Public Pension Reform in Japan

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Abstract: This paper aims to establish guidelines for public pension reform in Japan, using a numerical simulation approach. The paper introduces the example of a minimum guaranteed pension in the Swedish pension system and compares this with the basic pension in Japan’s public pension system, with regard to methods of income redistribution through a public pension scheme. Simulation results show that the switch from the basic pension to the guaranteed pension does not always generate favorable results. If we consider a public pension program with the same scale as the current Japanese program, the highest level of social welfare is attained when a public pension system consists of only a basic pension and is financed by a consumption tax.

I. INTRODUCTION

With a population that is rapidly aging, Japan faces serious public finance problems, particularly when it comes to tax and social security issues. Structural reforms are urgently needed to accommodate the impending demographic change. In particular, the sustainability of the public pension system is an important problem, and thus a reform of the public pension program was implemented in 2004 in Japan. However, this reform seems to be far from a radical reform. Hence, it was unable to dispel completely the suspicion with which the people view the public pension scheme. The necessity for a more drastic reform of the scheme is now becoming obvious.

The new Swedish pension system has attracted attention around world as a good example for pension reform. The public pension program in Sweden was drastically reformed in 1999. The program includes a fixed contribution rate, a one-to-one relation between contributions paid and pension credit awarded, and an automatic balancing mechanism for pension benefits to take account of changes in economic growth rates and life expectancy. Thus, the system offers many interesting suggestions for dealing with the problems of intergenerational disparity and for maintaining sound pension finance. The current Japanese public pension scheme has already partially adopted the Swedish pension system. For instance, a fixed contribution program and a mechanism for the automatic adjustment of benefits were newly introduced by Japan’s 2004 pension reform.

Although the Swedish pension system has many characteristics, this paper focuses on the fact that the system consists of an earnings-related pension and a minimum guaranteed pension. The earnings-related pension scheme has a one-to-one relation between contributions paid and pension credit awarded. The guaranteed pension is paid only to those with a low earnings-related pension benefit, and it is financed not by contributions but general tax revenue. In the current Japanese public pension program, the basic pension performs the function of income redistribution. This paper investigates whether it would enhance overall social welfare in Japan to switch from the current basic pension paid to everyone, rich and poor, to the minimum guaranteed pension adopted in Sweden.

Intuitively, one would expect the move to the earnings-related pension with a minimum guaranteed pension to promote economic growth, because the absence of income redistribution for the medium and high-income classes in this scheme is likely to stimulate the labor supply. In other words, the switch to the Sweden-type system is likely to enhance overall economic efficiency. Moreover, income redistribution may be implemented with greater equity under a minimum guaranteed pension than under a basic pension, because the guaranteed pension is paid only to the low-income class.

This paper examines whether the move from the current Japanese pension system to the Sweden-type system improves social welfare. Also, the paper compares a basic pension and a minimum guaranteed pension as methods of income redistribution. It also investigates the effects of different payment levels of the basic pension and the guaranteed pension, and examines different financial methods to adjust the accompanying changes in tax transfer to the pension sector. Furthermore, it explores what would make a desirable public pension scheme in Japan, and thus presents a concrete policy suggestion for pension reforms.

To analyze the problem, this paper looks at the Japanese tax and social security systems using an extended life-cycle general equilibrium model. Many papers have studied tax reforms using this kind of model; for instance, Auerbach and Kotlikoff (1983a, 1983b, 1987), Seidman (1983), Auerbach et al. (1989), Altig et al. (2001), Homma et al. (1987), and Ihori et al. (2006). Nearly all of them, however, have concentrated on analyzing the effects of an aging population on production and consumption, and thus on economic growth; but when dealing with pension reforms it is vital to evaluate not only efficiency but also equity.

This paper incorporates three representative households with different earning abilities in a life-cycle model of overlapping generations with an elastic labor supply. This enables us to examine equity issues in addition to efficiency issues. Thus, we are able to present some comprehensive and useful guidelines for pension reform. The macroeconomic and welfare effects of alternative pension policies are evaluated in the steady state with the 2005 age structure in Japan.

This paper is organized as follows. The next section identifies the basic model employed in the simulation analysis. Section 3 explains the method of simulation analysis and the assumptions adopted. Section 4 evaluates the simulation findings and discusses policy implications. Section 5 summarizes and concludes the paper.

Okamoto (2005a) also incorporated three representative households with different earning abilities. Altig et al. (2001) dealt with differences of lifetime earnings ability by incorporating 12 lifetime-income groups into a life-cycle model. Furthermore, Okamoto (2005b) introduced numerous representative households with continuous income distribution in each cohort.
II. THEORETICAL FRAMEWORK

The life-cycle growth model employed in this paper is grounded in the microeconomics of intertemporal choice, and the macroeconomics of savings and growth. The simulation model has three features. First, aggregate assets of the economy in each period consist of the assets of different generations that maximize their lifetime utility. This allows us to rigorously analyze changes in the supply of assets caused by demographic changes. Second, assets in the capital market, where aggregate assets appear as real capital, affect the production level. Third, it is possible to estimate realistic consumption-savings profiles for the elderly, by incorporating life-length uncertainty and unintended bequests into the model.

We calibrate the simulation of the Japanese economy by employing population data estimated by the National Institute of Population and Social Security Research in 2002. The model has 75 different overlapping generations. Three types of agents are considered: households, firms, and the government. The basic structure of households is as follows.

2.1 Household Behaviour

Households are divided into three income classes: low, medium, and high. A single household type represents each income class. Each household has the same mortality rate and the same utility function. Unequal labor endowments, however, create different income levels. Each household appears in the economy as a decision-making unit at the age of 21 and lives to a maximum of 95. Households face an age-dependent probability of death. Let $q_{j+1|j}$ be the conditional probability that a household of age $j$ lives to $j + 1$. Then the probability of a household of age 21 surviving until $s$ can be expressed by

$$p_s = \prod_{j=21}^{s-1} q_{j+1|j}.$$ (1)

The probability $q_{j+1|j}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2002).

The utility of each household depends on the levels of consumption and leisure. Each household works from age 21 to a maximum of $RE$, the compulsory retirement age. The labor supply is elastic but zero after (voluntary or compulsory) retirement. Each household that maximizes the expected lifetime utility makes lifetime decisions at age 21, concerning the choice between leisure and labor supply and the allocation of wealth between consumption and savings. The utility function of a representative household of income class $i$, the form of which is assumed to be time-separable, is

$$U^i = \frac{1}{1 - \delta} \sum_{s=21}^{95} p_s (1 + \delta)^{-(s-21)} \left\{ \left( C_s^i \right)^{1-\gamma} \frac{1}{p} + \phi \left( l_s^i \right)^{1-\gamma} \frac{1}{p} \right\}^{1-\frac{1}{\gamma}},$$ (2)

where $C_s^i$ represents consumption (or expenditure) at age $s$, $l_s^i$ leisure at age $s$, $\phi$ the utility weight on leisure, $\delta$ the adjustment coefficient for discounting the future, $\gamma$ the intertemporal elasticity of substitution in the consumption/leisure composite, and $p$ is the intratemporal
elasticity of substitution between consumption and leisure. The superscript \(i = l, m, h\) stands for low, medium, and high-income classes, respectively.

The flow budget constraint equation for each household at age \(s\) is

\[
A^i_{s+1} = (1 + r(1 - \tau))A^i_s + [1 - \tau_w \{wx^i e_s(1 - l^i_s)\} - \tau_{p} ]wx^i e_s(1 - l^i_s)
\]

\[+ b^i_s(\{l^i_s \}_{s=21}^{RH}) + a^i_s(1 + \tau_c)C^i_s \]

(3)

where \(A^i_s\) represents the amount of assets held by the household at the beginning of age \(s\), \(r\) the interest rate, \(w\) the wage rate per efficiency unit of labor, and \(e^i_s\) is the age profile of earnings ability. \(1 - l^i_s\) is the amount of labor supply, \(b^i_s(\{l^i_s \}_{s=21}^{RH})\) is the amount of public pension benefit, and \(a^i_s\) is the amount of bequest to be inherited at age \(s\), \(\tau_w \{wx^i e_s(1 - l^i_s)\}\) is the tax rate on labor income, \(\tau_c\) that on consumption, \(\tau_{p}\) that on interest income, and \(\tau_{p}\) is the contribution rate to the public pension scheme. \(x^i\) is the weight coefficient corresponding to the different levels of labor endowments across the three income classes.

The tax system consists of labor income, interest income, consumption, and inheritance taxes. Labor income is progressively taxed. The progressive tax schedule is incorporated in the same manner as in Auerbach and Kotlikoff (1987). If the tax base is \(wx^i e_s(1 - l^i_s)\), we choose two parameters labeled \(\alpha\) and \(\beta\), and set the average tax rate \(\tau_w\) equal to \(\alpha + 0.5 \beta \{wx^i e_s(1 - l^i_s)\}\) for all values of \(wx^i e_s(1 - l^i_s)\). The corresponding marginal tax rate \(\tau_w\) is \(\alpha + \beta \{wx^i e_s(1 - l^i_s)\}\).

Setting \(\beta = 0\) amounts to proportional taxation. One may make the tax system more progressive, holding revenue constant, by increasing \(\beta\) and decreasing \(\alpha\) simultaneously. The symbol \(\tau_w \{wx^i e_s(1 - l^i_s)\}\) in equation (3) means that \(\tau_w\) is a function of \(wx^i e_s(1 - l^i_s)\). The tax systems on interest income, consumption, and inheritances are proportional.

The public pension program is assumed to be a pay-as-you-go system that is close to the current Japanese system. The program consists of the basic pension (i.e., the flat part) and a part proportional to the average annual income from labor for each household. Variables related to the program are represented by

\[
b^i_s(\{l^i_s \}_{s=21}^{RH}) = \begin{cases} f + \theta H^i(\{l^i_s \}_{s=21}^{RH}) & (s \geq ST) \\ 0 & (s < ST) \end{cases},
\]

(4)

where

\[
H^i(\{l^i_s \}_{s=21}^{RH}) = \frac{1}{RH - 20} \sum_{s=21}^{RH} w^i e_s(1 - l^i_s),
\]

(5)

the age at which each household starts to receive public pension benefit is \(ST\), the average annual income from labor for each income class is \(H^i(\{l^i_s \}_{s=21}^{RH})\), \(RH\) (21 \(\leq RH \leq RE\)) is the

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3 To estimate the age profile of earnings ability, \(e^i\), the following equation is employed:

\[
Q = a_0 + a_1 A + a_2 A^2 + a_3 L + a_4 L^2
\]

where \(Q\) denotes average monthly cash earnings, \(A\) age, and \(L\) the length of one’s service for all workers. Table 9 presents the parameter values estimated using data in the Ministry of Health, Labor and Welfare (2005a). Because bonuses account for a large part of earnings in Japan, monthly cash earnings used here contain bonuses.

4 We conduct simulations for \(RH = 21, 22 \ldots RE\). Then we choose the retirement age at which the utility of income class \(i\) is maximized.
voluntary retirement age,\(^4\) the basic pension benefit per representative household is \(f\), and the weight coefficient of the part proportional to \(H^i(t_u^{R i} | u \geq 21)\) is \(\theta\). Thus, \(b_i((t_u^{R i} | u \geq 21))\) reflects different earnings abilities across the three income classes. \(b_i((t_u^{R i} | u \geq 21))\) signifies that the amount of public pension benefit is a function of leisure, \(l_i^t\).

There are accidental bequests caused by uncertainty over the length of life. The bequests, which were held as assets by deceased households, are handed to surviving 50-year-old households. Therefore \(a^t_i\) is positive if and only if \(s = 50\), and otherwise is zero. The inheritance is transferred within the same income class. When \(BQ_i^t\) is the sum of bequests inherited by 50-year-old households at period \(t\), \(a^t_{50}\) is defined by

\[
a^t_{50} = \frac{(1 - \tau_h)BQ_i^t}{N_t p_{50}(1 + n)^{-29}},
\]

where

\[
BQ_i^t = N_t \sum_{s=21}^{95} (p_s - p_{s+1})(1 + n)^{-(s-21)}A_i^{s+1}.
\]

\(N_t\) is the number of new households entering the economy for each income class as decision-making units at period \(t\), \(n\) is the common (gross) growth rate of successive cohorts, and \(\tau_h\) is the tax rate on inheritances of bequests. In the steady state of a life-cycle growth model, the amount of inheritances received is linked to the age profile of assets chosen by each household.

When we consider the utility maximization problem over time for each income class, in addition to the flow budget constraint represented by equation (3), the following constraint is imposed:

\[
\begin{cases}
0 \leq l_i^t \leq 1 & (21 \leq s \leq RE) \\
l_i^t = 1 & (RE + 1 \leq s \leq 95)
\end{cases}
\]

This is a constraint that labor supply is nonnegative, and that each household invariably retires after the compulsory retirement age \(RE\).

Let us consider the case in which each household maximizes expected lifetime utility under two constraints. Each household maximizes equation (2) subject to equations (3) and (8) (see Appendix A.1). From the utility maximization problem, the equation expressing evolutions of the consumption/leisure composite over time for each household is characterized by

\[
V_i^t = \left(\frac{p_{t-1}}{p_t}\right) \left[1 + \frac{\delta}{1 + r(1 - \tau_h)}\right] V_i^{t-1},
\]

where

\[
V_i^t = \left\{ (C_i^t)^{\frac{1}{\rho}} + \phi(l_i^t)^{\frac{1}{\rho}} \right\}^{\frac{1}{1 - \rho}} \left\{ (C_i^t)^{\frac{1}{\rho}} \right\}^{\frac{1}{1 - \rho}}.
\]

If the initial level, \(V_i^{21}\), is specified, the level of each age, \(V_i^t\), can be derived from equation (9). If \(V_i^t\) is specified, the levels of consumption, \(C_i^t\), and leisure, \(l_i^t\), at each age are obtained. The amount of assets held by each household at each age can be obtained from equation (3).
expected lifetime utility of each household is derived from equation (2).

The social welfare function, which takes account of different earnings abilities and thus provides different levels of consumption and leisure, is given by

\[ SW = U^l + U^m + U^h. \]  

This function includes the aspects of both efficiency and equity. It is derived from a summation of the expected lifetime utilities at age 21 for the three income classes. When comparing steady states, it is not necessary to take account of the utilities of all overlapping generations existing at period \( t \). A comparison of the lifetime utility of a single cohort is sufficient, because our aim is to compare the welfare level among simulation cases with alternative pension policies. The social welfare function is of the “Benthamite type,” but depends greatly on the utility of the low-income class.\(^5\) It is maximized if all income classes have the same level of the consumption/leisure composite.

With regard to the basic structure of firms, a single production sector is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. See Appendix B for the basic structures of firms and the government, and market equilibrium conditions.

III. SIMULATION ANALYSIS

3.1 Method of Simulation

The simulation model presented in the previous section is solved under the hypothesis of perfect foresight by households that correctly anticipate the interest, wage, tax, and contribution rates. If the tax and public pension systems are determined, the model can be solved using the Gauss-Seidel method (see Auerbach and Kotlikoff (1987) for the computation process).

3.2 Simulation Cases

This paper examines the effects of alternative pension policies on efficiency and equity in the 2005 current steady state. With regard to establishing parameters in the simulation cases, we make the total amount of public pension benefit fixed across cases to eliminate the effects of different pension size on the simulation results.\(^6\) Tax revenue neutrality is also assumed in order to allow a clearer evaluation of the effects of alternative pension policies. In all the simulation cases in our study, we hold that tax revenue is constant. The method we use is as

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\(^5\) The model employs a simple utilitarian social welfare function based on the steady state utility for a single generation. The function is most sensitive to income changes for the low-income class due to the concavity of the underlying utility function. As the parameter of intertemporal elasticity of substitution, \( y \), is lower, it depends more on the welfare of the low-income class. It becomes Rawlsian if \( SW = U^l \).

\(^6\) Under the hypothesis of perfect foresight by households, as in our simulation model, the level of social welfare is higher in the case without a pension scheme than in the case with a pension scheme. This is because, in the absence of a pension program, households that maximize their lifetime utility save relatively more to cover living expenses after retirement. This promotes capital accumulation, and thus enhances national income. However, a pension system has a variety of advantages that cannot be evaluated by a model analysis with perfect foresight. For example, one of the merits of a pension system often cited is that one can make provision for various uncertainties in the future and thus obtain a sense of security.
follows. Government expenditure per representative household ($g$) is exogenously given and fixed. Because the scale of the population is the same across all cases, government expenditure ($G_t$) is also exogenous and constant.

Case A is the current benchmark for 2005. We choose parameter values that are realistic for the Japanese economy. The public pension program consists of a basic pension and an amount proportional to the average annual income from labor for each household. Along the lines of the current Japanese system, general tax revenue covers one third of the flat basic pension, and contributions cover both the remaining two thirds and the overall proportional part (see Figure 1 for diagrams of the public pension system in each simulation case).

If the Sweden-type public pension system is introduced, then the basic model presented in Section 2 and Appendix B is modified as follows (see Appendix A.2 for the utility maximization problem for each income class, under the Sweden-type pension scheme). In the Sweden-type pension program, a minimum guaranteed pension, $b$, is introduced instead of a basic pension. By a minimum guaranteed pension, the low-income class is guaranteed the same amount of public pension benefit as the medium-income class. General tax revenue covers the balance, $S_t$, between the earnings-related pension benefit of the low-income class and the guaranteed pension benefit. The amount of public pension benefit for each income class is given by

$$b_{i}(\{l_{u}^{RH_{i}}\}_{u=21}) = \begin{cases} b & (s \geq ST, i = l) \\ \theta H^{i}(\{l_{u}^{RH_{i}}\}_{u=21}) & (s \geq ST, i = m, h) \\ 0 & (s < ST) \end{cases}$$

where

$$b = \theta H^{m}(\{l_{u}^{RH_{i}}\}) \quad \text{and} \quad H^{i}(\{l_{u}^{RH_{i}}\}) = \frac{1}{RH_{i} - 20} \sum_{u=21}^{RH_{i}} w_{x}^{i} e_{x}(1 - t'_{x}).$$

The budget constraint of the narrower government sector at time $t$ is defined by

$$T_{t} = G_{t} + S_{t},$$

where

$$S_{t} = N_{t} \sum_{u=ST}^{95} p_{u}(1 + n)^{(s-21)}[b - \theta H^{i}(\{l_{u}^{RH_{i}}\})].$$

The budget constraint of the public pension sector at time $t$ is represented by

$$R_{t} = P_{t}.$$

In Cases B and C, the Sweden-type pension program is introduced. The switch from the current Japanese pension scheme (Case A) to the Sweden-type pension scheme (Cases B and C) changes the amount of the tax transferred from the narrower government sector to the pension sector. In Case B, the accompanied change in the tax revenue is adjusted by a consumption tax. In Case C, it is adjusted by an interest income tax (see Tables 1 and 2 for the setting and simulation results for Cases A, B, and C).

Next, in Cases B and C, we investigate the effects of the move from a minimum guaranteed pension to a flat basic pension. Here, we compare a guaranteed pension and a basic pension as methods of income redistribution through a public pension system. Keeping the weight coefficient, $\theta$ given to the average annual income from labor the same as in Cases B and C, the
Figure 1: Diagrams of Public Pension System in Each Simulation Case
switch from a guaranteed pension to a basic pension is implemented, which gives Cases B-1 and C-1, respectively. Additionally, in Cases B and C, a guaranteed pension is abolished, and thus there exists only an earnings-related pension, which yields Cases B-2 and C-2, respectively (see Tables 3 and 4 for the setting and simulation results for Cases B-1, B-2, C-1, and C-2).

In Cases B-1 and C-1 the size of a basic pension is chosen to maximize social welfare, which gives Cases B-1* and C-1*, respectively. Moreover, for the simplicity of discussion, this paper assumes that the minimum guaranteed pension benefit for the low-income class is equal to the earnings-related pension benefit for the medium-income class. We evaluate the effects of different levels of the guaranteed pension benefit on simulation results. In Cases B and C the amount of the guaranteed pension benefit is decreased to 80% of the amount of the earnings-related pension benefit for the medium-income class, which yields Cases B-3 and C-3, respectively (see Tables 5 and 6 for the setting and simulation results for Cases B-1*, B-3, C-1*, and C-3). The following eleven simulation cases are now considered.

1) **Case A (Benchmark of the 2005 current state)**
The tax system on labor income has a realistic progressiveness, with an average tax rate of 7.44%. Tax rates on consumption, interest income, and inheritances are 5%, 20%, and 10%, respectively. Along the lines of the current Japanese public pension system, the general tax revenue covers one third of the basic pension.

2) **Case B (Sweden-type pension system and a consumption tax)**
In Case A, a minimum guaranteed pension is introduced instead of a basic pension. The low-income class is guaranteed the same level of pension benefit as the earnings-related pension benefit for the medium-income class. The accompanying change in tax revenue is adjusted by a consumption tax.

3) **Case C (Sweden-type pension system and an interest income tax)**
In Case A, a minimum guaranteed pension is introduced instead of a basic pension. The low-income class is guaranteed the same level of pension benefit as the earnings-related pension benefit for the medium-income class. The accompanying change in tax revenue is adjusted by an interest income tax.

4) **Case B-1 (Basic pension and a consumption tax)**
In Case B, keeping the weight coefficient given to the average annual income from labor constant, a basic pension is incorporated instead of a minimum guaranteed pension. The accompanying change in tax revenue is adjusted by a consumption tax.

5) **Case C-1 (Basic pension and an interest income tax)**
In Case C, keeping the weight coefficient given to the average annual income from labor constant, a basic pension is incorporated instead of a minimum guaranteed pension. The accompanying change in tax revenue is adjusted by an interest income tax.

6) **Case B-2 (Earnings-related pension and a consumption tax)**
In Case B, a minimum guaranteed pension is abolished, and thus there exists only an earnings-related pension. The accompanying change in tax revenue is adjusted by a consumption tax.
7) Case C-2 (Earnings-related pension and an interest income tax)
In Case C, a minimum guaranteed pension is abolished, and thus there exists only an earnings-related pension. The accompanying change in tax revenue is adjusted by an interest income tax.

8) Case B-1* (Optimal size of basic pension and a consumption tax)
In Case B-1, the size of a basic pension is chosen to maximize social welfare. The accompanying change in tax revenue is adjusted by a consumption tax.

9) Case C-1* (Optimal size of basic pension and an interest income tax)
In Case C-1, the size of a basic pension is chosen to maximize social welfare. The accompanying change in tax revenue is adjusted by an interest income tax.

10) Case B-3 (Decreased guaranteed pension and a consumption tax)
In Case B, the amount of minimum guaranteed pension benefit is decreased. The low-income class is guaranteed 80% of the amount of the earnings-related pension benefit for the medium-income class. The accompanying change in tax revenue is adjusted by a consumption tax.

11) Case C-3 (Decreased guaranteed pension and an interest income tax)
In Case C, the amount of minimum guaranteed pension benefit is decreased. The low-income class is guaranteed 80% of the amount of the earnings-related pension benefit for the medium-income class. The accompanying change in tax revenue is adjusted by an interest income tax.

3.3 Specification of Parameters
This paper examines the implications of several pension policies for the Japanese economy through comparing steady states. We choose parameter values that are realistic for the economy. Therefore, the values of the economic variables in Case A, such as the ratio of capital to labor (K/L), are close to those that are suggested by the Economic and Social Research Institute (2005). Parameter values are assigned with reference to empirical research, Hatano and Yamada (2007) in which the values are estimated using Japanese data. The parameter values used in the benchmark simulation are given in Table 7.

First, survival probabilities ($p_s$) are calculated from the National Institute of Population and Social Security Research (2002). Our model makes no distinction by sex, and thus this study uses male–female average values for 2005. Based on this data, the percentage of the aged population (65 or over) in the total population (21 or over) is 24.90% in 2005. The common growth rate of successive cohorts ($n$) is chosen so that the percentage in the simulation equals the estimated value.

Second, we choose the utility function’s leisure intensity parameter such that, on average, the medium-income class devotes approximately 50% of the available time endowment (of 16 hours per day) to labor during working years (roughly ages 21-61) in Case A.

Third, the method of assigning the weight given to labor endowments for the three income classes is explained. Table 8 shows the data from the Ministry of Finance (2005). This table presents the effective tax rates of wageworkers on a national income tax and a residence tax, with regard to a couple with two children. In our model, the three representative households, namely, low, medium, and high-income classes, have different earnings abilities. Table 8 suggests
that each income class, which accounts for one third of the total population, corresponds to the representative household earning 5, 7, or 10 million yen, respectively, on an annual base. The weight on labor endowments for each income class corresponds to the ratio of its amount of earned income. The medium-income class is used as a yardstick, that is, \( x^m = 1; x^l \) and \( x^h \) are assigned to reflect different earnings abilities across the three income classes.

Fourth, the method of assigning the parameter values that determine tax progressivity on labor income, namely, \( \alpha \) and \( \beta \), is described. Table 8 presents the effective tax rate calculated from a national income tax and a residence tax for each income class. The parameters on labor income are chosen so that the effective tax rate for each income class in the simulation is close to the estimated value, and that the average tax rate on labor income is the value suggested by this data (7.44%).

Finally, the public pension benefits consist of a basic pension and a part proportional to the income from labor for each income class. Under the current Japanese system, the general tax revenue covers one third of the flat part, and contributions cover both the remaining two thirds and the overall part proportional to the income from labor. Hence, the ratio of the part covered by taxes (\( \mu \)) in the basic pension is assigned to 1/3 in Case A.\(^7\) The basic pension benefit per representative household (\( f \)) is chosen so that, for the medium-income class, the ratio of the earnings-related pension benefit to the basic pension benefit is 0.777 in Case A. This value is estimated using data from the Ministry of Health, Labor and Welfare (2005b). The weight coefficient (\( \theta \)) on the earnings-related pension is adjusted so that the contribution rate (\( \tau_p \)) in Case A equals the actual rate of 14.29% in employee pension plans (Kosei Nenkin) in 2005.

IV. SIMULATION RESULTS

4.1 Findings and Policy Implications

4.1.1 Sweden-Type Pension System

In Case A, the public pension program consists of a basic pension and an amount proportional to the average annual income from labor for each household. General tax revenue covers one third of the basic pension, and contributions cover both the remaining two thirds and the overall amount proportional to the income from labor. In moving to the Sweden-type pension system (Cases B and C), the income redistribution is performed by a minimum guaranteed pension instead of a basic pension. General tax revenue covers the balance, \( S_t \), between the earnings-related pension benefit of the low-income class and the guaranteed pension benefit. The total amount of the minimum guaranteed pension benefit in Cases B and C is smaller than one third of the basic pension in Case A. Therefore, the amount of tax transfer, \( S_t \), from the narrower government sector to the public pension sector decreases to 0.59 in Case B and 0.58 in Case C (see Table 1). These values are about half of the amount of 1.19 in Case A. The total tax revenue, \( T_t \), diminishes from 8.86 in Case A to 8.26 in Case B and 8.25 in Case C. Because the tax rate on consumption is adjusted in Case B under tax revenue neutrality,\(^7\) Okamoto and Tachibanaki (2002) also included the flat basic pension in the public pension program. In that study, general tax revenue covered one third of the basic pension in the benchmark simulation, and the rate of tax transfer was raised from one third to a half (i.e., there is a rise in the basic pension by one sixth). That study examined the effects of an increase in the tax transfer on efficiency and equity.
it decreases from 5% to 3.57% (see Table 2). On the other hand, in Case C, the tax rate on interest income is adjusted and thus decreases from 20% to 15.15%.

In this paper the total amount of public pension benefits is kept constant across all simulation cases, to eliminate the effects of different sizes on results. The size of the minimum guaranteed pension in Cases B and C is substantially smaller than that of the basic pension in Case A. It is approximately one sixth the size of the basic pension. This creates a rise in the weight coefficient given to the annual income from labor, $\theta$. The coefficient becomes about twice as big, increasing from 0.220 in Case A to 0.449 in Case B and 0.444 in Case C. This implies that households can receive more pension benefit per unit of their labor supply. As a result, the total labor supply, $L_t$, increases from 141.5 in Case A to 143.3 in Case B and 142.6 in Case C. The labor supply of the low-income class, $L_l$, decreases from 33.17 in Case A to 32.33 in Case B and 32.24 in Case C. This is because, irrespective of the level of its labor supply, the low-income class can receive the same amount of pension benefit as the medium-income class, owing to a minimum guaranteed pension.

In switching to the Sweden-type pension system, the capital stock decreases in Case B (361.6 → 358.0) but it increases in Case C (361.6 → 374.9). A possible reason for this is as follows. In Case B the revenue-neutral tax rate on consumption decreases to 3.57%, and in Case C that on interest income decreases to 15.15%. A consumption tax promotes capital accumulation more than other tax regimes, whereas an interest income tax hinders capital accumulation.8 Thus, the ratio of tax revenue from consumption or interest income to total tax revenue determines the level of capital stock.

Social welfare deteriorates from −159.92 in Case A to −160.20 in Case B, where the capital stock diminishes. On the other hand, it ameliorates to −159.37 in Case C, where the capital stock increases. The move from the current basic pension to the minimum guaranteed pension might substantially reduce tax transfer to the pension sector (although it depends on the level of the guaranteed pension benefit). The simulation results show that the level of capital stock depends crucially on the choice of a tax regime to reduce tax transfer, and that this choice has a great influence on the level of social welfare.

4.1.2 Basic Pension and Minimum Guaranteed Pension

In Cases B and C, a minimum guaranteed pension is introduced instead of a basic pension. Simultaneously, the size of an earnings-related pension becomes bigger, and thus the weight coefficient, $\theta$, is about twice as much as that in Case A. There are the two effects in switching from Case A to Cases B and C. Here, we focus on the first effect, and investigate the effects of different methods of income redistribution on social welfare. Keeping the weight coefficient, $\theta$, the same as in Cases B and C, we consider the case in which a minimum guaranteed pension is abolished and a basic pension is introduced. To focus on the comparison with the

8 Chapter 5 in Okamoto (2004) examined the effects of several tax regimes on capital accumulation, employing a simulation model with proportional taxation. That study suggested that the ranking of different methods of taxation, according to their strength for promoting capital formation, was: consumption, inheritances, labor income, and interest income. Okamoto (2007) showed that, under conditions of revenue neutrality, an increase in the tax rate on consumption and a decrease in the tax rate on interest income improves social welfare because this combination substantially stimulates capital accumulation. Moreover, Okamoto (2005a) reported that a progressive expenditure tax is desirable with regard to both efficiency and equity.
guaranteed pension, we consider Cases B-1 and C-1 with the basic pension financed by general tax revenue. It should be noted that, with regard to size and composition, it is different from the current Japanese basic pension in Case A. In Case B-1 the accompanying change in tax revenue is adjusted by the tax rate on consumption, whereas in Case C-1 it is adjusted by that on interest income (see Table 3).

In the move from Case B to B-1 with a basic pension, the utility of the low-income class deteriorates from $-62.51$ in Case B to $-62.70$ in Case B-1 (see Table 4). Similarly, in the switch from Case C to C-1, the utility decreases from $-62.19$ in Case C to $-62.30$ in Case C-1. On the other hand, the labor supply of the low-income class increases from $32.33$ in Case B to $33.95$ in Case B-1. Similarly, it increases from $32.24$ in Case C to $33.78$ in Case C-1. A possible reason for this is as follows. Although the low-income class is guaranteed the same amount of pension benefit as the medium-income class in Cases B and C, the guarantee disappears in Cases B-1 and C-1. This fact may give the low-income class an incentive to work. Consequently, in the move from Case B-1 to B, the total labor supply increases from $143.3$ to $144.7$ and national income increases from $56.6$ to $57.1$. In the move from Case C to C-1, it increases from $142.6$ to $143.9$ and national income increases from $57.2$ to $57.8$.

The simulation results show that the switch from Cases B and C (with a minimum guaranteed pension) to Cases B-1 and C-1 (with a basic pension) improves social welfare. Social welfare improves from $-160.20$ in Case B to $-160.08$ in Case B-1, and from $159.37$ in Case C to $-159.11$ in Case C-1. This suggests that the level of social welfare is likely to be higher under a basic pension as in present-day Japan than under a minimum guaranteed pension as in the Swedish pension system. In particular, Case C-1, where the tax rate on interest income is decreased to adjust the accompanying change in tax revenue, generates a higher level of social welfare.

### 4.1.3 Cases with only an Earnings-Related Pension

The move from Case A to Cases B and C makes the weight coefficient given to the annual income from labor, $\theta$, approximately twice as much. To ascertain the effect of this change on the simulation results, we consider two extreme cases, namely, Cases B-2 and C-2. In these cases, a minimum guaranteed pension is completely abolished, and thus there exists only an earnings-related pension. Intuitively, one would expect the case where there is only an earnings-related pension to have an advantage with regard to efficiency. However, it has a disadvantage with regard to equity, due to the absence of a basic pension or a minimum guaranteed pension that fulfills the function of income redistribution. The balance of the merits and demerits determines whether the case with only an earnings-related pension improves social welfare.

In Cases B-2 and C-2, there is no tax transfer to the public pension sector. In Case B-2 the accompanying change in tax revenue is adjusted by a consumption tax, and in Case C-2 it is adjusted by an interest income tax. A fixed scale of public pension across the simulation cases gives rise to an increase in the weight coefficient, $\theta$. It increases from $0.449$ in Case B-1 to $0.494$ in Case B-2, and from $0.444$ in Case C-1 to $0.485$ in Case C-2 (see Table 3).

The level of social welfare is lower in Case B-2 with only an earnings-related pension ($-160.65$) than in Case B-1 with a basic pension ($-160.08$) (see Table 4). Similarly, the level of social welfare is lower in Case C-2 with only an earnings-related pension ($-159.18$) than in Case C-1 with a basic pension ($-159.11$). A low level of social welfare in the cases with
only an earnings-related pension is caused mainly by a deterioration of the utility of the low-income class, because of the absence of income redistribution. The utility of the low-income class is lower in Case B-2 (−62.97) than in Case B-1 (−62.70). Similarly, it is lower in Case C-2 (−62.36) than in Case C-1 (−62.30).

These results show that, in Cases B-2 and C-2 with only an earnings-related pension, the absence of income redistribution through a public pension program produces a decrease in the utility of the low-income class, resulting in a deterioration of social welfare. Therefore, Cases B-1 and C-1 with a basic pension may be more desirable than Cases B-2 and C-2 with only an earnings-related pension.

The effects of Cases B-2 and C-2, with only an earnings-related pension, on capital stock, labor supply, and national income are described below. The level of capital stock is lower in Case B-2 (355.0) than in Case B-1 (360.0), which could be seen as an unexpected result of the simulation. A possible reason for this is as follows. In Case B-2 there exists no tax transfer to the pension sector, because of the absence of a basic pension and a minimum guaranteed pension. Because the accompanying change in tax revenue is adjusted by a consumption tax, the tax rate on consumption is decreased to 2.32%. This implies a shift from a consumption tax (that promotes capital formation more than a labor income tax) to a labor income tax (i.e., pension contributions). This shift leads to a decrease in capital stock.

On the other hand, the level of capital stock is higher in Case C-2 (385.6) than in Case C-1 (380.2). A possible reason for this is as follows. Because, in Case C-2, the accompanying change in tax revenue is adjusted by an interest income tax, the tax rate on interest income is decreased to 11.12%. This indicates a shift from an interest income tax (that hinders capital accumulation more than a labor income tax) to a labor income tax (i.e., contributions). This shift creates an increase in capital stock.

Next, with regard to the effects of Cases B-2 and C-2 on labor supply, the absence of income redistribution through a public pension scheme gives the low-income class an incentive to work. The level of the labor supply of the low-income class is higher in Case B-2 (34.07) than in Case B-1 (33.95). Similarly, it is higher in Case C-2 (33.80) than in Case C-1 (33.78). As a result, the total labor supply increases in the cases with only an earnings-related pension. The level of the total labor supply is higher in Case B-2 (145.2) than in Case B-1 (144.7), whereas that of Case C-2 is the same as in Case C-1 (143.9).

Finally, the effects of Cases B-2 and C-2 on national income are described. In the switch from Case B-1 to B-2, the capital stock decreases but the total labor supply increases. As a result, the level of national income is slightly lower in Case B-2 (57.0) than in Case B-1 (57.1). In the move from Case C-1 to C-2, the capital stock increases and the total labor supply maintains the same level. Thus, the level of national income is higher in Case C-2 (58.0) than in Case C-1 (57.8).

4.1.4 Size of Basic Pension to Maximize Social Welfare

As described above, the level of social welfare is higher in Cases B-1 and C-1 with a basic pension than in the benchmark case (A), or in Cases B and C with a minimum guaranteed pension, or in Cases B-2 and C-2 with only an earnings-related pension. As the next step, in order to explore a desirable public pension program, we choose the size of the basic pension
to maximize social welfare with regard to Cases B-1 and C-1. In Case B-1* the accompanying change in tax transfer to the pension sector is adjusted by a consumption tax, whereas in Case C-1* it is adjusted by an interest income tax. The weight coefficient given to the annual income from labor, $\theta$, is adjusted to keep the scale of public pension constant across the simulation cases (see Table 5).

The simulation results show that, when a change in the tax revenue is adjusted by a consumption tax, a larger size of the basic pension (and thus a smaller size of an earnings-related pension) generates a higher level of social welfare (see Table 6). In other words, Case B-1*, where an earnings-related pension is completely abolished and there exists only a basic pension, produces the highest level of social welfare (−154.98). In this case, the tax rate on consumption rises to 14.06%. This indicates a shift from a labor income tax (i.e., contributions) to a consumption tax (that stimulates capital formation more than other tax regimes). As a result, capital stock increases substantially, from 360.0 in Case B-1 to 413.9 in Case B-1*. On the other hand, the total labor supply is lower in Case B-1* (138.9) than in Case B-1 (144.7), which may result mainly from an increase in the basic pension benefit. The level of national income is higher in Case B-1* (57.8) than in Case B-1 (57.1).

An increase in the basic pension benefit promotes income redistribution, resulting in a rise of the utility of the low-income class. The utility improves from −62.70 in Case B-1 to −60.19 in Case B-1*. If the basic pension benefit is increased and it is financed by a consumption tax, then the policy is desirable with regard to both efficiency and equity. Thus, Case B-1* generates the highest level of social welfare.

When the accompanying change in tax revenue is adjusted by an interest income tax, Case C-1*, where the size of the basic pension is a little less than twice as much as in Case C-1, gives rise to the highest level of social welfare (−159.09). The basic pension benefit per representative household, $f$, increases from 0.0162 in Case C-1 to 0.0303 in Case C-1*. Under the assumption of a constant scale of public pension across all the simulation cases, the weight efficient, $\theta$, reduces from 0.444 in Case C-1 to 0.410 in Case C-1*. In this case, the change in tax revenue is adjusted by an interest income tax. Thus, a smaller size of the basic pension creates a lower tax rate on interest income. A decrease in the tax rate stimulates capital accumulation, because an interest income tax hinders capital formation more than a labor income tax (i.e., contributions). However, lowering the basic pension benefit diminishes the utility of the low-income class, which reduces social welfare. Therefore, in this case, with regard to efficiency a smaller size of the basic pension is desirable, but with regard to equity a larger size is desirable. To balance the two effects, an optimal size for the basic pension must be derived.

To summarize, Case B-1*, where the accompanying change in tax revenue is adjusted by a consumption tax, produces the highest level of social welfare. If we consider a public pension system with the same scale as the current Japanese pension system, it is desirable that a public pension system consists of only a basic pension (namely, an earnings-related pension is completely abolished) and that it is financed by a consumption tax. Then the basic pension benefit is a little less than twice that in the current Japanese system. The basic pension benefit per representative household, $f$, increases from 0.1114 in Case A to 0.2012 in Case B-1*. Because the current basic pension benefit for a couple is approximately 130 thousand
yen per month, the benefit in Case B-1* would be about 230 thousand yen. In this case, the total amount of the basic pension benefit is financed by tax revenue, and thus the tax rate on consumption is approximately 14% (and pension contributions are abolished).

4.1.5. Reduction of Minimum Guaranteed Pension Benefit

For simplicity of discussion, in Cases B and C it is assumed that the minimum guaranteed pension benefit for the low-income class is equal to the earnings-related pension benefit for the medium-income class. This seems to be a sufficient guarantee for the low-income class. When the level of the guaranteed pension is lower, the simulation results will change. We conduct sensitivity analyses for evaluating the effects of lower guaranteed pensions. With regard to Cases B and C, we consider the cases in which the guaranteed pension benefit is decreased to 80% of the amount of the pension benefit for the medium-income class. This yields Cases B-3 and C-3, respectively.

In Case B-3 the accompanying change in tax revenue is adjusted by a consumption tax, and in Case C-3 it is adjusted by an interest income tax (see Table 5). In both cases, a reduction of the minimum guaranteed pension benefit reduces social welfare (see Table 6). The level of social welfare decreases from −160.20 in Case B to −160.91 in Case B-3, and from −159.37 in Case C to −159.60 in Case C-3. This result is caused mainly by a decrease in the utility of the low-income class. The level of the utility diminishes from −62.51 in Case B to −63.07 in Case B-3, and from −62.19 in Case C to −62.53 in Case C-3.

A reduction of the guaranteed pension benefit causes an increase in the labor supply of the low-income class. The labor supply increases from 32.33 in Case B to 32.44 in Case B-3, and from 32.24 in Case C to 32.28 in Case C-3. Total labor supply increases from 143.3 in Case B to 143.6 in Case B-3. Cases C and C-3 have the same level of total labor supply (142.6).

Furthermore, a reduction of the guaranteed pension increases the total assets of the low-income class, because the households that maximize lifetime utility save more to cover living expenses after retirement. The assets increases from 76.45 in Case B to 84.52 in Case B-3, and dramatically from 80.74 in Case C to 91.12 in Case C-3. Capital stock diminishes from 358.0 in Case B to 353.2 in Case B-3. This result is caused mainly by a shift from a consumption tax (that promotes capital formation more than a labor income tax) to a labor income tax (i.e., contributions). With a reduction of tax transfer to the pension sector, the tax rate on consumption decreases from 3.57% in Case B to 2.68% in Case B-3. On the other hand, capital stock increases from 374.9 in Case C to 379.9 in Case C-3. This result is caused mainly by a shift from an interest income tax (that hinders capital formation more than a labor income tax) to a labor income tax (i.e., contributions). With a reduction of the tax transfer, the tax rate on interest income diminishes from 15.15% in Case C to 12.25% in Case C-3.

Finally, the effects of Cases B-3 and C-3 on national income are described. The switch from Case B to B-3 increases the total labor supply but decreases the capital stock. Consequently, national income decreases from 56.6 in Case B to 56.4 in Case B-3. By contrast, the move from Case C to C-3 keeps the total labor supply unchanged and increases the capital stock. As a result, national income increases from 57.2 in Case C to 57.4 in Case C-3.
4.2 Comments

The following four comments need to be noted for interpreting the simulation results. First, our simulations are limited to the steady states for 2005. Thus, our study lacks a consideration of the transitional path. Tax and social security reforms have different effects on different generations. Specifically, current and future generations would experience different impacts of the reforms. Therefore, it is necessary to take account of not only steady states but also the transitional process in an aging society.

Second, the simulation model is solved under conditions of the hypothesis of perfect foresight. Households correctly anticipate the interest, wage, tax, and contribution rates. Without the assumption of perfect foresight, households that maximize their lifetime utility would have more assets, resulting in more capital accumulation. As households are more risk averse, this effect would be greater.

Third, our simulation model deals only with the unintended bequests consistent with uncertainty regarding the length of individual life. Horioka et al. (2000) suggested that unintended or strategic bequest motives make up the majority of bequests in Japan. Therefore, strategic bequest motives, which are one of intended bequest motives, should also be included in the model.

Finally, because the simulation results are dependent on the given parameters, we must be careful about the effects of any parameter changes. In particular, a slight change in the parameter of intertemporal elasticity of substitution ($\gamma$) has a great affect on capital formation.

V. CONCLUSIONS

This paper examined guidelines for public pension reforms in Japan, using an extended life-cycle general equilibrium simulation model. The paper investigated a desirable public pension system in Japan, making reference to the Swedish public pension program. In particular, it focused on the fact that the Swedish scheme consists of an earnings-related pension and a minimum guaranteed pension that is financed by general tax revenue.

First, we compared the basic pension in Japan’s current public pension system and the minimum guaranteed pension in the Swedish pension system. The simulation results show that the switch from a basic pension to a guaranteed pension does not always generate favorable results. The switch from a flat basic pension to a minimum guaranteed pension changes the amount of tax transfer to the public pension sector. Social welfare depends substantially on the choice of the tax regime that will be used to adjust the accompanying changes in tax revenue. A higher ratio of tax revenue from consumption to total tax revenue promotes capital accumulation and thus improves social welfare, because a consumption tax stimulates capital formation more than other tax regimes. On the other hand, a higher ratio of tax revenue from

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9 For example, a change from a labor income tax to a consumption tax creates income transfers among generations during the transition. At the onset of policy reform, the elderly who had already paid their labor income tax will have to pay an additional consumption tax. Because this generation would suffer from a double burden, the transition to a consumption tax is not Pareto improving. As this fact is well known, the conclusion that recommends a consumption tax will be required to provide further justification, that is, to suggest measures to avoid a double burden during the transition.
interest income decreases capital accumulation and thus reduces social welfare, because an interest income tax hinders capital formation more than other tax regimes.

Next, this paper compared a basic pension and a minimum guaranteed pension as methods of income redistribution through a public pension system. The level of social welfare was found to be higher under the case of a basic pension than under that of a minimum guaranteed pension. The paper also evaluated a public pension scheme that consists of an earnings-related pension only. This scheme did not improve social welfare. This is mainly because the utility of the low-income class is too low. Thus, this system has a problem with regard to equity. Furthermore, the system did not always promote capital formation, and thus it does not always produce favorable results with regard to efficiency either.

Finally, this paper explored a desirable public pension system in Japan. The simulation results suggest that, if we consider a public pension system with the same scale as the current Japanese pension system, the highest level of social welfare is attained when a public pension system consists of only a basic pension (i.e., an earnings-related pension is completely abolished) and it is financed by a consumption tax. Then, the basic pension benefit is a little less than twice that in the current Japanese system; the current pension amount for a couple is approximately 130 thousand yen per month, and thus the amount under this desirable system would be about 230 thousand yen. Because the total amount of the basic pension is financed by tax revenue (namely, pension contributions are completely abolished), the tax rate on consumption rises to approximately 14%.
### Table 1: Benchmark Case and Sweden-Type Pension

<table>
<thead>
<tr>
<th>Case A (Benchmark)</th>
<th>Case B (Sweden-type pension; consumption tax)</th>
<th>Case C (Sweden-type pension; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight coefficient in public pension, $\theta$</td>
<td>0.220</td>
<td>0.449</td>
</tr>
<tr>
<td>Minimum guaranteed pension, $b$</td>
<td>0</td>
<td>0.1785</td>
</tr>
<tr>
<td>Basic pension per representative household, $f$</td>
<td>0.1114</td>
<td>0</td>
</tr>
<tr>
<td>Contribution rate, $\tau_p$</td>
<td>14.29%</td>
<td>15.96%</td>
</tr>
<tr>
<td>Tax revenue, $T_t$</td>
<td>8.86</td>
<td>8.26</td>
</tr>
<tr>
<td>Tax transfer to pension sector, $S_t$</td>
<td>1.19</td>
<td>0.59</td>
</tr>
<tr>
<td>Capital stock, $K_t$</td>
<td>361.6</td>
<td>358.0</td>
</tr>
<tr>
<td>Total labor supply measured per effective labor unit, $L_t$</td>
<td>141.5</td>
<td>143.3</td>
</tr>
<tr>
<td>National income, $Y_t$</td>
<td>56.3</td>
<td>56.6</td>
</tr>
</tbody>
</table>

### Table 2: Simulation Results for Benchmark Case and Sweden-Type Pension System

<table>
<thead>
<tr>
<th>Case A (Benchmark)</th>
<th>Case B (Sweden-type pension; consumption tax)</th>
<th>Case C (Sweden-type pension; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax rate on labor income,$a \tau_w {wx^e(1 - l^e)}$</td>
<td>7.44%</td>
<td>7.53%</td>
</tr>
<tr>
<td>Tax rate on consumption, $\tau_c$</td>
<td>5%</td>
<td>3.57%</td>
</tr>
<tr>
<td>Tax rate on interest income, $\tau_r$</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Tax rate on inheritances, $\tau_h$</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Interest rate, $r$</td>
<td>4.67%</td>
<td>4.74%</td>
</tr>
<tr>
<td>Wage rate, $w$</td>
<td>0.260</td>
<td>0.256</td>
</tr>
<tr>
<td>Capital-labor ratio, $K/L$</td>
<td>2.556</td>
<td>2.499</td>
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<tr>
<td>(Low) Assets, $AS_l$</td>
<td>79.44</td>
<td>76.45</td>
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<tr>
<td>(High) Assets, $AS_h$</td>
<td>165.80</td>
<td>160.84</td>
</tr>
<tr>
<td>(Low) Labor supply, $L_l$</td>
<td>33.17</td>
<td>32.33</td>
</tr>
<tr>
<td>(High) Labor supply, $L_h$</td>
<td>62.86</td>
<td>64.39</td>
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<tr>
<td>(Low) Utility,$c U_l$</td>
<td>-62.42</td>
<td>-62.51</td>
</tr>
<tr>
<td>(High) Utility,$c U_h$</td>
<td>-44.66</td>
<td>-44.66</td>
</tr>
<tr>
<td>Social welfare,$c SW$</td>
<td>-159.92</td>
<td>-160.20</td>
</tr>
</tbody>
</table>

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*a The rate presented is an average tax rate on labor income. The parameter values that determine the tax progressivity on labor income are $\alpha = -0.0439, \beta = 0.52$, in all simulation cases.

*b The variable is endogenous.

*c Indexed with a value of 1/10,000.
### Table 3: Basic Pension and Earnings-Related Pension Only

<table>
<thead>
<tr>
<th></th>
<th>Case B-1 (Basic pension; consumption tax)</th>
<th>Case B-2 (Earnings-related pension only; consumption tax)</th>
<th>Case C-1 (Basic pension; interest income tax)</th>
<th>Case C-2 (Earnings-related pension only; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight coefficient in public pension, $\theta$</td>
<td>0.449</td>
<td>0.494</td>
<td>0.444</td>
<td>0.485</td>
</tr>
<tr>
<td>Minimum guaranteed pension, $b$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Basic pension per representative household, $f$</td>
<td>0.0166</td>
<td>0</td>
<td>0.0162</td>
<td>0</td>
</tr>
<tr>
<td>Contribution rate, $\tau_p$</td>
<td>15.97%</td>
<td>17.57%</td>
<td>15.80%</td>
<td>17.22%</td>
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<tr>
<td>Tax revenue, $T_t$</td>
<td>8.20</td>
<td>7.67</td>
<td>8.19</td>
<td>7.67</td>
</tr>
<tr>
<td>Tax transfer to pension sector, $S_t$</td>
<td>0.53</td>
<td>0</td>
<td>0.52</td>
<td>0</td>
</tr>
<tr>
<td>Capital stock, $K_t$</td>
<td>360.0</td>
<td>355.0</td>
<td>380.2</td>
<td>385.6</td>
</tr>
<tr>
<td>Total labor supply measured per effective labor unit, $L_t$</td>
<td>144.7</td>
<td>145.2</td>
<td>143.9</td>
<td>143.9</td>
</tr>
<tr>
<td>National income, $Y_t$</td>
<td>57.1</td>
<td>57.0</td>
<td>57.8</td>
<td>58.0</td>
</tr>
</tbody>
</table>

### Table 4: Simulation Results for Basic Pension and Earnings-Related Pension Only

<table>
<thead>
<tr>
<th></th>
<th>Case B-1 (Basic pension; consumption tax)</th>
<th>Case B-2 (Earnings-related pension only; consumption tax)</th>
<th>Case C-1 (Basic pension; interest income tax)</th>
<th>Case C-2 (Earnings-related pension only; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax rate on labor income, $a$ $\tau_w{wx^e(1-L_L^e)}$</td>
<td>7.53%</td>
<td>7.43%</td>
<td>7.68%</td>
<td>7.64%</td>
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<tr>
<td>Tax rate on consumption, $\tau_c$</td>
<td>3.29%$^b$</td>
<td>2.32%$^b$</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Tax rate on interest income, $\tau_r$</td>
<td>20%</td>
<td>20%</td>
<td>14.18%$^b$</td>
<td>11.12%$^b$</td>
</tr>
<tr>
<td>Tax rate on inheritances, $\tau_h$</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Interest rate, $r$</td>
<td>4.76%</td>
<td>4.81%</td>
<td>4.56%</td>
<td>4.51%</td>
</tr>
<tr>
<td>Wage rate, $w$</td>
<td>0.256</td>
<td>0.252</td>
<td>0.260</td>
<td>0.260</td>
</tr>
<tr>
<td>Capital-labor ratio, $K/L$</td>
<td>2.487</td>
<td>2.445</td>
<td>2.642</td>
<td>2.679</td>
</tr>
<tr>
<td>(Low) Assets, $A^L$</td>
<td>86.36</td>
<td>86.40</td>
<td>91.33</td>
<td>93.86</td>
</tr>
<tr>
<td></td>
<td>Case B-1*</td>
<td>Case B-3</td>
<td>Case C-1*</td>
<td>Case C-3</td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>(High) Assets, $A^h$</td>
<td>156.93</td>
<td>153.37</td>
<td>165.54</td>
<td>166.51</td>
</tr>
<tr>
<td>(Low) Labor supply, $L^l$</td>
<td>33.95</td>
<td>34.07</td>
<td>33.78</td>
<td>33.80</td>
</tr>
<tr>
<td>(High) Labor supply, $L^h$</td>
<td>64.30</td>
<td>64.52</td>
<td>63.89</td>
<td>63.88</td>
</tr>
<tr>
<td>(Low) Utility, $U^l$</td>
<td>-62.70</td>
<td>-62.97</td>
<td>-62.30</td>
<td>-62.36</td>
</tr>
<tr>
<td>(High) Utility, $U^h$</td>
<td>-44.53</td>
<td>-44.65</td>
<td>-44.27</td>
<td>-44.27</td>
</tr>
<tr>
<td>Social welfare, SW</td>
<td>-160.08</td>
<td>-160.65</td>
<td>-159.11</td>
<td>-159.18</td>
</tr>
</tbody>
</table>

\(a\) The rate presented is an average tax rate on labor income. The parameter values that determine the tax progressivity on labor income are \(\alpha = -0.0439, \beta = 0.52\) in all simulation cases.

\(b\) The variable is endogenous.

\(c\) Indexed with a value of 1/10,000.

Table 5: Optimal Size for Basic Pension and Reduced Minimum Guaranteed Pension

<table>
<thead>
<tr>
<th></th>
<th>Case B-1* (Optimal basic pension; consumption tax)</th>
<th>Case B-3 (Reduced guaranteed pension; consumption tax)</th>
<th>Case C-1* (Optimal basic pension; interest income tax)</th>
<th>Case C-3 (Reduced guaranteed pension; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight coefficient in public pension, (\theta)</td>
<td>0</td>
<td>0.492</td>
<td>0.410</td>
<td>0.483</td>
</tr>
<tr>
<td>Minimum guaranteed pension, (b)</td>
<td>0</td>
<td>0.1428</td>
<td>0</td>
<td>0.1428</td>
</tr>
<tr>
<td>Basic pension per representative household, (f)</td>
<td>0.2012</td>
<td>0</td>
<td>0.0303</td>
<td>0</td>
</tr>
<tr>
<td>Contribution rate, (\tau_p)</td>
<td>0.00%</td>
<td>17.47%</td>
<td>14.57%</td>
<td>17.17%</td>
</tr>
<tr>
<td>Tax revenue, (T_t)</td>
<td>14.11</td>
<td>7.76</td>
<td>8.64</td>
<td>7.76</td>
</tr>
<tr>
<td>Tax transfer to pension sector, (S_t)</td>
<td>6.44</td>
<td>0.09</td>
<td>0.97</td>
<td>0.09</td>
</tr>
<tr>
<td>Capital stock, (K_t)</td>
<td>413.9</td>
<td>353.2</td>
<td>375.1</td>
<td>379.9</td>
</tr>
<tr>
<td>Total labor supply measured per effective labor unit, (L_t)</td>
<td>138.9</td>
<td>143.6</td>
<td>143.9</td>
<td>142.6</td>
</tr>
<tr>
<td>National income, (Y_t)</td>
<td>57.8</td>
<td>56.4</td>
<td>57.5</td>
<td>57.4</td>
</tr>
</tbody>
</table>
Table 6: Simulation Results for Optimal Basic Pension and Reduced Guaranteed Pension

<table>
<thead>
<tr>
<th></th>
<th>Case B-1* (Optimal basic pension; consumption tax)</th>
<th>Case B-3 (Reduced guaranteed pension; consumption tax)</th>
<th>Case C-1* (Optimal basic pension; interest income tax)</th>
<th>Case C-3 (Reduced guaranteed pension; interest income tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax rate on labor income, $\tau_w$</td>
<td>8.64%</td>
<td>7.41%</td>
<td>7.70%</td>
<td>7.61%</td>
</tr>
<tr>
<td>Tax rate on consumption, $\tau_c$</td>
<td>14.06%$^b$</td>
<td>2.68%$^b$</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Tax rate on interest income, $\tau_r$</td>
<td>20%</td>
<td>20%</td>
<td>16.89%$^b$</td>
<td>12.25%$^b$</td>
</tr>
<tr>
<td>Tax rate on inheritances, $\tau_h$</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Interest rate, $r$</td>
<td>4.19%</td>
<td>4.79%</td>
<td>4.60%</td>
<td>4.53%</td>
</tr>
<tr>
<td>Wage rate, $w$</td>
<td>0.291</td>
<td>0.253</td>
<td>0.261</td>
<td>0.260</td>
</tr>
<tr>
<td>Capital-labor ratio, $K/L$</td>
<td>2.981</td>
<td>2.459</td>
<td>2.607</td>
<td>2.665</td>
</tr>
<tr>
<td>(Low) Assets, $AS^l$</td>
<td>84.62</td>
<td>84.52</td>
<td>89.01</td>
<td>91.12</td>
</tr>
<tr>
<td>(High) Assets, $AS^h$</td>
<td>196.75</td>
<td>153.38</td>
<td>164.55</td>
<td>164.82</td>
</tr>
<tr>
<td>(Low) Labor supply, $L^l$</td>
<td>32.57</td>
<td>32.44</td>
<td>33.77</td>
<td>32.28</td>
</tr>
<tr>
<td>(High) Labor supply, $L^h$</td>
<td>61.63</td>
<td>64.54</td>
<td>63.90</td>
<td>63.98</td>
</tr>
<tr>
<td>(Low) Utility, $U^l$</td>
<td>-60.19</td>
<td>-63.07</td>
<td>-62.27</td>
<td>-62.53</td>
</tr>
<tr>
<td>(High) Utility, $U^h$</td>
<td>-43.55</td>
<td>-44.72</td>
<td>-44.29</td>
<td>-44.38</td>
</tr>
<tr>
<td>Social welfare, $SW$</td>
<td>-154.98</td>
<td>-160.91</td>
<td>-159.09</td>
<td>-159.60</td>
</tr>
</tbody>
</table>

**Notes:**

- The rate presented is an average tax rate on labor income. The parameter values that determine the tax progressivity on labor income are $\alpha = -0.0439$, $\beta = 0.52$ in all simulation cases.
- The variable is endogenous.
- Indexed with a value of 1/10,000.
Table 7: Parameter Values used in the Benchmark Simulation

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate of successive cohorts</td>
<td>( n = 0.00693 )</td>
</tr>
<tr>
<td>New entrants for each income class in period ( t )</td>
<td>( N_t = 1 )</td>
</tr>
<tr>
<td>Utility weight on leisure</td>
<td>( \phi = 0.8005 )</td>
</tr>
<tr>
<td>Adjustment coefficient for discounting the future</td>
<td>( \delta = -0.008 )</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>( \gamma = 0.5 )</td>
</tr>
<tr>
<td>Intratemporal substitution elasticity</td>
<td>( \rho = 0.9312 )</td>
</tr>
<tr>
<td>Share of capital in production</td>
<td>( \varepsilon = 0.3 )</td>
</tr>
<tr>
<td>Scale parameter in production</td>
<td>( B = 0.3 )</td>
</tr>
<tr>
<td>Government expenditure per representative household</td>
<td>( g = 0.3049 )</td>
</tr>
<tr>
<td>Compulsory retirement age</td>
<td>( RE = 61 )</td>
</tr>
<tr>
<td>Starting age for receiving public pension benefit</td>
<td>( ST = 62 )</td>
</tr>
<tr>
<td>Ratio of the part covered by taxes in the basic pension</td>
<td>( \mu = 0.3333 )</td>
</tr>
<tr>
<td>Basic pension benefit per representative household</td>
<td>( f = 0.5532 )</td>
</tr>
<tr>
<td>Weight coefficient of the part proportional to income from labor in the public pension</td>
<td>( \theta = 0.2213 )</td>
</tr>
</tbody>
</table>

Table 8: Effective Tax Rates for National Income Tax and Residence Tax of Wageworkers

<table>
<thead>
<tr>
<th>Income class</th>
<th>Total amount of annual income (million yen)</th>
<th>Weight on labor endowments</th>
<th>Total amount of annual taxes: national income tax and residence tax (thousand yen)</th>
<th>Effective tax rates (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>5</td>
<td>( x^l = 0.7143 )</td>
<td>177</td>
<td>3.54</td>
</tr>
<tr>
<td>Medium</td>
<td>7</td>
<td>( x^m = 1 )</td>
<td>418</td>
<td>5.97</td>
</tr>
<tr>
<td>High</td>
<td>10</td>
<td>( x^h = 1.4286 )</td>
<td>1,041</td>
<td>10.41</td>
</tr>
</tbody>
</table>

Data given are for a couple with two children. Source: Ministry of Finance (2005).

Table 9: Estimation of the Age Profile of Earnings Ability

<table>
<thead>
<tr>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \alpha_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.202338</td>
<td>0.076286</td>
<td>-0.001038</td>
<td>0.127051</td>
<td>-0.001726</td>
</tr>
<tr>
<td>((-0.42627))</td>
<td>((2.32608))</td>
<td>((-3.53302))</td>
<td>((2.76281))</td>
<td>((-1.38994))</td>
</tr>
</tbody>
</table>

\[ S.E. = 0.07189 \]

\[ R^2 = 0.9929 \]
APPENDIX A

A.1 Benchmark Case

The utility maximization problem over time for each income class in Section 2 is regarded as the maximization of equation (2) subject to equations (3) and (8). Below, we consider the maximization problem when a retirement age is fixed as $RHi$. Let the Lagrange function be

$$L' = U^i + \sum_{s=21}^{95} \lambda_s^i \left[ - A_s^i + \{1 + r(1-\tau_r)\} A_s^i + \left[1 - \tau_w \{wx^i e_s (1-l_s^i)\} - \tau_p\} \right] wx^i e_s (1-l_s^i) \right]$$  \hspace{1cm} (A1)

$$+ b_s^i \left( \{l_{u_s}^{RH} \}_{u=21}^{RH} \right) + a_s^i - (1 + \tau_c) C_s^i \sum_{s=21}^{RH} \eta_s^i (1-l_s^i),$$

where $\lambda_s^i$ and $\eta_s^i$ represent the Lagrange multiplier for equations (3) and (8), respectively, and superscript $i \ (l, m, h)$ denotes low, medium, and high-income classes, respectively.

The first-order conditions on consumption $C_s^i$, leisure $l_s^i$, and assets $A_{s+1}^i$ for $s = 21, 22, \ldots, 95$ can be expressed by

$$p_s (1+\delta)^{(s-21)} \left\{ \frac{1-\frac{1}{\rho}}{\rho} \phi(l_s^i) \left( C_s^i \right)^{\frac{1}{\rho}} + \frac{1-\frac{1}{\gamma}}{\gamma} \phi(l_s^i) C_s^i \right\} = \lambda_s^i (1 + \tau_c), \hspace{1cm} (A2)$$

$$p_s (1+\delta)^{(s-21)} \left\{ \frac{1-\frac{1}{\rho}}{\rho} \phi(l_s^i) \left( C_s^i \right)^{\frac{1}{\rho}} + \frac{1-\frac{1}{\gamma}}{\gamma} \phi(l_s^i) C_s^i \right\} = \lambda_s^i (1 + \tau_c),$$

$$= \lambda_s^i \left[ (1 - \alpha - \tau_p) wx^i e_s - \beta (wx^i e_s)^2 (1-l_s^i) \right] + \sum_{k=ST}^{95} \lambda_s^i \frac{\theta wx^i e_s}{RH^i - 20} + \eta_s^i (s \leq RH), \hspace{1cm} (A3)$$

$$\lambda_s^i = \{1 + r(1-\tau_r)\} \lambda_{s+1}^i, \hspace{1cm} (A4)$$

$$\eta_s^i (1-l_s^i) = 0 \ (s \leq RH), \hspace{1cm} (A5)$$

$$1-l_s^i = 0 \ (s \leq RH), \hspace{1cm} (A6)$$

$$\eta_s^i \geq 0. \hspace{1cm} (A7)$$

The combination of equations (A2) and (A4) produces equations (9) and (10). If the initial value, $V_{21}^i$, is specified, the value of each age, $V_s^i$, can be derived from equation (9). If $V_s^i$ is specified, the values of consumption, $C_s^i$, and leisure, $l_s^i$, at each age are obtained in the method that follows.

For $s = 21, 22, \ldots, RH$, the combination of equations (A2) and (A3) yields the following expression:
\[ C_s^i = \left[ (1 - \alpha - \tau_p)wx^ie_s - \beta(wx^ie_s)^2(1 - l^i_s) + \frac{1}{\lambda_s^{i}} \sum_{k = ST}^{95} \lambda_k \frac{\theta wx^ie_s}{RH^i - 20} + \eta_s^i \right]^{\gamma_i} \]

\[ \phi(1 + \tau_c) \]

If the value of \( l^i_s \) is given under \( \eta_s^i = 0 \), the value of \( V_s^i \) can be obtained. The value of \( l^i_s \) is chosen so that the value of \( V_s^i \) obtained in the simulation is the closest to that calculated by evolution from \( V_s^i \) through equation (9). If the value of \( l^i_s \) chosen is unity or higher, the value of \( C_s^i \) is obtained from equation (10) under \( l^i_s = 1 \). If it is less than unity, the value of \( C_s^i \) is derived from equation (A8).

For \( s = RH^i + 1, RH^i + 2, \ldots, RH^i \), the condition of \( l^i_s = 1 \) leads to the following equation:

\[ V_s^i = \left\{ (C_s^i)^{\frac{1}{\rho}} + \phi \right\}^{\frac{1}{\rho}} \left( (C_s^i)^{\frac{1}{\rho}} \right)^{-\frac{1}{\rho}}. \]  

(A10)'

The value of \( C_s^i \) is chosen to satisfy this equation.

A.2 Sweden-Type Pension System Case

If the Sweden-type pension program is introduced, then the utility maximization problem in the benchmark case is modified as follows.

The first-order condition on leisure \( l^i_s \) for \( s = RH^i \) is expressed by

\[ p_s(1 + \delta)^{-(s-21)} \left\{ (C_s^i)^{\frac{1}{\rho}} + \phi(l_s^i)^{\frac{1}{\rho}} \right\}^{\frac{1}{1 - \rho}} \left( (C_s^i)^{\frac{1}{\rho}} \right)^{-\frac{1}{\rho}} \phi(l_s^i)^{\frac{1}{\rho}} \]

(A3)'

\[ = \lambda_s^i \left[ (1 - \alpha - \tau_p)wx^ie_s - \beta(wx^ie_s)^2(1 - l^i_s) \right] + z^i \sum_{k = ST}^{95} \lambda_k \frac{\theta wx^ie_s}{RH^i - 20} + \eta_s^i (s \leq RH^i), \]

where \( z^i = 0 \) and \( z^m = z^h = 1 \).

For \( s = 21, 22, \ldots, RH^i \), the combination of equations (A2) and (A3)’ yields the following expression:

\[ C_s^i = \left[ (1 - \alpha - \tau_p)wx^ie_s - \beta(wx^ie_s)^2(1 - l^i_s) + z^i \frac{1}{\lambda_s^i} \sum_{k = ST}^{95} \lambda_k \frac{\theta wx^ie_s}{RH^i - 20} + \eta_s^i \right]^{\gamma_i} \]

\[ \phi(1 + \tau_c) \]

(A8)’
APPENDIX B

Section 2 describes the basic structure of households in the simulation model. This appendix presents the basic structures of firms and the government, and market equilibrium conditions.

Firm Behaviour

The model has a single production sector that is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. Capital is homogeneous and nondepreciating, while labor differs only in its efficiency. All forms of labor are perfect substitutes. Households in different income classes or of different ages, however, supply different amounts of some standard measure per unit of labor input.

The aggregate production technology is the standard Cobb-Douglas form:

\[ Y_t = B K_t^\varepsilon L_t^{1-\varepsilon}, \]  

where \( Y_t \) is the total output (national income), \( K_t \) the total capital, \( L_t \) the total labor supply measured by the efficiency units, \( B \) a scaling constant, and \( \varepsilon \) is a share of capital. Using the property subject to a constant-returns-to-scale production function, we can obtain the following equation:

\[ Y_t = r K_t + w L_t. \]

Government Behaviour

The government sector consists of a narrower government sector and a public pension sector. The narrower government sector collects taxes, and spends them on general government expenditure and a transfer to the pension sector. There is no outstanding debt, and thus balanced budget policies are assumed.

The budget constraint of the narrower government sector at time \( t \) is given by

\[ T_t = G_t + \mu F_t, \]

where \( T_t \) is the total tax revenue from labor income, interest income, consumption, and inheritances, \( G_t \) is general government spending on goods and services, except for a transfer to the public pension sector, \( F_t \) is the total amount of basic pension benefit, and \( \mu \) is the ratio of the part covered by general tax revenues to \( F_t \).

The budget constraint of the public pension sector at time \( t \) is given by

\[ R_t = (1 - \mu) F_t + P_t, \]

where \( R_t \) is the total contribution to the pension program, and \( P_t \) is the total benefit of the part proportional to the income from labor. \( T_t, G_t, F_t, R_t, \) and \( P_t \) are defined by

\[ T_t = L X_t + \tau_r A S_t + \tau_v A C_t + \tau_h B Q_t, \]

\[ G_t = N_t \sum_{i=1}^{21} p_i (1+n)^{-(i-21)} 3g, \]
\[ F_t = N_t \sum_{s=21}^{95} \left( p_s (1+n)^{-s} \right) f, \] (B7)

\[ R_t = \tau_p w L_t, \] (B8)

\[ P_t = N_t \sum_{s=21}^{95} \left[ p_s (1+n)^{-s} \theta \{ H^l + H^m + H^h \} \right], \] (B9)

where \( g \) is annual government expenditure for each representative household, and

\[ BQ_t = BQ^l_t + BQ^m_t + BQ^h_t. \] (B10)

\( LX_t \) is the tax revenue from labor income, which is represented by

\[ LX_t = N_t \sum_{s=21}^{RE} p_s (1+n)^{-s} \left[ \alpha w x^l e_s (1-l_s^l) + \frac{1}{2} \beta \{ w x^l e_s (1-l_s^l) \}^2 + \alpha w x^m e_s (1-l_s^m) \right. \]

\[ + \left. \frac{1}{2} \beta \{ w x^m e_s (1-l_s^m) \}^2 + \alpha w x^h e_s (1-l_s^h) + \frac{1}{2} \beta \{ w x^h e_s (1-l_s^h) \}^2 \right]. \] (B11)

Aggregate variables can be obtained by a simple summation of the three income classes with the same weight, because each income group accounts for the same proportion of population. Similarly, aggregate assets supplied by households, \( AS_t \), and aggregate consumption, \( AC_t \), are obtained by

\[ AS_t = N_t \sum_{s=21}^{95} p_s (1+n)^{-s} \left\{ A_s^l + A_s^m + A_s^h \right\}, \] (B12)

\[ AC_t = N_t \sum_{s=21}^{95} p_s (1+n)^{-s} \left\{ C_s^l + C_s^m + C_s^h \right\}. \] (B13)

**Market Equilibrium**

Finally, equilibrium conditions for the capital, labor, and goods markets are described.

1 Equilibrium condition for the capital market

Because aggregate assets supplied by households are equal to real capital, we get

\[ AS_t = K_t. \] (B14)

2 Equilibrium condition for the labor market

Measured in efficiency units, because aggregate labor demand by firms is equal to aggregate labor supply by households, we get

\[ L_t = N_t \sum_{s=21}^{RE} p_s (1+n)^{-s} \left[ x^l e_s (1-l_s^l) + x^m e_s (1-l_s^m) + x^h e_s (1-l_s^h) \right]. \] (B15)
3 Equilibrium condition for the goods market

Because aggregate production is equal to the sum of consumption, investment, and government expenditures, we get

\[ Y_t = AC_t + (K_{t+1} - K_t) + G_t \]  \hspace{1cm} (B16)

An iterative program is performed to obtain the equilibrium values of the above equations.
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REFERENCES


DATA