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# Embodied Resource Flows and Product Flows: Combining the Absorbing Markov Chain with the Input-Output Model

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**Number 1002**

**January 2010**

# Embodied Resource Flows and Product Flows: Combining the Absorbing Markov Chain with the Input-Output Model

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**Abstract**

We develop the absorbing Markov chain (AMC) for describing in detail the network of paths through an industrial system taken by an embodied resource from extraction through intermediate products and finally consumer products. We refer to this as a resource-specific network. This work builds on a recent literature in industrial ecology that uses an AMC to quantify the number of times a resource passes through a recycling sector before ending up in a landfill. Our objective is to incorporate into that analysis an input-output (IO) table so that the resource paths explicitly take account of the interdependence of sectors through their reliance on intermediate products. This feature makes it possible to track multiple resources simultaneously and consistently and to represent both resources and products in mixed units. Hypothetical scenarios about technological changes and changes in consumer demand are analyzed using an IO model, and model solutions generate the AMC database. A numerical example is provided.

AMC analysis describes the resource-specific networks using matrices that are derived not from the Leontief inverse but from a generalized variant of the Ghosh inverse matrix. The Leontief inverse and especially the Ghosh inverse (although often not identified as such) have been used extensively to analyze ecological systems, and this paper extends these approaches for use in studying material cycles in industrial systems. Constructing the AMC formalizes the resource-specific network analysis and generalizes the content and interpretation of the Ghosh matrix. Path-based analyses derived from AMC theory are discussed in relation to the set of techniques called Structural Path Analysis (SPA).

The paper concludes by identifying the three most critical enhancements to the IO model needed for analyzing material cycles: the simultaneous incorporation of waste-processing sectors, stock and flow relationships, and international trade. The idea is to implement an AMC after each model extension. The modeling framework is intended for analyses such as: tracking a resource extracted in one region to landfills in other regions, evaluating ways to intensify secondary recovery at key junctures in-between. There are other ways, of course, to approach such an analysis, but the combination of an extended IO model and an AMC, representing both resources and products in mixed units, provides a comprehensive,

systematic and standardized approach that includes many features that are valued in industrial ecology and builds directly on a number of active research programs.

**Keywords:** industrial ecology, hybrid model, resource network, Leontief matrix, Ghosh matrix, material cycles

## Introduction

### *Objectives*

This paper presents a framework for quantifying various characteristics of the networks of paths through an economic system by which a resource is converted, process by process, into the products ultimately purchased by consumers. We will refer to this as a *resource-specific network*. The approach is based on an absorbing Markov chain (AMC) constructed from an input-output (IO) database and is intended for use with an IO model that generates the required database under alternative assumptions about future production technologies and consumption choices. The AMC takes as its starting point a small body of literature based on the use of Markov chains in material flow analysis (MFA).

In a recent paper Yamada et *al*. (2006) developed an AMC (although not identified as such; see Duchin and Levine 2008) to calculate the average number of times a particular resource is used before ending in a landfill. Matsuno et *al*. (2007) applied it to use of iron in Japan in 2000. The analysis is based on a state transition table, a square matrix that describes the flows of iron through 15 states: crude iron, a few steels and scraps, several product categories, exports and landfill. The authors found that the average ton of iron was used 2.67 times and that this figure is particularly sensitive to assumptions about the recovery rate of iron from products used in construction. Eckelman and Daigo (2008) extended this AMC model to study global use of copper in 2000 using material flow data to construct the state transition table. They found that a ton of copper is used on average 1.9 times before final disposal in a landfill of a product incorporating it. While these results are interesting in their own right, the authors point out (p. 266) that they are only early examples of what can be learned from the application of Markov chains to MFA.

In the studies cited, a separate transition table would be required for each resource to be tracked. Second, there is no mechanism to assure consistency among transition tables for different resources under alternative scenarios about changes in resource productivity or substitution among resources. These limitations can be overcome by including in the state transition table an IO table representing input requirements for intermediate products as well as any number of resources. All resources can be represented in the same transition table because they all make use of the same inter-industry relationships. When a new transition table is generated by analysis using an IO model, the flows of resources are consistent with each other.

Incorporating the inter-industry relationships characteristic of IO models requires altering some assumptions. While the transition tables in the studies cited above measure product flows in units of total mass of a physically incorporated resource, we measure products in quantity (or money value) of product. We represent the *direct* flows of resources into production sectors rather than *total* flows and use the IO model to estimate the total resource content. To avoid double-counting of the resource contents of products, resources flow as inputs only to the industries that extract them while the output of an extraction industry is treated as a product.

In this paper we make several simplifying assumptions and discuss ways to relax them in the concluding section. First, we do not deal with waste treatment because both costs and time delays need to be introduced to do so adequately. Second, we assume a one-region economy at one point in time. Third, we represent resources embodied in products, meaning resource requirements for production, by contrast with the physically incorporated resources measured in MFA studies and specified in the transition tables of the studies cited above. Only part of the embodied resource content is actually physically incorporated, and only a portion of the latter is potentially recoverable after the useful life of the product is exhausted. With additional parameters specifying the portions of required inputs that are physically incorporated in products and the portions of the latter that are potentially recoverable, it will be possible to deduce incorporated resources from required ones. Both concepts are needed in order to evaluate the ability to meet resource demand with minimal extraction of virgin resources.

By including an IO table, the transition table for our study provides comprehensive coverage of an economy. The resulting AMC is able to describe the network for each resource from extraction through embodiment in intermediate products and ultimately in consumer products. This network, as well as the individual paths of which it is composed, will be characterized by a number of measures including the average length of a path in the network. This AMC generalizes and systematizes a number of earlier efforts in the analysis of interdependence and in particular of pathways in industrial systems and in ecosystems, which are described in the remainder of this section. Subsequent sections describe the standard IO model with both product and resource flows and then the Markov chain, first in Markov notation and then rewritten in a notation more familiar from IO analysis, followed by a numerical illustration of the AMC. Then the relationship between the AMC and the IO model is described with a focus on the Ghosh matrix and SPA. The paper concludes with next steps for enhancing the proposed framework and suggests the broader set of questions that will then be addressable.

## ***Background***

Since the early 1990s IO models have been combined with other techniques of industrial ecology for life-cycle assessment of products, starting from Cobas *et al.* (1995), Hendrickson *et al.* (1998), Joshi (2000), Yutaka et al. (2001), and more recent studies including Lenzen (2007). Various chapters in Suh (2009) also describe the integration of MFA with IO. Waste input-output analysis (WIO), initiated by Nakamura *et al.* (2002, 2005, 2007, and 2009), uses material flow data to incorporate wastes and waste-treatment sectors into an IO model and will be revisited in the concluding section.

Both IO models and AMCs have been applied to study interdependence and flows of energy and materials through ecosystems, providing concepts and methods relevant to closing material cycles in an industrial system. Following Hannon's (1973) initial use of IO models to study interdependence in ecosystems, Patten (1982) developed environ analysis to study both upstream and downstream flows in ecosystems. While he called it an extension of IO analysis, it is actually closer to an AMC. Bailey *et al.* (2004a, b) demonstrated the applicability of environ analysis to industrial systems and recognized that "ecological input-output analysis, as applied to industrial systems, provides answers to different questions than does traditional economic input-output analysis (even when physical flows are integrated)." Markov chains also have a history of explicit applications in systems ecology to track energy and biomass flows (Higashi et al., 1993a; Leguerrier et al., 2006), including determining measures of nutrient recycling, of throughput (Barber, 1978a, b), and of trophic position (Levine 1980). The existence of a relationship between AMCs and IO models was noted by Higashi et al. (1993b) and made more explicit by Suh (2005), who developed a model that generalizes and formalizes seemingly disparate analyses of ecosystems and economic systems.

Structural Path Analysis (SPA) consists of a number of network algorithms that have been applied to the IO representation of an industrial system for evaluating the influence of one economic sector on another. Well-known applications are those of Defourny and Thorbecke (1984), Khan and Thorbecke (1989), Treloar (1997), Sonis and Hewings (1998), Peters and Hertwich (2006) and, as applied to ecosystems, Lenzen (2007). Suh (2005) points out some similarities between SPA and environ analysis. We discuss the relationship of SPA to the analysis based on an AMC of an economy in a later section.

## The Standard Input-Output Model

We start with an economy described in terms of its consumption pattern and its production technologies including resource requirements. We use the term resources to designate all factor inputs, including labor and capital as well as the natural resources that are our focus of attention. For simplicity, we assume that all domestic final demand corresponds to consumption, leaving the subjects of trade and investment to the concluding section. We also defer until the final section discussion of recycling sectors.

The vector $\mathbf{y}$ quantifies the consumption demand for each good, and two matrices of technical coefficients, $\mathbf{A}$ and $\mathbf{F}$, quantify intermediate inputs and resource requirements, respectively, per unit of output. Assuming values for these under some scenario, values are obtained for the vectors $\mathbf{x}$ of total output

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}, \tag{1}$$

and $\varphi$ of resources

$$\begin{aligned}\varphi &= \mathbf{F}\mathbf{x} \\ &= \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}.\end{aligned} \tag{2}$$

The vector $\varphi$ can be disaggregated to a matrix that describes the use of resources throughout the economy in two complementary ways,

$$\mathbf{\Phi} = \mathbf{F}\hat{\mathbf{x}} \tag{3}$$

and

$$\tilde{\mathbf{\Phi}} = \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{y}}\,, \tag{4}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are diagonal matrices based on **x** and **y** respectively. $\Phi$ describes the distributions of resources among total outputs while $\tilde{\mathbf{\Phi}}$ describes the distributions among consumption products. The vectors of row totals of (3) and (4) are equal to each other and to $\varphi$, while column entries distinguish individual resources. Columns cannot be added in either matrix as resources (in the numerators of the elements of **F**) will in general be measured in different units. Scenarios specifying changes in **y**, **A**, or **F** will produce new solution vectors, **x** and $\varphi$.

## Absorbing Markov Chains

For any system represented by n states, the parameters of an AMC are the probabilities of directly transitioning from one state to another; they are contained in an n x n transition matrix **M**. The entries in the $i^{th}$ row of **M** describe the likelihood of transitioning from state i to each other state, such that the row sum equals 1.0. Transition probabilities may also be interpreted in a non-stochastic manner (Kemeny and Snell, 1976, p 206). In our AMC, states represent resources, intermediate products, and consumption goods, and $m_{ij}$ is the proportion of the $i^{th}$ resource or product outflow going directly to state j. The results of the IO model, **x** and $\varphi$**,** as well as **A**, **F**, and **y**, are used to calculate a transition flow table, and the transition matrix is derived by taking row-wise percentages. (Thus the row for steel, for example, would show the portion of steel output delivered to construction, to the automobile industry*, etc.*) The AMC is a specific type of Markov chain containing absorbing states, which once entered cannot be exited, and configured such that one of these absorbing states must eventually be entered. Consumption goods are treated here as absorbing states, while resources and intermediate products are transient states.

### *The Fundamental Matrix*

The **M** matrix is put into canonical form with the transient states first and can be partitioned as follows:

$$\mathbf{M} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \tag{5}$$

The proportions of direct flows among the transient states (resources and intermediate products) are represented by the matrix designated **Q**, and those from transient to absorbing states (products to consumers) by the matrix **R**. If state i is an absorbing state, $m_{ij} = 0$ for $j \neq i$, and $m_{ii} = 1.0$, since the consumer goods by definition will not re-enter the economy. The **0** matrix and the identity matrix **I** (in the 2nd row of **M**) reflect these properties. Note that each row sum of **M** equals 1.0.

The analysis proceeds by calculating two key matrices. The *fundamental matrix* of an absorbing chain (Kemeny and Snell, 1976), **N**, is defined as

$$\mathbf{N} = \mathbf{I} + \mathbf{Q} + \mathbf{Q}^2 + \ \ldots \quad = (\mathbf{I} - \mathbf{Q})^{-1}. \tag{6}$$

Like the Leontief inverse matrix, $(\mathbf{I} - \mathbf{A})^{-1}$, the fundamental matrix of an AMC captures indirect as well as direct effects, as they are both examples of the principle of transitive closure (as observed by Harary *et al.*, 1965; Patten and Finn, 1979).

**N** is of particular interest for the analysis of resource-specific networks because its ij$^{th}$ element is the average number of times transient state j is encountered in transitioning from transient state i to an absorbing state, for example the average number of times a ton of crude iron passes through a transportation sector (embodied in some product) before it ends up in a consumer good. The resource-specific network for resource i consists of all the paths taken by any portion of that resource embodied in products. Each path has a length, the number of transitions from one state to another, or branches, that it contains. The *average* path length for resource i, $\sigma_i$ , is the sum over all paths of each path's length multiplied by the portion of resource it carries. From the description of **N** above, and noting that every time a transient state is encountered and then exited a branch must be transitioned, $\sigma_i$ can be conveniently computed as the i$^{th}$ row sum of **N**.

The other matrix of interest in an AMC analysis is **B**, where

$$\mathbf{B} = \mathbf{NR}, \tag{7}$$

where **R** contains the direct proportions of flows from each transient state to each absorbing state (i.e., from an intermediate product to a consumer good). **B** represents the ultimate distribution of every embodied resource (and every embodied intermediate product) among the products comprising the basket of goods delivered to consumers, and its row sums are equal to 1.0.

### ***Resource-Specific Networks***

The structure of a resource-specific network, composed of nodes joined by branches, determines the paths through the network traversed by some portion of the resource. Whole paths or individual nodes and branches can be characterized using the **Q**, **R, N** and **B** matrices. For example, note that $n_{ij}q_{jk}$ is the average number of times that the branch *jk*, where *j* and *k* are intermediate products, is traversed by the embodied resource *i* before it ends up in a consumption product, and $n_{ij}r_{jm}$ is the average number of transitions by embodied resource i of branch *jm* where *m* is a particular consumption good. Therefore, if

$\hat{\mathbf{n}}_\mathbf{i}$ is the diagonal matrix formed from the $i^{th}$ row of **N,** the average number of times that a unit of embodied resource i makes these particular transitions can be determined by calculating $\hat{\mathbf{n}}_\mathbf{i}\mathbf{Q}$ and $\hat{\mathbf{n}}_\mathbf{i}\mathbf{R}$. Particular transitions may be of interest for various reasons, for example to associate a geographic distance or a transport cost with them. Alternatively, comparing 2 resource-specific networks, possibly for the same raw material from two different sources, may be revealing.

It is also possible to describe the network for a given resource i that is eventually embodied in a specific consumer product j. For example, $q_{ij}b_{jk}$ is the portion of resource i that is ultimately embodied in consumer good k if it first is embodied in intermediate product j, and

$$q_{ij/k} = \frac{q_{ij}b_{jk}}{b_{ik}} \tag{8}$$

(derived in Kemeny and Snell, 1976) is that fraction of the flow of resource i ultimately embodied in consumer good k that first is directly embodied in intermediate product j, say the proportion of the coal eventually embodied in consumer cars that is first embodied in steel. Eq. 8 defines an element of the matrix

$$\mathbf{Q_k} = \hat{\mathbf{b}}_\mathbf{k}^{-1}\mathbf{Q}\mathbf{b_k}, \tag{9}$$

where the vector $\mathbf{b_k}$ is the $k^{th}$ column of **B**, measuring the portions of all resources that end up embodied in consumer good k. The corresponding fundamental matrix is

$$\begin{aligned}\mathbf{N_k} &= (\mathbf{I} - \mathbf{Q_k})^{-1} \\ &= \mathbf{b}_\mathbf{k}^{-1}\mathbf{N}\mathbf{b_k}\end{aligned} \tag{10}$$

with elements

$$n_{ij/k} = \frac{b_{jk}}{b_{ik}} n_{ij}. \tag{11}$$

The row sums of $\mathbf{N_k}$ form the vector $\boldsymbol{\sigma}_\mathbf{k}$, and its $i^{th}$ element provides the relevant measure for the $i^{th}$ resource and $k^{th}$ consumer product. If, for example, 20% of raw coal ends up embodied in consumer automobiles ($b_{jk}$) but only 5% of steel ($b_{ik}$) does, then the coal embodied in cars on average passes through the steel sector 4 times ($b_{jk}/b_{ik}$) as often as the average ton of coal.

The **Q**, **R**, **N** and **B** matrices are as central to AMC analysis as are **A**, **F**, $\mathbf{(I - A)^{-1}}$, and $\mathbf{\Phi}$ to IO models, and we next demonstrate the basic relationships between them.

### *The AMC Model in IO Notation*

We first define a matrix of inter-industry product flows denoted (using the notation of Suh 2005) as

$$\overline{\mathbf{A}} = \hat{\mathbf{x}}^{-1}\mathbf{X} \tag{12}$$

with coefficients $\bar{a}_{ij} = x_{ij}/x_i$, where **X**, the intersectoral flow matrix, is

$$\mathbf{X} = \mathbf{A}\hat{\mathbf{x}} \tag{13}$$

and $\hat{\mathbf{x}}$ is the diagonal matrix formed from **x.**

We also define

$$\overline{\mathbf{F}} = \hat{\boldsymbol{\varphi}}^{-1}\boldsymbol{\Phi} \tag{14}$$

with coefficients $\bar{f}_{rj} = \varphi_{rj}/\varphi_r$, and right side matrices as defined in (2) and (3). Both $\overline{\mathbf{A}}$ and $\overline{\mathbf{F}}$ are matrices of row-wise coefficients, by contrast with **A** and **F**. $\overline{\mathbf{A}}$ is called the Ghosh matrix to distinguish it from **A**, the Leontief matrix, to which it related to as

$$\begin{aligned} \overline{\mathbf{A}} &= \hat{\mathbf{x}}^{-1}\mathbf{X} \\ &= \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}} \end{aligned} \tag{15}$$

$\overline{\mathbf{F}}$, a new construction, is defined similarly relative to **F,** that is

$$\begin{aligned} \overline{\mathbf{F}} &= \hat{\boldsymbol{\varphi}}^{-1}\boldsymbol{\Phi} \\ &= \hat{\boldsymbol{\varphi}}^{-1}\mathbf{F}\hat{\mathbf{x}}. \end{aligned} \tag{16}$$

While $\overline{\mathbf{A}}$ and $\overline{\mathbf{F}}$ do not enter into the standard IO model, their definition is readily understood in an IO framework. The AMC can be rewritten in terms of these 2 matrices.

Ordering the transient states such that the resources precede the products reveals that both **R** and **Q** have well-defined, highly-sparse structures when applied to the industrial system. **R** measures in its rows the distribution of first the resources and then the products directly to consumers. Since resources are delivered only to extraction sectors, the top block of rows of **R** consists of zeroes. The lower block must be a diagonal matrix with the portions of output delivered directly to consumers down the diagonal. Thus $\mathbf{R}_x$ is defined in the following way:

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} \\ \hat{\mathbf{x}}^{-1}\hat{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{R}_x \end{bmatrix}. \tag{17}$$

**Q** must contain in each row the portions of the corresponding resource or product delivered directly as input to each resource or product. The two left-hand quadrants of **Q** must consist of all zeroes, as neither resources nor intermediate products flow to resources. The top right-hand quadrant shows the distribution of resources directly to extraction sectors, and thus is the matrix we called $\overline{\mathbf{F}}$, while the sub-matrix in the lower right represents the distribution of intermediate products, or $\overline{\mathbf{A}}$:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0} & \overline{\mathbf{F}} \\ \mathbf{0} & \overline{\mathbf{A}} \end{bmatrix}. \tag{18}$$

From **R** and **Q** so transcribed, we can determine

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1} = \begin{bmatrix} \mathbf{N}_{\varphi\varphi} & \mathbf{N}_{\varphi x} \\ \mathbf{N}_{x\varphi} & \mathbf{N}_{xx} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \overline{\mathbf{F}}(\mathbf{I} - \overline{\mathbf{A}})^{-1} \\ \mathbf{0} & (\mathbf{I} - \overline{\mathbf{A}})^{-1} \end{bmatrix}, \tag{19}$$

where **N** is a simple example, due to the sparseness of **Q**, of a Miyazawa (1976) inverse, and

$$\mathbf{B} = \mathbf{NR} = \begin{bmatrix} \mathbf{B}_{\varphi} \\ \mathbf{B}_{x} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{\varphi x}\mathbf{R}_{x} \\ \mathbf{N}_{xx}\mathbf{R}_{x} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{F}}(\mathbf{I} - \overline{\mathbf{A}})^{-1}\hat{\mathbf{x}}^{-1}\hat{\mathbf{y}} \\ (\mathbf{I} - \overline{\mathbf{A}})^{-1}\hat{\mathbf{x}}^{-1}\hat{\mathbf{y}} \end{bmatrix}. \tag{20}$$

We can readily rewrite $\tilde{\mathbf{\Phi}}$ (from Eq. 4) in terms of the $\mathbf{B}_{\varphi}$ matrix. Making use of Eq. (16) and (20), we obtain

$$\tilde{\mathbf{\Phi}} = \hat{\boldsymbol{\varphi}}\mathbf{B}_{\varphi} = \hat{\boldsymbol{\varphi}}\overline{\mathbf{F}}\mathbf{B}_{\mathbf{x}} = \mathbf{\Phi}\mathbf{B}_{\mathbf{x}}. \tag{21}$$

Thus we see that $\mathbf{B}_{x}$, the AMC matrix that distributes products to consumers, is also precisely the object that governs the transformation between the more familiar matrices $\mathbf{\Phi}$ and $\tilde{\mathbf{\Phi}}$ from IO Eq. (3) and (4). Likewise, $\mathbf{B}_{\varphi}$ distributes resources among consumer products.

For further insight into **N** note that from Eq. (20)

$$\mathbf{B}_{\varphi} = \mathbf{N}_{\varphi x}\, \mathbf{R}_{x} \text{ and } \mathbf{Bx} = \mathbf{N}_{xx}\, \mathbf{R}_{x}. \tag{22}$$

Let $b_{ij}$ and $n_{ij}$ represent the typical elements of $\mathbf{B}_{\varphi}$ (or $\mathbf{B}_{x}$) and $\mathbf{N}_{\varphi x}$ (or $\mathbf{N}_{xx}$), respectively. Then

$$b_{ij} = n_{ij}\frac{y_j}{x_j} \tag{23}$$

or

$$n_{ij} = b_{ij}\frac{x_j}{y_j}. \tag{24}$$

Each element of **B**, $b_{ij}$ , represents the fraction of resource i that is ultimately embodied in consumer product j, and $x_j/y_j$, is the output multiplier for industry j. Thus $n_{ij}$ is the product of two well-defined quantities, and the AMC row sum,

$$\sigma_i = \sum_j n_{ij} = \; = \sum_j b_{ij} \frac{x_j}{y_j} , \tag{25}$$

known to be the average path length of embodied resource i before it is ends up in a consumer good, can be expressed as the average output multiplier for products weighted by the fraction of i embodied in each consumer product. An increase in output multiplier, implying that a greater portion of output is retained within the production network, leads to an increase in average path length. This will be illustrated in a numerical example.

## Application of the Absorbing Markov Chain

We next carry out computations for a baseline and two alternative scenarios using an IO model of an economy with three resources and three products. A network model of the economy is depicted in Figure 1 in terms of the algebraic notation. Each branch has a weight determined by the coefficients of $\overline{\mathbf{F}}$ (distributing resources among sectors), $\overline{\mathbf{A}}$ (distributing intermediate products), and **R** (distributing products to consumers):

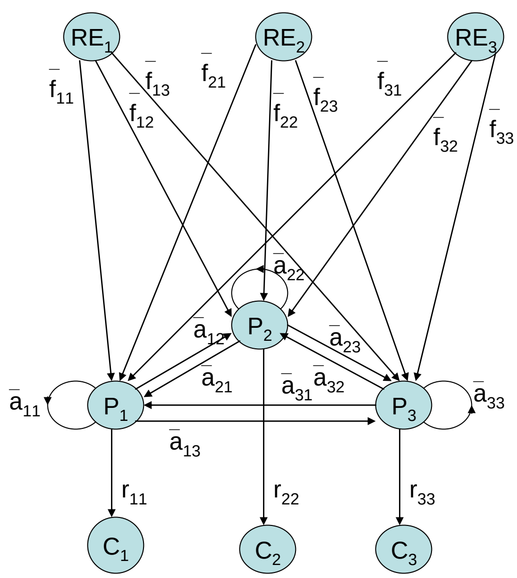


**Figure 1: AMC Coefficients for Resources and Products and Consumption Demand of Products In a Three-Product, Three-Resource Economy**

The first scenario assumes changes in consumer demand, **y**, while the second represents changes in the production technologies represented in **A** and **F**. Each calculation yields values for $\mathbf{x}$ and $\varphi$, from which new versions of $\overline{\mathbf{A}}$, $\overline{\mathbf{F}}$, and $\mathbf{R}$ are calculated. The AMC results for the scenarios are then compared.

***Baseline Scenario***

Consider an economy with three resources (RE), three intermediate products (P), and three consumption goods (C)

| | |
|---|---|
| $P_1$, $C_1$ agriculture | $RE_1$ labor |
| $P_2$, $C_2$ manufacturing | $RE_2$ ore |
| $P_3$, $C_3$ mining | $RE_3$ land |

and the following values for the baseline scenario:

$$\mathbf{y} = \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} 0.05 & 0.15 & 0 \\ 0.10 & 0.50 & 0.20 \\ 0 & 0.30 & 0.05 \end{bmatrix}, \text{ and } \mathbf{F} = \begin{bmatrix} 1.50 & 0.50 & 0.25 \\ 0 & 0 & 3.00 \\ 2.00 & 0 & 0.50 \end{bmatrix}.$$

Comparing columns of **A** shows that manufacturing (the sector producing the second product) is much more interconnected than the other sectors, and according to **F** all sectors use labor while only the mining sector inputs crude ore and only manufacturing requires no land.

The IO model applied to these data yields these results

$$\mathbf{x} = \begin{bmatrix} 18.5 \\ 50.5 \\ 17.0 \end{bmatrix} \text{ and } \boldsymbol{\varphi} = \begin{bmatrix} 57.25 \\ 51.0 \\ 45.5 \end{bmatrix},$$

which are used with the original data to compute $\overline{\mathbf{A}}$ and $\overline{\mathbf{F}}$ as well as **Q**, **R**, **N** and **B**.

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & | & 0.485 & 0.441 & 0.074 \\ 0 & 0 & 0 & | & 0 & 0 & 1.0 \\ 0 & 0 & 0 & | & 0.813 & 0 & 0.187 \\ -- & -- & -- & | & -- & -- & -- \\ 0 & 0 & 0 & | & 0.05 & 0.409 & 0 \\ 0 & 0 & 0 & | & 0.037 & 0.5 & 0.067 \\ 0 & 0 & 0 & | & 0 & 0.891 & 0.05 \end{bmatrix}, \ \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -- & -- & -- \\ 0.541 & 0 & 0 \\ 0 & 0.396 & 0 \\ 0 & 0 & 0.059 \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & | & 0.576 & 1.709 & 0.199 \\ 0 & 1 & 0 & | & 0.086 & 2.228 & 1.211 \\ 0 & 0 & 1 & | & 0.904 & 1.249 & 0.285 \\ -- & -- & -- & | & -- & -- & -- \\ 0 & 0 & 0 & | & 1.092 & 1.024 & 0.073 \\ 0 & 0 & 0 & | & 0.092 & 2.375 & 0.168 \\ 0 & 0 & 0 & | & 0.086 & 2.228 & 1.211 \end{bmatrix}, \boldsymbol{\sigma} = \begin{bmatrix} 3.484 \\ 4.524 \\ 3.438 \\ -- \\ 2.188 \\ 2.635 \\ 3.524 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0.311 & 0.677 & 0.012 \\ 0.046 & 0.882 & 0.071 \\ 0.489 & 0.495 & 0.017 \\ -- & -- & -- \\ 0.590 & 0.405 & 0.004 \\ 0.050 & 0.941 & 0.010 \\ 0.046 & 0.882 & 0.071 \end{bmatrix}.$$

The 6 rows and columns of **Q** and **N**, as well as the six rows of the **R** and **B** matrices, correspond to the three resources and the three intermediate products. The three columns of the **R** and **B** matrices correspond to the consumption demand for products. The rows of **B** indicate the portion of each resource and each intermediate product that ultimately is utilized in satisfying consumer demand for each product. The first column shows that 31.1% of labor but only 4.6% of ore goes toward satisfying the consumer demand for agricultural consumer products.

Figure 2 illustrates the resource-specific network for ore ($RE_2$) using values taken from the **N, Q,** and **R** matrices. In this figure each branch weight measures the number of times an average unit of ore traverses that branch. The node weights (underlined italics) sum to 4.524, as do the branch weights. Figure 2 indicates that the primary branches utilized by flows of embodied ore are $RE_2$ to $P_3$, $P_3$ to $P_2$, $P_2$ to $P_2$ and $P_2$ to $C_2$ -- ore from mining to manufacturing to manufactured consumer goods -- contributing 4.075/4.524 or 90% of the overall average path length for ore. Figure 2 also shows that 2.228/4.524 = 49% of the average path length for embodied ore is associated with node $P_2$, manufacturing. We could also quantify the network and paths for only that ore that is eventually embodied in manufactured consumer goods.

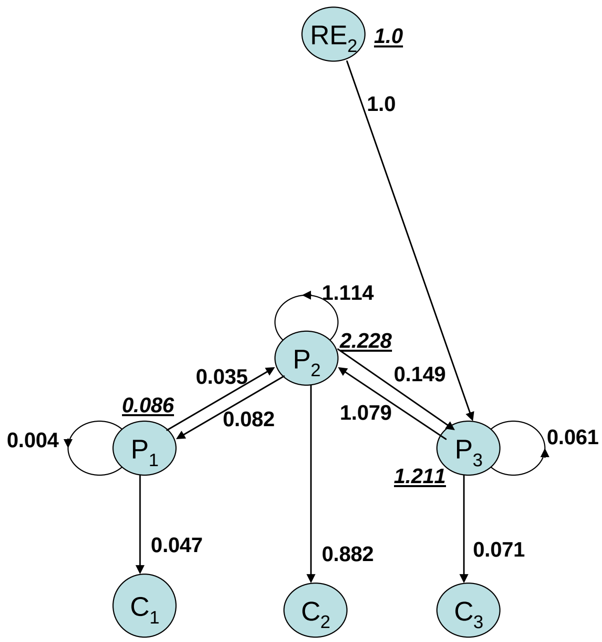


**Figure 2: Resource-Specific Network for Embodied Ore Showing Average Number of Times a Unit of Embodied Ore Enters or Exits Each Transient Node and Traverses Each Branch**

### *Alternative Scenarios*

Now we assume that consumer demand for manufactured goods quadruples to 80 units, compute new output levels and resource requirements (**x** and **φ**) using the IO model, compute the AMC transition table, and then calculate new **Q**, **R**, **N** and **B** matrices. As a result the vector of average path lengths is $\boldsymbol{\sigma} = [3.816 \;\; 4.597 \;\; 4.147 \mid 2.977 \;\; 2.589 \;\; 3.597]^T$, showing that all average path lengths increase relative to the baseline except for that of manufactured goods. This exception is explained by the fact that a larger share of manufactured goods is going directly to consumers. The branches $RE_2$ to $P_3$, $P_3$ to $P_2$, $P_2$ to $P_2$ and $P_2$ to $C_2$ now constitute 4.265/4.597 = 93% of the average path length for ore, and passing through node $P_2$ (manufacturing) represents 51% of that average path length, reflecting the increased amount of manufacturing activity.

Finally, assume all inputs to mining double except for ore, due to decreased ore quality. Computing the AMC for this scenario results in $\boldsymbol{\sigma} = [4.232 \;\; 5.330 \;\; 4.252 \mid 2.678 \;\; 3.394 \;\; 4.330]^T$, an increase in path lengths by an average of nearly 25% relative to the baseline. The decline in mining productivity requires that more work be done to satisfy the same consumption demand, which increases the amount of interdependency.

These examples demonstrate the mechanics of applying an AMC to results obtained with the IO model. The impacts naturally depend on the numerical values of the parameters and exogenous variables, but it will also be possible to state propositions and theorems generalizing some of the results independent of specific numerical values. In particular, reduced efficiency requires more work and therefore longer average path lengths, while increased consumer demand for a product shortens the average path length for delivering that product while increasing that for other products. The questions that can be addressed are much more interesting, of course, when the scope of the model is broader and the relation of output to demand is not linear, the subject of the concluding section.

## The AMC, the Ghosh Model, and Structural Path Analysis

The Ghosh matrix, $\overline{\mathbf{A}}$, the row-based matrix calculated from an IO flow table, describes the allocation of demand for a particular good across all using sectors and is used in the Ghosh model (Ghosh 1958), $\mathbf{x^T(I-\overline{A}) = v^T}$, or

$$\mathbf{x^T = v^T(I-\overline{A})^{-1}}, \tag{26}$$

where $\mathbf{v}$ is the vector of value-added, or the total money value of all resource inputs, $\mathbf{x}$ is the vector of total output in money values, and the right side matrix is the Ghosh inverse, the AMC matrix $\mathbf{N_{xx}}$ (see Eq. 19). The Markovian interpretation of $\mathbf{N_{xx}}$ naturally holds in the special case when all variables are measured in a common unit, here money value: in Eq. 26, $x_i$ is the sum of all sectors' resource costs, $v_j$, each weighted by the average number of times that sector's embodied output is used by sector i, $(n_{xx})_{ji}$ for sector j. However, the AMC provides more a general in that the interpretation of $\mathbf{N_{xx}}$ is not limited to cases where all products are measured in a single unit.

$\overline{\mathbf{F}}$ was not utilized by Ghosh, and neither it nor its product with the Ghosh inverse, $\mathbf{N_{\varphi x}}$ (see Eq. 19), has ever to our knowledge been used in either economic or ecosystem analysis. We have seen that $\mathbf{N_{\varphi x}}$ has an analogous interpretation to $\mathbf{N_{xx}}$: the $i^{th}$ row sum of $\mathbf{N_{\varphi x}}$ (minus 1.0, for its transition from the state of resource into the product of the relevant resource-extraction industry) measures the average number of transactions involving transitions from one sector to another by a unit of embodied resource i, for any number of resources measured in any units.

Suh (2005) recognized the use of both Ghosh and Leontief matrices in the ecological studies and the relationship of the former to AMCs. He also pointed out both the limitations of requiring all variables to be measured in a single unit and that SPA has been applied to the Ghosh inverse in ecosystem analysis, although not formalized by that name (Suh 2005, Suh and Kagawa 2005). SPA includes a number of different algorithms to identify the most influential paths by pruning less important ones and has been applied to decomposing the Leontief inverse to identify inputs with the greatest impact on the money value of a consumer good. In this paper the AMC characterizes paths downstream from resources to consumer products along resource-specific networks, and characterizing the transitions and path lengths for the entire network or individual paths, nodes, and branches. Of course, it would be possible to apply one or more of the existing SPA algorithms to the Markov $\mathbf{N}$ matrix, and some might wish to further extend the meaning of SPA to include the ways in which we have analyzed the $\mathbf{N}$ matrix.

## Next Steps

Industrial ecologists have expressed interest in analyzing scenarios about closing the material loops in an industrial system using IO models (MacLean et *al*. 2009), and the AMC analysis can be applied to more elaborated models than the standard IO framework used in our illustrative numerical example. We mention the 3 most important elaborations.

First, waste-processing sectors need to be incorporated into the IO model. This has been accomplished for the static, one-region case by the Waste Input-Output model of Nakamura *et al*. (2002, 2005, 2007, and 2009). Second, a model of the world economy is required since material cycles are global in scope. The World Trade Model (Duchin 2005, Stromman and Duchin 2006) can play this role but requires waste-processing sectors and dynamics. Third, a dynamic model is called for to characterize both landfills and the stocks of secondary sources of resources. A dynamic IO model exists (Duchin and Szyld 1998) with some but not all of the necessary features; it is for a single region and without waste-processing sectors.

The intention is to incorporate waste-processing sectors and stocks of durable goods and of infrastructure with an IO model of the world economy for scenario analysis, taking account of both resource requirements (embodied resources) and resources physically incorporated in products and potentially available for recovery and representing landfills (rather than delivery to consumers) as absorbing states. The AMC for this extended framework would then be analyzed to describe global, resource-specific networks. One could compare networks for different resources, or a single resource from different sources (such as iron from China and iron from Brazil), with regard to passage through waste-processing sectors and landfills in different regions of the world under alternative assumptions about technologies, consumer demand, and parameters governing the processing of wastes. For example, average path lengths over selected branches could be combined with external information, such as distances between regions, to determine the total distance traveled by a resource contained in internationally traded products between specific regions.

Many steps are needed to achieve the full integration of the models identified, but two next ones may be indicated: (1) applying an AMC analysis to scenario outcomes from the World Trade Model to examine the paths and path lengths of embodied resources across geographic regions and (2) introducing waste-processing sectors into this model. Increasingly detailed, environmentally-extended IO databases for past years also need to be compiled as a point of departure for building scenarios about the future, and fortunately that work, also, is in progress (see Tukker *et al*. 2009).

**Acknowledgments**

We are grateful for the detailed comments of three anonymous reviewers, which helped us clarify the motivation and exposition in this article.

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