calculate the a posteriori distribution of the upper bound of life expectancy and therefore (in light of available data) the likelihood of the bounds used in others researches.

The use of the expression (18) with A taking values in an interval, is complex and in this paper we propose a discretization of 0.5 years, thus, the distribution of future values of life expectancy are obtained using:

$$F_{LEB_{t+h}}(x) = \sum_{a \in \mathcal{A}} F_{LEB_{t+h},a}(x) \Pr\{A = a | \boldsymbol{LEB}\},\tag{19}$$

where $F_{LEB_{t+h},a}(\cdot)$ is the distribution function of LEB_{t+h} calculated assuming that the bound for life expectancy is a, and $\Pr\{A = a | LEB\}$ is the probability of A conditional to the observed sample. This probability can be approximated using:

$$\Pr\{A = a | \boldsymbol{LEB}\} = \frac{\alpha_a \exp(-1/2\text{BIC}(a))}{\sum_{a \in \mathcal{A}} \alpha_a \exp(-1/2\text{BIC}(a))},$$
(20)

where BIC(a) is the value of the BIC criteria in the model with bound a (see, Kass and Raftery, 1995).

In Figure 1 we present the estimated distribution of the upper bounds of life expectancy in men and women. We have used a circle to represent the a posteriori means of the bounds: 82.30 and 89.73, respectively. Figure 2 shows the prediction mean of life expectancy in men and women. For example, for the years 2025 and 2050 the prediction mean are: 78.63 and 80.29 in men and 86.23 and 97.98 in women.

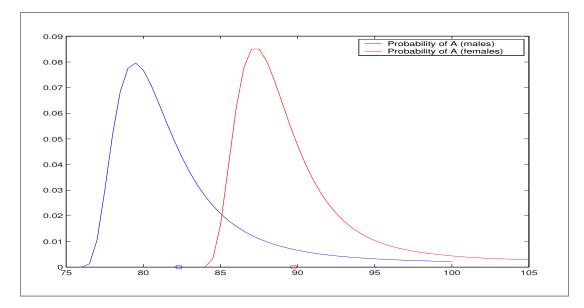


Figure 1. A posteriori probability of the upper bounds of life expectancy.

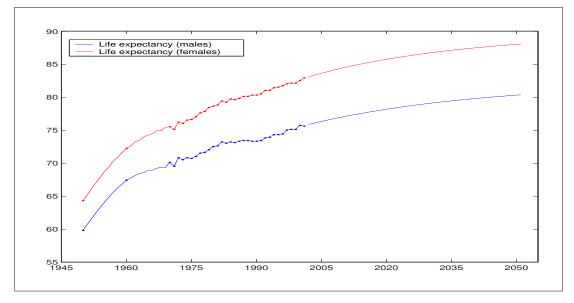


Figure 2. Forecasts using model averaging and the observed values in life expectancy at birth. Spain 1950 - 2050.

In Figure 3 we present the fan chart (see Wallis, 1999) of the bootstrap distribution of life expectancy forecasts. This method of representing uncertainty in forecasts is being employed more and more by institutions which carry out macroeconomic forecasting. A fan chart represents the forecast intervals of different levels. Specifically, in Figure 3 we represent the intervals at 20%, 40%, 60%, 80% and 90%, in addition to the median of the forecasts. First, we observe the asymmetry of these distributions; this element cannot be visualized

when using intervals which are symmetric with respect to the mean of the predictions, as in Keilman et al (2002) and Wilson and Bell (2004). Second, we can evaluate the assumptions or projections carried out in earlier works. Thus, for example, the projections carried out by the National Institute of Statistics (INE, 2004) fall in the 80% forecast interval for men and 70% for women.

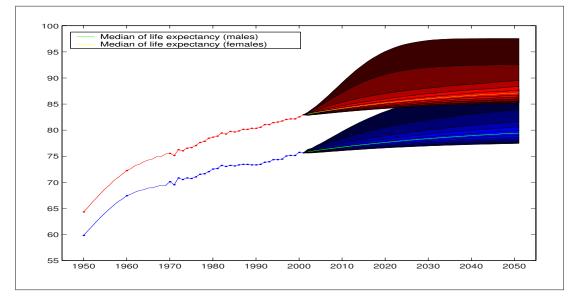


Figure 3. Fan chart using model averaging and the observed values of life expectancy at birth. Spain 1950 - 2050.

3.1.2 Forecasts of Mortality Rates by Age and Sex

Once we have obtained the predictions for life expectancy at birth we can find the corresponding predictions for mortality factors using model (16) and, using model (4) in which we allow the errors or specific age factors, $\varepsilon_{e,t}$, to follow an AR(p_{ε_e}) model, we obtain the forecast densities for mortality rates by age. To illustrate this further, in Figures 4 and 5 we show a fan chart and the forecast densities of mortality rates during the first year of life. In both, a clear reduction is observed in mortality in this age group. In Figure 6 we present the fan chart of the mortality curve for the years 2025 and 2050. The remaining ages and years can be obtained using the routines developed in this paper and which are available from the authors upon request.

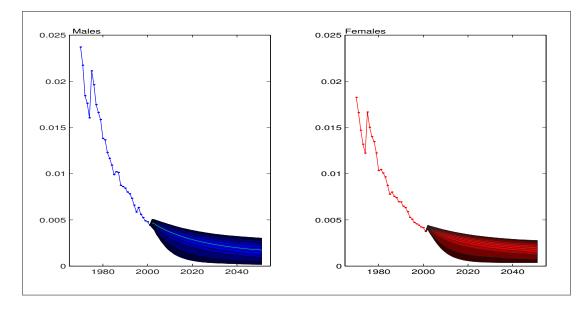


Figure 4. Fan chart using model averaging and the observed values of infant mortality rate. Spain 1970 - 2050.

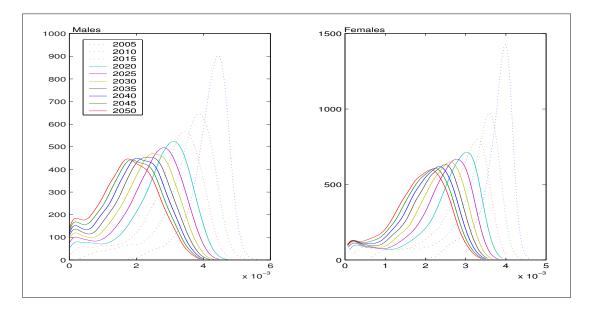


Figure 5. Forecast densities of infant mortality rates. Spain 2005 - 2050.

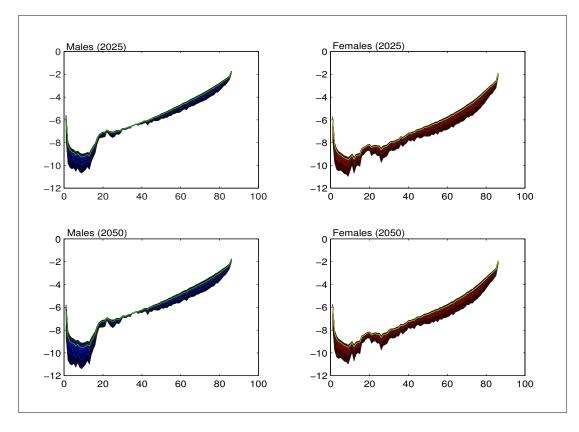


Figure 6. Fan chart using model averaging of the mortality curve. Spain 2025 - 2050. Fan chart using model averaging and the observed values of life expectancy at birth. Spain 1950 - 2050.
3.2 Application to Spanish Data: Fertility

In this section we obtain the projections for fertility rates by groups of age and sex for the Spanish population using a dynamic factor model. The first model considered was a unifactorial one used by Lee and Tuljapurkar (1994) and its goodness of fit was analyzed for the Spanish data. Later on a bifactorial model was considered where bimodality was clearly visible in the projections for fertility by ages.

On the other hand, we observed that the first common factor of the fertility rate model is highly correlated with the synthetic fertility index. This motivated the study of a model for this index similar to that proposed for life expectancy at birth. We also observed that the factors $f_t^{(M)}$ and $f_t^{(F)}$, as well as the weights, $d_e^{(M)}$ and $d_e^{(F)}$ have similar values. This suggests a joint modelling of births for both sexes as in Lee and Tuljapurkar (1994). Nevertheless, the relationship between the birth rate of males and females must be taken into account. In most countries this rate is between 105 and 107 males for every 100 females, and later this rate is modified owing to differences in patterns of mortality and migration in men and women. Finally, the existing relationship between the first factor and the synthetic index allows us to introduce restrictions in the projections of fertility rates by single-year ages.

3.2.1 Model for the Synthetic Fertility Index

In the following we propose a model for the synthetic fertility index which allows the distribution function of future values to be established. It is important to point out the close relationship between the synthetic fertility index and the first factor of fertility; the correlation is: 0.9737. This allows us to establish restrictions in the factor through the future values of the synthetic fertility index using model (16).

Similar to what was done in the previous section, we propose a transformation of the synthetic fertility index whose asymptotic behavior leads to an upper bound, B, of the number of children per women (similar to expression (17)). If, in the case of mortality, this bound can be explained by medical advances and the natural limits of the human organism, then in the case of fertility we understand that this bound can be explained by current socioeconomic conditions, the massive incorporation of women into the workforce and, therefore the opportunity costs that women considering maternity must face. As in the case of mortality we do not consider the bound as a single value but rather we assume an interval where the bound takes values. Specifically, we assume that B takes values in $\mathcal{B} \in [3, 8]$. We have set the lower limit of this interval according to the maximum indices reached during the years 1960 - 1970 and the upper limit according to the forecast intervals in an unrestricted model.

Figure 7 shows the estimated distribution of the upper bound of the synthetic fertility index. As in the above section we have used an interval discretization, specifically $\mathcal{B} = [3.0, 3.25, \ldots, 7.75, 8]$. A circle was used to indicate the mean a posteriori of the bound: 3.430.

Figure 8 shows the forecasts mean of the synthetic fertility index. These forecasts take

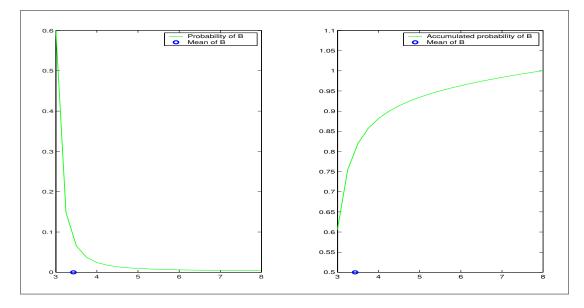


Figure 7. Probability and accumulated probability of the upper bound of the synthetic fertility index.

into account the uncertainty associated to the upper bound. For example, for the years 2025 and 2050 the mean forecasts are: 1.484 and 1.509, respectively. We also confirm that the forecasts tend towards an asymptote with a value nearing 1.510, slightly lower than the forecasts of the INE (2004).

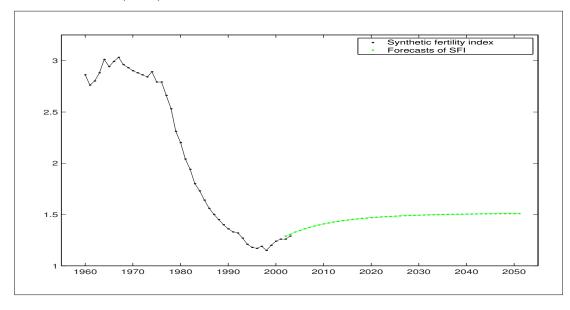


Figure 8. Forecasts using model averaging and the observed values of the synthetic fertility index. Spain 1960 - 2050.

An alternative to the transformations of type (17) used to establish the bound of these

indices is to impose restrictions on future trajectories of the index being studied in unrestricted models. For example, in Keilman et al (2002) for the SFI in Norway, the restriction was that it belonged to a prefixed interval, [0.5, 4]. In Figure 9 we show the fan chart of the bootstrap distribution for the synthetic fertility index forecast, imposing the restriction that the trajectories during the entire forecast horizon be less than 5.5. Note that the value 5.5 corresponds to the 95% percentile of the distribution of the upper limit for the SFI (see Figure 7). In the figure a slight recovery can be made out (in the central values of the projection) in future fertility. Nevertheless, values below one child per women have a considerable probability. Thus, for example, the probability that the SFI is below one child per woman is slightly lower than 30% in 2025 and higher than 30% in 2050. On the other hand, the probability of having fertility rates greater than or equal to the replacement level (2.1, at the current mortality rate) is around 20% in 2025 and 30% in 2050.

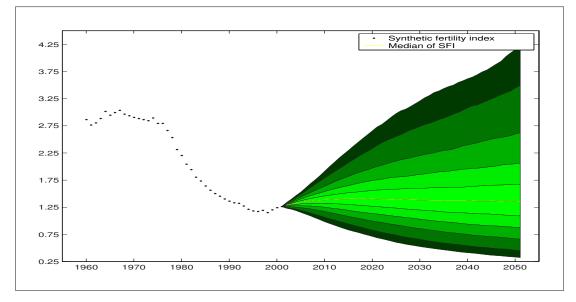


Figure 9. Fan chart using model averaging (with upper restricted trajectories) of the synthetic fertility index. Spain 1960 - 2050.

However, we see that the values used by Keilman et al (2002) for the SFI in Norway have a low probability in our case since (i) values lower than 0.5 have a probability below 5% until 2030, 10% to 2040 and only slightly above 10% in 2050, and (ii) values above 4 have a probability of less than 5% for almost the entire forecast horizon.

3.2.2 Forecast of Fertility Rates by Age

Once we have obtained the projection distributions of the synthetic fertility index we can obtain the corresponding distributions for the first factor of fertility using model (16). Then using the factorial model, in which we permit the errors or age specific factors, $\varepsilon_{e,t}$, to follow an AR(p_{ε_e}) model, we obtain the forecast densities for fertility rates by age. To further illustrate this point, in Figures 10 and 11 we depict a fan chart and the forecast densities of fertility rates for ages 20 and 30. In both we can see a tendency towards recovery in the central projections. Figure 12 depicts the fan chart of the fertility curve for the years 2025 and 2050 where, again, there is evidence of possible bimodality. The remaining ages and years can be obtained using the routines developed in this paper and are available from the authors upon request.

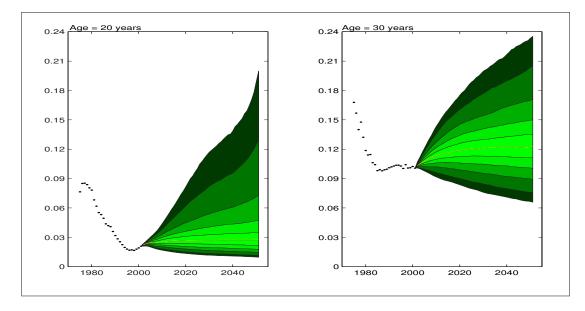


Figure 10. Fan chart using model averaging and the observed values of fertility rates for ages 20 and 30. Spain 1975 - 2050.

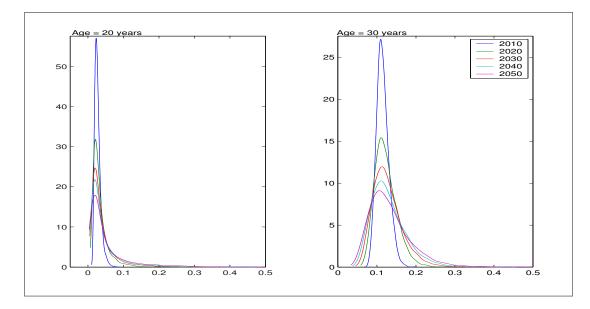


Figure 11. Forecast densities for ages 20 and 30 using model averaging of fertility rates. Spain 2005 - 2050.

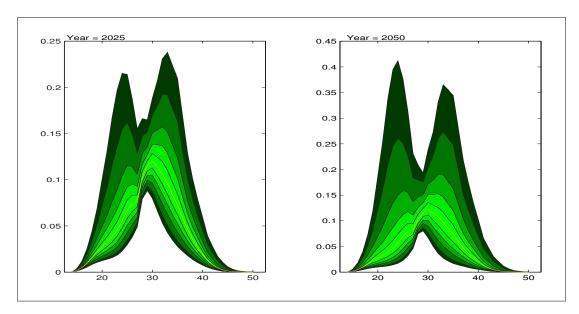


Figure 12. Fan chart using model averaging of the fertility curve. Spain 2025 - 2050.

3.3 Application to Spanish Data: Migrations

In this section we propose an evolutionary model of immigration in Spain for the next 50 years. This model picks up the sharp increase in the rate of immigrants over the last decade and allows for the assumption that in the long term the percentage of immigrants will stabilize, as has been the case in European countries with a longer history of taking in immigrants. In the case of birth and death rates as well as with immigration, Spain is no

different and should be studied within the European context. The majority of immigration in Spain is of an economic nature; the type of immigration received both in Spain and the European Union. Over the last several years, the flow of immigration to Spain has risen sharply but it still remains below the European mean of 5.1%.

We propose to model the flow of immigration using the methodology shown in Section 3.1.1, carrying out a transformation in the series, which in this case would be the number of immigrants INM_t obtained in the permanent observatory of immigration maintained by the Ministry of Labour and Social Services (Ministerio de Trabajo y Asuntos Sociales - www.extranjeros.mir.es).

• The following transformation of the series of the number of immigrants, INM_t is considered:

$$Y_{t,C} = \ln \frac{INM_t}{C - INM_t},\tag{21}$$

where C is the upper bound of number of immigrants.

In this first step, the key is to select the possible values of the parameter C. To do that, we make three different assumptions having as a reference the percentage of immigrant population in Europe:

- Assumption 1: $C \in (4000000, 8000000)$ corresponds to a percentage of the immigrant population in 2050 which will be between 8% and 15% of the Spanish population, based on population projections carried out by the National Statistical Institute (INE, 2004).
- Assumption 2: $C \in (4000000, 11000000)$ corresponds to a percentage of the immigrant population in 2050 which will be between 8% and 20% of the Spanish population, based on population projections carried out by the National Statistical Institute (INE, 2004).
- Assumption 3: $C \in (4000000, 15000000)$ corresponds to a percentage of the immigrant population in 2050 which will be between 8% and 28% of the Spanish population, based on population projections carried out by the National Statistical Institute (INE, 2004).

The next two steps are similar to the methodology proposed in Section 3.1.1. Figure 13 shows the growth curves in immigration to the year 2050 under the three above mentioned assumptions. Also incorporated into the graph is the future evolution of foreign immigration

suggested by the NSI. A rapid rise is observed in the number of immigrants until 2015 in assumption 1, until 2020 in assumption 2 and until 2025 in assumption 3. After this period of rapid increase the number of immigrants will begin to stabilize at around 6, 8, and 10 million in each of the assumptions. This stabilization in the number of immigrants does not imply that the rate of immigrant entry into the country will be zero but rather that the entry will be one of replacement. We assume for simplicity's sake in the model that the number of new immigrants is similar to the number of deaths occurring among the immigrants the year before.

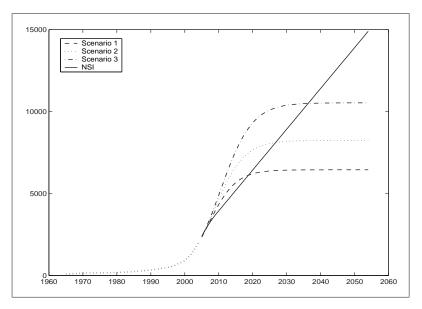


Figure 13. Evolution of immigration for the assumptions and NSI.

4 Projections for the Spanish Population

In the previous section we obtained, using bootstrap methods, the future distributions of the three demographic components which are basic for population forecasting using the cohort component method defined by the equations (1) and (2). In this section we apply this procedure in order to obtain projections of the Spanish population for the years 2005 to 2050. First we focus on the dynamic of certain aspects of the population, for example, total number of inhabitants, percentage of certain age groups (0 - 15, 16 - 64 and over 65) and the dependency ratios. Second we focus on the dynamic of the structure by single-year ages of the population. Tables 1 and 2, in the Appendix, contain the forecasts mean for female and male population by single-year ages in the period 2005 - 2050.

4.1 Evolution of the Population

In this subsection we analyze the evolution of basic elements of the population such as the total number of inhabitants, the percentage of the population of age groups 0 - 15, 16 - 64 and over 65 and the dependency ratios in those groups. We present our results for the three immigration scenarios obtained in the previous section.

In Figure 14 we present the projections of total inhabitants for the three immigration scenarios as well as the projections carried out by the NSI based on the 2001 census. We can observe that the three scenarios show a sustained increase in population during the first decade of the forecast, 2005 to 2014, with values similar among the three and to the forecasts from the NSI. Later, stagnation can be seen in population growth under the low and medium scenarios in the years 2023 - 2024 and 2029 - 2030, respectively, and finally a reduction in the population. In the high scenario the population growth does not stop during the entire forecast period, although around the year 2025 the growth rate decreases. These projections differ from those carried out by the NSI where stable growth is shown for almost the entire period.

Figure 15 shows the projected population percentages of broad age groups: (i) infant and school age, 0 - 15 years of age; (ii) economically active, 16 - 64; (iii) retirement age, 65 and over. First, we observe that under the three assumptions the projected percentages are similar. Only in the proportion of the population 0 - 15 years of age, and in the decade 2025 to 2035 can notable differences be seen between the low scenario and the other two. As far as the projections from the NSI, we observe similarities during the first decade for all groups, but in the following years a large difference can be seen with respect to the proportion of the 0 - 15 population. This difference may be due to the different immigrant entry dynamic

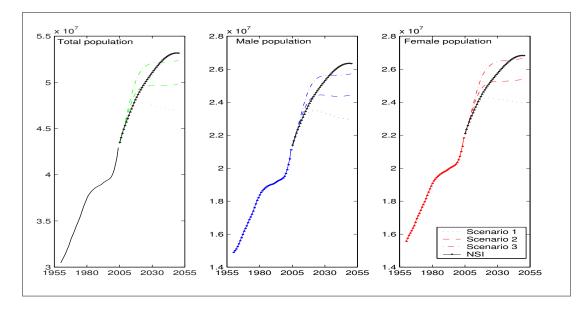


Figure 14. Forecasts and the observed values of total population using the cohort component model. Spain 1960 - 2050.

assumed by the NSI and the proposed scenarios. For the 16 - 64 age group the projections are quite similar until the year 2040; in the following decade the NSI forecasted a proportion 1% - 2% above that of the projections for the scenarios suggested in this paper. Finally, for the retirement age group very similar values were obtained by the NSI and by the scenarios, showing a clear increase in this projection which leads to the possibility that migratory flows are not a solution to the problem of an aging population.

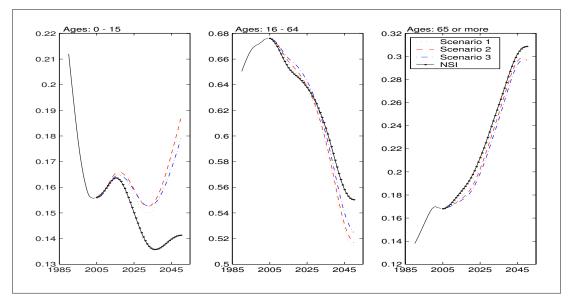


Figure 15. Forecasts and the observed values of population proportions of age groups 0 - 15, 16 - 64 and over 65 using the cohort component model. Spain 1990 - 2050.

4.2 Structure of the Population

In this section we analyze the evolution of the structure by single-year ages and sex during the period 2005 - 2050. To do this, we study the behavior of population pyramids in the years of the forecast. Figure 16 shows the forecast of the population pyramids by sex and age for the three proposed scenarios. We observe several elements which we would like to point out:

- The three scenarios lead to similar population pyramids, where the number of individuals is the only element which clearly differentiates them.
- In the year 2005 the majority of individuals of both sexes (salient in the pyramids) are found in the age group of 20 50, in 2025 they are between 30 70 years of age, and in 2050 they are between 50 and 80. This progressive aging of the population is observed as an upward movement in the most prominent age groups.
- In the final years of the forecast a slight widening of the base of the pyramids is observed.

5 Conclusions

In this paper, projections of the Spanish population are obtained by age and sex for the period 2005-2050. This analysis is carried out by combining two various sources of information, that is historical evolution of the birth, mortality and immigration rates for the different ages and sexes and the evolution of life expectancy and the synthetic fertility index. The predictions are based on a factorial model which extents to the model proposed by Lee Carter. In the analysis of mortality we observe a strong linear relation between the first factor and the life expectancy. In the second step of the analysis we incorporate the information of this index into the factorial model predictions. Based on biological considerations, we assume that life expectancy must have an upper (but unknown) bound. We obtain

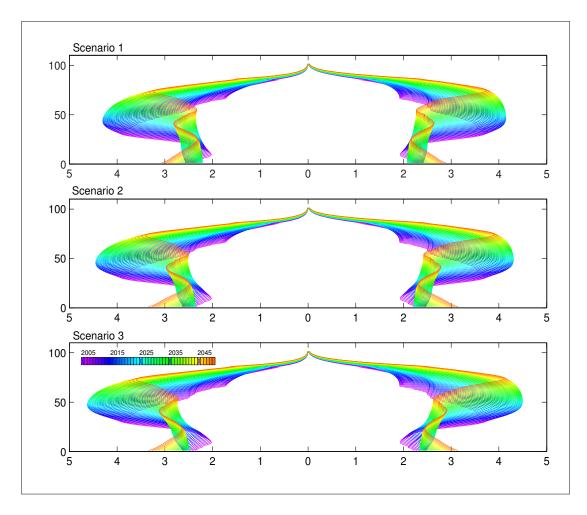


Figure 16. Forecasts of population pyramids using the cohort component model. Spain 2005 - 2050.

a predictive distribution of the life expectancy that incorporates this non linear restriction in its evolution. The predictive distribution of this index is used to obtain predictions of the mortality with a horizon of 45 years. These ideas are extended to the analysis of fecundity and immigration. Combining these three sources of information and using a cohort component model, we obtain prediction for the Spanish population by sex and age. Some remarkable conclusions can be derived from these projections:

- The female and male life expectancies seem to tend to 98 and 90 years, respectively.
- The synthetic fertility index tends to a value of 1.5 children.
- None of the three scenarios of immigration, which suppose immigrant proportions of 12%, 17% and 22% in average, seem to be enough to correct the aging of the Spanish population.

Acknowledgements

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Appendix

Table 1

Forecasts	mean	for	female	population	by	single-year	ages.	Spain	2005 -	2050.

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Table 2 $\,$

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